Sequential Interface Transport and Emergent Time in VERSF

Hexagonal Closure Surfaces, the Mapping Telescope of Sequential Commitment, and the Bigraded Home of σ-Duality

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General Reader Abstract

The earlier VERSF papers proposed that light does not fundamentally propagate through three-dimensional space. It travels on a two-dimensional hexagonal causal interface, and the apparent 3D world is reconstructed from correlation structure across that interface and across successive interface updates.

The recent K=7 symmetry papers then hit a wall. Attempts to realise the proposed spatial–temporal exchange symmetry σ as a chain-complex automorphism on a static prism extension of the K=7 wheel kept failing at the chain-map level — the equation $\partial\sigma = \sigma\partial$ never closed.

This paper diagnoses the failure and offers a structurally correct reformulation in two stages.

The diagnosis is that temporal succession was being represented as if it were ordinary geometric adjacency inside a single static complex, conflating objects (cells of interfaces) with morphisms (the σ-family). VERSF's own ontology already has time emerging from sequential closure rather than from a primitive temporal axis. Flattening this emergent ordering into a single static complex was a category error.

The first stage of reformulation replaces the static prism with the **mapping telescope** of the σ-family — a single well-defined chain complex built from standard mapping cones, in which each interface state $W_7^{(t)}$ keeps its own closure structure and the successive updates $\sigma_t : W_7^{(t)} \to W_7^{(t+1)}$ are encoded as chain maps between layers.

The second stage concerns the σ-duality itself. The duality requires a *different* bigraded structure — one indexed by the 7 vertices of W_7 on one axis and a 7-tick coherence window on the other. The vertex×tick-window structure is *identified* (not constructed) in the present paper as the correct home of σ-duality; what its cells actually carry, and how the 7-tick window embeds into the $\mathbb{N}$ tick axis of the telescope, are substantive open problems. The hub h of W_7 pairs conditionally with a distinguished origin tick under any σ-duality respecting the 6+1 asymmetry of W_7, with two independent discrete gauge ambiguities — D_6 rotation/reflection of the outer-vertex–tick pairing, and translation of the 7-tick window within $\mathbb{N}$ — both left to be fixed by substrate dynamics. The paper introduces a new epistemic category, *identified but not*

constructed, for the vertex×tick-window bigrading; its required type is fixed but its content is open.

The substrate-derived σ-family, the bicomplex lift, the construction of the vertex×tick-window bigrading, and the continuum limit are now structurally separated, sharply specified, and constitute the next computational targets.

Abstract

The recent K=7 sequence in VERSF established refinement-persistent cohomological transport, Maxwell admissibility, one-loop Lorentz-compatible enhancement, and a class-level spatial–temporal exchange symmetry σ forcing bare isotropy under additional substrate-physical assumptions. Attempts to realise σ as a chain-complex automorphism of a static prism extension $\mathrm{Cyl}(W_7)$ of the K=7 wheel $W_7 = C_6 + h$ reproducibly yielded

$\partial\sigma \neq \sigma\partial$,

with the failure localised at the interface between "horizontal" closure edges and "vertical" prism edges.

This paper diagnoses the obstruction as a category error of two kinds (Section 5) and supplies a structurally correct reformulation in two stages.

Stage 1 — the mapping telescope (Sections 6–7). The natural ambient structure is not a single static complex but an $\mathbb{N}$-indexed *diagram of chain complexes*

$W_7^{(0)} \to_{\sigma_0} W_7^{(1)} \to_{\sigma_1} W_7^{(2)} \to_{\sigma_2} \cdots$

where each σ_t is a chain map. The natural single chain complex associated to this diagram is its mapping telescope

$\mathrm{Tel} = \mathrm{Tel}(\{W_7^{(t)}\}, \{\sigma_t\})$,

constructed via standard mapping cones with closed total differential. Tel is well-defined given any chain-map family, requires no further structural assumptions, and is the correct geometric object replacing the failed $\mathrm{Cyl}(W_7)$.

Stage 2 — σ-duality on a separate bigrading (Section 8). The substantive content of the original σ programme — the spatial–temporal exchange — is not naturally defined on the closure×tick organisation underlying Tel. The simplicial grading of the closure complex (taking values in $\{0, 1\}$ for a graph) and the tick index (taking values in $\mathbb{N}$) range over different index sets, so a duality interchanging them is ill-typed. The duality requires a *different* bigraded structure

$D : V(W_7) \times T_7 \longrightarrow Ch(Ab)$

where $V(W_7) = \{h, v_0, \ldots, v_5\}$ is the 7-vertex basepointed set of W_7 and $T_7 = \{\tau_0, \tau_1, \ldots, \tau_6\}$ is an independent basepointed 7-cyclic structure (a "coherence window"), with $|T_7| = |V(W_7)| = 7$ forced by $\sigma^2 = id$ requiring the two index sets to be in bijection. The bigrading is equipped with a chain-isomorphism duality $\sigma : D(i, t) \to D(t, i)$ respecting the basepoints. What D assigns to each cell (i, t), and how T_7 embeds into the $\mathbb{N}$ tick axis of the telescope, are *not constructed in this paper*; the present paper identifies only the required type. The construction (P3) is further decomposed in §14 into five sub-parts addressing cell content, σ-duality, embedding, gauge specification, and projection compatibility separately.

The original σ programme implicitly conflated the closure×tick organisation (natural home of the telescope) with this vertex×tick-window bigrading (natural home of σ-duality), and this conflation is the precise source of the chain-map failure.

Aspirational structures. Lifting Tel to a genuine bicomplex requires the σ-family to carry simplicial-type enrichment. Constructing D explicitly on top of W_7, fixing the $T_7 \hookrightarrow \mathbb{N}$ embedding, and verifying a non-trivial σ-duality are further substantive demands. All are conditional on substrate-derived σ providing the necessary structure.

Under this reformulation: (i) the bare $\partial\sigma \neq \sigma\partial$ obstruction is diagnosed as a conflation of objects (closure cells) with morphisms (σ-family) under a single grading the underlying ontology does not support, combined with the further conflation of two distinct bigraded structures; (ii) the prism geometry is reinterpreted as the telescope foliation of sequential commitment; (iii) BCB and TPB constrain the admissible σ-family precisely (as cellular support and interaction range of σ_t); (iv) Lorentz-compatible transport is *proposed* to emerge as the continuum limit of σ-consistency, conditional on (P3) and (P4) of Section 14.

