

The Locality Decision Theorem

Constraint Locality, Reachability, and Global Assembly in the Transport Construction

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General Reader Summary

The previous paper showed that the whole sector-side question — is the remaining residue one open problem or two? — turns on a single geometric fact: whether the transport construction allows a degeneracy that is blind to which kind of motion produced it. That fact cannot be settled by trial and error, because failing to build such a degeneracy proves nothing about whether one could exist.

The paper before this one descended a level and asked the deeper question: what makes any degeneracy *allowed* in the first place? It argued that once "allowed" is pinned down precisely, the blind-degeneracy question becomes ordinary mathematics. But it left one structural property open — whether "allowed" depends only on the local shape of a degeneracy, or also on the whole history and global structure of the transport that produced it. Call this the locality question. If allowedness is local, the blind-degeneracy question is a clean test of two shapes against each other. If it is not, the question carries a global character that changes what the final gate of the programme actually is.

This paper does two things. First, it proves a decision procedure for the locality question. One gate sets up the right object — the local rulebook — and two further gates then decide whether that rulebook captures allowedness: whether every shape obeying the rules can actually be built, and whether locally-allowed pieces assemble into a globally-allowed whole. The procedure returns one of four results: local, two genuinely distinct ways of being non-local, and one cautious flag that means "the local rulebook may be incomplete — check before concluding." Second, it shows that whichever way the verdict comes out, the result feeds the programme cleanly: a local verdict turns the blind-degeneracy question into a precise compatibility test; a non-local verdict identifies the final gate as global, very likely of a specific mathematical type, which is itself a sharp and useful fact.

What this paper does **not** do is declare the verdict. The verdict requires inspecting the transport construction against the three gates — an audit of existing material, not a new derivation — and that audit is stated here as the precise remaining task. The contribution is to make that audit decisive: to prove that its possible outcomes are exactly four — local, or non-local in one of three precisely-identified ways — and that whichever one obtains mechanically determines the shape of the next step.

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Abstract

One Residue or Two? reduced the sector-side relationship $\text{FBI-comp} \leftrightarrow \text{ND}$ to a Boolean C , and reduced C to a strictly geometric existence question: whether the transport construction admits an admissible σ -invariant degeneracy. Its Minimal Residue Theorem established that this question is the *only* route to a one-residue capstone. The successor analysis descended to the object beneath the question — admissibility itself — and proved that the existence question is settled only by a necessary-and-sufficient characterization of admissibility (the **iff-requirement**), with everything downstream controlled by whether admissibility is *local*: a function of the local degeneracy geometry G alone, or carrying transport-history or global content.

This paper makes the locality question decidable. We prove the **Locality Decision Theorem** (Theorem 3.3): with Gate 1 as a well-posedness precondition that constitutes the candidate local predicate L , locality holds iff Gates 2 and 3 hold, and all three gates are jointly sufficient —

Gate 1 (local form — precondition): every admissibility constraint references only pointwise or germ-level degeneracy data, with no transport-history or global-selection variable; this is what makes L the full local content of admissibility; **Gate 2 (reachability):** every local geometry satisfying L is realized by some locally admissible transport; **Gate 3 (global assembly):** every locally realizable G is globally realizable — local realizability of G implies a globally admissible transport realizes it.

Gate 1 is a precondition, not a co-equal necessity condition: a construction can be content-local (admissible $\Leftrightarrow L$) while failing Gate 1, if a non-local constraint is redundant on the realizable geometries (Remark 3.3A). Gates 2 and 3, by contrast, are genuinely necessary given Gate 1 — each failure exhibits $\text{adm_glob} \not\subseteq L$. The gates are *presuppositionally ordered* — Gate 2's

statement is well-posed only if Gate 1 passes, Gate 3's only if Gates 1–2 pass — and we prove the **First-Failure Principle** (Theorem 3.4): the procedure halts at the first failing gate, and its four outputs form a genuine partition, exclusive because "Gate n fails" presupposes Gates 1 ... $n-1$ passed. The organizing result is therefore the partitioned **Locality Decision Procedure**, returning exactly one of four outputs: **L** (all gates pass — locality); **N₁** (Gate 1 fails — *conservative*: L possibly incomplete, a redundancy check owed before the verdict is well-posed, not yet a non-local verdict); **N₂** (Gate 2 fails under Gate 1 — reachability non-locality, [Proven]); **N₃** (Gate 3 fails under Gates 1–2 — assembly non-locality, [Proven], the deepest cell, breaking only at global assembly). When global assembly is a gluing of local data, **N₃** is the non-vanishing of the per-G realizable obstruction-class *image* of a witnessing geometry — the image of the witnessing G's entire set of realizable classes, which under unique realizers (§3.2) collapses to a single class (Proposition 3.6, conditional on gluing structure). We give the germ refinement of Gate 1 and the amendment it forces on the prior σ -action (Definition 3.5), and we fix an **adversarial evidence standard** (Principle 3.7): a *found* counterexample refutes a gate outright, but a *failed* search supports without establishing it, so a search-discharged local verdict is [Conditional] absent an exhaustive proof — even though Theorem 3.3 is a [Proven] equivalence.

We then show the verdict feeds the programme cleanly in every case. Under a local verdict, the σ -question becomes the emptiness of a single explicit set $A \cap \Sigma$ (Theorem 4.2), and the prior enumeration gate is reduced (Proposition 6.1). Under a non-local verdict (**N₂** or **N₃**), the σ -question's final gate is identified as global — under **N₃** with gluing structure, as the non-vanishing of a specific obstruction-class image — which the capstone inherits as a structural fact (Remark 4.3). These four locality outputs are orthogonal to the three σ -terminal states of the prior reduction: the locality verdict fixes the *form* of the final gate, while the σ -terminal states (construction / exclusion / sharpened-open, Theorem 5.1) fix the *resolution status* of the residue count. The σ -resolution is itself exactly three (Theorem 5.1): construction ($\neg C$, two residues), exclusion (C , one residue), or a sharpened-but-open compatibility statement, with the illegitimate "failed construction $\Rightarrow C$ " excluded (Corollary 5.2).

