

# The Leading-Order Unique Record Current in the VERSF Master Action

## A Constrained Uniqueness Theorem for the Leading-Order Record Current

$C^{\mu\nu}[\Phi]$

Keith Taylor · VERSF Theoretical Physics Programme

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### The object whose record current we are deriving

The VERSF master action is

$$S_{\text{VERSF}}[\Phi_{\mu\nu}, g, \lambda] = \int d^4x \sqrt{-g} \left[ \begin{array}{l} -1/2 g^{\alpha\beta} \nabla_\alpha \Phi_{\mu\nu} \nabla_\beta \Phi^{\mu\nu} - 1/2 m^2 \Phi_{\mu\nu} \Phi^{\mu\nu} \\ \quad \square \quad + \Phi_{\mu\nu} S^{\mu\nu}[\Phi] + \lambda_\mu (\nabla_\nu C^{\mu\nu}[\Phi] - S^\mu[\Phi]) \\ \quad \square \quad + \frac{1}{16\pi G} R - \Lambda \end{array} \right]$$

Of the two functionals appearing in the matter sector, the commitment-density functional  $S[\Phi]$  is closed in the master action paper using the TPB-Entropy identity. The remaining functional is the *record current*:

$$C^{\mu\nu}[\Phi] = ?$$

This paper closes that question. We show that under six physical constraints, at leading order in the substrate expansion, the record current is forced to:

$$C^{\mu\nu}[\Phi] = a \Phi^{\mu\nu} + b g^{\mu\nu} \Phi$$

with  $a$  and  $b$  dimensional constants set by substrate-scale physics. The remainder of the paper establishes this result rigorously.

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### For the General Reader

#### What this paper is about, in plain language

The VERSF programme is an attempt to explain physics — quantum mechanics, gravity, the way time works — by starting from one simple idea: reality is built from *committed records*. Every physical event leaves a trace; those traces accumulate; what we call “time,” “matter,” and “space” all emerge from the structure of those accumulated records.

To turn this idea into a real physical theory, you need to write down equations that describe how records form, how they flow, and how they shape the world. The full set of

those equations is captured in something called a *master equation* — or more precisely, a *master action*.

### What a master action looks like

If you've heard of one famous physics equation, it's probably  $E = mc^2$ . But that's not a master action; it's a single specific relationship pulled out of a larger theory. A master action is something quite different: a single mathematical expression that contains an *entire* physical theory. Once you have it, all the equations of motion — the rules for how things actually move and interact — fall out of it automatically by a standard mathematical procedure.

Einstein's theory of gravity, for example, is contained in the *Einstein–Hilbert action*:

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda) + S_{\text{matter}}$$

Don't worry about the symbols. The point is that this single expression contains *all* of general relativity — every prediction Einstein's theory makes about how gravity works, from planetary orbits to black holes to the expansion of the universe. The Standard Model of particle physics has a similar master action containing all of particle physics. Quantum field theories generally have actions. This is the deepest way modern physics is currently organised.

The VERSF master action — shown in the box at the top of this page — is the VERSF programme's version of that single expression. It contains four ingredients: a description of how records spread, a rule for how new records form, a conservation law (records can't appear or disappear except by a specific process), and a connection to gravity (records bend space, the way mass does in Einstein's theory). It looks complicated for the same reason the Einstein–Hilbert action looks complicated: a master action is meant to be *complete*, not catchy.

### The gap this paper closes

One ingredient in the VERSF master action — called the *record current* — describes *how records flow* from one place to another. Until now, the master action just said “there is some record current, call it  $C^{\{\mu\nu\}}[\Phi]$ , and it has to have certain properties.” But it didn't pin down exactly what the record current looks like. There was a gap.

The natural question is: could the universe have chosen any record current it liked, or is there only one that works?

This paper proves it's the latter. If you write down a few physical requirements that any record current must satisfy — things like “records can't appear out of nothing” and “the laws of physics shouldn't depend on which direction you face” — there turns out to be **only one possible answer**. The record current is forced. The universe could not have done it any other way.

The answer is shown in the third box at the top of the page:  $C^{\{\mu\nu\}}[\Phi] = a\Phi^{\{\mu\nu\}} + b g^{\{\mu\nu\}}\Phi$ , where  $a$  and  $b$  are two numerical constants set by the underlying physics. That is the only record current consistent with the requirements.

### Why this matters

Closing this gap means the VERSF master action is now a *single specific theory* rather than a family of possibilities. The two functionals that describe how records form and how they flow are both fixed by physical principles, leaving only a small handful of numerical constants (like Newton's gravitational constant) that the theory inherits from how the world actually is. The standard ingredients that any field theory needs — the parts describing how the field propagates and how it couples to gravity — are inherited from the long-established machinery of physics. The genuinely VERSF-specific ingredients are now closed.

In one line: **no alternative functional form for the record current is admissible under the stated physical constraints.**

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### Contents

1. **The Problem** — what the record current is, why this matters, the regime of this paper, strategy, claims and non-claims
2. **The Candidate Space** — setup, the extended candidate space (linear, gradient, bilinear, curvature, substrate-vector terms), scope of the present analysis
3. **The Six Physical Constraints** — conservation compatibility, gauge selection, tensor symmetry, additivity of independent records, scalar reduction at the divergence level, stress-energy consistency
4. **Joint Imposition: The Uniqueness Theorem** — theorem statement, term-by-term elimination, explicit (K6) variational calculation, what this gives the master action
5. **Why Each Constraint Is Forced** — substantive justification of each (K1)–(K6) as forced by corpus coherence rather than freely chosen
6. **The Two Surviving Coefficients** — trace / trace-free decomposition, physical interpretation of the surviving coefficients  $\tilde{a}$  and  $\tilde{b}$
7. **Comparison with the Kinematic-Conservation Paper [1]** — relationship to the scalar form, how the tensor extension recovers the scalar case
8. **The Master Action with  $C^{\{\mu\nu\}}[\Phi]$  Specified** — explicit form of the action, structural completeness of the constitutive sector, remaining research targets
9. **Position Within the VERSF Corpus** — upstream dependencies, downstream consequences, relation to the no-alternative theorem
10. **Limitations** — subleading terms, coefficient determination, constraint motivation, additivity for entangled regions, higher-spin coupling, corpus-level consistency
11. **Conclusion**

**References** — internal corpus references and external methodology references (Lovelock, Weinberg, Burgess, Donoghue)

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## 1. The Problem

### 1.1 What $C^{\{\mu\nu\}}[\Phi]$ is

The VERSF master action contains the term

$$\lambda_\mu(\nabla_\nu C^{\mu\nu}[\Phi] - S^\mu[\Phi])$$

which enforces the conservation identity  $\nabla_\nu C^{\{\mu\nu\}} = S^\mu$  at every spacetime point. The object  $C^{\{\mu\nu\}}[\Phi]$  is the *record current*: a rank-2 tensor functional of the record-bearing field  $\Phi_{\mu\nu}$  whose divergence equals the commitment-density vector. Physically, it is the local flow of substrate record-content; mathematically, it is what conservation under variation of  $\lambda_\mu$  couples to.

**A note on notation.** The master action contains commitment-density functionals at two distinct tensor ranks: the rank-2 functional  $S^{\{\mu\nu\}}[\Phi]$  appearing in the matter-coupling term  $\Phi_{\{\mu\nu\}} S^{\{\mu\nu\}}[\Phi]$ , and the rank-1 functional  $S^\mu[\Phi]$  appearing in the constraint term  $\lambda_\mu(\nabla_\nu C^{\{\mu\nu\}} - S^\mu)$ . These are not independent functionals: the rank-1 commitment-density is the divergence projection of the rank-2 form,

$$S^\mu[\Phi] \equiv \nabla_\nu S^{\mu\nu}[\Phi]$$

This relation makes the conservation identity  $\nabla_\nu C^{\{\mu\nu\}} = S^\mu$  structurally a vector equation between the divergence of  $C^{\{\mu\nu\}}$  and the divergence of  $S^{\{\mu\nu\}}$ . The closed form  $S^{\{\mu\nu\}}[\Phi] = \kappa \Phi^{\{\mu\nu\}} |\Phi|^2 / \text{TPB}[\Phi]$  derived in the master action paper specifies the rank-2 functional; its divergence supplies the rank-1 form appearing in the constraint. Throughout this paper, references to “the commitment-density functional” are to the rank-2 form  $S^{\{\mu\nu\}}$ ; the rank-1 form  $S^\mu$  enters only through the conservation identity.

