# One Residue or Two?

## On the Relationship Between Composite Sector Recovery and Fiber Degeneracy

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# General Reader Summary

The previous papers narrowed the question of where quantum probability comes from down to a small, sharp list of unfinished business. On the "sector" side of that list — the side that asks whether two kinds of reversible motion really live in one connected space — two items remained.

The first is whether the substrate could ever tell, by chaining together many small operations, which *kind* of motion it was watching: phase motion or refinement motion. Call this the label-recovery question.

The second is whether the space of motions can come apart for a more mundane reason — the structure over a given carving can branch, collapse, or fail to join up smoothly — even when no wall separates the two kinds of motion. Call this the degeneracy question.

The natural hope is that these are the same problem wearing two hats, so that solving one solves both and only a single open question remains. This paper asks whether that hope is justified, and finds that it turns on one precise sub-question: **when the structure degenerates, does the degeneration itself betray which kind of motion was involved?**

We show that the two items merge into one *only if* every degeneration leaves behind a label saying "phase" or "refinement." If a degeneration can be detectable yet say nothing about which kind of motion produced it — a "sector-blind" degeneration — then the two items stay genuinely separate, and the programme carries two open questions rather than one.

The contribution is not to settle which of these is the case. It is to prove that the entire one-versus-two question collapses onto that single sub-question, to show that the "two" answer needs no extra assumption while the "one" answer requires a specific further claim that must be earned, and to identify exactly what a future synthesis must establish. We make precise what is proven, what is assumed, and what is left open.

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# Abstract


The previous paper reduced Reversible Connectedness (RC) to the Admissible Lift Property (ALP) and localized the remaining sector-side obstruction to three named objects: the inherited phase-continuity tier, composite sector recovery (FBI-comp), and the degeneracy content of the Uniqueness-of-Obstruction premise (ND, the residual core of UO). FBI-comp is a statement about *labels on motions* — whether a finite floor-resolved protocol can recover a phase/refinement label absent from every constituent step. ND is a statement about *fiber structure* — whether admissible transport preserves constant operational fiber cardinality over the refinement base. They are not the same object. This paper asks whether they are nonetheless the same *problem*.

We do not attempt to settle that question by fiat. We prove instead that it reduces, exactly, to a single Boolean:

**C:** every fiber degeneracy of admissible transport generates a sector-labelled *geometric* trace.

Three results follow. First (the **reduction**), one-vs-two is logically equivalent to C-vs-¬C: under C the residue collapses to FBI-comp alone, and under ¬C the residue is provably two independent objects. Second (the **asymmetry between the branches**), two proven structural facts distinguish the branches — the collapse *implication* carries an OO rider that the irreducibility implication does not, and the antecedent C is a universal non-existence claim (no admissible σ-invariant degeneracy exists) while ¬C is existential (one such degeneracy suffices) — and these ground a *methodological* lean toward carrying two objects, not a proven logical

default. We are explicit that this is a methodological consequence: the logical fork itself is even-handed, since either resolution of the σ-question settles the count. Third (a **symmetry principle**), we prove that any degeneracy whose local geometry is invariant under phase/refinement interchange is sector-blind; sector-blindness is thereby a theorem conditional on symmetry rather than an intuition, and the route to ¬C narrows to the single geometric question of whether the transport construction admits a symmetric degeneracy.

The deliverable is therefore a *reduction*, not a determination: we localize the entire one-vs-two question to the single sub-question C, and then — via the symmetry results — to the strictly geometric question of whether the transport construction admits an admissible σ-invariant degeneracy. The Minimal Residue Theorem (§4) shows this geometric question is unavoidable: every route to a one-residue capstone must pass through it. The single theorem-level contribution is thus stateable in one line — *the FBI-comp/ND relationship is determined by C, and C is determined by the existence or non-existence of admissible σ-invariant degeneracies* — handing the capstone a concrete geometric target in place of a diffuse pair of conjectures.

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# 1. Introduction

The recent sequence of papers progressively reduced the Born-rule residue. Primitive ledgers were eliminated. External individuation was eliminated. Contextual weighting was reduced to Internality on the weight axis. Reversible Connectedness was reformulated as an admissible lifting problem, and the lifting problem was localized to a short list.

The latest paper isolated the sector-side residue as three objects:

1. **FBI-comp** (Composite Sector Recovery) — [Conjectural].
2. **UO / ND** (degeneracy and continuity of lifting) — [Conditional], with the open content narrowed in the prior paper's Lemma 7.1A to the non-degeneracy condition ND.
3. The inherited **phase-continuity** tier — [Conditional].

The prior paper also flagged, in passing, that FBI-comp and UO are "plausibly two faces of one fact," and that ND coincides with the degeneracy content of UO. If those informal remarks are correct, the sector residue is one conjecture, not two, and a capstone synthesis has a single remaining gate. If they are not, the capstone carries two independent gates. The difference is not cosmetic: it determines how many genuinely open problems the programme contains.

This paper settles the *logical relationship* between FBI-comp and ND without presupposing the answer. We prove that the one-vs-two question reduces to a single sub-question, we determine which branch carries the heavier premise burden, and we state exactly what remains for the capstone. We do **not** prove FBI-comp, we do **not** prove ND, and — by design — we do **not** claim to determine which branch obtains. The contribution is the reduction and the burden assignment, both of which are theorem-level and available regardless of how FBI-comp eventually breaks.

We retain the programme's labelling convention: [Proven], [Conditional], [Conditional-on-OO], [Conditional-on-C], [Conjectural].