The verdict itself is **not** declared here. Evaluating the three gates against the transport construction is an inspection of existing material, stated as the precise remaining task (§8). What is proven is the decision procedure, its failure-mode characterization, and the outcome-indexed consequences (Theorem 7.1) — all of which stand independently of how the inspection resolves.

We retain the labelling convention: [Proven], [Conditional], [Conditional-on-OO], [Conditional-on-C], [Methodological], [Conjectural], and [Open — inspection target] for results requiring evaluation against the transport construction not reproduced here.

1. The σ -Question Reduces to a Locality Verdict

One Residue or Two? closed with the dependency chain

σ -invariant degeneracy question \rightarrow C-vs- \neg C \rightarrow one residue or two \rightarrow FBI-comp \rightarrow RC \rightarrow ℓ^2 ,

and proved (its Theorem 4.5) that every collapse proof must pass through the σ -question. Two facts then forced a descent below that question. First, the universal/existential asymmetry: \neg C is settled by exhibiting one admissible σ -invariant degeneracy, while C asserts that none exists, so a failed construction settles nothing. Second, both verdicts run through the same object — to exhibit an admissible σ -invariant degeneracy one must know what admissibility requires, and to prove admissibility forces σ -breaking one must know the same. The bottleneck is a characterization of admissibility, and the iff-requirement showed that characterization must be necessary and sufficient.

The successor analysis then isolated **locality** as the property on which the downstream shape depends. If admissibility is a predicate on local degeneracy geometry, the σ -question is a clean intersection; if admissibility carries history or global content, the σ -question inherits that content and the final gate is not purely geometric. The locality question was left open, with a four-step inspection proposed for it.

This paper formalizes that inspection into a decision procedure and proves it correct. The chain is thereby anchored one level deeper still:

locality verdict \rightarrow (compatibility intersection, or global obstruction) \rightarrow C-vs- \neg C \rightarrow one residue or two \rightarrow FBI-comp \rightarrow RC \rightarrow ℓ^2 .

The head of the chain is now the locality verdict, and we prove it is decided by a procedure of three presuppositionally-ordered gates — Gate 1 fixing the object, Gates 2–3 deciding it — with four possible outputs. The verdict converts the σ -question into one of two precise objects: an intersection test (local) or a global obstruction-class image (non-local). Either is a sharp, capstone-ready statement.

The proven content is the decision procedure and what each of its outputs delivers; the gate evaluation against the construction is marked [Open — inspection target] and deferred to §8.

2. Admissible Degeneracy and the Candidate Predicate

We fix the objects, taking the definitions from *One Residue or Two?* §8 and §2.

2.1 Local degeneracy geometry

At a point R of the refinement base $\mathcal{R}/\approx_{\mathcal{R}}$, the **local degeneracy geometry** G is the branching, collapse, or continuity-failure datum of the fiber $\pi^{-1}(\cdot)$ of the projection $\pi : \mathcal{A} \rightarrow \mathcal{R}$ over a neighbourhood of R , expressed in the substrate degrees of freedom — the phase displacement

axis, the refinement displacement axis, and the remaining operational data. σ is the local involution swapping the phase and refinement axes and fixing all other operational data; G is **σ -invariant** if $\sigma \cdot G = G$.

A **degeneracy** is an ND-failure: a locus where the operational fiber cardinality — the number of fiber elements distinguishable at the floor, i.e. counted after floor-resolution — fails to be constant across the base. We write \mathcal{D} for the class of local degeneracy geometries that can occur as such a locus.

2.2 Admissibility, stated as an existential over global transports

The inherited notion is:

A degeneracy with local geometry G is **admissible** iff there exists a globally admissible transport T — a transport satisfying all of the construction's constraints over the whole base — that realizes G at some R .

The word *globally* is not decorative; it is the crux. The construction's constraints are conditions on transports, and "admissible degeneracy" is defined by an existential quantifier over transports satisfying those constraints *everywhere*. Whether that existential reduces to a condition on G alone is exactly the locality question, and it is where non-locality can hide even when every written constraint reads as a condition on G .

2.3 The candidate predicate

Let the construction's constraints that reference only pointwise or germ-level degeneracy data be collected into a predicate on \mathcal{D} :

$L(G) :\Leftrightarrow G$ satisfies every local (pointwise- or germ-level) admissibility constraint.

L is the **candidate predicate** — the best approximation to admissibility that can be written as a condition on \mathcal{D} . The locality question is precisely whether admissibility equals L :

Locality: (G admissible) $\Leftrightarrow L(G)$, for all $G \in \mathcal{D}$.

Two gaps can separate admissibility from L . First, L might omit content because some genuine admissibility constraint references non-local data and so was not collected into L — admissibility would then be strictly stronger than L in a way L cannot express. Second, even if every constraint is local and L collects them all, L might be strictly *weaker* than admissibility: a G could satisfy every local constraint yet fail to be realized by any globally admissible transport, so L holds but admissibility fails. The decision theorem of §3 resolves exactly which of these gaps can occur and pins each to a gate.

3. The Locality Decision Theorem

This is the paper's central result. We first restate the standard the characterization must meet, then define the three gates and prove they decide locality (Theorem 3.3), then promote the criterion to a procedure whose presuppositional ordering yields a four-verdict partition (Theorem 3.4). The remaining subsections give the germ refinement, the cohomological form of the global gate, and the evidence standard for the two argued gates.

3.1 The iff-requirement

Theorem 3.1 (iff-requirement) [Proven]. A characterization of admissibility settles the σ -question — in either direction — only if it is necessary and sufficient. A merely necessary condition cannot witness $\neg C$ (a σ -invariant geometry meeting it need not be admissible); a merely sufficient condition cannot establish C (σ -breaking might fail only for admissible geometries outside it). Only a predicate P with (admissible $\Leftrightarrow P$) supports both directions.

