Substrate Dynamics and the Higgs Ratio

A Unified VERSF Formulation: From Commitment Structure to Finite Electroweak Curvature

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For the General Reader

This paper asks a simple question with a quantitative answer: **why does the Higgs boson have the mass that it does, relative to the vacuum energy scale that sets the masses of the other particles in the Standard Model?**

The observed ratio is $m_H / v \approx 0.5085$. No accepted theory derives this number; in the Standard Model it is a measured input, one of about twenty free parameters. The framework presented here predicts the value:

$$m_H / v = 32 / 63 \approx 0.5079$$

agreeing with observation to roughly one part in a thousand (a residual of about 1.2 standard deviations, well within current measurement uncertainty).

The prediction does not come from a clever fit. It emerges, in a single line, from a more basic claim about physical reality: **the world is not fundamentally continuous, but built up out of discrete commitment events** — moments at which a piece of physical information becomes definite and cannot be undone. The numbers 32 and 63 are not chosen; they fall out of the algebra of how such commitment events can be organised consistently.

The structure of the argument.

Part I develops the underlying "substrate" — the discrete pre-physical layer on which everything else rests. It introduces a small number of objects: events (where commitments happen), a partial ordering on events (recording what comes before what), and a vector-valued quantity ρ at each event (recording the substrate's local state). Five axioms govern how these objects interact. From these axioms, the framework derives the geometric structures of physics — distance, time, light cones — as *emergent* properties of how events relate to one another, rather than as input assumptions about a smooth space-time. Lorentzian causal structure (the cause-effect distinction of relativity) is the natural way this geometry organises itself; full curved space-time emerges in a continuum limit.

A particular structural number appears: $K = 7$. This is the number of distinct ways the substrate can be locally consistent, derived elsewhere in the VERSF programme through six independent routes. K + 1 = 8 directions emerge naturally at each event, with a symmetry group SU(8) acting on them.

Part II applies the framework to the Higgs ratio. The Higgs field lives along one of the 8 directions. The other 7 generate a 63-dimensional space of consistent deformations (this is where 63 comes from: it is the dimension of the algebra of traceless 8×8 matrices, the algebra of SU(8)). When the framework averages over these 63 deformations evenly — which a uniqueness theorem about Killing forms forces it to do — the Higgs ratio shifts from a baseline value of 1/2 to:

$(1/2) \times (1 + 1/63) = 32/63 \approx 0.5079.$

That is the prediction. Three substrate parameters appear in the underlying theory; **none of them appears in the predicted ratio**. The number 32/63 is purely structural.

Part III extends the picture to a statistical layer (a Boltzmann-style probability measure on substrate states) connecting Part I's deterministic substrate to the statistical regularities observed in nature.

Why this matters. A precise prediction for a Standard Model parameter — derived from a discrete substrate rather than fitted to data — is unusual. The framework commits to specific structural claims that are individually checkable, and to specific failure modes (listed explicitly in §18.1). It is intended to be falsifiable: if precision measurement places the Higgs ratio outside the predicted band [0.5064, 0.5095] at 3σ, the prediction fails. If checkable structural results (like the basin-uniqueness assumption on small examples) fail, named parts of the framework collapse. The framework is structured so that hostile review has clear targets.

Companion material. Two related papers, *Admissible Coarse-Graining Theory* and *Exact Lorentz Invariance under Coarse-Graining*, address Open Problems #9 and #10 of this paper. The candidate substrate microdynamics — the specific update rule governing how the substrate evolves — is developed in Part I (§§5.9.10, 5.9.11, Appendix F). The full VERSF programme catalogue is at versf-eos.com.

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Abstract

We present a unified VERSF formulation linking substrate commitment dynamics to a candidate prediction for the dimensionless Higgs-to-vacuum ratio.

Part I. Five axioms; ten theorems; twenty propositions; seventeen definitions; eight conjectures. §5.7.5's signature-selection programme is justified by Propositions 15, 17. §5.9.3 establishes refinement existence and confluence (Propositions 14, 16, 18). §5.9.7 supplies the Minimal Microdynamical Skeleton (Definition 13) with concrete realisation (Definition 14). §5.9.8 supplies the Invariant Continuum-Limit Propagation Speed (Theorem 9). §5.9.9 supplies the Non-Abelian Defect Suppression Proposition (Proposition 20): under admissible edge-bisection refinement, leading non-Abelian holonomy defects decay as A_C^{(n)} ~ 16^{-n} A_C^{(0)}.

§5.9.10 supplies the candidate substrate microdynamics with Theorem 1′ (Minimal Admissible Refinement Dynamics): the candidate update rule $\Psi_{n+1} = \Pi_{adm}^{GF} \cdot (S_m \otimes \exp(i\varepsilon_m H_{cl})) \cdot \Psi_n$ is the minimal operator expression of seven structural commitments + strong continuity of closure transport. The closure Hamiltonian is identified as $H_{cl} = G|_{\mathfrak{su}(8)}$, the $\mathfrak{su}(8)$-restricted Hessian of F at the admissible manifold. A Schur's-lemma commutativity argument establishes that the three admissibility-component Hessians G_C, G_T, G_R commute within each SU(8) isotype. Conjecture 8 (Uniform Admissibility Hessian Scaling) characterises parameter-independence of eigenvalue ratios. §5.9.11 supplies Theorem 10 (Gradient-Flow Selector Existence for Analytic Coercive Single-Basin Substrates): under standard analytic-coercive-bounded-below hypotheses, the gradient-flow admissibility selector Π_{adm}^{GF} exists by the Łojasiewicz-Simon convergence theorem, with basin-uniqueness as the only substrate-specific residual hypothesis.

Part II. $m_H/v = 32/63 \approx 0.50794$ against observed 0.50849 ± 0.00045 ($\approx 1.2\sigma$ residual).

Part III. Statistical layer (Axiom 5, Proposition 10, Conjecture 3).

Appendices D, E, F, G. Four worked refinement models on the four-point diamond: scalar (D), uniform-direction vector (E), non-uniform vector with Abelian transport including substrate microdynamics worked example with $H_{cl} = L_{12}$ and reflection-symmetry-reduction argument (F), non-Abelian transport with explicit suppression-rate computation (G).

A quantitative 3σ falsification line is given at [0.50639, 0.50949], tightening to ≈ [0.50734, 0.50854] under HL-LHC precision.

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1. Introduction

1.1 Epistemic discipline

The framework distinguishes four levels of claim: **Axiom** (primitive assumption, not derived); **Theorem** (proved structural claim); **Proposition** (auxiliary result, usually proved); **Conjecture** (claim believed but not yet proved). Section 1.5 details what this paper does *not* claim, so a reader has a clear map of the framework's scope.

1.2 Notation

- E: set of commitment events.
- $\Lambda = (E, \preceq)$: the substrate poset (events with partial order).
- $\rho : E \to \mathbb{C}^{K+1}$: vector-valued commitment density at each event; $u(e) = \|\rho(e)\|^2$ is the local saturation.
- $A[\rho]$: admissibility functional (non-negative; $A = 0$ defines the admissible manifold $\mathcal{M}_{adm}$).
- $F[\rho]$: free-energy functional $= \int V_{sub}\, d\mu + A$.

- σ: gradient-flow parameter ordering commitment events along Axiom 2 dynamics.
- K: closure cardinality. The framework's central structural integer; K = 7 (Conjecture 2 here; six independent derivation routes elsewhere in the VERSF programme).
- SU(K+1) = SU(8): symmetry group of A acting on the closure-normalised direction sector $\mathbb{C}^{K+1} = \mathbb{C}^8$.

1.3 Numbering conventions

Theorems. Ten total: Theorems 1–10. Theorem 1 is Traceless Selection (§9); Theorem 1′ is the substrate microdynamics minimality theorem (§5.9.10), distinguished by the prime.

Axioms. Five total.

Propositions. Twenty total: Propositions 1–20.

Conjectures. Eight total: Conjectures 1–8.

Definitions. Seventeen total: Definitions 1–17.

1.4 Outline

Part I (§§2–5) develops the substrate-dynamical foundation: primitives (§2), admissibility decomposition (§3), commitment dynamics (§4), and emergent geometric, refinement, and microdynamical structure (§5). Part II (§§6–16) derives the Higgs ratio $m_H/v = 32/63$. Part III (§17) introduces the statistical substrate layer. §18 enumerates remaining open problems and their failure modes; §19 concludes.

1.5 What this paper does not claim

The framework operates within disciplined scope. The following are explicitly *not* claimed by this paper:

- **A full derivation of General Relativity.** Part I supplies substrate dynamics with conditional continuum reconstruction (Theorem 8) and proto-Lorentzian structure (Axiom 4, Proposition 11). It does not derive the full Einstein equations, the Einstein-Hilbert action, diffeomorphism invariance, or matter coupling to curvature.
- **A derivation of the Standard Model.** Part II predicts $m_H/v = 32/63$ from K = 7 closure structure and the renormalisation results of Theorems 1, 2, 5, 6 + Proposition 13. It does not derive the $SU(3) \times SU(2) \times U(1)_Y$ gauge group, fermion content, mass hierarchies, mixing angles, or any other Standard Model parameter. The identification rule of §8.1.1 (Conjecture 4) is supplied as a structural commitment rather than derived from first principles (Open Problem #7).
- **Exact Lorentz invariance.** Proposition 11 establishes leading-order Lorentzian-interval stability under admissible coarse-graining; Theorem 9 establishes invariant continuum-limit propagation speed. Exact Lorentz invariance — the full Lorentz group as an exact symmetry of the continuum theory — is open (Open Problem #10).

- **Full quantum emergence.** §5.8 connects to the VERSF Quantum Reconstruction Programme but does not derive the Born rule, Hilbert-space structure, projective measurement, or operator algebras within this paper. The candidate microdynamics of §§5.9.10–5.9.11 supplies discrete-time unitary refinement structure, not quantum emergence (Open Problem #5).
- **Uniqueness of the candidate microdynamics.** Theorem 1′ establishes the candidate's form as the minimal operator expression of seven explicit structural commitments + strong continuity of closure transport. The candidate is *not* claimed unique in the absence of these commitments. Alternative microdynamics accepting different combinations of the seven commitments would yield different operator structures and are not excluded by Theorem 1′. Higher-order non-factorised couplings (Open Problem #11(c)) are explicitly allowed.
- **A derivation of consciousness, observers, or measurement.** Bit formation (§5.9.11) refers strictly to refinement-stable observables in the technical sense of Definition 11. The framework does not address what it would mean for substrate Bits to be observed by an external agent, nor does it derive any structure of observers from the substrate.
- **Derivation of substrate parameters.** λ_{sub}, ρ_c, β are free parameters (Open Problem #6). The principal predictions ($m_H/v = 32/63$) are shown to be *independent* of these parameters; their individual values are not derived.

What this paper does claim: a structurally constrained substrate architecture (Part I) with five axioms, ten theorems, twenty propositions, seventeen definitions, and eight conjectures; a candidate admissible refinement microdynamics (§§5.9.10, 5.9.11, Appendix F) with explicit structural commitments and named conditional hypotheses; conditional continuum reconstruction (Theorem 8) from substrate refinement; a candidate finite electroweak curvature relation (Part II) $m_H/v = 32/63 \approx 0.50794$; refinement-group irrelevance of non-Abelian commutator content (Proposition 20, Appendix G); selector existence theoremically (Theorem 10) for analytic-coercive-single-basin substrate functionals.

Where this paper crosses the line between "claim" and "do not claim" is governed by named conditional hypotheses (M-Convexity, Refinement Extension, basin uniqueness, strong continuity, Conjecture 8). Each is identified explicitly within the text; §18.1 enumerates them.

Part I — Deterministic Substrate Dynamics

2. Primitive Objects

Definition 1 (Commitment events). $E = \{ e_i \}$.

Definition 2 (Commitment ordering). $\Lambda = (E, \preceq)$, reflexive, antisymmetric, transitive.

Definition 3 (Commitment density, vector-valued). $\rho : E \to \mathbb{C}^{K+1}$; $u(e) = \|\rho(e)\|^2$.

Definition 4 (Admissibility functional). $A[\rho]$; $\mathcal{M}_{adm} = \{ \rho : A[\rho] = 0 \}$.

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3. The Admissibility Decomposition

3.1 The decomposition axiom

Axiom 1. $A[\rho] = A_C[\rho] + A_T[\rho] + A_R[\rho]$ with non-negative components.

3.2 Closure consistency

The closure-inconsistency operator measures the failure of three transport operators along a triangle to compose to the identity:

$$C(e_i, e_j, e_k) = U(e_i \to e_j) \cdot U(e_j \to e_k) \cdot U(e_k \to e_i) - \mathbb{1}$$

valued in the adjoint of SU(K+1). The closure-admissibility functional is:

$$\mathbf{A_C[\rho] = \Sigma_\triangle \|C(e_i, e_j, e_k)\|^2 \cdot u(e_i) \cdot u(e_j) \cdot u(e_k).}$$

Proposition 1. $A_C = 0 \Rightarrow$ local flatness (substrate transport is path-independent on admissible configurations).

3.3 Transport consistency

The transport-inconsistency content measures the failure of the substrate current to be consistent with the gradient-flow:

$$\mathbf{A_T[\rho] = \int \|\nabla \cdot J_\rho + \partial_\sigma \rho\|^2 \, d\mu.}$$

J_ρ is the substrate current built linearly from ρ, valued in the fundamental of SU(K+1).

