Structural Inevitability of Quantum Kinematics Under Admissibility

A Conditional Uniqueness Theorem from Finite Distinguishability and Irreversible Commitment

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Plain-Language Summary

Why does the world come equipped with quantum mechanics rather than some other equally consistent theory? The standard answer in physics textbooks is: it just does. We measure that it does, and the postulates are written down to fit. The deeper question — *could it have been otherwise* — is usually left aside.

This paper sharpens an answer that has been building in the VERSF programme. The answer it offers is *conditional* and we are explicit about three layers of condition: (i) the admissibility primitives that describe what physical reality is doing — finite distinguishability, irreversible commitment with explicit record locality, channel uniqueness, compositional consistency, observer invariance; (ii) one supplementary axiom that does specific load-bearing work and is explicitly identified — continuous reparametrisation; (iii) a methodological principle, not a theorem, asserting that any kinematic theory must specify five structural dimensions that we analyse.

Given (i), (ii), and (iii), the standard quantum apparatus is the unique architecture left standing. The state space must be a separable Hilbert space. The numbers must be the complex numbers. Composition must be the tensor product. Probabilities must obey the Born rule. Closed-system evolution must be unitary. Together these are exactly the standard quantum apparatus.

The strength of this rewrite over earlier drafts is that what was previously assumed is now derived, what cannot be derived is explicitly labelled, and what is methodological rather than provable is correctly framed as a principle. Local tomography — the condition driving much of the elimination — is now derived from admissibility primitives plus an explicit substrate-architecture commitment to record locality, with an independent supplementary route as backup. The probability rule is derived through four conditions native to VERSF, with the standard mathematical analysis (Gleason's theorem) clearly labelled as the imported step that completes the selection. The dynamics rule converts the previous interpretive claim "decoherence equals commitment" into a structural theorem via a rank-reduction argument. Quaternionic Hilbert space — the scalar-field alternative most resistant to elimination — is ruled out by Adler's tensor-product-existence argument, which establishes that quaternionic Hilbert spaces do not admit a consistent compositional structure.

The structural-completeness step is the most honest framing change. We do not claim to have *proven* that any future kinematic theory must specify exactly the five dimensions we analyse. We claim instead that any such theory currently conceivable does, and that the elimination strategy generalises to any theory specifying these five dimensions. We frame this as a *principle* — methodological scope-fixing, not a theorem.

What remains: the result depends on physical inputs (the admissibility primitives, including record locality) and on continuous reparametrisation. We are explicit about each.

Within the structurally specifiable space, under admissibility-with-record-locality plus continuous reparametrisation, the constraint system has a unique fixed point: standard quantum mechanics.

In other words: if you accept that physical reality is doing certain specific things — that only finitely many distinguishable states fit into any bounded region, that certain moments fix one possibility as a fact, that this fixing is the only way the world's stock of facts changes, that records of those fixings inhere in local substrate, that composite systems can be analysed through their parts, that physics doesn't depend on the labels used to describe it, and that descriptions can be smoothly reparametrised — then standard quantum mechanics is the only physical theory that holds together. Every alternative fails at least one of these conditions.

Abstract

We establish a conditional structural inevitability theorem: under the admissibility framework of the VERSF programme — finite distinguishability (A1), irreversible commitment (A2) with explicit record locality (A2′), channel uniqueness (P), compositional consistency (A3), and continuous observer invariance (A4) — together with the structural completeness principle (⋆) of §11, the standard quantum kinematic architecture

$A_{QM} = (\mathcal{H}, \mathbb{C}, \otimes_{tensor}, p_{Born}, U_{unitary})$

is the unique fixed point of the admissibility constraint system within the space of kinematic architectures specifiable by the five dimensions ($\mathcal{S}$, $\mathbb{F}$, $\otimes$, p, U).

The argument is reorganised relative to earlier drafts to address vulnerabilities identified in successive rounds of referee analysis. (i) **Local tomography is derived in §4 by a non-circular argument using (A2′) — record locality — explicitly labelled as a substrate-architecture commitment**. Earlier drafts contained a residual circularity in which "record" was implicitly identified with "locally readable record"; v6 replaces this with explicit (A2′), grounded in VERSF's substrate ontology in [7, 8]. A supplementary route via the information-additivity axiom (A1+) provides independent confirmation but is not required. (ii) **The quaternionic elimination is replaced by Adler's tensor-product-existence argument**. Earlier drafts attempted an explicit probability calculation that did not in fact show the asserted non-invariance under quaternionic phase reparametrisation; v6 replaces this with the structural argument from

Adler [12, Ch. 4] establishing that quaternionic Hilbert spaces do not admit a consistent tensor product, violating (A3) directly. The elimination of (F-ℍ) no longer depends on continuous (A4). (iii) **Born rule emergence is explicit about scaffolding**: §9 establishes the four conditions (C1)–(C4) from admissibility, then *explicitly states* the reduction to Gleason's domain. (iv) **Dynamics is sharpened by an explicit rank-distinguishability lemma (Lemma 5.0)** carefully phrased as "rank provides the saturated upper bound on perfectly distinguishable orthogonal pure-state alternatives," with explicit dependence on Theorem 4. (v) **Structural completeness is correctly framed as a principle, not a theorem**: §11 asserts the five-dimensional decomposition as a methodological scope-fixing principle.

Two further v6 changes worth noting in the abstract. First, **(P) channel uniqueness is acknowledged explicitly as the strongest of the basic primitives** — an exclusive-mechanism claim, not just an existence claim — with its substrate-level grounding flagged. Second, **(A1+) information additivity is demoted from "supplementary-but-needed" to "robustness backup"**: the primary route of Theorem 0 now uses (A2′) instead, and (A1+) provides an independent confirmation route rather than a load-bearing input.

The Master Theorem (§12) consolidates the elimination theorems plus structural completeness into a fixed-point statement: the admissibility framework (with (A2′) record locality folded in) plus continuous (A4) defines a constraint system whose unique fixed point in the structurally specifiable space is A_QM.

The result is conditional on three explicitly flagged inputs beyond the basic admissibility primitives: record locality (A2′) for the primary derivation of local tomography, continuous (A4) for §7.3, and the structural completeness principle (⋆) for §12. Each is identified, each is physically motivated, each is isolated. Section 13 separates VERSF-original contributions from imported lemmas; sections 14–15 situate the result in the reconstruction literature and enumerate residual limitations. The companion paper [1] handles the dynamical content of admissibility; the present paper handles the kinematic content.

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1. Introduction

1.1 The reconstruction programme and what's left to do

Quantum mechanics is conventionally presented as a set of postulates: a separable complex Hilbert space $\mathscr{H}$, states as rays in $\mathscr{H}$ (or density operators), observables as self-adjoint operators, the Born rule $p_i = |\langle\psi|\varphi_i\rangle|^2$ for measurement probabilities, the tensor product $\mathscr{H}_{AB} = \mathscr{H}_A \otimes \mathscr{H}_B$ for composite systems, and unitary evolution between measurements. Each postulate is mathematically sharp; together they predict everything we presently know how to calculate. None of the postulates explains why it is the right one.

The reconstruction programme — Hardy [2], Masanes and Müller [3], Chiribella, D'Ariano, and Perinotti [4], Dakić and Brukner [5], among others — addresses this gap by deriving quantum structure from operational or informational primitives. These derivations are valuable. They typically leave open a residual question: are the derived structures *the only* structures consistent with the chosen primitives, or merely *a* consistent solution? Most reconstruction theorems require explicit auxiliary axioms — continuous reversibility, local tomography, simplicity — sometimes criticised for building elements of the conclusion into the inputs.

The VERSF programme has independently shown [1, 6, 7, 8] that key elements of quantum kinematics — separable complex Hilbert representation, U(1) phase symmetry, and the Born rule

— arise from admissibility constraints governing finite distinguishability and irreversible record formation. The present paper addresses the residual question for the VERSF derivation: under the admissibility constraints plus explicitly-flagged supplementary axioms, every catalogued alternative to standard quantum kinematics fails, and within the structurally specifiable space (§11) the standard quantum apparatus is the unique fixed point.

1.2 What this paper claims, precisely

We prove the following:

Master Theorem (informal). The admissibility framework — finite distinguishability, irreversible commitment with explicit record locality, channel uniqueness, compositional consistency, continuous observer invariance — together with the structural completeness principle (⋆) of §11, defines a constraint system on the space of kinematic architectures. The unique fixed point of this constraint system is the standard quantum kinematic architecture A_QM = ($\mathscr{H}$, ℂ, ⊗_tensor, p_Born, U_unitary).

The formal statement, with all its scope conditions and conditional dependencies, is in §12.

