The K = 7 Closure Symmetry

Spatial–Temporal Exchange Symmetry of the Hexagonal Interface and Bare Lorentz-Compatibility from Closure Symmetry plus Minimality

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General Reader Abstract

The previous paper in this programme — the K = 7 Wilson Limit — established a one-loop result: the substrate's coarse-grained electromagnetic transport is more Lorentz-compatible than its bare microscopic action by a small factor (about one part in 10^4). That paper also showed, honestly, that this small factor is far too small to explain the observed precision of Lorentz invariance in nature (one part in 10^{17} or better). The paper therefore left a load-bearing question for the programme: *why is the substrate already very nearly Lorentz-compatible at the bare microscopic level?*

This paper addresses that question. We show that the K = 7 hexagonal closure structure admits a class-preserving involution σ — an automorphism of the closure-counting structure that exchanges spatial and temporal plaquette classes. Under two substrate-physical conditions — that σ is the spatial-temporal exchange (an identification grounded in the cell's directional structure and the substrate-physical feature-identity content of channels, structurally underwritten if σ admits an explicit chain-complex realisation on a labelled cell) and that the bare action respects σ as a substrate-physical symmetry (a minimality principle on the coupling structure) — σ-invariance of the bare Wilson action forces *exactly* equal spatial and temporal couplings: β_s = β_t at the bare scale. The bare action is therefore exactly Lorentz-compatible under these two conditions.

The empirical Lorentz invariance observed in laboratory and astrophysical electromagnetism is consistent with this result. The one-loop matching of the Wilson Limit paper becomes a *robustness* result: small perturbations away from the σ-invariant bare action are additionally suppressed in the IR by the small factor (1 − δ).

A stronger version — one in which σ is constructed as an explicit chain-complex automorphism on a labelled K = 7 cell, replacing both conditions with a derived equivariance — is identified as open combinatorial work in §10 and Appendix A.

Abstract

We carry out the derivation deferred to this paper by the K = 7 Wilson Limit paper §9.3 and §11(L4): the substrate-physical origin of bare Lorentz-compatibility on the K = 7 hexagonal closure substrate.

Setup. The K = 7 paper (companion) established that the bare microscopic Wilson coupling $\beta_{K=7} = 2^7 \cdot 15/14 \approx 137.143$ is fixed by the closure-counting integers K = 7 and $N_{loop} = 14$. The Wilson Limit paper established that the anisotropic Wilson action on the K = 7 substrate has spatial and temporal couplings β_s, β_t with a coupling anisotropy $\zeta_{bare} = \sqrt{(\beta_s/\beta_t)}$ that determines, modulo a one-loop matching factor $(1 - \delta) \sim 1 - 10^{-4}$, the IR effective Lorentz-compatibility of electromagnetic transport. Both papers identified the value of ζ_{bare} as an open substrate-physical question.

Central result. We construct a class-preserving involution σ on the K = 7 hexagonal closure structure and establish:

(i) Theorem 1 (cardinality level, §4). σ exists as a class-preserving bijection: the $N_{loop} = 14$ channels per cell are bijected between spatial and temporal classes with the 11 + 2 + 1 partition preserved class-by-class.

(ii) σ-selection (§5.3a, substrate-physical channel-feature-matching). Among the class-preserving bijections that satisfy (i), σ is identified specifically as the spatial-temporal exchange — the bijection that pairs each spatial channel with the temporal channel encoding the same substrate-physical feature (the same boundary edge in its temporal-axis embedding, the same interior-vertex role, the same global mode). At the cardinality level the orbit of class-preserving bijections has order $11! \cdot 2! \cdot 1! \approx 8 \times 10^7$; substrate-physical feature-matching collapses this to a single bijection (modulo orientation choices) *provided* channels carry feature-identity content inherited from the K = 7 paper's channel construction (the second inheritance point of §2.3). The chain-complex realisation of §10(L1), if available, verifies that this channel-feature-matching extends consistently to the cell's chain-complex structure.

(iii) Minimality Premise (§5.3b, substrate-physical). Once σ is identified, the bare Wilson action functional should not distinguish channels that σ identifies. Structurally parallel to BCB (minimal closure-consistent dynamics) and TPB (minimal finite-speed propagation scale) once σ is given.

(iv) Theorem 2 (§6). Under (i), (ii), and (iii), $\beta_s = \beta_t = \beta_{K=7} = 2^7 \cdot 15/14 \approx 137.143$ at the bare scale.

The bare coupling anisotropy is therefore $\zeta_{bare} = 1$ under (i), (ii), (iii). The IR effective coupling anisotropy $\zeta_{eff} = 1$ follows from the Wilson Limit paper's matching theorem.

A dependency surfaced in v7. The substrate-physical content of σ-selection (ii) — that σ is uniquely determined by channel-feature-matching rather than picked from an $\sim 10^8$-element orbit — depends on the K = 7 paper's channel construction carrying feature-identity content directly

(the second inheritance point of §2.3). Under that inheritance, the v6 framing of σ-selection as content-bearing channel-feature-matching is operative. Without it, σ-selection reduces to picking an element from the cardinality orbit and becomes a substrate-physical postulate without the structural support of channel-by-channel feature-matching. This dependency is one of the key reconciliation items between the present paper and the K = 7 paper (§10 L6).

What this establishes. Bare Lorentz-compatibility of the substrate's electromagnetic transport under closure symmetry plus substrate-physical assumptions. Empirical Lorentz invariance at observed precision ($\lesssim 10^{-17}$) is consistent with this result. The one-loop matching of the Wilson Limit paper is re-interpreted as a *robustness* check.

What this does not establish. The chain-complex realisation of σ on a labelled K = 7 cell (open, §10 L1, Appendix A). Spatial rotation invariance, matter coupling, and non-abelian extensions are not addressed. The orientation-symmetric cardinality decomposition (i) and the channel feature-identity content underlying (ii) are inherited from the K = 7 paper with inheritance questions flagged in §2.3.

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1. Introduction

The VERSF programme has derived a sequence of integer-counting results from the K = 7 hexagonal closure structure. The K = 7 paper established the bare electromagnetic coupling $\beta_{K=7} = 2^7 \cdot 15/14$ from a constraint-counting argument on the hexagonal interface. The Wilson Limit paper established the one-loop matching rate $\delta \sim \alpha/100$ from a perturbative calculation around the same structure. Each result is exact in the sense that the integer-counting argument produces a definite rational value, not an approximation.

The Wilson Limit paper's §9 and §11(L4) identified one remaining substrate-physical question: the bare coupling anisotropy ζ_{bare}. The matching theorem reduced bare anisotropy by a factor $(1 - \delta) \sim 1 - 10^{-4}$ in the IR effective theory — far too small to explain empirical Lorentz-invariance bounds at $\sim 10^{-17}$. The paper therefore left open whether the substrate's microscopic action is bare-isotropic exactly, bare-isotropic approximately with calculable corrections, or bare-anisotropic with a separate mechanism for the empirical bound.

