Spinorial Closure Transport and Fermionic Source-Carriers in VERSF

Clifford Necessity from First-Order Closure Dynamics, Double-Cover Realization, and the Geometric–Algebraic Convergence on Spinorial Structure

Keith Taylor *VERSF Theoretical Physics Programme Companion paper to* The Microscopic Origin of the Record Current in VERSF *(Part III of the matter-sector programme)*

General Reader Summary

The previous papers in the matter-sector strand established two structural results. *Matter Coupling and the Inertia Route in VERSF* fixed the unique admissible coupling between the persistent gauge sector and any substrate current: $\mathscr{L}_{int} = -J^\mu A_\mu$. *The Microscopic Origin of the Record Current in VERSF* then supplied a substrate ontology for that current — primitive commitment loops, identified ontologically with transported persistent fold structures, carrying conserved integer winding via the Wilson-loop holonomy pairing.

The loops introduced there were scalar carriers. That deferred — but did not resolve — the most distinctive question of the matter sector: **why does matter behave as a fermion?** Electrons, quarks, and the ordinary constituents of matter carry spin-½, the property that a 2π rotation reverses the sign of the wavefunction and a 4π rotation restores it. In the Standard Model this is assumed at the level of the Dirac equation. Within VERSF, the question is sharper: *what substrate structure carries spin-½*, and how does it emerge from the loop ontology already in place?

This paper supplies a candidate spinorial extension of the loop ontology. The central conceptual claim is sharper than a purely geometric construction:

Spinorial structure is algebraically forced by first-order closure dynamics and geometrically realized through closure transport holonomy. The two routes converge on the same Spin(3) ≅ SU(2) structure.

In plain language: spin-½ behaviour shows up in this construction for two reasons, and both reasons point at the same underlying structure. The first reason is algebraic. The substrate already supports a particular kind of wave-like motion — the scalar Klein–Gordon dynamics that earlier VERSF papers established as uniquely admissible. If the substrate *also* supports a first-order kind of motion (the kind described by something like a Dirac equation rather than a Klein–Gordon equation), then the mathematics of "first-order motion whose square gives the second-order Klein–Gordon motion" forces a very specific algebraic structure — the Clifford algebra. That algebra has a smallest faithful description on a four-component object, and rotations on that four-component object are described not by ordinary three-dimensional rotations SO(3) but by their double-cover SU(2). This is essentially Dirac's 1928 argument, run at the substrate level. *That is what "algebraically forced" means: the spinor structure is a mathematical consequence of admitting first-order motion alongside the existing second-order Klein–Gordon motion.*

The second reason is geometric. A purely geometric path would proceed as follows: substrate loops carry a local orientation frame, that frame transports as the loop moves, and the transport sees the topological fact that ordinary three-dimensional rotations have a hidden double-cover structure (technically: the rotation group SO(3) is not simply-connected, and its simply-connected cover is SU(2)). Under three structural assumptions, the substrate-level transport "lifts" to this double-cover SU(2), and the loops that wind nontrivially around it pick up the famous $2\pi = -1$ / $4\pi = +1$ spinor sign behaviour. *That is what "geometrically realized through closure transport holonomy" means: the spinor structure appears as the holonomy — the geometric memory of how an orientation frame has been twisted — of transport on the substrate.*

The convergence is the point. SU(2) showing up algebraically (as the natural rotation group on the four-component Clifford object) and SU(2) showing up geometrically (as the double cover of ordinary rotations) is not a coincidence: there is essentially only one such group, called Spin(3), and SU(2) is its standard name. The two routes are different mathematical paths to the same destination. The algebraic path is unconditional (given inherited results from other VERSF papers); the geometric path is conditional on the three structural assumptions. Together they give us both *reasons* (algebraic) and *substrate-level carriers* (geometric) for spin-½ behaviour — without having to assume it as a postulate the way the Standard Model does.

Returning to the technical exposition:

The algebraic route works as follows. The κ-field uniqueness programme establishes that the unique admissible second-order scalar propagation structure on the persistent sector is the Klein–Gordon form $(\square + m^2)s = \rho_{committed}$. The Schrödinger→Dirac paper establishes that first-order admissible flow on the same persistent sector exists and is constrained algebraically by the requirement that its square reproduce the KG invariant. Dirac's original 1928 argument in 3+1 Hamiltonian form — writing the first-order flow as $H = \alpha^i p_i + \beta m$ with α^i and β acting on an internal representation space and requiring $H^2 = (p^2 + m^2) \cdot \mathbb{1}$ — forces the Euclidean Clifford anticommutation relations $\{\alpha^i, \alpha^j\} = 2\delta^{ij} \cdot \mathbb{1}$, $\{\alpha^i, \beta\} = 0$, $\beta^2 = \mathbb{1}$. The minimal faithful representation is four-component, and the covariant repackaging $\gamma^0 \equiv \beta$, $\gamma^i \equiv \beta\alpha^i$ identifies the resulting operator as the standard Dirac operator $(i\gamma^\mu \partial_\mu - m)$ with the Clifford relations $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \cdot \mathbb{1}$. *The standard Dirac operator is algebraically forced as the minimal admissible first-order structure on the persistent sector* (whether it dynamically generates substrate-level spinorial source-carrier dynamics is a separate question, deferred to §22 item 1). Spinorial structure is therefore *algebraically unavoidable* given first-order admissible flow on the same persistent sector as the established scalar KG dynamics.

The geometric route is the closure-frame bundle construction. Orientation frames transport on a candidate bundle B(P) under a refinement-stable connection. Under three explicitly stated structural conjectures C1–C3, the admissible transport group lifts to SU(2), the double cover of the spatial rotation group SO(3). Spinorial loop sectors exhibit $U(2\pi) = -\mathbb{1}$ and $U(4\pi) = +\mathbb{1}$.

The convergence is the central new claim. The minimal Clifford representation of the spatial Clifford algebra Cl(3) carries the action of the spin group Spin(3) = SU(2) on the 2-component Weyl spinor space $\mathbb{C}^2$. The geometric SU(2) of the closure-frame bundle and the algebraic SU(2) ≅ Spin(3) acting on the spinor representation are *the same group acting in the same way on the same representation space* — both target the fundamental (2-dimensional) representation of SU(2), the minimal spinor lift. The two routes converge because, at the level of Spin(3) on its fundamental representation, there is only one structure to target. The construction therefore acquires both *algebraic inevitability* (from Clifford necessity, conditional only on inherited first-order existence) and *geometric realisation* (from closure-frame transport, conditional on C1–C3).

The convergence is at the level of *group structure*: both routes target SU(2)/Spin(3). It is not at the level of *sector decomposition*: the spinorial-vs-trivial-holonomy decomposition is geometric, requiring C1–C3 for the bundle-level decomposition into $U(2\pi) = -\mathbb{1}$ and $U(2\pi) = +\mathbb{1}$ sectors. The algebraic route gives universal Clifford structure on all first-order admissible flow; the geometric route gives the holonomy-based two-sector decomposition that bridges to physical fermion identification. Both are needed for the full spinorial picture.

The K=7 minimal fact architecture supplies the dimensional bridge: the K=7 architecture includes three spatial transport generators, which underlie Cl(3) with SU(2) ≅ Spin(3) action on 2-component Weyl spinors. Adding the temporal generator yields Cl(1,3) with 4-component Dirac spinors.

A productive sidebar on chiral mass coupling. A separate algebraic observation, recorded as a sidebar in §6.7: the covariant first-order operator $\Sigma = \Gamma^\mu p_\mu + \beta m$ produced by *direct covariant squaring*, distinct from Dirac's 3+1 Hamiltonian-form derivation, has β forced (up to phase) to be γ^5, producing a *chiral-mass* operator $\Sigma = \gamma^\mu p_\mu + \gamma^5 m$. This is structurally distinct from the standard Dirac operator $(i\gamma^\mu \partial_\mu - m)$ produced by Theorem 1. The §6.7 sidebar interprets the two derivational routes as possibly corresponding to different physical stages — the 3+1 Hamiltonian-form derivation governing the effective low-energy spinorial sector, the chiral-mass structure possibly reflecting a deeper substrate-level pre-electroweak organisation, with electroweak symmetry breaking as the structural bridge between the two. This interpretation is *conjectural* and is explicitly deferred to §22 item 12. The standard Dirac operator remains the primary effective spinorial structure throughout the present work; the chiral-mass sidebar identifies a working conjecture, not a derived result.

The construction is the geometric–algebraic precursor to fermionic physics, not its derivation. It does not derive the Dirac equation as a substrate evolution equation, canonical anticommutation relations $\{\psi(x), \psi^\dagger(y)\} = \delta(x - y)$, the spin-statistics theorem, quantum electrodynamics, or the Standard Model fermion spectrum. What it does supply is: the spinorial transport structure required for fermionic source-carriers, the algebraic forcing argument that makes this structure unavoidable, the geometric realisation that grounds it in closure-frame bundle topology, and the convergence claim that ties algebra and geometry into a single coherent ontology.

The remaining matter-sector gaps — Dirac operator as substrate evolution equation, canonical anticommutation, species decomposition, full QED, master-action construction, substrate-to-electroweak bridge — become concrete next-paper deliverables with explicit target theorems and named discharge paths.

Abstract

The Microscopic Origin paper established the substrate ontology of the conserved record current $J^\mu = \rho_{pers} u^\mu = \Pi_{pers} C^\mu$ as the coarse-grained transport of refinement-persistent topologically protected commitment loops. The loops were scalar carriers. The dominant remaining matter-sector gap is the spinorial structure required to recover fermionic source-carriers.

This paper supplies a candidate spinorial extension constructed through two convergent routes:

Algebraic route (Clifford necessity). Inheriting (i) κ-field uniqueness Theorem U establishing the Klein–Gordon form $(\Box + m^2)s = \rho_{committed}$ as the unique admissible second-order scalar propagation structure, and (ii) the Schrödinger→Dirac paper establishing the existence of first-order admissible closure flow on the persistent sector, Dirac's 3+1 Hamiltonian-form derivation (Theorem 1) forces any such first-order operator $H = \alpha^i p_i + \beta m$ with $H^2 = (p^2 + m^2) \cdot \mathbb{1}$ to satisfy the Euclidean Clifford anticommutation relations $\{\alpha^i, \alpha^j\} = 2\delta^{ij} \cdot \mathbb{1}$, $\{\alpha^i, \beta\} = 0$, $\beta^2 = \mathbb{1}$. The minimal faithful representation is 4-dimensional. Under covariant repackaging, *the standard Dirac operator $(i\gamma^\mu \partial_\mu - m)$ is algebraically forced as the minimal admissible first-order structure on the persistent sector*; whether this algebraic form generates substrate-level dynamics is deferred to §22 item 1.

Geometric route (closure-frame bundle lift). Oriented commitment loops carry closure-orientation frames on the candidate closure-frame bundle B(P), transporting under a refinement-stable connection. Under three structural conjectures C1–C3 of §10.1, the admissible transport group lifts to SU(2) (Structural Inference SI). The spinorial sectors exhibit $U(2\pi) = -\mathbb{1}$ and $U(4\pi) = +\mathbb{1}$.

Convergence (Theorem 2). The minimal Clifford representation of the spatial Clifford algebra Cl(3) carries the action of the spin group Spin(3) ≅ SU(2) on the 2-component Weyl spinor space $\mathbb{C}^2$. The algebraic SU(2) acting on this fundamental representation and the geometric SU(2) emerging from the minimal spinor lift of the closure-frame bundle are *the same group acting in the same way on the same fundamental (2-dimensional) representation*. The two routes converge at the level of group structure on the spin group of 3-space.

We establish one definition, four operational theorems, one structural inference, one minimality result, and one provisional proposition. (Definition 1, Theorem 1, Theorem 3, and the Minimality Theorem are *proven* given inheritance from κ-field uniqueness, Schrödinger→Dirac first-order existence, and the candidate bundle construction. Theorem 2 is a *structural correspondence* by standard Clifford / spin-group representation theory plus minimal-representation selection. Theorem 4 and Structural Inference SI are *conditional* on the three structural conjectures C1–C3. Proposition 6 is *conjectural*. The Fold integration of §15 is *interpretive*.)

Explicit clarification of scope. The present paper derives the spinorial transport structure and Clifford-compatible internal representations required for fermionic source-carriers. It does not derive canonical anticommutation relations, the full spin-statistics theorem, or canonical fermionic quantisation. ***The algebraic form algebraically forced by Theorem 1 is the standard Dirac operator (3+1 Hamiltonian-form derivation), inherited as the minimal admissible first-order structure on the persistent sector*****; whether this algebraic form is the substrate evolution equation generating spinorial source-carrier dynamics is the target of §22 item 1's Dirac-emergence theorem, deferred to future work. The covariant chiral-mass operator forced by direct covariant squaring (§6.7) is interpreted as a possible substrate-level pre-electroweak organisation of the persistent sector — a working conjecture explicitly deferred to §22 item 12. The standard Dirac operator remains the primary effective spinorial structure throughout. No claims regarding Standard Model electroweak reconstruction are established here.** Items left open are explicitly deferred to subsequent papers.

Contents

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1. Setting: What Previous Papers Established and What They Left Open

The Matter Coupling paper closed three structural problems of the electromagnetic programme — coupling uniqueness, gauge ⇔ conservation, and Maxwell-form dynamics — and identified the substrate origin of J^μ as the dominant remaining gap. The Microscopic Origin paper closed that gap operationally and ontologically.

Parallel to the matter-sector strand, the **κ-field uniqueness programme** has established complementary results on the substrate-level scalar sector. The κ-field uniqueness theorem (κ-field paper, Theorem U) establishes that the unique admissible second-order scalar propagation structure on the persistent sector takes the Klein–Gordon form $(\square + m^2)\, s = \rho\_committed$, with s the unique admissible commitment scalar field and m the substrate-derived mass coupling. The no-alternative theorem (κ-field paper, Theorem N) excludes all admissible second-order scalar alternatives. The identification $\kappa \equiv s$ ties this construction to the κ-field of the broader programme. These results constrain the scalar propagation sector unconditionally.

A third strand — the **Schrödinger→Dirac paper** — establishes two results inherited as joint preconditions to Theorem 1 of the present paper:

1. *First-order existence.* First-order admissible flow operators exist on the persistent sector, satisfying locality, refinement persistence, Hermiticity (from the Hamiltonian admissibility paper), and consistency with the inherited second-order scalar invariant. This existence is a non-trivial structural result, not an algebraic constraint, and it underwrites everything that follows in §6.
2. *Algebraic forcing.* Any first-order admissible flow operator on the persistent sector whose square equals the inherited second-order KG invariant satisfies the Clifford anticommutation relations. The minimal faithful representation is 4-dimensional.

The present paper inherits both as Theorem 1's preconditions. Without (1), Theorem 1 is vacuously true (no operators to constrain); with (1) and (2), Theorem 1 is a *derived* result conditional on inheritance.

The remaining matter-sector gaps following these strands are:

1. **Spinorial structure.** Why does matter carry spin-½?
2. **Species decomposition.** Which loop topologies map to which observed particles?
3. **Quantum-field promotion.** Operator-valued promotion to Fock structure with anticommutation relations.

The present paper addresses item 1. Items 2–3 are explicitly deferred.

The conceptual approach is the central framing: spinorial structure emerges from the *convergence* of an algebraic forcing (conditional on inherited first-order existence + κ-field uniqueness) and a geometric realisation (conditional on three structural conjectures). A purely geometric construction — through orientation transport on the closure-frame bundle, with SU(2) emerging from SO(3) double-cover topology under three structural conjectures — is retained here (§§5, 10) but is not the sole structural pillar; the algebraic route (§6) supplies independent unconditional support for the same SU(2)-spinor structure.

The paper inherits and extends the source-carrier-vs-matter discipline of Microscopic Origin §13.2. Throughout, "spinorial source-carrier" means *source-carrier endowed with spinorial transport and Clifford-compatible internal structure*, not "fermionic matter particle."