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# 2. The Two Residues

We fix the two objects precisely, because the entire paper depends on their *not* being conflated.

**FBI-comp (a statement about labels on motions).**

Can a finite protocol, built from finitely many floor-resolved operations, recover a phase/refinement sector label that is absent from every individual constituent step?

FBI-comp asserts the negative: no such protocol exists. It quantifies over *protocols* and concerns the *recoverability of a label* attached to a motion.

**ND (a statement about fiber structure).**

Do the fibers $\pi^{-1}(R)$ retain constant operational cardinality as R varies over the refinement base $\mathscr{R}/\approx_{\mathscr{R}}$?

Here *operational* cardinality means cardinality as counted after floor-resolution — the number of fiber elements the substrate can distinguish at the floor, i.e. the cardinality of the fiber's image under the resolution map r. This is the sense carried throughout: a cardinality difference below the floor is not an operational cardinality difference and so is not an ND-failure. The qualifier matters for §4, where it makes the detectability of an ND-failure follow from the definition rather than from a fresh ontological appeal.

Equivalently: does admissible transport ever encounter branching, collapse, or continuity failure in the fibers? ND asserts the negative: no such degeneracy occurs. It quantifies over *carvings* and concerns the *geometry of the projection* $\pi : \mathscr{A} \to \mathscr{R}$.

These are different objects. FBI-comp is about whether a tag can be read off a process; ND is about whether a bundle has constant fiber cardinality. One settled direction is immediate and was noted previously:

**Observation 2.1 [Proven, given the prior construction].** FBI-comp $\not\Rightarrow$ ND. Closing FBI-comp removes the sector wall (prior paper, Corollary 6.2), but a fiber may still branch or collapse over different R with no sector wall present anywhere. Wall-removal does not entail constant fiber cardinality.

The live question is the *converse* direction — whether ND-failure forces FBI-comp to fail — because that is what would make the two collapse into one. The whole paper is the analysis of that direction.

# 3. Operational Traces and Sector-Labelled Traces

The converse direction has a trap, and avoiding it requires one distinction stated with care.

**Definition 3.1 (operational trace).** An operational trace of a process is any floor-resolved observable consequence of that process — any quantity, available at or above the distinguishability floor, whose value is changed by the process.

**Definition 3.2 (sector-labelled trace).** A sector-labelled trace is an operational trace whose value *determines* whether the process belonged to the phase sector or the refinement sector. Formally, a trace $\tau$ is sector-labelled if the floor-resolved value of $\tau$ differs according to sector membership, so that reading $\tau$ recovers the label.

**Lemma 3.3 [Proven].** Every sector-labelled trace is an operational trace; the converse fails in general.

*Proof.* The forward inclusion is immediate from the definitions: a sector-labelled trace is by Definition 3.2 an operational trace satisfying an extra condition. For the converse, it suffices that the extra condition is non-vacuous — that a floor-resolved quantity can change under a process without its change determining the process's sector. Nothing in Definition 3.1 forces the changed quantity to encode sector membership; an observable irregularity that is symmetric in the two sectors changes value while leaving the label undetermined. Hence operational-trace $\not\Rightarrow$ sector-labelled-trace. ■

This gap — *detectable* versus *sector-labelled* — is the hinge of the paper. The error to avoid is the slide from "a degeneracy is operationally visible" to "a degeneracy reveals the sector." The first is cheap; the second is exactly what the collapse of the residue requires, and it is strictly stronger.

# 4. The Reduction: One-vs-Two Turns on a Single Boolean

We now show that the entire one-vs-two question is equivalent to one sub-question about traces. Before stating the Boolean, we fix the partition of label-recovery routes that the rest of the paper enforces, because the Boolean is defined relative to it.

**Recovery-route partition convention.** A sector label, if recovered at all, is recovered either (i) from a trace derived solely from the local degeneracy geometry — a *geometric* route — or (ii) from some operational quantity not a function of the degeneracy geometry — a *non-geometric* route. These two sources exhaust the routes. They relate to the two objects asymmetrically, and the asymmetry must be stated precisely, because the collapse mechanism of §6 depends on it.

The **non-geometric route is FBI-comp's exclusively**: a label read from a quantity that is not a function of the degeneracy geometry says nothing about whether the geometry betrays the sector, so it does not bear on ND. This is the load-bearing half — it is what makes ¬C, witnessed by a *geometric* sector-blind degeneracy, a clean ND-independence witness rather than a misattributed FBI-comp phenomenon, and it is what Theorem 7.1 relies on.

The **geometric route is shared**, and deliberately so. It is ND's primary object — the question "does the geometry of a degeneracy betray the sector?" is exactly a question about the geometric route. But when a geometric degeneracy *does* carry a label, the procedure "execute the transport, then read that geometric trace" is simultaneously a finite floor-resolved protocol of the kind FBI-comp quantifies over. The geometric route is therefore not walled off from FBI-comp; it is precisely the locus where a labelled degeneracy and an FBI-comp-violating protocol coincide — and that coincidence *is* the collapse implication (§6). Far from being a leak, this overlap is the mechanism by which C forces ND.

So the convention is: non-geometric label recovery is FBI-comp's alone and irrelevant to ND; geometric label recovery is ND's object and, when present, an FBI-comp violation as well. Every Boolean below is defined over the *geometric* route — the shared locus — because that is where the FBI-comp/ND relationship is decided; the non-geometric route, being ND-irrelevant, does not enter the relationship at all.

With the partition fixed, we proceed: first the near-trivial half, then the substantive half.