This paper addresses the question through a three-component argument:

(i) Theorem 1 (§4) — cardinality-level: the K = 7 closure structure admits a class-preserving involution σ on the spatial and temporal plaquette channels, with the 11 + 2 + 1 decomposition matching between orientation classes.

(ii) σ-selection (§5.3a) — substrate-physical channel-feature-matching: among the class-preserving bijections satisfying Theorem 1, σ is the one that pairs each spatial channel with the temporal channel encoding the same substrate-physical feature. This is one specific bijection (modulo orientation choices), not a member of an undifferentiated orbit — provided channels carry feature-identity content from the K = 7 paper's channel construction.

(iii) Minimality Premise (§5.3b) — substrate-physical: once σ is selected, the bare action does not distinguish channels that σ identifies.

Theorem 2 (§6) combines these: under all three, $\beta_s = \beta_t = \beta_{K=7}$ at the bare scale.

The split between σ-selection (substrate-physical-content-bearing or substrate-physical-postulate, depending on inheritance) and σ-imposition via minimality is doing real work. An earlier draft (v3) conflated the two into a single "Minimality Premise" parallel to BCB and TPB; the conflation overstated the parallelism. In v6 the substrate-physical content of σ-selection was sharpened: it is *not* "pick this one bijection out of many" but rather "pair channels by their substrate-physical feature identity", which is a structural-consistency claim with much more content than orbit-membership suggests. In v7 the inheritance dependency of σ-selection's content-bearing character is made explicit: the channel-feature-matching framing of σ-selection rests on the K = 7 paper carrying feature-identity content in its channel construction. If yes, σ-selection is the structurally-rich step described above. If no, σ-selection reduces to selecting an arbitrary class-preserving bijection from the $\sim 10^8$-element orbit, and becomes a substrate-physical postulate of the v3 type.

This dependency affects what the chain-complex resolutions of §10(L1) mean:

- Under (L1.a)/(L1.b) — chain-complex realisation exists on a natural K = 7 cell — σ-selection's channel-feature-matching is verified to extend consistently to the cell complex (assuming the feature-identity inheritance is in place), and the architecture reduces to cardinality plus structural-consistency plus minimality, cleanly parallel to BCB and TPB.

- Under (L1.c) — no chain-complex realisation exists on any natural K = 7 cell — the interpretation depends on the feature-identity inheritance. Under the inheritance, (L1.c) is a substantive structural finding: the K = 7 closure structure supports class-level channel-feature-matching but not chain-complex-level feature-matching — itself a structural property of the K = 7 system worth characterising. Without the inheritance, (L1.c) is the v3-style "no chain-complex realisation exists at all", with σ-selection as a substrate-physical postulate at the channel level.

The paper is organised as follows. §2 collects the inherited framework, with two inheritance questions surfaced. §3 constructs σ at the level of plaquette classes. §4 verifies that σ preserves the K = 7 closure-counting structure at the cardinality level (Theorem 1). §5 distinguishes the labelling-consistency content of σ-invariance from the substrate-physical content via the two-step σ-selection + minimality framework. §6 states the bare coupling theorem. §7 re-interprets the Wilson Limit paper's one-loop matching. §8 addresses empirical Lorentz-invariance bounds. §9–§11 catalogue what is and is not established. §12 concludes. Appendix A is a status note on the open chain-complex construction; Appendix B is the cross-paper notation index.

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2. Inherited Framework

2.1 The K = 7 Hexagonal Closure Structure (from K = 7 paper)

The K = 7 paper established:

- the closure integer K = 7 from a rank-nullity argument on the constraint matrix of the hexagonal interface;
- the loop count $N_{loop} = 14 = 2K$ from reversibility and channel democracy;
- the bare coupling $\beta_{K=7} = 2^K \cdot (2K + 1)/(2K) = 2^7 \cdot 15/14 \approx 137.143$ from constraint-counting on the closure structure.

The substrate consists of a hexagonal tiling in the spatial directions and a tick-axis temporal direction with spacing $a_t = N_b \tau_s$.

Each minimal interface cell I supports $N_{loop} = 14$ closed loops contributing to the bare Wilson action. We label these channels by their orientation class: *spatial plaquettes* $\square_s$ and *temporal plaquettes* $\square_t$.

2.2 The Anisotropic Wilson Action (from Wilson Limit paper)

$\mathcal{S}_W[\theta] = \beta_s \cdot \mathcal{S}_s[\theta] + \beta_t \cdot \mathcal{S}_t[\theta]$,

with

$\mathcal{S}_\alpha[\theta] = \Sigma_{\text{cells } I} \Sigma_{\ell \in \square\alpha(I)} (1 - \cos F_{I,\ell}^{(\alpha)})$, $\alpha \in \{s, t\}$, and $\zeta \equiv \sqrt{(\beta_s / \beta_t)}$ the coupling anisotropy.

2.3 Closure-Counting Structure — and Two Inheritance Questions

The N_loop = 14 per cell decomposes as:

N_loop = 11 (B-class: boundary-edge channels) + 2 (V-class: interior-vertex channels) + 1 (G-class: global closure mode) = 14.

The 14 are *constraint channels* arising from reversibility-pairing of K = 7 closure constraints (rank 6 + nullity 1), not 2-faces of the cell complex.

The present paper rests on two inheritance points from the K = 7 paper, both worth flagging.

First inheritance: orientation-symmetric cardinality decomposition. Theorem 1 (§4) requires the 14 = 11 + 2 + 1 decomposition to hold separately and with matching cardinalities in each orientation class — 11 spatial B-channels and 11 temporal B-channels, etc. Whether the K = 7 paper establishes this orientation-symmetric decomposition directly (with the rank-nullity analysis manifestly orientation-symmetric) or whether the orientation-symmetric split is an additional structural claim layered on top of an unoriented total count, is a reconciliation point we flag here and revisit in §10(L6) and §11.4.

Second inheritance: feature-identity content of channels. σ-selection (§5.3a) depends on each channel having an identifiable substrate-physical feature — specifically, a boundary-edge identity, an interior-vertex role, or the unique global-mode role — that is shared between its spatial and temporal-axis embeddings. This feature-matching is the substrate-physical content of σ-selection and is what makes the v6 framing of σ-selection content-bearing rather than orbit-picking. If the K = 7 paper's channel construction carries this feature-identity content directly (with each channel arising from a specific closure constraint that retains a substrate-physical identity under temporal-axis embedding), σ-selection's content is well-defined and §3.2's collapse from $\sim 10^8$ to ~1 goes through. If it does not (if the K = 7 paper's channel construction is purely count-based without feature-identity content), σ-selection's substrate-physical content reduces, the orbit-size framing of earlier drafts becomes operative, and σ-selection becomes a substrate-physical postulate of the v3 type.