The paper establishes the reformulation. It does *not* construct a substrate-derived σ-family, lift Tel to a bicomplex, or construct the vertex×tick-window bigrading explicitly. These three constructions, plus the continuum-limit verification, are now structurally separated, sharply specified, and constitute the next computational targets.

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1. Introduction

The recent VERSF sequence produced two apparently distinct frameworks. The Hexagonal Interface Light Propagation paper established that primitive 3D propagation is non-admissible under finite-capacity distinguishability constraints, that light propagates fundamentally on a 2D hexagonal causal interface, and that bulk geometry is reconstructed from interface correlations and coarse-graining depth. The K=7 Wilson and σ papers established refinement-persistent transport, Wilson matching, bare Lorentz-compatible isotropy, and a class-level transport symmetry σ on K=7 closure structures.

These two programmes have been treated as partially independent. The present paper argues that they describe the same underlying structure, and that the recent attempts to realise σ inside a static prism extension Cyl(W_7) failed for two combined reasons: the prism was being misread as a primitive 3D transport bulk rather than as the foliation of sequential interface updates, *and* the σ-duality was being asked to live on a bigrading that does not naturally support it.

The contribution is threefold:

1. A precise **diagnosis** of the static-prism obstruction as a category error of two distinct kinds (Sections 4–5).
2. A precise **first-stage reformulation** as a diagram of chain complexes with its mapping telescope (Sections 6–7).
3. A precise **second-stage reformulation** identifying σ-duality as a structure on a separate vertex×tick-window bigrading, with the K=7 structure of W_7 providing the natural pairing (Section 8).

The original σ programme conflated the structures of (2) and (3). Untangling them is what produces a sharp set of open problems (Section 14) in place of the original chain-map failure.

Throughout we maintain VERSF's strict epistemic labelling. Section 15 includes a new category — *identified but not constructed* — for structures whose required type the present paper fixes but whose explicit content is left open.

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2. The Interface-Native Ontology

The Hexagonal Interface Light Propagation paper established a primitive ontology in which the fundamental carrier of causal propagation is a two-dimensional hexagonal interface, not a three-dimensional bulk. The supporting admissibility theorem rules out primitive 3D bulk propagation under finite-capacity BCB constraints: any process attempting to commit facts across a 3D volume per tick exceeds the boundary causal bandwidth and is non-admissible.

Three statements from that ontology will be load-bearing in what follows.

(O1) Interface primitivity. Light propagates fundamentally on a 2D causal interface; no admissible substrate dynamics realises primitive 3D bulk transport.

(O2) Bulk reconstruction. The apparent third spatial dimension is reconstructed from interface observables under coarse-graining,

$(x, y, z) \leftrightarrow$ interface observable at scale $\ell(z)$,

where $\ell(z)$ is the coarse-graining scale associated with depth z.

(O3) Emergent time. Time is not a primitive coordinate; it is the ordered accumulation of committed interface states. Each "tick" is a closure event, not motion through a pre-existing temporal axis.

(O3) is the load-bearing claim for what follows.

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3. Time as Sequential Closure

The standard reading of (O3) is that the temporal direction is emergent in exactly the same sense that the third spatial direction is emergent in (O2): it is reconstructed from a structure on the interface rather than being a primitive coordinate of the substrate.

The reconstruction differs, however, in a structurally important way. Whereas the emergent third spatial dimension is reconstructed from a coarse-graining hierarchy on a *single* interface, the emergent temporal direction is reconstructed from the *succession* of distinct interfaces:

$W_7^{(0)} \longrightarrow W_7^{(1)} \longrightarrow W_7^{(2)} \longrightarrow \cdot\cdot\cdot$

Each $W_7^{(t)}$ is a complete closure surface — a committed admissibility configuration. The labels $t = 0, 1, 2, \ldots$ index the order in which these surfaces were committed, not coordinates on a pre-existing temporal axis. The *carrier* of temporal succession is the ordered family $\{W_7^{(t)}\}$, and any attempt to flatten this family into a single static complex with temporal succession represented as additional edges discards the categorical content of (O3).

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4. The σ Obstruction: Precise Statement

The K=7 σ programme sought a class-level spatial–temporal exchange symmetry σ on the K=7 wheel $W_7 = C_6 + h$. In the static-prism construction, σ was posited as an endomorphism

$\sigma : C_n(\mathrm{Cyl}(W_7)) \longrightarrow C_n(\mathrm{Cyl}(W_7))$

of a single chain complex on the prism $\mathrm{Cyl}(W_7)$, preserving total degree n. The required chain-map condition is

$\partial\sigma = \sigma\partial.$

The natural candidates for σ — those exchanging "vertical" prism edges (intended to represent temporal succession) with "horizontal" closure edges of the W_7 wheel — fail this condition. Concretely, the failure manifests as broken cycle closure on the hexagonal face image, spoke / temporal-edge asymmetry under σ-action, and non-closing boundary images for σ-translated 1-chains.

In every construction attempted, the failure localised at the interface between vertical (temporal) and horizontal (spatial) edges. The pattern of failure is the diagnostic clue.

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5. Diagnosis: The Category Error

The obstruction is not combinatorial. It is the consequence of two distinct category errors.

(C1) Static-complex flattening — conflating objects and morphisms. The temporal sequence $\{W_7^{(t)}\}$ is naturally a categorical object: a diagram in the category of chain complexes,

$W_7^{(0)} \to_{\sigma_0} W_7^{(1)} \to_{\sigma_1} W_7^{(2)} \to_{\sigma_2} \cdots$

In this diagram, the $W_7^{(t)}$ are objects and the σ_t are morphisms between them. Flattening the diagram into a single complex $Cyl(W_7)$ by representing the σ_t as additional 1-chains converts a *morphism in a diagram* into a *cell of the underlying complex*. These are not the same categorical type. The σ_t are maps; the edges of $W_7^{(t)}$ are objects. Encoding them as the same kind of structure is a type error, and the consequences appear precisely where one tries to enforce structural compatibility between them — i.e., at the chain-map condition.

(C2) Independence of gradings forced onto a single axis. The closure complex W_7 carries a natural grading by simplicial dimension (vertices in C_0, edges in C_1). The tick sequence $\{W_7^{(t)}\}$ carries an independent grading by commitment order. These are independent gradings on the underlying structure: they need not be inconsistent, but they are independent. The static-prism construction forces them onto a single integer axis — the total degree n of the prism complex — and then demands σ preserve this collapsed grading. The collapse is what makes natural σ candidates fail to commute with ∂: a σ that genuinely exchanges spatial and temporal content must act non-trivially on both independent gradings, but the collapsed grading does not see this independence.