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2. Notation and Conventions

We adopt the conventions of Matter Coupling §2 and Microscopic Origin §2 throughout, extended for the spinorial and Clifford-algebraic structure introduced here.

Spacetime and indices. Four-dimensional Lorentzian continuum with metric signature (+, −, −, −). Greek indices μ, ν, … run over 0, 1, 2, 3; Latin indices i, j, … over 1, 2, 3; spinor indices α, β, … over 1, 2 for two-component Weyl spinors and A, B, … over 1, 2, 3, 4 for four-component Dirac spinors. Natural Heaviside–Lorentz units ($\hbar = c = \varepsilon_0 = 1$).

Persistent sector. $H^1(\mathcal{G}(\Lambda))$ for real coefficients and $H^1(\mathcal{G}(\Lambda); \mathbb{Z})$ for integer coefficients. Persistent transport potential A_μ with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Full commitment four-current $C^\mu = (\rho, J_c^i)$; projected record current $J^\mu = \Pi_{pers} C^\mu = \rho_{pers} u^\mu$. Persistent transport manifold P (Microscopic Origin §6.3).

Scalar commitment field. The unique admissible commitment scalar field of the κ-field uniqueness programme is s(x), satisfying $(\Box + m^2)\, s = \rho_{committed}$, with $\Box = \eta^{\mu\nu} \partial_\mu \partial_\nu$ the d'Alembertian and m the substrate-derived mass coupling. The κ ≡ s identification means the κ-field of earlier VERSF papers and the scalar commitment field s of the present paper are notationally interchangeable; we write s for clarity.

Closure-orientation frame. A *closure-orientation frame* at a point x ∈ P is a triple $F(x) = (e_1(x), e_2(x), e_3(x))$ on the admissible closure-tangent sector at x, satisfying closure-orthonormality inherited from the triangular orientation programme. F(x) takes values in the candidate closure-frame bundle B(P) of §5.

Clifford algebra and gamma matrices. The Clifford algebra Cl(1,3) is the algebra over $\mathbb{C}$ generated by four elements γ^μ (μ = 0, 1, 2, 3) satisfying

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \cdot \mathbb{1},$$

where {A, B} = AB + BA is the anticommutator and $\eta^{\mu\nu} = \mathrm{diag}(+, -, -, -)$. The minimal faithful complex representation of Cl(1,3) is on the carrier space $\mathbb{C}^4$ (4-component Dirac spinors), realised concretely by the 4 × 4 Dirac matrices. We use $M(n, \mathbb{C})$ to denote the algebra of n × n complex matrices; $\mathbb{C}^n$ denotes its standard carrier space. So Cl(1,3) acts on $\mathbb{C}^4$ via its minimal representation in M(4, $\mathbb{C}$).

The spatial subalgebra Cl(3) is generated by γ^i (i = 1, 2, 3) with $\{\gamma^i, \gamma^j\} = -2\delta^{ij} \cdot \mathbb{1}$. Its minimal faithful complex representation is on the carrier space $\mathbb{C}^2$ (2-component Weyl spinors), realised concretely by the Pauli matrices (up to factors of i). The Cl(3) action on $\mathbb{C}^2$ carries the standard SU(2) action on Weyl spinors.

The chirality operator $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ satisfies $\{\gamma^5, \gamma^\mu\} = 0$ for all μ and $(\gamma^5)^2 = \mathbb{1}$. The projectors $P_\pm = (\mathbb{1} \pm \gamma^5)/2$ split the Dirac representation $\mathbb{C}^4$ into left- and right-handed Weyl components.

Dirac 3+1 Hamiltonian-form matrices. The Dirac 3+1 Hamiltonian form uses matrices α^i (i = 1, 2, 3) and β acting on $\mathbb{C}^4$, satisfying

$$\{\alpha^i, \alpha^j\} = 2\delta^{ij} \cdot \mathbb{1},\ \{\alpha^i, \beta\} = 0,\ \beta^2 = \mathbb{1},\ (\alpha^i)^2 = \mathbb{1}.$$

The relation to covariant Cl(1,3) generators is $\gamma^0 \equiv \beta$ and $\gamma^i \equiv \beta\alpha^i$, with γ^μ satisfying the covariant Clifford relations $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \cdot \mathbb{1}$. In this convention $\beta = \gamma^0$ and $\alpha^i = \gamma^0\gamma^i$.

Spin group. The spin group Spin(n) is the unique connected double cover of the rotation group SO(n). For n = 3, Spin(3) ≅ SU(2). For n = 1,3 (Lorentzian), Spin(1,3) ≅ SL(2, $\mathbb{C}$).

First-order closure flow operator (Hamiltonian form). The first-order admissible closure flow operator on the persistent sector is, in 3+1 Hamiltonian form,

$$H = \alpha^i p_i + \beta m,$$

with α^i and β substrate-level operators on a representation space to be determined by admissibility constraints (§6). Theorem 1 establishes that admissibility forces α^i and β to satisfy the Euclidean Clifford relations on a 4-dimensional minimal representation, identifying H with the Dirac Hamiltonian and the covariant first-order operator $\gamma^\mu p_\mu - m$ with the standard Dirac operator (after the standard γ^0 multiplication).

Structural conjectures C1–C3. The conditional results of §10 (Theorem 4 and Structural Inference SI) depend on three structural conjectures made explicit in §10.1: C1 (frame bundle structure group is SO(3)), C2 (admissible refinement-stable transport requires single-valued lifts), C3 (persistent sector selects the connected double cover). Under §11's geometric–algebraic convergence, the *group-structure* load on these conjectures is reduced; the *sector-decomposition* load is unchanged.

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3. Structural Dependencies

This section states without re-derivation the prior VERSF results on which the construction depends.

3.1 Persistent cohomological transport

Refinement-persistent observable sector $H^1(\mathcal{G}(\Lambda))$ with continuum transport reducing uniquely to Maxwell-form structure. Wilson-loop construction $W(\gamma) = \exp(i \oint_\gamma A_\mu dx^\mu)$ provides the holonomy pairing.

3.2 Commitment continuity

Full commitment four-current satisfies $\partial_\mu C^\mu = 0$ ontologically. Inherited unchanged.

3.3 Topological threshold $\beta_1 \geq 1$

Irreversible commitment requires nontrivial cycle structure.

3.4 The loop-current ontology (Microscopic Origin)

$J^\mu(x) = \rho_{pers}(x) u^\mu(x) = \Pi_{pers} C^\mu(x)$, with the dust-fluid normalisation $u^\mu u_\mu = 1$, the primitive commitment loops satisfying (L1)–(L4), and the source-admissibility framework SA1–SA5 (Microscopic Origin Lemma 1). Inherited unchanged.

3.5 The κ-field uniqueness programme

The κ-field uniqueness programme establishes:

1. **KG uniqueness (Theorem U).** The unique admissible second-order scalar propagation structure on the persistent sector takes the Klein–Gordon form $(\square + m^2)s = \rho_{committed}$.
2. **No-alternative theorem (Theorem N).** All admissible second-order scalar alternatives are excluded.
3. **κ ≡ s identification.** The κ-field of earlier VERSF papers and the scalar commitment field s of this construction are identified.

These results are *load-bearing for §6*. Theorem 1 requires that the second-order invariant on the persistent sector be the KG operator $\square + m^2$; Theorem U supplies this. Without κ-field uniqueness, Theorem 1 would be conditional on a separate uniqueness assumption.

3.6 The Schrödinger→Dirac paper

The Schrödinger→Dirac paper establishes two results inherited as Theorem 1's joint preconditions:

1. **First-order existence (Schrödinger→Dirac, Theorem E).** First-order admissible flow operators exist on the persistent sector, satisfying locality, refinement persistence, Hermiticity, and consistency with the inherited second-order scalar KG dynamics.
2. **Algebraic forcing (Schrödinger→Dirac, Theorem F).** Any first-order admissible flow operator whose square equals the inherited second-order KG invariant satisfies the

Clifford anticommutation relations. Dirac's 3+1 Hamiltonian-form argument supplies the algebraic mechanism.

Inheritance status. Theorem 1 of §6 below is *conditional on inheritance from Schrödinger→Dirac Theorems E and F*. Within the present paper this inheritance is treated as discharged, but the conditionality is recorded explicitly in §21's status table. If the Schrödinger→Dirac paper establishes only the algebraic forcing (Theorem F) and not the existence (Theorem E), the present paper's Theorem 1 inherits the existence as a *working assumption* rather than a discharged precondition; this would shift Theorem 1 from "derived under named inheritance" to "derived under one named inheritance and one stated assumption." The §21 status table is calibrated accordingly.

3.7 The K=7 minimal fact architecture

The K=7 architecture papers establish that K=7 minimal fact dimensions characterise the substrate. The *spatial-transport-generator count d=3* — the only piece of the K=7 decomposition load-bearing for the present paper — is inherited from the K=7 architecture. The full decomposition of K=7 into temporal, mass-coupling, spatial, and internal/gauge sectors is a richer structural result of those papers; for the spinorial construction here, only the d=3 spatial-generator count is required. §8 develops the d=3 → Cl(3) → Cl(1,3) consequences.

3.8 The Fold programme (with orientation and parity structure)

The Fold programme as consolidated in *The Fold and the Record* (v31) establishes the standard Fold-programme results (irreversible substrate evolution, topological trapping at $\beta_1 \geq 1$, fold as minimal irreversible distinction) and additionally develops (σ, ω) state-pair structure at the fold level, a $Z_2 \times Z_2$ grading combining orientation parity and closure-flow parity, interface transport between fold states, and loop holonomy with closure parity. These structures are used in §15.

3.9 The triangular closure programme

The local closure tangent at a substrate point inherits a triangular orientation structure carrying SO(3) rotational degeneracy, supplying the structural origin of the orientation frames of §5 and conjecture C1 of §10.1.

3.10 Microscopic Origin: source-admissibility framework

The source-admissibility framework SA1–SA5 (Microscopic Origin §4) constrains the substrate-level current J^μ. The spinorial extension introduces orientation-admissibility OA1–OA4 and Clifford-compatibility CC1–CC2 (both in §4 below).

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4. The Joint Admissibility Constraints: Source + Orientation + Clifford Compatibility

The Microscopic Origin paper's source-admissibility framework constrains the current J^μ. The spinorial extension introduces two additional families:

- **Orientation-admissibility (OA1–OA4)**, constraining the substrate orientation frame F(x).
- **Clifford-compatibility (CC1–CC2)**, constraining the substrate-level Clifford structure arising algebraically from first-order closure dynamics.

The joint framework SA + OA + CC is the structural constraint any spinorial source-carrier must satisfy.

4.1 Source-admissibility (inherited unchanged)

SA1–SA5 of Microscopic Origin §4.1 are inherited unchanged. The spinorial extension does not modify the current's source-admissibility constraints.

4.2 Orientation-admissibility

Proposition 1 (Orientation admissibility). An orientation frame F(x) on the persistent transport sector is admissible iff:

1. **Closure-locality (OA1).** F(x) depends only on substrate field values at x and finitely many derivatives in the continuum refinement limit.
2. **Refinement persistence (OA2).** F(x) survives the refinement coarsening map δ^*.
3. **Closure-orthonormality (OA3).** F(x) satisfies the closure-orthonormality relation inherited from the triangular orientation programme.
4. **Admissible transport (OA4).** $F(x + \delta x)$ is obtained from F(x) by an admissible transport operation on B(P) preserving OA1–OA3.

Lemma 1 (Bundle-level equivalence). F(x) satisfies OA1–OA4 iff it is a refinement-stable section of B(P) under an admissible connection.

Proof. As Microscopic Origin Lemma 1. ■

4.3 Clifford-compatibility

The algebraic route of §6 produces a Clifford-algebraic structure on the substrate-level first-order flow operator. For this structure to participate consistently in the substrate framework, it must satisfy:

1. **Algebraic locality (CC1).** The Clifford generators (in either the Hamiltonian form α^i, β or the covariant form γ^μ) act locally on the substrate representation.
2. **Representation-refinement persistence (CC2).** The minimal faithful representation of Cl(1,3) on $\mathbb{C}^4$ descends consistently under refinement coarsening.

Lemma 2 (Clifford-bundle equivalence). The Clifford structure satisfies CC1–CC2 iff it arises as the structure algebra of a refinement-stable spinor bundle S(P) over P whose typical fibre is the minimal Cl(1,3) representation $\mathbb{C}^4$.

Proof. Standard spinor-bundle construction applied to P, with refinement persistence inherited from the refinement functor. Structurally parallel to Lemma 1 with B(P) replaced by S(P). ■

4.4 Joint admissibility

Proposition 2 (Joint admissibility framework). A spinorial source-carrier structure on the persistent sector is admissible iff it simultaneously satisfies SA1–SA5 (on the underlying current), OA1–OA4 (on the orientation frame), and CC1–CC2 (on the Clifford structure).

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5. Oriented Commitment Loops and the Candidate Closure-Frame Bundle

5.1 Definition

Definition 1 (Oriented commitment loop). An *oriented commitment loop* $\mathcal{C}_i$ is a primitive commitment loop (Microscopic Origin Definition 2, satisfying (L1)–(L4)) equipped with a local closure-orientation frame $F_i(x)$ satisfying OA1–OA4 of §4.2.

An oriented commitment loop carries four attributes: commitment weight q_i, admissible transport four-velocity u_i^μ with $u_{i\mu}u_i^\mu = 1$, winding number $w_i \in \mathbb{Z}$, and closure-orientation frame $F_i(x)$ transporting on B(P).

Reading. Oriented commitment loops are *candidate spinorial source-carrier sectors* — substrate structures satisfying the admissibility-and-conservation requirements any future fully dynamical construction must respect. The joint (L1)–(L4) + (OA1)–(OA4) conditions are boundary conditions on candidate spinorial source-carrier sectors, not independent postulates.

5.2 The candidate closure-frame bundle B(P)

The candidate closure-frame bundle B(P) is defined provisionally pending triangular-closure-programme completion, paralleling Microscopic Origin §6.3's provisional Π_{pers}.

Candidate construction. Let P be the persistent transport manifold. B(P) has fibre at $x \in P$ the space of orientation frames F(x) satisfying OA1, OA3, and compatibility with the local triangular closure structure at x. An admissible section additionally satisfies OA2 and is transported by an OA4-admissible connection. Rigorous explicit construction is §22 item 6.

5.3 The closure-orientation connection ω_μ

The connection ω_μ on B(P) is the unique admissible connection compatible with OA1–OA3 and refinement persistence:

- ω_μ takes values in the Lie algebra of B(P)'s structure group (under C1 of §10.1, this is $\mathfrak{so}(3)$);
- preserves closure-orthonormality under parallel transport;
- descends consistently under refinement coarsening;
- unique up to admissible gauge transformations.

Parallel transport along closed $\gamma \subset P$: $U(\gamma) = \mathcal{P} \exp(i \oint_\gamma \omega_\mu dx^\mu)$.

5.4 Joint admissibility of oriented loops

Proposition 3 (Joint admissibility of oriented loops). An oriented commitment loop $\mathcal{C}_i$ contributes to a current satisfying SA1–SA5, an orientation frame satisfying OA1–OA4, and (once the Clifford structure of §6 is established on the bundle) satisfies CC1–CC2 on the associated spinor bundle S(P).

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6. Clifford Structure from First-Order Closure Dynamics — *The Algebraic Route*

This section establishes the algebraic route to spinorial structure: first-order admissible closure flow on the persistent sector, combined with the κ-field uniqueness theorem, *forces* Clifford-algebraic structure on the substrate-level transport. The argument is the substrate-level reconstruction of Dirac's 1928 derivation, inherited from the Schrödinger→Dirac paper.

The central result is that the spinorial sector is not merely *suggested* by closure-frame geometry — it is *algebraically unavoidable* once first-order admissible flow is admitted on the same persistent sector as the established scalar KG dynamics. This algebraic forcing is the structural pillar that the geometric construction of §10 realises in substrate-level transport carriers.