This second inheritance point determines whether the v6/v7 framing of σ-selection (as content-bearing channel-feature-matching) or the v3 framing (as substrate-physical postulate) is operative for the present paper. It is among the most important reconciliation items between the present paper and the K = 7 paper (§10 L6), and we flag the dependency throughout.

2.4 The Question

The Wilson Limit paper §9.3 framed the open substrate-physical question as: *Does the K = 7 closure structure force $\beta_s = \beta_t$ at the bare scale?*

We answer in the affirmative under (i) Theorem 1, (ii) σ-selection, and (iii) the Minimality Premise — with the inheritance dependencies of (i) and (ii) on the K = 7 paper flagged explicitly per §2.3.

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3. The Spatial–Temporal Exchange Involution σ

3.1 Construction

We define σ as the involution on plaquette classes that exchanges spatial and temporal orientation while preserving the closure-counting decomposition of §2.3.

Let $\square_s(I)$ be the spatial plaquettes in cell I and $\square_t(I)$ the temporal plaquettes. Both have cardinality $N_{loop} = 14$.

Definition (σ). The involution $\sigma : \square_s(I) \cup \square_t(I) \to \square_s(I) \cup \square_t(I)$ is the map that:

(σ.i) bijects $\square_s(I) \leftrightarrow \square_t(I)$ preserving the closure-counting decomposition: B-class to B-class (11 ↔ 11), V-class to V-class (2 ↔ 2), G-class to G-class (1 ↔ 1);

(σ.ii) satisfies $\sigma^2 = \mathrm{id}$;

(σ.iii) **pairs each spatial channel with the temporal channel encoding the same substrate-physical feature** — the boundary edge in its temporal-axis embedding, the interior-vertex role in its temporal-axis embedding, the global mode in its temporal-axis embedding.

3.2 The Cardinality Orbit and Channel-Feature-Matching

At the bare cardinality level, the class-preserving involutions of (σ.i) and (σ.ii) form an orbit determined by the number of class-preserving bijections $\mathcal{S}_s \to \mathcal{S}_t$ (each extending uniquely to an involution on $\mathcal{S}_s \cup \mathcal{S}_t$):

$|\mathrm{orbit}| = 11! \cdot 2! \cdot 1! \approx 8 \times 10^7$.

(Earlier drafts of this paper cited 10^{15}; that figure double-counted by treating spatial-to-temporal and temporal-to-spatial directions as independent, which they are not for an involution. The correct order is 10^8, not 10^{15}.)

The cardinality orbit is large but not the operative content of σ-selection. The substrate-physical content of σ-selection is *channel-by-channel feature-matching*. Each channel in $\mathcal{S}_s$ has a substrate-physical identity inherited from the K = 7 paper's closure-counting construction (the second inheritance point of §2.3):

- Each B-channel corresponds to a specific boundary edge of the hexagonal cell (which closure constraint of the rank-6 boundary it represents).
- Each V-channel corresponds to a specific interior-vertex role (which of the 2 nullity-1-paired hub channels it represents).
- The unique G-channel corresponds to the global closure mode (the topological invariant of the cell boundary).

Concrete example. Consider the spatial B-channel encoding boundary edge ε_1 of the hexagonal cell — the channel arising from the closure constraint at that edge in the cell's spatial 2D structure. Under temporal-axis embedding (with the tick-axis direction substituting for one spatial axis of the hexagonal tiling), this constraint has a unique temporal counterpart: the B-channel arising from the same closure constraint at the same hexagonal-edge position but with the tick-axis playing the role of the chosen spatial axis. σ pairs *these specifically* — not, for instance, spatial ε_1 with temporal ε_3, which would mismatch the substrate-physical feature-identity. The feature-identity is the operative identifier, not the channel index in some arbitrary ordering.

Under the feature-identity inheritance, σ is uniquely determined by channel-feature-matching, modulo orientation/handedness choices on the cell. The collapse from $\sim 10^8$ to ~ 1 is the substantive content of (σ.iii).

The dependency. This collapse depends on the second inheritance point of §2.3: that the K = 7 paper's channel construction carries feature-identity content directly. Under that inheritance, channel-feature-matching uniquely determines σ. Without it — if the K = 7 paper's channel construction is purely count-based — the orbit-size figure 8×10^7 is the operative content of (σ.i)–(σ.ii) and σ-selection reduces to a substrate-physical postulate (v3 framing) without the structural support of channel-feature-matching.

The chain-complex realisation of §10(L1). If available, it verifies that the substrate-physical channel-feature-matching of (σ.iii) extends consistently to the cell's chain-complex structure (vertices, edges, 2-faces). Under (L1.a)/(L1.b), the verification succeeds. Under (L1.c) — assuming the feature-identity inheritance is in place — the channel-feature-matching is found inconsistent with the chain structure of any natural K = 7 cell.

(L1.c) under the feature-identity inheritance is a substantive structural finding rather than an arbitrary selection failure. It would imply that the K = 7 closure structure supports class-level orientation symmetry at the channel-counting level but not at the cell-complex level — itself a structural property of the K = 7 system worth characterising. Such a finding would point toward investigating which chain-level constraints obstruct the extension of channel-feature-matching to the cell complex, and what substrate-physical content the obstruction carries. This is a research direction in its own right, not just a degenerate resolution to (L1).

3.3 Geometric Interpretation, Calibrated

Intuitively, σ is the operation "rotate the spatial-temporal plane by 90°", acting on the discrete closure structure rather than on continuous spacetime. On a hypercubic lattice, the analogous operation is the exchange of one spatial axis with the temporal axis — an exact SO(4) Euclidean rotation. On the hypercubic lattice the cardinality/feature-matching/chain-complex distinctions all collapse: the Euclidean rotation supplies the chain-complex lift for free, σ-invariance is a derived constraint, and the channel-feature-matching is automatic from the geometric symmetry.

On the K = 7 hexagonal interface, the situation is structurally different. The spatial directions form a hexagonal tiling rather than an orthogonal lattice; σ would exchange the tick-axis with a hexagonal-axis direction, but no simple Euclidean rotation realises this exchange on the discrete structure. The chain-complex lift — explicit σ_vert, σ_edges, and equivariance — is non-trivial to construct and is the open question of §10(L1). The cardinality match of Theorem 1 is what we can establish; σ-selection (σ.iii) is the substrate-physical channel-feature-matching (subject to the §2.3 second inheritance); the chain-complex lift would verify the feature-matching at the cell-complex level.

3.4 What σ Is and Is Not

σ is *not* a continuous Lorentz transformation.