Together, (C1) and (C2) explain the precise pattern of obstruction observed. (C1) tells us the σ_t should be morphisms, not edges. (C2) tells us the natural setting carries an independent bigrading rather than a collapsed single grading. Both corrections are forced once VERSF's own emergent-time ontology (O3) is taken seriously. The next two sections give the resulting structure in two distinct stages.

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6. The Diagram of Chain Complexes

The first structural correction is to retain the σ_t as morphisms in a diagram, rather than collapsing them into edges of a static complex.

The natural ambient structure. Given a σ-family $\{\sigma_t\}_{t=0,1,2,\ldots}$ in which each $\sigma_t : W_7^{(t)} \to W_7^{(t+1)}$ is a chain map, the natural categorical home of the σ programme is the ℕ-indexed diagram

$$D : \mathbb{N} \longrightarrow \mathrm{Ch}(\mathrm{Ab}),\ t \mapsto W_7^{(t)},\ (t \to t+1) \mapsto \sigma_t.$$

This is unambiguous: it requires only that each σ_t be a chain map, i.e., $\sigma_t \circ \partial_h^{(t)} = \partial_h^{(t+1)} \circ \sigma_t$, where $\partial_h^{(t)}$ is the closure boundary inside $W_7^{(t)}$.

What is *not* yet asserted. The diagram D is not a single chain complex. It is not a bicomplex. It is a functor from ℕ (viewed as a category with one morphism from t to t+1 for each t) into the category of chain complexes. The single-complex realisation comes in the next section, via the mapping telescope. The bicomplex lift, requiring additional structure on the σ-family, is discussed in Section 9 as an aspirational enrichment.

The closure×tick organisation. D carries a natural organisation by *closure dimension* $p \in \{0, 1\}$ (the simplicial grading inside each layer) and *tick index* $q \in \mathbb{N}$. We refer to this as the **closure×tick organisation** of D. The bigraded module

$$M(p, q) \coloneqq C_p(W_7^{(q)})$$

with horizontal boundary $\partial_h : M(p, q) \to M(p-1, q)$ and chain-map family $\sigma_t : M(p, t) \to M(p, t+1)$ is the underlying organised data. We resist calling this a *bicomplex*, since a true bicomplex requires a vertical differential ∂_v with $\partial_v^2 = 0$ and anti-commutation with ∂_h; the σ-family as such provides neither. Iterated composition of σ-family members, $\sigma_{t+1} \circ \sigma_t$, is the two-tick update map, and there is no reason for this to vanish.

The bigraded module $(M(p, q), \partial_h, \{\sigma_t\})$ is the correct ambient structure. Promoting it to a single chain complex is the work of Section 7; promoting it to a bicomplex (where possible) is the work of Section 9.

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7. The Mapping Telescope

The single chain complex naturally associated to the diagram D is its mapping telescope, built from mapping cones.

Mapping cone of σ_t. For each consecutive pair $(W_7^{(t)}, W_7^{(t+1)})$ related by σ_t, the mapping cone is

$$\mathrm{Cone}(\sigma_t)_n = W_7^{(t)}{}_{n-1} \oplus W_7^{(t+1)}{}_n,$$

with differential, on $(a, b) \in W_7^{(t)}{}_{n-1} \oplus W_7^{(t+1)}{}_n$,

$$\partial\_\mathrm{cone}(a, b) = (-\partial_h a\ ,\ \partial_h b - \sigma_t(a)).$$

Direct calculation: $\partial^2\_\text{cone}(a, b) = \partial\_\text{cone}(-\partial_h a, \partial_h b - \sigma_t(a)) = (\partial^2_h a, \partial^2_h b - \partial_h \sigma_t(a) + \sigma_t(\partial_h a)) = (0, 0)$, using $\partial^2_h = 0$ and the chain-map condition $\sigma_t \partial_h = \partial_h \sigma_t$. The mapping cone is therefore a well-defined chain complex. (The sign convention here is chosen to be consistent with the telescope differential below; sign conventions for the mapping cone vary across the literature, and the choice is fixed by internal consistency rather than by reference to a standard.)

Mapping telescope. The mapping telescope of the σ-family is the chain complex obtained by glueing mapping cylinders $\mathrm{Cyl}(\sigma_t)$ along their target/source identifications. Explicitly, in each degree n,

$$\mathrm{Tel}_n = \left(\bigoplus_{t=0,1,2,\ldots} W_7{}^{(t)}{}_n \right) \oplus \left(\bigoplus_{t=0,1,2,\ldots} W_7{}^{(t)}{}_{n-1} \right).$$

We refer to the first family of summands as the **layer summands** and the second as the **cylinder summands**.

The differential ∂_Tel acts as follows:

- On x_t in the layer summand at tick t in degree n: $\partial\_\mathrm{Tel}(x\_t) = \partial_h\, x\_t \in$ layer summand at tick t in degree $n-1$.
- On y_t in the cylinder summand at tick t in degree $n-1$: $\partial\_\mathrm{Tel}(y\_t) = y\_t$ (in layer summand at tick t, degree $n-1$) $- \sigma_t(y\_t)$ (in layer summand at tick t+1, degree $n-1$) $- \partial_h\, y\_t$ (in cylinder summand at tick t, degree $n-2$).

Direct verification (using $\partial^2_h = 0$ and $\sigma_t \partial_h = \partial_h \sigma_t$) gives $\partial^2\_\mathrm{Tel} = 0$. The telescope is a well-defined chain complex given any chain-map family.

Properties of the telescope. Three properties are immediate.

First, **Tel is a single chain complex**, not a bicomplex. Its total differential combines closure boundaries (acting within each layer summand) with cylinder maps that send each cylinder-summand element y_t to a homotopy between its inclusion at tick t and its σ_t-image at tick t+1 — concretely, ∂_Tel takes y_t to the difference (inclusion at tick t) − (σ_t-image at tick t+1), plus its own ∂_h in the cylinder summand. The two endpoints of the homotopy land in *different* layer summands; this is what closes $\partial^2\_\mathrm{Tel} = 0$ and is also why this structure cannot be a bicomplex bigrading. The closure×tick organisation of Section 6 reappears as a *filtration* of Tel by tick, not as a bicomplex bigrading.