6.1 The scalar KG sector (inherited)

By κ-field uniqueness Theorem U (§3.5), the persistent sector carries the unique admissible scalar propagation equation

$(\square + m^2)\, s = \rho_{committed}$,

with the second-order Klein–Gordon invariant

$H_{KG}^2 \equiv \square + m^2$

(stated directly in terms of ∂_μ, with $\Box = \eta^{\mu\nu} \partial_\mu \partial_\nu$, to avoid operator-momentum convention dependence). Theorem N excludes alternatives.

6.2 First-order admissible closure flow (existence inherited)

A first-order admissible closure flow operator H on the persistent sector is a first-order linear differential operator satisfying:

- **Locality**: depends on field values at x and first derivatives only.
- **Refinement persistence**: descends consistently under refinement coarsening.
- **Hermiticity**: H is self-adjoint, generating unitary evolution on the persistent sector (inherited from the Hamiltonian admissibility paper).
- **KG² consistency**: H^2 equals the on-shell version of the inherited KG dispersion, i.e., $H^2 = (p^2 + m^2) \cdot \mathbb{1}$ with $p^2 = \delta^{ij} p_i p_j = -\nabla^2$, on each component of the representation space. This is the Hamiltonian-squared form of the inherited differential KG operator $H\_KG^2 = \Box + m^2$: a positive-energy plane-wave solution of $(\Box + m^2)\varphi = 0$ satisfies $E^2 = p^2 + m^2$ with $E = i\partial_t$, so $H^2 = (p^2 + m^2) \cdot \mathbb{1}$ is the standard on-shell statement of the KG dispersion (see §6.3 for the explicit matching in the proof of Theorem 1).

Existence inheritance. *Existence* of such operators on the persistent sector is inherited from the Schrödinger→Dirac paper (Theorem E, §3.6). Without this inheritance, the theorems of §6 are vacuously true. With it, the theorems are derived results conditional on the inheritance. The §21 status table is calibrated accordingly.

6.3 Theorem 1 — Clifford necessity from first-order closure flow (3+1 Hamiltonian form)

Theorem 1 (Clifford necessity, 3+1 Hamiltonian form). Inheriting from κ-field uniqueness Theorem U and Schrödinger→Dirac Theorem E (existence of first-order admissible flow), any first-order admissible closure flow operator H on the persistent sector whose square equals the inherited KG invariant in the standard Hamiltonian-form sense — i.e., $H^2 = (p^2 + m^2) \cdot \mathbb{1}$ where $p^2 = \delta^{ij} p_i p_j = -\nabla^2$ and the squaring reproduces the relativistic dispersion of the KG sector — takes the form

$$H = \alpha^i p_i + \beta m \quad (i = 1, 2, 3;\ p_i = -i\partial_i),$$

with α^i and β substrate-level operators on a representation space satisfying the **Euclidean Clifford anticommutation relations**

$$\{\alpha^i, \alpha^j\} = 2\delta^{ij} \cdot \mathbb{1},\ \{\alpha^i, \beta\} = 0,\ \beta^2 = \mathbb{1},\ (\alpha^i)^2 = \mathbb{1}.$$

The minimal faithful representation of this algebra is 4-dimensional. The substrate representation space is therefore (at minimum) a 4-component spinor structure $\mathbb{C}^4$.

Proof. Write the candidate first-order Hamiltonian-form operator as

$H = A^i p_i + Bm,\ i = 1, 2, 3,$

with A^i and B linear operators on an internal representation space to be determined. Computing H^2:

$H^2 = (A^i p_i + Bm)(A^j p_j + Bm) = A^i A^j p_i p_j + (A^i B + B A^i)\, m\, p_i + B^2 m^2 = \tfrac{1}{2} \{A^i, A^j\}\, p_i p_j + (A^i B + B A^i)\, m\, p_i + B^2 m^2,$

where the first term is symmetrised (the antisymmetric part vanishes against the symmetric $p_i p_j$).

The Hamiltonian-form KG² consistency requires $H^2 = (p^2 + m^2) \cdot \mathbb{1} = \delta^{ij} p_i p_j \cdot \mathbb{1} + m^2 \cdot \mathbb{1}$ (with all components of the representation space satisfying the same KG dispersion). Matching coefficients:

- **Coefficient of $p_i p_j$**: $\tfrac{1}{2} \{A^i, A^j\} = \delta^{ij} \cdot \mathbb{1}$, equivalently $\{A^i, A^j\} = 2\delta^{ij} \cdot \mathbb{1}$. Setting $A^i \equiv \alpha^i$ gives the Euclidean Clifford anticommutation relations on three generators.
- **Coefficient of $m\, p_i$**: $A^i B + B A^i = 0$, i.e. $\{A^i, B\} = 0$. Setting $B \equiv \beta$ gives $\{\alpha^i, \beta\} = 0$.
- **Coefficient of m^2**: $B^2 \cdot \mathbb{1} = \mathbb{1}$, i.e. $\beta^2 = \mathbb{1}$. Combined with the $\{\alpha^i, \beta\} = 0$ relations and the requirement that all components satisfy the same dispersion: $(\alpha^i)^2 = \mathbb{1}$.

These are precisely the relations Dirac used in 1928 to derive the gamma matrices. The minimal faithful representation of the algebra generated by $\{\alpha^1, \alpha^2, \alpha^3, \beta\}$ with these relations is 4-dimensional, realised concretely by 4×4 matrices acting on $\mathbb{C}^4$. Standard representation theory (Dirac matrices, Jordan-Wigner argument) shows no lower-dimensional faithful representation exists. The substrate representation space is therefore at minimum $\mathbb{C}^4$ — 4-component spinor structure. ■

Covariant repackaging. Setting $\gamma^0 \equiv \beta$ and $\gamma^i \equiv \beta\alpha^i$, the four operators γ^μ ($\mu = 0, 1, 2, 3$) satisfy the covariant Lorentzian Clifford relations

$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \cdot \mathbb{1}$

with $\eta^{\mu\nu} = \mathrm{diag}(+, -, -, -)$. Verification:

- $\{\gamma^0, \gamma^0\} = 2\beta^2 = 2 \cdot \mathbb{1} = 2\eta^{00} \cdot \mathbb{1}$ ✓
- $\{\gamma^i, \gamma^j\} = \{\beta\alpha^i, \beta\alpha^j\} = \beta(\alpha^i \beta \alpha^j + \alpha^j \beta \alpha^i)\beta + \ldots$ (use $\{\alpha^i, \beta\} = 0$ and $\{\alpha^i, \alpha^j\} = 2\delta^{ij}$) Direct computation: $\gamma^i \gamma^j + \gamma^j \gamma^i = \beta\alpha^i \beta\alpha^j + \beta\alpha^j \beta\alpha^i = \beta(\alpha^i \beta)\alpha^j + \beta(\alpha^j \beta)\alpha^i = \beta(-\beta \alpha^i)\alpha^j + \beta(-\beta \alpha^j)\alpha^i = -\beta^2 \alpha^i \alpha^j - \beta^2 \alpha^j \alpha^i = -(\alpha^i \alpha^j + \alpha^j \alpha^i) = -2\delta^{ij} \cdot \mathbb{1} = 2\eta^{ij} \cdot \mathbb{1}$ ✓ (with $\eta^{ij} = -\delta^{ij}$).
- $\{\gamma^0, \gamma^i\} = \{\beta, \beta\alpha^i\} = \beta^2 \alpha^i + \beta\alpha^i \beta = \alpha^i + \beta(-\beta \alpha^i) = \alpha^i - \alpha^i = 0 = 2\eta^{0i} \cdot \mathbb{1}$ ✓.

The covariant Dirac operator is then

$D = i\gamma^\mu \partial_\mu - m \cdot \mathbb{1}$,

i.e., *the standard Dirac operator with non-chiral mass term $m \cdot \mathbb{1}$, algebraically forced as the minimal admissible first-order structure on the persistent sector*. The Dirac equation is $D\psi = 0$, equivalently $(i\gamma^\mu \partial_\mu - m)\psi = 0$ — at this stage as the algebraic form forced by Theorem 1, not yet as a substrate evolution equation. The KG invariant is recovered via $\bar{D}D$ where $\bar{D} = i\gamma^\mu \partial_\mu + m$:

$$\bar{D}D = (i\gamma^\mu \partial_\mu + m)(i\gamma^\mu \partial_\mu - m) = -\gamma^\mu \gamma^\nu \partial_\mu \partial_\nu - m^2 \cdot \mathbb{1} = -\tfrac{1}{2}\{\gamma^\mu, \gamma^\nu\}\partial_\mu \partial_\nu - m^2 \cdot \mathbb{1} = -\eta^{\mu\nu} \partial_\mu \partial_\nu \cdot \mathbb{1} - m^2 \cdot \mathbb{1} = -(\Box + m^2) \cdot \mathbb{1}$$

(after the symmetrisation step). So $\bar{D}D = -(\Box + m^2) \cdot \mathbb{1}$, equivalently $D\bar{D} = -(\Box + m^2) \cdot \mathbb{1}$, recovering the KG operator (up to overall sign) on each spinor component.

Why 3+1 form and not direct covariant squaring. The 3+1 Hamiltonian-form derivation is Dirac's historically correct argument. It produces the standard Dirac operator $(i\gamma^\mu \partial_\mu - m)$ with non-chiral mass term, which is the operator inherited by the effective spinorial sector and the coupled action of §12 below. Direct covariant squaring of $\Sigma = \Gamma^\mu p_\mu + \beta m$ (with β anticommuting with all four Γ^μ) forces β to be γ^5 rather than γ^0 — producing a *covariant chiral-mass* operator $\gamma^\mu p_\mu + \gamma^5 m$, structurally distinct from the standard Dirac operator. The two operators arise from different derivational routes and may correspond to different physical stages: §6.7 records the covariant chiral-mass observation as a sidebar and interprets it as a possible substrate-level pre-electroweak organisation of the persistent sector, with the standard Dirac operator governing the effective low-energy spinorial sector. The interpretation is conjectural and is recorded as §22 item 12. For the present paper's primary aim — deriving the algebraic form of admissible first-order closure flow inherited by the effective spinorial sector — the 3+1 Hamiltonian-form derivation is the primary derivation, and Theorem 1 above states it as such.

Structural reading. Theorem 1 establishes the algebraic forcing *conditional on inheritance from κ-field uniqueness Theorem U and Schrödinger→Dirac Theorem E*. The conditions OA1–OA4 do not enter; the structural conjectures C1–C3 of the geometric route do not enter. Only the two named inheritance results from independent programmes underwrite Theorem 1.

Within those inheritances, the result is *derived*: the Euclidean Clifford relations on three α^i generators plus the anticommuting β are forced by the Hamiltonian-form squaring; the covariant Lorentzian Clifford relations on four γ^μ are forced by the algebraic repackaging. Spinorial structure on $\mathbb{C}^4$ is forced; the substrate evolution equation that emerges is the standard Dirac equation $(i\gamma^\mu \partial_\mu - m)\psi = 0$ with non-chiral mass.

This is a structural result of central importance: the spinorial sector acquires algebraic support conditional only on inheritance from κ-field uniqueness and Schrödinger→Dirac, not on the three structural conjectures of the geometric route.

6.4 Identification with the Dirac operator

By the covariant repackaging of §6.3, the operator $H = \alpha^i p_i + \beta m$ is in 1-to-1 correspondence with the covariant Dirac operator $(i\gamma^\mu \partial_\mu - m)$ acting on 4-component spinors $\mathbb{C}^4$, via the standard Hamiltonian-to-covariant transition:

$$(i\partial_t - H)\psi = 0 \Leftrightarrow (i\gamma^\mu \partial_\mu - m)\gamma^0\psi = 0 \Leftrightarrow D\psi' = 0 \text{ with } \psi' = \gamma^0\psi,\ D = i\gamma^\mu \partial_\mu - m.$$

(Multiplication by γ^0 converts the Hamiltonian-form equation to the covariant Dirac equation; the substitution $\psi' = \gamma^0\psi$ absorbs the γ^0 on the left.)

What Theorem 1 + the covariant repackaging establish: the algebraic form of any first-order admissible closure flow operator on the persistent sector, in covariant form, is the standard Dirac operator on 4-component spinors. This is the algebraic *form* of the spinorial dynamics. Whether this operator actually generates substrate-level spinorial source-carrier dynamics — i.e., whether ψ' satisfies $D\psi' = 0$ as an effective wave equation arising from substrate dynamics — is the target of §22 item 1's Dirac-emergence theorem. Theorem 1 establishes the algebraic form; §22 item 1 elevates this form to a substrate evolution equation.

The structural reading: *the algebraic form of the spinorial dynamics is forced by Theorem 1 (conditional on inheritance); whether this algebraic form is the dynamical equation governing spinorial source-carriers requires the additional Dirac-emergence work of §22 item 1.*

6.5 Chirality and Cl_even / Cl_odd grading

The Clifford algebra Cl(1,3) admits a Z_2 grading separating Cl_even (generated by even products of gamma matrices: $\mathbb{1}$, $\gamma^\mu\gamma^\nu$, γ^5) from Cl_odd (generated by odd products: γ^μ, $\gamma^\mu\gamma^\nu\gamma^\rho$). The chirality operator $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ satisfies $\{\gamma^5, \gamma^\mu\} = 0$ for all μ and $(\gamma^5)^2 = \mathbb{1}$, generating the chirality projection

$$P_\pm = (\mathbb{1} \pm \gamma^5)/2$$

decomposing the 4-component Dirac representation $\mathbb{C}^4$ into left- and right-handed Weyl components $\mathbb{C}^2_L \oplus \mathbb{C}^2_R$.

This Z_2 grading and chirality structure are inherent to the algebraic structure forced by Theorem 1 and are not additional constructions. Their substrate-level interpretation is supplied in §15 by identification with the Fold programme's $Z_2 \times Z_2$ grading.

6.6 Structural status of the algebraic route

Three results are established in §6.3–§6.5, all conditional on κ-field uniqueness Theorem U and Schrödinger→Dirac Theorem E (existence) and Theorem F (algebraic forcing):

- **Algebraic form forced:** any first-order admissible closure flow on the persistent sector takes the form of the Dirac operator (in covariant form) acting on $\mathbb{C}^4$.
- **Representation dimensionality forced:** the minimal faithful representation of the algebra is 4-dimensional; lower-dimensional faithful representations do not exist.

- **Chirality structure inherent:** the Z_2 chirality grading via γ^5 is automatic from the Cl(1,3) structure.

These are derived results within the named inheritance; they are not additional postulates.

6.7 Sidebar — Covariant chiral-mass form and the substrate-to-electroweak bridge

The primary result of Theorem 1 is that the standard Dirac operator is *algebraically forced as the minimal admissible first-order structure* on the persistent sector, via Dirac's original 3+1 Hamiltonian-form derivation:

$$H = \alpha^i p_i + \beta m$$

with covariant repackaging identifying the resulting operator as the standard Dirac operator

$$(i\gamma^\mu \partial_\mu - m) \psi = 0.$$

This is the operator inherited by the effective spinorial sector of the present paper and by the coupled action of §12 — at this stage as the algebraic form forced by Theorem 1, with the substrate evolution equation deferred to §22 item 1.

The present subsection records a separate algebraic observation arising from a different derivational route: direct covariant squaring of a first-order operator on the persistent sector.

The purpose of this subsection is not to introduce an alternative fermionic dynamics for the present paper. Rather, it identifies a potentially important structural bridge between the substrate-level spinorial sector and electroweak chirality.

The standard Dirac operator remains the primary effective spinorial structure throughout the present work.

The technical observation. Consider the candidate covariant first-order operator

$$\Sigma = \Gamma^\mu p_\mu + \beta m, \quad \mu = 0, 1, 2, 3,$$

with Γ^μ and β acting on an internal representation space and requiring Σ^2 to reproduce the inherited KG invariant.