3.4 Record consistency

Record morphisms $\Phi_{ij} \in End(\mathbb{C}^{K+1})$ capture how the substrate transfers ρ-data between events. Record consistency requires that two-step morphisms agree with one-step morphisms across compositional paths:

$$\mathbf{A_R[\rho] = \Sigma \|\Phi_{jk} \circ \Phi_{ij} - \Phi_{ik}\|^2 \cdot u(e_i) \cdot u(e_k).}$$

Φ-defects are valued in the adjoint of SU(K+1).

3.5 Three identified compositional modes

Propositions 2, 3. The three admissibility modes C, T, R are structurally distinct and explicitly identified; exhaustiveness of the three-mode decomposition is conjectural (Open Problem #8 — a boundary-inconsistency candidate fourth mode has not been ruled out). The three components are logically independent.

3.6 Quadratic structure

Theorem 4 (Quadratic Admissibility from SU(K+1) Symmetry). Each admissibility component is quadratic in its inconsistency operator's natural SU(K+1) representation: C in adjoint, J_ρ in fundamental, Φ in adjoint + trivial.

3.7 Additivity

Proposition 7 (Additivity). $A = A_C + A_T + A_R$ is uniquely characterised, up to overall normalisation, by zero-on-mode-intersection, linearity, and absence of cross-coupling.

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4. Commitment Dynamics

4.1 Gradient flow

Axiom 2. $F[\rho] = \int V_{sub}\, d\mu + A[\rho]$; $\partial_\sigma \rho_i = -\delta F/\delta\bar{\rho}_i$.

4.2 Substrate potential

Axiom 3. $V_{sub}(\rho) = (\lambda_{sub}/4)(\|\rho\|^2 - \rho_c^2)^2$. Higher-order invariants are closure-scale-suppressed.

4.3 Logistic dynamics

Theorem 3 (Logistic Dynamics). On $\mathcal{M}_{adm}$: $\partial_\sigma r^2 = \lambda_{sub} \cdot r^2 \cdot (\rho_c^2 - r^2)$, where $r = \|\rho\|$. Fixed points: $r = 0$ (unstable), $r = \rho_c$ (stable).

4.4 Growth and decay

Propositions 4, 5. Substrate magnitude exhibits monotonic growth toward saturation under Axiom 2; non-admissible perturbations decay onto $\mathcal{M}_{adm}$.

4.5 Parameters

The substrate has three free parameters: $\boldsymbol{\lambda_{sub}}$ (coupling strength), $\boldsymbol{\rho_c}$ (saturation magnitude), and $\boldsymbol{\beta}$ (the statistical-layer inverse temperature, introduced in Part III). Numerical determination of these parameters is Open Problem #6. The principal predictions of this paper depend on *none* of them.

5. Emergent Structure

5.1 Operational time

Definition 5 (Operational time). Along a chain C of substrate events: $\tau(C) = \int_C \|\rho(e)\| \, d\sigma$.

5.2 Triangle-degree localisation

Definition 9 (Triangle degree). $\deg_\triangle(e) = \#\{$ triples of events containing e $\}$.

Theorem 7 (Triangle-Degree Localisation). Substrate observables concentrate on minimum-triangle-degree events; under regularity hypotheses, the concentration follows an inverse-power form.

Proposition 8. In standard poset families, minimum-triangle-degree events form a codimension-1 boundary.

Conjecture 1 (Universal Codimension-1 Localisation). Minimum-triangle-degree localisation produces codimension-1 interface concentration universally across admissible poset families (Open Problem #1).

5.3 Correlation-kernel geometry

Definitions 6, 7. $G(e_i, e_j) = \langle\langle \rho(e_i), \rho(e_j) \rangle\rangle$ (correlation kernel); $d(e_i, e_j) = -\log|G(e_i, e_j)|$ (correlation-kernel distance).

Proposition 6. d satisfies the triangle inequality.

5.4 Lorentzian causal structure

Proposition 9. Comparable event pairs are timelike-related; incomparable pairs are spacelike-related. The partial order Λ supplies a substrate-level cause-effect distinction.

5.5 The Lorentzian Interval Ansatz

Axiom 4 (Lorentzian Interval Ansatz). For event pairs:

$$s^2(e_i, e_j) = c_\rho^2 \cdot \tau(e_i, e_j)^2 - d(e_i, e_j)^2.$$

The substrate forces a direction; Axiom 4 commits to the Lorentzian sign convention. Derivation of signature from substrate structure alone is Open Problem #2′; §5.7.5 supplies a candidate variational derivation route.

5.6 Admissibility transport operator

Definition 8. L_A: the admissibility transport operator on substrate observables. Field modes are eigenvectors of L_A: $L_A \psi_n = \lambda_n \psi_n$.

5.7 Lorentzian Interval Stability under Admissible Coarse-Graining

5.7.1–5.7.4 Setup and leading-order stability

The substrate supplies a distinguished direction through the partial order Λ; Axiom 4 commits this direction to Lorentzian signature; admissible coarse-graining respects the directional structure (Definition 10); under admissible Γ, the interval is stable at leading order (Proposition 11). The framework's relation to causal-set continuum limits is structurally distinguished: vector-valued density, admissibility functionals, and correlation-kernel geometry are framework-specific features absent from standard causal-set approaches.

Definition 10 (Admissible Coarse-Graining). $\Gamma : \Lambda \to \Lambda'$ preserving (i) ordering, (ii) admissibility closure, (iii) operational distinguishability above the coarse-graining scale.

Proposition 11 (Lorentzian Interval Stability). Given Axiom 4, under admissible Γ: $s'^2 \sim s^2$ up to corrections suppressed by inverse coarse-graining scale.

5.7.5 The Signature Selection Programme

Axiom 4 commits to Lorentzian signature; this subsection supplies a candidate variational derivation route.

The interval admissibility functional. For a candidate interval form s on the substrate:

$$\mathcal{I}[s] = A[\rho] + \gamma \cdot \mathcal{C}_{causal}[s] + \eta \cdot \mathcal{C}_{hyperbolic}[s]$$

with $\gamma, \eta > 0$.

Causal-cone penalty. Let $Cone_s(e)$ denote the set of events causally connected to e under interval form s (events e′ with $s^2(e, e') \geq 0$ in the substrate's timelike-positive convention):

$$\mathcal{C}_{causal}[s] = \Sigma_{e \in E} \left(|Cone_s(e)| / |E| - \chi \right)^2$$

where $\chi \in (0, 1)$ is the target causal-cone fraction reflecting finite causality consistent with substrate admissibility. Under Euclidean signature, every event is causally connected to every other ($|Cone(e)|/|E| \to 1$), and $\mathcal{C}_{causal}$ is maximised. Under degenerate signatures, $Cone_s$ is ill-defined and the penalty diverges. Under Lorentzian signature, $|Cone(e)|/|E|$ takes a finite intermediate value tied to the substrate's local-distinguishability structure.

χ is itself substrate-derived (rather than freely adjustable): the natural identification is χ = (operational-distinguishability scale) / (substrate diameter), inherited from Definition 10(iii). Derivation of χ from substrate first principles is part of the open work for Conjecture 5.

Hyperbolic-propagation penalty. Let Δ_s be the Laplace-type operator induced by s through the correlation-kernel metric, and $\square_s$ the substrate's natural hyperbolic wave operator inherited from Axiom 2 and operational-time additivity:

$$\mathcal{C}\_\text{hyperbolic}[s] = \|\Delta_s - \square_s\|^2$$

Under Euclidean signature Δ_s is elliptic and differs maximally from the hyperbolic $\square_s$; under degenerate signatures Δ_s is parabolic; **under Lorentzian signature $\Delta_s = \square_s$ exactly** — the Laplacian of a Lorentzian metric *is* the d'Alembertian, so $\mathcal{C}$_hyperbolic = 0.

Among second-order differential operators induced by quadratic forms with K+1 eigenvalue signs, only Lorentzian signature produces an operator coinciding with the substrate hyperbolic operator. This is sharp: $\mathcal{C}$_hyperbolic alone selects Lorentzian signature uniquely.

Conjecture 5 (Hyperbolic Admissibility Selection). Among admissible interval forms — quadratic assignments consistent with $\tau > 0$ on ordered pairs and $d > 0$ on unordered pairs — Lorentzian signature uniquely minimises $\mathcal{J}[s] = A[\rho] + \gamma \cdot \mathcal{C}\_\text{causal}[s] + \eta \cdot \mathcal{C}\_\text{hyperbolic}[s]$, with $\mathcal{C}$_hyperbolic = 0 achieved iff signature is Lorentzian.

The functionals $\mathcal{C}$_causal and $\mathcal{C}$_hyperbolic have explicit mathematical content above. Their derivation as the *unique* admissibility-preserving forms from substrate axioms, and the proof of Conjecture 5's uniqueness claim, are part of the open derivation work for Open Problem #2′.

Propositions 15, 17. Minimal Quadratic Penalty Principle and Inconsistency-Operator Selection Principle: $\mathcal{J}[s]$ is the minimal admissibility-penalising quadratic functional on candidate signatures consistent with the operator structure of A, justifying the form of the signature-selection functional.

5.7.6 Status

Established: substrate-distinguished direction; τ, d as positive quantities on respective event-pair classes; given Axiom 4, leading-order stability under Γ (Proposition 11).

Not yet established: choice of Lorentzian signature from substrate axioms 1–3 alone (Axiom 4 commits; Conjecture 5 supplies the candidate variational derivation); exact Lorentz invariance under Γ (Open Problem #10); full metric-tensor dynamics.

5.8 Relation to the VERSF Quantum Reconstruction Programme

The substrate's discrete commitment structure connects to quantum mechanics through the VERSF Quantum Reconstruction Programme. Five structural connections: (i) TPB refinement at the substrate level produces continuity in the coarse-grained limit; (ii) simplex rigidity in the closure-normalised sector gives rise to linear extension of substrate observables to a Hilbert-space structure; (iii) admissible coarse-graining is the unifying mechanism connecting substrate-level discrete dynamics to continuum-level quantum-mechanical descriptions; (iv) Part III's statistical layer connects to quantum emergence via Conjecture 3; (v) the present paper supplies the substrate-side effective-continuum-limit framework, with full quantum emergence developed in the companion *Quantum Reconstruction* paper.

5.9 Admissible Coarse-Graining and Continuum Reconstruction

5.9.1 Principle

Physical continuum structure is an admissible coarse-grained representation of discrete irreversible substrate dynamics.

5.9.2 Effective locality

$|G(e_i, e_j)| \sim \exp(-d/\xi) \Rightarrow$ distant regions are effectively independent at any finite coarse-graining scale.

5.9.3 Continuum reconstruction from refinement stability

Definition 11 (Refinement Stability). A substrate observable $\mathcal{O}[\Lambda_n]$ is refinement-stable iff $\lim_{n\to\infty} \mathcal{O}[\Lambda_n]$ exists and is independent of the admissible refinement sequence.

Definition 12 (Refinement Operator). $R_{n\to n+1} : \mathcal{O}[\Lambda_n] \to \mathcal{O}[\Lambda_{n+1}]$ with $R(\mathcal{O}_\_) = \mathcal{O}_\_$ defining refinement fixed points. Linearised around a fixed point: $\varepsilon_{n+1} = L_R \cdot \varepsilon_n + O(\|\varepsilon_n\|^2)$, with relevant ($|\lambda_{L_R}| > 1$), irrelevant ($< 1$), and marginal ($= 1$) eigendirections.

Proposition 12 (Continuum Reconstruction Criterion). A continuum-limit geometry exists iff (i) $A[\Gamma_n(\rho)] = 0$ for all n; (ii) $s_n^2 \sim s^2$; (iii) refinement-stable observables exist.

Theorem 8 (Conditional Continuum Reconstruction). If (1) $A[\Gamma_n(\rho)] = 0$; (2) $s_n^2 \to s_\_^2$; *(3)* $\mathcal{O}[\Lambda_n] \to \mathcal{O}_\_$; (4) limits are sequence-independent, then $(\Lambda_n, \rho_n, d_n, \tau_n) \to (\mathcal{M}, g_{\mu\nu}^{eff}, \rho_{eff})$ up to admissible-coarse-graining equivalence.

Hypothesis (4) requires sequence-independence over *every* admissible refinement sequence. Proposition 14 reduces this to a testable pairwise condition.

Proposition 14 (Refinement Confluence Criterion). Let $\Gamma_n : \Lambda \to \Lambda_n$ and $\tilde{\Gamma}_n : \Lambda \to \tilde{\Lambda}_n$ be two admissible refinement sequences. If there exists a common admissible refinement subsequence $\hat{\Gamma}_n$ with subsequences $\{\hat{\Gamma}_{n_k}\}$ and $\{\hat{\Gamma}_{m_k}\}$ such that $\hat{\Gamma}_{n_k}$ factorises through Γ_{n_k} and $\hat{\Gamma}_{m_k}$ factorises through $\tilde{\Gamma}_{m_k}$, then the two continuum limits are equivalent:

$$\lim_{n\to\infty} \mathcal{O}[\Gamma_n(\Lambda)] \sim \lim_{n\to\infty} \mathcal{O}[\tilde{\Gamma}_n(\Lambda)]$$

up to admissible-coarse-graining equivalence.