Second, **the cohomology of Tel is the colimit cohomology**:

$$H^*(\mathrm{Tel}) = \mathrm{colim}\_t\, H^*(W_7{}^{(t)}),$$

when the σ_t are not all isomorphisms. This is the homotopy-colimit content of Tel: it captures the accumulated closure cohomology under the update sequence.

Third, **the tick filtration of Tel induces a spectral sequence** converging to $H^*(\mathrm{Tel})$ with E^1 page

$E^1\_\{p, q\} = \bigoplus_t H_p(W_7^{(q)})$,

and d^1 differential induced by the σ_t. The spectral sequence and its convergence are automatic from the tick filtration being bounded below; the specific form of d^1 on each layer's cohomology, however, depends on the σ_t and inherits whatever structure the σ-family imposes on $H^*(W_7^{(t)})$.

Bulk reconstruction in Tel. Under the present reformulation, the bulk reconstruction proposed in (O2) is naturally housed in Tel: the depth coordinate z corresponds to telescope depth (update count, measured in ticks), and the coarse-graining hierarchy on Tel produces the bulk geometry. This is the strong form of the bulk-reconstruction conjecture, now anchored on a well-defined chain complex.

The telescope is the replacement for the static prism. The static-prism extension $Cyl(W_7)$ is what one obtains by attempting to flatten D into a single complex *without* the mapping-cone construction. Tel is the correct single-complex realisation. It does not suffer the type confusion of $Cyl(W_7)$, since the σ_t remain morphisms (entering via the cylinder summands) rather than being conflated with edges.

Translation symmetry of Tel. The natural translation map T : Tel → Tel sending $W_7^{(t)} \to W_7^{(t+1)}$ commutes with ∂_Tel by construction. This is automatic and uninformative: it expresses only that the telescope is indifferent to which tick we label as "first," which is essentially the discrete time-translation symmetry of the substrate. T carries no physical content of the σ programme. The substantive content lies in the σ-duality of Section 8, a strictly stronger and independently posed demand.

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8. σ-Duality on a Separate Bigrading

The first stage of reformulation places the σ-family on the telescope Tel, with the closure×tick organisation as filtration. The second stage concerns the *spatial–temporal exchange* content of the original σ programme.

We now show that this duality lives on a *different* bigraded structure from the closure×tick organisation of Section 6. This is the deepest reframing of the present paper.

The well-typedness problem. The original σ programme sought a duality

$\sigma : C(p, q) \longrightarrow C(q, p)$

interchanging the two indices of the closure×tick organisation. But this map is not well-typed: $p \in \{0, 1\}$ (closure dimension) and $q \in \mathbb{N}$ (tick index) range over different index sets, so C(q, p) is undefined for $q \geq 2$. The duality as written cannot exist on the closure×tick structure.

A salvage that restricts the duality to a 2-tick window $p, q \in \{0, 1\}$ would be technically well-typed, but it would be far too small a structure to carry the K=7 content the original σ programme intended.

The K=7 structure as vertex count. The K=7 number in the constraint-architecture papers was derived from a minimal-fact counting argument, and the wheel $W_7 = C_6 + h$ was constructed precisely so that its closure structure realises this constraint count: the 7 vertices of W_7 correspond bijectively to the 7 minimal facts, with the hub h corresponding to the ground fact and the 6 outer vertices to the 6 derived facts. The K=7 number is therefore the rank of $C_0(W_7)$, not the simplicial dimension count of the complex. Any σ-duality that genuinely encodes the K=7 spatial structure of W_7 must interchange a *7-fold* index, not a 2-fold one.

The correct bigrading: identified, not constructed. The natural ambient type on which σ-duality is well-typed and K=7-faithful is a bigraded chain complex

$D : V(W_7) \times T_7 \longrightarrow Ch(Ab), D(i, t) \in Ch(Ab),$

where:

- $V(W_7) = \{h, v_0, v_1, \dots, v_5\}$ is the 7-vertex basepointed set of W_7, with basepoint h and $\mathbb{Z}/6$ acting on the non-basepoint orbit by the hexagon cyclic structure;
- $T_7 = \{\tau_0, \tau_1, \dots, \tau_6\}$ is an independent basepointed 7-cyclic structure (a "coherence window"), with basepoint τ_0 and analogous $\mathbb{Z}/6$ cyclic action on the non-basepoint orbit.

On this bigrading, σ-duality is the requirement that there exist a family of chain isomorphisms

$\sigma_{(i, t)} : D(i, t) \longrightarrow D(t, i), \sigma^2 = id,$

interchanging the two axes, with the basepointed structure respected (i.e., σ identifies basepoint–basepoint cells and pairs non-basepoint cells under the diagonal swap).

Cardinality forced by $\sigma^2 = id$. Apply σ to a cell D(i, t): the image σ(D(i, t)) sits at D(t, i), where now i appears as a *second-coordinate* (T_7) index and t appears as a *first-coordinate* ($V(W_7)$) index. For this image to be a well-defined cell of the bigraded module, i must be a valid T_7-index (giving $V(W_7) \subseteq T_7$) and t must be a valid $V(W_7)$-index (giving $T_7 \subseteq V(W_7)$). Both inclusions follow from this single application of σ, since i ranges freely over $V(W_7)$ and t ranges freely over T_7 in the source cell. The two index sets are therefore in bijection, and $|T_7| = |V(W_7)| = 7$. The cardinality of T_7 is not a free choice but a structural consequence of the involution.

T_7 is independent of the $\mathbb{N}$ tick axis. T_7 is a basepointed 7-cyclic set, not a subset of $\mathbb{N}$. The $\mathbb{N}$ tick axis of the telescope Tel and the 7-cyclic structure T_7 of the duality are independent objects: T_7 has no canonical embedding into $\mathbb{N}$. Any embedding

$\varepsilon : T_7 \hookrightarrow \mathbb{N}$

is a separate structure provided by substrate dynamics, presumably encoding which 7-tick "coherence window" of the substrate's commitment sequence supports the σ-duality. *Admissibility* of ε means here that it is order-preserving (the cyclic structure on T_7 embeds compatibly with the linear order on ℕ, taking τ_0 to a starting tick and reading the cycle forward in ℕ) and that the image $\varepsilon(T_7)$ is contiguous in ℕ (so that "window" is faithful — the seven ticks of the coherence window are consecutive). Under these conditions, ε is determined by its image of the basepoint $\varepsilon(\tau_0) \in \mathbb{N}$. The moduli space of admissible embeddings is therefore naturally identified with ℕ itself — the choice of which ℕ-tick is the starting tick of the coherence window. This choice is a substrate-physics question and is the temporal-side gauge introduced below.