1.3 What this paper does not claim

We are explicit about the boundary. The theorem does **not** establish:

1. **Unconditional uniqueness across all conceivable future alternatives.** The structural completeness principle (§11) is a *principle*, not a theorem of the paper: it asserts methodologically that any kinematic theory must specify the five dimensions analysed. A future proposal genuinely orthogonal to this decomposition forces re-examination.
2. **Independence from record locality (A2′).** Theorem 0's primary route uses (A2′) — the substrate-architecture commitment that records of independent-region commitments are accessible to coordinated local measurements. A reader rejecting (A2′) rejects the local-substrate ontology of VERSF and therefore the primary route.
3. **Independence from continuous (A4).** The continuity of observer reparametrisation is load-bearing for §7.3.
4. **Derivation of the admissibility framework itself.** (A1)–(A4), (P) are physical inputs justified in [6, 7, 8] but not derived here.
5. **Originality of the imported technical lemmas.** Solèr [17], Gleason [19], Wigner [9], Stinespring [21], Adler [12] are imported; §13 makes this separation explicit.
6. **The dynamical content of admissibility.** The conservation identity for the record current and its scalar closure are in [1].

The honest scope: *given* admissibility primitives including (A2′), *with* continuous (A4), and *within* structural completeness, the kinematic architecture is the unique fixed point of the constraint system. The supplementary axiom (A1+) provides an independent backup route in Theorem 0 but is not required.

1.4 What's new in v6

Strengthenings relative to v5 addressing referee analysis:

- **§2.1 (record locality (A2′)).** The previous draft's (A2′) was framed as a "clarification" of (A2)'s content, closing a "global-record loophole." A careful reader pointed out that this missed the case of records that are *jointly* readable but not *coordinated-locally* readable — exactly the structure of an entangled state in standard QM. v6 strengthens (A2′) to specifically rule out this case, on substrate-architecture grounds: records of independent regions are accessible to coordinated local measurements of those regions, because the substrate itself is local. This is no longer a clarification but a substantive substrate-architecture commitment, acknowledged as such.
- **§2.1 ((P) honesty note).** Added a note acknowledging that (P) is the strongest of the basic primitives — an exclusive-mechanism claim, not just an existence claim — with its substrate-level grounding flagged.
- **§4.4 (Theorem 0 primary route).** Step 5 is rewritten to use the strengthened (A2′) explicitly. The previous draft's three-case trilemma is replaced with a substrate-localisation analysis that uses (A2′) directly rather than the implicit assumption that "record" means "locally readable record."
- **§7.2 (quaternionic elimination).** Replaced the previous draft's attempted explicit probability calculation — which on close examination did not show the asserted non-invariance — with the structural argument from Adler [12, Ch. 4] establishing non-existence of the quaternionic tensor product. Decisive in a way the previous calculation was not. Reduces dependence on continuous (A4): (F-ℍ) now falls under (A3).
- **§7.4, §12 (architecture changes).** With (A2′) now load-bearing for Theorem 0's primary route, (A1+) is demoted from "supplementary-but-needed" to "robustness backup." The Master Theorem rests on basic admissibility + (A2′) + continuous (A4) + (⋆).
- **§9 (density operator bridge).** Added a paragraph at the start of §9 explaining how density operators enter the framework — via convex extension for classical mixtures, and via Theorem 5.2 for subsystem reductions.
- **§10.1 (Lemma 5.0).** Added "relies on Theorem 4 for the perfect-distinguishability claim" parenthetical, flagging the Born-rule dependency.
- **§11.4 (categorical QM acknowledgement).** Added a paragraph addressing the most foreseeable objection to (⋆): that categorical formulations of QM specify (Q1)–(Q5) as derived rather than primary.
- **Tier-3 polish.** (A1+) clause clarifying what N counts (cardinality of maximal antichains); explicit parameter-count numbers in §7.1 (15 vs 6 for d_A=d_B=2); bridge sentence added to Theorem 5.2's conclusion; Solèr forward reference to [8, §§3.4–3.5].

1.5 Strategy of the proof

For each of the five structural dimensions, we eliminate every catalogued alternative by exhibiting a direct admissibility violation. The structural completeness principle (§11) extends the eliminations from "every catalogued alternative" to "every alternative within the five-dimensional space." The Master Theorem assembles the eliminations into the fixed-point statement.

Each elimination is *local*: it uses one or two admissibility conditions to rule out one alternative. Locality is diagnostically useful and scales to future alternatives within the structurally specifiable space.

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2. The Admissibility Framework

We adopt the primitives developed in prior VERSF work [6, 7, 8]. We collect them here in the form needed.

2.1 The basic admissibility primitives

(A1) Finite distinguishability. For any bounded resource budget — equivalently, any bounded causal region R — the number of pairwise distinguishable states accessible to operations confined to R is finite. There is a finite cardinality bound N(R) on antichains in the distinguishability lattice of R.

(A2) Irreversible commitment. There exist physical processes c: $\mathcal{S}$_admissible → $\mathcal{S}$_records mapping multiple admissible prior states to a single posterior record state, with no left-inverse.

(A2′) Record locality. A commitment event produces a record borne by some substrate region. When the recording substrate is independent across spatial regions, the record is accessible to *coordinated local measurements* of those regions — not merely to joint operations across them.

This is a substantive substrate-architecture commitment, not a clarification of (A2). A record that existed but was readable *only* through joint operations across multiple independent regions would be borne by non-local substrate structure: substrate degrees of freedom not localised to any single region. VERSF's local-substrate ontology (developed in [7, 8]) forbids this. (A2′) is therefore an explicit statement of that ontology at the level of records.

Earlier drafts treated record locality as implicit in (A2), and a sharp reader would correctly note that "record" without further qualification leaves open the case of records jointly readable but not coordinated-locally readable — exactly the structure of an entangled state in standard QM. Closing this loophole requires a positive commitment to record locality, which we make explicit here. A reader who rejects (A2′) is rejecting the local-substrate ontology of VERSF, which is justified independently in the substrate-construction papers.

(P) Channel uniqueness. Distinguishability changes — additions to or removals from the count of accessible distinct alternatives — occur *only* through commitment events of type (A2).

Honesty note. (P) is the strongest of the basic primitives. (A1) is a finiteness claim and (A2) an existence claim; (P) is an *exclusive-mechanism* claim — it asserts that the universe has only one way of changing accessible distinguishability counts. A reader sympathetic to (A1) and (A2) might still question (P) on the grounds that physics could in principle admit other distinguishability-changing mechanisms (continuous distinguishability "drift," for instance, between commitment events). The justification for (P) is substrate-level: in [7, 8], commitment events are argued to be the unique mechanism by which the substrate's record content changes, with non-commitment dynamics constrained to be record-preserving by the structure of the substrate itself. The reader should know that (P) is doing more work than (A1) and (A2) individually, and that its justification lies outside the present paper.

(A3) Compositional consistency. For independent systems A, B with admissible state spaces $\mathcal{S}$_A, $\mathcal{S}$_B:

- **(A3a) No-creation.** Distinct local pairs yield distinct joint states.
- **(A3b) No-destruction.** Local distinguishability is preserved jointly.
- **(A3c) Local tomography.** Every joint state is determined by joint statistics of local measurements alone. *(Now derived in Theorem 0, §4.)*

(A4) Observer invariance. Physical structure is invariant under continuous reparametrisation of the description: there exists a continuous group G of transformations of the description under which all admissibility primitives commute.

2.2 The supplementary information-additivity axiom

(A1+) Information additivity for independent regions. For independent regions R_A and R_B with bounded distinguishability counts N(R_A) and N(R_B) — where N counts the cardinality of maximal antichains in the regional distinguishability lattice (equivalently, the dimension of a maximal orthogonal frame in the corresponding state space) — the information content of the joint region satisfies

$$I(R\_A \cup R\_B) = I(R\_A) + I(R\_B), \text{ equivalently } N(R\_A \cup R\_B) = N(R\_A) \cdot N(R\_B),$$

where $I = \log_2 N$. This is information extensivity for independent composition.

Status. (A1+) is *not* derivable from (A1) alone. (A1) bounds local information; (A1+) asserts joint information *equals* the sum of local information for independent composition. The supplementary axiom is physically motivated — it is what "independence" means at the information level — but it is *not* a logical consequence of the basic admissibility primitives.

Why independence supports additivity. Without additivity, independent composition would permit either sub-additive joint information (the joint system carries *less* information than the sum of its parts, implying interaction or correlation that contradicts independence) or super-additive joint information (the joint system carries *more* information than the sum of its parts, implying joint structure not present in either independently — again contradicting independence). Additivity is therefore the unique resource-arithmetic compatible with the interpretation of independent composition as non-interacting resource combination. We adopt (A1+) on this physical motivation while flagging that it remains a supplementary axiom rather than a derived consequence of (A1).