**Proposition 4.1 (a surviving degeneracy is detectable) [near-definitional; see §2].** ND-failure implies the existence of an operational trace.

*Proof.* ND is defined (§2) in terms of *operational* fiber cardinality — cardinality as resolved at the floor. An ND-failure is therefore, by the definition of ND itself, a change in a floor-resolved quantity: the operational cardinality of the fiber differs across the base. That change *is* an operational trace in the sense of Definition 3.1. The only residual content is the boundary case of a cardinality change too fine to resolve; but such a change is not an *operational* cardinality change and so is not an ND-failure in the sense §2 defines — by OO it has been quotiented away by the resolution map r. Hence every ND-failure, as §2 means it, is detectable, and the step from "operational cardinality change" to "floor-resolved trace" is carried by the word *operational* already present in the definition, not by a fresh appeal to OO. ■

Proposition 4.1 secures only the *weak* horn — that a surviving degeneracy is *visible*. It does not, and cannot by itself, secure the *strong* horn that the visible trace is sector-labelled. That is the content of the Boolean we now name, over the geometric route fixed above.

**Definition (the central Boolean C).**

**C:** every fiber degeneracy of admissible transport generates a *sector-labelled geometric trace* — a trace, derived solely from the local degeneracy geometry, whose value determines the sector.

C is defined over the geometric route only, per the partition convention. This is the single sense of C used everywhere below; there is no separate "unrestricted C" in play.

**Proposition 4.2 (C is non-trivial) [Proven].** C does not follow from Proposition 4.1, nor from the definitions of FBI-comp and ND. Proposition 4.1 delivers an operational trace; C demands a *sector-labelled* one; by Lemma 3.3 the latter is strictly stronger. C is therefore a substantive additional claim, neither definitionally true nor a consequence of the trace-existence established so far.

*Proof.* Immediate from Lemma 3.3: the inference (operational trace exists) $\Longrightarrow$ (sector-labelled trace exists) is exactly the converse that Lemma 3.3 shows can fail. Any derivation of C from Proposition 4.1 alone would close that gap, contradicting Lemma 3.3. ■

Proposition 4.2 establishes only the non-freeness of C. It does **not** assert ¬C (that sector-blind degeneracies exist); establishing ¬C requires the construction of §8. The fork between collapse and irreducibility is therefore genuinely open and rests entirely on C.

**Theorem 4.3 (the reduction) [Proven; collapse direction Conditional-on-OO].** The one-vs-two question is logically equivalent to C-vs-¬C:

— If **C** holds, then (under OO) FBI-comp $\Longrightarrow$ ND, and the sector residue collapses to the single object FBI-comp.

— If **¬C** holds, then FBI-comp and ND are independent, and the sector residue is two objects.

The detailed implications are proven in §6 (collapse) and §7 (irreducibility). Because C is defined over the geometric route only, ¬C is witnessed by a *geometric* sector-blind degeneracy — which is genuine ND territory, not a non-geometric route misattributed to ND — so the implication ¬C $\Longrightarrow$ two-independent-objects is valid as stated, with no leakage from FBI-comp's domain. The point established here is structural: because C and ¬C are exhaustive and exclusive, the collapse and irreducibility scenarios are exhaustive and exclusive, and *no third possibility for the FBI-comp/ND relationship exists* once attention is restricted to the named obstruction modes (the scope restriction is made precise in §9). The one-vs-two question has therefore been reduced — exactly, and with no remainder relative to the named obstruction modes — to the single Boolean C.

Two labelling points. First, the collapse direction (C $\Longrightarrow$ one object) runs through Theorem 6.1, which is Conditional-on-OO; the equivalence therefore inherits that OO-dependence on its collapse leg, which the theorem label records. The irreducibility direction (¬C $\Longrightarrow$ two objects) carries no OO, resting only on Observation 2.1 and the definitions. Second, "no remainder" is

now literal rather than reconstructed: C means one thing — the geometric Boolean — throughout, so there is no second Boolean tracked under the same name. ■

The Boolean C is itself reducible one step further. The trace-theoretic content of C is fully discharged by the symmetry results of §8, leaving a statement purely about admissible geometry.

**Corollary 4.4 (C reduces to a geometric existence question) [Proven, given §8].** The truth value of C is determined by a single geometric question about the transport construction:

Does the transport construction admit an admissible σ-invariant fiber degeneracy?

Specifically: if such a degeneracy exists, then ¬C holds; if no such degeneracy exists, then C holds.

*Proof.* By Corollary 8.3 (proven in §8), an admissible σ-invariant degeneracy is operational yet sector-blind, hence a counterexample to "every degeneracy generates a sector-labelled geometric trace," i.e. a witness for ¬C. Conversely, by Corollary 8.2A the only geometric traces that carry a label are σ-breaking ones; so if no admissible σ-invariant degeneracy exists, every admissible degeneracy is σ-breaking and its geometric trace is sector-labelled — which is C. ■

Because C is defined over the geometric route (per the partition convention of §4), the equivalence is exact, not approximate: C is the geometric Boolean and the σ-invariant degeneracy question is its geometric content. A label recovered from a non-geometric quantity is, by the partition convention, FBI-comp's territory and does not bear on C or on the FBI-comp/ND relationship. The biconditional C ⟺ (no admissible σ-invariant degeneracy) is therefore clean.

The compression is genuine: the residue is no longer the trace-theoretic Boolean C but the strictly geometric question *existence of σ-invariant admissible degeneracies*. The trace reasoning has been fully spent in §8, and what remains for the capstone is a question about the admissibility constraints of the transport construction alone — a much cleaner target, examinable without any further appeal to traces, protocols, or OO.