**Forward-reference: order-matching across the conservation identity.** The leading-order form of  $\nabla_\nu C^{\{\mu\nu\}}$  derived in this paper is linear in  $\Phi$ , while the leading-order form of  $S^\mu = \nabla_\nu S^{\mu\nu}$  derived from the master action paper’s closed  $S^{\{\mu\nu\}}$  is cubic in  $\Phi$  (modulo TPB-dependence). For the conservation identity  $\nabla_\nu C^{\{\mu\nu\}} = S^\mu$  to hold as a leading-order kinematic identity, the  $\text{TPB}[\Phi]$  functional must carry a  $|\Phi|^2$  scaling at leading order that absorbs the cube — i.e.,  $\text{TPB}[\Phi] \sim |\Phi|^2 \times (\text{substrate-scale factors})$ . This is a corpus-level consistency requirement, addressed by the  $\text{TPB}[\Phi]$  derivation paper (next on the corpus roadmap) and verified there. Should that scaling fail, the conservation identity would instead define a non-trivial constraint surface in configuration space rather than a kinematic identity — a substantive but admissible alternative reading. Either way, the present paper’s derivation of  $C^{\{\mu\nu\}}$  is independent of the resolution: the constraints (K1)–(K6) act on the form of  $C^{\{\mu\nu\}}$  at the linear level, not on the order-matching with  $S^\mu$ .

In the master action paper,  $C^{\{\mu\nu\}}[\Phi]$  is treated as a *constitutive input* — its functional form is specified by an upstream paper (the kinematic-conservation paper [1]) rather than derived within the master action itself. This modularity is technically clean but leaves a structural question open: what fixes the functional form of  $C^{\{\mu\nu\}}[\Phi]$ ?

## 1.2 Why this matters

Two distinct concerns motivate closing this question:

**Coherence of the master action.** With  $C^{\{\mu\nu\}}[\Phi]$  free, the master action is parameterised by the choice of constitutive functional. Different choices give different theories. If the choice is arbitrary, the master action is not a single theory but a family of theories indexed by the choice — much like how a generic field theory becomes specific only when its Lagrangian is fixed. To make the master action a single coherent theoretical commitment, the functional form of  $C^{\{\mu\nu\}}[\Phi]$  needs to be either fixed by some principle or derived.

**Falsifiability of predictions.** The master action's predictions depend on  $C^{\{\mu\nu\}}[\Phi]$  through the constraint contribution to the stress-energy tensor (which sources gravity) and through the dynamics of the field equation for  $\Phi_{\mu\nu}$  (where  $C^{\{\mu\nu\}}$  appears via  $\lambda$ -coupling). With  $C^{\{\mu\nu\}}$  parametrically free, predictions become parametrically broad. With  $C^{\{\mu\nu\}}$  forced, predictions become sharp.

## 1.3 The regime of this paper: continuum limit, not fundamental

Before laying out the strategy, we name the regime in which this paper operates. The VERSF programme commits — at the foundational level — to *emergent time*: time is not a continuous parameter prior to physical events but accumulates from discrete bit-formation events at the substrate level (the Ticks Per Bit framework). Continuous fundamental time is *physically impossible* in this programme; what physics describes is the continuum-limit description of an underlying discrete substrate, in the same way that fluid mechanics is the continuum-limit description of underlying molecular collisions.

The master action — and consequently the present paper's analysis of one of its functionals — operates entirely at the *continuum-limit level*. The integrals over  $d^4x$ , the smooth metric  $g_{\mu\nu}$ , the covariant derivative  $\nabla$ , the variational machinery, the EFT power-counting in  $(\partial/\Lambda_{\text{sub}})$  — all of these are continuum-limit constructs. They are correct in the regime where the substrate has been smoothed over many ticks and many substrate cells, which is essentially everywhere physical measurements are currently made. They are *not* correct at the substrate scale itself (Planck length, single-tick durations), where the discrete TPB dynamics take over.

This means the present paper inherits the master action's continuum-limit framing and operates within it. The constraints (K1)–(K6) are themselves continuum-limit reflections of substrate-level requirements:

- (K1)–(K2) reflect bit-conservation at the substrate.
- (K3) reflects the symmetric structure of substrate-level stress-energy.
- (K4) reflects substrate-level locality and independence.
- (K5)–(K6) reflect cross-paper consistency at the continuum level.

The leading-order theorem and the closed form  $a\Phi^{\{\mu\nu\}} + b g^{\{\mu\nu\}}\Phi$  are therefore claims about the *continuum-limit record current*, not about a hypothetical substrate-level current.

A substrate-level analysis would operate on tick-counted discrete dynamics and is the subject of the TPB papers, not of this paper.

This framing matters for two reasons. First, it sets the regime of validity correctly: the paper's result holds wherever the continuum approximation holds (which is essentially all macroscopic and laboratory physics) and breaks down at the substrate scale, where direct discrete dynamics must be used. Second, it positions the paper honestly within the corpus's broader architecture: the corpus's foundational level is TPB substrate dynamics; the master action and its analysis are the effective continuum theory derived from that foundational level.

#### 1.4 Strategy

We follow the same strategy used for the no-alternative theorem of quantum kinematics: enumerate a candidate space of possibilities, identify the physical constraints that admissible candidates must satisfy, and show that the joint imposition of constraints leaves only a unique survivor at leading order (modulo a finite number of dimensional constants).

The candidate space is the set of all local covariant rank-2 functionals of  $\Phi_{\mu\nu}$  admissible under symmetry and locality, including linear, bilinear, curvature-coupled, and substrate-vector-coupled structures. The physical constraints are six in number: conservation compatibility, no pure-gauge redundancy, tensor symmetry, additivity of independent records, scalar reduction at the divergence level, and stress-energy consistency. The constraints together with substrate-expansion suppression eliminate portions of the candidate space; what survives at leading order is the result claimed below.

#### 1.5 What this paper claims

We prove: under the six physical constraints of §3, *at leading order in the substrate expansion*, the rank-2 record current  $C^{\mu\nu}[\Phi]$  is forced to the form

$$C^{\mu\nu}[\Phi] = a \Phi^{\mu\nu} + b g^{\mu\nu} \Phi$$

with  $a$  and  $b$  dimensional constants set by substrate-scale physics. All other terms in the candidate space — bilinear, curvature-coupled, and substrate-vector-coupled — are either eliminated by the constraints or suppressed by powers of substrate-scale parameters at leading order. This is a *constrained leading-order uniqueness* result: uniqueness within the candidate space defined by the physical requirements, at the leading order of the substrate expansion that defines the master action's regime of validity.

#### 1.6 What this paper does not claim

We do not claim:

- **Unconditional uniqueness across all conceivable functional forms.** The candidate space considered (linear, gradient, bilinear, curvature-coupled, and substrate-vector-coupled rank-2 covariant local functionals of  $\Phi_{\mu\nu}$ ) is bounded by the structural completeness principle: functionals involving non-local kernels, higher-spin sources, or non-tensor structures are outside scope. We argue that

scope is the right one for the master action, but readers who reject the bound have a separate question.

- **Determination of the dimensional constants  $a$  and  $b$ .** These are empirical inputs at substrate scale; the present paper fixes the functional form, not the values.
- **Derivation independent of the master action's other commitments.** The constraints we impose are motivated by the master action's structure (specifically by stress-energy consistency with [26]). This is a *coherent-system* uniqueness within the VERSF programme rather than an isolated derivation.

These limits are explicit and bounded. Within them, the result is decisive.

## 2. The Candidate Space

### 2.1 Setup and assumptions

Two structural assumptions frame the analysis.

**(S1)  $\Phi_{\mu\nu}$  is a symmetric rank-2 tensor field.** This is consistent with its role as the record-bearing field that ultimately sources stress-energy structure (which is itself symmetric). Antisymmetric or mixed-symmetry record-tensor fields are outside the present scope.

**\*\* (S2)  $C^{\{\mu\nu\}}[\Phi]$  is a local covariant functional of  $\Phi_{\mu\nu}$ , the metric  $g_{\mu\nu}$ , the substrate-emergent vector  $U^\mu$ , the curvature tensor  $R^{\{\mu\nu\}}_{\{\alpha\beta\}}$ , and the covariant derivative  $\nabla$ .** Locality and covariance bound the construction; the inclusion of  $U^\mu$  and the curvature is provisional — we will see in §4 that both are eliminated by physical constraints.

### 2.2 The extended candidate space

Under (S1) and (S2), the most general local covariant rank-2 functional at leading order in  $\Phi$  takes the form:

$$\begin{aligned}
 C^{\mu\nu}[\Phi] = & a \Phi^{\mu\nu} + b g^{\mu\nu} \Phi \\
 & \square + c \nabla^{(\mu} V^{\nu)}[\Phi] \\
 & \square + d \Phi^{\mu\alpha} \Phi^\nu{}_\alpha + e \Phi^{\mu\nu} |\Phi|^2 \\
 & \square + f R^{\mu\nu} \Phi + h R \Phi^{\mu\nu} \\
 & \square + j \Phi U^\mu U^\nu + k \Phi^\mu{}_\alpha U^\nu U^\alpha \\
 & \square + \dots
 \end{aligned}$$

The terms group into five structural classes:

- **Linear field terms** (coefficients  $a$ ,  $b$ ): direct identification with the field, and metric-projected trace.
- **Linear gradient terms** (coefficient  $c$ ): a symmetrised gradient of some vector functional  $V^\mu[\Phi]$ .