σ is *not* the full Lorentz group. Spatial rotation invariance is treated elsewhere.

σ *is* — substrate-physically, under the §2.3 second inheritance — the channel-by-channel feature-matching of (σ.iii). The chain-complex realisation that would verify this feature-matching at the cell-complex level is open (§10 L1).

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4. Closure-Counting Invariance under σ

4.1 The Cardinality Statement

We verify that σ as defined in §3 preserves the K = 7 closure-counting structure at the cardinality level.

4.2 The Closure-Counting Decomposition

From §2.3:

|□_s(I)| = 11 (spatial B-class) + 2 (spatial V-class) + 1 (spatial G-class) = 14, |□_t(I)| = 11 (temporal B-class) + 2 (temporal V-class) + 1 (temporal G-class) = 14.

4.3 σ-Invariance of the Decomposition at the Class Level

σ : (11 spatial B-channels) ↔ (11 temporal B-channels), σ : (2 spatial V-channels) ↔ (2 temporal V-channels), σ : (1 spatial G-channel) ↔ (1 temporal G-channel).

4.4 σ-Invariance Theorem (Cardinality Level)

Theorem 1 (existence of a class-preserving bijection). The K = 7 closure structure admits a class-preserving involution $\tau : \square\_s(I) \cup \square\_t(I) \to \square\_s(I) \cup \square\_t(I)$ satisfying:

- $\tau(B\_s) = B\_t$, $\tau(V\_s) = V\_t$, $\tau(G\_s) = G\_t$ (class preservation);
- $\tau^2 = \mathrm{id}$ (involution);
- $|B\_s| = |B\_t| = 11$, $|V\_s| = |V\_t| = 2$, $|G\_s| = |G\_t| = 1$ (cardinality match, inherited from §2.3's first inheritance point).

The orbit of class-preserving involutions has order $11! \cdot 2! \cdot 1! \approx 8 \times 10^7$. The specific σ of §3 is identified within this orbit by the substrate-physical channel-feature-matching of (σ.iii), which under the §2.3 second inheritance point reduces the orbit to a unique element modulo orientation choices.

Proof. Cardinality matching is inherited from §2.3 (first inheritance point). Any pairwise bijection of the matched classes yields a class-preserving involution; this gives the mathematical orbit of order $\approx 8 \times 10^7$. The substrate-physical feature-matching of (σ.iii) — under the §2.3 second inheritance point — picks σ specifically, reducing the orbit to one element (modulo orientation). The chain-complex realisation of §10(L1) verifies that this substrate-physical reduction extends consistently to the cell's chain structure. ■

4.5 What This Theorem Does Not Establish

Theorem 1 establishes the existence of a class-preserving involution and identifies σ specifically by channel-feature-matching, subject to the §2.3 inheritance points. It does *not* establish:

- That the channel-feature-matching of σ extends consistently to the chain-complex structure of any natural K = 7 cell (open, §10 L1).
- An explicit chain-complex automorphism σ_vert, σ_edges on a labelled K = 7 cell.
- σ-equivariance of plaquette holonomies $F\_\{\sigma(P)\}(\theta) = F\_P(\sigma^{-1}\theta)$.
- That demanding σ-invariance of the bare action functional is, by itself, a substantive substrate-physical constraint on (β_s, β_t).

The chain-complex realisation question is the principal open item (§10 L1). The substrate-physical content of σ-invariance of the action — what makes Theorem 1 imply $\beta\_s = \beta\_t$ — requires the σ-selection step (§5.3a) and the minimality step (§5.3b).

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5. From Cardinality to Substrate-Physical Constraint

5.1 σ as a Labelling Operation: The Algebraic Identity

Under Theorem 1, σ bijects $\mathcal{S}_s$ with $\mathcal{S}_t$ at the index-set level. Treating σ as a permutation on plaquette labels (with no action on the gauge field θ):

$$\sigma(\mathcal{S}_W)(\theta) = \beta_s \Sigma_{P \in \mathcal{S}_s} (1 - \cos F_{\sigma(P)}(\theta)) + \beta_t \Sigma_{P \in \mathcal{S}_t} (1 - \cos F_{\sigma(P)}(\theta)) = \beta_s \cdot \mathcal{S}_t(\theta) + \beta_t \cdot \mathcal{S}_s(\theta).$$

Demanding $\sigma(\mathcal{S}_W) = \mathcal{S}_W$ as functions of θ then requires

$$(\beta_s - \beta_t) \cdot (\mathcal{S}_s(\theta) - \mathcal{S}_t(\theta)) = 0 \text{ for all } \theta.$$

The functionals $\mathcal{S}_s$ and $\mathcal{S}_t$ are linearly independent as functions of θ on the U(1) configuration space — the spatial and temporal plaquette index sets are disjoint, and the plaquette holonomies F are functionally independent across these disjoint sets. The constraint therefore forces

$$\beta_s = \beta_t$$

as a labelling-consistency requirement.

5.2 What the Cardinality Argument Gives, and What It Doesn't

The §5.1 derivation runs identically with any element of the cardinality orbit of Theorem 1 — *any* class-preserving bijection on the index set yields $\beta_s = \beta_t$ under labelling-consistency demand. Two interpretive caveats apply:

(C1) σ-selection. The specific selection of σ as the channel-feature-matching spatial-temporal exchange (§3, σ.iii) is what makes the conclusion $\beta_s = \beta_t$ substrate-physically meaningful as a *Lorentz-compatibility statement* rather than as the consequence of an arbitrary index-set bijection. Under the §2.3 second inheritance (feature-identity content), channel-feature-matching uniquely determines σ; the chain-complex realisation of §10(L1) verifies this at the cell-complex level under (L1.a)/(L1.b).

(C2) σ-imposition. Demanding "$\sigma(\mathcal{S}_W) = \mathcal{S}_W$ under labelling" is a labelling-consistency requirement, not a dynamical-symmetry constraint. The Minimality Premise of §5.3b is what converts this labelling-consistency requirement into a substrate-physical constraint.

5.3 The Substrate-Physical Content: σ-Selection plus Minimality

5.3a σ-Selection (Channel-Feature-Matching)

σ-Selection. Among the class-preserving bijections of Theorem 1, σ is the bijection that pairs each spatial channel with the temporal channel encoding the same substrate-physical feature. Each B-channel has a specific boundary-edge identity, each V-channel has a specific interior-vertex role, and the G-channel is unique; σ pairs the spatial and temporal-axis embeddings of each feature.

This is *not* orbit-picking from ~10^8 class-preserving bijections — provided the K = 7 paper's channel construction carries feature-identity content (§2.3 second inheritance point). Under that inheritance, channel-feature-matching uniquely determines σ (modulo orientation/handedness choices), and the substantive content of σ-selection is the substrate-physically motivated channel-by-channel pairing rather than selection-from-orbit. Without the inheritance, σ-selection reduces to substrate-physical postulate (v3 framing); the present paper proceeds under the inheritance.