Sign-convention note. Conventions vary across the literature in handling the relative signs between $p_\mu p^\mu$, $\partial_\mu \partial^\mu$, and the metric signature in covariant squaring of Dirac-type operators. The convention used here is that direct covariant squaring of $\Sigma = \gamma^\mu p_\mu + \beta m$ with $p_\mu = i\partial_\mu$ produces Σ^2 that matches the inherited KG invariant up to the standard overall sign appropriate to the chosen signature, with the gamma matrices satisfying $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \cdot \mathbb{1}$ as forced by the leading-order squaring. The substantive structural content of the matching — the

Clifford relations on the Γ^μ, the anticommutation of β with all four Γ^μ, and $\beta^2 = \pm\mathbb{1}$ — is invariant under choice of signature convention.

Repeating the squaring argument of Theorem 1's proof:

$$\Sigma^2 = \tfrac{1}{2}\{\Gamma^\mu, \Gamma^\nu\} p_\mu p_\nu + (\Gamma^\mu \beta + \beta \Gamma^\mu) m p_\mu + \beta^2 m^2.$$

Matching against the inherited KG invariant:

- $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu} \cdot \mathbb{1}$: forces the *covariant Lorentzian Clifford relations* on four Γ^μ generators.
- $\{\Gamma^\mu, \beta\} = 0$ for all $\mu \in \{0, 1, 2, 3\}$: forces β to anticommute with *all four* covariant Clifford generators.
- $\beta^2 = \pm\mathbb{1}$ (sign depending on convention).

Critical structural observation. The element of Cl(1,3) anticommuting with all four γ^μ is *not* γ^0. (Verification: $\{\gamma^0, \gamma^0\} = 2(\gamma^0)^2 = 2\eta^{00} \cdot \mathbb{1} = +2 \cdot \mathbb{1} \neq 0$.) The unique element (up to phase) anticommuting with all four γ^μ is $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. The covariant squaring forces

$\beta = \gamma^5$ (up to phase),

producing the *covariant chiral-mass operator*

$$\Sigma = \gamma^\mu p_\mu + \gamma^5 m.$$

This is structurally distinct from the standard Dirac operator $D = i\gamma^\mu \partial_\mu - m$: the latter has mass term $m \cdot \mathbb{1}$ (non-chiral, scalar mass); the former has mass term $\gamma^5 m$ (chiral, coupling left- and right-handed components with a chirality flip).

Interpretive framing. The covariant chiral-mass form should be interpreted cautiously. *The present paper does not claim that the substrate-level fermionic dynamics is governed directly by the operator*

$$\Sigma = \gamma^\mu p_\mu + \gamma^5 m.$$

Rather, the significance of the observation is structural:

- *Direct covariant squaring naturally produces a chiral mass coupling*, while
- *the 3+1 Hamiltonian-form derivation produces the standard non-chiral Dirac mass structure*,

suggesting that chirality may enter the substrate-level fermionic sector prior to effective low-energy mass formation. The two derivational routes are not in conflict; they may correspond to different physical stages of the substrate-to-effective-theory bridge.

The working interpretation proposed here is that:

- the standard Dirac operator ($i\gamma^\mu \partial_\mu - m$) governs the *effective low-energy spinorial sector* identified in Theorem 1,

while:

- the chiral-mass structure may reflect a deeper *substrate-level pre-electroweak organisation* of the persistent sector.

Under this interpretation, electroweak symmetry breaking acts as the structural bridge between:

- substrate-level chiral organisation,

and:

- effective non-chiral mass-eigenstate fermions.

This interpretation remains conjectural and is explicitly deferred to the substrate-to-electroweak bridge programme identified in §22 item 12. **No claims regarding Standard Model electroweak reconstruction are established in the present paper.**

The sidebar is not load-bearing for the primary Theorem 1 conclusion: Theorem 1 establishes the standard Dirac operator directly via the 3+1 Hamiltonian-form derivation, and that is the operator inherited by §§12 and 22. The covariant chiral-mass observation is a structurally interesting parallel derivation whose interpretation as substrate-level pre-electroweak structure is a working conjecture for future work, not a derived result of the present paper.

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7. Minimality of the Spinorial Sector

The Clifford forcing of §6 establishes that 4-component spinor structure is the *minimal* admissible representation given first-order closure flow + KG^2 consistency. This section makes the minimality explicit by excluding simpler substrate transport structures.

The minimality argument follows the exclusion-style structural discipline established in other VERSF papers. Spinorial structure becomes the *unique* admissible solution given the inherited constraints, not merely *one possible* solution.

7.1 Scalar transport: insufficient

A scalar substrate field satisfying first-order admissible flow would require a first-order operator Σ_{scalar} with $(\Sigma_{scalar})^2 = \square + m^2$ where Σ_{scalar} acts on a one-dimensional scalar representation. The most general first-order scalar operator is

$$\Sigma_{scalar} = c^\mu \partial_\mu + d \cdot m$$

with c^μ and d real (or complex) c-numbers. Direct computation:

$(\Sigma_{scalar})^2 = c^\mu c^\nu \partial_\mu \partial_\nu + 2dc^\mu m \partial_\mu + d^2 m^2$.

Matching with the requirement $(\Sigma_{scalar})^2 = \eta^{\mu\nu} \partial_\mu \partial_\nu + m^2$ (the scalar KG invariant) requires:

- $c^\mu c^\nu = \eta^{\mu\nu}$ (the LHS is a symmetric rank-2 tensor with $c^\mu c^\nu \geq 0$ on the diagonal in any signature; the RHS has indefinite signature with $\eta^{00} = +1$, $\eta^{ii} = -1$: impossible for c-number c^μ);
- $2dc^\mu = 0$ (forces $d = 0$ or $c^\mu = 0$);
- $d^2 = 1$ (incompatible with $d = 0$).

No scalar solution exists. The scalar sector is restricted to second-order propagation (the inherited KG sector). Scalar transport cannot supply first-order closure flow consistent with the scalar KG dynamics.

7.2 Vector transport: representationally obstructed

A vector substrate field V^α satisfying first-order admissible flow would require a first-order operator Σ_{vector} acting on the 4-dimensional vector representation of the Lorentz group. The minimal vector operator is $\Sigma_{vector} = A^\mu \partial_\mu + B m$ with A^μ and B acting on the vector representation. For $(\Sigma_{vector})^2 = (\Box + m^2) \cdot \mathbb{1}_4$, A^μ must satisfy the Clifford anticommutation relations $\{A^\mu, A^\nu\} = 2\eta^{\mu\nu} \cdot \mathbb{1}_4$ in the vector representation.

The obstruction is representation-theoretic. The Lorentz group $SL(2, \mathbb{C})$ has irreducible representations labelled by (j_L, j_R) with $j_L, j_R \in \{0, ½, 1, ...\}$. Two distinct 4-dimensional representations are relevant:

- The **vector representation** is (½, ½), which is 4-dimensional, *real* (under the natural reality structure compatible with Lorentz signature), and acts as ordinary 4-vectors transforming under Lorentz boosts and rotations.
- The **minimal spinor representation** of Cl(1,3) is (½, 0) ⊕ (0, ½), which is 4-dimensional, *complex*, and decomposes into left- and right-handed Weyl spinors under chirality projection.

These representations have the same complex dimension but are *inequivalent as representations of the Lorentz group*:

- The vector (½, ½) is a single irreducible representation of complex dimension 4 with a natural real structure (mass-shell vectors are real).
- The Dirac (½, 0) ⊕ (0, ½) is a reducible representation, a direct sum of two inequivalent complex 2-dimensional Weyl representations.

The Clifford algebra Cl(1,3), with its anticommutation relations $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \cdot \mathbb{1}$, admits a faithful complex representation only on (½, 0) ⊕ (0, ½) and its tensor multiples. The vector representation (½, ½) does *not* admit a faithful representation of Cl(1,3): attempting to embed Cl(1,3) generators in 4 × 4 matrices that transform as the (½, ½) representation produces inconsistencies with the Lorentz structure (the resulting "Γ^μ" cannot simultaneously be Lorentz vectors *and* satisfy the anticommutation relations on the (½, ½) representation).

The vector sector is therefore *representationally obstructed*: the required Clifford algebraic structure cannot be realised on the vector representation of the Lorentz group. Vector transport cannot supply spinorial source-carrier structure.

7.3 Rank-2 tensor transport: reducible, with minimality selection

A rank-2 tensor substrate field $T^{\mu\nu}$ satisfying first-order admissible flow would require an operator Σ_{tensor} acting on a rank-2 tensor representation: the full 16-dimensional $T^{\mu\nu}$, or the 10-dimensional symmetric $T^{(\mu\nu)}$, or the 6-dimensional antisymmetric $T^{[\mu\nu]}$.

These representations are *reducible* as representations of Cl(1,3) augmented by Lorentz structure. Each decomposes as:

- $T^{\mu\nu}$ (16-dim) $\cong (1, 1) \oplus 2 \cdot (½, ½) \oplus (0, 0)$ under Lorentz, with multiple copies of the minimal spinor representation embedded after tensoring with the chirality-decomposed Cl(1,3) structure.
- $T^{(\mu\nu)}$ (10-dim) decomposes similarly.
- $T^{[\mu\nu]}$ (6-dim, like $F^{\mu\nu}$) decomposes as $(1, 0) \oplus (0, 1)$.

In each case, attempting to embed a faithful Cl(1,3) representation produces a decomposition into copies of the minimal 4-dimensional spinor representation plus orthogonal Lorentz sectors not carrying the Clifford structure faithfully. The Clifford algebra acts non-trivially only on the spinor sub-representations; the remaining sectors carry trivial or non-faithful Clifford action.

The exclusion is by minimality, not by impossibility. A rank-2 antisymmetric tensor representation can contain copies of the minimal spinor representation as sub-representations. The minimality principle excludes the larger reducible representations in favour of the smallest irreducible faithful representation: the 4-dimensional spinor.

The minimality principle is structural, not preferential: substrate admissibility (specifically Matter Coupling's (P5) lowest-order closure consistency) selects the *minimal* irreducible substrate representation supplying the required dynamics, since admissibility selects the *least* additional substrate structure consistent with the required dynamics. Within this principle, the 4-dimensional spinor is the minimal irreducible faithful Cl(1,3) representation, and higher reducible representations (rank-2 tensor and above) are excluded.

The structural distinction is:

- §7.1 (scalar): excluded by *impossibility* — no scalar solution exists.
- §7.2 (vector): excluded by *representational obstruction* — the required Clifford structure cannot be realised on the vector representation.

- §7.3 (rank-2 tensor and above): excluded by *minimality* — reducible representations contain copies of the minimal spinor representation, and minimality selects the smallest irreducible.

7.4 Theorem 5 — Minimality of the spinorial sector

Theorem 5 (Minimality). Among substrate transport structures supplying first-order admissible closure flow consistent with the inherited scalar KG dynamics (§3.5), the minimal admissible substrate representation is the 4-component spinor representation of Cl(1,3). Scalar transport is impossible (§7.1), vector transport is representationally obstructed (§7.2), and rank-2 tensor (and higher) transport is excluded by minimality (§7.3).

Structural reading. Theorem 5 upgrades the Clifford forcing of Theorem 1 from "spinorial structure is *one* admissible solution" to "spinorial structure is *the unique* admissible solution at minimal representation order." This is the exclusion-style structural strengthening characteristic of the more mature VERSF papers (κ-field no-alternative, closure-uniqueness). The spinorial sector emerges as algebraically forced *and* representation-theoretically minimal.

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8. The K=7 → d=3 → Cl(1,3) Dimensional Bridge

This section establishes the dimensional chain from the K=7 minimal fact architecture (inherited from the K=7 papers, §3.7) to the 4-component spinor representation of Cl(1,3). The chain supplies a structural prediction: the dimensionality of the spinor representation is *forced* by the K=7 spatial-generator count combined with the standard 3+1 spacetime decomposition.

8.1 The K=7 architecture — what is and is not inherited

The K=7 architecture papers establish that the substrate carries K=7 minimal fact dimensions. *The only K=7 content load-bearing for the present paper is the inclusion of three spatial transport generators.* This is a weaker (and structurally simpler) claim than the full decomposition of K=7 into temporal, mass-coupling, spatial, and internal/gauge sectors.

A natural decomposition consistent with VERSF's broader architecture reads:

- 1 temporal dimension (commitment-time direction),
- 1 mass-coupling dimension (substrate-derived scalar mass scale),
- 3 spatial transport generators (spatial commitment-flow directions),
- 2 internal/gauge dimensions (remaining structure, contributing to gauge sector).

The *full* decomposition is a richer structural result of the K=7 papers. The *spatial-generator count d=3* — the only piece required for the Cl(3) construction below — is inherited from the K=7 architecture and is the load-bearing inheritance for §8.

Inheritance status. Theorem of §8 below is *conditional on the K=7 architecture inheritance supplying d=3 spatial transport generators*. The §21 status table records this conditionality. If the K=7 papers derive the full 7 = 1 + 1 + 3 + 2 decomposition, the inheritance is clean. If they derive only the K=7 count without the full decomposition, the §8 chain inherits the spatial-generator-count step as an additional structural assumption.

8.2 d=3 → Cl(3) → SU(2) Weyl spinors

With d=3 spatial transport generators inherited from K=7, the spatial Clifford algebra is Cl(3), generated by three matrices γ^i (i = 1, 2, 3) satisfying $\{\gamma^i, \gamma^j\} = -2\delta^{ij} \cdot \mathbb{1}$ (spatial signature convention, matching the Cl(1,3) restriction). The minimal faithful complex representation of Cl(3) is on the carrier space $\mathbb{C}^2$ — 2-component Weyl spinors — realised concretely by Pauli matrices (up to factors of i: $\gamma^i = i\sigma^i$ in one common convention). The Cl(3) action on $\mathbb{C}^2$ carries:

- a representation of the spin group Spin(3) ≅ SU(2),
- 2-component Weyl spinors as the *fundamental (2-dimensional) representation* of SU(2),
- the standard SU(2) action on Weyl spinors as the spinorial transport structure.

8.3 1+3 → Cl(1,3) → 4-component Dirac spinors

Adding the temporal generator γ^0 to the spatial Cl(3) produces the Lorentzian Clifford algebra Cl(1,3) with covariant relations $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \cdot \mathbb{1}$. The minimal faithful complex representation of Cl(1,3) is on the carrier space $\mathbb{C}^4$ — 4-component Dirac spinors — a doubling of the spatial Cl(3) representation. Cl(1,3) representation carries:

- a representation of the spin group Spin(1, 3) ≅ SL(2, $\mathbb{C}$),
- 4-component Dirac spinors as the carrier space $\mathbb{C}^4$,
- the chirality decomposition Dirac = (Weyl_L ⊕ Weyl_R) via γ^5 projection (§6.5),
- the Lorentz action on Dirac spinors as the relativistic spinorial transport structure.

8.4 The full chain

The dimensional chain reads:

**K=7 → 3 spatial transport generators → Cl(3) → SU(2) ≅ Spin(3) → 2-component Weyl spinors ($\mathbb{C}^2$)
→ +1 temporal generator → Cl(1,3) → SL(2, $\mathbb{C}$) ≅ Spin(1, 3) → 4-component Dirac spinors ($\mathbb{C}^4$).**

This establishes that the spinor representation dimensionality is *not freely chosen*. It is forced by:

1. The K=7 architecture supplying d=3 spatial transport generators (§8.1, inherited).
2. The Clifford forcing of §6 requiring admissible first-order flow to take Clifford-algebraic form.
3. The 1+3 spacetime decomposition extending Cl(3) to Cl(1,3).

The 4-component spinor structure observed at the level of Dirac fields acquires a *substrate-level structural origin*: it is the minimal admissible representation of the Clifford algebra arising from the K=7 spatial-generator count.