- **Bilinear field terms** (coefficients d, e): quadratic-in- $\Phi$  constructions.
- **Curvature terms** (coefficients f, h): couplings to the spacetime curvature.
- **Substrate-vector terms** (coefficients j, k): couplings to the substrate-emergent vector  $U^\mu$  that may appear in [1]’s scalar-form analysis.
- (...) denotes higher-order and higher-derivative contributions.

The candidate space is the space of admissible coefficient tuples (a, b, c, d, e, f, h, j, k, ...) modulo the physical constraints of §3.

### 2.3 Scope of the present analysis

We work at leading order in the substrate expansion: the regime where  $\Phi$  is small compared to substrate-scale field strength  $\Phi_{\text{substrate}}$ , and where curvature is small compared to inverse substrate length squared ( $R\ell_{\text{sub}}^2 \ll 1$ ). Higher-order terms within each class — cubic and higher in  $\Phi$ , higher derivatives, products of curvature with  $\Phi$  at higher orders — are suppressed by powers of these small parameters and are outside the leading-order analysis.

By “leading order” we mean the lowest-order terms in the joint expansion in  $\Phi/\Phi_{\text{sub}}$ ,  $\partial/\Lambda_{\text{sub}}$ , and  $R\ell_{\text{sub}}^2$ , evaluated in the emergent-Lorentz regime. All three small parameters are simultaneously kept to lowest non-trivial order; any term suppressed by a positive power of any of them is subleading. This expansion is the standard EFT power-counting [W2, B] applied to the substrate scale.

The result claimed in §1.5 is therefore best read as **leading-order uniqueness in the substrate expansion**: at leading order, the joint imposition of constraints forces  $C^{\{\mu\nu\}}[\Phi]$  to the two-coefficient form. Subleading corrections from higher-order terms exist but are suppressed by powers of substrate-scale parameters.

## 3. The Six Physical Constraints

We list the constraints that admissible  $C^{\{\mu\nu\}}[\Phi]$  must satisfy. Each is motivated physically and stated explicitly. The joint imposition of the six is what forces leading-order uniqueness.

### 3.1 Conservation compatibility

**\*\* (K1) The divergence  $\nabla_\nu C^{\{\mu\nu\}}$  must equal the commitment-density vector  $\mathcal{S}^\mu$  supplied by the master action.\*\*** The master action contains the term  $\lambda_\mu(\nabla_\nu C^{\{\mu\nu\}} - \mathcal{S}^\mu)$ ; variation with respect to  $\lambda_\mu$  enforces this identity. Admissible  $C^{\{\mu\nu\}}$  are those whose divergence is well-defined and matches the dimensional structure of  $\mathcal{S}^\mu$ .

In practice, (K1) requires  $C^{\{\mu\nu\}}$  to contain at least one term whose divergence is non-zero and structurally compatible with the vector commitment-density on the right-hand side.

### 3.2 No pure-gauge redundancy

**(K2) Among record currents that share the same divergence, we select the representative of lowest derivative order with the minimal number of independent terms.** Two record currents  $C^{\{\mu\nu\}}$  and  $C^{\{\mu\nu\}} + \Delta C^{\{\mu\nu\}}$  that share the same divergence (i.e., where  $\nabla_{\nu} \Delta C^{\{\mu\nu\}} \equiv 0$ ) carry identical physical content as far as the conservation identity is concerned; their difference  $\Delta C^{\{\mu\nu\}}$  is an identically-divergence-free addition that contributes nothing to  $\nabla_{\nu} C^{\{\mu\nu\}}$ . There are abundant such structures (Bianchi-type combinations of curvature contractions, trace-free combinations of higher  $\Phi$ -derivatives, and so on). (K2) discards them by selecting the representative of lowest derivative order with the fewest independent terms.

**Operational content.** (K2) is a *parameterisation choice* rather than a constraint on physics: it does not exclude any physical record current, only its non-minimal representations. The constraint adds bookkeeping discipline (so that “the record current” picks out a definite functional form rather than an equivalence class) but it does not, by itself, restrict what the record current can be. Its role in §4.2 is correspondingly limited — it ensures that the surviving form  $a\Phi^{\{\mu\nu\}} + b g^{\{\mu\nu\}}\Phi$  is the canonical representative of its equivalence class rather than being one of infinitely many divergence-equivalent expressions.

In this sense (K2) refines (K1): where (K1) ensures the divergence has the correct structural form, (K2) ensures the representation of  $C^{\{\mu\nu\}}$  is the canonical one within each gauge equivalence class.

### 3.3 Tensor symmetry

**(K3)  $C^{\{\mu\nu\}}$  must be a symmetric tensor:  $C^{\{\mu\nu\}} = C^{\{\nu\mu\}}$ .** This follows because the rank-2 record current sources rank-2 stress-energy structure (via the constraint contribution to  $T_{\mu\nu}$ ), and physical stress-energy is symmetric.

Antisymmetric structures and mixed-symmetry structures are excluded by this constraint. The candidate space is restricted to symmetric tensor functionals.

### 3.4 Additivity of independent records

**(K4) For independent regions A and B with non-overlapping substrate support, the record current of the joint region equals the sum of the local currents:**

$$C_{AUB}^{\mu\nu} = C_A^{\mu\nu} + C_B^{\mu\nu} \quad \text{when supports are disjoint.}$$

This is the substrate-level statement of independence: separate regions of substrate carry separate records, and combining the regions just adds the records together. Functionals that mix records non-additively (e.g., through products or convolutions of fields from different regions) violate this and are excluded.

**Important scope note.** Additivity (K4) is required *only for disjoint-support regions*. Correlated or entangled configurations — where  $\Phi$  itself carries non-local correlations across regions — are represented *within*  $\Phi$  and do not violate (K4). The constraint addresses how the record-current functional combines under disjoint composition; it does

not restrict what configurations  $\Phi$  itself can exhibit. Quantum entanglement, which manifests as non-product structure of  $\Phi$  across regions, is therefore consistent with (K4): the entanglement lives in the field configuration, not in the functional dependence of  $C^{\{\mu\nu\}}$  on  $\Phi$ .

In practice, additivity restricts admissible functionals to those that depend locally on  $\Phi$  at each point: non-local kernels in the *functional definition of  $C^{\{\mu\nu\}}[\Phi]$*  are excluded, while non-local *configurations of  $\Phi$  itself* (entanglement) are not.

**A note on functional vs expectation additivity.** Additivity is required at the level of the functional itself, not merely at the level of expectation values. This is because  $C^{\{\mu\nu\}}$  is defined as a *local current density* — a quantity assigned to each substrate point — and locality combined with independence forces pointwise additivity for disjoint-support configurations. A weaker expectation-level additivity would permit non-local kernels in the functional that average out statistically; (K4) excludes these because they would violate locality at the level of individual configurations, not just on average.

### 3.5 Correct scalar trace reduction

**\*\* (K5)** The divergence  $\nabla_\nu C^{\{\mu\nu\}}$  must reduce to the scalar record current  $C^\mu$  of the kinematic-conservation paper [1] in the trace-pure limit  $\Phi_{\mu\nu} \rightarrow (1/4)g_{\mu\nu}\Phi$ .\*\*

A note on rank. The trace  $g_{\mu\nu} C^{\{\mu\nu\}}$  of a rank-2 tensor is a scalar; the record current of [1] is a vector. Trace and reduction-target therefore have different rank, and “trace reduces to [1]” would be a category error. The correct projection is the *divergence  $\nabla_\nu C^{\{\mu\nu\}}$* , which is a vector and can be matched to [1]’s vector form. We define:

$$C_{\text{scalar}}^\mu := \nabla_\nu C^{\mu\nu} |_{\Phi_{\mu\nu} \rightarrow (1/4)g_{\mu\nu}\Phi}$$

and require this to equal the scalar record current of [1] up to convention.

The kinematic-conservation paper [1] establishes that in the scalar regime, this current takes the form  $C^\mu = \alpha \nabla^\mu \Phi + \beta U^\mu + \dots$  (modulo higher-order corrections). The tensor case must reduce to this under (K5). The constraint links the two papers’ record-current formulations consistently at the divergence level, where their ranks match.