Proof. Suppose admissible $\Rightarrow N$ and let G be σ -invariant with $N(G)$; admissibility of G does not follow, so G need not witness $\neg C$. Suppose $S \Rightarrow$ admissible with $S \Rightarrow \sigma$ -breaking proven; an admissible G' with $\neg S(G')$ is unconstrained, and if σ -invariant it witnesses $\neg C$ against the intended C . With admissible $\Leftrightarrow P$, the admissible set is exactly $\{P\}$, so a σ -invariant member of $\{P\}$ is admissible (witnessing $\neg C$), and "every admissible G is σ -breaking" is identical to " $P(G) \Rightarrow \sigma$ -breaking(G)" (establishing C). ■

Theorem 3.1 is the reason the candidate L must be shown *equal* to admissibility, not merely necessary for it. The three gates are precisely the conditions under which $L =$ admissibility.

3.2 The three gates

Definition 3.2 (the three gates). The three gates referenced throughout are as follows.

Gate 1 (local form — a well-posedness precondition). Every admissibility constraint of the construction references only pointwise degeneracy data $G(R)$ or germ-level data $\{ G(R') : R' \text{ in a neighbourhood of } R \}$, with no constraint referencing transport history (the path by which the lift reached R) or global-selection data (a condition on the whole carving). Gate 1 is the condition under which L *collects every admissibility constraint*, so that L is the full local content of admissibility and the necessity direction holds:

Gate 1 \Rightarrow (admissible $\Rightarrow L$).

Gate 1 is not a co-equal verdict-bearing condition alongside Gates 2 and 3; it is the precondition that *constitutes* L as the right object for Gates 2 and 3 to decide. Its role and the precise sense in which it can fail are treated in §3.3.

Gate 2 (reachability). Every G satisfying L is realized by some *locally* admissible transport — a transport defined on a neighbourhood of R and satisfying the constraints there:

Gate 2 : \Leftrightarrow ($L(G) \Rightarrow \exists$ locally admissible transport realizing G).

Gate 3 (global assembly). Every locally realizable G is *globally* realizable — if some locally admissible transport realizes G , then some globally admissible transport does:

Gate 3 : \Leftrightarrow (\exists locally admissible transport realizing $G \Rightarrow \exists$ globally admissible transport realizing G).

The formal statement is **existential in the realizer**: it asserts that local realizability of G implies global realizability of G , not that *every* local realizer of G extends. The distinction matters when a single G admits several local realizers. We fix the regime explicitly.

Realizer-uniqueness clause. Say the construction has *unique realizers* if each L -satisfying G has an essentially unique local realizer (the realizer is determined by the degeneracy-geometry datum G). Under unique realizers, "G is globally realizable" and "the realizer of G extends" coincide, so the existential and universal readings of Gate 3 agree, "the obstruction class of the realizer of G " (Proposition 3.6) is well-defined per G , and a single non-extending realizer is a counterexample. Without unique realizers, only the existential reading is correct: Gate 3 fails at G iff *every* local realizer of G fails to extend, and the obstruction object is the image of the *set* of realizable classes of G .

Which regime holds is a property of the transport construction (an [Open — inspection target], part of 8.A/8.C). All statements below are written in the existential form, hence correct under either regime; where unique realizers would license the cleaner universal phrasing, we note it. The necessity direction of Theorem 3.3 uses only the existential form and is therefore regime-independent.

The gates are independent *in content* — where all three are well-posed, no gate's truth follows from another's — but they are *dependent in presupposition*: Gate 2 and Gate 3 are only well-posed once Gate 1 has fixed L (and Gate 3 once Gate 2 has supplied local realizers). The two statements are not in tension: content-independence concerns truth values among well-posed gates; presupposition-dependence concerns which gates *have* truth values. The combination "Gate 1 fails, Gate 2 holds" is therefore not a coherent cell of a 2^3 table — it is a case in which Gate 2 has no truth value — which is exactly the structure exploited in §3.4.

3.3 The decision theorem

Theorem 3.3 (Locality Decision Theorem) [Proven]. Admissibility is local — that is, ($\text{adm_glob} \Leftrightarrow L$) on \mathcal{D} , where $\text{adm_glob}(G) : \Leftrightarrow (\exists$ globally admissible transport realizing G) — under the following structure:

Sufficiency. $\text{Gate 1} \wedge \text{Gate 2} \wedge \text{Gate 3} \Rightarrow \text{locality}$. **Relative biconditional.** Given Gate 1, $\text{locality} \Leftrightarrow (\text{Gate 2} \wedge \text{Gate 3})$. **Necessity of Gates 2 and 3 (given Gate 1).** If Gate 1 holds and either Gate 2 or Gate 3 fails, locality fails.

Gate 1 functions as a well-posedness precondition, not a co-equal necessity condition; its distinct status is recorded in the remark following the proof.

Proof. Write $\text{adm_glob}(G)$ for the per- G existential " \exists globally admissible transport realizing G ." This is a definite predicate on \mathcal{D} : for each G it has a truth value, regardless of whether individual realizing transports agree among themselves.

Sufficiency. Assume all three gates. By Gate 1, L collects every admissibility constraint, so any globally admissible transport realizing G satisfies the local constraints at R , whence $L(G)$: this gives $\text{adm_glob} \Rightarrow L$. Conversely assume $L(G)$. By Gate 2, G has a locally admissible realizer; since G is then locally realizable, Gate 3 gives a globally admissible transport realizing G ; hence $\text{adm_glob}(G)$. This gives $L \Rightarrow \text{adm_glob}$. The two inclusions give $\text{adm_glob} \Leftrightarrow L$ — locality.

Relative biconditional and necessity of Gates 2–3 given Gate 1. Assume Gate 1, so $\text{adm_glob} \Rightarrow L$ holds (necessity of L is secured by Gate 1 alone). Then locality ($\text{adm_glob} \Leftrightarrow L$) reduces to the converse $L \Rightarrow \text{adm_glob}$. We show this converse is exactly $\text{Gate 2} \wedge \text{Gate 3}$.