In Theorem 0, we use (A1+) for the *supplementary* route only. The primary route does not require (A1+) and stands without it. This isolates the supplementary axiom and makes its role explicit rather than implicit.

2.3 Status of the primitives

(A1) is the substrate's capacity bound, derived from Bekenstein-style arguments in [6]. (A2) makes facts *facts* and grounds the irreversibility of the second law in [7]; (A2′) is its substrate-architecture clause asserting record locality, justified in [7, 8]. (P) asserts commitment as the *only* mechanism of distinguishability change. (A3a), (A3b) follow from independence; (A3c) is now derived in §4. (A4) in continuous form is load-bearing for §7.3. (A1+) is supplementary and not load-bearing for the Master Theorem.

2.4 Dependencies summary

Axiom	Role	Status
(A1)	Finite distinguishability	Basic admissibility
(A1+)	Information additivity	Supplementary, providing independent confirmation in Theorem 0's supplementary route; not required for the primary route
(A2)	Irreversible commitment	Basic admissibility
(A2′)	Record locality (substrate-architecture commitment)	Load-bearing in Theorem 0 primary route; grounded in substrate ontology of [7, 8]
(P)	Channel uniqueness	Basic admissibility (strongest of the basic primitives — see honesty note in §2.1)
(A3a), (A3b)	Compositional independence	Basic admissibility
(A3c)	Local tomography	Derived in Theorem 0 (primary route uses basic primitives + (A2′))
Continuous (A4)	Continuous reparametrisation	Load-bearing for §7.3 (disconnected fields); (F-$\mathbb{H}$) elimination via Adler no longer requires it
(⋆)	Structural completeness principle	Methodological scope-fixing (§11)

The basic admissibility primitives are physical; (A2′) is a substrate-architecture commitment grounded in [7, 8]; continuous (A4) is explicitly flagged supplementary; (A1+) is supplementary and non-load-bearing; (⋆) is a principle, not a theorem.

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3. The Inevitability Problem

3.1 Definition

A **kinematic architecture** is a tuple A = ($\mathcal{S}$, $\mathbb{F}$, ⊗_A, p_A, U_A) where $\mathcal{S}$ is the space of pure states, $\mathbb{F}$ is the scalar field, ⊗_A is the compositional law, p_A is the probability functional, and U_A is closed-system inter-commitment dynamics. The structural completeness principle (§11) asserts that any kinematic architecture is fully specified by these five components.

An **admissible kinematic architecture** is one satisfying the admissibility primitives (A1)–(A4), (P), with (A3c) following from Theorem 0.

3.2 The constraint-system formulation

The admissibility framework defines a constraint system $\mathcal{C}$ on the space of kinematic architectures: $\mathcal{C}$(A) = ⊤ iff A is admissible. The Master Theorem of §12 establishes:

Master Theorem. Within the space of architectures satisfying the structural completeness principle (⋆), under admissibility primitives — including (A2′) record locality — plus continuous (A4), $\mathcal{C}$ has a unique fixed point: A_QM.

3.3 Strategy

Per-dimension eliminations (§§6–10) plus the structural completeness principle (§11) feed into the fixed-point statement (§12). Each elimination cites the specific admissibility condition that rules out the specific alternative.

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4. Theorem 0: Derivation of Local Tomography

4.1 Why this section is now central

Local tomography (A3c) was the principal load-bearing assumption of earlier reconstruction approaches and earlier VERSF drafts. The criticism: "you assumed the condition that selects quantum theory." This section addresses the criticism by deriving (A3c) from more basic admissibility primitives along two independent routes.

The two routes have different scope:

- **Primary route** uses the basic admissibility primitives (A1), (A2), (P), (A3a), (A3b) plus the substrate-architecture commitment (A2′) — record locality. It establishes (A3c) from admissibility-with-record-locality.
- **Supplementary route** uses the explicit information-additivity axiom (A1+). It provides an independent confirmation but is not required if the primary route is accepted.

A reader rejecting (A2′) — equivalently, rejecting VERSF's local-substrate ontology — is referred to the supplementary route. A reader rejecting (A1+) has the primary route as a backup. Both routes together are robust to single-axiom rejection.

4.2 Statement

Theorem 0 (Local tomography from admissibility primitives). Under (A1), (A2), (P), (A3a), (A3b), every joint state of an independent composite system $A \otimes B$ is determined by joint statistics of local measurements alone. That is: if two joint states ρ_{AB}, σ_{AB} yield identical joint statistics under all coordinated local measurements, then $\rho_{AB} = \sigma_{AB}$.

The supplementary route additionally establishes: under (A1+) — information additivity — joint distinguishability is bounded above by $N_A \cdot N_B$, with equality saturated by tensor-product composition.

This is local tomography in its full probabilistic form, equivalent to the (A3c) of earlier drafts.

4.3 Setup: information cost of distinguishability

Two states are *distinguishable* if there is a measurement procedure separating them with non-trivial probability. By (A1), the count of pairwise distinguishable states in any bounded region is finite, with cardinality bound N(R).

By the Bekenstein-style argument grounding (A1) in [6], this corresponds to an *information capacity* $I(R) = \log_2 N(R)$ bits. By (P), information content is conserved between commitments and changes only at commitment events.

4.4 Primary route: channel-uniqueness argument

We derive (A3c) using only the basic admissibility primitives.

Hypothesis for contradiction. Suppose local tomography fails: there exist joint states $\rho_{AB} \neq \sigma_{AB}$ yielding identical statistics under every coordinated local measurement.

Step 1: a hidden distinguishable alternative exists. $\rho_{AB} \neq \sigma_{AB}$ means there is *some* measurement separating them — by definition of state inequality. By hypothesis, no coordinated local measurement separates them. So the measurement separating them is non-local: it does not factorise into coordinated local operations. Call this measurement M_{global}.

Step 2: the hidden alternative carries information. M_{global} produces a distinguishable outcome on the difference $\delta = \rho_{AB} - \sigma_{AB}$. By (A1), M_{global}'s outcome corresponds to a distinguishable alternative carrying at least one bit of information. Call this bit b_{global}.

Step 3: b_{global} must enter through commitment. The composite system $A \otimes B$ was formed by joining independent A and B. Any product state $\rho_A \otimes \rho_B$ is fully determined by its local marginals (this is what "product state" *means*); product states therefore carry no hidden non-product information.

If ρ_AB and σ_AB are distinguishable by M_global but not by any local measurement, then ρ_AB and σ_AB are not both product states. At least one of them is a non-product (entangled or correlated) state. The non-product structure must have entered the description at some moment.

Step 4: appeal to (P). Non-product structure carrying b_global is a distinguishable alternative beyond what local measurements can resolve. Its appearance in the joint description is a *distinguishability change* — a change in the count of distinguishable joint alternatives. By (P), distinguishability changes occur only through commitment events.

Step 5: contradiction. A commitment event produces a record (by (A2)). The record is borne by some substrate region. By **(A2′)** (record locality), if that substrate region is independent across the A/B partition — equivalently, if the record is borne by substrate localised to A, to B, or partitioned across A and B — then the record is accessible to coordinated local measurements of A and B.

Three cases exhaust the substrate possibilities. (a) The record is borne by substrate within A. By (A2′), it is accessible to local measurements on A; coordinated local measurements detect b_global, contradicting hiddenness. (b) Symmetric: record borne by substrate within B. (c) The record is borne by *joint* substrate distributed in a partitioned way across A and B. By (A2′), accessibility is preserved through coordinated local measurements of A and B together; coordinated local measurements detect b_global, contradicting hiddenness.

The case earlier drafts missed — *record exists, is real, is jointly readable, but is not readable by any coordinated local measurement* — is precisely what (A2′) excludes. Such a record would require non-local substrate structure: substrate degrees of freedom not localised to any independent region. VERSF's local-substrate ontology forbids this; (A2′) is the explicit statement of that ontology at the level of records.

A sharp reader will recognise that the "joint-but-not-coordinated-locally-readable" structure is precisely an entangled-state record in standard quantum mechanics. The point is not to deny that such structures exist in QM — they manifestly do — but to recognise that (A2′)'s record locality, applied to *independent* regions, places strong constraints on what commitment-event records of independent-region commitments can be: they cannot be hidden in non-local substrate, because the substrate itself is local.

The hypothesis of failed local tomography is therefore inconsistent with the conjunction of (A1), (A2), (A2′), (A3a), (A3b), (P). The primary route's load-bearing axiom is (A2′) — explicitly labelled, substrate-grounded, and doing the work that earlier drafts smuggled into the implicit meaning of "record."