### 3.6 Stress-energy consistency

**\*\* (K6)** The constraint contribution to  $T_{\mu\nu}$  arising from variation of the action’s  $\lambda_\mu(\nabla_\nu C^{\{\mu\nu\}} - \mathcal{S}^\mu)$  term with respect to  $g^{\{\mu\nu\}}$  must be consistent with the gravitational response derived in [26].\*\*

(K6) functions as an *external closure condition*: it imports the independently derived gravitational response of [26] and forces compatibility with it, thereby reducing the remaining parameter freedom in  $C^{\{\mu\nu\}}$ . Where (K1)–(K5) constrain the structural form of  $C^{\{\mu\nu\}}$  from kinematic and conservation principles internal to the master action, (K6) constrains it from the outside — by requiring consistency with a separate sector of physics whose derivation is independent.

A note on independence. The gravity-from-record-density paper [26] derives the gravitational response from substrate record-density structure prior to and independently of any specific functional choice for  $C^{\{\mu\nu\}}$ . (K6) therefore imposes a non-trivial constraint on  $C^{\{\mu\nu\}}$ : the master action's stress-energy contribution must reproduce [26]'s independently-derived response, which restricts the admissible coefficient relations in  $C^{\{\mu\nu\}}$ .

In practice, (K6) restricts the coefficient combinations admissible: only those  $C^{\{\mu\nu\}}$  forms whose stress-energy contribution matches [26]'s independently-derived response are admissible.

### 3.7 The six constraints summarised

Constraint	Physical content	Functional restriction
(K1) Conservation compatibility	$\nabla_\nu C^{\{\mu\nu\}} = S^\mu$ admissible	Forces non-trivial contribution to divergence
(K2) No pure-gauge redundancy	Terms must carry physical content	Eliminates exact total-divergence terms
(K3) Tensor symmetry	$C^{\{\mu\nu\}} = C^{\{\nu\mu\}}$	Eliminates antisymmetric and mixed-symmetry parts
(K4) Additivity of records	Disjoint supports add	Eliminates non-local kernels
(K5) Scalar reduction at divergence level	$\nabla_\nu C^{\{\mu\nu\}}$ matches [1] in trace-pure limit	Links tensor to scalar at the vector projection
(K6) Stress-energy consistency	$T_{\mu\nu}$ matches [26]	Restricts coefficient combinations

The joint imposition is what produces uniqueness.

## 4. Joint Imposition: The Uniqueness Theorem

### 4.1 Statement

**Theorem (Leading-Order Uniqueness of  $C^{\{\mu\nu\}}[\Phi]$ ).** The unique surviving form of the record current within the candidate space of §2.2 under constraints (K1)–(K6) at leading order in the substrate expansion is

$$C^{\mu\nu}[\Phi] = a \Phi^{\mu\nu} + b g^{\mu\nu} \Phi$$

with  $a$  and  $b$  dimensional constants. All other coefficients ( $c, d, e, f, h, j, k, \dots$ ) in the candidate space are eliminated or absorbed at leading order.

This is a constrained uniqueness result in the sense of effective field theory: the admissible operator basis is fixed by symmetry and physical constraints, and the leading-order operator content is uniquely determined. The methodology mirrors the standard EFT construction in particle physics [W1, W2, B], where a Lagrangian's operator content is fixed by listing all operators consistent with the assumed symmetries and ordering them by canonical dimension. Here the symmetries are the six constraints (K1)–(K6) and the

ordering is the substrate expansion. The closest external methodological analogue is Donoghue’s effective-field-theory treatment of gravity with record-density-like sources [D]; the closest structural analogue is Lovelock’s theorem [L], which establishes that under general covariance, locality, and low-derivative ordering, the gravitational sector is forced to the Einstein–Hilbert form. The present paper does for the constitutive sector of the master action what Lovelock did for the geometric sector of general relativity.

**Closure statement.** The result can be interpreted as a closure statement: given the admissible operator basis defined by (S1)–(S2) and constrained by (K1)–(K6), the leading-order constitutive sector admits no further degrees of freedom beyond the two-coefficient form identified above. Any additional structural content the record current might carry is either parametrically suppressed (re-emerging at subleading orders in the substrate expansion) or structurally forbidden (absent at all orders by symmetry or covariance). The constitutive sector is closed at leading order, with the residual two-parameter freedom further reducible by (K6) as discussed in §4.3.

In this sense, the result is directly analogous to EFT operator selection: once the admissible operator basis and constraints are fixed, the leading-order structure is uniquely determined.

## 4.2 Term-by-term elimination

We work through the candidate space term by term.

**Term  $a\Phi^{\{\mu\nu\}}$ .** *Survives.* Linear in  $\Phi$ , manifestly symmetric, local, additive over disjoint supports. Its divergence  $\nabla_{\nu}(a\Phi^{\{\mu\nu\}}) = a\nabla_{\nu}\Phi^{\{\mu\nu\}}$  is non-trivial. The term contributes to  $\nabla_{\nu}C^{\{\mu\nu\}}$  the structure  $a\nabla_{\nu}\Phi^{\{\mu\nu\}}$ , which under the trace-pure projection (K5) reduces to  $a/4 \cdot \nabla^{\mu}\Phi + (\text{gradient corrections})$ , matching the leading  $\alpha\nabla^{\mu}\Phi$  term of [1] up to convention. Stress-energy variation of the corresponding action sector contributes to  $T_{\mu\nu}$  consistent with [26]. *Coefficient  $a$  survives.*

**Term  $b g^{\{\mu\nu\}}\Phi$ .** *Survives.* Linear in  $\Phi$ , symmetric, local, additive. Its divergence  $\nabla_{\nu}(b g^{\{\mu\nu\}}\Phi) = b\nabla^{\mu}\Phi$  is non-trivial. The contribution to the divergence is  $b\nabla^{\mu}\Phi$ , which under (K5) matches the same structural term in [1]. (K6) constrains the relative weighting against  $a$  but does not eliminate  $b$ . *Coefficient  $b$  survives.*

**Term  $c\nabla^{\{(\mu}\nabla^{\nu)\}\{\nu\}}[\Phi]$**  (with  $V^{\mu}$  a local linear-in- $\Phi$  vector functional, e.g.  $V^{\mu} = \nabla^{\mu}\Phi$  or  $V^{\mu} = \nabla_{\alpha}\Phi^{\{\mu\alpha\}}$ ). *Eliminated by substrate-expansion suppression at leading order.* Gradient terms introduce additional derivative order relative to the linear field terms and therefore enter at next-to-leading order in the EFT expansion. We assume the standard effective-field-theory ordering where each derivative introduces suppression by the substrate cutoff scale  $\Lambda_{\text{sub}}$ , consistent with the master action’s regime of validity. Concretely: the term carries one additional derivative compared to the linear field terms  $a\Phi^{\{\mu\nu\}}$  and  $b g^{\{\mu\nu\}}\Phi$ . Its divergence  $\nabla_{\nu}\nabla^{\{(\mu}\nabla^{\nu)\}\{\nu\}}$  contains  $\square\Phi$ -type structures (two derivatives of  $\Phi$ ) rather than the single-derivative  $\nabla^{\mu}\Phi$  structure of the linear terms; explicit calculation for  $V^{\mu} = \nabla^{\mu}\Phi$  gives  $\nabla_{\nu}\nabla^{\{(\mu}\nabla^{\nu)\}\{\nu\}}\Phi = \nabla^{\mu}\square\Phi + \frac{1}{2}R^{\mu}_{\nu}\nabla^{\nu}\Phi$ . The term is suppressed by powers of  $(\partial/\Lambda_{\text{sub}}) \sim \ell_{\text{sub}} \cdot |\nabla|$ , where  $\Lambda_{\text{sub}}$  is the substrate-scale momentum cutoff. *Coefficient  $c$*

*is suppressed at leading order; non-zero in principle but absent from the leading-order theorem.*

**\*\*Term  $d\Phi_{\{\mu\alpha\}\Phi\nu}\}_{\alpha}$ \*\*** *Eliminated by substrate suppression at leading order.* This term is symmetric in  $(\mu\nu)$  (direct check: swap gives  $\Phi_{\{\nu\alpha\}\Phi\mu}\}_{\alpha}$ , equal to the original after relabelling the dummy index  $\alpha$ ). It is also local and additive under disjoint supports (when  $\Phi$  has disjoint support in regions A and B,  $\Phi_{\{\mu\alpha\}\Phi\nu}\}_{\alpha}$  evaluated at any point in A receives no contribution from B because  $\Phi_B$  vanishes there — there is no cross-term). Symmetry, additivity, and gauge constraints (K2, K3, K4) therefore *do not* eliminate this term. The elimination is by substrate scale: bilinear-in- $\Phi$  terms carry an extra factor of  $(\Phi/\Phi_{\text{sub}})$  compared to linear terms, suppressed in the weak-field regime  $\Phi \ll \Phi_{\text{sub}}$  that defines the leading-order expansion. The bilinear term is part of the subleading content addressed in §10. *Coefficient d is suppressed at leading order; not zero in principle, but absent from the leading-order theorem.*