Argument. Factorisation through a common refinement $\hat{\Gamma}_{n_k}$ means both $\Gamma_{n_k}(\Lambda)$ and $\tilde{\Gamma}_{m_k}(\Lambda)$ admit a further admissible coarse-graining producing the same $\hat{\Gamma}(\Lambda)$. By Definition 10, this third refinement is admissible, and its limit coincides with the limits of both subsequences. Since $\mathcal{O}[\Gamma_n(\Lambda)]$ and $\mathcal{O}[\tilde{\Gamma}_n(\Lambda)]$ both have subsequences converging to $\mathcal{O}_*$, the full continuum limits coincide. ■

Proposition 14 is the substrate analogue of the Church-Rosser property: two reduction paths are equivalent iff they admit a common continuation. Admissible coarse-grainings play the role of admissible reductions; refinement-stable observables play the role of normal forms.

Proposition 16 (Existence for Diamond-like Posets). For substrate posets locally resembling the four-point diamond, admissible refinement sequences exist and Theorem 8's hypotheses are simultaneously satisfiable (verified explicitly in Appendix D).

Proposition 18 (Edge-Bisection Existence for Finite DAG Posets). For every finite acyclic poset Λ, the edge-bisection refinement operator S_m is well-defined and admissible.

Conjecture 6 (Admissible Refinement Existence). For every admissible substrate poset, there exists at least one admissible refinement sequence satisfying Theorem 8's hypotheses.

5.9.4 Relation to renormalisation

Definition 12's eigenvalue classification (relevant/irrelevant/marginal) parallels Wilsonian renormalisation. Substrate-level admissible refinement is the discrete analogue of Wilsonian RG flow; refinement-stable observables are the analogue of RG fixed points. Proposition 20 (§5.9.9 below) makes this analogy quantitative for non-Abelian commutator content.

5.9.5 Convergence with quantum reconstruction

Admissible coarse-graining is the unifying mechanism: it produces the geometric continuum limit of §5.9.3 *and* the Hilbert-space limit of the Quantum Reconstruction Programme (§5.8) from the same substrate-level discrete structure. The two limits converge in the continuum description.

5.9.6 Status

Established: Definitions 11, 12; Propositions 12, 14, 16, 18; Theorem 8 (conditional). Open: full continuum-limit existence for non-diamond posets (Conjecture 6); exact Lorentz invariance under Γ (Open Problem #10); admissible-coarse-graining theory as a separate companion paper (Open Problem #9).

5.9.7 The Minimal Microdynamical Skeleton

Definition 13 (Minimal Microdynamical Skeleton). Substrate microdynamics is the triple ($\mathcal{H}_{sub}$, T_m, Π_{adm}) with update rule $\rho_{n+1} = \Pi_{adm} \cdot T_m \cdot \rho_n$, satisfying four algebraic identities:

(i) $T_m^m = T$ (refinement composition); (ii) $\Pi_{adm}^2 = \Pi_{adm}$ (idempotence); (iii) $\Pi_{adm} \cdot T_m \cdot \Pi_{adm} = \Pi_{adm} \cdot T_m$ (admissibility-preserving commutation); (iv) $A[\Pi_{adm} \Psi] = 0$ (admissibility filtering).

Definition 14 (Concrete Microdynamical Realisation). $\mathcal{H}_{sub} = \ell^2(E) \otimes \mathbb{C}^{K+1}$; $T_m = S_m \otimes U$ for $U \in SU(K+1)$; Π_{adm}: $\mathcal{H}_{sub} \to \mathcal{M}_{adm}$. Verified to satisfy identities (i)–(iv) under structural hypotheses (Refinement Extension; admissibility-selector existence).

Conjecture 7 (Microdynamical Closure). Any microdynamics satisfying Definition 13's identities generates an admissible refinement flow compatible with R of Definition 12.

The candidate microdynamics satisfying Definition 13 is constructed explicitly in §§5.9.10–5.9.11.

5.9.8 Invariant Propagation Structure

The substrate's effective propagation speed $c_n = d_n / \tau_n$ at refinement level n is well-defined for chain pairs at all admissible refinement scales.

Proposition 19 (Propagation-Speed Stability). Under edge-bisection refinement preserving operational-time density: $c_n = c_0$ for all n.

Theorem 9 (Invariant Continuum-Limit Propagation Speed). Under all admissible refinement sequences satisfying Definition 10's three conditions, the continuum-limit propagation speed $c_\infty = \lim_n c_n$ exists and is independent of the refinement sequence.

Theorem 9 supplies the invariant continuum-limit speed of light from substrate-level refinement.

5.9.9 Non-Abelian Refinement Suppression

§5.9.7's Concrete Microdynamical Realisation uses single-generator SU(8) transport — an Abelian one-parameter sub-flow within SU(8). Appendix F instantiates this Abelian case on the diamond. The structural question for full electroweak gauge structure (which requires non-Abelian $SU(2)\times U(1)_Y$ generators with $[T_a, T_b] \neq 0$) is: under admissible refinement, how rapidly do non-Abelian commutator defects between non-commuting SU(8) transport sectors decay?

Proposition 20 (Non-Abelian Defect Suppression). Let $H_1, H_2 \in \mathfrak{su}(8)$ with $[H_1, H_2] \neq 0$, and let $A_C^{(0)}$ be the closure-admissibility defect for a substrate configuration with mixed-generator transport along path-comparison chains, computed at leading order:

$$\mathbf{A_C^{(0)} = \varepsilon^4 \cdot \|[H_1, H_2]\|^2 \cdot \rho_c^4 + O(\varepsilon^5).}$$

Under admissible edge-bisection refinement at refinement-phase scaling $\varepsilon \to \varepsilon/2$ per segment, the defect decays exponentially:

$$A_C^{(n)} \sim 16^{-n} \cdot A_C^{(0)}.$$

Proof. By Baker-Campbell-Hausdorff expansion to second order in ε:

$$e^{i\varepsilon H_2} \cdot e^{i\varepsilon H_1} = \mathbb{1} + i\varepsilon(H_1 + H_2) - (\varepsilon^2/2)(H_1^2 + H_2^2 + 2H_2H_1) + O(\varepsilon^3)$$
$$e^{i\varepsilon H_1} \cdot e^{i\varepsilon H_2} = \mathbb{1} + i\varepsilon(H_1 + H_2) - (\varepsilon^2/2)(H_1^2 + H_2^2 + 2H_1H_2) + O(\varepsilon^3)$$

Subtracting:

$$U_b - U_c = \varepsilon^2 \cdot [H_1, H_2] + O(\varepsilon^3).$$

The closure-admissibility functional evaluated on this defect:

$$A_C^{(0)} = \|U_b - U_c\|^2 \cdot u(a) \cdot u(d) = \varepsilon^4 \cdot \|[H_1, H_2]\|^2 \cdot \rho_c^4 + O(\varepsilon^5).$$

Under one admissible edge-bisection refinement, each segment's phase scales as $\varepsilon \to \varepsilon/2$, giving $A_C^{(1)} = A_C^{(0)}/16 + O(\varepsilon^5)$. Iterating: $A_C^{(n)} = 16^{-n} \cdot A_C^{(0)} + O(\varepsilon^5)$. ■

Interpretation. The factor $1/16 < 1$ places non-Abelian commutator-defect content among the *irrelevant* eigendirections of the linearised refinement operator L_R (Definition 12). In renormalisation-group language: non-Abelian commutator defects are irrelevant operators in the substrate's RG flow, decaying exponentially toward $\mathcal{M}_{adm}$ with refinement.

Structural consequence. The substrate tolerates genuine non-Abelian SU(8) transport without violating admissibility under refinement. Non-Abelian content is structurally compatible with the framework, supporting the framework's invocation of $SU(2)\times U(1)_Y$ gauge structure in §8.1.1.

The explicit computation with specific generator assignments is given in Appendix G.

5.9.10 Substrate Microdynamics: The Candidate Update Rule

§5.9.7 introduced the Minimal Microdynamical Skeleton (Definition 13) and the Concrete Microdynamical Realisation (Definition 14) with $T_m = S_m \otimes U$ for $U \in SU(K+1)$ abstract. §5.9.10 supplies the **explicit candidate update rule** addressing Open Problem #11.

The candidate update rule:

$$\Psi_{n+1} = \Pi_{adm}^{GF} \cdot (S_m \otimes \exp(i\varepsilon_m H_{cl})) \cdot \Psi_n$$

operating on substrate states $\Psi \in \mathcal{H}_{sub} = \ell^2(E) \otimes \mathbb{C}^8$, with S_m the edge-bisection refinement operator (Proposition 18), $\exp(i\varepsilon_m H_{cl}) \in SU(8)$ the closure transport generated by Hermitian $H_{cl} \in \mathfrak{su}(8)$, and Π_{adm}^{GF} the gradient-flow admissibility selector (Definition 15a below).

5.9.10.1 The seven structural commitments and Theorem 1′

The candidate update rule is **minimal under seven structural commitments** (each listed below), all present in earlier parts of this paper. The word "forced" is used in the conditional sense throughout: the candidate's factorised operator form follows from the seven commitments + strong continuity, not from any unstated assumption. Alternative microdynamics that accept different combinations of these commitments yield different operator structures and are not excluded by Theorem 1′ below. **Five of the seven commitments are direct inheritances from earlier parts of this paper; two are bridged applications** that invoke earlier results in the substrate-dynamics context.

1. **State localisation** (Definitions 1, 2): substrate information lives on events. The substrate Hilbert space carries an event-space factor $\ell^2(E)$. *Direct inheritance.*
2. **Closure-normalised content** (Conjecture 2: K = 7 cardinality universality): each event carries K + 1 = 8 closure-normalised directions, giving $\mathbb{C}^8$. *Direct inheritance.*
3. **Admissible refinement** (Definition 10, Proposition 18): substrate evolution generates finer admissible commitment structure via the refinement operator S_m. *Direct inheritance.*
4. **Norm preservation under closure transport**: closure transport preserves the L^2 norm, giving U_m unitary on $\mathbb{C}^8$, i.e., $U_m \in U(8)$. *Direct inheritance from the L^2-norm structure of $\mathcal{H}_{sub}$.*
5. **Factorisation locality**: closure transport at each event does not depend on the poset's global structure beyond what S_m captures, giving $T_m = S_m \otimes U_m$ with U_m a single operator on $\mathbb{C}^8$.

   Bridged application. Non-factorised T_m where closure transport depends on poset position would also preserve norm; factorisation is a structural commitment beyond bare norm preservation. The bridge: closure-normalised directions are *intrinsic to each event* — an event carries K + 1 closure directions as part of its event-data (Definition 3), not as a function of its place in the poset — and the SU(K+1) symmetry of A (Theorem 4) acts on the closure-direction sector *uniformly across the substrate*. There is no poset-dependent deformation of SU(K+1)'s action built into the substrate structure. Factorisation locality expresses the substrate principle that closure-direction structure is event-intrinsic and uniform. Non-factorised couplings would correspond to *additional substrate physics* not in the current formulation; they are deferred to higher-order corrections (Open Problem #11(c)).

6. **Traceless restriction**: $U_m \in SU(8)$, not the full U(8).

   Bridged application. This invokes Theorem 1 (Traceless Selection, §9). Theorem 1 establishes that non-trivial contributions to the Higgs mass-ratio renormalisation live in the traceless sector $\mathfrak{su}(K+1)$; the trace direction $\mathfrak{u}(1)$ is the radial-magnitude sector, which V_{sub} fixes at $\|\rho\| = \rho_c$. Commitment 6 is the *substrate-dynamics* counterpart: U_m must preserve the same admissibility-algebra structure that Theorem 1 analyses. The two claims are related but not identical — Theorem 1 is a renormalisation statement, commitment 6 a transport-symmetry statement — and the bridge is that both rest on the structural fact that admissibility is encoded in the traceless deformation sector. $U_m \in U(8)$ with non-trivial $\mathfrak{u}(1)$ component would generate uniform phase rotations of ρ that do not preserve the orientation of closure directions in $\mathfrak{su}(K+1)$; restricting to SU(8) keeps U_m within the algebra that A is built from.

7. **Admissibility selection** (Axiom 1, Definition 4): substrate states are mapped to $\mathcal{M}_{adm}$ by an admissibility selector Π_{adm}. *Direct inheritance.*

Definition 15 (Admissibility Selector). An admissibility selector $\Pi_{adm} : \mathcal{H}_{sub} \to \mathcal{M}_{adm}$ is a map satisfying: (i) admissibility filtering: $A[\Pi_{adm} \Psi] = 0$ for all $\Psi \in \mathcal{H}_{sub}$; (ii) admissibility fixing: $\Pi_{adm} \Phi = \Phi$ for all $\Phi \in \mathcal{M}_{adm}$.

Two realisations:

- **15a (Gradient-flow selector, primary)**: $\Pi_{adm}^{GF} \Psi := \lim_{\sigma\to\infty} \rho_\Psi(\sigma)$, where $\rho_\Psi(\sigma)$ is the Axiom 2 gradient flow starting from Ψ. Theorematic for analytic-coercive-single-basin substrates by Theorem 10 (§5.9.11).
- **15b (L²-projection selector, alternative)**: $\Pi_{adm}^{L^2} \Psi := \arg\min_{\Phi \in \mathcal{M}_{adm}} \|\Psi - \Phi\|^2$. Well-defined when $\mathcal{M}_{adm}$ is closed and convex (M-Convexity hypothesis, currently unverified for the substrate's $\mathcal{M}_{adm}$).

Theorem 1′ (Minimal Admissible Refinement Dynamics). Given the seven structural commitments above, plus **strong continuity of closure transport in the transport parameter ε**, the lowest-order refinement-compatible update has the form:

$$\boldsymbol{\Psi_{n+1} = \Pi_{adm} \cdot (S_m \otimes \exp(i\varepsilon_m H_{cl})) \cdot \Psi_n}$$

where $H_{cl} \in \mathfrak{su}(8)$ is a Hermitian traceless generator.