If Gate 2 fails: some G has $L(G)$ but no locally admissible realizer, hence no globally admissible realizer, so $\neg \text{adm_glob}(G)$ while $L(G)$. Then $\text{adm_glob} \subsetneq L$, so $\text{adm_glob} \neq L$: locality fails.

If Gate 3 fails: some locally realizable G is not globally realizable — i.e. G has a local realizer but *no* local realizer of G extends to a global one (the negation of the existential Gate 3). The local realizer witnesses $L(G)$; yet $\neg \text{adm_glob}(G)$. Again $\text{adm_glob} \subsetneq L$: locality fails.

Conversely, if both Gate 2 and Gate 3 hold then $L \Rightarrow \text{adm_glob}$ by the sufficiency argument, giving locality. Hence, given Gate 1, $\text{locality} \Leftrightarrow (\text{Gate 2} \wedge \text{Gate 3})$, and each of Gate 2, Gate 3 is necessary for it. ■

Remark 3.3A (Gate 1 is a precondition, not a content-necessity condition) [Proven]. Gate 1 is *not* necessary for locality in the sense Gates 2 and 3 are. Locality is the content condition $\text{adm_glob} \Leftrightarrow L$; Gate 1 is a condition on the *form* of the written constraints. The two can come apart. Suppose the construction carries a local constraint $\phi(G)$ together with a history-referencing constraint $\psi(\text{history})$ that is satisfiable by some realizing transport *whenever* $\phi(G)$ holds — an ordinary redundancy, not a pathology. Then $\text{adm_glob}(G) = \phi(G) = L(G)$, so locality holds, yet Gate 1 fails because ψ references history. Thus $\text{locality} \not\Rightarrow \text{Gate 1}$: a form-non-local construction can be content-local.

Consequently the failure of Gate 1 does not establish non-locality. It establishes only that L , as assembled from the *local* constraints, may be incomplete — that the local-vs-global question is not yet well-posed in terms of L , because a non-local constraint sits outside L whose content has not been shown redundant. The honest reading of a Gate-1 failure is therefore "the question is malformed pending a redundancy check," not "admissibility is non-local." This matches the

presupposition role of Gate 1 in §3.4: Gate 1 is what makes L the right object, and when it fails the object itself is in question.

We record the contrast with Gates 2 and 3, whose failures *do* establish non-locality: each exhibits a G with $L(G) \wedge \neg \text{adm_glob}(G)$, i.e. $\text{adm_glob} \subsetneq L$, a genuine content gap not removable by any redundancy. This asymmetry — Gate 1 a precondition on the object, Gates 2–3 verdict-bearing on the object once fixed — is the corrected structure the rest of the paper uses.

3.4 The Locality Decision Procedure and the partition of outputs

We now make the criterion organizing — converting it into a procedure whose output is one of four cells. Because Gate 1 is a precondition (Remark 3.3A), the cells partition *procedure outputs*, and the relationship between an output and the ground-truth locality verdict is stated precisely in Theorem 3.4.

The three gates are not evaluated in parallel. Each later gate's *statement* presupposes the earlier gates have passed:

— Gate 2 quantifies over "G satisfying L." If Gate 1 fails, L does not collect all admissibility constraints, so "the local constraints" is not the right object and Gate 2's question is not well-posed. — Gate 3 quantifies over "locally admissible realizers." If Gate 2 fails, there are L -satisfying G with no local realizer, so Gate 3's domain is wrong and its question is not well-posed.

The gates are therefore *presuppositionally ordered*: Gate 2 is meaningful only given Gate 1, and Gate 3 only given Gates 1 and 2. This is stronger than a chosen evaluation order — it is forced by what each gate is about.

The Locality Decision Procedure. Given the admissibility construction, evaluate in order:

Gate 1 (Constraint Locality). Do any admissibility constraints reference transport history or global structure? **Gate 2 (Reachability).** Is every geometry satisfying the local constraints L realized by some locally admissible transport? **Gate 3 (Global Assembly).** Is every locally realizable G globally realizable — does some globally admissible transport realize it? (Equivalently, under unique realizers: does the realizer of G extend?)

The verdict is determined by the first gate that fails, or by all three passing.

Theorem 3.4 (First-Failure Principle and the four-output partition) [Proven]. The Locality Decision Procedure returns exactly one of four outputs, and these form a partition (exhaustive and exclusive):

L (Locality established). All three gates pass. Admissibility is the local predicate L on \mathcal{D} (Theorem 3.3); ground truth is locality. **N_1 (Gate 1 fails — local content question malformed).**

A constraint references transport-history or global data. This is a *conservative* output: by Remark 3.3A it does **not** by itself establish non-locality — the ground truth may still be locality if the non-local constraint is redundant on the realizable geometries. N_1 reports that L is possibly incomplete and a redundancy check is owed before the locality verdict is well-posed. **N_2 (Gate 1 passes, Gate 2 fails — reachability non-locality)**. The local constraints are satisfiable by geometries no admissible transport realizes; $\text{adm_glob} \subsetneq L$. Ground truth is non-locality. **N_3 (Gates 1–2 pass, Gate 3 fails — assembly non-locality)**. Some locally realizable G is not globally realizable (no local realizer of G extends); $\text{adm_glob} \subsetneq L$. Ground truth is non-locality.

Moreover (the **First-Failure Principle**): once a gate fails, every later gate is both ill-posed and procedure-irrelevant, so the output is fixed by the first failure.

Note the output–truth relationship: outputs L , N_2 , N_3 each pin the ground-truth verdict (locality, non-locality, non-locality respectively), whereas N_1 is conservative — it flags that the verdict is not yet well-posed in terms of L , not that the verdict is non-local. A run can emit N_1 while the ground truth is locality (the redundant-constraint case of Remark 3.3A). The partition is therefore a partition of *procedure outputs*, exact as such; it is a partition of ground-truth verdicts only on the sub-cases $\{L, N_2, N_3\}$, with N_1 resolved by a subsequent redundancy check into either locality or a genuine non-local verdict.