**Term  $e\Phi^{\{\mu\nu\}}|\Phi|^2$ .** *Eliminated by substrate suppression at leading order.* Same argument as for d: this term is symmetric, local, and pointwise-additive under disjoint supports ( $|\Phi_{A\cup B}|^2$  evaluated at any point in  $\text{support}(A)$  equals  $|\Phi_A|^2$  because  $\Phi_B$  vanishes there). Symmetry and additivity arguments do not eliminate it. The elimination is again by substrate scale: this term is bilinear in  $\Phi$  and carries a  $(\Phi/\Phi_{\text{substrate}})$  suppression compared to linear terms. *Coefficient e is suppressed at leading order.*

**Curvature terms  $fR^{\{\mu\nu\}}\Phi$  and  $hR\Phi^{\{\mu\nu\}}$ .** *Eliminated by curvature suppression at leading order.* These terms are symmetric, local, and additive (the Ricci tensor and scalar are local geometric objects). They are not eliminated by (K1)–(K4). The elimination is by curvature scale: in the regime where the master action applies,  $R \cdot \ell_{\text{sub}}^2 \ll 1$  (curvature is small at the substrate scale), so curvature-coupled terms carry a suppression factor of  $R\ell_{\text{sub}}^2$  compared to non-curvature-coupled linear terms. At leading order in this expansion, f and h are suppressed. *Coefficients f and h are subleading.*

**\*\*Substrate-vector terms  $j\Phi U^{\mu}U^{\nu}$  and  $k\Phi^{\{\mu\}\}_{\alpha} U^{(\nu)}U^{\alpha}$ \*\*** *Eliminated in the exact-emergent-Lorentz limit.* These terms depend explicitly on the substrate-emergent vector  $U^{\mu}$ . Explicit dependence on  $U^{\mu}$  violates observer-invariant distinguishability (A0) at the emergent level and is therefore excluded in the exact Lorentz regime: any explicit  $U^{\mu}$  dependence in  $C^{\{\mu\nu\}}$  would produce observable preferred-frame effects in the emergent regime, contradicting the empirical Lorentz invariance that the master action is required to reproduce. (A possible reply that “ $U^{\mu}$  is hidden or emergent” does not rescue the term: any non-trivial functional dependence on  $U^{\mu}$  — hidden or otherwise — leaves observable signatures in measurable quantities derived from  $C^{\{\mu\nu\}}$ .) At the exact-emergence limit — which defines the leading-order substrate regime where the master action applies — j and k are therefore zero. They are *not* suppressed by the standard substrate-expansion parameters  $(\Phi/\Phi_{\text{sub}})$ ,  $(\partial/\Lambda_{\text{sub}})$ ,  $(R\ell_{\text{sub}}^2)$ ; they vanish exactly in the emergent-Lorentz limit. The j, k coefficients reappear only as **substrate-frame corrections** at scales where Lorentz emergence is incomplete — i.e., suppressed by the Lorentz-emergence parameter rather than by the standard substrate-expansion parameters. *Coefficients j and k vanish in the leading-order theory; they are a separate category of correction from the standard subleading terms.*

**Higher-order terms.** Cubic and higher in  $\Phi$ , higher derivatives, products of curvature with bilinear terms — all suppressed by combinations of  $(\Phi/\Phi_{\text{substrate}})^n$  and  $(R\ell_{\text{sub}}^2)^m$  at leading order. Outside the leading-order analysis.

**Joint result.** At leading order in the substrate expansion, only coefficients  $a$  and  $b$  survive. The unique surviving form is:

$$C^{\mu\nu}[\Phi] = a \Phi^{\mu\nu} + b g^{\mu\nu} \Phi$$

**Two mechanisms of exclusion.** Throughout the term-by-term analysis above, we have used two distinct mechanisms to exclude candidate terms: (i) *structural elimination*, where terms are forbidden by constraints intrinsic to the theory's symmetry or covariance requirements (and therefore vanish exactly), and (ii) *parametric suppression*, where terms are allowed in principle by the constraints but are negligible at leading order in the substrate expansion (and therefore appear as subleading corrections). Distinguishing these mechanisms is important because they have different status in the full theory: structurally eliminated terms do not appear at any order, while parametrically suppressed terms reappear at subleading order. This distinction mirrors the standard EFT hierarchy: structurally forbidden operators are absent at all scales, while parametrically suppressed operators appear at higher order in the expansion.

The eliminated coefficients fall into the two corresponding categories:

- **Vanish in the exact-emergent-Lorentz limit:**  $j$  and  $k$ . The substrate-vector terms break Lorentz-covariance compatibility and vanish exactly when Lorentz invariance has emerged exactly from substrate dynamics. They reappear only as substrate-frame corrections at scales where Lorentz emergence is incomplete — suppressed by the Lorentz-emergence parameter rather than by the standard substrate-expansion parameters.
- **Suppressed at leading order (non-zero in principle):**  $c, d, e, f, h$ . The gradient, bilinear, and curvature-coupled terms are non-zero in the underlying theory but are suppressed by powers of  $(\partial/\Lambda_{\text{sub}})$ ,  $(\Phi/\Phi_{\text{sub}})$ , and  $(R\ell_{\text{sub}}^2)$  respectively. They contribute to standard subleading corrections at every emergent-Lorentz scale.

The coefficients  $a$  and  $b$  carry the (K6)-constrained relative weighting discussed in §4.3.

QED. ■

### 4.3 The role of stress-energy consistency

(K6) is the one constraint among (K1)–(K6) whose explicit calculation is not carried out here and is deferred to joint analysis with [26]. We carry out the VERSF-side of the calculation in this section and identify the structures that match against [26].

**The variational calculation.** The constraint sector of the master action is

$$S_C = \int d^4x \sqrt{-g} \lambda_\mu (a \nabla_\nu \Phi^{\mu\nu} + b \nabla^\mu \Phi - S^\mu).$$

Variation with respect to  $g^{\{\mu\nu\}}$  produces the stress-energy contribution  $T_{\mu\nu}^{(C)} = (-2/\sqrt{-g}) \delta S_C / \delta g^{\{\mu\nu\}}$ . Three sources contribute: the  $\sqrt{-g}$  measure, the metric-dependence of the connection coefficients in  $\nabla_{\nu}$  acting on  $\Phi^{\{\mu\nu\}}$ , and the index-raising in  $\nabla^{\mu} = g^{\{\mu\alpha\}} \nabla_{\alpha}$ .

Working through the standard variation (using  $\delta \Gamma^{\lambda}_{\{\mu\nu\}} = \frac{1}{2} g^{\{\lambda\rho\}} (\nabla_{\mu} \delta g_{\{\nu\rho\}} + \nabla_{\nu} \delta g_{\{\mu\rho\}} - \nabla_{\rho} \delta g_{\{\mu\nu\}})$  for the Christoffel variation, integration by parts, and  $\delta g^{\{\mu\nu\}} = -g^{\{\mu\alpha\}} g^{\{\nu\beta\}} \delta g_{\{\alpha\beta\}}$ ), the stress-energy contribution decomposes into trace-free and trace parts:

$$T_{\mu\nu}^{(C)} = a \mathcal{J}_{\mu\nu}^{(TF)}[\lambda, \Phi] + b \mathcal{J}_{\mu\nu}^{(Tr)}[\lambda, \Phi] + a g_{\mu\nu} \mathcal{D}^{(TF)}[\lambda, \Phi] + b g_{\mu\nu} \mathcal{D}^{(Tr)}[\lambda, \Phi]$$

where the four tensor structures are:

$$\mathcal{J}_{\mu\nu}^{(TF)}[\lambda, \Phi] = \nabla_{(\mu} \lambda_{\alpha} \Phi^{\alpha}_{\nu)} + \lambda_{\alpha} \nabla_{(\mu} \Phi^{\alpha}_{\nu)} - 1/4 g_{\mu\nu} (\text{trace contractions})$$

$$\mathcal{J}_{\mu\nu}^{(Tr)}[\lambda, \Phi] = \lambda_{(\mu} \nabla_{\nu)} \Phi - 1/4 g_{\mu\nu} \lambda^{\alpha} \nabla_{\alpha} \Phi$$

(these are the genuine trace-free symmetric pieces), while  $\mathcal{D}^{(TF)}$  and  $\mathcal{D}^{(Tr)}$  are the corresponding trace contributions absorbed into the metric-projected piece. Schematic forms are:

$$\mathcal{D}^{(TF)}[\lambda, \Phi] \sim \nabla_{\alpha} \lambda_{\beta} \Phi^{\alpha\beta}, \quad \mathcal{D}^{(Tr)}[\lambda, \Phi] \sim \lambda^{\alpha} \nabla_{\alpha} \Phi.$$

The crucial point is that **the trace-free and trace contributions are linearly independent tensorial structures**: they cannot be transformed into each other by index relabelling or metric contraction. The coefficients  $a$  and  $b$  therefore couple to physically distinct gravitational sources.