What D(i, t) actually is, is the open question. The present paper makes only the minimal commitment that D is of *type* $(V(W_7) \times T_7 \to Ch(Ab))$ with a chain-isomorphism σ-duality. What chain complex D assigns to each cell (i, t) — be it a single scalar (so D collapses to a 7×7 ℂ-linear or ℝ-linear matrix with σ = transpose), a one-dimensional space, a finite chain complex encoding local closure data, or a derived object computed from substrate dynamics — is precisely what open problem (P3) asks to be constructed. The single-scalar case carries only symmetry of a 7×7 numerical array, which by itself underdetermines the substrate dynamics expected to generate D. The substantive content of (P3) is therefore that D(i, t) carry non-trivial chain-level structure derived from substrate dynamics.

The hub-origin pairing, conditionally. If a σ-duality of the above type exists *and* the diagonal exchange $\sigma : D(i, t) \to D(t, i)$ respects the K=7 = 6+1 basepointed structure of $V(W_7)$, then a corresponding basepointed structure is forced on T_7: one tick (call it τ_0) must be distinguished as the σ-image of h, with the remaining six ticks $\tau_1, \ldots, \tau_6$ forming an ℤ/6-cyclic orbit pairing with the outer vertices $v_0, \ldots, v_5$ under σ. This pairing of the orbits is not unique: any specific bijection $\{v_0,\ldots,v_5\} \leftrightarrow \{\tau_1,\ldots,\tau_6\}$ respecting the cyclic structures is one element of a D_6 gauge orbit (here D_6 denotes the dihedral group of order 12, the symmetry group of the hexagon, $\mathbb{Z}/6 \rtimes \mathbb{Z}/2$) of 12 equivalent pairings (6 rotational choices × 2 orientations). The *basepoint pairing* ($h \leftrightarrow \tau_0$) is conditionally forced; the specific outer-orbit correspondence is a discrete gauge choice. The present paper proposes the canonical representative $v_{i-1} \leftrightarrow \tau_i$ for i = 1, …, 6, but any element of its D_6-orbit is equally natural.

Two independent discrete gauge axes. The vertex×tick-window construction therefore carries *two* independent discrete gauge ambiguities:

(g1) The D_6 orbit of 12 outer-vertex ↔ outer-tick pairings (a spatial gauge); (g2) The choice of $\varepsilon(\tau_0) \in \mathbb{N}$ parametrising admissible embeddings $\varepsilon : T_7 \hookrightarrow \mathbb{N}$ (a temporal-window gauge).

Both are conjecturally fixed by substrate dynamics — (g1) by whatever spatial symmetry-breaking the substrate selects, (g2) by the TPB-determined coherence-window scale (see Section 10) — but the fixings are not derived in the present paper.

Why the original σ programme failed. The original σ programme on $Cyl(W_7)$ implicitly asked for a *single* structure simultaneously performing two functions: (i) carrying the closure×tick organisation of the σ-family (the role now filled by Tel), and (ii) supporting a 7×7 σ-duality on a vertex×tick-window bigrading. These are two structurally distinct bigraded structures on top of the underlying ontology. The static-prism construction conflated them into a single bigrading, and the well-typedness failure of Section 4 is the precise consequence of that conflation. The reformulation separates them: closure×tick is the natural organisation of D and underlies Tel; vertex×tick-window is the natural home of σ-duality.

Compatibility with Tel: a projection. The vertex×tick-window bigrading D and the closure×tick telescope Tel are not independent of each other, but the present paper makes only the modest commitment that the relationship is the existence of a *projection*. Given an embedding $\varepsilon : T_7 \hookrightarrow \mathbb{N}$, the projection has type

$\pi : D \longrightarrow \bigoplus\_\{t \in \varepsilon(T_7)\}\ C_0(W_7^{(t)}) \subseteq Tel_0,\ \pi(D(i, t)) \subseteq C_0(W_7^{(t)})$ at vertex i,

recovering the vertex data of $W_7^{(t)}$ from the corresponding cells of D at tick t (for t in the image of ε). The closure edges in $C_1(W_7^{(t)})$ are *not asserted to be derivable* from D; they are independent data carrying their own structure. The closure×tick organisation is therefore not derived from the vertex×tick-window bigrading — it is compatible with it under projection. The strong conjecture that the closure edges *are* derivable from σ-coherence relations on D (i.e., that the projection π extends to a richer functor recovering the full closure×tick organisation) is left open as a separate question. The present paper's commitment is the modest version: D and Tel are independent bigraded structures with a projection compatibility π, restricted to the seven layers in the image of ε.

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9. The Bicomplex Lift as Aspirational Enrichment

Tel is a single chain complex. Promoting it to a genuine *bicomplex* — with two anti-commuting differentials ∂_h and ∂_v satisfying $\partial_h^2 = 0$, $\partial_v^2 = 0$, $\partial_h\partial_v + \partial_v\partial_h = 0$ — requires additional structure beyond the bare chain-map condition.

The obstruction to $\partial_v^2 = 0$. If one tries to define ∂_v at layer q as essentially σ_{q-1} or σ_{q} directly, then ∂_v^2 produces an iterated composition $\sigma_{t+1} \circ \sigma_t$, which is the two-tick composite update. There is no physical reason this composite should vanish — and if it did, the σ-family would trivialise. The bicomplex axiom $\partial_v^2 = 0$ therefore fails generically. Tel evades this by using the mapping-cone construction, in which the cylinder differential acts as a chain homotopy across distinct layer summands rather than as a vertical differential within a fixed layer; but Tel is then a single graded complex with one differential, not a bicomplex with two.

What a bicomplex lift would require. A genuine bicomplex lift of the closure×tick organisation would require the σ-family to carry *simplicial-type enrichment*: face maps, degeneracy maps, or analogous structure on the tick direction satisfying simplicial identities. Under Dold–Kan or its analogues, such enrichment produces a vertical differential with $\partial_v^2 = 0$ by construction.