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9. From Discrete Oriented Loops to Continuum Spinorial Transport

9.1 The spinorial extension of the microscopic current

The Microscopic Origin paper's microscopic transport current is unchanged in the spinorial extension. The extension adds an *orientation-and-Clifford-tagged transport structure* alongside the current:

$$\mathcal{T}_{micro}(x) = \Sigma_i \, q_i \int d\tau \, u_i^\mu(\tau) \, \psi_i(\tau) \otimes F_i(x_i(\tau)) \, \delta^{(4)}(x - x_i(\tau)),$$

with $\psi_i(\tau) \in \mathbb{C}^4$ the Clifford-internal spinor state of loop i (an element of the minimal Cl(1,3) representation) carried along the worldline. The full microscopic transport $\mathcal{T}_{micro}$ therefore carries indices in three sectors: the spacetime 4-vector index from u_i^μ, the spinor indices from ψ_i, and the frame indices from F_i.

The current J^μ is recovered from $\mathcal{T}_{micro}$ by tracing out the spinor structure and the frame indices, leaving only the spacetime 4-vector index. Concretely, the spinor sector contributes $\bar{\psi}\gamma^\mu\psi$ (the Dirac current 4-vector, with the γ^μ index matching the kept spacetime index), and the orientation-frame sector contributes a scalar weight at leading order (the frame indices are traced out, leaving an SO(3)-invariant scalar). Combining:

$$J^\mu(x) = \langle \bar{\psi}(x) \, \gamma^\mu \, \psi(x) \rangle_{local} \cdot w_F(x),$$

with $\langle \cdot \rangle_{local}$ the ensemble average over the persistence-filtered substrate population in the local Eckart rest frame and $w_F(x)$ the scalar frame-weight from tracing out F. The structure decomposes as: scalar four-current J^μ (Microscopic Origin) + orientation frame structure (§5) + Clifford internal structure (§6), with J^μ recovered by tracing out the latter two.

9.2 Spinorial hydrodynamic limit

Theorem 6 (Hydrodynamic limit with spinor and orientation). Under the separation-of-scales conditions (H1)–(H2) of Microscopic Origin Theorem 3, extended by (H3) local orientation coherence (well-defined local mean orientation frame F(x)) and (H4) local spinor coherence (Clifford-covariant ensemble average in the local Eckart rest frame), the spinor-orientation-tagged microscopic transport coarse-grains to

$$\mathcal{T}(x) = \rho_{pers}(x) \, u^\mu(x) \otimes \psi(x) \otimes F(x),$$

with $\psi(x) \in \mathbb{C}^4$ the coarse-grained spinor field and F(x) the coarse-grained orientation frame.

Argument. (H1) and (H2) inherited from Microscopic Origin Theorem 3 establish the dust-fluid coarse-graining of the scalar four-current. (H3) supplies a well-defined local mean orientation frame as the persistence-filtered ensemble average. (H4) supplies a well-defined local mean spinor field as the Clifford-covariant ensemble average in the local Eckart rest frame. Combining the four hydrodynamic conditions, the orientation-and-spinor-tagged microscopic transport coarse-grains to the tensor-product structure stated. ■

9.3 The spinorial conservation chain

The conservation chain inherited from Microscopic Origin propagates through the spinorial extension unchanged at the level of the underlying current. Additional bundle-level conserved quantities arise from the spinor and frame structure:

- spinor current conservation: $\partial_\mu(\bar{\psi}\gamma^\mu \psi) = 0$ under $D\psi = 0$ (Dirac-equation conservation; conditional on the Dirac-emergence theorem of §22 item 1),
- frame transport invariants on B(P) (Theorem 3).

The leading-order source-current conservation $\partial_\mu J^\mu = 0$ is unchanged; bundle-level invariants are *additional* substrate-level structure.

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10. The Double-Cover Transport Group — *The Geometric Route*

This section establishes the geometric route to spinorial structure: SU(2) emergence from SO(3) double-cover topology on the candidate closure-frame bundle, conditional on three structural conjectures C1–C3. With the algebraic route of §6 supplying support for spinor *structure*, the geometric route's role is to supply the *substrate-level transport carriers* with their *sector decomposition* into spinorial and trivial-holonomy components — a structural feature the algebraic route alone does not provide.

10.1 The three structural conjectures

Structural Conjecture C1 (Closure-frame structure group is SO(3)). The rotational degeneracy of the closure-orientation frame F(x) inherited from the triangular orientation programme is precisely SO(3). *Plausibility.* Three-dimensional orthogonal closure with orientation-preservation, consistent with §8's K=7 → d=3 decomposition. Discharge: §22 item 7.

Structural Conjecture C2 (Refinement-stable transport requires single-valued lifts). Admissible refinement-stable transport on B(P) requires single-valued representation of parallel transport around closed curves. *Plausibility.* Refinement-stability of transport observables requires well-defined transport outcomes; multi-valued representations fail this. Discharge: §22 item 8.

Significance of C2 — calibrated. The §11 geometric–algebraic convergence partially reduces the load on C2, but the reduction is at the level of *group structure* not *sector decomposition*. Specifically:

- *Group structure.* The algebraic Clifford forcing of §6 produces Spin(3) ≅ SU(2) acting on the minimal Cl(3) representation *without* requiring C2. The existence of SU(2) as the relevant group is independently established by Theorem 1 and the standard Spin(3) ≅ SU(2) identification.
- *Sector decomposition.* The decomposition of persistent oriented loops into trivial-holonomy ($U(2\pi) = +\mathbb{1}$) and spinorial ($U(2\pi) = -\mathbb{1}$) sectors is *intrinsically geometric*. It requires the SU(2) lift of bundle transport on B(P), which depends on C2 (single-valuedness) to force the lift in the first place. The algebraic route gives universal Cl(1,3)-internal structure on *every* first-order admissible flow operator; the geometric route is what distinguishes loops that *carry* spinorial transport behaviour at the holonomy level (i.e., that exhibit $U(2\pi) = -\mathbb{1}$) from loops that do not.

If C2 fails — that is, if multi-valued bundle transport is admissible after all — the *structural identification* of the spinorial sector remains valid algebraically (every first-order admissible operator still has Clifford structure on $\mathbb{C}^4$), but the *physical identification* of which loops correspond to fermionic source-carriers via the $U(2\pi) = -\mathbb{1}$ transport signature breaks down. The bridge between substrate-level Clifford structure and observed fermion species (Channel C of §18) relies on the geometric sector decomposition for its observational content. So C2 carries reduced but non-trivial structural weight.

Structural Conjecture C3 (Persistent sector selects the connected double cover). Among covering spaces of SO(3), the persistent sector selects the unique connected double cover. *Plausibility.* The persistent sector is connected by the structure of the refinement-stable limit; SU(2) is the unique connected covering space of SO(3) other than SO(3) itself. Discharge: §22 item 9.

10.2 Theorem 3 — Closure-frame transport holonomy

Theorem 3 (Closure-frame transport holonomy). Transport of oriented closure frames around non-contractible admissible loops in P generates nontrivial geometric holonomy on the candidate closure-frame bundle B(P), under the candidate bundle construction of §5.

Proof. Non-trivial base topology ($\beta_1 \geq 1$) plus generic non-flatness of the closure-orientation connection ω_μ gives nontrivial holonomy for non-contractible $\gamma \subset P$. ■

10.3 Why SO(3) transport alone is insufficient

If admissible transport were SO(3) without double-cover lift, then $U(2\pi) = \mathbb{1}$ in SO(3) and no spinorial sign reversal would occur. Spinorial behaviour requires $U(2\pi) \neq \mathbb{1}$ and $U(4\pi) = \mathbb{1}$, forcing the lift to SU(2).

The algebraic Clifford forcing of §6 supplies *independent* support for the SU(2) group: the minimal Cl(3) representation carries the SU(2) action, not the SO(3) action. The geometric and algebraic routes agree on the SU(2) target.

10.4 Structural Inference SI — Double-cover transport group is SU(2)

Structural Inference SI (Double-cover transport group, geometric route). *Conditional on C1–C3.* The admissible transport group on the persistent oriented sector is SU(2).

Argument. By C1 the structure group is SO(3) with $\pi_1(SO(3)) = \mathbb{Z}_2$. By C2 admissible transport requires single-valued lifts. By C3 the persistent sector selects the connected double cover. The unique connected double cover of SO(3) is SU(2). $U(2\pi) = -\mathbb{1}$ and $U(4\pi) = +\mathbb{1}$ follow from the lift structure. ■

10.5 Theorem 4 — Spinorial persistence-sector decomposition

Theorem 4 (Spinorial persistence-sector decomposition). *Conditional on C1–C3.* Persistent oriented commitment loops decompose into trivial-holonomy and spinorial sectors. Spinorial sectors exhibit $U(2\pi) = -\mathbb{1}$ and $U(4\pi) = +\mathbb{1}$.

Argument. Structural Inference SI of §10.4 lifts admissible transport to the SU(2) double cover under C1–C3. Loops in P decompose by their image under the $U(2\pi)$ holonomy of the lift: loops with trivial $U(2\pi) = +\mathbb{1}$ form the trivial-holonomy sector; loops with non-trivial $U(2\pi) = -\mathbb{1}$ form the spinorial sector. The signature $U(4\pi) = U(2\pi)^2 = +\mathbb{1}$ follows. The decomposition is well-defined on persistent loops because admissible refinement preserves the $U(2\pi)$ class (single-valuedness of the lift under C2 guarantees that the holonomy class is refinement-stable). ■

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11. Convergence of the Algebraic and Geometric Routes

This section establishes that the algebraic Clifford forcing of §6 and the geometric SU(2) lift of §10 target the *same group structure* — the spin group $Spin(3) \cong SU(2)$ on its fundamental representation. The convergence is at the level of group structure (both routes target SU(2) on $\mathbb{C}^2$) but *not* at the level of sector decomposition (which remains geometric-only).

11.1 The algebraic SU(2): Spin(3) on the Cl(3) representation

The Clifford algebra Cl(3) is the algebra generated by three γ^i ($i = 1, 2, 3$) with $\{\gamma^i, \gamma^j\} = -2\delta^{ij} \cdot \mathbb{1}$. Its minimal faithful complex representation acts on the carrier space $\mathbb{C}^2$, realised by 2×2 matrices in $M(2, \mathbb{C})$ (the algebra of 2×2 complex matrices). To be precise about the distinction between algebra and carrier space: M(2, $\mathbb{C}$) is the *algebra* of 2 × 2 complex matrices, while $\mathbb{C}^2$ is its *standard carrier* (2-dimensional column-vector space). The Cl(3) representation in M(2, $\mathbb{C}$) acts on $\mathbb{C}^2$ by matrix-vector multiplication.

The *spin group* Spin(3) is the unique connected double cover of SO(3), realised concretely as the unit-norm subgroup of the even subalgebra $Cl_0(3)$ under Clifford multiplication. Standard Clifford representation theory establishes:

Spin(3) ≅ SU(2),

as a Lie group isomorphism. The action of Spin(3) on the Cl(3) carrier space $\mathbb{C}^2$ is the *fundamental (2-dimensional) representation* of SU(2), realised concretely as the standard SU(2) action on 2-component Weyl spinors.

The algebraic Clifford forcing of §6 therefore produces SU(2) as the structure group acting on $\mathbb{C}^2$ via its fundamental representation. This SU(2) is the *spin group* of 3-space, arising algebraically from the Clifford anticommutation relations.

11.2 The geometric SU(2): closure-frame bundle lift

The geometric SU(2) of Structural Inference SI (§10.4) arises from the SO(3) structure group of the closure-frame bundle B(P) lifted to its connected double cover. The unique connected double cover of SO(3) is SU(2) (the universal cover; $\pi_1(SO(3)) = \mathbb{Z}_2$). The geometric SU(2) is therefore Spin(3) ≅ SU(2) — the same group, by uniqueness, as the algebraic SU(2) of §11.1.

The lift acts on an *associated spinor bundle* S(P) whose typical fibre carries the *minimal* faithful representation of SU(2). The minimal spinor lift of an SO(3)-structured frame bundle is by the *fundamental (2-dimensional) representation of SU(2)* — i.e., by 2-component Weyl spinors on $\mathbb{C}^2$. Higher-dimensional representations of SU(2) (the adjoint (3-dim), the (4-dim) representation, etc.) would produce non-minimal lifts; minimality selects the smallest faithful lift, which is the fundamental 2D representation.

Note on the minimality principle here. This minimality principle is structurally akin to §7's exclusion of higher-dimensional representations but is not literally the same principle. §7's minimality is about *substrate representation order* — selecting the smallest internal representation supplying the required dynamics among scalar, vector, spinor, rank-2 tensor, etc. §11.2's minimality is about *representation choice within a fixed group* — given that SU(2) is the structure group, selecting the fundamental 2D representation over higher irreducibles (adjoint 3D, the 4D representation, ...) as the carrier of the spinor lift. Both are natural structural principles within the VERSF programme and share the common flavour "smallest faithful representation supplying the required structure"; they are not identical but they cohere as members of a broader minimality discipline.

11.3 Theorem 2 — Geometric–algebraic convergence at the group-structure level

Theorem 2 (Geometric–algebraic convergence on the fundamental SU(2) representation). *Conditional on (a) inheritance from κ-field uniqueness Theorem U and Schrödinger→Dirac Theorems E and F (algebraic side, via Theorem 1), and (b) the three structural conjectures C1–C3 (geometric side, via Structural Inference SI).* Under these joint conditions, the geometric SU(2) of Structural Inference SI (§10.4) and the algebraic SU(2) ≅ Spin(3) acting on the minimal Cl(3) representation (§11.1) are *isomorphic as Lie groups* and *act in the same way on the same representation*: specifically, both act on the 2-component Weyl spinor space $\mathbb{C}^2$ as the fundamental (2-dimensional) representation of SU(2).

The abstract group-theoretic fact underlying the convergence — that the connected double cover of SO(3) is unique up to isomorphism and is SU(2), and that the minimal faithful representation of this group is the fundamental 2D representation — is *unconditional*: it follows from standard topology and Clifford / Lie-group representation theory. What is conditional is that both routes are *applicable* in the present construction: Theorem 1 requires the algebraic inheritances, and SI requires the geometric conjectures. Theorem 2's content is that *when both routes apply, they target the same structure*; it does not by itself discharge either set of conditions.

Proof. By §11.1, the algebraic SU(2) is the spin group Spin(3), acting on $\mathbb{C}^2$ via the fundamental representation by standard Clifford representation theory.

By §11.2, the geometric SU(2) is the connected double cover of SO(3), forced by Structural Inference SI (conditional on C1–C3). By the topological characterisation of Spin(n) as the unique connected double cover of SO(n) for $n \geq 3$, the geometric SU(2) is Spin(3) ≅ SU(2). The minimal spinor lift acts on $\mathbb{C}^2$ via the fundamental representation, by minimality of the lift.

The two groups are isomorphic by uniqueness of Spin(3) as the connected double cover of SO(3): any construction producing the connected double cover produces (up to isomorphism) the same group. The two actions are on the *same fundamental representation* of SU(2): both are the standard SU(2) action on 2-component Weyl spinors on $\mathbb{C}^2$. ■

Structural reading. Theorem 2 makes the convergence explicit *at the group-structure level on the fundamental representation*. The two routes target the same SU(2) on $\mathbb{C}^2$ because (i) the spin group of 3-space is unique up to isomorphism, and (ii) the fundamental representation is the minimal faithful representation of SU(2), selected on both sides by minimality.

Important calibration: convergence is at the level of group structure, not sector decomposition. Theorem 2 establishes that the two routes target the same SU(2) acting on the same Weyl-spinor representation. It does *not* establish that the two routes produce the same *sector decomposition* of persistent oriented loops. Specifically:

- The *algebraic route* (§6) gives universal Clifford structure on *every* first-order admissible flow operator. Every such operator carries the Cl(1,3) algebra and acts on the 4-component Dirac representation $\mathbb{C}^4$. There is no algebraic decomposition into "spinorial vs non-spinorial" sectors at the operator level — every admissible first-order flow is spinorial in the algebraic sense.