The reduction has a further consequence that constrains *every* future attempt at collapse, not merely this paper's analysis of it.

**Theorem 4.5 (Minimal Residue Theorem) [Proven, given §8].** Any proof that the sector residue collapses to one object must establish the single condition:

no admissible σ-invariant fiber degeneracy exists.

(This condition can obtain in two ways — no σ-invariant degeneracy exists at all, or some exist but all are inadmissible — but these are not distinct logical targets: both are the assertion that the admissible σ-invariant degeneracies are absent, which holds vacuously when there are no σ-invariant degeneracies at all.) Equivalently, every route to a one-residue capstone passes through the σ-invariant degeneracy question; there is no collapse proof that avoids it.

*Proof.* Collapse is C (Theorem 4.3). By Corollary 4.4, C holds iff the transport construction admits no admissible σ-invariant degeneracy. Suppose a collapse proof did not establish this condition: then an admissible σ-invariant degeneracy exists, and by Corollary 8.3 it witnesses ¬C, contradicting the proof's conclusion of C. Hence any valid collapse proof establishes the condition. ■

Theorem 4.5 is what makes the σ-question not merely *sufficient* but *necessary* for collapse. A future worker cannot establish one residue by some unrelated argument that sidesteps degeneracy geometry; the only door to collapse is the demonstration that admissible σ-invariant degeneracies do not exist. This sharpens the capstone target from "settle C somehow" to "settle the σ-invariant degeneracy question, because nothing else can settle C."

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# 5. The Asymmetry Between the Two Branches

The reduction of §4 is symmetric in appearance — C gives one object, ¬C gives two. The *implications* are not equally costly in premises, and the *proof obligations* C and ¬C impose are not equally heavy. We state both asymmetries precisely, and we are careful not to overclaim either: an antecedent is a cost on both sides of a fork or on neither, so the asymmetry cannot be located in treating C as a cost while treating ¬C as free. The genuine asymmetries lie elsewhere, and we isolate them.

We separate two genuine asymmetries.

**Theorem 5.1 (the OO rider sits only on collapse) [Proven].** Consider the two *implications* of Theorem 4.3, holding their respective antecedents fixed. The collapse implication (C ⟹ one object) requires OO; the irreducibility implication (¬C ⟹ two objects) does not.

*Proof.* The collapse implication runs through Theorem 6.1, whose derivation invokes OO at the step certifying a sector-labelled trace as a readable admissible protocol — the bridge from "a sector-labelled trace exists" to "the label is recoverable by a protocol FBI-comp quantifies over." Remove OO and that step fails. The irreducibility implication runs through Theorem 7.1, which derives independence from ¬C using only Observation 2.1 (itself OO-free) and the definitions of FBI-comp, ND, and sector-labelled trace; no step appeals to OO. Hence OO is a premise of the collapse implication alone. This is an asymmetry between the two *implications*, with the antecedents C and ¬C held fixed on their respective sides — it does not treat either antecedent as free. ■

**Proposition 5.2 (the proof obligations are universal vs existential) [Proven].** The antecedent C is a universal/non-existence claim; its negation ¬C is existential, dischargeable by a single witness.

*Proof.* By Corollary 4.4, C ⇔ (no admissible σ-invariant fiber degeneracy exists) — a non-existence statement, quantifying universally over the admissible-transport configurations and asserting none is a σ-invariant degeneracy. Its negation ¬C ⇔ (there exists an admissible σ-invariant degeneracy) is existential, established by exhibiting one such configuration. ■

These two are the honest content of "the branches are not on equal footing." Note what is *not* claimed: not that collapse costs "C+OO" while irreducibility costs nothing — that bookkeeping would treat the fork's antecedent as a cost on one side only. The claims are the two above: the OO rider on the collapse implication, and the universal-vs-existential character of the proof obligations. Their joint upshot is a *methodological* lean, not a logical default:

**Corollary 5.3 (methodological lean toward two) [Methodological].** Establishing ¬C requires producing one admissible σ-invariant degeneracy; establishing C requires excluding all of them. Exhibiting a single witness is generically a lighter proof obligation than a universal exclusion, and the collapse implication additionally carries the OO rider that irreducibility does not. Pending resolution of the σ-invariant degeneracy question, it is therefore methodologically prudent to carry **two** open objects, economizing to one only upon a proof that no admissible σ-invariant degeneracy exists.

This is consistent with §10–§11, which state the remaining work even-handedly as "construct an admissible σ-invariant degeneracy, or prove none exists." The even-handedness there is the *logical* symmetry of the fork (either resolution settles the count); the lean here is the *methodological* observation that the two resolutions are not equally easy to attempt. There is no contradiction: the fork is logically two-sided, while the proof effort it demands is lopsided. We claim only the latter, and only as methodology.

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# 6. The Collapse Branch and Its True Conditionality

We give the collapse implication and assign its conditionality correctly, separating the substantive geometric premise from the thinner ontological one.

**Theorem 6.1 (collapse) [Conditional-on-C, and separately Conditional-on-OO].** Assume C. Then, under OO,

FBI-comp ⟹ ND.

*Proof.* Assume FBI-comp: no finite floor-resolved protocol recovers the phase/refinement label. Suppose, for contradiction, that ND fails — a fiber degeneracy of admissible transport occurs. By C, this degeneracy generates a sector-labelled trace τ. By Definition 3.1, τ is floor-resolved; by Definition 3.2, reading τ recovers the sector label. The operation "execute the transport, then read τ" is a finite procedure composed of floor-resolved operations — and by OO a floor-

resolved observable is genuine readable structure, not surplus — so it is a finite floor-resolved protocol of exactly the kind FBI-comp quantifies over. This protocol recovers the label, contradicting FBI-comp. Hence ND fails to fail: ND holds. ■

Two points on the labelling.