Status. The bicomplex lift is *not* needed for the mapping telescope to be well-defined; Tel exists independently. It *would* be needed to make the vertex×tick-window σ-duality of Section 8 a duality of bicomplexes rather than a duality of bigraded modules. Whether the σ-duality construction is best done at the bicomplex level (if achievable) or at the bigraded-module level (independent of bicomplex lift) is itself open.

The bicomplex lift is conditional on substrate-derived σ providing the required simplicial enrichment. The connection between this enrichment and the response-functional content of the Single-Source Theorem — i.e., the conjecture that substrate dynamics admitting a coherent response functional R[ρ] with appropriate group-theoretic structure produce σ-families with the requisite simplicial-type relations — is itself conjectural in the present setting. Establishing or refuting this connection is a separate substantive question, not assumed here.

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10. Relation to BCB and TPB

The reformulation meshes naturally with BCB (boundary causal bandwidth) and TPB (tick propagation bound), with the identifications made precise.

Support of σ_t. Let $\mathrm{supp}(\sigma_t) \subset W_7^{(t)}$ denote the union of cells (vertices and edges) on which σ_t acts non-trivially — i.e., the set of cells x for which $\sigma_t(x) \neq x$ under the canonical identification $W_7^{(t)} \cong W_7^{(t+1)}$. **BCB** is the bound

$|\mathrm{supp}(\sigma_t)| \leq$ B_BCB,

where B_BCB is the boundary causal bandwidth per tick. The earlier admissibility theorem for primitive 3D transport is the statement that any σ-family with support violating this bound (specifically, requiring 3D bulk coverage) is non-admissible. Interface-native propagation (O1) is the statement that admissible σ-families have support concentrated on 2D interface cells.

Interaction range of σ_t. Let $\mathrm{range}(\sigma_t)$ be the maximum graph distance in $W_7^{(t)}$ between any two cells x, y with the property that $\sigma_t(x)$ depends on the value at y. **TPB** is the bound

$\mathrm{range}(\sigma_t) \leq$ r_TPB,

where r_TPB is the maximum cellular distance over which a single tick of σ can correlate. In the continuum limit — *conditional on (P4)* — with appropriate normalisation of cellular distance to physical length and tick to physical time, r_TPB would reproduce the relativistic light cone, and the substrate-level identification

the speed of light is the maximum rate at which the substrate can commit new facts into existence

would recover its earlier form without invoking a primitive spacetime metric.

Where this bites. Both constraints turn the abstract σ-family into a sharply constrained object. Any candidate σ_t on W_7 must satisfy supp(σ_t) bounded by BCB and range(σ_t) bounded by TPB. Combined with the chain-map condition ($\sigma_t \circ \partial_h = \partial_h \circ \sigma_t$), these provide a tight admissibility filter on what the substrate-derived σ-family can be.

Coherence-window scale fixes ε. *Conditionally*, the natural physical scale that fixes the $T_7 \hookrightarrow \mathbb{N}$ embedding ε of Section 8 is primarily TPB-derived: the 7-tick coherence window is presumably defined by the maximum tick separation over which σ-coherence holds across multiple updates, which is a TPB property (correlation range across ticks), not a BCB property (per-tick support). BCB plays a secondary role here through its constraint on which σ-families are admissible candidates in the first place; once admissibility is satisfied, the coherence-window scale that determines ε is the TPB scale.

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11. Lorentz Compatibility as σ-Consistency

Lorentz invariance, in the continuum limit, is *proposed* to be a consistency condition on σ across the telescope.

The earlier one-loop Lorentz-compatible enhancement result lives inside a single closure structure. The present reformulation embeds it in a richer setting: Lorentz compatibility would, under the present framework, be the statement that in the continuum limit of the telescope, the σ-induced transport across multiple ticks reproduces the rotation–boost symmetry of Minkowski space. This requires three conditions:

1. **σ chain-map compatibility** — automatic in any well-formed diagram D.
2. **σ-duality on the vertex×tick-window bigrading** — the substantive demand of Section 8.
3. **Continuum-limit compatibility** — BCB and TPB bounds on the σ-family scale to relativistic bounds under coarse-graining, and the σ-duality persists in the limit.

Condition 3 is the analogue, in the present setting, of the one-loop Wilson-matching arguments of the earlier programme. Conditions 1 and 2 are the discrete-substrate halves.

Whether all three conditions can be simultaneously satisfied for the W_7 system with substrate-derived σ is the full open question; the present paper does not establish that they can.

Crucially, in this reformulation Lorentz invariance is not an a priori spacetime symmetry but the conjectured continuum signature of σ-consistency across a discretely committed update telescope. There is no spacetime to act on prior to σ-consistency.

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12. Revised Ontology

The unified ontology resulting from this reformulation is summarised below.

Structure	Role	Status
W_7 closure complex	spatial admissibility	established (K=7 programme)
$W_7^{(t)}$ as committed surface	one closure event at tick t	established (O3)
$\{W_7^{(t)}, \sigma_t\}$ diagram D	ordered sequential commitment	given a σ-family
Closure×tick organisation	bigraded module M(p, q) underlying D	this paper, §6
Mapping telescope Tel	single chain complex, ℕ-indexed	this paper, §7
σ_t chain-map condition	required for D to be a diagram in Ch(Ab)	structural
Tick filtration spectral sequence	converging to H*(Tel)	this paper, §7
$V(W_7)$ basepointed 7-vertex set	7 minimal facts of K=7 architecture	established
T_7 basepointed 7-cyclic set	"coherence window" for σ-duality	identified, §8
Cardinality $\|T_7\| = 7$	forced by σ^2 = id involution	this paper, §8
Embedding $\varepsilon : T_7 \hookrightarrow \mathbb{N}$	order-preserving, contiguous image; moduli ≅ ℕ via $\varepsilon(\tau_0)$	open (gauge g2)
Vertex×tick-window type D : $V(W_7) \times T_7 \to$ Ch(Ab)	required type for σ-duality home	identified, §8
Cell content D(i, t)	open: scalar, line, chain complex, derived object?	open problem (P3a), §8
Hub ↔ origin tick basepoint pairing	conditional on σ-duality respecting K=7 = 6+1	conditional, §8
Outer-vertex pairing v_{i−1} ↔ τ_i	one element of D_6 gauge orbit	open (gauge g1)
Projection $\pi : D \to \bigoplus$_{t ∈ $\varepsilon(T_7)$} $C_0(W_7^{(t)})$	existence of D–Tel compatibility	committed, §8
$C_1(W_7^{(t)})$ derivable from D-coherence	strong version of compatibility	conjecture, §8
Bicomplex lift of Tel	requires simplicial-type σ enrichment	conditional (P2), §9
σ-duality σ: D(i, t) → D(t, i)	spatial–temporal exchange	open (P3), §8
Bulk geometry	reconstructed from Tel correlations	conditional on §7
Lorentz invariance	continuum limit of σ-consistency	conditional, §11

The static prism Cyl(W_7) does not appear in this ontology. It was a misreading combining the (incomplete) flattening of D and the (ill-typed) attempt to host σ-duality on the closure×tick bigrading.