- The *geometric route* (§10) gives a *two-sector decomposition* of persistent oriented loops into trivial-holonomy ($U(2\pi) = +\mathbb{1}$) and spinorial ($U(2\pi) = -\mathbb{1}$) sectors. This decomposition is intrinsically a *bundle-level* feature, requiring the SU(2) lift of frame transport on B(P), and depends on C1–C3.

The convergence on group structure means: when a loop *is* in the spinorial sector (by the geometric decomposition), the SU(2) acting on its lifted frame is the same SU(2) as the algebraic SU(2) on the spinor's Clifford-internal structure. But the *identification* of which loops are in the spinorial sector — the bridge to physical fermion species — remains a geometric question, requiring C1–C3.

This means the §18 falsifiability discussion of C2's reduced significance must be calibrated: if C2 fails, the structural existence of Clifford spinor structure on first-order operators (algebraic, universal) survives, but the bundle-level sector decomposition (geometric, identifying which substrate loops carry the fermionic transport signature) breaks down.

11.4 The synthesis: algebraic structure + geometric sector decomposition

The geometric–algebraic convergence supplies the synthesis statement that is the structural backbone of the present construction:

Spinoriality is algebraically forced by first-order closure flow and geometrically realised through closure transport holonomy. The two routes converge on the same Spin(3) ≅ SU(2) group structure on the fundamental representation. The sector decomposition into spinorial and trivial-holonomy components is geometric-only.

Each route contributes:

- **Algebraic route (§6, Theorem 1):** Universal Clifford structure on every first-order admissible flow operator, conditional on inheritance from κ-field uniqueness Theorem U and Schrödinger→Dirac Theorems E and F. The spinorial structure is structurally inevitable given the inherited results; no C1–C3-type structural conjectures of the geometric route enter.
- **Geometric route (§10, Structural Inference SI):** Conditional substrate-level realisation. Under C1–C3, the candidate closure-frame bundle B(P) lifts to SU(2) transport, supplying the *substrate-level transport carriers* and the *two-sector decomposition* that bridges to physical fermion identification.
- **Convergence (§11, Theorem 2):** Identity of the targets at the group-structure level. Both routes produce SU(2) ≅ Spin(3) acting on the same fundamental representation $\mathbb{C}^2$. The convergence follows from uniqueness of Spin(3) plus minimality of the fundamental representation.

11.5 What the convergence does and does not establish

Does establish:

- That the *group structure* produced by algebraic forcing (universal Clifford action of SU(2) on $\mathbb{C}^2$) and by geometric lift (SU(2) double cover of SO(3) on $\mathbb{C}^2$) is the *same group on the same fundamental representation*.
- That the algebraic route supplies independent (inheritance-conditional, not C1–C3-conditional) support for the *existence* of spinorial structure.
- That the geometric route supplies the substrate-level *sector decomposition* into spinorial and trivial-holonomy components.
- That the spinorial structure is structurally inevitable given inherited results.

Does not establish:

- The Dirac equation as a substrate evolution equation (§22 item 1).
- Canonical anticommutation relations (§22 item 4).
- The spin-statistics theorem (§22 item 5).
- The assignment of spinorial sectors to observed fermion species (§22 item 2).
- Discharge of structural conjectures C1–C3 (§22 items 7–9).
- That the algebraic and geometric routes produce the same *sector decomposition* — they don't; algebraic gives universal structure, geometric gives the bundle-level two-sector decomposition.

The convergence is a structural cross-check on the spinorial-*structure* conclusion, not on the spinorial-*sector* conclusion or on the full fermionic field theory.

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12. Coupling to the Persistent Gauge Sector

12.1 Leading-order coupling unchanged

By inheritance from Matter Coupling Theorem 1 and Microscopic Origin §7.1, the leading-order admissible interaction is $\mathscr{L}_{int} = -J^\mu A_\mu$. The spinorial extension does not modify the leading-order coupling structure.

12.2 Spinorial coupling at the next admissibility order

At the next admissibility order, the spinor field ψ couples to A_μ through the spinorial current $\bar{\psi}\gamma^\mu\psi$:

$$\mathscr{L}_{int}^{(1)} = -e\,(\bar{\psi}\,\gamma^\mu\,\psi)\,A_\mu$$

with e the spinorial coupling constant (to be identified with the elementary electric charge through species decomposition, §22 item 2). This is the standard QED matter coupling.

12.3 The full electromagnetic-spinorial coupled action

The full coupled action at admissible order, using the *standard Dirac operator* forced by Theorem 1 via the 3+1 Hamiltonian-form derivation, takes the form

$$S[A, \psi] = \int (-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - J^\mu A_\mu + \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi - e \bar{\psi}\gamma^\mu\psi A_\mu) d^4x.$$

The spinor sector is the *standard Dirac Lagrangian* $\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$ with non-chiral mass term — the operator algebraically forced by the 3+1 Hamiltonian-form Theorem 1 of §6.3, after the standard Hamiltonian-to-covariant transition. (The covariant chiral-mass form $\Sigma = \gamma^\mu p_\mu + \gamma^5 m$ discussed in §6.7's sidebar is a structurally distinct operator interpreted as a possible substrate-level pre-electroweak organisation; the effective action presented here uses the standard Dirac form inherited from Theorem 1, with §22 item 12 deferring the substrate-to-electroweak bridge to future work.)

The variational structure reproduces (conditional on the Dirac-emergence theorem of §22 item 1) the standard QED action *as an effective theory at admissibility order on the persistent sector*, not as a derived substrate dynamics.

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13. Multi-Loop Transport and Exchange Structure (Provisional)

13.1 The exchange-statistics question

Spinorial transport of single loops (§10) is one of the two structural signatures of fermionic matter. The other is antisymmetric exchange statistics. The substrate-level analogue of the spin-statistics theorem remains open.

13.2 Proposition 6 — Antisymmetric exchange (provisional)

Proposition 6 (Antisymmetric exchange — provisional). Multi-loop transport on the spinorial sector admits an antisymmetric exchange structure: $\Psi(\mathcal{C}_2, \mathcal{C}_1) = -\Psi(\mathcal{C}_1, \mathcal{C}_2)$ for odd exchange parity.

Status. Conjectural pending (i) a candidate exchange-transport construction and (ii) a configuration-space topology argument paralleling Leinaas–Myrheim and Wilczek. Validation is §22 item 3.

13.3 Candidate exchange-transport structure

Configuration space $C_n(P)$ of n distinct oriented loops admits a fibration $C_n(P) \to P^n / S_n$ with fibre the n-fold product of orientation-frame and spinor bundles modulo permutations. Multi-loop transport states are sections of an associated vector bundle whose fibres are tensor products of single-loop Cl(1,3) representations. Under interchange, configuration-space exchange combines with spinor-frame interchange to produce the conjectured −1 sign on spinorial sectors.

13.4 What §13 does not establish

Spin-statistics theorem (§22 item 5), canonical anticommutation relations (§22 item 4), Dirac operator as substrate evolution equation (§22 item 1). Proposition 6 supplies a structural framework, not a derivation.

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14. Relationship to the Unique Commitment Field

This section makes explicit the sectoral relationship between the spinorial source-carrier programme and the scalar κ-field uniqueness programme.

Sector	Propagation order	Unique admissible dynamics	Carrier structure
Scalar	Second-order	$(\Box + m^2)\, s = \rho\_committed$ (κ-field Theorem U)	Scalar commitment field s
Spinorial	First-order	$(i\gamma^\mu \partial_\mu - m)\, \psi = 0$ (target, §22 item 1)	4-component Dirac spinor ψ on Cl(1,3)

The two sectors are *complementary admissible sectors* of the persistent transport manifold:

- The scalar sector is the *unique second-order* admissible propagation structure (κ-field Theorem U).
- The spinorial sector is the *first-order* propagation structure with the standard Dirac form (algebraically forced by Theorem 1 via the 3+1 Hamiltonian-form derivation).
- The two sectors are related by $H^2 = (p^2 + m^2) \cdot \mathbb{1}$ (or equivalently $\bar{D}D = -(\Box + m^2) \cdot \mathbb{1}$ covariantly): the square of the first-order spinorial Hamiltonian reproduces the second-order scalar KG dynamics on each spinor component.

14.1 Structural reading

The scalar sector is unique by κ-field uniqueness; the spinorial sector is unique (Clifford forcing, Theorem 1) and minimal (Theorem 5). Higher-order propagation is excluded by lowest-order admissibility (Matter Coupling (P5)); higher-dimensional internal representations are excluded by minimality. The two sectors are therefore the complete leading-order admissible propagation structure on the persistent sector.

14.2 Consistency between sectors

The KG² consistency requirement of §6.2 ties the sectors together. The spinorial sector cannot exist independently of the scalar sector: its first-order dynamics is *defined* by the requirement that its square equal the unique scalar second-order dynamics. The scalar sector grounds the spinorial sector algebraically.

Conversely, the spinorial sector allows the persistent sector to carry first-order admissible dynamics — the scalar sector being intrinsically second-order. The two sectors are mutually constraining.

14.3 Cross-sector cross-check

The mass m appearing in $(\square + m^2)s = \rho_{committed}$ and in $(i\gamma^\mu \partial_\mu - m)\psi = 0$ must be the *same* substrate-derived parameter. This follows from the $H^2 = (p^2 + m^2) \cdot \mathbb{1}$ consistency: the m^2 term in $p^2 + m^2$ comes from the $\beta^2 \cdot m^2$ term in H^2, so the mass parameters are identified by construction ($\beta^2 = \mathbb{1}$ in the Hamiltonian-form derivation, with the covariant repackaging preserving this identification through γ^0).

Structural prediction: spinorial source-carriers and scalar commitment fields with the same substrate mass parameter share the same physical mass. The species-decomposition deliverable (§22 item 2) must respect this constraint.

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15. Fold-Origin of Spinorial Source-Carriers (Ontological Integration)

The operational construction of §§5–13 introduced oriented commitment loops carrying Clifford-internal spinor structure, with spinorial transport emerging algebraically (§6) and geometrically (§10) with convergence on the same group structure (§11). This section provides the ontological bridge to the Fold programme.

Two layers of Fold integration. As before: ontological alignment (§15) and technical coincidence (§22 item 9). This section is *interpretive integration*, not new derivation.

15.1 The Fold programme with (σ, ω) state-pair structure

The Fold programme as consolidated in *Fold* v31 develops fold states as (σ, ω) state pairs at the fold level, with σ representing the closure-orientation parity and ω representing the closure-flow parity, generating a $Z_2 \times Z_2$ grading on fold states.

The natural question is whether this fold-level $Z_2 \times Z_2$ has a counterpart at the spinor level, and if so what the pairing is. The answer is structurally subtler than a naive identification would suggest.

The Cl(1,3) Z_2 grading is not independent of chirality. The Clifford algebra Cl(1,3) carries a natural Z_2 grading separating Cl_{even} (generated by even products of γ^μ: $\mathbb{1}$, $\gamma^\mu\gamma^\nu$, γ^5) from Cl_{odd} (generated by odd products: γ^μ, $\gamma^\mu\gamma^\nu\gamma^\rho$). The chirality operator $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ is a product of four γ^μ, hence $\gamma^5 \in Cl\_even$. The chirality projectors $P\_\pm = (\mathbb{1} \pm \gamma^5)/2$ are therefore both even-graded. As a result, the chirality grading and the Cl_even/Cl_odd grading are *correlated*, not independent: they do not form an honest $Z_2 \times Z_2$ on the spinor side.

A natural candidate pairing would be σ ↔ chirality (via γ^5) and ω ↔ Cl_even/Cl_odd. The σ ↔ chirality identification is structurally natural and is retained below. The ω ↔ Cl_even/Cl_odd identification is *not* admissible, because the Cl-grading is not independent of chirality ($\gamma^5 \in$ Cl_even) and therefore cannot supply the second factor of a genuine $Z_2 \times Z_2$ at the spinor level.

Natural candidates for the second Z_2 at the spinor level. The fermionic-theory Z_2's actually independent of chirality are:

- **Parity P** (spatial inversion), which acts on Dirac spinors via $\psi(x^0, x) \to \gamma^0\psi(x^0, -x)$. Parity commutes with the Lorentz subgroup but is independent of chirality (it interchanges left-handed and right-handed components rather than projecting onto them).
- **Charge conjugation C**, which acts via $\psi \to C\,\bar{\psi}^T$ for a charge-conjugation matrix C and is independent of chirality (it interchanges particle and antiparticle sectors).
- The combined CPT discrete symmetries form a richer grading structure on fermionic theory.

The right physical counterpart of the fold-level ω-parity depends on the *physical content* of ω in Fold v31. If ω-parity represents a discrete spatial-inversion-like structure in fold transport, the natural pairing is ω ↔ P. If it represents a particle/antiparticle-like structure (e.g., distinguishing forward-time-committed folds from backward-time-committed folds, or distinguishing fold-creation from fold-annihilation events), the natural pairing is ω ↔ C. Without the explicit Fold v31 definitions of ω, both candidates remain on the table.

Conclusion and verification item. The σ ↔ chirality pairing is retained as the structurally natural identification of orientation parity. The second Z_2 at the spinor level — paired with ω — is left as ω ↔ {P or C, subject to Fold v31 verification of ω's physical content}. The verification is recorded as §22 item 9.

15.2 The σ ↔ chirality pairing

Under the σ ↔ chirality pairing motivated by §15.1 (orientation parity at fold level corresponds naturally to chirality grading at spinor level, both having the character of rotation-like parities):

- $\sigma = +$ corresponds to right-handed Weyl spinor sector $P\_+ \psi$.
- $\sigma = -$ corresponds to left-handed Weyl spinor sector $P\_- \psi$.

The chirality projection $(\mathbb{1} \pm \gamma^5)/2$ is the substrate-level reading of the fold-level σ-parity projection. The handedness of fermionic matter (left-handed neutrinos, etc.) acquires a Fold-theoretic origin in the σ-parity of fold commitment, conditional on σ being correctly identified with chirality rather than with the alternative second Z_2 at the spinor level (parity P or charge conjugation C).

15.3 The ω-pairing (placeholder pending Fold v31 verification)

The pairing of ω-parity (closure-flow parity) with a spinor-level Z_2 is left as a verification item, since (per §15.1) the previously-proposed ω ↔ Cl_even/Cl_odd identification is not correct (the Cl-grading is not independent of chirality, so it cannot supply an honest second Z_2).

The natural candidates for the second Z_2 at the spinor level — independent of chirality — are parity P and charge conjugation C. The pairing then depends on the physical content of ω in Fold v31:

- *If ω is a spatial-inversion-like parity at the fold level* (e.g., distinguishing forward-spatial-direction closure flow from backward-spatial-direction closure flow under spatial inversion): the natural pairing is ω ↔ P, with ω-parity propagating under refinement coarsening to spinor-level parity acting via $\gamma^0\psi(x^0, -x)$.
- *If ω is a particle/antiparticle-like parity at the fold level* (e.g., distinguishing fold-creation from fold-annihilation events, or forward-time-committed from backward-time-committed folds): the natural pairing is ω ↔ C, with ω-parity propagating to spinor-level charge conjugation acting via $\psi \to C\bar{\psi}$^T.
- *If ω is yet another structure not directly identifiable with either P or C*: the pairing requires alternative discrete symmetry candidates — for example, time reversal T (the third independent Z_2 of the standard Dirac-field discrete-symmetry group ⟨C, P, T⟩), or a substrate-level discrete structure not represented in ⟨C, P, T⟩.

Without the explicit definitions and physical interpretation of ω in Fold v31, no pairing can be committed to here. The verification is recorded as §22 item 9: *check the explicit (σ, ω) definitions of Fold v31 to identify the correct spinor-level pairing for ω*. This is a substantive Fold-integration deliverable, not a presentational matter; until it is discharged, the §15 Fold integration supplies σ ↔ chirality cleanly and leaves the ω pairing open.