First, the *primary* conditionality of this branch is **C**, not OO. The contradiction is driven by C — it is C that turns the degeneracy into a label-bearing trace. Labelling the collapse merely "Conditional-on-OO" would hide the strongest assumption (C, a substantive geometric claim) behind a programme-wide axiom that reads as free. We label it **[Conditional-on-C]** primarily, with OO in its proper, thinner role.

Second, OO's role is *exactly* the bridge "sector-labelled trace exists ⟹ label recoverable by an admissible protocol." Without OO one could imagine a trace that is structurally present but operationally unreadable; OO forecloses that by denying structural status to anything with no operational witness, so a *sector-labelled* trace (which by Definition 3.2 is operational) is ipso facto readable. This is the same role OO plays for OSP and OIP in the prior paper, and it carries the same [Conditional-on-OO] standing — but it is a *second*, weaker premise layered on top of C, not the carrier of the branch.

**Corollary 6.2 [Conditional-on-C + OO].** Under C and OO, the sector residue collapses to FBI-comp:

FBI-comp ⟹ ND ⟹ (with the prior paper's Lemma 7.1A) UO.

The chain reads: FBI-comp closes the wall and, via C, also forces non-degeneracy, which is the residual content of UO; so a single conjecture discharges the entire sector side.

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# 7. The Irreducibility Branch

We give the complementary implication, and note honestly that it is near-definitional once ¬C is granted — the substance is the *existence* of sector-blind degeneracies (¬C), addressed in §8, not the consequence drawn from them here.

**Theorem 7.1 (irreducibility) [Proven, given ¬C].** Assume ¬C: there exists a fiber degeneracy of admissible transport that generates an operational trace but no sector-labelled trace. Then FBI-comp and ND are independent — neither implies the other.

*Proof.* Observation 2.1 already gives FBI-comp ⇏ ND unconditionally. For the converse, ND ⇏ FBI-comp: ND constrains fiber cardinality and says nothing about label recovery, so ND may hold while a finite protocol nonetheless recovers a label by some non-degeneracy route; ND is simply silent on FBI-comp. It remains to show the collapse implication of §6 *fails* under ¬C, so that the two are not secretly linked through it. By hypothesis there is a degeneracy whose trace is

sector-blind; the contradiction in Theorem 6.1's proof cannot be run on it, because reading its (sector-blind) trace does not recover the label and so violates no instance of FBI-comp. Thus FBI-comp can hold in the presence of an ND-failure: FBI-comp $\not\Rightarrow$ ¬(ND-failure), i.e. FBI-comp does not force ND. With neither direction holding, the objects are independent. ■

**Remark 7.2 (where the substance lives).** Theorem 7.1 unpacks what "sector-blind" means more than it discovers anything; granting ¬C, independence is close to immediate. The genuinely substantive claim is ¬C itself — the *existence* of a sector-blind admissible degeneracy. That existence is not proven in this paper. §8 reduces it to a clean geometric condition: a degeneracy whose local geometry is invariant under phase/refinement interchange *is* sector-blind (Theorem 8.2), so ¬C follows if the transport construction admits such a symmetric degeneracy (Corollary 8.3). Until that admissibility is settled, ¬C is [Conjectural]. By the asymmetry of §5, ¬C is the existential horn — discharged by exhibiting a single admissible σ-invariant degeneracy (Proposition 5.2) — whereas C is the universal horn requiring the exclusion of all of them, and the collapse implication additionally carries the OO rider that irreducibility does not (Theorem 5.1). These facts ground the methodological lean of Corollary 5.3 toward carrying two objects pending resolution; they do not make "two" a proven default. The paper's honest position is therefore: methodologically prudent to carry two objects unless C is proven, with ¬C reduced to a single geometric existence question awaiting resolution.

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# 8. The Symmetry Principle: σ-Invariance Implies Sector-Blindness

This section does not prove ¬C. But it does better than intuition: it isolates a clean geometric hypothesis — local symmetry under phase/refinement interchange — from which sector-blindness follows as a *theorem*. The remaining gap is then narrowed to a single, sharply stated question about whether the transport construction realizes such a symmetric degeneracy. Sector-blindness is thereby established as a proven conditional, with one explicit open premise — the admissibility of a symmetric degeneracy — rather than rested on a heuristic.

We first fix the relevant symmetry.

**Definition 8.1 (phase/refinement interchange).** Let σ denote the local involution on the substrate degrees of freedom at a point R that swaps the phase displacement axis with the refinement displacement axis (and is the identity on all other operational data). A local degeneracy geometry at R is *σ-invariant* if its description — the branching, collapse, or continuity-failure datum of the fiber over a neighbourhood of R — is fixed by σ.

The content of σ-invariance is that the degeneracy "does not privilege either axis": the way the fiber branches is the same whether one calls the locally-active displacement phase or refinement.

Definition 8.1 fixes σ as an involution on substrate degrees of freedom; we extend its action to the traces that read those degrees of freedom. For a *geometry-derived* trace $\tau = f(G)$ — one whose value is a function of the local degeneracy geometry G alone — define the induced action

$\sigma\cdot\tau := f(\sigma\cdot G)$, equivalently $\sigma\cdot\tau = \tau \circ \sigma$ on the geometry.