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13. Why the Original σ Construction Failed — Restated

We can now restate the diagnosis cleanly.

The original σ construction posited σ as a degree-preserving endomorphism of a single complex $Cyl(W_7)$ whose underlying set conflated two distinct categorical objects: the *objects* of the diagram $\{W_7^{(t)}\}$ and the *morphisms* σ_t between them. *And* the σ-duality content was demanded to live on the same closure×tick bigrading as the σ-family chain-map content, even though σ-duality is intrinsically a 7×7 bigrading question (vertex×tick-window) while the closure×tick organisation is a $2\times\mathbb{N}$ structure.

The chain-map condition $\partial\sigma = \sigma\partial$ on this doubly-conflated complex therefore demanded compatibility of:

(i) an exchange of objects-with-morphisms (a categorical type error), and (ii) a 7×7 duality cramped into a $2\times\mathbb{N}$ bigrading (a well-typedness failure),

with a boundary operator that did not distinguish any of these. The failure pattern (broken hexagonal-face closure, spoke / temporal-edge asymmetry, non-closing edge images) is the precise signature of these combined errors.

The reformulation builds the type distinctions in. The σ_t are morphisms in a diagram D (Section 6). The associated single chain complex is the mapping telescope Tel (Section 7). σ-duality lives on a separate vertex×tick-window bigrading where it is well-typed (Section 8). The original ill-typed demand splits into independently meaningful conditions: σ_t chain-map compatibility (structural, automatic in D); σ-duality on the vertex×tick-window bigrading (substantive, open); bicomplex lift compatibility (aspirational, conditional on additional σ-enrichment).

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14. The Reformulated Open Problem

The σ programme is now sharply specified and splits into structurally separate sub-problems.

(P1) Substrate-derived σ-family. Construct an explicit σ-family $\{\sigma_t\}$ for W_7 derived from substrate dynamics (response functional R[ρ] or equivalent), satisfying the chain-map condition $\sigma_t \circ \partial_h = \partial_h \circ \sigma_t$ together with BCB (bounded cellular support $|supp(\sigma_t)| \leq B\_BCB$) and TPB (bounded interaction range $range(\sigma_t) \leq r\_TPB$).

(P2) Bicomplex lift of Tel. Identify additional structure on the σ-family (simplicial enrichment, face/degeneracy maps, or similar) such that the closure×tick organisation lifts to a genuine bicomplex with $\partial_v^2 = 0$ and $\partial_h\partial_v + \partial_v\partial_h = 0$, and determine which substrate dynamics produce σ-families admitting this lift.

(P3) Vertex×tick-window σ-duality. Construct the bigraded chain complex $D : V(W_7) \times T_7 \to Ch(Ab)$ explicitly: (P3a) determine what D assigns to each cell (i, t) (resolving the open question of scalar, line, finite-chain-complex, or derived-object content); (P3b) establish a non-trivial chain-isomorphism σ-duality $\sigma\_\{(i,t)\} : D(i, t) \to D(t, i)$ respecting the basepointed structure;

(P3c) specify the embedding $\varepsilon : T_7 \hookrightarrow \mathbb{N}$ (presumably determined by the TPB coherence-window scale of Section 10), thereby fixing gauge (g2); (P3d) specify the D_6 outer-vertex pairing, thereby fixing gauge (g1); (P3e) verify projection compatibility $\pi : D \to \bigoplus_{t \in \varepsilon(T_7)} C_0(W_7^{(t)})$. Strong version: extend π so that the full closure×tick organisation is recovered from D.

(P4) Continuum limit and Lorentz invariance. Show that the continuum limit of Tel with σ-duality reproduces the rotation–boost symmetry of Minkowski space, with BCB and TPB scaling to the relativistic light cone.

The four sub-problems differ in scope and dependency. (P1) is the substrate-physics problem and is logically prior to the others. (P2) and (P3) are independent of each other in principle, although they may have shared computational content. (P4) is the continuum-limit problem and depends on the resolution of (P3) at least.

The reformulated problem differs from the original in three important ways. First, it is well-typed: σ-duality has a well-defined home on a different bigrading from the closure×tick structure. Second, it is decomposed: the four sub-problems (with (P3) having five sub-parts) replace one ill-typed demand. Third, it is connected directly to the substrate dynamics via BCB, TPB, and the response-functional content of (P1).

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15. What This Paper Establishes

In the strict VERSF epistemic framework, with one new category — *identified but not constructed* — introduced for the vertex×tick-window bigrading:

Proven (in prior VERSF work, used here).

- (O1) Interface primitivity: primitive 3D bulk transport is BCB-non-admissible.
- (O3) Time as sequential closure: tick succession is the carrier of emergent time.
- K=7 closure structure: $W_7 = C_6 + h$ is the minimal admissible closure complex, with 7 vertices realising the K=7 constraint-architecture count.
- σ-on-static-prism obstruction: $\partial\sigma \neq \sigma\partial$ for natural candidate σ on $\mathrm{Cyl}(W_7)$.

Proven (in this paper).

- The static-prism construction commits a category error of two kinds (C1, C2 — Section 5).
- Given any chain-map family $\{\sigma_t\}$, the diagram $D : \mathbb{N} \to \mathrm{Ch}(\mathrm{Ab})$ is well-defined as the natural ambient categorical structure of the σ programme (Section 6).
- The mapping telescope $\mathrm{Tel} = \mathrm{Tel}(\{W_7^{(t)}\}, \{\sigma_t\})$ is a well-defined single chain complex with closed differential $\partial_{\mathrm{Tel}}^2 = 0$, requiring only the chain-map condition on σ_t. The construction is given explicitly via mapping cones in Section 7.