15.4 Interface transport between fold states → spinor transport

The Fold v31 programme develops interface transport between (σ, ω) fold states. At the substrate level, this propagates to:

- transport on the orientation-frame bundle B(P) (geometric route, §10);
- transport on the associated spinor bundle S(P) (algebraic route, §6).

The two are connected by Theorem 2 at the group-structure level. The Fold interface transport supplies the substrate-level dynamical mechanism behind both, modulo the σ↔chirality pairing (committed) and the ω↔{P, C, …} pairing (pending §22 item 9 verification).

15.5 Loop holonomy with closure parity → spinorial winding sectors

Fold v31's loop holonomy with closure parity propagates to:

- Trivial-holonomy sectors (Theorem 4) ↔ loops with even closure-parity holonomy at fold level.
- Spinorial sectors (Theorem 4) ↔ loops with odd closure-parity holonomy at fold level.

The Theorem 4 two-sector decomposition therefore acquires a fold-level origin.

15.6 The unified ontological chain

Void → Fold (with (σ, ω) state pairs and $Z_2 \times Z_2$ grading)
→ Irreversible commitment (with orientation and closure parity)
→ Closure topology + closure orientation + parity
→ Persistent oriented loops with Clifford-internal structure
→ J^μ + spinorial transport + chirality (from σ) + second Z_2 pairing (from ω, pending verification)
→ Gauge transport + spinorial source-carriers (with chirality and exchange structure).

The chain is committed at the σ ↔ chirality step; the ω-pairing step remains pending §22 item 9 verification against Fold v31. The structural integrity of the chain does not depend on the specific ω-pairing; what depends on it is the *physical identification* of the second spinor-level Z_2 (parity, charge conjugation, or other) with a fold-level structural element.

15.7 Status and scope

Interpretive integration, not new derivation. §15 is not an independent proof; algebraic forcing of §6 supplies that independently.

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16. Connection to the Master-Action Programme

The Fold v31 programme develops a master-action framework with RG flow, universality classes, closure kernels, and coarse-graining structures.

16.1 Spinorial sector as first-order fermionic sector

The spinorial transport sector should ultimately emerge as the first-order fermionic sector of the VERSF master action under coarse-grained closure flow.

Conjectured master-action structure:

- Scalar sector $S_{scalar} = \int[\tfrac{1}{2} \partial_\mu s \, \partial^\mu s - \tfrac{1}{2}m^2 s^2 + \text{interactions}]\, d^4x$ (KG sector).
- Spinorial sector $S_{spinor} = \int[\bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi + \text{interactions}]\, d^4x$ (Dirac sector forced by Theorem 1).
- Gauge sector $S_{gauge} = \int[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \text{interactions}]\, d^4x$ (Maxwell sector).

16.2 Universality classes and species decomposition

Under closure-RG flow, distinct loop topology classes converge to distinct universality classes corresponding to observed particle species. Fermionic species (leptons, quarks) ↔ specific spinorial universality classes.

16.3 Status

Conjectural. The §16 master-action connection is a structural alignment, not a derivational support. The master-action programme is in development. The master-action explicit construction (§22 item 10) is a *programmatic target* rather than a next-paper deliverable in the literal sense.

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17. Structural Consequences

17.1 Spinorial structure is algebraically forced. *Derived (conditional on inheritance from κ-field Theorem U and Schrödinger→Dirac Theorems E, F).* By Theorem 1, first-order admissible closure flow forces Clifford-algebraic structure with minimal 4-component spinor representation. No C1–C3 conjectures enter.

17.2 Spinorial transport is geometrically realised through closure-frame holonomy. *Conditional on C1–C3.* Structural Inference SI supplies the geometric realisation; Theorem 4 supplies the two-sector decomposition.

17.3 Spinoriality is uniquely the minimal admissible structure for first-order flow. *Derived.* By Theorem 5: scalar impossible (§7.1), vector representationally obstructed (§7.2), rank-2 tensor excluded by minimality (§7.3).

17.4 The geometric and algebraic routes converge on the same Spin(3) ≅ SU(2) at the group-structure level. *Derived.* By Theorem 2.

17.5 Half-integer spin is the substrate-level signature of double-cover transport / Clifford representation. *Derived (algebraically, group-structure) + conditional on C1–C3 (geometrically, sector identification).*

17.6 Chirality arises from fold-level σ-parity; the second fold-level Z_2 pairing is pending Fold v31 verification. *Conditional on σ↔chirality (§15.2); ω-pairing open between parity P and charge conjugation C pending Fold v31 verification (§15.3); naive ω↔Cl_even/Cl_odd inadmissible because the Cl-grading is not independent of chirality.*

17.7 Stable fermionic matter requires spinorial topology. *Conditional on C1–C3 for sector identification.*

17.8 Antisymmetric exchange is a candidate consequence. *Provisional.*

17.9 The K=7 architecture forces 4-component spinor representations. *Conditional on K=7 architecture inheritance supplying d=3 spatial generators.* Via the K=7 → d=3 → Cl(1,3) chain of §8.

17.10 The covariant chiral-mass form vs. standard Dirac form — different derivational routes, potentially different physical stages. *Derived (algebraic observation); interpretive framing as substrate-level pre-electroweak organisation; substrate-to-electroweak bridge open.* Direct covariant squaring (§6.7) produces a chiral-mass operator $\gamma^\mu p_\mu + \gamma^5 m$, structurally distinct from the standard Dirac operator produced by the 3+1 Hamiltonian-form derivation of Theorem 1. The two are interpreted as different derivational routes that may correspond to different physical stages: the 3+1 Hamiltonian-form derivation governs the effective low-energy spinorial sector; the chiral-mass structure may reflect a deeper substrate-level pre-electroweak organisation, with electroweak symmetry breaking as the structural bridge. The interpretation is conjectural and is §22 item 12.

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18. Falsifiability Channels

Channel A: Closure-frame bundle structure fails to be SO(3). Tests C1.

Channel B: Admissible transport admits multi-valued representations. Tests C2. *Calibrated significance:* if C2 fails, the *algebraic group structure* survives (every first-order admissible flow operator still carries Cl(1,3) structure on $\mathbb{C}^4$ via Theorem 1), but the *sector decomposition* into spinorial and trivial-holonomy components — and therefore the bridge to physical fermion species identification — breaks down. The structural Clifford framework is preserved; the substrate-to-particle correspondence requires geometric repair.

Channel C: Spinorial sectors do not correspond to observed fermions. Tests the species-decomposition assignment. Depends on the geometric sector decomposition (and therefore on C1–C3).

Channel D: Antisymmetric exchange does not emerge from spinorial transport. Tests Proposition 6.

Channel E: Spinorial source-carriers do not promote to Dirac matter. Tests the Dirac-emergence pathway of §22 item 1.

Channel F: The κ-field uniqueness theorem fails or admits non-KG admissible scalar dynamics. Tests the inheritance underlying Theorem 1.

Channel G: Geometric and algebraic SU(2) fail to be isomorphic on the fundamental representation. Tests Theorem 2.

Channel H: First-order admissible flow does not exist on the persistent sector. Tests the Schrödinger→Dirac Theorem E inheritance. If first-order admissible flow turns out not to be available on the persistent sector, Theorem 1 becomes vacuously true and the algebraic route's content disappears. The geometric route (§10) would survive, but the synthesis statement of §11.4 would degrade to "spinoriality is geometrically realised through closure transport holonomy" without the algebraic-inevitability backbone. Observational signature: any persistence-sector phenomenon requiring first-order admissible dynamics for its description (e.g., quasiparticle dispersion at low energies, finite-mass propagation modes consistent with Dirac form) failing to admit a first-order admissible description.

Channel I: Substrate-to-electroweak bridge fails. The §6.7 sidebar interprets the covariant chiral-mass operator as possibly reflecting a substrate-level pre-electroweak organisation of the persistent sector, with electroweak symmetry breaking as the bridge between this organisation and effective non-chiral mass-eigenstate fermions. If no such substrate-level pre-electroweak organisation exists — or if it exists but does not bridge to the Standard Model's chirality structure through electroweak symmetry breaking — then the §6.7 interpretive framing fails. Observational signature: failure to reconstruct any substrate-level structural precursor to the Standard Model's chirality structure (left-handed-only SU(2)_L, V−A weak interactions, etc.). *Channel I is a long-horizon falsifiability channel rather than a near-term observational test; its content is structural rather than empirical at the present stage of the programme, and operational falsification awaits the substrate-to-electroweak bridge construction itself.* The primary spinorial-structure construction (3+1 Hamiltonian-form Theorem 1) is unaffected by Channel I; what is at stake is the §22 item 12 working conjecture.

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19. What This Paper Achieves, and What It Does Not

19.1 What is achieved (diagnostic)

Seven results:

1. Joint admissibility framework SA + OA + CC.
2. Oriented commitment loops satisfy joint admissibility within current catalogues.
3. Spinorial structure algebraically forced by Theorem 1, conditional only on inheritance from κ-field uniqueness and Schrödinger→Dirac (not on C1–C3).
4. Spinorial structure uniquely minimal by Theorem 5: scalar impossible, vector representationally obstructed, rank-2 tensor excluded by minimality.
5. Algebraic and geometric routes converge on the same Spin(3) ≅ SU(2) on the fundamental representation (Theorem 2). Convergence at group-structure level; sector decomposition remains geometric.
6. Fold integration extended with (σ, ω) substrate ontology and σ ↔ chirality pairing (§15); the second Z_2 pairing remains open pending Fold v31 verification.
7. Covariant chiral-mass observation (§6.7) identifies a working conjecture about substrate-level pre-electroweak organisation as a structural deliverable (§22 item 12). The chiral-mass and standard Dirac forms arise from different derivational routes and may correspond to different physical stages (substrate-level pre-electroweak vs. effective low-energy).

19.2 What is not achieved

The present paper derives the spinorial transport structure, Clifford-compatible internal representations, and geometric–algebraic convergence at the group-structure level required for fermionic source-carriers. It does not derive:

- **The Dirac equation as a substrate evolution equation (§22 item 1).**
- **Canonical anticommutation relations $\{\psi(x), \psi^\dagger(y)\} = \delta(x - y)$ (§22 item 4).**
- **The full spin-statistics theorem (§22 item 5).**
- **Fermionic Fock-space quantisation, path integral measure, renormalised QFT.**
- **The Standard Model fermion spectrum (§22 item 2).**
- **The substrate origin of chirality-distinguishing structure as a pre-electroweak organisation of the persistent sector (§22 item 12, working conjecture).**
- **The Standard Model electroweak mechanism. The present paper does not derive the Standard Model electroweak mechanism or establish that the substrate-level chiral structure of §6.7 corresponds directly to physical electroweak chirality.**

Deliverable table:

Fermionic-Matter Deliverable	Status here
Spinorial transport structure on source-carriers	**Substantially supplied.** Algebraic conditional on inheritance; geometric conditional on C1–C3.
Geometric–algebraic group-structure convergence on Spin(3)	**Established** by Theorem 2.
Minimality of the spinorial sector	**Established** by Theorem 5.
Dirac operator emergence as substrate evolution equation	**Not supplied.** §22 item 1.
Antisymmetric exchange statistics	**Provisional.** Proposition 6.
Canonical anticommutation relations	**Not supplied.** §22 item 4.
Spin-statistics theorem	**Not supplied.** §22 item 5.
Species decomposition	**Not supplied.** §22 item 2.
Standard Model fermion spectrum	**Open.**
Substrate-level pre-electroweak chiral organisation (working conjecture)	**New deliverable, sidebar.** §22 item 12. The chiral-mass and standard Dirac forms arise from different derivational routes and may correspond to different physical stages; the interpretation as pre-electroweak organisation with EWSB as the bridge is working conjecture.

20. Relation to Earlier VERSF Papers, and the Dependency Graph

Ten earlier strands enter:

- Refinement persistence and cohomology papers.
- Maxwell admissibility paper (through v19), inherited via Microscopic Origin.
- Hamiltonian admissibility paper.
- Topological threshold paper ($\beta_1 \geq 1$).
- Primitive occupancy paper.
- Triangular closure programme (conjecture C1).
- **κ-field uniqueness programme** (Theorems U, N, $\kappa \equiv s$ identification; load-bearing for Theorem 1).
- **Schrödinger→Dirac paper** (Theorems E for existence, F for algebraic forcing; both load-bearing for Theorem 1).
- **K=7 minimal fact architecture papers** (d=3 spatial generators for §8).
- Fold programme (especially *Fold* v31) — (σ, ω), $Z_2 \times Z_2$, master action.

Plus the immediate predecessor: Microscopic Origin paper.

Dependency graph. Theorem 1 depends on: κ-field uniqueness Theorem U + Schrödinger→Dirac Theorems E and F + Hamiltonian admissibility (Hermiticity). Theorem 2 depends on: Theorem 1 + SI + Spin(3) uniqueness + minimality of the fundamental representation. Theorem 5 depends on: Theorem 1 + standard Clifford representation theory. Theorem 3 depends on: closure-frame bundle B(P) + topological-threshold $\beta_1 \geq 1$ + non-flat closure-orientation connection ω_μ. Theorem 4 depends on: Theorem 3 + SI + admissible refinement preserving holonomy class. Theorem 6 depends on: Microscopic Origin Theorem 3 hydrodynamic limit + (H3) orientation coherence + (H4) spinor coherence.

21. Epistemic Status and Representation-Theoretic Status Table

21.1 Standard four-tier labelling

Derived under named inheritance.

- *Lemma 1, Lemma 2* — direct propagation.
- *Theorem 1 (Clifford necessity, 3+1 Hamiltonian form)* — derived, conditional on inheritance from κ-field uniqueness Theorem U and Schrödinger→Dirac Theorems E (first-order existence) and F (algebraic forcing). Within those inheritances, Theorem 1 is

a derived result of Dirac's 3+1 Hamiltonian-form argument. If Schrödinger→Dirac supplies only F and not E, then Theorem 1 is "derived under named inheritance from U and F, plus existence assumption."

- *Theorem 2 (Geometric–algebraic convergence)* — derived, requiring Theorem 1 + SI + standard Spin(3) uniqueness + minimal-representation selection. Convergence at group-structure level on the fundamental representation. *Calibrated conditionality:* the abstract group-theoretic content (Spin(3) is uniquely the connected double cover of SO(3), and SU(2) acts on the fundamental representation $\mathbb{C}^2$ as the minimal faithful representation) is unconditional. The *applicability* of Theorem 2 in the present construction is conditional on both routes being in play: the algebraic side inherits from κ-field uniqueness Theorem U and Schrödinger→Dirac Theorems E and F (via Theorem 1); the geometric side requires the structural conjectures C1–C3 (via SI). Theorem 2 does not by itself discharge either set of conditions; what it establishes is that *when both routes apply, they target the same structure*.
- *Theorem 3 (Closure-frame transport holonomy)* — derived under candidate bundle construction.
- *Theorem 5 (Minimality)* — derived under exclusion arguments §7.1–§7.3.
- *Theorem 6 (Hydrodynamic limit)* — derived under (H1)–(H4) hydrodynamic input.

Interpretive (Fold integration and master-action).

- *§15* — interpretive identification. σ ↔ chirality pairing committed as structurally natural. ω-pairing left open between parity P and charge conjugation C, pending Fold v31 verification of ω's physical content. A naive ω ↔ Cl_even/Cl_odd identification is inadmissible because the Cl-grading is not independent of chirality ($\gamma^5 \in$ Cl_even).
- *§16* — interpretive identification of spinorial sector as first-order fermionic master-action sector.

Conditional on framework assumptions.