(Numbering note: this paper's Theorem 1 is Traceless Selection (§9); Theorem 1' is the substrate microdynamics minimality theorem, distinguished by the prime.)

Proof sketch. Commitments 1, 2 give $\mathcal{H}_{sub} = \ell^2(E) \otimes \mathbb{C}^8$. Commitment 3 gives S_m acting on the $\ell^2(E)$ factor. Commitments 4, 6 give $U_m \in SU(8)$ acting on the $\mathbb{C}^8$ factor. Commitment 5 gives the factorised $T_m = S_m \otimes U_m$ structure. Commitment 7 gives Π_{adm} onto $\mathcal{M}_{adm}$. The minimal factorised update rule is therefore $\Pi_{adm} \cdot (S_m \otimes U_m) \cdot \Psi_n$. Strong continuity of closure transport in ε then gives $U_m(\varepsilon) = \exp(i\varepsilon H_{cl})$ by Stone's theorem for one-parameter unitary groups, with H_{cl} Hermitian; restriction to SU(8) gives $H_{cl} \in \mathfrak{su}(8)$. ■

On the continuity hypothesis. Stone's theorem requires a real continuous parameter ε with $U_m(\varepsilon)$ varying continuously in the strong operator topology. The candidate's parameter is $\varepsilon = \varepsilon_{coarse}/m$, real-valued. Closure transport could in principle be discrete (U_m a fixed unitary at each refinement level, with no continuous-ε family) without violating commitments 1–7; the continuous-ε form is therefore an additional structural commitment.

5.9.10.2 Identification of H_{cl}

The Hessian G of the merged paper's free-energy functional F at the admissible manifold is **Hermitian** — F is real-valued, and the Hessian of a real-valued functional on a complex configuration space (treating ρ and $\bar{\rho}$ as independent variables) is Hermitian intrinsically: $G^\dagger = G$.

Definition 16 (Closure Hamiltonian). H_{cl} is the $\mathfrak{su}(8)$-sector restriction of the Hessian of F at $\mathcal{M}_{adm}$:

$$H_{cl} = G|_{\mathfrak{su}(8)}, \quad G = \delta^2 F / \delta\bar{\rho}\,\delta\rho \,|_{\mathcal{M}_{adm}}.$$

H_{cl} is obtained by projecting G's action on the closure-normalised configuration space $\mathbb{C}^8$ onto the $\mathfrak{su}(8)$ sector — equivalently, removing the $\mathfrak{u}(1)$ trace contribution (which lies in V_{sub}'s radial direction at saturation).

On the dual interpretation of G. The Hermitian Hessian G appears in two distinct dynamical structures: the dissipative gradient flow $\partial_\sigma \delta\rho = -G\cdot\delta\rho$ of Axiom 2 (the substrate's *physical* dynamics), and a formal unitary evolution $\delta\rho(\tau) = \exp(-i\tau G)\delta\rho(0)$ under imaginary parameter $\tau = -i\sigma$. The "imaginary-σ" interpretation has no established substrate-physics meaning. The candidate's discrete-time unitary update rule uses G in this formal unitary interpretation; it is a **mathematical structure useful for refinement analysis** rather than an alternative physical picture of substrate dynamics. Axiom 2's gradient flow is the substrate's physical dynamics; the candidate microdynamics is the discrete-time Tick-level layer interfacing with the refinement-operator framework (Definition 12).

Proposition 21 (Spectral content of H_{cl}). $H_{cl} \in \mathfrak{su}(8)$ is a Hermitian traceless operator on $\mathbb{C}^8$ with at most 8 eigenvalues summing to zero. The eigenvalues are determined by the collective (substrate-integrated) action of the A-Hessian on closure-normalised directions, projected onto the $\mathfrak{su}(8)$ sector. The $\mathfrak{u}(1)$ trace direction lies outside H_{cl} by construction.

5.9.10.3 Schur's-lemma commutativity and SU(8) representation content

The Hessian decomposes as $G = G_V + G_C + G_T + G_R$ from $F = \int V_{sub} + A_C + A_T + A_R$. After the $\mathfrak{su}(8)$ restriction:

$$H_{cl} \simeq (G_C + G_T + G_R)|_{\mathfrak{su}(8)}.$$

Proposition 22 (Schur's-lemma commutativity of admissibility Hessians). At the saturated admissible configuration, G_C, G_T, G_R commute within each SU(8) isotype of the tangent space. Within each SU(8)-irreducible isotype R:

$$G_X|_{\text{isotype } R} = \alpha_X^{(R)} \cdot \mathbb{1}_R, \quad X \in \{C, T, R\},$$

for some scalar $\alpha_X^{(R)}$. Their sum G is block-diagonal under the SU(8) isotype decomposition with eigenvalues $\Sigma_X \alpha_X^{(R)}$ per isotype R.

Proof. Each A_X is SU(8)-invariant (Theorem 4), so each Hessian G_X commutes with the SU(8) action on tangent vectors at the saturated configuration. By Schur's lemma, an SU(8)-invariant linear operator acts as a scalar within each SU(8)-irreducible isotype. Scalar operators commute. ■

SU(8) representation content of inconsistency operators.

- **C (closure inconsistency, adjoint of SU(8)).** $C(e_i, e_j, e_k) = U_{ij} \cdot U_{jk} \cdot U_{ki} - \mathbb{1}$ with each $U \in SU(8)$; first-order variation lies in $\mathfrak{su}(8)$ = adjoint.
- **J_ρ (transport current, fundamental of SU(8)).** $A_T = \int \|\nabla \cdot J_\rho + \partial_\sigma \rho\|^2 \, d\mu$ with J_ρ built linearly from $\rho \in \mathbb{C}^8$ (fundamental).
- **Φ (record morphisms, adjoint + trivial of SU(8)).** $\Phi_{ij} \in \mathrm{End}(\mathbb{C}^8)$ = adjoint ⊕ trivial. Φ-defects are adjoint-valued; the trivial component (overall magnitude scaling) is fixed at ρ_c at saturation and does not contribute to $G|_{\mathfrak{su}(8)}$.

5.9.10.4 Conjecture 8: Uniform Admissibility Hessian Scaling

The three admissibility components have different scalings with substrate parameters: $A_C \sim \rho_c^6$, $A_R \sim \rho_c^4$; A_T's scaling depends on how J_ρ scales with ρ. After non-dimensionalisation, define $\alpha_X^{(R)} = \lambda_{sub}^{p_X^{(R)}} \cdot \rho_c^{q_X^{(R)}} \cdot \hat{\alpha}_X^{(R)}$ with $\hat{\alpha}_X^{(R)}$ dimensionless.

Conjecture 8 (Uniform Admissibility Hessian Scaling). The eigenvalue ratios of H_{cl} across distinct SU(8) isotypes of the tangent space at the saturated configuration are substrate-parameter independent if and only if, after non-dimensionalisation, the dimensionless ratios $\hat{\alpha}_C^{(R)} : \hat{\alpha}_T^{(R)} : \hat{\alpha}_R^{(R)}$ are R-dependent but substrate-parameter-independent.

Status. Conjecture 8's resolution may require substrate-physics input rather than mathematical derivation. Axiom 1 specifies non-negative components but does not fix their relative coefficients. If the relative coefficients are *additional substrate-physics inputs* not derivable from λ_{sub}, ρ_c, β alone, Conjecture 8 is a parameter-relation claim rather than a derivable theorem.

5.9.11 Selector Existence and Tick/Bit Structure

§5.9.11 supplies the gradient-flow admissibility selector with theoremic existence under standard hypotheses, and positions the candidate microdynamics within the Tick/Bit ontology of §5.8.

5.9.11.1 Gradient-flow selector existence

Proposition 23 (Gradient-Flow Selector Existence, four sub-hypotheses). The gradient-flow selector Π_{adm}^{GF} (Definition 15a) exists as a valid admissibility selector under: (i) global existence $\rho_\Psi(\sigma)$ $\forall \sigma \in [0, \infty)$; (ii) convergence $\lim_{\sigma\to\infty} \rho_\Psi(\sigma) \in \mathcal{M}_{adm}$; (iii) basin uniqueness; (iv) regularity (at least measurable).

Theorem 10 (Gradient-Flow Selector Existence for Analytic Coercive Single-Basin Substrates). Let $F : \mathcal{H}_{sub} \to \mathbb{R}_{\geq 0}$ be real analytic, coercive, and bounded below, with

$\mathcal{M}_{adm} = \{\Psi : F(\Psi) = 0\}$. Assume each connected basin of the gradient flow $\partial_\sigma \rho = -\delta F/\delta\bar{\rho}$ contains exactly one admissible limit point. Then the gradient-flow selector

$$\Pi_{adm}^{GF}\, \Psi = \lim_{\sigma\to\infty} \rho_\Psi(\sigma)$$

exists and satisfies the admissibility-selector axioms of Definition 15.

Proof sketch. Coercivity prevents trajectories from escaping to infinity (the sublevel set $\{\rho : F(\rho) \le F(\Psi)\}$ is bounded; $\partial_\sigma F = -\|\delta F/\delta\bar{\rho}\|^2 \le 0$). Boundedness below plus monotonic decrease ensures $F[\rho_\Psi(\sigma)]$ has a limit. Real analyticity allows application of the **Łojasiewicz-Simon convergence theorem**: for analytic functionals, gradient-flow trajectories in compact sets converge to a single critical point rather than oscillating between critical values. Boundedness from coercivity provides compactness. The critical set with F-value zero is exactly $\mathcal{M}_{adm}$; the trajectory's limit is therefore in $\mathcal{M}_{adm}$. Single-basin assumption ensures uniqueness. By analyticity, the selector map is measurable. At admissible $\Phi \in \mathcal{M}_{adm}$, $\delta F/\delta\bar{\rho} = 0$, so the flow is stationary: $\Pi_{adm}^{GF}\, \Phi = \Phi$. ■

What Theorem 10 establishes. Three of Proposition 23's four sub-hypotheses become derived consequences under standard analytic-coercive conditions: only basin-uniqueness remains as substrate-specific input. The merged paper's $F = \int V_{sub} + A$ satisfies analyticity (polynomial in $\rho, \bar{\rho}$), coercivity ($V_{sub} \sim \|\rho\|^4$), and bounded-below ($F \ge 0$). Theorem 10 therefore establishes selector existence theorematically for the framework's F, with basin-uniqueness the only substrate-specific structural hypothesis.

5.9.11.2 Tick/Bit structural positioning

Definition 17 (Tick). A Tick is one application of the candidate microdynamical update rule: $\Psi_{n+1} = \Pi_{adm}^{GF} \cdot (S_m \otimes \exp(i\varepsilon_m H_{cl})) \cdot \Psi_n$. Equivalently, a Tick is the discrete substrate evolution step $n \to n+1$.

Tick-level structure (reversible admissible evolution): S_m (admissible refinement, information-preserving), $\exp(i\varepsilon_m H_{cl})$ (closure transport within SU(8), norm-preserving), and Π_{adm}^{GF} (the bridge between reversible Tick-level evolution and irreversibility). The first two are reversible; Π_{adm}^{GF} is the discrete information-reducing step.

Bit-level structure (irreversible committed distinctions): a Bit corresponds to a refinement-stable observable in the sense of Definition 11. Under iterated Ticks, refinement-stable observables retain their values and constitute the substrate record. Irreversible accumulation of such distinctions generates record growth and temporal ordering (§5.8).

The candidate microdynamics supplies the admissible Tick-level evolution underlying Bit formation, rather than the irreversible commitment mechanism itself.

TPB consistency. The refinement-composition identity $T_m^m = T$ (Definition 13(i)) is the algebraic content of multi-scale admissibility consistency: TPB acts as a multi-scale admissibility constraint preventing refinement-path ambiguity.

Conceptual hierarchy:

Structure	Role
Tick (Definition 17)	Reversible admissible substrate evolution
TPB ($T_m^m = T$)	Multi-scale transport consistency constraint
Π_{adm}^{GF} (Definition 15a)	Admissibility enforcement
Bit (refinement-stable observables)	Irreversible committed distinction
Record	Accumulated committed information
Time	Ordering of irreversible record growth

5.9.11.3 Verification of Definition 13's algebraic identities

- **(i) Refinement composition $T_m^m = T$.** With $\varepsilon_m = \varepsilon_{coarse}/m$: $T_m^m = S_m^m \otimes \exp(i\varepsilon_{coarse} H_{cl})$. Reduces to $S_m^m = S$ by associativity of edge-bisection (Proposition 18). ✓
- **(ii) Idempotence $\Pi_{adm}^2 = \Pi_{adm}$.** For $\Pi_{adm} = \Pi_{adm}^{GF}$, by Definition 15a (selector axioms). Theorematic via Theorem 10 for analytic-coercive-single-basin substrates; otherwise conditional on Proposition 23 sub-hypotheses. ✓ (conditional)
- **(iii) Admissibility-preserving commutation.** Closure side preserves admissibility (Theorem 4). Poset side under Refinement Extension hypothesis (verified for the diamond; part of Open Problem #9 for general substrates). ✓ (conditional)
- **(iv) Admissibility filtering $A[\Pi_{adm} \Psi] = 0$.** By Definition 15a. ✓ (conditional)

5.9.11.4 Predictions and falsifiability

Prediction (Algebraic spectral pattern, conditional on Conjecture 8). H_{cl} has at most 8 eigenvalues on $\mathbb{C}^8$ with representation content fixed by the SU(8) isotype decomposition of §5.9.10.3: adjoint contributions from C, fundamental from J_ρ, adjoint from Φ-defects.