4.5 Supplementary route: information-additivity argument

Under the supplementary axiom (A1+), an independent and quantitative argument for the same conclusion.

By (A1+), N(R_AB) = N_A · N_B for independent A, B. By (A3a), distinct local pairs are distinct joint states, providing N_A · N_B distinguishable joint states. By (A3b), all of them are accessible to coordinated local measurements (specifically, to product measurements with classical post-processing).

A hidden global degree of freedom would be a distinguishable joint alternative *beyond* the N_A · N_B accessible to coordinated local measurements. By (A1+)'s cardinality bound, no such additional alternative is permitted: the joint cardinality is *exactly* N_A · N_B.

Therefore no hidden global degree exists, and every distinguishable joint state is resolvable by coordinated local measurements.

4.6 What the two routes establish jointly

The primary route uses basic admissibility primitives plus the substrate-architecture commitment (A2′), and is robust to questioning of information additivity. The supplementary route uses (A1+) and is robust to questioning of (A2′) — i.e., to a reader who does not accept VERSF's local-substrate ontology. Together, the two routes are robust to single-axiom rejection: any escape from the primary route requires rejecting (A2′) or one of (A2), (P); any escape from the supplementary route requires rejecting (A1+).

A reader needing extreme robustness can rely on either route alone. A reader satisfied with admissibility plus (A2′) has the primary route; a reader who prefers (A1+) over (A2′) has the supplementary route. We treat (A2′) as the load-bearing primary input while keeping (A1+) as independent confirmation.

4.7 Where the residual content sits

The primary route's load-bearing components are the basic admissibility primitives (A1), (A2), (P), (A3a), (A3b) plus the substrate-architecture commitment (A2′) — record locality. Earlier drafts treated record locality as implicit in (A2); v6 makes it explicit and acknowledges its substantive content.

(A2′) is not derivable from (A2) alone. (A2) asserts that commitments produce records; (A2′) asserts that records of independent-region commitments are accessible to coordinated local measurements of those regions. The latter is a substantive substrate-architecture commitment: it asserts that the VERSF substrate is local in a strong sense (records inhere in regions of substrate; substrate is partitioned across independent regions; non-local substrate structure carrying records is forbidden).

The trade is honest. (A2′) replaces the earlier draft's hidden assumption that "record = locally readable record" with an explicit substrate-locality axiom. A reader who rejects (A2′) is rejecting the local-substrate ontology of VERSF, justified independently in [7, 8]. A reader who accepts (A2′) gets a derivation of (A3c) that is no longer circular.

Theorem 0's primary route therefore reduces (A3c) to: basic admissibility primitives + (A2′). The supplementary route via (A1+) provides an *independent* confirmation under a different supplementary commitment. With (A2′) doing genuine work in the primary route, (A1+) is no longer required — it becomes a robustness backup rather than a load-bearing input.

From this point forward, (A3c) is treated as derived from admissibility plus (A2′). Where (A3c) is invoked in §§6–10, the underlying derivation traces to admissibility + (A2′) via Theorem 0's primary route.

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5. Classification of Catalogued Alternatives

We classify candidate kinematic architectures by their structural deviations from A_QM. Section 11 (structural completeness principle) frames the five-dimensional decomposition as the methodological scope within which the catalogue operates.

5.1 State-space geometry (Theorem 1, §6)

- **(S-N)** Nonlinear state spaces (Weinberg [10]).
- **(S-J)** Pure Jordan-algebraic state spaces (exceptional Jordan algebra [11, 18]).
- **(S-LS)** Linear-but-non-orthomodular spaces.
- **(S-H)** Hilbert spaces over normed division algebras (the surviving class).

5.2 Scalar field (Theorem 2, §7)

- **(F-ℝ)** Real Hilbert space.
- **(F-ℂ)** Complex Hilbert space.
- **(F-ℍ)** Quaternionic Hilbert space.
- **(F-p)** p-adic Hilbert space.
- **(F-fin)** Galois-field Hilbert space.

5.3 Compositional law (Theorem 3, §8)

- **(C-T)** Tensor product.
- **(C-D)** Direct sum.
- **(C-J)** Jordan-product compositions.
- **(C-W)** Weakened tensor (composition without (A3c)).
- **(C-N)** Super-quantum compositions [16].

5.4 Probability functional (Theorem 4, §9)

- **(P-1)** Linear amplitude rule.
- **(P-2)** Quadratic Born rule.
- **(P-q)** Power-law rule with $q \neq 2$.

- **(P-NL)** Nonlinear functionals.
- **(P-G)** Non-quadratic GPT effects.

5.5 Inter-commitment dynamics (Theorem 5, §10)

- **(D-U)** Closed-system unitary evolution.
- **(D-CP)** Completely positive non-unitary on closed systems.
- **(D-NL)** Nonlinear deterministic on closed systems.
- **(D-Stoch)** Stochastic non-unitary on closed systems.

Open-system Lindblad / GKSL dynamics are addressed in §10.5.

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6. Theorem 1: State-Space Geometry

Theorem 1. Under admissibility, the state space $\mathcal{S}$ must carry the structure of a separable Hilbert space over a normed division algebra. (S-N), (S-J), (S-LS) are eliminated.

6.1 Eliminating (S-N)

Weinberg [10]: fundamental nonlinearity in composite systems generically permits faster-than-light signalling. In our terms, faster-than-light signalling is a distinguishability change at B not mediated by a commitment event at B, violating (P).

6.2 Eliminating (S-J)

Albert [18]: J_3(𝕆) does not admit an associative bilinear product closing under composition. Therefore (S-J) violates (A3) — the joint admissible space falls outside the admissibility class.

6.3 Eliminating (S-LS)

Without orthomodularity, additivity over orthogonal partitions fails. Commitment outcomes form a complete partition by (A1) and (P); their probabilities must sum to 1. Without orthomodularity this sum is ill-defined. (S-LS) violates (A1) at the probability-assignment level.

6.4 Reduction to Hilbert via Solèr's theorem

Among linear, orthomodular, inner-product spaces with infinite-dimensional orthonormal basis, Solèr's theorem [17] reduces to Hilbert space over $\mathbb{R}$, $\mathbb{C}$, or $\mathbb{H}$. The VERSF contribution at this step is to establish that admissibility forces Solèr's hypotheses (linearity, orthomodularity, inner product, separability, and crucially the existence of an infinite orthonormal sequence whose pairwise inner products take a constant value — the Solèr condition proper). Linearity follows from the structure of the admissibility class; orthomodularity follows from §6.3's elimination of (S-LS); the inner product structure follows from probability assignment via Theorem 4;

separability and the Solèr condition specifically are established in [8, §§3.4–3.5] from finite-distinguishability arguments at the substrate level. Section 13 makes this scaffolding structure explicit.

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7. Theorem 2: Scalar Field

Theorem 2. Under the conditions of Theorem 1 plus continuous (A4), the scalar field $\mathbb{F}$ must be $\mathbb{C}$. (F-$\mathbb{R}$), (F-$\mathbb{H}$), (F-p), (F-fin) are eliminated.

7.1 Eliminating (F-$\mathbb{R}$): hidden parameters via Theorem 0

In $\mathscr{H}_\mathbb{R}$ of dimension d, a pure state carries d−1 real parameters (after normalisation). The joint pure-state space $\mathscr{H}_\mathbb{R}^{d_A d_B}$ carries $d_A d_B - 1$ real parameters. Coordinated local measurements on independent subsystems access at most $d_A^2 + d_B^2 - 2$ real parameters jointly: $d_A^2 - 1$ from local measurements on A (specifying a real density operator on $\mathscr{H}_A$), $d_B^2 - 1$ from local measurements on B, with normalisation accounting for the −2. This is the Wootters [13] count.

For $d_A = d_B = 2$ (the simplest non-trivial case): the joint pure-state space carries $4 \cdot 4 - 1 = 15$ real parameters, while coordinated local measurements access $4 + 4 - 2 = 6$. The factor-of-two-and-a-half excess is structural, not an artefact of accounting: real Hilbert space genuinely has joint-state degrees of freedom that no coordinated local measurement can resolve.

The excess parameters carry information distinguishable in joint statistics but inaccessible to coordinated local measurements — a hidden global degree of freedom in the sense of Theorem 0. By Theorem 0 (primary route, using (A2′) record locality), this is forbidden under admissibility.

(F-$\mathbb{R}$) is therefore eliminated by Theorem 0, derived from admissibility primitives.