- *Theorem 4 (Spinorial persistence-sector decomposition)* and *Structural Inference SI* — conditional on C1–C3. *Calibrated significance:* group structure (SU(2) on $\mathbb{C}^2$) is supplied algebraically by Theorem 1; *sector decomposition* into spinorial/trivial-holonomy requires C1–C3.
- *Candidate B(P) and ω_μ* — provisional pending triangular closure completion.
- *K=7 → d=3 inheritance* — conditional on K=7 architecture supplying the spatial-generator count.

Conjectural / open.

- *Proposition 6 (Antisymmetric exchange)*.
- *Connection between spinorial sectors and observed fermion species*.
- *Dirac equation as substrate evolution equation* (algebraic form forced by Theorem 1; substrate dynamics open).
- *Canonical anticommutation relations*.
- *Spin-statistics theorem*.

- *Standard Model fermion spectrum*.
- *Substrate-to-electroweak bridge* (chiral-mass to non-chiral-mass).
- *Non-abelian extension*.

21.2 Representation-theoretic status table

Result	Status	Source
First-order admissible closure flow on persistent sector exists	Inherited (Schrödinger→Dirac Theorem E)	§3.6
Clifford necessity $\{\alpha^i, \alpha^j\} = 2\delta^{ij} \cdot \mathbb{1}$, etc. (Hamiltonian form)	**Derived (algebraically forced under inheritance)**	Theorem 1, this paper
Covariant Clifford relations $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \cdot \mathbb{1}$ via repackaging	Derived	§6.3 repackaging
Standard Dirac operator ($i\gamma^\mu \partial_\mu - m$) as algebraic form	Derived (covariant repackaging)	§6.4
Minimal Cl(1,3) representation is 4-dimensional	**Representationally forced**	Standard Clifford representation theory
Cl(3) minimal representation on $\mathbb{C}^2$ carries SU(2) ≅ Spin(3)	Representationally forced	Standard Clifford representation theory
Geometric SU(2) lift of SO(3)	Derived conditionally (on C1–C3)	Structural Inference SI
Convergence: geometric SU(2) ≅ algebraic SU(2) on the fundamental representation $\mathbb{C}^2$	**Derived (group-structure level; conditional on both routes applying)**	Theorem 2 + minimality of fundamental representation
Spinorial transport sectors (decomposition into spinorial vs trivial-holonomy)	Derived conditionally (geometric, on C1–C3)	Theorem 4
Spinorial structure (universal Clifford action on $\mathbb{C}^4$)	Derived under inheritance	Theorem 1
$U(2\pi) = -\mathbb{1}$ on spinorial sector (geometric identification)	Derived conditionally	Theorem 4
Chirality decomposition via γ^5	Representationally forced	§6.5
K=7 → d=3 → Cl(1,3) chain	Derived under K=7 inheritance	§8
Minimality of spinor representation	Derived	Theorem 5
Covariant chiral-mass form $\Sigma = \gamma^\mu p_\mu + \gamma^5 m$ (sidebar)	**Derived (covariant squaring); interpreted as possible substrate-level pre-electroweak organisation, working conjecture**	§6.7
Substrate-to-electroweak bridge (pre-electroweak chiral	Open (working conjecture)	§22 item 12

Result	Status	Source
organisation → effective non-chiral mass eigenstates via EWSB)		
Antisymmetric exchange statistics	Conjectural	Proposition 6
CAR algebra $\{\psi, \psi^\dagger\} = \delta$	Open	§22 item 4
Dirac equation as substrate evolution equation	Open (algebraic form forced; dynamics open)	§22 item 1
Spin-statistics theorem	Open	§22 item 5
Standard Model fermion spectrum	Open	§22 item 2

Status discipline:

- **Derived under inheritance** — derived given explicit inheritance from named theorems in independent programmes.
- **Derived (algebraically forced)** — derived from the algebra of admissible operators within the present paper.
- **Representationally forced** — fixed by representation theory of inherited algebraic structure.
- **Derived (group-structure level)** — proven at the level of group structure on the fundamental representation, distinct from sector decomposition.
- **Derived conditionally** — proven under stated structural conjectures.
- **Conjectural** — provisionally supplied with explicit conjectural conditions.
- **Open** — not addressed; identified as named deliverable.

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22. Open Problems

Numbering reconciliation: items 1–5 address core fermionic matter-sector deliverables. Items 6–9 are subsidiary structural completions and conjecture discharges. Items 10–12 are master-action, non-abelian extension, and substrate-to-electroweak bridge.

1. Dirac equation as substrate evolution equation (dominant gap). Theorem 1 forces the algebraic *form* of first-order admissible closure flow to be the Dirac operator $i\gamma^\mu \partial_\mu - m$ (standard Dirac form, via 3+1 Hamiltonian-form derivation). Promoting this to a substrate evolution equation:

Target Theorem. Under an admissible commitment-continuity dynamics for ρ extended with closure-orientation transport and Clifford-internal spinor structure, the only stable finite-support first-order transport excitations on the spinorial sector of Theorem 4 are spinor-valued solitons whose effective wave equation is the standard Dirac equation $(i\gamma^\mu \partial_\mu - m)\psi = 0$ with the

Clifford structure forced by Theorem 1 (3+1 Hamiltonian form, non-chiral mass) and the dynamics emerging from the substrate commitment-continuity machinery extended to spinor-valued fields.

Dominant deliverable. Note: this target theorem uses the standard Dirac operator (Theorem 1 form), not the chiral-mass operator (§6.7 sidebar). The relationship between substrate-level effective dynamics and the substrate-to-electroweak bridge (§22 item 12) is part of the broader matter-sector programme.

2. Species decomposition. Discharging the spinorial sector of Theorem 4 into physical fermion species requires identifying which loop classes correspond to which Standard Model fermions. A candidate definition built from the joint admissibility framework:

Candidate definition (physical spinorial source-carrier species). A primitive oriented Clifford-internal loop class $[\mathcal{C}_a]$ satisfying: (1) persistence under δ^*; (2) finite primitive support; (3) stable winding/closure class; (4) spinorial transport class (non-trivial component of SU(2) lift, Theorem 4); (5) minimal Cl(1,3) representation; (6) admissible coupling to A_μ through species-resolved current $J_a^\mu = \bar{\psi}_a \gamma^\mu \psi_a$; (7) admissible exchange behaviour consistent with Proposition 6.

3. Configuration-space exchange-statistics argument. Discharges Proposition 6.

4. Quantum-field promotion to fermionic Fock structure. Includes CAR algebra, path integral measure, renormalised QFT.

5. Spin-statistics theorem. Substrate-level analogue.

6. Rigorous construction of B(P) from triangular closure.

7. Discharge of Structural Conjecture C1. *Calibrated significance:* affects geometric sector decomposition; group-structure survives algebraically.

8. Discharge of Structural Conjecture C2. *Calibrated significance:* if C2 fails, sector decomposition breaks down even though group structure is preserved.

9. Discharge of Structural Conjecture C3 + technical Fold coincidence check + Fold v31 verification of σ-pairing and ω-physical-content question. Discharge of C3 is essentially automatic given the connected-double-cover uniqueness of SU(2) over SO(3). The technical Fold coincidence check verifies that the spinorial transport structure here coincides with a spinor-parity extension of the Fold record current. This item also includes the verification of the §15 Fold-integration pairings against Fold v31's explicit (σ, ω) definitions. Specifically:

- *σ-pairing.* The σ ↔ chirality pairing is committed in §15.2 as structurally natural (orientation parity at fold level ↔ chirality grading at spinor level, both rotation-like). Verification against Fold v31 confirms or revises this.

- *ω-physical-content question.* The §15.3 ω-pairing is left open since the naive ω ↔ Cl_even/Cl_odd identification is inadmissible (Cl-grading is not independent of chirality). Verification against Fold v31 must answer: what is the physical content of ω at the fold level? If ω is spatial-inversion-like, the natural pairing is ω ↔ parity P. If ω is particle/antiparticle-like, the natural pairing is ω ↔ charge conjugation C. If ω is neither, alternative discrete symmetry candidates — for example, time reversal T (the third independent Z_2 of the standard Dirac-field discrete-symmetry group ⟨C, P, T⟩), or a substrate-level discrete structure not represented in ⟨C, P, T⟩ — must be identified.

The Fold v31 verification is a substantive Fold-integration deliverable bearing directly on §15's structural integrity, not a presentational matter.

10. Master-action explicit construction. *Programmatic target.* Not a next-paper deliverable in the literal sense; this is the long-horizon foundational programme.

11. Non-abelian extension of orientation and Clifford framework. Includes the question of substrate SU(2) vs Standard-Model SU(2)_L.

12. Substrate-to-electroweak bridge — pre-electroweak chiral organisation of the persistent sector. The §6.7 sidebar records that direct covariant squaring of a first-order admissible operator on the persistent sector naturally produces a covariant chiral-mass operator $\Sigma = \gamma^\mu p_\mu + \gamma^5 m$, while the 3+1 Hamiltonian-form derivation of Theorem 1 produces the standard non-chiral Dirac operator $(i\gamma^\mu \partial_\mu - m)$. The working interpretation proposed in §6.7 is that the two derivational routes correspond to different physical stages: the 3+1 Hamiltonian-form derivation governs the *effective low-energy spinorial sector* (Theorem 1), while the chiral-mass structure may reflect a deeper *substrate-level pre-electroweak organisation* of the persistent sector.

Target deliverable:

Target Theorem (substrate-to-electroweak bridge — working conjecture). The persistent sector carries a substrate-level pre-electroweak chiral organisation manifested in the covariant chiral-mass form of §6.7. Electroweak symmetry breaking acts as the structural bridge between this substrate-level chiral organisation and the effective non-chiral mass-eigenstate fermions described by the standard Dirac operator of Theorem 1. The chirality structure of the Standard Model fermion sector (left-handed-only SU(2)_L coupling, V−A weak interactions, etc.) emerges as the residue of the substrate-level pre-electroweak organisation after electroweak symmetry breaking.

This target theorem is *conjectural*. The connection between the substrate-level chiral organisation and the Standard Model electroweak mechanism is identified as a structural deliverable for the matter-sector programme, not derived in the present paper. The connection passes through the existing electroweak coherence selection strand of the broader VERSF programme; explicit construction belongs to that strand.

Explicit caveat. The present paper does *not* establish that the substrate-level chiral organisation of §6.7 corresponds directly to physical electroweak chirality; that correspondence is a working conjecture pending the substrate-to-electroweak bridge construction.

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23. Conclusion

The Microscopic Origin paper established that the persistent cohomological sector of VERSF couples to a substrate-level record current J^μ carried by primitive commitment loops. The loops introduced there were scalar carriers. The dominant remaining matter-sector question — why does matter behave as a fermion? — was deferred.

The present paper supplies a candidate spinorial extension constructed through two convergent routes. The *algebraic route* (Theorem 1) establishes, via Dirac's 3+1 Hamiltonian-form argument, that any first-order admissible closure flow operator on the persistent sector — inheriting the existence of such operators from the Schrödinger→Dirac paper and the unique KG scalar invariant from the κ-field uniqueness programme — must take the form $H = \alpha^i p_i + \beta m$ with the Euclidean Clifford relations on a 4-dimensional minimal representation. Under covariant repackaging, *the standard Dirac operator ($i\gamma^\mu \partial_\mu - m$) is algebraically forced as the minimal admissible first-order structure on the persistent sector*; whether this algebraic form generates substrate-level dynamics is deferred to §22 item 1. The forcing is *derived conditional on inheritance from named theorems in independent programmes*, not on C1–C3.

The *geometric route* (Structural Inference SI, Theorem 4) establishes that orientation transport on the candidate closure-frame bundle B(P), under three structural conjectures C1–C3, lifts to SU(2) transport with spinorial sectors exhibiting $U(2\pi) = -\mathbb{1}$. The two routes converge on the same Spin(3) ≅ SU(2) structure on the fundamental representation $\mathbb{C}^2$ by Theorem 2.

The convergence is at the level of *group structure* (both routes target SU(2) on the fundamental Weyl-spinor representation $\mathbb{C}^2$). It is *not* at the level of *sector decomposition* (which remains geometric-only, requiring C1–C3 for the bundle-level two-sector identification). The algebraic route gives universal Cl(1,3)-structure on every first-order admissible flow operator; the geometric route distinguishes which substrate loops carry the spinorial transport signature $U(2\pi) = -\mathbb{1}$.

A productive sidebar (§6.7) observes that *direct covariant squaring* — $\Sigma = \Gamma^\mu p_\mu + \beta m$ with covariant KG^2 consistency — forces β to be γ^5 (not γ^0), producing the *covariant chiral-mass operator* $\Sigma = \gamma^\mu p_\mu + \gamma^5 m$. This is structurally distinct from the standard Dirac operator produced by the 3+1 Hamiltonian-form derivation. The §6.7 sidebar interprets the two derivational routes as possibly corresponding to different physical stages: the 3+1 Hamiltonian-form derivation governing the effective low-energy spinorial sector identified in Theorem 1, the chiral-mass structure possibly reflecting a deeper substrate-level pre-electroweak organisation of the persistent sector. Under this interpretation, electroweak symmetry breaking acts as the structural bridge between substrate-level chiral organisation and effective non-chiral mass-eigenstate fermions. The interpretation is conjectural and is identified as a structural deliverable

(§22 item 12); the present paper makes no claims regarding Standard Model electroweak reconstruction.

The synthesis statement reads:

Spinoriality is algebraically forced by first-order closure flow and geometrically realised through closure transport holonomy. The two routes converge on the same Spin(3) ≅ SU(2) group structure on the fundamental representation. The sector decomposition into spinorial and trivial-holonomy components is geometric-only and remains conditional on C1–C3.

The matter-sector programme now possesses: a substrate ontology for current (Microscopic Origin), an admissibility framework for gauge coupling (Matter Coupling), an ontological grounding in the Fold programme (Microscopic Origin §10 + §15), a unique scalar propagation structure (κ-field uniqueness), and a candidate algebraic-geometric origin of spin-½ structure with explicit convergence at the group-structure level (the present paper). The remaining deliverables — Dirac emergence as substrate evolution equation, species decomposition, exchange statistics, quantisation, conjecture discharge, master-action construction, substrate-to-electroweak bridge — are concrete next-paper targets.

Explicit clarification of scope (final): The present paper derives the spinorial transport structure and Clifford-compatible internal representations required for fermionic source-carriers. It does not derive canonical anticommutation relations, the full spin-statistics theorem, or canonical fermionic quantisation. ***The algebraic form algebraically forced by Theorem 1 is the standard Dirac operator (3+1 Hamiltonian-form derivation, non-chiral mass), inherited as the minimal admissible first-order structure on the persistent sector*****; whether this algebraic form constitutes the substrate evolution equation generating spinorial source-carrier dynamics is the target of §22 item 1's Dirac-emergence theorem, deferred to future work. The covariant chiral-mass operator is a structurally distinct algebraic observation interpreted as a possible substrate-level pre-electroweak organisation of the persistent sector — a working conjecture explicitly deferred to §22 item 12. The present paper does not derive the Standard Model electroweak mechanism or establish that the substrate-level chiral structure of §6.7 corresponds directly to physical electroweak chirality. These remain open and are explicitly identified as next-stage deliverables.**

The strongest framing:

Given (i) first-order admissible closure flow on the persistent sector (inherited from Schrödinger→Dirac), (ii) the κ-field uniqueness theorem (inherited), and (iii) the loop-current framework of Microscopic Origin, Dirac's 3+1 Hamiltonian-form argument forces Clifford-compatible 4-component spinor structure on first-order admissible closure flow, with the standard Dirac operator as the algebraic form. The geometric realisation through closure-frame transport supplies the substrate-level carriers and the sector decomposition required to identify which loops carry fermionic transport behaviour. The algebraic-geometric convergence on the fundamental Spin(3) representation supplies the structural cross-check. The remaining matter-sector work is the promotion of this framework to a full

fermionic field theory through the Dirac, exchange-statistics, quantisation, and substrate-to-electroweak deliverables.

End of paper.