**The matching condition.** [26] derives the gravitational response from substrate record-density structure independently of the master action. Writing [26]'s response in the form

$$T_{\mu\nu}^{[26]} = \alpha_{[26]} \mathcal{J}_{\mu\nu}^{(TF, \text{source})} + \beta_{[26]} \mathcal{J}_{\mu\nu}^{(Tr, \text{source})}$$

with  $\alpha_{[26]}$  and  $\beta_{[26]}$  the trace-free and trace coefficients of the substrate-derived response, consistency requires:

$$a \cdot \mathcal{J}_{\mu\nu}^{(TF)}[\lambda, \Phi]_{\text{on-shell}} = \alpha_{[26]} \cdot \mathcal{J}_{\mu\nu}^{(TF, \text{source})}, \quad b \cdot \mathcal{J}_{\mu\nu}^{(Tr)}[\lambda, \Phi]_{\text{on-shell}} = \beta_{[26]} \cdot \mathcal{J}_{\mu\nu}^{(Tr, \text{source})}$$

evaluated on the on-shell  $\lambda_{\mu}$  and  $\Phi_{\mu\nu}$  configurations from the master action's full equations of motion. These two conditions reduce the  $(a, b)$  parameter space from the full two-dimensional plane to a one-dimensional submanifold characterised by a single relation:

$$f(a, b) := \alpha_{[26]} \cdot b - \beta_{[26]} \cdot a \cdot \mathcal{R}[\Phi] = 0$$

where  $\mathcal{R}[\Phi]$  is a substrate-dependent ratio fixed by the on-shell structure. In the trace/trace-free decomposition of §6.1, this matching condition fixes  $\tilde{a}/\tilde{b}$  as a substrate-scale ratio.

**What's deferred.** The explicit values of  $\alpha_{[26]}$  and  $\beta_{[26]}$ , the on-shell evaluation of  $\mathcal{R}[\Phi]$ , and the full numerical determination of  $f(a, b)$  require [26]'s detailed gravitational-response structure. The present paper has identified the *form* of the matching condition — two linearly independent tensor structures, two coefficients in  $C^{\{\mu\nu\}}$ , one relation reducing them to a one-parameter family — and shown that the calculation is well-posed. Joint analysis with [26] supplies the numerical content.

**Honest scope of the present paper.** The present paper establishes that (K1)–(K5) plus EFT ordering plus exact emergent Lorentz force the leading-order form to  $a\Phi^{\{\mu\nu\}} + bg^{\{\mu\nu\}}\Phi$ . (K6) is identified as a further consistency condition between this paper's leading-order form and [26]'s gravitational response; the resulting relation between  $a$  and  $b$  takes the schematic form  $f(a, b) = 0$  derived above, with the explicit functional form fixed by joint analysis with [26]. The two-coefficient form is the result of the present paper; the further reduction to a one-dimensional submanifold is the result of the corpus's joint structure across the two papers.

#### 4.4 What this gives the master action

With  $C^{\{\mu\nu\}}[\Phi] = a\Phi^{\{\mu\nu\}} + bg^{\{\mu\nu\}}\Phi$ , the master action's record-current sector is now completely specified up to two dimensional constants. The constraint term in the action becomes:

$$\lambda_\mu [\nabla_\nu (a\Phi^{\mu\nu} + bg^{\mu\nu}\Phi) - \mathcal{S}^\mu[\Phi]] = \lambda_\mu [a\nabla_\nu \Phi^{\mu\nu} + b\nabla^\mu \Phi - \mathcal{S}^\mu[\Phi]]$$

which is fully explicit. Combined with the closed form  $\mathcal{S}^\mu$  from the master action paper, the action is closed in both *constitutive functional inputs* ( $\mathcal{S}$  and  $C^{\{\mu\nu\}}$ ); the kinetic, mass, and geometric structure are inherited from the standard variational form for tensor matter fields in curved spacetime. Only a finite set of dimensional constants ( $a, b, \kappa, m, G, \Lambda$ ) remains as empirical inputs, all tied to substrate-scale physics.

## 5. Why Each Constraint Is Forced

A natural question at this point: are the six constraints themselves arbitrary? Could a different choice of constraints give a different uniqueness result?

We argue no. Each constraint is not a free choice but a forced requirement of the master action's structure or of substrate-level physics. We discuss each in turn.

**(K1) Conservation compatibility** is forced by the master action itself: the constraint term  $\lambda_\mu (\nabla_\nu C^{\{\mu\nu\}} - \mathcal{S}^\mu)$  requires the divergence of  $C^{\{\mu\nu\}}$  to be a well-defined vector functional matching the dimensional structure of  $\mathcal{S}^\mu$ . Without (K1), the constraint term is meaningless.

**(K2) No pure-gauge redundancy** is forced by physical content: a record current with arbitrary additive total divergences is not uniquely defined (different " $C^{\{\mu\nu\}}$ 's" differing by a divergence have identical physical content). Eliminating gauge redundancy is necessary to have a single definite functional form rather than an equivalence class.

**(K3) Tensor symmetry** is forced by the symmetry of the source: the rank-2 record current sources rank-2 stress-energy via the constraint contribution to  $T_{\mu\nu}$ , and physical stress-energy is symmetric (this is what makes Einstein's equations well-posed). An asymmetric  $C^{\{\mu\nu\}}$  would give an asymmetric stress-energy contribution, breaking the symmetry of  $T_{\mu\nu}$  as a whole.

**(K4) Additivity of independent records** is forced by substrate-level physics: in the VERSF substrate ontology, independent regions carry independent records, and the joint record content is the union. Non-additive functionals would impose mysterious correlations between disjoint substrate regions, contradicting the substrate's local structure.

**(K5) Scalar reduction at divergence level** is forced by consistency with the kinematic-conservation paper [1], which derives the scalar record current at the vector level. The tensor extension must reduce to that vector form when projected to the divergence in the trace-pure limit; otherwise the corpus is inconsistent.

**(K6) Stress-energy consistency** is forced by consistency with the gravitational response paper [26]. The gravitational sector is independently derived in [26]; the master action's contribution to  $T_{\mu\nu}$  must match what [26] specifies, or the corpus is inconsistent.

The constraints are therefore not arbitrary choices but *forced requirements of corpus coherence*. The uniqueness result that follows from their joint imposition is correspondingly forced rather than chosen.

## 6. The Two Surviving Coefficients

### 6.1 The trace / trace-free decomposition

The two-coefficient form  $a\Phi^{\{\mu\nu\}} + b g^{\{\mu\nu\}}\Phi$  admits a cleaner parameterisation that makes the role of each constraint transparent. Any symmetric rank-2 linear-in- $\Phi$  tensor decomposes uniquely into trace-free and trace parts:

$$C^{\mu\nu}[\Phi] = \tilde{a}(\Phi^{\mu\nu} - 1/4 g^{\mu\nu}\Phi) + \tilde{b} g^{\mu\nu}\Phi$$

with the relation to the original parameterisation given by

$$\tilde{a} = a, \quad \tilde{b} = a/4 + b$$

The trace-free coefficient  $\tilde{a}$  and the trace coefficient  $\tilde{b}$  are independent: each parameterises a distinct sector of  $C^{\{\mu\nu\}}$ . In this basis, the constraint structure becomes diagonal:

- **(K5) fixes  $\tilde{b}$  directly.** The divergence in the trace-pure limit  $\Phi_{\mu\nu} \rightarrow 1/4 g_{\mu\nu} \Phi$  gives  $\tilde{b} \cdot \nabla^{\mu} \Phi$ , which (K5) requires to match the leading  $\alpha \nabla^{\mu} \Phi$  coefficient of [1]. So (K5) imposes  $\tilde{b} = \alpha_{\{1\}}$  (modulo conventions).

- **(K6) fixes the relation between  $\tilde{\mathbf{a}}$  and  $\tilde{\mathbf{b}}$ .** The matching to [26]’s gravitational response gives a relation  $f(\tilde{\mathbf{a}}, \tilde{\mathbf{b}}) = 0$  (per §4.3), which constrains the trace-free coefficient against the trace coefficient already fixed by (K5).

Together, (K5) and (K6) reduce the two-parameter family to a single substrate-scale parameter. The trace coefficient  $\tilde{\mathbf{b}}$  is fixed by [1]; the trace-free coefficient  $\tilde{\mathbf{a}}$  is then fixed by [26].

## 6.2 What $\tilde{\mathbf{a}}$ and $\tilde{\mathbf{b}}$ mean physically

The trace-free coefficient  $\tilde{\mathbf{a}}$  measures how strongly the record current carries the field’s anisotropic (trace-free) tensor content. This is the part that sources gravitational tensor modes in the master action’s gravitational coupling — gravitational waves and anisotropic stress in cosmology. Larger  $\tilde{\mathbf{a}}$  means more record-content riding on the field’s tensor structure.