- If Conjecture 8 holds, eigenvalue *ratios* between distinct isotypes are parameter-independent.
- If Conjecture 8 fails, only the representation-block structure is parameter-independent.

Higgs-sector compatibility (not derivation). The candidate microdynamics does not derive $m_H/v = 32/63$. The Higgs radial mass lives in the $\mathfrak{u}(1)$ trace direction; $H_{cl} \in \mathfrak{su}(8)$ is the traceless restriction and does not contain the radial mode. Theorems 1, 2, 5, 6 + Proposition 13 (Part II) supply the Higgs prediction independently of the candidate microdynamics.

Part II — Finite Electroweak Curvature

6. Closure Cardinality Universality

Conjecture 2 (K = 7 Universality). The substrate-level closure cardinality is K = 7; sectorial cardinalities inherit this from the substrate.

K = 7 is derived elsewhere in the VERSF programme through six independent routes (see *K = 7 Convergence* in the references). This paper takes K = 7 as input from Conjecture 2.

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7. The Higgs Ratio as a Curvature Quantity

For the Standard Model potential $V(\varphi) = \lambda(\varphi^\dagger\varphi - v^2)^2$ with vacuum at $\varphi = v$:

$m_H^2 = 2\lambda v^2 \Rightarrow m_H / v = \sqrt{2\lambda}$.

The Higgs ratio is therefore the curvature of V along the radial direction, normalised by the vacuum scale. Part II computes $\sqrt{2\lambda}$ from the substrate.

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8. The Closure-Normalised Baseline

8.1 The identification rule

$\rho = \rho_c \cdot \hat{e}$ for an 8-component closure-normalised direction $\hat{e} \in \mathbb{C}^8$ with $\|\hat{e}\| = 1$. The Higgs field is identified with $\hat{e}$ by:

$\hat{e} = (\varphi/\rho_c, 0, \ldots, 0)$.

That is, the Higgs field occupies one of the 8 closure-normalised directions; the remaining 7 are non-Higgs sectors. Derivation of this identification from substrate first principles is Open Problem #7; §8.1.1 supplies a candidate variational derivation.

8.1.1 Sector-Selection Criterion

The §8.1 identification rule selects one closure-normalised direction; this subsection makes the selection rule mathematically concrete.

Sector-admissibility functional. For closure-normalised subspace $\mathcal{S} \subset \mathbb{C}^{K+1}$:

$$\mathbf{A_{sector}[\mathcal{S}] = A_{SU(2)\times U(1)}[\mathcal{S}] + A_{radial}[\mathcal{S}] + A_{fermion}[\mathcal{S}]}$$

with the following first-pass forms.

SU(2)×U(1) closure-preserving transport. Let L_A be the admissibility transport operator (Definition 8); $\Pi_{\mathcal{S}}$ the projection onto $\mathcal{S}$; D_{EW} the electroweak generator algebra

($SU(2)\times U(1)_Y$ generators) on the substrate via the $K = 7 \to$ SM gauge-group reduction (see *Derivation of the Standard Model Gauge Group within VERSF*):

$$A_{SU(2)\times U(1)}[\mathcal{S}] = \| [L_A, \Pi_{\mathcal{S}}] - D_{EW} \|^2$$

Generic $\mathcal{S}$ produces generic $[L_A, \Pi_{\mathcal{S}}]$ with no special relation to electroweak structure; the unique minimiser is $\mathcal{S}_{EW}$ where the substrate-transport commutator equals D_{EW}.

Radial-mode stability. With $r_{\mathcal{S}} = \|\Pi_{\mathcal{S}} \rho\|$ and m_{rad}^2 the target Higgs mass-squared:

$$A_{radial}[\mathcal{S}] = \left(\frac{d^2V_{sub}}{dr_{\mathcal{S}}^2}\Big|_{r_{\mathcal{S}} = \rho_c} - m_{rad}^2 \right)^2$$

By Axiom 3, $d^2V_{sub}/dr^2|_{\rho_c} = \lambda_{sub} \cdot \rho_c^2$ (giving the substrate-derived radial mass). The condition selects $\mathcal{S}$ whose radial direction matches the observed Higgs mass.

Fermion-coupling admissibility. With $Y_f(\mathcal{S})$ the Yukawa coupling of fermion species f to the Higgs field on $\mathcal{S}$, and Y_f^{adm} the admissible coupling derived from substrate closure geometry (see *Deriving Flavour Mixing from Closure Geometry*):

$$A_{fermion}[\mathcal{S}] = \Sigma_f \| Y_f(\mathcal{S}) - Y_f^{adm} \|^2$$

The criterion.

$$\mathcal{S}_{EW} = \arg\min_{\mathcal{S} \subset \mathbb{C}^{K+1}} A_{sector}[\mathcal{S}]$$

Conjecture 4 (Electroweak Sector Selection). A_{sector} has a unique minimiser $\mathcal{S}_{EW} \subset \mathbb{C}^{K+1}$ subject to $A[\rho] = 0$. The minimiser coincides with §8.1's identification.

What remains open: derivation of D_{EW}, m_{rad}^2, Y_f^{adm} from substrate first principles (each ties to a companion paper); existence and uniqueness of the minimiser; verification on explicit calculation.

8.2 The Closure Quartic Democracy theorem

Theorem 5 (Closure-Quartic Democracy). Under Axiom 3 + closure-unit normalisation + §8.1 (refined by §8.1.1): the baseline quartic coupling is

$\boldsymbol{\lambda_0 = 1 / (K + 1)}$.

For $K = 7$: $\lambda_0 = 1/8$, giving $(m_H/v)_{baseline} = \sqrt{2 \cdot 1/8} = \sqrt{1/4} =$ **1/2**.

---

9. The Admissible Deformation Algebra

Theorem 1 (Traceless Selection). Non-trivial renormalisations of m_H / v live on the traceless deformation algebra $\mathfrak{su}(K+1)$ — not on the full $\mathfrak{u}(K+1)$. The $\mathfrak{u}(1)$ trace direction corresponds to radial-magnitude scaling of ρ, which V_{sub} fixes at saturation and which decouples from the deformation-algebra renormalisation.

The dimension of this algebra:

$D_{corr} = \dim \mathfrak{su}(8) = 8^2 - 1 = 63$.

---

10. The Uniform Renormalisation Theorem

Theorem 2 (Uniform Renormalisation). Under closure-unit normalisation, the renormalisation contribution from each direction of $\mathfrak{su}(K+1)$ is uniform:

$\delta = 1 / D_{corr} = 1 / 63$.

---

11. Killing-Form Uniqueness

Theorem 6 (Killing-Form Uniqueness). Every SU(K+1)-invariant bilinear form on $\mathfrak{su}(K+1)$ is proportional to the Killing form. Under the closure-unit normalisation, the proportionality constant is fixed: each deformation direction is weighted by $1 / D_{corr}$.

The uniform weighting of Theorem 2 is therefore not a choice but a consequence of SU(8)-invariance plus normalisation.

---

12. Linear Action on the Curvature Ratio

Proposition 13 (Linear Action on Curvature). Closure deformations rotate the closure-normalised direction ê without changing its norm. The Higgs ratio m_H / v shifts linearly under these rotations; the underlying coupling λ is unchanged at first order.

This is the substrate-level statement of how $\mathfrak{su}(8)$ deformations act on the §8.1 identification.

---

13. The Effective Closure Potential

Combining the baseline (Theorem 5: 1/2) with the uniform 𝔰𝔲(8) renormalisation (Theorem 2: 1/63) by linear action (Proposition 13):

$(m_H / v)_{eff} = (1/2) \cdot (1 + 1/63) = 32/63 \approx 0.50794$.

For the underlying coupling:

$\lambda_{eff} = (1/2) \cdot (32/63)^2 \approx 0.1290$ (compare $\lambda_{SM} \approx 0.1293$).

---

14. Why 63 and Not Some Other Number

Alternative numerator candidates (64, 56, 49, 28, 21) are excluded on algebra-intrinsic grounds. 64 = dim 𝔲(8) includes the 𝔲(1) trace direction, excluded by Theorem 1 (Traceless Selection). 56, 49, 28, 21 correspond to other natural-looking SU(8) representation dimensions (such as antisymmetric 3-tensor or symmetric 2-tensor) but violate the Killing-form uniqueness of Theorem 6: the Casimir invariant, Dynkin index, and rank of 𝔰𝔲(8) jointly select dim 𝔰𝔲(8) = 63 as the unique admissible weighting.

---

15. Scheme Commitment and Falsification

Prediction: $m_H / v = 32/63 = 0.50794$.

Observation (PDG 2024, ATLAS+CMS): 0.50849 ± 0.00045.

Residual: $\approx 1.22\sigma$.

3σ falsification band: [0.50639, 0.50949].

HL-LHC projected band: [0.50734, 0.50854].

The prediction is conditional on Conjecture 2 (K = 7), the §8.1 identification rule (refined by Conjecture 4 / §8.1.1), and Proposition 13's linear action. If HL-LHC precision pushes the measurement outside the 3σ band, the prediction fails and the framework's structural commitments require revision; Appendix B's logical dependency graph identifies which structural element has failed.

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16. Closure Renormalisation vs Continuum Renormalisation

The framework's renormalisation operates on a *finite* admissible deformation algebra (dim $\mathfrak{su}(8) = 63$), not on a continuum of momentum modes. There are no UV divergences in the closure-renormalisation calculation. Standard continuum renormalisation is an emergent low-energy approximation: the substrate's discrete admissible coarse-graining (§5.9) supplies the continuum-level RG flow as a refinement-stable observable structure (Definition 12).

Part III — The Statistical Substrate Layer

17. The Substrate Probability Measure

17.1 Axiom 5 and the Gibbs-Boltzmann form

Axiom 5 (Substrate Probability Measure). $\mathcal{D}\rho = (1/Z) \cdot \exp(-\beta \cdot F[\rho]) \cdot \mathcal{D}_0\rho$.

The substrate's statistical layer takes a Gibbs-Boltzmann form with inverse-temperature parameter $\beta > 0$ and a reference measure $\mathcal{D}_0\rho$ (specification of which is Open Problem #4).

17.2 Max-entropy derivation

Proposition 10 (Gibbs-Boltzmann from Max-Entropy). Among substrate probability measures with fixed expected free-energy $\langle F \rangle$, the Gibbs-Boltzmann form of Axiom 5 is the unique maximum-entropy measure.

This justifies Axiom 5's specific exponential form as a structural consequence of the maximum-entropy principle applied to substrate states.

17.3 $\beta \to \infty$ limit

In the $\beta \to \infty$ limit, the Gibbs-Boltzmann measure concentrates on the global minimum of F. The substrate becomes deterministic and reduces to Axiom 2's gradient-flow dynamics. The statistical layer thus interpolates between thermal noise (small β) and the deterministic-substrate limit (large β).

17.4 Conjecture 3: Born-compatible marginal

Conjecture 3 (Born-Compatible Marginal). Marginal probabilities for direction-only observables on the substrate take the form $P(i) \propto |\rho_i|^2$, consistent with the Born rule of quantum mechanics.

Quantum emergence from the substrate is Open Problem #5.

17.5 Parameter dependence

The framework has three substrate parameters: λ_sub, ρ_c, β. **The Higgs ratio prediction $m\_H/v = 32/63$ depends on none of them.** The prediction is purely structural, governed by $K = 7$ closure cardinality and the dimension of $\mathfrak{su}(8)$. Numerical determination of λ_sub, ρ_c, β is Open Problem #6.

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18. Remaining Open Problems

1. **Universal codimension-1 identification.** Minimum-triangle-degree localisation produces interface concentration in standard poset families (Proposition 8); universality across admissible poset families is Conjecture 1.
2. **Full continuum Lorentzian dynamics.** Theorem 9 establishes invariant continuum-limit propagation speed; Proposition 20 establishes that non-Abelian Lorentzian corrections refine away exponentially. Full continuum-limit Lorentzian dynamics — including curvature, full diffeomorphism invariance, and matter coupling — remains open.

2′. **Derivation of signature from substrate structure.** §5.7.5's Signature Selection Programme supplies a candidate variational derivation route (Conjecture 5 with concrete $\mathcal{C}\_causal$, $\mathcal{C}\_hyperbolic$). Remaining: derivation of these functionals as the unique admissibility-preserving forms; proof of Lorentzian uniqueness.

3. **Single-constraint structure of Axiom 5.** Whether the substrate probability measure can be derived from a single underlying admissibility constraint (rather than introduced as an additional axiom) is open.
4. **Reference measure $\mathcal{D}_0\rho$.** Specification of the reference measure in Axiom 5 from substrate first principles.
5. **Quantum emergence.** Conjecture 3 supplies the Born-compatible marginal; full quantum emergence (Hilbert spaces, operators, measurement) is developed in companion *Quantum Reconstruction* paper.
6. **Numerical determination of λ_sub, ρ_c, β.** The framework has three free parameters; the principal predictions are independent of them, but their numerical values are not determined within this paper.
7. **Self-contained derivation of the identification rule.** §8.1.1's Sector-Selection Criterion (Conjecture 4) supplies a candidate variational derivation route. Remaining: derivation of D_EW, m_rad^2, $Y\_f^{\wedge}adm$ from companion papers; existence and uniqueness of A_sector-minimiser. Proposition 20 establishes that non-Abelian gauge structure is admissibility-compatible.
8. **Three modes exhaustiveness.** Propositions 2, 3 identify three admissibility modes (C, T, R); a boundary-inconsistency candidate fourth mode has not been ruled out.
9. **Admissible coarse-graining and continuum reconstruction.** Propositions 11, 12, 14, 16, 18 + Theorem 8 + Theorem 9 + Proposition 20 = current progress. Full theory of admissible coarse-graining is a separate companion paper.
10. **Exact Lorentz invariance under coarse-graining.** Proposition 11 establishes leading-order interval stability. Full Lorentz invariance — including all higher-order terms — is a separate companion paper.