The chain-complex realisation of §10(L1), if available, verifies that the channel-feature-matching extends consistently to the cell's chain-complex structure. Under (L1.a)/(L1.b), the verification succeeds. Under (L1.c) — and under the feature-identity inheritance — the channel-feature-matching is found inconsistent with any natural cell complex's chain structure, which is a substantive structural finding about the K = 7 system rather than an arbitrary selection failure. Such a finding would point to which chain-level constraints obstruct the extension of channel-feature-matching to the cell complex, and what substrate-physical content the obstruction carries.

5.3b The Minimality Premise (Substrate-Physical)

Minimality Premise. Once σ is identified (per §5.3a), the bare Wilson action functional realises the minimal admissible structure compatible with σ: it does not separately weight channels that σ identifies.

Structurally parallel to:

- **BCB** — minimal closure-consistent transport form. Among admissible dynamics on the refinement-persistent sector, BCB selects the minimal one (Wilson action).
- **TPB** — minimal finite-speed propagation scale. Among admissible propagation bounds consistent with one-tick atomic update, TPB selects the minimal one (c_c).
- **Minimality Premise** — minimal action structure consistent with σ-symmetry. Once σ is selected by channel-feature-matching (§5.3a), the substrate realises the minimal admissible action that respects σ.

For BCB and TPB, the distinguished structure (Wilson form; c_c) is selected by structural derivations independent of minimality, and the minimality move operates cleanly on top. For the Minimality Premise here, σ-selection is the analogue of "the distinguished structure is selected". Under (L1.a)/(L1.b) the channel-feature-matching is verified at the cell-complex level and the parallel is structurally even; under (L1.c) the channel-feature-matching is found inconsistent with the cell complex and the parallel becomes aspirational in the content-rich sense of §3.2 — itself a sharper structural claim than "we postulate σ".

5.4 Theorem 1 plus σ-Selection plus Minimality: The Derivation

Combining Theorem 1 (§4) with σ-Selection (§5.3a) and the Minimality Premise (§5.3b), the §5.1 derivation forces $\beta_s = \beta_t$. Theorem 2 (§6) records the consequence.

5.5 What Would Strengthen the Argument

The chain-complex realisation of §10(L1) would:

- Verify that σ-selection's channel-feature-matching (§5.3a) extends consistently to the cell complex;
- Replace the labelling-consistency demand of §5.1 with σ-equivariance of holonomies, making σ-invariance of the action a dynamical-symmetry constraint;
- Resolve whether the architectural parallelism with BCB and TPB is structurally even (L1.a/b) or aspirational (L1.c).

This is the principal open item.

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6. The Bare Coupling Theorem

6.1 Statement

Theorem 2 (Bare Lorentz-Compatibility under Cardinality plus σ-Selection plus Minimality). Under (i) the K = 7 closure-counting structure of the K = 7 paper (Theorem 1, with the orientation-symmetric cardinality decomposition of §2.3's first inheritance point), (ii) σ-selection of σ as the channel-by-channel feature-matching spatial-temporal exchange (§5.3a, with the feature-identity content of §2.3's second inheritance point), and (iii) the Minimality Premise (§5.3b), the bare Wilson action satisfies

$\beta\_s = \beta\_t = \beta\_K{=}7 = 2^7 \cdot 15/14 \approx 137.143$

at the bare scale.

Proof. By Theorem 1, σ exists as a class-preserving bijection. By §5.3a (under the §2.3 second inheritance), σ is identified by channel-feature-matching as the spatial-temporal exchange. By §5.3b, the bare action realises the minimal admissible structure compatible with σ. By §5.1, this forces β_s = β_t. The common value is the K = 7 paper's β_K=7. ■

6.2 The Conditional Structure

Theorem 2 holds under three substrate-physical conditions:

- **(i) Cardinality (Theorem 1, §4):** the orientation-symmetric 11 + 2 + 1 decomposition. Inherited from the K = 7 paper (§2.3 first inheritance, pending reconciliation).
- **(ii) σ-selection (§5.3a):** σ is the channel-by-channel feature-matching spatial-temporal exchange. Channel-feature-matching is supported by the K = 7 paper's channel construction (§2.3 second inheritance, pending reconciliation). Under (L1.a)/(L1.b) of §10, it is structurally underwritten by chain-complex consistency on the cell. Under (L1.c), the cell-level chain consistency fails, and the channel-feature-matching stands as a substrate-physical postulate at the channel level without cell-complex verification.
- **(iii) Minimality (§5.3b):** the bare action does not separately weight what σ identifies.

The strengthening that would convert (ii) from "supported by the K = 7 channel construction + chain-complex realisation if available" to "structurally derived" is the chain-complex realisation of σ on a labelled K = 7 cell (§10 L1). The strengthening that would resolve the v6/v7 framing of (ii) versus the v3 framing is the reconciliation of the §2.3 second inheritance point with the K = 7 paper (§10 L6).

6.3 The IPR Correction — a Separate Empirical-Comparison Conditional

The K = 7 paper's bare derivation gives $\beta_{K=7} \approx 137.143$. The empirical fine-structure constant is $\alpha^{-1} \approx 137.036$. The difference is the IPR correction under derivation in the K = 7 paper.

Theorem 2 concerns the *bare equality* $\beta_s = \beta_t$ at the bare scale, and is independent of the IPR correction. For the empirical comparison — comparing $\beta_s = \beta_t$ against an experimental probe sensitive to spatial-vs-temporal anisotropy in α — the further question is whether the IPR correction is itself σ-symmetric. We flag this in §11.3 as a conditional for the empirical comparison only; it is not a hypothesis of Theorem 2.

6.4 What the Theorem Says and Does Not Say

Theorem 2 says (under (i), (ii), (iii)):

- The substrate's bare action is exactly isotropic under spatial-temporal exchange.
- The bare coupling anisotropy ζ_{bare} is exactly 1.
- This is the substrate-physical content of the K = 7 closure structure's channel-feature-matching combined with the Minimality Premise.

Theorem 2 does *not* say:

- That the substrate is exactly Lorentz-invariant under all Lorentz transformations.
- That the chain-complex realisation of σ has been established.
- That σ-selection's channel-feature-matching has been verified at the chain-complex level.
- That the IPR correction preserves the empirical equality (separate claim, §6.3).
- That continuum QED emerges from the substrate.

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7. Consequences for the Wilson Limit Paper

7.1 Re-Interpretation of the One-Loop Matching

The Wilson Limit paper established

$$\zeta_{eff} - 1 = (1 - \delta)(\zeta_{bare} - 1) + \mathcal{O}((\zeta_{bare} - 1)^2), \quad \delta \sim 10^{-4}.$$

Under Theorem 2, $\zeta_{bare} = 1$ exactly. $\zeta_{eff} = 1$ exactly to leading order.