7.2 Eliminating (F-$\mathbb{H}$): non-existence of the quaternionic tensor product

We replace earlier drafts' attempted "explicit probability calculation" with the decisive structural argument from Adler [12, Ch. 4, §§4.2–4.3]: quaternionic Hilbert spaces do not admit a consistent tensor product, and therefore violate (A3) at the architectural level.

The Adler obstruction. For two quaternionic Hilbert spaces $\mathscr{H}_A^\mathbb{H}$ and $\mathscr{H}_B^\mathbb{H}$, one would like to construct a joint quaternionic Hilbert space $\mathscr{H}_{AB}^\mathbb{H}$ that:

(i) carries a quaternionic linear structure; (ii) accommodates product states $\psi_A \otimes \psi_B$ for $\psi_A \in \mathscr{H}_A^\mathbb{H}$, $\psi_B \in \mathscr{H}_B^\mathbb{H}$; (iii) is bilinear in its factors: $(q\psi_A) \otimes \psi_B = \psi_A \otimes (q\psi_B)$ for $q \in \mathbb{H}$; (iv) carries a consistent inner product respecting the quaternionic structure.

Adler establishes that no construction satisfies all four simultaneously. The obstruction is non-commutativity of ℍ. If both factors carry left-ℍ-module structure, bilinearity in the form (iii) requires q to commute through the tensor symbol independently of which factor it acts on. For two scalars q, q′ acting on different factors, one route gives $(q\psi_A) \otimes (q'\psi_B) = qq'(\psi_A \otimes \psi_B)$, and another gives $q'q(\psi_A \otimes \psi_B)$; these differ for non-commuting q, q′. Bilinearity collapses. Mixed module structures (left on one factor, right on the other) fail bilinearity already at single-factor scalar multiplication. Quotient constructions designed to enforce bilinearity break the quaternionic linear structure (i): the quotient space is at most a real or complex Hilbert space, not a quaternionic one.

The conclusion is that any object that could serve as a quaternionic tensor product $\mathscr{H}_{AB}^{\mathbb{H}}$ either fails to be a quaternionic Hilbert space (collapse to a smaller scalar field) or fails to be bilinear in its factors (loss of the compositional structure needed to represent independent systems).

Consequence for admissibility. Composition of independent quaternionic systems requires constructing a joint admissible state space $\mathcal{S}_{AB}$. By the Adler obstruction, no such joint space exists in the quaternionic-Hilbert-space class: the natural quaternionic tensor product fails to satisfy the linear-Hilbert-space conditions of Theorem 1, and any quotient or modified construction either drops out of the quaternionic class or fails bilinearity.

(F-ℍ) therefore violates **(A3)** directly: the joint admissible space does not exist within the quaternionic admissibility class. Composition of independent quaternionic systems is structurally impossible; quaternionic Hilbert space is not a closed admissibility class under composition.

Why this argument is decisive. Adler's obstruction is a *structural* failure of (A3) for quaternionic Hilbert spaces, well-documented in the quaternionic-quantum-mechanics literature [12]. It does not depend on any specific measurement or state. Earlier drafts attempted an "explicit probability calculation" showing non-invariance under quaternionic phase reparametrisation; on careful examination, that calculation does not in fact show the asserted non-invariance — in standard left-module sesquilinear conventions, one-sided global quaternionic phase reparametrisations preserve probabilities just as U(1) phases do in the complex case. The genuine pathology of quaternionic QM lies upstream of any probability calculation; it lies in the architecture of composition itself.

Where dependence sits. The Adler argument uses (A3) — compositional consistency — and is independent of continuous (A4). This *strengthens* the elimination relative to earlier drafts, which depended on continuous (A4). (F-ℍ) now fails on architectural grounds without invoking continuity.

7.3 Eliminating (F-p) and (F-fin)

p-adic [14] and Galois-field [15] scalars fail continuous (A4) directly: the field is disconnected (totally disconnected for p-adic, discrete for Galois). No continuous reparametrisation group exists., (F-fin)

7.4 Selection of ℂ

Among ℝ, ℂ, ℍ admitted by Solèr's theorem, the eliminations of §§7.1 and 7.2 leave only ℂ:

- (F-ℝ) fails by hidden-parameter excess via Theorem 0 — using (A1), (A2), (A2′), (A3a), (A3b), (P).
- (F-ℍ) fails by non-existence of the quaternionic tensor product (Adler [12]) — using (A3) at the architectural level.
- (F-p) and (F-fin) fail continuous (A4) directly.

Note that the elimination of (F-ℍ) in v6 no longer invokes continuous (A4); it now rests on (A3) instead. This *strengthens* Theorem 2 by reducing its dependence on continuity. Continuous (A4) remains load-bearing for §7.3 (eliminating disconnected scalar fields), but the structurally most challenging case — quaternionic — falls under (A3) instead.

The scalar field is ℂ.

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8. Theorem 3: Compositional Law

Theorem 3. Under the conditions of Theorems 1 and 2 plus Theorem 0, the compositional law $\otimes_A$ must be the tensor product. (C-D), (C-J), (C-W), (C-N) are eliminated.

8.1 Eliminating (C-D)

$\mathscr{H}_{AB} = \mathscr{H}_A \oplus \mathscr{H}_B$ has dimension $\dim(\mathscr{H}_A) + \dim(\mathscr{H}_B)$ but (A3a) requires capacity for $\dim(\mathscr{H}_A) \cdot \dim(\mathscr{H}_B)$ distinct joint pairs. Direct sum lacks the capacity.

8.2 Eliminating (C-J)

By Theorem 1, Jordan-algebraic compositions inherit the closure failure of the exceptional Jordan algebra (Albert [18]).

8.3 Eliminating (C-W)

Weakened-tensor compositions fail (A3c). By Theorem 0, (A3c) is *derived* from basic admissibility primitives via the channel-uniqueness route; therefore (C-W) violates the basic admissibility primitives via Theorem 0's primary route. The dependence on (A3c) as an axiom is gone.

8.4 Eliminating (C-N)

Super-quantum compositions [16] require joint distinguishability exceeding what is allowed by the combinatorial bounds derivable from admissibility. Under (A1+), the bound is $N_A \cdot N_B$

(Theorem 0 supplementary route). Even without (A1+), the channel-uniqueness route eliminates super-quantum compositions: their excess joint correlations are hidden global degrees forbidden by Theorem 0's primary route.

8.5 Selection of tensor product

The unique compositional law surviving Theorems 0, 1, 2 plus admissibility is the tensor product.

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9. Theorem 4: Probability Functional

Theorem 4. The probability functional p_A must be the Born rule $p_i = |\langle\psi|\varphi_i\rangle|^2$. The four conditions characterising p_A are derived from admissibility; the problem then reduces to Gleason's domain, where Gleason's theorem completes the selection.

A note on density operators. From §9 onwards we work with density operators ρ on $\mathscr{H}$ as well as pure states. Density operators enter the framework as the natural extension of the pure-state architecture (Theorem 1) along two routes: (i) classical mixtures of pure preparations — a probabilistic ensemble $\{(p_i, |\psi_i\rangle)\}$ corresponds to $\rho = \Sigma_i\, p_i\, |\psi_i\rangle\langle\psi_i|$ — which follows from convex extension of the pure-state structure with no further axioms; and (ii) reduced states of subsystems of pure entangled states, $\rho_A = \mathrm{Tr}_E(|\Psi\rangle\langle\Psi|_{AE})$, which is the content of Theorem 5.2 (§10.5). Until §10.5, statements involving density operators ρ should be read as applying to pure states or to convex mixtures of preparations; the subsystem-reduction reading becomes available only after Theorem 5.2 is established.

9.1 The four conditions from admissibility

We give a VERSF-native derivation of the four conditions an admissible probability functional must satisfy.

Probability as the consistent extension of commitment counting. For orthogonal alternatives, the probability is the relative frequency of commitment to each alternative — this is the empirical content of probability. For non-orthogonal cases, the probability functional must extend commitment counting in a way consistent with admissibility.

(C1) Reduction to commitment frequency. On orthogonal alternatives, p reduces to the empirical commitment frequency. *Derivation.* By (A2), commitments produce records; by (A1), commitment outcomes are well-defined; their relative frequency is empirical. The probability functional must coincide with this empirical quantity on orthogonal alternatives — anything else would conflict with what probability measures.

(C2) Compositional factorisation. For product states $|\psi\rangle_A \otimes |\varphi\rangle_B$ and product measurements, joint probability factorises: $p_{AB} = p_A \cdot p_B$. *Derivation.* By (A3), independent systems must have factorising joint probabilities for product states. Failure of factorisation would make A and B not-independent, contradicting independence.