11. **Substrate microdynamics.** §§5.9.10, 5.9.11, and Appendix F supply the candidate update rule (Theorem 1′), closure Hamiltonian identification (Definition 16), gradient-flow selector existence (Theorem 10), Tick/Bit positioning, diamond worked example with $H_{cl} = L_{12}$, and reflection-symmetry-reduction argument. Open sub-targets: (a) basin-uniqueness verification beyond the symmetric diamond; (b) full Hessian computation on the diamond extracting $\mu(\theta)$, $\alpha_X^{(R)}$ prefactors, testing Conjecture 8; (c) explicit multi-generator non-Abelian update rule; (d) Conjecture 8 epistemic status (theorem vs substrate-physics input); (e) Tick-to-Bit transition mechanism.

18.1 Failure modes

The framework is constructed to be falsifiable structurally, not just phenomenologically. Each conditional hypothesis has a specific consequence if false:

Structural element	If false, consequence
Basin uniqueness (Theorem 10 hypothesis (iii); §5.9.11)	Gradient-flow selector Π_{adm}^{GF} becomes multi-valued. Theorem 10's selector-existence result inapplicable; selector existence falls back to Proposition 23's four-sub-hypothesis conditional structure. Definition 15a no longer supplies a well-defined map.
Conjecture 8 (Uniform Admissibility Hessian Scaling; §5.9.10.4)	Eigenvalue ratios of H_{cl} become substrate-parameter-dependent. Prediction (algebraic spectral pattern) loses parameter-independence content; only SU(8) representation-block structure remains parameter-independent.
Refinement Extension (§5.9.11.3)	Admissibility-preserving refinement fails. Definition 13(iii) breaks. Candidate microdynamics's compatibility with the Minimal Microdynamical Skeleton becomes substrate-dependent.
Proposition 20 extension to specific generators	Non-Abelian transport with specific SU(8) generators (e.g., $SU(2)\times U(1)_Y$ generators identified by Conjecture 4) fails to refine with the 16^{-n} rate. Compatibility with full electroweak gauge structure becomes problematic.
Strong continuity of closure transport (Theorem 1′ hypothesis)	Exponential form $U_m(\varepsilon) = \exp(i\varepsilon_m H_{cl})$ invalid. Stone's theorem doesn't apply; H_{cl} generator structure replaced by discrete unitary structure. Candidate's specific operator form requires reformulation.
M-Convexity (Definition 15b hypothesis)	L^2-projection selector $\Pi_{adm}^{L^2}$ not well-defined. Affects only the alternative selector; primary gradient-flow selector unaffected.
K = 7 (Conjecture 2)	Closure cardinality ≠ 8. Substrate Hilbert-space factor $\mathbb{C}^{K+1} \neq \mathbb{C}^8$; $SU(K+1) \neq SU(8)$. Higgs prediction requires reformulation; under K = 7 the result is 32/63, but a different K gives a different number and the agreement with observation is generically lost.
Theorem 4 (Quadratic admissibility from SU(K+1) symmetry)	A not quadratic in inconsistency operators in their SU(K+1) representations. Schur's-lemma commutativity (§5.9.10.3) loses its SU(8)-invariance premise; G_C, G_T, G_R need not commute within isotypes. H_{cl} identification structure requires alternative analysis.

Structural element	If false, consequence
Single-basin verification on the symmetric diamond (Appendix F.5)	Even the diamond loses theorematic selector existence. Theorem 10 applicability on the diamond fails; selector existence on the diamond falls back to Proposition 23 sub-hypotheses.

Each row identifies a specific empirical or mathematical fact that, if established to fail, narrows the framework's claims precisely. Vulnerability is the price of structural specificity.

18.2 Next calculational target

Among the open problems above, the most immediate next calculational target is:

Explicit extraction of the Hessian coefficients α_X^{(R)} on the symmetric diamond configuration to test Conjecture 8 (Open Problem #11(b)).

Concretely: compute G = δ²F/δρ̄δρ at the diamond's saturated configuration with directional structure (Appendix F.1); decompose G|*{𝔰𝔲(8)} into the three isotypes identified in Appendix F.8 ({L*{12} antisymmetric, (e_1, e_2)-plane reflection-even longitudinal, e_3–e_8 collective}); extract per-isotype, per-component scalars α_C^{(R)}, α_T^{(R)}, α_R^{(R)} from the explicit forms of A_C, A_T, A_R (§§3.2–3.4); non-dimensionalise to α̂_X^{(R)}; test whether the ratios α̂_C^{(R)} : α̂_T^{(R)} : α̂_R^{(R)} are parameter-independent across the three isotypes.

This calculation: tests Conjecture 8 directly on the simplest non-trivial substrate; determines whether Conjecture 8 is a theorem or substrate-physics input; yields the scalar μ(θ) of Appendix F.7; refines Open Problem #11(b) to substantive content. It requires no new theoretical apparatus, only careful Hessian computation on the diamond using §§3.2–3.4's explicit functionals.

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19. Conclusion

The framework has three structural parts.

Part I (deterministic substrate dynamics). Five axioms produce ten theorems, twenty propositions, eight conjectures, and seventeen definitions. §5.9 identifies admissible coarse-graining as the central structural mechanism, with: §5.9.3's continuum reconstruction framework (Theorem 8, Proposition 14, Propositions 16, 18); §5.9.7's Minimal Microdynamical Skeleton; §5.9.8's invariant propagation speed (Theorem 9); §5.9.9's non-Abelian defect suppression (Proposition 20); and §§5.9.10–5.9.11's substrate microdynamics specification (Theorem 1′, Theorem 10, Conjecture 8, Definitions 15–17).

Part II (Higgs prediction). From K = 7 closure cardinality (Conjecture 2), the §8.1 identification rule (refined by §8.1.1's Sector-Selection Criterion), and the renormalisation results of Theorems 1, 2, 5, 6 + Proposition 13:

$m_H / v = 32 / 63 \approx 0.50794$

against observed 0.50849 ± 0.00045 — residual ≈ 1.2σ. The prediction is independent of all three substrate parameters λ_{sub}, ρ_c, β.

Part III (statistical substrate layer). Gibbs-Boltzmann substrate measure (Axiom 5, Proposition 10) with the Born-compatible marginal conjecture (Conjecture 3).

The framework's structural commitments. Six principal structural commitments are summarised in Appendix B's dependency graph: substrate primitives (Definitions 1–4); admissibility decomposition (Axiom 1, Theorem 4); commitment dynamics (Axioms 2, 3); emergent geometry (Definitions 5–9, Theorem 7, Axiom 4, Proposition 11); admissible coarse-graining and continuum reconstruction (Definitions 10–12, Theorem 8, Theorem 9, Proposition 20, Theorems 1′ and 10); statistical layer (Axiom 5).

Operational consequences. The framework is constructed to be refuted on identifiable grounds. The Higgs prediction is structural and quantitatively testable: if HL-LHC pushes the measurement outside [0.50734, 0.50854], the prediction fails. The candidate microdynamics is structurally vulnerable: if direct refinement-stability calculations fail to yield exponential commutator-defect suppression on candidate non-Abelian substrate posets, Proposition 20 is falsified; if H_{cl}'s representation content fails to match the SU(8) isotype structure of §5.9.10.3 on calculable substrate examples, Theorem 1′ and Definition 16 face a structural problem. §18.1's failure-mode table identifies each conditional commitment's specific narrowing consequence.

Relation to companion papers. *Admissible Coarse-Graining Theory* (Open Problem #9) and *Exact Lorentz Invariance under Coarse-Graining* (Open Problem #10) are separate companion papers. *Derivation of the Standard Model Gauge Group within VERSF* and *Deriving Flavour Mixing from Closure Geometry* supply structural inputs to §8.1.1's Sector-Selection Criterion. *Quantum Reconstruction* develops Conjecture 3's connection to quantum mechanics. The substrate microdynamics specification is internal to this paper (§§5.9.10, 5.9.11, Appendix F).

If HL-LHC lands within the predicted band, the implication is that dimensionless Standard Model constants encode the finite admissible deformation structure of a discrete substrate. If outside, Appendix B's dependency chain identifies which element has failed. If the conditional structural commitments fail on calculable examples, §18.1's failure-mode table identifies which part of the framework narrows.

Appendix A — Numerical Summary

Quantity	Symbol	Value
Closure cardinality	K	7
Closure-normalised dimension	K + 1	8

Quantity	Symbol	Value
Baseline coupling (Theorem 5)	$\lambda_0 = 1 / (K + 1)$	0.1250
Baseline ratio	$(m_H / v)_{baseline}$	0.5000
Admissible deformation algebra	—	$\mathfrak{su}(8)$
Admissible algebra dimension	$D_{corr} = \dim \mathfrak{su}(8)$	63
Per-mode weighting (Theorem 6)	$\delta = 1 / D_{corr}$	0.01587
Curvature shift	$1 + \delta$	1.01587
Predicted ratio	$(m_H / v)_{eff} = (1/2)(1 + 1/63)$	$32/63 \approx 0.50794$
Observed ratio (PDG 2024)	m_H / v	0.50849 ± 0.00045
Residual	—	$\approx 1.22\sigma$
3σ falsification band	—	[0.50639, 0.50949]
HL-LHC projected band	—	[0.50734, 0.50854]
Effective coupling	λ_{eff}	≈ 0.1290
SM coupling (PDG)	λ_{SM}	≈ 0.1293
Non-Abelian RG-suppression rate (Prop. 20)	$A_C^{(n)} / A_C^{(0)}$	16^{-n}

Appendix B — Logical Dependency Graph

```
PART I — Deterministic Substrate

[DEFINITIONS 1–4, 9]       Commitment events, ordering, density,
admissibility, triangle-degree
        ↓
[AXIOM 1]                  Admissibility Decomposition
[PROPOSITIONS 2, 3]        Three modes; exhaustiveness conjectural
[THEOREM 4]                Quadratic admissibility from SU(K+1)
[PROPOSITION 7]            Additivity
        ↓
[AXIOMS 2, 3]              Gradient flow; quartic potential
[THEOREM 3]                Logistic dynamics
[PROPOSITIONS 4, 5]        Growth, decay
        ↓
[THEOREM 7]                Triangle-Degree Localisation
[PROPOSITION 8]            Codimension-1 boundary
[CONJECTURE 1]             Universal interface localisation
        ↓
[DEFINITIONS 6–8]          Correlation kernel, distance, transport operator
[PROPOSITIONS 6, 9]        Triangle inequality; Lorentzian causal structure
        ↓
[AXIOM 4]                  Lorentzian Interval Ansatz
[§5.7.5, PROPOSITION 15]   Minimal Quadratic Penalty Principle
[§5.7.5, PROPOSITION 17]   Inconsistency-Operator Selection Principle
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```
[CONJECTURE 5]             Hyperbolic Admissibility Selection
        ↓
[DEFINITION 10]            Admissible coarse-graining Γ
[PROPOSITION 11]           Lorentzian Interval Stability (leading order)
        ↓
[§5.8]                     Relation to VERSF Quantum Reconstruction
        ↓
[DEFINITIONS 11, 12]       Refinement stability; refinement operator R
[PROPOSITION 12]           Continuum Reconstruction Criterion
[THEOREM 8]                Conditional Continuum Reconstruction
[PROPOSITION 14]           Refinement Confluence Criterion
[CONJECTURE 6]             Admissible Refinement Existence
[PROPOSITION 16]           Existence for Diamond-like Posets
[PROPOSITION 18]           Edge-Bisection Existence for Finite DAG Posets
        ↓
[DEFINITION 13 + (i)-(iv)]   Minimal Microdynamical Skeleton + identities
[DEFINITION 14]            Concrete Microdynamical Realisation
[CONJECTURE 7]             Microdynamical Closure
        ↓
[§5.9.8, PROPOSITION 19]  Propagation-Speed Stability
[THEOREM 9]                Invariant Continuum-Limit Propagation Speed
        ↓
[§5.9.9, PROPOSITION 20]  Non-Abelian Defect Suppression
        ↓
[§5.9.10, DEFINITIONS 15, 16]   Admissibility selector; closure Hamiltonian
H_cl
[§5.9.10, THEOREM 1′]     Minimal Admissible Refinement Dynamics
[§5.9.10, PROPOSITIONS 21, 22]  Spectral content; Schur's-lemma commutativity
[§5.9.10, CONJECTURE 8]   Uniform Admissibility Hessian Scaling
        ↓
[§5.9.11, DEFINITION 17]  Tick
[§5.9.11, PROPOSITION 23] Gradient-flow selector existence (four sub-hyps)
[§5.9.11, THEOREM 10]     Selector existence for analytic-coercive-single-
basin


PART II — Higgs Curvature

[CONJECTURE 2, §8.1, §8.1.1 + CONJECTURE 4]
[THEOREM 5]                Closure Quartic Democracy
[BASELINE]                 (m_H / v)_baseline = 1/2
[THEOREMS 1, 6, 2]         Traceless Selection; Killing-form; Uniform
Renormalisation
[PROPOSITION 13]           Linear Action on Curvature Ratio
[PREDICTION]               m_H / v = 32 / 63 ≈ 0.50794


PART III — Statistical Substrate Layer

[AXIOM 5; PROPOSITION 10; CONJECTURE 3]


APPENDICES
APPENDIX D                 Minimal Scalar Toy Refinement Model (diamond)
APPENDIX E                 Uniform-Direction Vector Refinement Model
APPENDIX F                 Non-Uniform Vector / Substrate Microdynamics Worked
Example
```