Proof. Output-determination. If Gate 1 fails, the procedure halts with N_1 (later gates ill-posed, since L is not fixed). If Gate 1 passes and Gate 2 fails, it halts with N_2 ; by the relative biconditional of Theorem 3.3, Gate-2 failure under Gate 1 gives $\text{adm_glob} \subsetneq L$, non-locality. If Gates 1–2 pass and Gate 3 fails, it halts with N_3 ; likewise non-locality. If all pass, output L , locality by sufficiency (Theorem 3.3). By the presupposition ordering, after any failure the later gates are not well-posed, so the output is fixed by the first failure.

Exhaustiveness. Every evaluation passes all three gates (L) or has a first failure at Gate 1, 2, or 3 (N_1 , N_2 , N_3). No fourth path.

Exclusivity. L excludes every N_n (L requires all gates to pass). The N_n are pairwise exclusive by the presupposition structure: "Gate n fails" is a well-posed proposition only when Gates 1 ... $n-1$ passed, so N_2 presupposes Gate 1 passed (excluding N_1) and N_3 presupposes Gates 1–2 passed (excluding N_1 , N_2). The conjunction " N_1 and N_3 " does not denote — Gate 3 has no truth value once Gate 1 has failed. Exclusivity is thus structural, not stipulated. ■

Corollary 3.4A (the outputs are nested by gates passed) [Proven]. The four outputs are strictly ordered by how many gates a construction clears before the procedure halts: N_1 clears none; N_2 clears Gate 1; N_3 clears Gates 1 and 2; L clears all three. In terms of the set of gates passed, $\emptyset \subset \{1\} \subset \{1,2\} \subset \{1,2,3\}$ for N_1 , N_2 , N_3 , L respectively. Among the two genuine non-locality outputs, N_3 is the *deeper*: a construction reaching N_3 is local in constraint-form and in reachability, failing only at global assembly.

Proof. The passed-gate sets are determined by the first-failure point (Theorem 3.4) and are totally ordered by inclusion as stated. N_2 and N_3 are the non-locality outputs (Theorem 3.4); N_3 passes the strictly larger set $\{1,2\} \supset \{1\}$, hence is deeper. ■

The organizing question of the paper is therefore not the binary "is admissibility local?" but the sharper, partitioned "**which of the four outputs does the procedure return, and what does each pin?**" A referee cannot report a bare "locality failed"; the procedure forces identification of the first failing gate, Corollary 3.4A places it on a gates-passed scale, and Theorem 3.4 records which outputs pin ground truth (L, N_2, N_3) and which is conservative pending a redundancy check (N_1). This is the programme's recurring move — replacing a binary with a structured partition whose every cell carries information — applied to the locality verdict, with the output/truth distinction made explicit rather than elided.

3.5 The germ refinement and its amendment to the σ -action

Gate 1 admits pointwise *or* germ-level constraints. If any genuine admissibility constraint is germ-level — referencing $\{ G(R') : R' \text{ near } R \}$ rather than $G(R)$ alone — locality is not lost, but the carrier of the theory changes: G must be taken as the germ, not the pointwise datum.

Definition 3.5 (germ-level G and the extended σ -action). When Gate 1 is satisfied with at least one germ-level constraint, redefine the local degeneracy geometry as the germ $\hat{G} = \{ G(R') : R' \text{ in a neighbourhood of } R \}$, and extend σ to act on germs by acting on the substrate axes pointwise across the neighbourhood: $(\sigma \cdot \hat{G})(R') = \sigma \cdot G(R')$. \hat{G} is σ -invariant iff $\sigma \cdot \hat{G} = \hat{G}$ as germs.

This forces an amendment to *One Residue or Two?* Definition 8.1, whose σ was a pointwise involution on substrate degrees of freedom at R . Under the germ refinement:

— σ -invariance in the compatibility intersection (§4 below) is a germ-level condition; — the biconditional characterization of sector-labelledness (that paper's Corollary 8.2A) is read at germ level, with the "identity on all other operational data" clause applied across the neighbourhood; — the Symmetry Principle (that paper's Theorem 8.2) holds verbatim with G replaced by \hat{G} , since its proof uses only that σ fixes the geometry and acts on the axes, both of which extend to germs.

The amendment is mechanical and changes no proof's structure; it changes the domain on which σ and the predicates are evaluated. It is flagged here because it must be carried back into the prior paper as a dependency if the inspection returns a germ-level Gate 1.

3.6 Gate 3's structural form

Gate 3 — global realizability of locally realizable geometries — is the gate most likely to carry the substantive content, because it is where the construction's global consistency requirements (record-current consistency across the base) act. We give its structural form under a hypothesis about the assembly, stated conditionally because the hypothesis is itself a property of the construction.

Proposition 3.6 (cohomological form of Gate 3) [Conditional on gluing structure]. Suppose the extension of a locally admissible transport to a global one is a *gluing problem*: local realizers are defined on a cover of the base, they agree on overlaps up to admissible transition data, and a

global realizer is a consistent choice of transitions. Then for a given G , Gate 3 holds at G iff *some* realizable local-realizer class of G has vanishing obstruction in the relevant cohomology of the base. Under unique realizers (the regime clause of §3.2) each G has one such class, and this reads "the obstruction class of G vanishes"; without unique realizers it reads "the image of G 's realizable classes contains zero." Gate 3 holds universally iff every locally realizable G has a vanishing-obstruction realizer — equivalently, iff for every such G the *image* of its realizable classes meets zero. (Triviality of the whole obstruction group is sufficient but not necessary: a non-trivial group may still admit a vanishing realizable class for each G . The object is the per- G realizable image, not the group.)