(C3) Continuous reparametrisation invariance. p is invariant under the continuous symmetry group of the description. *Derivation.* Direct from continuous (A4): physical structure (including p) is invariant under continuous reparametrisation.

(C4) Additivity over orthogonal partitions. For any orthogonal decomposition $\Sigma_i p_i = 1$. *Derivation.* By (A1), commitment outcomes form a complete partition; by (P), the probability of "some outcome" is 1.

These four conditions are derived from the admissibility framework. Each cites a specific admissibility primitive.

9.2 Reduction to Gleason's domain — explicit statement

With (C1)–(C4) established from admissibility, we **explicitly state** what comes next.

Reduction-to-Gleason statement. With the four conditions (C1)–(C4) in hand, the problem of identifying the unique admissible probability functional on a complex Hilbert space (Theorems 1–2) with tensor-product composition (Theorem 3) reduces to the classical problem solved by Gleason's theorem [19] and its CFMR generalisation [22]: the unique probability measure on closed subspaces of a Hilbert space of dimension ≥ 3 satisfying additivity (C4) is $p = \mathrm{Tr}(\rho P)$, the Born functional.

We are *importing* Gleason's theorem to complete the selection. The VERSF contribution is to derive (C1)–(C4) from physical primitives so that Gleason's theorem applies; the technical step of selecting the Born functional given (C1)–(C4) is Gleason's, not ours.

This reduction-to-Gleason statement is the honest framing. A reader sceptical of any step can identify which: the VERSF derivation of (C1)–(C4) (the present paper's contribution), or Gleason's theorem and CFMR extension (imported, well-established).

9.3 Eliminating catalogued probability alternatives

Each catalogued alternative violates at least one of (C1)–(C4):

- **(P-1)** linear: violates (C4) — $\Sigma |c_i| \neq 1$ for normalised states.
- **(P-q)** for $q \neq 2$: violates (C4) on dim ≥ 3 by CFMR [22].
- **(P-NL)** nonlinear: violates (C2) — joint factorisation fails.
- **(P-G)** non-quadratic GPT: requires failure of (A3c), contradicting Theorem 0.

The catalogued alternatives fail by direct condition violation, before Gleason is invoked. Gleason's theorem then certifies that *no other* probability functional on dimension ≥ 3 Hilbert spaces satisfies (C4).

9.4 Cross-check from the Double Square Rule paper

The VERSF Double Square Rule paper [24] derives the Born rule by an independent route: an informational-cost reformulation $p_i = \exp(-\Delta I_i)$. The agreement of two VERSF-internal derivations from different starting points is non-trivial structural evidence.

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10. Theorem 5: Inter-Commitment Dynamics

Theorem 5. Closed-system inter-commitment dynamics must be unitary. (D-CP), (D-NL), (D-Stoch) on closed systems are eliminated. Open-system effective dynamics are reductions of A_{QM} via Theorem 5.2.

10.1 Lemma 5.0: rank and operational distinguishability

We make explicit the relationship between rank reduction and operational distinguishability — a step the previous draft treated as obvious but which deserves a formal lemma.

Lemma 5.0 (Rank-distinguishability correspondence; relies on Theorem 4 for the perfect-distinguishability claim). For a density operator ρ on a Hilbert space $\mathscr{H}$: (i) rank(ρ) = dim(supp ρ), the dimension of the support of ρ. (ii) supp ρ is spanned by orthogonal pure states $\{|\varphi_i\rangle\}$ with non-zero weight in ρ. (iii) These orthogonal pure states are *perfectly distinguishable* by projective measurement onto $\{|\varphi_i\rangle\}$. (iv) **rank(ρ) provides an upper bound on the number of mutually orthogonal — and therefore perfectly distinguishable — pure-state alternatives present in ρ**, with this upper bound saturated by the spectral decomposition of ρ.

Proof. (i) is the definition of rank for self-adjoint operators. (ii) follows from the spectral theorem: any density operator has a diagonal decomposition $\rho = \Sigma_i p_i |\varphi_i\rangle\langle\varphi_i|$ with $\{|\varphi_i\rangle\}$ orthogonal and $p_i > 0$ on the support. (iii): orthogonal pure states are perfectly distinguishable by the projective measurement onto their span — a direct consequence of the Born rule applied to orthogonal states. (iv): by (i), the dimension of supp ρ is rank(ρ); by (ii), this dimension is the maximum number of mutually orthogonal pure states with non-zero weight; by (iii), these orthogonal pure states are perfectly distinguishable. The spectral decomposition saturates this upper bound. Non-orthogonal alternatives within supp ρ may exist (they are not perfectly distinguishable from each other), but the count of *perfectly distinguishable* orthogonal alternatives is bounded by rank(ρ) and saturated.

This careful phrasing — *upper bound on perfectly distinguishable orthogonal alternatives, saturated by the spectral decomposition* — is what justifies treating rank changes as changes in the count of perfectly distinguishable alternatives in subsequent arguments. A reader familiar with standard QM should note that we are not identifying rank with all operationally meaningful distinguishability (which would be too strong) but with the saturated upper bound on *perfectly distinguishable orthogonal* alternatives — a precise and standard quantum-mechanical concept.

10.2 Lemma 5.1: rank reduction without commitment violates (P)

Lemma 5.1. Let U: $\mathcal{S} \to \mathcal{S}$ be a closed-system inter-commitment evolution map. If rank(U(ρ)) < rank(ρ) for some ρ, then U violates (P).

Proof. By Lemma 5.0(iv), rank(ρ) is the saturated upper bound on the number of mutually orthogonal, perfectly distinguishable pure-state alternatives in ρ. If rank(U(ρ)) < rank(ρ), this saturated upper bound has decreased: the maximum number of perfectly distinguishable orthogonal alternatives extractable from U(ρ) is strictly less than from ρ.

A decrease in the count of perfectly distinguishable orthogonal alternatives is a *distinguishability change* in the sense of (P): the set of distinct alternatives accessible to the system has strictly contracted. By (P), distinguishability changes occur only at commitment events.

If the rank reduction occurs *between* commitment events — as posited for closed-system inter-commitment dynamics — then a distinguishability change occurs without a commitment event. This violates (P) directly.

10.3 Eliminating closed-system non-unitary candidates

Lemma 5.1 directly eliminates the catalogued alternatives.

(D-CP) on closed systems. CP non-unitary maps generically reduce rank (this is precisely what makes them non-unitary on the same space). By Lemma 5.1, this violates (P).

(D-NL). Nonlinear deterministic dynamics violate the linear state-space structure of Theorem 1.

(D-Stoch). Stochastic non-unitary dynamics introduce phase-information loss without explicit commitments. The phase-information loss reduces off-diagonal density-matrix elements, leading to rank reduction in the limit and to measure-decreasing intermediate dynamics. Lemma 5.1 (or its measure-theoretic refinement) applies.

10.4 Closed-system unitarity from Wigner's theorem

By Theorem 1, the state space is a Hilbert space. By (P) and Lemma 5.1, closed-system inter-commitment dynamics preserve rank, hence preserve the saturated upper bound on perfectly distinguishable orthogonal pure-state alternatives.

Distinguishability preservation translates to inner-product modulus preservation: $|\langle\psi_t|\varphi_t\rangle|^2 = |\langle\psi_0|\varphi_0\rangle|^2$ for all states and times between commitments. (For pure states, two states are perfectly distinguishable iff their inner product vanishes; preservation of perfect distinguishability is preservation of the orthogonality structure, which extends to preservation of inner-product moduli for general state pairs.)

Wigner's theorem [9]: any inner-product-preserving bijection on a Hilbert space is unitary or anti-unitary. Continuous time evolution under continuous (A4) excludes the anti-unitary branch.

Closed-system inter-commitment dynamics are unitary.

10.5 Theorem 5.2: decoherence equals commitment

Theorem 5.2. Effective non-unitarity on an open subsystem A of a closed system $A \otimes E$ corresponds structurally to commitment events at the system–environment boundary.

Proof. By §10.4, $A \otimes E$ evolves unitarily between commitments. Let $\rho_{AE}(t) = U(t)\, \rho_{AE}(0)\, U(t)^\dagger$. The reduced state $\rho_A(t) = \mathrm{Tr}_E\, \rho_{AE}(t)$ generically has decreasing rank — the standard decoherence phenomenon.

By Lemma 5.1, rank decrease of $\rho_A(t)$ requires a commitment event. By the hypothesis that $A \otimes E$ is closed and unitarily evolving, no commitment occurs at the closed-system level.

The commitment must therefore occur at the *boundary* — that is, at the act of *tracing out* E. Physically, this corresponds to A being unable to access environmental information without E registering the access as a commitment. The act of restricting attention to A is, at the substrate level, the act of treating E's degrees of freedom as having registered (committed to) a definite outcome from A's perspective; the partial trace formalises this restriction.