This is well-defined precisely for geometry-derived traces, since for those the trace value is a function of G and σ acts on G. We do *not* assign σ an action on traces outside this class; all results below that write $\sigma(\tau)$ are confined to geometry-derived τ, which is the only class Theorem 8.2 and Corollary 8.2A use.

Before showing σ-invariance implies sector-blindness, we show the converse direction at the level of geometry-derived traces — that σ is not an arbitrary symmetry but the *unique* one whose breaking is necessary for any label carried by the geometry. This forecloses the referee question "why σ?": a geometric sector label is, by its very meaning, a σ-breaking datum.

**Lemma 8.1A (sector labels require symmetry breaking) [Proven].** Let τ be a *geometry-derived* sector-labelled trace. Then τ is not invariant under the induced action of the phase/refinement interchange σ:

τ geometry-derived and sector-labelled $\Longrightarrow \sigma\cdot\tau \neq \tau$.

*Proof.* By Definition 3.2, a sector-labelled trace assigns different floor-resolved values according to whether the motion belonged to the phase sector or the refinement sector. The involution σ exchanges those two labels (Definition 8.1). If τ were σ-invariant ($\sigma\cdot\tau = \tau$), then τ would assign the same value to a motion and to its σ-image — i.e. the same value to the phase-sector reading and the refinement-sector reading — and so could not distinguish them, contradicting sector-labelledness. Hence $\sigma\cdot\tau \neq \tau$. ■

Lemma 8.1A establishes that σ-symmetry breaking is *necessary* for label recovery: there is no sector-labelled trace that respects σ. The choice of σ is therefore forced — it is precisely the symmetry that any label must break, because "carrying a sector label" and "distinguishing the two things σ swaps" are the same property. This is the answer to "why is σ the right symmetry?": it is the only symmetry relative to which sector-labelling is even definable.

We now prove the converse implication, which together with Lemma 8.1A yields a clean biconditional.

**Theorem 8.2 (symmetry principle: σ-invariance ⟹ sector-blindness) [Proven].** If a fiber degeneracy at R is σ-invariant, then any operational trace derived *solely* from the local degeneracy geometry is sector-blind.

*Proof.* Let $\tau = f(G)$ be a geometry-derived trace, with induced action $\sigma\cdot\tau = f(\sigma\cdot G)$. By Lemma 8.1A, τ can be sector-labelled only if $\sigma\cdot\tau \neq \tau$. By σ-invariance, $\sigma\cdot G = G$, hence $\sigma\cdot\tau = f(\sigma\cdot G) = f(G) = \tau$: the trace is fixed by σ. By the contrapositive of Lemma 8.1A, a σ-invariant geometry-derived trace is not sector-labelled. Therefore τ is sector-blind. ■

**Corollary 8.2A (biconditional characterization) [Proven].** For traces derived solely from the local degeneracy geometry, sector-labelledness and σ-breaking coincide:

$\tau$ sector-labelled $\Leftrightarrow \sigma\cdot\tau \neq \tau$.

*Proof.* Forward is Lemma 8.1A. Reverse: suppose $\sigma\cdot\tau \neq \tau$ for a geometry-derived $\tau$. By Definition 8.1, σ acts as the identity on all operational data *other than* the phase/refinement axes; so the only feature of G that σ can alter is the relative assignment of the phase and refinement axes. A change in $\tau$ under σ is therefore a change driven *solely* by the phase/refinement exchange — not by σ disturbing some unrelated structure $\tau$ reads, since σ disturbs none. And since σ is precisely the phase↔refinement swap (Definition 8.1), the two values σ relates, $\tau$ and $\sigma\cdot\tau$, are exactly the phase-sector and refinement-sector readings of $\tau$; their being distinct ($\sigma\cdot\tau \neq \tau$) is therefore $\tau$ assigning distinct values to the two sectors, which is sector-labelledness. The "identity on all other operational data" clause supplies that the difference is purely sectoral, and σ's being the label-swapping involution supplies that the difference *is* a difference across the two sectors rather than mere non-invariance. ■

The biconditional is the structural payoff. Label recovery from a degeneracy is now *exactly* σ-symmetry breaking in the degeneracy geometry — not merely implied by it, but equivalent to it. Sector-blindness at a σ-invariant degeneracy is therefore not a suggestive coincidence; it is forced, and the only way a degeneracy can carry a label is by breaking the very symmetry that defines the label.

The hypothesis "trace derived solely from the local degeneracy geometry" is worth one comment. A trace that also reads *non-geometric* data — some independent operational quantity correlated with sector — could in principle break sector-blindness even at a σ-invariant degeneracy. Theorem 8.2 says only that the *degeneracy itself* supplies no label; it does not exclude a label arriving from elsewhere. But "elsewhere" is exactly a non-degeneracy route to label recovery, which is the province of FBI-comp, not of ND. So Theorem 8.2 cleanly separates the two: a σ-invariant degeneracy contributes nothing to label recovery, and any label recovery that does occur is charged to FBI-comp. This is precisely the independence the irreducibility branch needs.

**Corollary 8.3 (conditional witness for ¬C) [Conditional on admissibility of a σ-invariant degeneracy].** If the transport construction admits a σ-invariant fiber degeneracy, then ¬C holds: that degeneracy is operational (detectable, by Proposition 4.1) yet sector-blind (by Theorem 8.2), so not every degeneracy is sector-labelled.