The one-loop matching is no longer the *mechanism* of IR Lorentz-compatibility but a *robustness check*: small perturbations $\delta\zeta$ around $\zeta\_bare = 1$ are suppressed by $(1 - \delta)$ in the IR.

7.2 The Three K = 7 Results in Order

1. **$\beta\_K{=}7 = 2^7 \cdot 15/14 \approx 137.143$** (K = 7 paper): the bare electromagnetic coupling.
2. **$\beta\_s = \beta\_t = \beta\_K{=}7$ exactly** (this paper, Theorem 2, under (i)+(ii)+(iii)): the bare coupling is isotropic.
3. **$(1 - \delta) \sim 1 - 10^{-4}$** (Wilson Limit paper): the one-loop matching robustness factor.

Each is fixed by the same K = 7 closure-counting integers.

7.3 The BCB / TPB Parallel, Calibrated

The Wilson Limit paper §10.4 framed BCB (minimal closure-consistent transport form) and TPB (minimal finite-speed propagation scale) as substrate-physical minimality principles operating at two structural levels — both with their distinguished objects (Wilson form; c_c) derived independently and minimality operating cleanly on top.

The present paper adds σ-selection plus the Minimality Premise as a third principle, operating on the coupling structure. Under (L1.a)/(L1.b), σ-selection's channel-feature-matching is verified consistent with the cell complex; the parallel with BCB and TPB is structurally even — each principle is the minimal admissible structure compatible with a structurally distinguished feature.

Under (L1.c) — and under the §2.3 second inheritance — the channel-feature-matching is found inconsistent with any natural cell complex's chain structure, which is a substantive structural claim about the K = 7 closure structure rather than an arbitrary postulate-failure. The parallel becomes aspirational in this content-rich sense: not "we postulated σ without derivation", but "the K = 7 structure supports class-level channel-feature-matching but not chain-complex-level feature-matching — a structural property worth characterising". Resolving (L1) would either confirm the structurally-even parallel (L1.a/b) or yield this substantive structural finding (L1.c).

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8. Comparison with Empirical Bounds

8.1 The Wilson Limit Paper's Tension under Theorem 2

Under Theorem 2, $\zeta\_bare = 1$ exactly, so $\zeta\_eff - 1 = 0$ to leading order. The empirical bound $(\Delta c/c) \lesssim 10^{-17}$ is satisfied trivially.

8.2 Sources of Deviation

(D1) **Symmetry-breaking corrections to σ at sub-leading order** (§10 L3).

(D2) **Coupling to matter and to non-abelian sectors** (§10 L4).

(D3) **Higher-loop corrections to the Wilson Limit matching.** $\zeta\_eff = 1 + \mathcal{O}(10^{-8})$, well below 10^{-17}.

(D4) **Chain-complex realisation of σ (§10 L1):**

- Under (L1.a)/(L1.b): σ-selection's channel-feature-matching is verified at the cell-complex level. Theorem 2 is upgraded to derived equivariance.
- Under (L1.c) — and under the §2.3 second inheritance: the channel-feature-matching is found inconsistent with any natural cell complex. The leading-order $\zeta\_bare = 1$ is preserved but the structural character of the derivation is weaker. The deeper question — why the K = 7 closure structure supports class-level but not chain-complex-level channel-feature-matching — becomes substantive in its own right.

Current empirical bounds are consistent with all sources at the order-of-magnitude level.

8.3 What the Empirical Data Now Tests

Empirical bounds constrain (D1)–(D4) combined to be below 10^{-17}, which the present paper's content satisfies.

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9. What This Establishes and What It Does Not

9.1 Established

- Theorem 1: the K = 7 closure structure admits a class-preserving involution σ at the cardinality level, identified specifically by channel-feature-matching (§4.4), subject to §2.3's two inheritance points.
- The two-step substrate-physical structure (σ-selection + minimality) is coherently distinguished and serves its derivational role (§5).

9.2 Inherited

- The K = 7 closure structure (K = 7 paper).
- The orientation-symmetric cardinality decomposition (§2.3 first inheritance, pending reconciliation per §10 L6).
- **The substrate-physical feature-identity content of channels (§2.3 second inheritance, pending reconciliation per §10 L6).** Determines whether the v6/v7 content-bearing framing of σ-selection or the v3 postulate framing is operative.
- The bare coupling $\beta\_K{=}7 = 2^7 \cdot 15/14$ (K = 7 paper).
- The Wilson Limit paper's matching theorem.
- The σ-symmetry of the IPR correction for empirical comparison (§6.3).

9.3 Conditional

- **(ii) σ-selection** (§5.3a) — σ is the channel-by-channel feature-matching spatial-temporal exchange. Under the §2.3 second inheritance, σ-selection has substrate-physical content (channel-feature-matching); without it, σ-selection reduces to substrate-physical postulate (v3 framing). Under (L1.a)/(L1.b) of §10, the channel-feature-matching is verified at the cell-complex level.
- **(iii) The Minimality Premise** (§5.3b).
- **Theorem 2** (§6.1).
- The re-interpretation of the Wilson Limit matching as robustness (§7.1).
- The empirical-bound consequences (§8).
- The σ-symmetry of the IPR correction for empirical comparison (§6.3).
- Both inheritance points of §2.3.

9.4 Open

- **The chain-complex realisation of σ on a fully labelled K = 7 cell** (§10 L1, Appendix A).
- Sub-leading symmetry-breaking corrections to σ (§10 L3).
- Coupling to matter and to non-abelian sectors (§10 L4).
- Reconciliation of inherited claims with the K = 7 paper (§10 L6) — particularly the second inheritance point, which determines whether σ-selection is content-bearing or postulated.

9.5 What Cannot Be Concluded

This paper does not establish:

- That full Lorentz invariance is exact at the bare scale (only the spatial-temporal exchange part, under (i)+(ii)+(iii)).
- That continuum QED emerges from the substrate.
- That the closure structure is derived from more primitive substrate axioms.
- That σ-selection's channel-feature-matching has been verified at the chain-complex level (open, §10 L1).
- An explicit chain-complex realisation of σ.

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10. Limitations and Open Questions

(L1) The explicit chain-complex realisation of σ on a natural K = 7 cell.

"Natural" means a cell model derived directly from the K = 7 closure structure's vertex-and-edge specification, as distinct from ad hoc constructions chosen to admit a chain map. The notion is partly programme-internal.

Three resolutions:

(L1.a) σ admits a chain-complex realisation on the hub-plus-hexagon cell verifying the channel-feature-matching.

(L1.b) σ admits a chain-complex realisation on a different natural cell model.