The trace-out is therefore the formal expression of an environmental commitment event. (R1) — "decoherence equals commitment" — is now a structural consequence of admissibility, not an interpretive choice.

10.6 Selection of unitary

Closed-system dynamics are unitary (Wigner via Lemma 5.1 via (P)). Open-system dynamics are unitary closed-system dynamics with environmental commitments traced out (Theorem 5.2). The standard quantum dynamical apparatus is the unique admissible dynamical structure.

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11. The Structural Completeness Principle

11.1 Why this is now framed as a principle, not a theorem

Earlier drafts presented the structural completeness of the five-dimensional decomposition as a theorem of the paper. This was the wrong framing: a strictly proven theorem requires either an exhaustive enumeration of all conceivable kinematic architectures (which we cannot give) or a metaphysical guarantee that no future architecture will introduce a structural dimension we have not anticipated (which we cannot prove).

The honest framing is *methodological*: any kinematic theory currently conceivable, and any kinematic theory specifiable by the operational requirements of physics (predictions, compositions, dynamics), specifies the five dimensions we analyse. We frame this as a *principle* with justification, not a theorem, and the Master Theorem is correspondingly conditional on this principle.

This framing is more defensible because it makes the methodological commitment explicit. A reader objecting "but a future theory might escape your decomposition" is correct in principle and is met by the explicit acknowledgement that the result is conditional on (⋆); they then have the specific objection that some structural component is missing from (Q1)–(Q5), which can be addressed concretely if articulated.

11.2 Statement of the principle

Structural Completeness Principle (⋆). Any operationally meaningful kinematic theory specifies, and is fully specified by, answers to:

(Q1) What is the space of pure states $\mathcal{S}$? **(Q2)** What is the scalar field $\mathbb{F}$ (when $\mathcal{S}$ has linear structure)? **(Q3)** What is the compositional law $\otimes$? **(Q4)** What is the probability functional p? **(Q5)** What is the dynamical law U?

These five questions are, by the operational content of physics, exhaustive. Within architectures specified by (Q1)–(Q5), the elimination strategy of §§6–10 applies.

11.3 Justification

(Q1) — *necessity*: there must be states. *Sufficiency*: given the rest, $\mathcal{S}$ fixes what is being predicted about.

(Q4) — *necessity*: there must be empirical predictions, hence probabilities. Without (Q4), no empirical content.

(Q5) — *necessity*: states must evolve, or no temporal physics is possible. *Sufficiency*: given the rest, U fixes how predictions evolve.

(Q3) — *necessity*: composite systems must be describable, or the architecture is local-only.

(Q2) — conditional on (Q1)'s linear structure; if $\mathcal{S}$ is non-linear, (Q2) does not apply (and Theorem 1 then eliminates the architecture).

The five questions cover the operational requirements of physics: what is, how predictions arise, how systems compose, how time enters. Any property of an architecture affecting empirical predictions enters through one of these five.

11.4 What the principle does and does not claim

Claims. Currently conceivable kinematic architectures fall within the five-dimensional space; the elimination strategy of §§6–10 applies to any architecture in this space; novel architectures within the space can be tested using the same per-dimension elimination.

Does not claim. Future architectures with structural components outside (Q1)–(Q5) cannot exist; the principle is exhaustive across all metaphysically possible theories.

Status. Methodological scope-fixing. The Master Theorem of §12 is conditional on (⋆).

A reader proposing a kinematic architecture genuinely orthogonal to (Q1)–(Q5) forces re-examination. We claim such proposals are unlikely to arise within operational physics but acknowledge they cannot be definitively excluded.

One foreseeable objection. Categorical and process-theoretic formulations of quantum mechanics (Abramsky–Coecke and successors) take the basic objects of the theory to be symmetric monoidal categories with dagger structure rather than state spaces and probability functionals. Such frameworks specify (Q1)–(Q5) as *derived* notions extracted from the categorical structure, rather than as primary specifications. The principle (⋆) is not invalidated by this: the kinematic content extracted from a categorical formulation still answers (Q1)–(Q5), and the eliminations of §§6–10 still apply to that extracted content. Readers wedded to categorical primacy may prefer to read this paper as a constraint on the kinematic content extracted from any admissible categorical framework.

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12. The Master Theorem: Fixed-Point Statement

12.1 Statement

Master Theorem. Under the basic admissibility primitives (A1), (A2), (P), (A3a), (A3b), (A4) of §2 *together with the substrate-architecture commitment (A2')* (record locality), the continuous form of (A4), and the structural completeness principle (⋆) of §11, the constraint system $\mathcal{C}$ has exactly one fixed point in the structurally specifiable space:

A_QM = ($\mathscr{H}$, ℂ, ⊗_tensor, p_Born, U_unitary).

The supplementary axiom (A1+) provides an independent confirmation route in Theorem 0 but is not load-bearing for the primary derivation.

12.2 Proof

By Theorem 0 (primary route, using basic admissibility + (A2′)), (A3c) is established. By (⋆), every architecture in scope is specified by ($\mathcal{S}$, 𝔽, ⊗, p, U).

- Theorem 1: $\mathcal{S}$ is a Hilbert space over a normed division algebra.
- Theorem 2: 𝔽 = ℂ. (F-ℝ) eliminated by Theorem 0; (F-ℍ) eliminated by (A3) via Adler's tensor-product-existence argument; (F-p), (F-fin) eliminated by continuous (A4).
- Theorem 3: ⊗ = tensor product (using Theorem 0).
- Theorem 4: p = Born functional (using (C1)–(C4) plus Gleason routing).

- Theorem 5: U = unitary (closed-system) plus open-system reduction (Theorem 5.2).

The unique architecture satisfying all five fixings is A_QM. Modulo representational equivalence, this is unique.

12.3 Fixed-point reading

The constraint system $\mathcal{C}$ acts on the structurally specifiable space; A_QM is its unique fixed point. This makes the result *structural*: A_QM is the closure-fixed-point of the admissibility constraint system, not a contingent solution among many.

Within the structurally specifiable space, under admissibility-with-record-locality plus continuous reparametrisation, the constraint system has a unique fixed point: quantum mechanics.

12.4 Scope and conditions

Condition 1 — Structural completeness (⋆) of §11. Methodological scope-fixing. A future architecture genuinely orthogonal to (Q1)–(Q5) forces re-examination.

Condition 2 — Record locality (A2′) of §2.1. Substrate-architecture commitment. Used in Theorem 0's primary route to derive (A3c). Justified at substrate level in [7, 8].

Condition 3 — Continuous (A4). Load-bearing for §7.3 (disconnected scalar fields). v6's elimination of (F-ℍ) no longer uses continuity — the tensor-product-existence argument from Adler [12] applies through (A3) directly.

Condition 4 — Admissibility primitives. Physical inputs justified in [6, 7, 8].

Information additivity (A1+) is not a Master Theorem condition. It provides an independent confirmation route for Theorem 0 but is not required for the primary derivation. Demoted from earlier drafts' status as a load-bearing supplementary axiom.

After Theorem 0's primary route, (A3c) is no longer an independent assumption. After Theorem 5.2, decoherence-as-commitment is no longer interpretive choice. The previously load-bearing axioms of earlier drafts are now structurally derived or correctly framed as principles.

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13. What Is VERSF and What Is Imported

13.1 Why this section exists

A fair question: given the imports (Solèr, Wigner, Gleason, Stinespring), what work does the VERSF framework do? This section answers explicitly.

13.2 Schematic separation

- **§4 (Theorem 0).** *Fully VERSF-original.* The primary-route channel-uniqueness argument with explicit (A2′) record locality is new. The supplementary-route argument uses (A1+) explicitly labelled. Earlier reconstructions assumed (A3c).
- **§6 (state-space geometry).** VERSF: linear structure, orthomodularity, separability, inner product all from admissibility. Imported: Solèr's theorem [17] reducing to Hilbert space.
- **§7 (scalar field).** VERSF: parameter-count violation for (F-ℝ) via Theorem 0; application of Adler's tensor-product-existence obstruction to admissibility framework for (F-ℍ); disconnectedness for (F-p), (F-fin) via continuous (A4). Imported: parameter-counting arithmetic [13]; Adler's quaternionic obstruction [12]; quaternionic algebra [20].
- **§8 (compositional law).** VERSF: elimination of (C-D), (C-J), (C-W), (C-N) using Theorem 0 and basic admissibility. Imported: tensor-product uniqueness given (A3c).
- **§9 (probability functional).** VERSF: derivation of (C1)–(C4) from admissibility; explicit *reduction-to-Gleason statement*. Imported: Gleason's theorem [19] and CFMR generalisation [22].
- **§10 (dynamics).** VERSF: Lemma 5.0 (rank-distinguishability correspondence); Lemma 5.1 (rank reduction violates (P)); Theorem 5.2 (decoherence equals commitment). Imported: Wigner's theorem [9]; Stinespring [21] for open-system reduction.
- **§11 (structural completeness principle).** VERSF-original framing as methodological principle. Underlying five-dimensional decomposition is implicit in the reconstruction literature.
- **§12 (Master Theorem).** VERSF-original. Fixed-point assembly.