Appendix C — Notation

Symbol	Meaning
E	Set of commitment events
$\Lambda = (E, \preceq)$	Substrate poset
$\rho : E \to \mathbb{C}^{K+1}$	Vector-valued commitment density
$u(e) = \|\rho(e)\|^2$	Local saturation
$A[\rho] = A_C + A_T + A_R$	Admissibility functional
$\mathcal{M}_{adm} = \{\rho : A[\rho] = 0\}$	Admissible manifold
$V_{sub} = (\lambda_{sub}/4)(\|\rho\|^2 - \rho_c^2)^2$	Substrate potential
$F[\rho] = \int V_{sub}\, d\mu + A[\rho]$	Free-energy functional
σ	Gradient-flow parameter (Axiom 2)
$\lambda_{sub}, \rho_c, \beta$	Substrate parameters
$K = 7$	Closure cardinality (Conjecture 2)
$K + 1 = 8$	Closure-normalised direction count
$SU(K+1) = SU(8)$	Admissibility symmetry group
$\mathfrak{su}(8)$	Traceless deformation algebra; dim = 63 = D_{corr}
$\tau(C)$	Operational time along chain C
$G(e_i, e_j)$	Correlation kernel
$d(e_i, e_j) = -\log\|G\|$	Correlation-kernel distance
$s^2(e_i, e_j) = c_\rho^2 \tau^2 - d^2$	Lorentzian interval (Axiom 4)
L_A	Admissibility transport operator
C, J_ρ, Φ	Closure, transport, record inconsistency operators
$\Gamma : \Lambda \to \Lambda'$	Admissible coarse-graining
$R_{n\to n+1}$	Refinement operator on observables
S_m	Edge-bisection refinement operator
$\mathcal{H}_{sub} = \ell^2(E) \otimes \mathbb{C}^8$	Substrate Hilbert space
Π_{adm}	Admissibility selector (Definition 15)
Π_{adm}^{GF}	Gradient-flow selector (Definition 15a)
$\Pi_{adm}^{L^2}$	L^2-projection selector (Definition 15b)
$H_{cl} = G\|_{\mathfrak{su}(8)}$	Closure Hamiltonian (Definition 16)
$G = \delta^2 F / \delta\bar{\rho}\delta\rho\|_{\mathcal{M}_{adm}}$	Hessian of F at admissible manifold
G_C, G_T, G_R	Admissibility-component Hessians
$\alpha_X^{(R)}, \hat{\alpha}_X^{(R)}$	Dimensionful / dimensionless prefactors per isotype
L_{12}	(e_1, e_2)-plane rotation generator in $\mathfrak{su}(8)$
$H_1, H_2 \in \mathfrak{su}(8)$	Non-commuting SU(8) generators (Appendix G)

Symbol	Meaning
Tick	One application of the update rule (Definition 17)
$A_C^{(n)}$	Closure-admissibility defect at refinement level n

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Appendix D — Minimal Toy Refinement Model

A concrete example demonstrating that the framework's admissible-coarse-graining structure operates as §5.9 claims. The example is small enough to verify by hand and structured to exercise each of Theorem 8's four hypotheses.

D.1 The base poset

$E_0 = \{a, b, c, d\}$ with partial order generated by:

$a \prec b$, $a \prec c$, $b \prec d$, $c \prec d$

This is the standard diamond poset: a at the past, d at the future, b and c at intermediate spacelike-separated positions. Transitive closure gives $a \prec d$; b and c remain incomparable: $b \parallel c$.

D.2 Substrate structures on E_0

Following Definition 3, scalar reduction (suppressing the $K + 1 = 8$ vector structure): $\rho : E_0 \to \mathbb{R}$ with all events at saturation $\rho(a) = \rho(b) = \rho(c) = \rho(d) = \rho_c$ (the stable fixed point of Theorem 3).

Operational time τ. Each step of σ-length 1, $\rho = \rho_c$ throughout:

- $\tau(a, b) = \tau(a, c) = \tau(b, d) = \tau(c, d) = \rho_c$
- $\tau(a, d) = 2\, \rho_c$ (along either chain)
- τ undefined for incomparable $b \parallel c$

Correlation kernel G. $G(e_i, e_j) = \exp(-d_{ij}/\xi)$ with substrate scale ξ:

pair	d_{ij}	G = e^{-d/ξ}
(a,b), (a,c), (b,d), (c,d)	ξ	e^{-1}
(a,d)	2ξ	e^{-2}
(b,c)	$\sqrt{2} \cdot \xi$	$e^{-\sqrt{2}}$

The b-c value reflects spacelike separation through a.

Lorentzian interval. $s^2(e_i, e_j) = c_\rho^2\, \tau^2 - d^2$ with $c_\rho = 1$:

pair	τ	d	s^2	classification
(a,b)	ρ_c	ξ	$\rho_c^2 - \xi^2$	timelike if $\rho_c > \xi$
(a,d)	$2\rho_c$	2ξ	$4(\rho_c^2 - \xi^2)$	timelike if $\rho_c > \xi$
(b,c)	—	$\sqrt{2}\,\xi$	$-2\xi^2$	spacelike

The diamond is timelike-coherent when $\rho_c > \xi$ (the operationally relevant regime).

D.3 The refinement map

Insert an event between each direct covering pair in E_0: covers are $a \prec b$, $a \prec c$, $b \prec d$, $c \prec d$, giving four new events m_{ab}, m_{ac}, m_{bd}, m_{cd}:

$E_1 = \{a, m_{ab}, b, m_{ac}, c, m_{bd}, m_{cd}, d\}$

with order generated by:

$a \prec m_{ab} \prec b \prec m_{bd} \prec d$ $a \prec m_{ac} \prec c \prec m_{cd} \prec d$

$|E_1| = 8 = 2\,|E_0|$. The map $\Gamma : \Lambda_0 \to \Lambda_1$ is inclusion.

D.4 Verification of Theorem 8's four hypotheses

Hypothesis (1): $A[\Gamma(\rho)] = 0$. Extend saturation to the new events ($\rho = \rho_c$ on all of E_1). Since A vanishes on saturated configurations under refinement-consistent extension, $A[\rho_1] = 0$. ✓

Hypothesis (2): $s_1^2 \sim s_0^2$. For the (a, d) pair along $a \prec m_{ab} \prec b \prec m_{bd} \prec d$: each refined step has σ-length 1/2 (the original 1 is bisected), giving:

$\tau_1(a, d) = 4 \times (1/2) \times \rho_c = 2\rho_c = \tau_0(a, d)$.

Graph distance on E_1: $d_1(a, d) = 4 \times (\xi/2) = 2\xi = d_0(a, d)$. Therefore:

$s_1^2(a, d) = (2\rho_c)^2 - (2\xi)^2 = 4(\rho_c^2 - \xi^2) = s_0^2(a, d)$.

Interval preserved exactly (not just at leading order) for this toy model. ✓

Hypothesis (3): Refinement-stable observables exist. Consider the substrate diameter $D[\Lambda] = \max_{e_i, e_j \in \Lambda} d(e_i, e_j)$. On E_0: $D[\Lambda_0] = 2\xi$. On E_1 with the refinement above, the (a, d) graph distance is $4 \times (\xi/2) = 2\xi$. **$D[\Lambda_n] = 2\xi$ for all n.**

Iterating to refinement level n with 2^{n+1} events per chain, each segment $\xi/2^n$: total $d_n(a, d) = 2^{n+1} \times \xi/2^n = 2\xi$. The diameter is exactly refinement-stable. ✓

Hypothesis (4): Sequence independence (Proposition 14 / confluence). Consider $\tilde{\Gamma} : E_0 \to E_1'$ inserting a single event m_{ad}' directly between a and d (bypassing b, c). E_1 and E_1' differ. **Common refinement E_2:** include all original events {a, b, c, d}, edge-bisection events $\{m_{ab}, m_{ac}, m_{bd}, m_{cd}\}$, and the $\tilde{\Gamma}$ event $\{m_{ad}'\}$. E_2 admissibly extends both E_1 and E_1'. By Proposition 14, their continuum limits coincide. ✓

D.5 What the toy model demonstrates

For the four-point diamond, all four Theorem 8 hypotheses are simultaneously verifiable:

- Order preserved under refinement.
- Triangle inequalities preserved (Proposition 6 + additivity of d along refined chains).
- s^2 sign structure is stable (timelike pairs remain timelike; spacelike remain spacelike).
- Refinement-stable observables exist (diameter is exactly stable).
- Sequence independence (Proposition 14) holds for at least one alternative refinement pair.

The continuum-limit triple ($\mathcal{M}$, $g_{\mu\nu}^{eff}$, ρ_{eff}) is constructed explicitly: one timelike (a-to-d) and one spacelike (b-to-c) Lorentzian direction.

D.6 What the toy model does not demonstrate

- Non-trivial gauge structure ($K = 7$ vector $\rho \in \mathbb{C}^{K+1}$ is not exercised by the scalar reduction).
- Higgs prediction (Part II operates on the $SU(K+1)$ sector, absent here).
- Universality across poset families.
- Quantitative numerical predictions.

The diamond is one worked example; substantive realisations require the $K + 1 = 8$ vector structure (Appendices E, F, G).

Appendix E — Uniform-Direction Vector Refinement Model

Appendix E extends Appendix D's scalar diamond to the $K + 1 = 8$ vector setting with *uniform* directional structure across events: $\rho(e) = \rho_c \cdot \hat{e}$ for a single fixed $\hat{e} \in \mathbb{C}^8$ at all events.

E.1 Setup

$E_0 = \{a, b, c, d\}$ (diamond as in Appendix D). Take $\hat{e}(a) = \hat{e}(b) = \hat{e}(c) = \hat{e}(d) = e_1$ (the same closure-normalised direction at all events).

E.2 Substrate structures

Closure inconsistency C vanishes trivially: with uniform direction, all transport operators $U(e_i \to e_j)$ are identity, so $C = 𝟙·𝟙·𝟙 − 𝟙 = 0$. Similarly J_ρ vanishes (no directional variation to transport) and Φ-defects vanish. **A = 0 trivially.**

Operational time and correlation-kernel structure inherit from Appendix D (scalar magnitude controls all). Refinement extension to E_1 preserves uniformity: $ê(m_{ab}) = ê(m_{ac}) = ê(m_{bd}) = ê(m_{cd}) = e_1$.

E.3 Verification of Theorem 8 hypotheses

All four hypotheses are verified trivially: A = 0 throughout (extending Hypothesis (1) to the vector case); s_n^2 and refinement-stability follow from Appendix D unchanged.

E.4 What Appendix E adds beyond Appendix D

Uniform-direction vector structure shows the framework operates consistently on the K + 1 = 8 vector configuration space, with admissibility preservation under refinement maintained when directional structure is included. But uniform-direction substrates are *trivially* admissible (A = 0 by construction); the interesting test of admissibility under refinement requires *non-uniform* directional structure where C, J_ρ, Φ generically have non-vanishing values.

E.5 Beyond uniform direction

Non-uniform directional structure on the diamond (e.g., ê(b) and ê(c) tilted into the (e_1, e_2) plane symmetrically) introduces non-trivial substrate admissibility content. Single-generator Abelian transport in such a configuration is worked through in Appendix F. Multiple non-commuting generators introduce Baker-Campbell-Hausdorff corrections to admissibility under refinement; their suppression rate is computed in Appendix G.

Appendix F — A First Nontrivial Vector Microdynamics

Appendix F instantiates the candidate substrate microdynamics of §§5.9.10–5.9.11 on the four-point diamond with non-uniform directional structure. The closure Hamiltonian is identified as $H_{cl} = L_{12}$ (the (e_1, e_2)-plane rotation generator) supported by a reflection-symmetry-reduction argument; the full update rule is verified concretely; the diamond's relevant SU(8) isotype set for testing Conjecture 8 is identified.

F.1 The diamond with non-uniform directional structure

$E_0 = \{a, b, c, d\}$ with partial order $a \prec b$, $a \prec c$, $b \prec d$, $c \prec d$. Saturated configuration $\rho = \rho_c \cdot ê(e)$ with non-uniform symmetric directional structure:

$ê(a) = e_1$ $ê(b) = \cos\theta \cdot e_1 + \sin\theta \cdot e_2$ $ê(c) = \cos\theta \cdot e_1 - \sin\theta \cdot e_2$ $ê(d) = e_1$

for $\theta \in [0, \pi/4)$. The b and c events carry directions tilted symmetrically into the (e_1, e_2) plane.

F.2 The closure Hamiltonian on the diamond: $H_{cl} = L_{12}$

For the diamond, the candidate update rule takes the closure Hamiltonian as:

$H_{cl} = L_{12} = i(e_1 \otimes e_2 - e_2 \otimes e_1) \in \mathfrak{su}(8)$**

This choice is supported by the reflection-symmetry-reduction argument of §F.6 below, not by a full Hessian computation. The full Hessian extracting $\mu(\theta)$ and $\alpha_X^{(R)}$ is the open derivation target identified in §18.2.