(L1.c) σ-selection's channel-feature-matching (under the §2.3 second inheritance) is inconsistent with the chain structure of any natural K = 7 cell. A substantive structural finding rather than an arbitrary selection failure.

(L2) Spatial rotation invariance. Treated in the corpus's hexagonal closure geometry paper.

(L3) Sub-leading corrections to σ.

(L4) Coupling to matter and to non-abelian sectors.

(L5) Higher-loop robustness. Inherited from the Wilson Limit paper.

(L6) Reconciliation with the K = 7 paper. Three inherited claims that should be reconciled with the K = 7 paper's specific derivations:

- The orientation-symmetric cardinality decomposition (§2.3 first inheritance).
- **The substrate-physical feature-identity content of channels (§2.3 second inheritance).** Among the inheritance items, this one is the most consequential for the present paper's framing: it determines whether σ-selection (§5.3a) is content-bearing channel-feature-matching (v6/v7 framing) or substrate-physical postulate from the cardinality orbit (v3 framing). It also determines the interpretation of (L1.c) — substantive structural finding (under the inheritance) versus generic no-chain-map-exists resolution (without it).
- The σ-symmetry of the IPR correction (§6.3).

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11. Epistemic Status

11.1 Established

- Theorem 1: a class-preserving σ exists at the cardinality level (§4.4), identified by channel-feature-matching under the §2.3 inheritances.
- The two-step distinction between σ-selection (§5.3a) and Minimality Premise (§5.3b) of §5.

11.2 Inherited

- K = 7 closure structure and bare coupling β_K=7 (K = 7 paper).
- Orientation-symmetric cardinality decomposition (K = 7 paper, §2.3 first inheritance, pending reconciliation per §10 L6).
- **Substrate-physical feature-identity content of channels (K = 7 paper, §2.3 second inheritance, pending reconciliation per §10 L6).** Whether this inheritance holds determines the operative framing of σ-selection: content-bearing channel-feature-matching (v6/v7) or substrate-physical postulate (v3).
- σ-symmetry of the IPR correction (K = 7 paper, per §6.3).
- Anisotropic Wilson framework and one-loop matching theorem (Wilson Limit paper).

11.3 Conditional (the substantive ones)

- **σ-selection** (§5.3a) — σ is the channel-by-channel feature-matching spatial-temporal exchange. The present paper adopts this under the §2.3 second inheritance. Structurally underwritten under (L1.a)/(L1.b); standing as substrate-physical postulate at the channel level (with channel-feature-matching support) under (L1.c).
- **The Minimality Premise** (§5.3b).
- **Theorem 2** (§6.1) — conditional on Theorem 1 plus σ-selection plus the Minimality Premise. *If the reader does not accept σ-selection (§5.3a) or the Minimality Premise (§5.3b), and the chain-complex realisation of §10(L1) is not established, Theorem 2 does not hold.*
- The re-interpretation of the Wilson Limit matching as robustness (§7.1).
- The empirical-bound consequences (§8).
- The σ-symmetry of the IPR correction for empirical comparison (§6.3).
- The orientation-symmetric cardinality decomposition inheritance (§2.3 first inheritance).
- **The substrate-physical feature-identity content of channels (§2.3 second inheritance).** *Under this inheritance, σ-selection is the content-bearing channel-feature-matching of §5.3a; without it, σ-selection reduces to a substrate-physical postulate (v3 framing) and the interpretation of (L1.c) shifts from "substantive structural finding" to "generic no-chain-map-exists".*

11.4 Open

- **The chain-complex realisation of σ on a labelled K = 7 cell** (§10 L1).
- Sub-leading symmetry-breaking corrections to σ (§10 L3).
- Coupling to matter and to non-abelian sectors (§10 L4).
- Reconciliation of inherited claims with the K = 7 paper (§10 L6) — especially the second inheritance point.

11.5 What Cannot Be Concluded

This paper does not establish full Lorentz invariance, only the spatial-temporal exchange part under the (i)+(ii)+(iii) conditional structure.

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12. Conclusion

The Wilson Limit paper of this programme established the one-loop matching theorem on the K = 7 substrate. That paper acknowledged the one-loop factor was insufficient to explain empirical Lorentz invariance at observed precision, and identified the bare coupling anisotropy ζ_{bare} as the open substrate-physical question.

This paper has addressed that question by a three-component substrate-physical argument:

(i) Theorem 1 (§4) establishes that the K = 7 closure structure admits a class-preserving involution σ at the cardinality level.

(ii) σ-selection (§5.3a) identifies σ specifically as the channel-by-channel feature-matching spatial-temporal exchange, *under the §2.3 second inheritance point that the K = 7 paper's channel construction carries feature-identity content*. The cardinality orbit of class-preserving bijections has order ≈ 8×10^7; channel-feature-matching under the inheritance collapses this to a unique element (modulo orientation choices).

(iii) The Minimality Premise (§5.3b) is the substrate-physical principle that the bare action realises the minimal admissible structure consistent with σ-symmetry. Parallel to BCB and TPB.

Theorem 2 (§6) combines these: under (i), (ii), and (iii), $\beta_s = \beta_t = \beta_{K=7}$ exactly at the bare scale.

The dependency between §2.3's second inheritance point and §5.3a's content-bearing framing of σ-selection is surfaced explicitly in v7: under the feature-identity inheritance, σ-selection's substrate-physical content is channel-feature-matching, and the chain-complex realisation question of §10(L1) becomes a verification question (does the channel-level feature-matching extend to the cell complex?); without the inheritance, σ-selection reduces to selecting an element from the cardinality orbit and becomes a substrate-physical postulate (v3 framing). This dependency is among the most important reconciliation items between the present paper and the K = 7 paper (§10 L6).

The Wilson Limit paper's matching theorem is now re-interpreted: under Theorem 2 it is a robustness result. The substantive substrate-physical mechanism for bare Lorentz-compatibility is the K = 7 closure structure's channel-feature-matching combined with the Minimality Premise — with one-loop matching as additional IR robustness.

The principal open item is the chain-complex realisation of σ on a labelled K = 7 cell (§10 L1, Appendix A). Resolving (L1) — whether by verifying σ-selection's channel-feature-matching at the cell-complex level (L1.a/b) or by establishing that the feature-matching is inconsistent with any natural cell complex (L1.c) — would either upgrade Theorem 2 to derived equivariance or yield a substantive structural finding about the K = 7 closure structure: that it supports class-level channel-feature-matching without supporting it at the chain-complex level, which is a structural property worth characterising in its own right.

The three K = 7 results — $\beta\_K=7 = 2^7 \cdot 15/14$, $\beta\_s = \beta\_t$ (under (i)+(ii)+(iii)), and $(1 - \delta) \sim 10^{-4}$ — are three faces of the same K = 7 closure structure under the substrate-physical principles of the corpus.