The argument is now incomplete in exactly *one* place, and it is a clean one. We no longer assume sector-blindness; we have proven it from σ-invariance. What remains open is solely whether the transport construction of the prior paper *realizes* a σ-invariant degeneracy, or instead forbids all of them by some constraint not yet examined. Constructing an admissible σ-invariant degeneracy settles ¬C (two objects); proving the construction excludes every σ-invariant degeneracy removes this route to ¬C and returns C to contention. This is the concrete, geometric form of "prove or refute C," and it is a question about the transport construction alone — no longer entangled with the trace-theoretic reasoning, which Theorem 8.2 has discharged.

# 9. Residual Modes and the Scope of "Two"

The reduction of §4 and the dichotomy of §11 concern the FBI-comp/ND relationship *within the named obstruction modes*. The prior paper's Lemma 7.1A enumerated three obstruction types — sector, holonomy, degeneracy — and tagged the *exhaustiveness* of that enumeration as [Conditional], explicitly flagging residual modes: a failure of π to be open, or a global selection obstruction that is neither pure holonomy nor pure fiber-collapse.

This matters for the headline count. "One residue or two" is shorthand for "one or two *among {sector, degeneracy}*," with holonomy already reduced (it presupposes lifts) and phase continuity carried separately. If a residual mode exists and is itself sector-blind, the residue could be **three**: FBI-comp, ND, and the residual obstruction, with the last independent of both.

We bracket residual modes explicitly:

**Scope convention.** All counts in this paper are conditional on the prior paper's three-type enumeration being exhaustive. Under that condition the residue is one (if C) or two (if ¬C). If a residual mode survives the enumeration, the count is incremented accordingly, and whether the residual mode is sector-labelled or sector-blind is a further instance of the same C-style question applied to that mode.

This is the honest scope. The reduction is clean *given* the enumeration; it does not silently close an enumeration the prior paper left open.

# 10. Consequences for ALP and the Capstone

We first state, as a theorem, exactly what the capstone inherits — making the dependence of the entire remaining sector-side architecture on the single status of C explicit rather than implied.

**Theorem 10.1A (Capstone Dependency Theorem) [Proven, given the reduction and §6–§7].** The number of independent sector-side conjectures entering the capstone is determined entirely by the truth value of C:

C true ⟹ sector-side input is { FBI-comp }.

C false ⟹ sector-side input is { FBI-comp, ND }.

Equivalently, by Corollary 4.4, the sector-side conjecture count is determined by whether the transport construction admits an admissible σ-invariant degeneracy.

*Proof.* If C holds, Corollary 6.2 gives FBI-comp ⟹ ND ⟹ UO, so ND is not an independent input — it is discharged by FBI-comp — and the sole sector-side conjecture is FBI-comp. If C fails, Theorem 7.1 gives FBI-comp ⊥ ND, so each is an independent input that must be discharged separately. The two cases are exhaustive and exclusive (Theorem 4.3), and the count (one, resp. two) is fixed in each. The geometric restatement is Corollary 4.4. ■

The theorem is what makes the present paper a determinant of the remaining architecture rather than a survey of possibilities: the capstone's sector-side logical structure is not partially constrained but *completely fixed* by a single Boolean, and that Boolean is now a concrete geometric existence question. The propagation into the lifting programme then follows the branch.

**Collapse branch (C true).** FBI-comp ⟹ ND ⟹ UO (Corollary 6.2). The Admissible Lift Property then has a single conjectural input on the sector side: FBI-comp. ALP holds once FBI-comp is established (modulo OO and the standing constructions), and RC follows by the prior paper's Theorem 8.1 with the phase-continuity tier. The capstone carries **one** sector gate.

**Irreducibility branch (¬C true).** FBI-comp ⊥ ND. ALP then depends on two independent sector-side inputs — FBI-comp *and* ND — each of which must be discharged separately. RC inherits both. The capstone carries **two** sector gates.

In both branches the phase-continuity tier remains as a third, separately inherited item; it is untouched by C and is not part of the one-vs-two count, which concerns only the FBI-comp/ND pair.

The capstone input is therefore stated with full precision, and — by the Minimal Residue Theorem (§4) — in its sharpest possible form: **resolve the σ-invariant degeneracy question.** Construct an admissible σ-invariant degeneracy (settling ¬C, two gates), or prove the transport construction admits none (settling C, one gate). By Theorem 4.5 there is no other door: every collapse proof must establish non-existence of admissible σ-invariant degeneracies, so this is not merely *a* sufficient input but the *only* one. This is the single, well-posed geometric question the present paper hands forward, in place of the diffuse pair "are FBI-comp and ND related?"

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# 11. The Main Dichotomy, Stated Honestly

We collect the architecture. The dichotomy is **complete** (the branches are exhaustive and exclusive) and **conditional on the §9 scope**; it is *not* a determination of which branch obtains.

**Collapse branch (C) — delivers one object; collapse implication carries the OO rider.**

C ⟹ ( FBI-comp ⟹ ND ⟹ UO ).

The sector residue is one conjectural object, FBI-comp. The capstone carries one sector gate.

**Irreducibility branch (¬C) — delivers two objects; implication OO-free.**

¬C ⟹ ( FBI-comp ⊥ ND ).

The sector residue is two independent objects. The capstone carries two sector gates.

**The discriminating Boolean.**

C: every fiber degeneracy of admissible transport generates a sector-labelled *geometric* trace — one derived solely from the local degeneracy geometry, whose value determines the sector.

**Asymmetry.** Two honest asymmetries distinguish the branches (§5): the collapse implication carries an OO rider that irreducibility does not (Theorem 5.1), and C is a universal non-existence claim while ¬C is existential (Proposition 5.2). These ground a *methodological* lean toward two (Corollary 5.3), consistent with the even-handed logical fork stated here. The symmetry principle of §8 reduces ¬C — and hence the two-object outcome — to a single geometric existence question: whether the transport construction admits a degeneracy invariant under phase/refinement interchange (Theorem 8.2, Corollaries 4.4 and 8.3).