13.3 Six contributions of the present paper

1. **Theorem 0's primary route** — local tomography from basic admissibility primitives plus the substrate-architecture commitment (A2′) — non-circular derivation that earlier reconstructions lacked.
2. **The (A1+) supplementary route** — independent confirmation under explicit additivity, providing robustness without being load-bearing.
3. **VERSF-native (C1)–(C4)** — probability conditions derived from admissibility rather than imported.
4. **Lemmas 5.0 and 5.1 plus Theorem 5.2** — converting "decoherence equals commitment" from interpretive claim to structural theorem.
5. **The structural completeness principle** — methodological scope-fixing correctly framed.
6. **The fixed-point Master Theorem** — assembly into a single uniqueness statement.

The present paper makes substantial structural contributions while clearly labelling imported lemmas and supplementary axioms.

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14. Comparison with Reconstruction Literature

14.1 Hardy [2]

Hardy's "simplicity" plays a role similar to combinatorial local tomography. We avoid simplicity by deriving local tomography from admissibility (Theorem 0). Hardy's framework is operational; ours is physical.

14.2 Masanes–Müller [3]

Masanes–Müller use local tomography as an axiom. We derive it (Theorem 0 primary route), the principal technical advance.

14.3 Chiribella–D'Ariano–Perinotti [4]

CDP's purification ≈ our (A2). The dynamics treatment differs: CDP work with channels operationally; we derive closed-system unitarity from rank-preservation under (P).

14.4 Dakić–Brukner [5]

Different motivation through entanglement; same conclusions. Born-rule selection differs: they use entanglement principles; we use (C1)–(C4) plus Gleason.

14.5 What's new

1. Local tomography is a theorem, not an axiom.
2. Born-rule conditions are derived from admissibility.
3. Decoherence-as-commitment is a structural theorem.
4. Structural completeness is correctly framed as a principle.
5. The Master Theorem provides a unified fixed-point statement.

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15. Limitations

We list residual issues honestly.

1. **Structural completeness principle (⋆) is methodological, not proven.** Future architectures genuinely orthogonal to (Q1)–(Q5) force re-examination. Categorical formulations are addressed in §11.4.
2. **Record locality (A2′) is a substrate-architecture commitment.** Earlier drafts treated record locality as implicit in (A2). v6 makes it explicit and acknowledges it as a substantive substrate-architecture axiom. Its justification lies in the substrate-construction papers [7, 8] rather than in the basic admissibility primitives. A reader rejecting the local-substrate ontology of VERSF will reject (A2′) and therefore the primary route of Theorem 0.

3. **(A1+) supplementary axiom.** Theorem 0's primary route does not require it; the supplementary route does. (A1+) is now demoted from "supplementary-but-needed" to "robustness backup": rejection of (A1+) does not invalidate the primary route, which now rests on (A2′) instead.
4. **Continuity of (A4).** Load-bearing for §7.3 (eliminating disconnected scalar fields). v6's elimination of (F-ℍ) via Adler's tensor-product-existence argument no longer uses continuous (A4) — so continuity's role is reduced relative to v5. Whether continuity follows from a deeper VERSF principle is open.
5. **(P) is the strongest of the basic primitives.** Honest acknowledgement noted in §2.1. (P) is an exclusive-mechanism claim, not just an existence claim, and a reader sympathetic to (A1) and (A2) might still question (P). Its justification lies in [7, 8].
6. **Imported lemmas.** Solèr [17], Wigner [9], Gleason [19], CFMR [22], Albert [18], Stinespring [21], Adler's tensor-product obstruction [12] are imported. The VERSF framework provides setup, not re-derivation.
7. **Dimensional and separability subtleties.** Solèr requires infinite-dimensional separable spaces; Gleason fails in dimension 2.
8. **Kinematic only.** Dynamical content beyond inter-commitment unitarity is in [1].
9. **Density operator ordering.** §9 introduces density operators (mixed states and subsystem reductions) before §10 derives the open-system reduction structure. The ordering is addressed by the bridge note at the start of §9, which restricts ρ to convex mixtures of preparations until §10.5.
10. **No constructive content.** Fixed-point statement, not constructive derivation.

None undermines the Master Theorem within its stated scope.

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16. Position Within the VERSF Corpus

This paper provides:

- the Master Theorem (§12) — fixed-point statement of admissibility constraint system,
- per-dimension eliminations (Theorems 1–5),
- Theorem 0 derivation of local tomography (primary + supplementary routes),
- Lemmas 5.0, 5.1 and Theorem 5.2 on dynamics,
- structural completeness principle (§11),
- explicit VERSF/imported separation (§13).

This paper does not provide:

- dynamical content of admissibility (record current, scalar closure) — see [1],
- action-principle realisation — see [25, §9.5],
- Born-rule informational-cost derivation — see [24],
- gravity sector — see [26],
- substrate-level emergence of homogeneity, isotropy, Lorentz invariance — see [27],
- derivation of admissibility primitives from substrate physics — see [6, 7, 8].

The intended use: conditional fixed-point uniqueness theorem strengthening earlier VERSF derivations, with all load-bearing assumptions explicitly identified.

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17. Conclusion

We have established that, within the structurally specifiable space (structural completeness principle, §11), under the admissibility framework — including record locality (A2′) — and continuous (A4), the constraint system $\mathcal{C}$ has a unique fixed point: standard quantum mechanics

A_QM = ($\mathscr{H}$, ℂ, ⊗_tensor, p_Born, U_unitary).

v6 strengthenings relative to v5:

Theorem 0 derives local tomography by a non-circular argument using (A2′) — record locality — explicitly labelled as a substrate-architecture commitment. v5's primary route had a residual circularity: "record" was implicitly identified with "locally readable record." v6 closes this by strengthening (A2′) to specifically state that records of independent regions are accessible to coordinated local measurements of those regions, on substrate-architecture grounds. (A1+) information additivity is correspondingly demoted from load-bearing to robustness backup.

Quaternionic Hilbert space is decisively eliminated by Adler's tensor-product-existence argument [12, Ch. 4], replacing v5's attempted probability calculation that did not in fact show non-invariance. The new argument uses (A3) directly and reduces dependence on continuous (A4).

(P) is explicitly acknowledged as the strongest of the basic primitives, with substrate-level grounding flagged.

The Born rule emerges through explicit Gleason routing with a clear reduction-to-Gleason statement.

The dynamics theorem rests on a carefully phrased rank-distinguishability lemma explicitly noted as relying on Theorem 4.

Structural completeness is correctly framed as a principle, not a theorem, with categorical-QM acknowledgement in §11.4.

The result is conditional on three explicitly flagged inputs beyond the basic admissibility primitives: (A2′) record locality, continuous (A4), and (⋆) structural completeness. Each is identified, each is physically motivated, each is isolated. The previously load-bearing axioms (A3c) and (R1) are now derived as theorems.

The strength of the result is what is no longer assumed. The honesty of the result is what remains an input — clearly labelled, methodologically motivated, not pretending to be derived.

Within the structurally specifiable space, under admissibility-with-record-locality plus continuous reparametrisation, the constraint system has a unique fixed point. The fixed point is quantum mechanics. Each condition is explicit. Each elimination is local. Each imported lemma is labelled.

This is, we argue, the strongest defensible claim the framework currently supports. The result is conditional in clearly-bounded ways and structural within those bounds — a fixed-point uniqueness theorem properly scoped, with all scaffolding visible.

Acknowledgements

This paper consolidates earlier VERSF working notes on the no-alternative content into a fixed-point uniqueness statement, with substantial strengthenings addressing referee analysis of previous drafts. The reconstruction literature [2, 3, 4, 5, 17] is essential precursor work. Companion papers in the VERSF Theoretical Physics Programme — on the record current and scalar closure [1], the admissibility closure framework [8], the Born rule from informational geometry [24], the BCB Lagrangian [25], gravity from record density [26], and proto-time and emergent Lorentz invariance [27] — supply the upstream and downstream material referenced throughout.

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[7] Taylor, K. *Irreversible Commitment and the Origin of Facts.* VERSF Programme working paper.

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