F.3 One iteration of the candidate update rule

Apply $T_1 = S_1 \otimes \exp(i\varepsilon_1 L_{12})$ with $\varepsilon_1 = \theta/2$ to:

$\Psi_0 = \rho_c \cdot (|a\rangle \otimes e_1 + |b\rangle \otimes (\cos\theta \cdot e_1 + \sin\theta \cdot e_2) + |c\rangle \otimes (\cos\theta \cdot e_1 - \sin\theta \cdot e_2) + |d\rangle \otimes e_1)$.

S_1 action. Inserts intermediate events m_{ab}, m_{ac}, m_{bd}, m_{cd}, with interpolated directions:

$\hat{e}(m_{ab}) = \cos(\theta/2) \cdot e_1 + \sin(\theta/2) \cdot e_2$ $\hat{e}(m_{ac}) = \cos(\theta/2) \cdot e_1 - \sin(\theta/2) \cdot e_2$
$\hat{e}(m_{bd}) = \cos(\theta/2) \cdot e_1 + \sin(\theta/2) \cdot e_2$ $\hat{e}(m_{cd}) = \cos(\theta/2) \cdot e_1 - \sin(\theta/2) \cdot e_2$

$\exp(i\varepsilon_1 L_{12})$ action. Rotates closure-normalised directions by phase $\varepsilon_1 = \theta/2$ in the (e_1, e_2) plane.

Admissibility projection. The refined state lies on $\mathcal{M}_{adm}$: $A_C = 0$ exactly for the symmetric rotational transport. Π_{adm}^{GF} acts as identity (the gradient-flow long-σ limit of an admissible state is the state itself).

Result. Ψ_1 = the admissible refined state on E_1; admissibility preserved exactly.

F.4 Iterating to the continuum limit

Repeated iteration produces Ψ_n on E_n with 2^{n+1} events per chain, each carrying rotation phase $\theta/2^n$. By Proposition 16, the iterated refinement satisfies all four Theorem 8 hypotheses. The continuum limit produces ($\mathcal{M}$, $g_{\mu\nu}^{eff}$, ρ_{eff}) with one timelike (a-to-d) and one spacelike (b-to-c) Lorentzian direction, consistent with Proposition 11.

F.5 Theorem 10 applicability on the diamond

$F = \int V_{sub} + A$ satisfies analyticity (polynomial), coercivity ($V_{sub} \sim \|\rho\|^4$), bounded-below ($F \geq 0$) globally. For the symmetric diamond at the saturated configuration, basin-uniqueness holds: the rotational-transport configuration is a single connected basin of the gradient flow with the saturated configuration as the admissible attractor. **Therefore Theorem 10 applies, and gradient-flow selector existence on the diamond is theorematic.**

F.6 Reflection-symmetry-reduction argument supporting $H_{cl} = L_{12}$

The choice $H_{cl} = L_{12}$ in §F.2 is supported by a rigorous reflection-symmetry-reduction argument.

Setup. The symmetric diamond configuration (§F.1) is invariant under the combined reflection:

$b \leftrightarrow c$, $e_2 \mapsto -e_2$,

with $e_1, e_3, \ldots, e_8$ fixed.

Argument. $F = \int V_{sub} + A$ is built from SU(8)-invariant ingredients. The reflection $b \leftrightarrow c$, $e_2 \mapsto -e_2$ is a specific SU(8) action (it lies in the stabiliser of the saturated configuration). F is therefore invariant under this reflection on the entire reflection-symmetric submanifold of configuration space — not just at $\theta = 0$, but at all $\theta \in [0, \pi/4)$.

Consequently the Hessian G at any reflection-symmetric saturated configuration $\rho_*(\theta)$ commutes with the reflection operator: $G(\theta) \cdot \mathcal{R} = \mathcal{R} \cdot G(\theta)$ for all θ. This forces $G(\theta)$ to block-decompose into reflection-even and reflection-odd sectors **exactly at every θ**, not just at leading order. The decoupling is a θ-independent algebraic consequence.

The leading antisymmetric mode. The reflection-odd sector at lowest order in θ is generated by the antisymmetric directional mode $\delta\hat{e}(b) = +e_2$, $\delta\hat{e}(c) = -e_2$ (with $\delta\hat{e}(a) = \delta\hat{e}(d) = 0$). Under $b \leftrightarrow c$, this mode maps to $\delta\hat{e}(b) = -e_2$, $\delta\hat{e}(c) = +e_2$; combined with $e_2 \mapsto -e_2$, the mode returns to itself with one sign change — confirming reflection-odd character.

This antisymmetric mode is the infinitesimal rotation in the (e_1, e_2) plane generated by *$L_{12} = i(e_1 \otimes e_2 - e_2 \otimes e_1)$* $\in \mathfrak{su}(8)$**.

By contrast, perturbations into $e_3, \ldots, e_8$ are reflection-*even* and block-decouple from L_{12} **exactly at all θ**.

Conclusion. In the symmetric diamond configuration, $G|_{\mathfrak{su}(8)}$ *block-decomposes under the reflection symmetry exactly at all θ. The leading antisymmetric block is generated by* L_{12}. This supports $H_{cl} = L_{12}$ as the leading-order closure Hamiltonian.

F.7 What §F.6 establishes vs what remains open

Established (exact, θ-independent): $G(\theta)$ on the symmetric diamond block-decomposes into reflection-odd + reflection-even sectors. The odd block is generated at lowest order by L_{12}. The even block (containing $e_3, \ldots, e_8$ contributions) is decoupled from the odd block at all θ.

Conjectural (θ-dependent, leading-order):

$$G(\theta)|_{\mathfrak{su}(8)} = \mu(\theta) \cdot L_{12} + G_{even}(\theta) + O(\theta^2)$$

with G_{even} small relative to $\mu(\theta) \cdot L_{12}$ at small θ. The scalar $\mu(\theta)$, finite-θ corrections, relative size of G_{even}, and per-component prefactors $\alpha_X^{(R)}$ (testing Conjecture 8) require the full Hessian computation.

F.8 Concretised isotype set for Conjecture 8 testing

The diamond's directional structure (localised in the (e_1, e_2) plane; $e_3, \ldots, e_8$ not participating) and the reflection symmetry reduce the relevant SU(8) isotype set substantially. For the diamond, the relevant isotypes for H_{cl} are approximately:

{L_{12} antisymmetric, (e_1, e_2)-plane reflection-even longitudinal, e_3–e_8 collective} — three isotypes.

Operational target for Conjecture 8 testing. Extract $\hat{\alpha}_X^{(R)}$ for {C, T, R} components across these ~3 isotypes — approximately 9 dimensionless prefactor values — by explicit Hessian computation. Test whether $\hat{\alpha}_C^{(R)} : \hat{\alpha}_T^{(R)} : \hat{\alpha}_R^{(R)}$ ratios are parameter-independent across the three isotypes. This is the bounded, well-defined calculation identified in §18.2 as the next calculational target.

F.9 What Appendix F demonstrates

The candidate update rule of §5.9.10 with $H_{cl} = L_{12}$:

- Operates concretely on the four-point diamond with non-uniform directional structure.
- Generates the refinement sequence of §F.4, satisfying Theorem 8 hypotheses.
- Produces a continuum-limit triple consistent with Proposition 11.
- Has H_{cl} supported by a rigorous symmetry argument (§F.6), not bare assertion.
- Has gradient-flow selector existence theorematic via Theorem 10 (§F.5).

The remaining open work — full Hessian computation extracting $\mu(\theta)$ and $\alpha_X^{(R)}$, basin-uniqueness verification on non-symmetric configurations, explicit multi-generator non-Abelian extension — is identified in Open Problem #11's sub-targets (a)–(e) and §18.2's next calculational target.

Appendix G — First Non-Abelian Refinement Test

Appendix F instantiated the candidate microdynamics with single-generator SU(8) transport — an Abelian one-parameter sub-flow. Appendix G supplies the first explicit non-Abelian computation, establishing that non-Abelian commutator defects between two non-commuting SU(8) transport sectors decay exponentially under admissible edge-bisection refinement.

G.1 Setup

Take the diamond as in Appendix F. Choose two non-commuting SU(8) generators:

$H_1, H_2 \in \mathfrak{su}(8)$, with $[H_1, H_2] \neq 0$.

Assign transport along the two diamond chains with alternating generator content:

$U_{ab} = e^{i\varepsilon H_1}$, $U_{bd} = e^{i\varepsilon H_2}$ (chain $a \to b \to d$) $U_{ac} = e^{i\varepsilon H_2}$, $U_{cd} = e^{i\varepsilon H_1}$ (chain $a \to c \to d$)

The cumulative transports:

$U_b = e^{i\varepsilon H_2} \cdot e^{i\varepsilon H_1}$ ($a \to b \to d$) $U_c = e^{i\varepsilon H_1} \cdot e^{i\varepsilon H_2}$ ($a \to c \to d$)

For Abelian generators, $U_b = U_c$ trivially. For non-commuting H_1, H_2: **$U_b \neq U_c$.**

G.2 Leading-order non-Abelian defect

Baker-Campbell-Hausdorff expansion to second order:

$e^{i\varepsilon H_2} \cdot e^{i\varepsilon H_1} = \mathbb{1} + i\varepsilon(H_1 + H_2) - (\varepsilon^2/2)(H_1^2 + H_2^2 + 2 H_2 H_1) + O(\varepsilon^3)$ $e^{i\varepsilon H_1} \cdot e^{i\varepsilon H_2} = \mathbb{1} + i\varepsilon(H_1 + H_2) - (\varepsilon^2/2)(H_1^2 + H_2^2 + 2 H_1 H_2) + O(\varepsilon^3)$

The path-comparison defect:

$\Delta_{ad} = U_b - U_c = \varepsilon^2 \cdot [H_1, H_2] + O(\varepsilon^3)$.

The 0th and 1st order terms cancel; the 2nd order terms differ only in $H_2 H_1$ vs $H_1 H_2$, giving the commutator.

The closure-admissibility functional evaluated on this defect:

$A_C^{(0)} = \|\Delta_{ad}\|^2 \cdot u(a) \cdot u(d) = \varepsilon^4 \cdot \|[H_1, H_2]\|^2 \cdot \rho_c^4 + O(\varepsilon^5)$.

For non-commuting H_1, H_2, $A_C^{(0)} > 0$: admissibility is violated at order ε^4, the base level of non-Abelian commutator defect prior to refinement.

G.3 Refinement suppression

Apply admissible edge-bisection refinement. Each transport segment's phase scales as $\varepsilon \to \varepsilon/2$ to preserve operational-time density. After one refinement step:

$\Delta_{ad}^{(1)} = (\varepsilon/2)^2 \cdot [H_1, H_2] + O((\varepsilon/2)^3) = (\varepsilon^2/4) \cdot [H_1, H_2] + O(\varepsilon^3)$.

The refined-level admissibility defect:

$A\_C^{(1)} = (\varepsilon^4/16) \cdot \|[H\_1, H\_2]\|^2 \cdot \rho\_c^4 + O(\varepsilon^5) = A\_C^{(0)} / 16 + O(\varepsilon^5)$.

After n iterated refinements (phase per segment $\varepsilon \to \varepsilon/2^n$):

$A\_C^{(n)} = (\varepsilon/2^n)^4 \cdot \|[H\_1, H\_2]\|^2 \cdot \rho\_c^4 + O(\varepsilon^5) =$ **$16^{-n} \cdot A\_C^{(0)} + O(\varepsilon^5)$.**

G.4 Interpretation

The factor 1/16 < 1 places non-Abelian commutator-defect content among the **irrelevant** eigendirections of the linearised refinement operator L_R (Definition 12). In renormalisation-group language: non-Abelian commutator defects are irrelevant operators in the substrate's RG flow, decaying exponentially toward $\mathcal{M}$_adm.

Structural consequence. The substrate tolerates genuine non-Abelian SU(8) transport without violating admissibility under refinement. Non-Abelian content per se is not destabilising; only non-Abelian content combined with absence of admissible refinement would be a problem. The framework operates throughout in the admissible-refinement category, so non-Abelian content is structurally benign.

G.5 Note on the specific factor 16

The factor 16 depends on the edge-bisection refinement convention with phase halving per segment. For alternative schemes (edge-trisection, phase rescaling 1/3, etc.), the suppression base differs. The *structurally important* claim is *exponential* suppression of non-Abelian defects, which holds for any scheme $\varepsilon\_{n+1} = \varepsilon\_n / \kappa$ with $\kappa > 1$. For edge-bisection, the suppression base is $\kappa^4 = 16$; for other admissible schemes, the base is κ^4 generally. The exponential character is universal.

G.6 What Appendix G establishes

- **Genuine non-Abelian content exhibited.** $\Delta\_{ad} = \varepsilon^2 \cdot [H\_1, H\_2] + O(\varepsilon^3)$.
- **Refinement suppression exponential.** $A\_C^{(n)} \sim 16^{-n} A\_C^{(0)}$.
- **RG-irrelevance.** Non-Abelian commutator content flows toward zero exponentially.
- **Compatibility with §8.1.1's Sector-Selection Criterion.** Non-Abelian SU(2)×U(1)_Y structure (Conjecture 4) is admissibility-compatible under refinement.
- **Compatibility with the substrate microdynamics of §§5.9.10–5.9.11.** Non-Abelian closure Hamiltonians do not obstruct the candidate update rule.

G.7 What Appendix G does not establish

- Universality across substrate poset families beyond the diamond.
- Specific-generator identification (e.g., the SU(2)×U(1)_Y generators of Conjecture 4) and verification of the same suppression rate for them.
- Higher-order BCH corrections ($O(\varepsilon^3)$ terms from nested commutators).
- Full electroweak gauge structure verification.

These extensions are natural companion-paper work.

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