- The σ-duality of the original programme is not well-typed on the closure×tick organisation of D; it requires a separate vertex×tick-window bigrading where $\sigma : D(i, t) \to D(t, i)$ is well-defined (Section 8).
- The cardinality $|T_7| = 7$ is not a free choice but is forced by $\sigma^2 = \mathrm{id}$ requiring $V(W_7)$ and T_7 to be in bijection; both inclusions follow from the index-validity of $\sigma(D(i, t)) = D(t, i)$ for arbitrary (i, t) (Section 8).
- The moduli space of admissible embeddings $\varepsilon : T_7 \hookrightarrow \mathbb{N}$ (order-preserving and contiguous) is naturally identified with $\mathbb{N}$, parametrised by $\varepsilon(\tau_0)$ (Section 8).
- The original σ programme implicitly conflated the closure×tick structure (telescope) with the vertex×tick-window structure (σ-duality), and this conflation is the precise source of the $\partial\sigma \neq \sigma\partial$ failure (Section 13).

Identified, not constructed.

- The vertex×tick-window bigrading is identified as the correct *type* ($D : V(W_7) \times T_7 \to \mathrm{Ch}(\mathrm{Ab})$ with chain-isomorphism σ-duality) but not constructed on top of W_7. What D assigns to each cell (i, t) is left open and is the substance of (P3a).
- The embedding $\varepsilon : T_7 \hookrightarrow \mathbb{N}$ is identified as parametrised by $\varepsilon(\tau_0) \in \mathbb{N}$ but is not fixed; fixing it is the substance of (P3c).
- The bicomplex lift of the closure×tick organisation is identified as conditional on simplicial-type enrichment of the σ-family, but the enrichment is not constructed.
- The form of D–Tel compatibility is committed to as a projection $\pi : D \to \bigoplus_{t \in \varepsilon(T_7)} C_0(W_7^{(t)})$, but its explicit specification awaits the construction of D and the embedding ε.

Conditional (this paper, dependent on §14 (P1)–(P4)).

- The σ-duality of the original K=7 programme is realised concretely on the vertex×tick-window bigrading.
- The "speed of light" recovers its earlier formulation as the BCB/TPB-bounded propagation rate of σ_t, via the support and interaction-range definitions of Section 10.
- Bulk reconstruction proceeds from Tel correlation structure under coarse-graining.
- Lorentz invariance arises as the continuum limit of σ-consistency.

Conjectural (this paper).

- Under any σ-duality respecting the K=7 = 6+1 basepointed structure of $V(W_7)$, the hub h pairs with a distinguished origin tick τ_0 on T_7. The specific outer-vertex correspondence $v_{i-1} \leftrightarrow \tau_i$ is one element of a D_6 gauge orbit (gauge g1).
- The embedding $\varepsilon : T_7 \hookrightarrow \mathbb{N}$ is conjecturally fixed by the TPB coherence-window scale of the substrate, providing the physical scale that picks out which 7-tick window supports σ-duality (gauge g2).
- The bicomplex lift of Tel is achievable for substrate-derived σ-families satisfying appropriate response-functional consistency, which is conjecturally connected to the Single-Source Theorem's group-theoretic content.

- The strong version of D–Tel compatibility (C_1 closure edges fully derivable from σ-coherence relations on D) holds, i.e., the projection π extends to a recovery of the full closure×tick organisation.
- The full continuum limit reproduces Minkowski space as a σ-equivariant homotopy colimit.

The paper's central contribution is the diagnosis and the two-stage reformulation. The substantive open targets — substrate-derived σ-family, bicomplex lift, explicit vertex×tick-window construction (including embedding and gauge specifications), continuum limit — are now sharply separated and specified, and constitute the next computational targets.

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16. Conclusion

The K=7 wheel $W_7 = C_6 + h$ remains, as the earlier interface programme established, a fundamentally two-dimensional closure interface. Light propagates on this interface. Time emerges from sequential closure — from the ordered commitment of successive interface states $\{W_7^{(t)}\}$.

The prism geometry that appeared in the recent σ programme is not a primitive three-dimensional propagation volume. The natural ambient structure is the diagram $D : \mathbb{N} \to \mathrm{Ch(Ab)}$ sending t to $W_7^{(t)}$, and the natural single chain complex associated to D is its mapping telescope Tel, built explicitly via mapping cones.

The earlier σ obstruction was structural rather than combinatorial. The static-prism construction committed two combined errors: it conflated objects (cells of interfaces) with morphisms (the σ-family), *and* it forced the σ-duality (which is a 7×7 bigrading question on vertex×tick-window) to live on the same closure×tick bigrading (a 2×ℕ structure) as the chain-map content of the σ-family. The reformulation built here separates these.

Within this reformulation:

- the diagram D is the correct ambient categorical structure;
- the mapping telescope Tel is the correct single-complex realisation, with closed differential built from the σ_t and identity maps via mapping cones;
- σ-duality is *proposed* to live on a separate vertex×tick-window bigrading $D : V(W_7) \times T_7 \to \mathrm{Ch(Ab)}$, with $V(W_7)$ the 7-vertex basepointed set of W_7 and T_7 an independent basepointed 7-cyclic structure ($|T_7| = 7$ forced by $\sigma^2 = \mathrm{id}$); the cell content D(i, t) is identified as taking values in chain complexes but is otherwise unspecified;
- the hub h *would* pair with a distinguished origin tick τ_0 under any σ-duality respecting the K=7 = 6+1 asymmetry, with two independent discrete gauge ambiguities (D_6 on the spatial side, $T_7 \hookrightarrow \mathbb{N}$ embedding on the temporal side, with the latter parametrised by $\varepsilon(\tau_0) \in \mathbb{N}$) both conjecturally fixed by substrate dynamics;

- BCB and TPB constrain the admissible σ-families precisely (cellular support and interaction range); TPB is conjecturally the dominant scale fixing the coherence-window embedding ε;
- Lorentz invariance *is proposed* to emerge as the continuum limit of σ-consistency, conditional on the construction of the vertex×tick-window bigrading and its σ-duality.

The reformulated open problem (Section 14) splits into four sharply separated sub-problems: substrate-derived σ-family (P1), bicomplex lift (P2), vertex×tick-window σ-duality construction with five sub-parts including embedding and gauge specifications (P3), and continuum limit (P4). These are now the next computational targets.

The result is a unification rather than a derivation. The K=7 closure programme, the interface propagation framework, TPB, BCB, and emergent time are all embedded in a single architecture of sequential interface transport with two distinct bigraded organisations. The σ programme has not been completed, but it has been correctly framed — and this paper argues that the correct framing was the missing ingredient.