In a sentence: *the K = 7 closure structure admits a class-preserving spatial-temporal exchange involution σ identified by channel-by-channel feature-matching (§5.3a, under the §2.3 second inheritance); under the Minimality Premise that the bare action does not separately weight what σ identifies (§5.3b), σ-invariance forces β_s = β_t exactly; the substrate is therefore exactly Lorentz-compatible at the bare scale modulo the open chain-complex realisation question of §10(L1) — which would verify the channel-feature-matching at the cell-complex level (L1.a/b) or yield a substantive structural finding that the feature-matching is inconsistent with any natural cell complex (L1.c)*.

Appendix A — Status of the Explicit Combinatorial Realisation of σ

This appendix records the status of an open piece of programme work: the explicit construction of σ as a chain-complex automorphism on a fully labelled K = 7 cell.

"Natural cell model" means a cell model derived directly from the K = 7 closure structure's vertex-and-edge specification, as distinct from ad hoc constructions chosen to admit a chain map. The notion is partly programme-internal: what counts as a natural derivation of the cell depends on the substrate axiomatisation. Future tests of (L1.c) should make the cell-model class explicit.

A.1 What the Explicit Construction Would Establish

(a) A specific cell model with labelling convention.

(b) $\sigma\_vert : V \to V$, $\sigma\_vert^2 = id$.

(c) $\sigma\_edges : E \to E$ (modulo orientation), $\sigma\_edges^2 = id$.

(d) $\partial\sigma\_edges = \sigma\_vert\ \partial$ verified for every edge.

(e) Descent to the cycle space: σ_edges induces a permutation realising the channel-feature-matching of §5.3a.

(f) Holonomy equivariance $F\_\{\sigma(P)\}(\theta) = F\_P(\sigma^{-1}\theta)$.

If all six are established, σ is a chain-complex automorphism. σ-selection extends from channel-level feature-matching to cell-complex-level verification. The Minimality Premise of §5.3b is no longer needed for the derivation.

A.2 What Was Attempted (v2) and Why It Was Retracted

A previous draft attempted the construction on the hub-plus-hexagon cell. Reviewer-identified issues:

(M1) Loops not closed under the proposed edge set.

(M2) Conflation of 2-faces with cycle-space basis.

(M3) Proposed σ_edges was not a well-defined involution.

(M4) Worked example did not close as a cycle.

(M5) Verification used only the bijection property, not edge-level structure.

The v2 appendix was retracted.

A.3 What Would Be Required

(i) Constructing σ_vert : $V \to V$ — possibly sending lower-slice spatial vertices to upper-slice vertices.

(ii) Deriving σ_edges from σ_vert by chain-map compatibility. When multiple edges share endpoints (the $K = 7$ hub's spoke-temporal pairing), σ_vert determines σ_edges only up to a choice within that subset.

(iii) Verifying $\sigma\_vert^2 = id$ ($\sigma\_edges^2 = id$ follows).

(iv) Computing the induced cycle-space action; verifying it realises the channel-feature-matching of §5.3a.

(v) Verifying holonomy equivariance row by row.

(vi) Filling in the verification table.

The substantive question is whether σ_vert can be chosen so that all subsequent steps go through — equivalently, whether the channel-feature-matching of §5.3a (under the §2.3 second inheritance) extends to the cell's chain-complex structure.

A.4 Status

Open (Appendix A, §10 L1). Construct σ as an explicit chain-complex automorphism on a fully labelled natural $K = 7$ cell, or establish that no such construction exists on any natural $K = 7$ cell model.

Appendix B — Cross-Paper Notation Index

Symbol	Meaning	Source
$K = 7$	Closure integer	K = 7 paper
$N_{loop} = 14 = 2K$	Loop count per cell	K = 7 paper
$14 = 11 + 2 + 1$	Closure-counting decomposition	K = 7 paper, two inheritance points flagged §2.3
$\beta_{K=7} = 2^7 \cdot 15/14$	Bare coupling at isotropic point	K = 7 paper
$\square_s$, $\square_t$	Spatial and temporal plaquette classes	Wilson Limit paper
$\mathcal{S}_s$, $\mathcal{S}_t$	Spatial and temporal Wilson sums	Wilson Limit paper
β_s, β_t	Spatial and temporal couplings	Wilson Limit paper
$\zeta = \sqrt{(\beta_s/\beta_t)}$	Coupling anisotropy	Wilson Limit paper
σ	Spatial-temporal exchange involution (channel-feature-matching, §3)	This paper
σ-selection	Identification of σ by channel-by-channel feature-matching	This paper §5.3a (under §2.3 second inheritance)
Minimality Premise	The bare action realises the minimal admissible structure consistent with σ	This paper §5.3b
σ_{vert}, σ_{edges}	Vertex- and edge-level realisations of σ (open)	This paper Appendix A
Cardinality orbit $\approx 8 \times 10^7$	The $11! \cdot 2! \cdot 1!$ class-preserving bijections	This paper §3.2, §4.4
$(1 - \delta)$, $\delta \sim 10^{-4}$	One-loop matching robustness factor	Wilson Limit paper

Key Identifications

- Theorem 1: σ exists as a class-preserving bijection at the cardinality level, identified specifically by channel-feature-matching under the §2.3 inheritances.
- σ-selection (under §2.3 second inheritance): channel-by-channel feature-matching — one bijection (modulo orientation choices) rather than one of $\approx 8 \times 10^7$ in the cardinality orbit.
- Minimality Premise: the bare action does not separately weight what σ identifies (§5.3b).
- Under (i) Theorem 1 + (ii) σ-selection + (iii) Minimality Premise: $\beta_s = \beta_t = \beta_{K=7}$ exactly (Theorem 2).
- Combined with Wilson Limit paper's matching: $\zeta_{eff} = \zeta_{bare} = 1$ exactly to leading order.
- The chain-complex realisation of σ on a labelled cell is the principal open item; would verify σ-selection's channel-feature-matching at the cell-complex level.

References

The present paper is internal to the VERSF Theoretical Physics Programme. The two principal companion papers are:

- The K = 7 paper (β derivation): *The Fine Structure Constant from K = 7 Closure Geometry*, VERSF Theoretical Physics Programme.
- The Wilson Limit paper (one-loop matching): *The K = 7 Wilson Limit*, VERSF Theoretical Physics Programme.

Additional corpus references: the synthesis paper, the four substrate refinement papers (I–IV), and the hexagonal closure geometry paper (spatial rotation invariance treatment). The corpus index at versf-eos.com gives the authoritative list.

Standard Physics References (Inherited from Wilson Limit Paper)

- Karsch, F. (1982). SU(N) gauge theory couplings on asymmetric lattices. *Nuclear Physics B* **205**, 285–300.
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