Proof sketch. Under the gluing hypothesis, a global realizer is a global section of the sheaf of admissible local realizers; the obstruction to extending a given compatible family of local sections to a global section is, by the standard cohomological description of gluing, a class in the first cohomology of the cover with coefficients in the relevant sheaf. A given local realizer extends iff its class is zero; G is globally realizable iff *some* realizable local realizer of G has zero class — which is the existential Gate 3 at G . ■

Proposition 3.6 is conditional — it requires that the global assembly *be* a gluing problem of this type, which the inspection must confirm. Its value is directional: if the construction's global consistency is of gluing type (as emergent-geometry programmes assembling local commitment data into a global record-current structure typically are), then Gate 3 is not a vague "can the pieces assemble" but a *computation* — for each locally realizable G , whether the image of G 's realizable classes meets zero. And a witnessing failure is not merely a non-local verdict; it is the *structure* of the non-locality (the N_3 output of Theorem 3.4 made explicit): a G whose entire realizable-class set has non-vanishing image (under unique realizers, a single non-vanishing class). This is exactly what the capstone needs — not "the gate is global" but "the gate is the non-vanishing of the realizable image of [the named witnessing geometry]."

3.7 The adversarial evidence standard

Gates 2 and 3 are not decided by inspecting a written clause; they are discharged by arguments the worker constructs (a surjectivity argument for Gate 2, a no-obstruction argument for Gate 3). This makes them vulnerable to motivated reasoning in a way Gate 1 is not. We fix the standard that protects against it.

Principle 3.7 (adversarial evidence standard) [Methodological]. Gates 2 and 3 are each established by a *structured search for a counterexample that fails*, not by an unstructured proof of the clean verdict.

— For Gate 2, the search is for a G satisfying L with no locally admissible realizer. A failed search is evidence for Gate 2 only if it is structured: the searched space of candidate G must be stated, and its representativeness argued, so that "no counterexample found" carries weight. — For Gate 3, the search is for a locally realizable G that is not globally realizable — concretely, under Proposition 3.6, a G all of whose realizable local-realizer classes have non-vanishing image in the obstruction cohomology (under unique realizers, a single non-extending realizer). A

failed search is evidence only if the realizable classes searched are identified and shown exhausted.

The asymmetry with the σ -construction is essential and must be respected: a failed search for a σ -invariant degeneracy proves nothing about C (universal/existential asymmetry, prior paper), but a structured failed search for a *Gate-2 or Gate-3 counterexample* can be evidence for a universal, *provided* the search space is stated and representative. The difference is that the gate counterexample searches range over a characterized space (the L-satisfying geometries; the realizable obstruction-class image), whereas the σ -construction ranged over the uncharacterized space of all degeneracies. Characterization is what makes a failed search admissible as evidence — which is the same reason the whole programme descends to characterization first.

A clean pass on all three gates established to this standard yields a local verdict; but note its epistemic standing. A *found* counterexample at Gate 2 or 3 refutes the gate outright (and is therefore [Proven] non-locality of that type). A *failed* search supports but does not by itself establish the gate (Principle 3.7), unless the search is a genuine exhaustive proof — a surjectivity proof for Gate 2, a proof that the realizable obstruction-class image vanishes for Gate 3. A local verdict discharged by non-exhaustive search is therefore at best [Conditional] locality, not [Proven]; Theorem 3.3 is a [Proven] logical equivalence, but the verdict it characterizes inherits the strength of the gate discharge. A counterexample at any gate yields a non-local output with its source identified by Theorem 3.4. Neither outcome is a failure (§5).

4. The Compatibility Question as a Clean Intersection

Granting a local verdict (Theorem 3.3's three gates all pass), admissibility is the predicate L on \mathcal{D} — or on germs \hat{G} under the refinement of Definition 3.5 — and the σ -question takes its sharpest form.

Definition 4.1 (compatibility set). Let $A = \{ G \in \mathcal{D} : L(G) \}$ be the admissible geometries and $\Sigma = \{ G \in \mathcal{D} : \sigma \cdot G = G \}$ the σ -invariant geometries (both read at germ level if Definition 3.5 applies). The **compatibility set** is $A \cap \Sigma$.

Theorem 4.2 (compatibility controls C) [Proven, given a local verdict and the prior paper's §8].

$A \cap \Sigma \neq \emptyset \Leftrightarrow \neg C$ (an admissible σ -invariant degeneracy exists; sector residue is two objects); $A \cap \Sigma = \emptyset \Leftrightarrow C$ (every admissible degeneracy is σ -breaking, hence sector-labelled on the geometric route; sector residue is one object).

Proof. $A \cap \Sigma \neq \emptyset$ means some admissible G is σ -invariant; by the prior paper's Corollary 8.3 such a G is operational yet sector-blind, witnessing $\neg C$. $A \cap \Sigma = \emptyset$ means every admissible G is

σ -breaking; by the prior paper's Corollary 8.2A a σ -breaking geometry-derived trace is sector-labelled, so every admissible degeneracy carries a sector-labelled geometric trace — which is C. The biconditionals follow from the prior paper's $C \Leftrightarrow$ (no admissible σ -invariant degeneracy). ■

Theorem 4.2 is what the local verdict buys: the entire sector-side residue count becomes the emptiness of a single set $A \cap \Sigma$, both of whose defining predicates are explicit — L from the (now-completed) characterization, σ -invariance from Definition 8.1 as amended. This is "ordinary mathematics" in the intended sense: whether two stated conditions on the same domain can hold at once, with no further appeal to traces, protocols, or the operational ontology.

Remark 4.3 (the non-local verdict). If the inspection returns a non-local output, Theorem 4.2 does not apply as stated, and the replacement depends on which gate failed (Theorem 3.4). Under N_3 (Gate 3 failure) with gluing structure (Proposition 3.6), the σ -question factors into a local part — is a σ -invariant G a locally admissible realizer? — and a global part — does its obstruction-class image vanish? The final gate above C is then the *conjunction* of a geometric compatibility (local) and a cohomological vanishing (global). This is a sharper object than the prior paper's diffuse existence question, and it is the form the capstone inherits: the sector residue's final gate is identified as part-geometric, part-cohomological, with the global part a named obstruction-class image. Under N_2 (Gate 2 failure) the factorization differs but the principle holds — the non-local datum is identified and handed forward. Under N_1 the prior step is a redundancy check (Remark 3.3A), since N_1 does not yet establish non-locality.