The value of the result is independent of which branch holds. If C is proven, the programme economizes to a single remaining sector conjecture. If ¬C is proven, the programme is shown to contain two genuinely independent open questions — itself a sharp structural fact, not a failure. In neither case is the present paper's deliverable contingent on FBI-comp yielding: the reduction (Theorem 4.3) and the branch asymmetries (Theorem 5.1, Proposition 5.2) are [Proven] now.

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# 12. Limitations

This paper does not prove C, and does not prove ¬C. It proves only that the one-vs-two question is equivalent to C-vs-¬C (Theorem 4.3) and that the branches differ by the two asymmetries of §5 (the OO rider and the universal-vs-existential proof obligation). The discriminating question C is handed to the capstone unresolved.

This paper does not prove FBI-comp or ND. Their *relationship* is the object here; their individual truth is not addressed.

Proposition 4.1 is near-definitional given the §2 reading of ND in terms of *operational* (floor-resolved) fiber cardinality: an ND-failure in that sense is already a change in a floor-resolved quantity. OO enters only at the boundary, to confirm that a cardinality change too fine to resolve is not structure and so not an ND-failure; against an opponent who rejects OO, that boundary stipulation is unavailable, but the main content of 4.1 rests on the definition of ND rather than on a fresh ontological appeal. This is a thinner OO-dependence than the collapse branch's.

The collapse branch (Theorem 6.1) is primarily [Conditional-on-C] and secondarily [Conditional-on-OO]. We separate these because they are load-bearing in distinct ways and conflating them would mislabel a C-driven implication as merely OO-driven, hiding the substantive geometric premise behind a programme-wide axiom. If C is false the branch is vacuous; if OO is denied the trace-read is uncertified even granting C.

The irreducibility branch (Theorem 7.1) is [Proven] *given* $\neg C$, but ¬C is itself unproven. The near-definitional character of Theorem 7.1 means the real work is the existence question of §8.

The symmetry principle (§8, Theorem 8.2) is [Proven]: σ-invariance implies sector-blindness for any trace built from the degeneracy geometry. What is *not* proven is its hypothesis — that the transport construction admits a σ-invariant degeneracy (Corollary 8.3 is [Conditional] on exactly this). The construction might forbid all σ-invariant configurations by some constraint not yet examined, in which case this route to ¬C closes and C regains contention. We have not examined the transport construction at the level of detail required to decide this. Note also that Theorem 8.2 governs only traces derived *solely* from the degeneracy geometry; a label arriving from a non-geometric quantity is a non-degeneracy route to recovery, charged to FBI-comp rather than to ND, and is outside the theorem's scope by design.

All counts are conditional on the prior paper's three-type obstruction enumeration being exhaustive (§9). A surviving residual mode increments the count and reopens a C-style question for that mode.

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# 13. Conclusion

The previous paper left two objects on the sector side — composite sector recovery (FBI-comp) and fiber degeneracy (ND) — with an informal hope that they were "probably one fact." This paper tests that hope and replaces it with structure.

We proved that the question "one residue or two?" is not vague: it reduces, exactly and with no remainder, to a single Boolean —

does every fiber degeneracy of admissible transport betray the sector of the motion that produced it?

If it does (C), FBI-comp absorbs degeneracy and the sector side is one conjecture. If it does not (¬C), the two are independent and the sector side is two. The branches are not symmetric in proof obligation, though they are symmetric as a logical fork: the collapse implication carries an OO rider the irreducibility implication does not, and C is a universal non-existence claim while ¬C is existential, discharged by a single witness. These two proven facts ground a methodological lean toward carrying two objects pending resolution of the σ-question — a lean about proof effort, not a proven default.

The symmetry principle sharpens what remains: we proved that any degeneracy invariant under phase/refinement interchange is sector-blind, so ¬C — and the two-object outcome — reduces to whether the transport construction admits such a symmetric degeneracy. But the paper's deliverable does not rest on resolving that. It rests on the reduction and the burden assignment, both of which stand as theorems now, whatever FBI-comp eventually does.

The capstone therefore receives a single, sharply stated input in place of a diffuse pair:

Construct an admissible σ-invariant degeneracy (settling ¬C, two gates), or prove the transport construction admits none and that every admissible degeneracy is sector-labelled (settling C, one gate).

The first answer gives the programme two independent sector gates; the second gives it one. Either way, the logical architecture of the remaining sector-side programme is now reduced to that one constructible-or-refutable question — which is the most a pre-capstone paper can honestly deliver, and exactly what the capstone needs.

Stated in one line, the contribution is: **the FBI-comp/ND relationship is determined by C, and C is determined by the existence or non-existence of admissible σ-invariant degeneracies.** The remaining sector-side dependency chain is correspondingly concentrated:

σ-invariant degeneracy question → C-vs-¬C → one residue or two → FBI-comp → RC → $\ell^2$.

Each arrow denotes a dependency settled by a result of this paper or a prior one: the topmost node *determines* the truth value of C (existence gives ¬C, non-existence gives C — so the arrow is the determination, not an implication in either fixed direction), C-vs-¬C determines the residue count, and the rest is the established descent. The single geometric question at the head — whether the transport construction admits an admissible σ-invariant degeneracy — is now the sole open input on which the entire descent to the $\ell^2$-norm depends.