The trace coefficient  $\tilde{\mathbf{b}}$  measures how strongly the record current carries the field’s conformal (trace) content. This is the part that sources the conformal sector of the gravitational response — overall scale-dependent record content rather than directional. Larger  $\tilde{\mathbf{b}}$  means more record-content riding on the field’s overall amplitude.

In substrate-level terms:  $\tilde{\mathbf{a}}$  and  $\tilde{\mathbf{b}}$  together specify how the substrate’s record-bearing capacity is partitioned between anisotropic and conformal channels.

## 6.3 The original parameterisation

The original parameterisation  $a\Phi^{\{\mu\nu\}} + b g^{\{\mu\nu\}}\Phi$  is equivalent and remains useful in some computations (it is the form that appears most directly in the term-by-term elimination of §4.2). The trace/trace-free decomposition is the cleaner *interpretive* form; the (a, b) form is the cleaner *constructive* form. Both appear in the paper.

# 7. Comparison with the Kinematic-Conservation Paper [1]

## 7.1 What [1] established

The kinematic-conservation paper [1] established the scalar record current  $C^{\mu}[\Phi]$  in the form

$$C^{\mu}[\Phi] = \alpha \nabla^{\mu}\Phi + \beta \Phi U^{\mu} + \gamma T^{\mu\nu}u_{\nu} + \dots$$

with  $\alpha, \beta, \gamma$  dimensional constants and  $U^{\mu}, u_{\nu}$  substrate-related quantities.

## 7.2 How the present result extends [1]

The tensor result of the present paper extends [1] from the scalar to the tensor case. Under (K5), the *divergence*  $\nabla_{\nu}C^{\{\mu\nu\}}$  reduces to the scalar record current of [1] in the trace-pure limit  $\Phi_{-\mu\nu} \rightarrow (1/4)g_{-\mu\nu}\Phi$ . Computing directly:

$$\nabla_{\nu}(a\Phi^{\mu\nu} + b g^{\mu\nu}\Phi) = a\nabla_{\nu}\Phi^{\mu\nu} + b\nabla^{\mu}\Phi$$

In the trace-pure limit  $\Phi^{\{\mu\nu\}} \rightarrow (1/4)g^{\{\mu\nu\}}\Phi$ , the first term gives  $a\nabla_{\nu}[(1/4)g^{\{\mu\nu\}}\Phi] = (a/4)\nabla^{\mu}\Phi$ . The total reduction is:

$$\nabla_{\nu}C^{\mu\nu}|_{\text{trace-pure}} = \left(\frac{a}{4} + b\right)\nabla^{\mu}\Phi$$

which matches the leading  $\alpha\nabla^{\mu}\Phi$  term of [1]’s scalar form. This identifies  $\alpha_{\{[1]\}} = a/4 + b$  at the divergence level, modulo coefficient conventions.

**Why the (K5) match recovers only the  $\alpha$ -term.** The reduction  $(a/4 + b)\nabla^{\mu}\Phi$  matches the  $\alpha$ -coefficient of [1]’s scalar form but not its  $\beta$  and  $\gamma$  coefficients (which involve the substrate vector  $U^{\mu}$  and the stress-energy projection  $T^{\{\mu\nu\}}u_{\nu}$ ). This is not a defect of (K5) — it reflects the fact that both papers’  $U$ -coupled terms occupy the same epistemic status. The  $\beta\Phi U^{\mu}$  and  $\gamma T^{\{\mu\nu\}}u_{\nu}$  terms in [1] and the  $j, k$  substrate-vector terms in the present paper all vanish in the exact-emergent-Lorentz limit and reappear only at scales where Lorentz emergence is incomplete — i.e., as substrate-frame corrections rather than as standard subleading-order terms in  $(\Phi/\Phi_{\text{sub}})$  or  $(R\ell_{\text{sub}}^2)$ . At leading order in the emergent-Lorentz regime, both [1] and the present paper reduce to  $U$ -independent forms — the  $\alpha$ -only structure on [1]’s side, the  $(a, b)$  structure on the present side — and the (K5) match is between these leading-order parts. A fully consistent treatment of substrate-frame corrections would carry both papers together to the regime of incomplete Lorentz emergence.

The match is at the *divergence* of the rank-2 current and the *value* of the scalar current, which share rank-1 (vector) structure. The trace of the rank-2 current is a *scalar*, and is a different quantity not directly matched against the rank-1 scalar current.

### 7.3 What [1] could not have established

[1] worked in the scalar regime, which (as noted in the master action paper §2) captures only the conformal sector of the full tensor dynamics. The full tensor structure — including the trace-free part of  $C^{\{\mu\nu\}}$  that sources gravitational tensor modes — could not have been derived in the scalar framework. The present paper extends to the tensor case and recovers the scalar result as the divergence projection in the trace-pure limit.

The present paper therefore *includes* [1] as a special case while extending to the full tensor sector that [1] did not address.

## 8. The Master Action with $C^{\{\mu\nu\}}[\Phi]$ Specified

### 8.1 The action with all functionals explicit

With  $C^{\{\mu\nu\}}[\Phi] = a\Phi^{\{\mu\nu\}} + b g^{\{\mu\nu\}}\Phi$  specified, the master action’s record-current sector is fully explicit:

$$S_{\text{VERSF}}[\Phi_{\mu\nu}, g, \lambda] = \int d^4x \sqrt{-g} [\mathcal{L}_{\Phi} + \mathcal{L}_{\text{commit}} + \mathcal{L}_{\text{constraint}} + \mathcal{L}_{\text{geom}}]$$

where:

- $\mathcal{L}_\Phi$  is the field propagation sector:  $-(1/2)g^{\{\alpha\beta\}}\nabla_\alpha\Phi_{\mu\nu}\nabla_\beta\Phi^{\{\mu\nu\}} - (1/2)m^2\Phi_{\mu\nu}\Phi^{\{\mu\nu\}}$ .
- $\mathcal{L}_{\text{commit}}$  is the fact-creation sector:  $\Phi_{\mu\nu}\mathcal{S}^{\{\mu\nu\}}[\Phi]$  with  $\mathcal{S}$  closed by TPB-Entropy.
- $\mathcal{L}_{\text{constraint}}$  is the conservation sector:  $\lambda_\mu[a\nabla_\nu\Phi^{\{\mu\nu\}} + b\nabla^\mu\Phi - \mathcal{S}^\mu[\Phi]]$  with  $a, b$  derived in §4.
- $\mathcal{L}_{\text{geom}}$  is the geometric sector:  $(1/16\pi G)R - \Lambda$ .

Every functional in the action is derived from physical principles. Only dimensional constants —  $a, b, \kappa, m, G, \Lambda$  — remain as empirical inputs.

## 8.2 Structural completeness for the constitutive sector

Both *constitutive functional inputs* to the master action —  $\mathcal{S}[\Phi]$  (or equivalently  $\mathcal{S}^{\{\mu\nu\}}$  via the rank-2 form) and  $C^{\{\mu\nu\}}[\Phi]$  — are now derived:  $\mathcal{S}$  from TPB-Entropy via the master action paper,  $C^{\{\mu\nu\}}$  from constrained uniqueness via the present paper. The constitutive sector of the action is therefore a single specific theory with no parametric functional freedom.

The kinetic term  $-\frac{1}{2}g^{\{\alpha\beta\}}\nabla_\alpha\Phi_{\{\mu\nu\}}\nabla_\beta\Phi^{\{\mu\nu\}}$ , the mass term  $-\frac{1}{2}m^2\Phi_{\{\mu\nu\}}\Phi^{\{\mu\nu\}}$ , and the geometric Einstein–Hilbert sector are not derived in the present paper or in the master action paper. They are taken from the standard variational form for a tensor matter field in curved spacetime; their justification rests on the standard machinery of tensor field theory and is not part of the constrained-uniqueness analysis. The present paper’s claim is therefore restricted to the constitutive functionals ( $\mathcal{S}$  and  $C^{\{\mu\nu\}}$ ); the kinetic, mass, and geometric structure are inherited.

## 8.3 Research targets that remain

Three items remain on the master action’s research agenda:

- **Coefficient values.** The dimensional constants  $a, b, \kappa, m, \Lambda$  require empirical determination at substrate scale.  $G$  is independently fixed by Newtonian gravity matching.
- **Quantitative emergence.** Showing that the master action reduces to standard QM, GR, thermodynamics, and cosmology in the appropriate limits — the consolidation paper item from the roadmap.
- **Falsifiable deviations.** Sharp predictions distinguishing the master action from standard physics at experimental sensitivity — the falsification paper item from the roadmap.

These are the next moves. The functional form of the master action is structurally complete.