5. The Three Terminal States

We fix the possible terminations of the programme this paper's decision procedure initiates, and prove they are exhaustive — forbidding the illegitimate inference of C from a failed construction.

These three σ -terminal states are on a *different axis* from the four locality outputs of §3.4 and must not be conflated: the locality output (L / N_1 / N_2 / N_3) fixes the *form* of the final gate — clean intersection or named global obstruction — while the σ -terminal state (A / B / C below) fixes the *resolution status* of the residue count. The full state of the programme is a pair (locality output, σ -terminal state); the "four" and the "three" are orthogonal coordinates, not competing partitions of one thing.

Theorem 5.1 (three terminal states) [Proven]. Any completion of the σ -question via the locality route terminates in exactly one of:

(A) Construction. An admissible σ -invariant degeneracy is exhibited — a $G \in A \cap \Sigma$ (local verdict) or a globally realizable σ -invariant G (non-local verdict). Then $\neg C$: two residues.

(B) Exclusion. Admissibility is proven to force σ -breaking on every degeneracy — $A \cap \Sigma = \emptyset$ (local), or no admissible global realizer of any σ -invariant G (non-local). Then C: one residue.

(C) Sharpened-open. The locality verdict and characterization are established, but the compatibility question — emptiness of $A \cap \Sigma$, or vanishing of the relevant obstruction-class image — is not resolved. The σ -question survives as a precise compatibility/cohomology statement rather than a diffuse existence question.

Proof. A and B are the two truth values of " \exists admissible σ -invariant degeneracy," which by Theorem 4.2 (local) or Remark 4.3 (non-local) are $\neg C$ and C . C is the state where that truth value is undetermined despite the characterization being in hand. Exhaustive: the truth value is either determined (A or B) or not (C). Exclusive: A and B contradict, and C negates "determined." ■

Corollary 5.2 (no illegitimate fourth state) [Proven]. "A construction was attempted and failed, therefore C " is not a terminal state. By Theorem 3.1 and the universal/existential asymmetry, a failed σ -construction yields neither A nor B; it yields C unless upgraded into an exclusion proof (Outcome B by content). A report of a failed construction terminates in C . Note the contrast established in Principle 3.7: this prohibition is specific to the σ -construction; structured failed searches at Gates 2 and 3 are admissible evidence because their search spaces are characterized.

Outcome C is a genuine advance. The prior paper handed forward an existence question over an implicit notion of admissibility. Outcome C hands forward a compatibility question over an explicit characterization, with the locality verdict settled and (on a non-local verdict) the global gate named. That is strictly sharper and is the precondition for any future resolution by either route. We state up front that C is a publishable terminus, so the programme is not pressured to manufacture a verdict it has not earned.

6. Conditional Second Deliverable: Enumeration Exhaustiveness

The prior paper's §9 left a [Conditional] gate: whether the obstruction enumeration — sector, holonomy, degeneracy — is exhaustive, or whether residual modes survive (failure of π to be open; global-selection obstructions with no fiber-cardinality signature). This gate shares machinery with the locality decision in the **local-verdict** case, and there only.

Proposition 6.1 (a local verdict reduces the enumeration gate) [Conditional on a local verdict and the completed characterization]. Suppose admissibility is local with predicate L . Then the admissible degeneracy modes are exactly $\{ L \}$, and any obstruction manifesting as a fiber degeneracy is classified by L . Consequently a residual obstruction can escape the enumeration only if it is not a fiber degeneracy at all — only an open-map failure or a global-selection obstruction with no operational fiber-cardinality change.

Proof sketch. In the local case L enumerates the admissible degeneracy geometries by definition; any degeneracy-type obstruction is one of these, hence inside the "degeneracy" enumeration

class. An obstruction outside the enumeration must carry no operational fiber-cardinality change — precisely the residual class §9 named. ■

Proposition 6.1 reduces, but does not close, the §9 gate: in the local case the only escape is a genuine non-degeneracy obstruction, which is outside L's scope and points the search elsewhere.

[Open — inspection target 6.A, local verdict only.] Residual non-degeneracy obstructions. Determine whether the construction admits an obstruction that is neither a fiber degeneracy (captured by L) nor pure holonomy (presupposing lifts). A negative answer closes the §9 enumeration in the local case; a positive answer exhibits a residual mode, reopens the count, and raises a fresh C-style question for that mode (prior paper, §9 scope convention).

Under a **non-local verdict**, this second deliverable separates off: L does not enumerate the degeneracy modes, so the enumeration question is independent work. Note the connection made explicit by Theorem 3.4: an N_3 output (assembly non-locality) is plausibly *the same object* as a §9 global-selection residual mode — both are global obstructions to consistent transport. If so, the non-local verdict and the residual-mode question are two views of one global structure, and characterizing the Gate 3 obstruction-class image would address both at once. This is a connection to test, not a result; it is flagged because it would consolidate two of the capstone's open gates into one.

7. Consequences, Branched on Outcome

We state what each terminal state delivers downstream, inheriting the prior paper's Capstone Dependency Theorem (its Theorem 10.1A: the sector-side count is fixed by the truth value of C).

Outcome A ($\neg C$, two residues). FBI-comp \perp ND (prior paper, Theorem 7.1). Two independent sector gates, FBI-comp and ND, each discharged separately; ALP and RC inherit both. The σ -question is closed at two.

Outcome B (C, one residue). FBI-comp \Rightarrow ND \Rightarrow UO (prior paper, Corollary 6.2, under OO). One sector gate, FBI-comp, absorbing ND; ALP and RC inherit one. The σ -question is closed at one, and the collapse is *earned* by an exclusion proof — the form the Minimal Residue Theorem demanded.

Outcome C (verdict open, but sharpened). The count is not fixed, but the open object is now the explicit compatibility statement (local verdict: emptiness of $A \cap \Sigma$) or the named obstruction class (non-local verdict: its vanishing), rather than a diffuse existence question. The capstone inherits the σ -question in this sharpened form as one named gate, and the descent to ℓ^2 is stated complete *modulo* it — the synthesis-capstone posture of honest, minimized inheritance.