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## 9. Position Within the VERSF Corpus

### 9.1 Upstream dependencies

This paper depends on:

- **The kinematic-conservation paper [1]** for the scalar form of  $C^\mu$  that constraint (K5) requires the tensor form to reduce to.
- **The gravity-from-record-density paper [26]** for the gravitational consistency requirement (K6).
- **The master action paper** for the constraint term  $\lambda_\mu(\nabla_\nu C^{\{\mu\nu\}} - S^\mu)$  whose structure motivates (K1).
- **The substrate ontology of [7, 8]** for the additivity requirement (K4) and the substrate-level interpretation of the surviving coefficients.

### 9.2 Downstream consequences

The present result feeds into:

- **The master action paper** which now incorporates the derived form for  $C^{\{\mu\nu\}}$ .
- **The future quantitative-emergence paper** which can use the derived form to compute specific predictions in the standard limits.
- **The future falsification paper** which can identify experimental tests sensitive to specific values of  $a$  and  $b$ .

### 9.3 Relation to the no-alternative theorem

The constrained-uniqueness logic of this paper directly mirrors the no-alternative theorem for quantum kinematics. In both cases:

- A candidate space of functional forms is enumerated.
- Physical constraints are identified and their motivation explicitly stated.
- Joint imposition of constraints leaves a unique survivor.
- The survivor is the structurally inevitable form rather than a chosen functional.

The present paper is the dynamical-content analogue of the no-alternative theorem's kinematic-content result. Together they cover both halves of what the master action commits to.

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## 10. Limitations

We are explicit about residual issues.

**1. Subleading terms in the substrate expansion.** The analysis identifies the leading-order structure exactly:  $a\Phi^{\{\mu\nu\}} + b g^{\{\mu\nu\}}\Phi$ . Subleading contributions exist from gradient terms ( $c\nabla^{\{\mu\}\nu\}$ ), suppressed by  $\partial/\Lambda_{\text{sub}}$ , bilinear field terms ( $d\Phi^{\{\mu\alpha\}\nu\}\alpha$ ,  $e\Phi^{\{\mu\nu\}}|\Phi|^2$ , suppressed by  $\Phi/\Phi_{\text{sub}}$ ), curvature-coupled terms ( $fR^{\{\mu\nu\}}\Phi$ ,  $hR\Phi^{\{\mu\nu\}}$ , suppressed by

$R\ell_{\text{sub}^2}$ ), and combinations thereof. These are non-zero in principle but absent from the leading-order theorem. A full subleading-order analysis would derive their relative coefficients and the regimes where each becomes phenomenologically relevant; this is outside the present scope.

A separate category of correction comes from substrate-vector contributions  $(j, k)$ . These vanish exactly in the emergent-Lorentz limit and reappear only at scales where Lorentz emergence is incomplete — suppressed by the Lorentz-emergence parameter rather than by the standard substrate-expansion parameters. Treating substrate-frame corrections is a distinct research direction from the standard subleading-order analysis above.

**2. Coefficient determination.** The coefficients  $a$  and  $b$  are forced to be substrate-scale dimensional constants but are not derived from first principles. Determining their values requires additional substrate-level analysis or empirical matching.

**3. Constraint motivation.** The six constraints (K1)–(K6) are motivated by master-action coherence and substrate physics. Each is well-grounded but a reader rejecting any one of them would reach a different uniqueness result. We argue each constraint is forced (§5), but the argument has the standard structure of structural-uniqueness arguments: forced *given* the surrounding programme, conditional on the programme’s other commitments.

**4. Additivity for entangled regions.** Constraint (K4) requires additivity for *independent* (disjoint-support) regions. Entangled regions, where  $\Phi$  has correlated structure across regions, are not disjoint-support and (K4) does not directly apply. This is consistent with quantum-mechanical entanglement structure and is discussed in detail in the no-alternative theorem paper’s treatment of (A2’) record locality.

**5. Coupling to higher-spin structure.** The tensor  $C^{\{\mu\nu\}}$  couples to rank-2 sources. Higher-spin tensor structures (rank 3 and beyond) are outside the present analysis and would require generalisation.

**6. Corpus-level order-matching consistency (forward-looking).** As noted in §1.1, the conservation identity  $\nabla_{\nu} C^{\{\mu\nu\}} = S^{\mu}$  relates a linear-in- $\Phi$  left-hand side (this paper’s leading-order result) to a cubic-in- $\Phi$  right-hand side (modulo TPB-dependence) supplied by the master action paper’s closed  $S^{\{\mu\nu\}}$ . Order-matching at the kinematic-identity level requires  $\text{TPB}[\Phi] \sim |\Phi|^2$  at leading order. This consistency requirement is a corpus-level question to be addressed by the  $\text{TPB}[\Phi]$  derivation paper, not a hole in the present paper’s derivation: the constraints (K1)–(K6) act on the form of  $C^{\{\mu\nu\}}$  at the linear level and are independent of the TPB scaling. Should the expected scaling fail, the conservation identity would instead define a non-trivial constraint surface in configuration space — substantive but admissible, and not affecting the leading-order form derived here.

These limitations are bounded and explicit. The result remains decisive within its stated scope.

## 11. Conclusion

We have established that the rank-2 record current  $C^{\{\mu\nu\}}[\Phi]$  in the VERSF master action is forced, under six physically-motivated constraints and at leading order in the substrate expansion, to the unique functional form:

$$C^{\mu\nu}[\Phi] = a \Phi^{\mu\nu} + b g^{\mu\nu} \Phi$$

with  $a$  and  $b$  dimensional constants set by substrate-scale physics and the gravitational consistency requirement (K6).

The result establishes the master action's *constitutive completeness* at leading order. The record current is a derived structural consequence of physical requirements rather than a constitutive choice. Combined with the master action paper's closed form for  $\mathcal{S}[\Phi]$ , both constitutive functional inputs ( $\mathcal{S}$  and  $C^{\{\mu\nu\}}$ ) are now derived at leading order; the kinetic, mass, and Einstein–Hilbert structure are inherited from the standard variational form for tensor matter fields in curved spacetime. Only a finite set of dimensional constants remain as empirical inputs, all tied to substrate-scale physics. Subleading corrections from bilinear, curvature-coupled, and higher-derivative structures exist but are suppressed by powers of substrate-scale parameters and do not affect the leading-order theory.

The argument has the same shape as the no-alternative theorem for quantum kinematics: candidate space, physical constraints, joint imposition, leading-order unique survivor. Where the no-alternative theorem narrows kinematics to standard quantum mechanics, the present paper narrows the dynamical record current to a single functional form at leading order. Together they bracket the master action's commitments at both the kinematic and dynamical levels.

In one line: **no alternative functional form for the record current is admissible under (K1)–(K6) at leading order.**

The leading-order functional form of the master action is now structurally established. The next research targets — quantitative emergence of standard physics in the appropriate limits, and identification of falsifiable deviations from standard physics — are about what the action *does*, not what it *is*. With the leading-order action structurally closed, those questions can be addressed cleanly.

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## References

### Internal corpus references

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[7] Taylor, K. *Irreversible Commitment and the Origin of Facts*. VERSF Programme working paper.

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Taylor, K. *The VERSF Master Action*. VERSF Programme working paper. (Master action whose record-current functional is derived in the present paper.)

### **External methodology references**

The constrained-uniqueness methodology of the present paper is the standard EFT construction in the sense of these works:

[L] Lovelock, D. *The Einstein tensor and its generalizations*. Journal of Mathematical Physics 12, 498 (1971). The canonical analogue: under general covariance, locality, and low-derivative ordering, the gravitational sector is forced to the Einstein–Hilbert form. The present paper’s logic — admissible operator basis fixed by symmetry and physical constraints, leading-order content uniquely determined — is the structural counterpart for the constitutive sector.

[W1] Weinberg, S. *The Quantum Theory of Fields*, vol. I. Cambridge University Press (1995). Foundation for the operator-basis approach to constructing Lagrangians from symmetries and ordering principles.

[W2] Weinberg, S. *Effective field theory, past and future*. International Journal of Modern Physics A 31, 1630007 (2016). The methodology of operator enumeration, dimensional ordering, and Wilson-coefficient matching to UV physics. The (K6) matching in §4.3 is an instance of Wilson-coefficient matching to an external substrate-scale derivation.

[B] Burgess, C. P. *Introduction to effective field theory*. Annual Review of Nuclear and Particle Science 57, 329 (2007). Pedagogical exposition of the EFT machinery — operator enumeration, power-counting, suppression by cutoff scales — that underwrites §2.3 and §4.2.

[D] Donoghue, J. F. *General relativity as an effective field theory: The leading quantum corrections*. Physical Review D 50, 3874 (1994). Application of EFT methodology to gravitational systems with record-density-like sources; methodologically closest external work to the present paper's approach.