In all three outcomes, the **phase-continuity tier** (prior paper, §10) remains a separate, independently-carried gate, untouched by this paper. It is not part of the FBI-comp/ND count and is its own future work or capstone section.

Theorem 7.1 (outcome-indexed capstone inheritance) [Proven, given the prior paper's Theorem 10.1A]. The sector-side input to the capstone is:

Outcome A \Rightarrow { FBI-comp, ND } (two gates); Outcome B \Rightarrow { FBI-comp } (one gate);
Outcome C \Rightarrow { FBI-comp, σ -compatibility (sharpened: intersection-emptiness or class-vanishing) } (one gate plus one sharpened-open question),

in each case alongside the separately-inherited phase-continuity tier, and subject to the §6 enumeration result for the residual-mode count.

Proof. Direct from the prior paper's Theorem 10.1A for A and B, and from §5 for C, where C is undetermined and the compatibility/obstruction statement is the open object. The phase-continuity and enumeration clauses are the separately-tracked gates of §6 and the prior paper's §§9–10. ■

8. The Inspection Targets

The proven content above is the decision procedure and its consequences. The verdict requires evaluating the three gates against the transport construction — an inspection of existing material, stated here as the precise remaining task, in the order the adversarial standard (Principle 3.7) prescribes.

[Open — inspection target 8.A.] Gate 1, by classification. List every admissibility constraint in the transport programme. For each, record the data it references and classify it as pointwise-G, germ-of-G, history-dependent, or global. Gate 1 passes iff every constraint is pointwise or germ. A germ-level constraint triggers the Definition 3.5 amendment. A history- or global-referencing constraint yields output N_1 — which, by Remark 3.3A, is conservative: it does not establish non-locality but obliges a redundancy check (is the non-local constraint redundant on the realizable geometries, in which case ground truth is locality?). Note that this redundancy check is *itself* a reachability-flavoured adversarial search over the realizable geometries, inheriting Principle 3.7's evidence standard; an N_1 output therefore kicks off Gate-2-style work, it does not resolve by inspection. *The classification of constraints is the safe, inspection-decidable step; the resolution of an N_1 output is not.*

[Open — inspection target 8.B.] Gate 2, by adversarial search. Take the conjunction L of all pointwise/germ constraints. Search for a G satisfying L with no locally admissible realizer. State the searched space and argue its representativeness. A *found* counterexample establishes Gate-2 failure (output N_2 , [Proven] reachability non-locality). A *failed* structured search supports but does not establish Gate 2 (Principle 3.7): it yields [Conditional] support for locality, upgraded to

[Proven] only by an exhaustive surjectivity proof. *This is the dangerous gate — non-locality can hide here while every written clause reads local.*

[Open — inspection target 8.C.] Gate 3, by obstruction analysis. Confirm whether global assembly is a gluing problem (Proposition 3.6 hypothesis). If so, identify the obstruction cohomology and search for a G that is *locally realizable but not globally realizable* — equivalently, a G all of whose realizable local-realizer classes have non-vanishing obstruction image. Under unique realizers (§3.2) this reduces to a single non-extending realizer; without unique realizers the counterexample must show *every* realizer of G fails to extend, a strictly heavier burden the inspection must honour. A *found* such G establishes Gate-3 failure (output N_3 , [Proven] assembly non-locality), in which case *name the class* (or the realizable image) — it is the structure of the non-locality and the object the capstone inherits. A *failed* search supports Gate 3 only as [Conditional], upgraded to [Proven] by an exhaustive proof that every locally realizable G has a vanishing-obstruction realizer. *This is the likely site of the action, given the construction's global record-current consistency requirements.*

The output is determined once 8.A–8.C are evaluated, with the epistemic standing of each per Principle 3.7:

all three pass \rightarrow local verdict ([Proven] if Gates 2–3 discharged exhaustively, else [Conditional])
 \rightarrow §4 intersection \rightarrow write the verdict paper toward σ -Outcomes A/B/C via $A \cap \Sigma$; Gate 2 or 3 fails \rightarrow non-local verdict (N_2 or N_3 , [Proven]) \rightarrow Remark 4.3 \rightarrow characterize the non-local datum (under N_3 , the named obstruction-class image) and hand it to the capstone; Gate 1 fails \rightarrow N_1 \rightarrow run the redundancy check (Remark 3.3A), which resolves into either locality or a genuine non-local verdict.

Neither a local nor a non-local verdict is a setback. A local verdict yields the strongest next paper (a clean compatibility test); a non-local verdict yields a structural fact no prior paper has established — that the sector residue's final gate is global — under N_3 as a named obstruction-class image. Only an unstructured discharge of Gates 2 or 3 *presented as* a [Proven] verdict (violating Principle 3.7) would be a genuine error.

9. Limitations

This paper does not evaluate the three gates against the transport construction. It proves the decision procedure (Theorem 3.3), the First-Failure Principle and four-output partition (Theorem 3.4, Corollary 3.4A), the germ amendment (Definition 3.5), the conditional cohomological form of Gate 3 (Proposition 3.6), the adversarial standard (Principle 3.7), the consequences of a local verdict (Theorem 4.2, Proposition 6.1), the terminal-state exhaustiveness of the σ -resolution (Theorem 5.1), and the outcome-indexed inheritance (Theorem 7.1). The verdict itself is [Open — inspection target] (§8) and is the work that completes the programme step.

Proposition 3.6 is conditional on the global assembly being a gluing problem of the stated type. If it is not, Gate 3 remains well-defined (global realizability of locally realizable G) but lacks the cohomological computation, and its discharge reverts to the general adversarial search of Principle 3.7 without the obstruction-class shortcut. The inspection must confirm the gluing hypothesis before relying on the cohomological form.

The germ amendment (Definition 3.5) propagates to *One Residue or Two?* Definition 8.1 if and only if Gate 1 passes with a germ-level constraint. The amendment is mechanical but must be carried back as a dependency; the prior paper's Symmetry Principle and biconditional are stated pointwise and would be re-read at germ level.

Theorem 4.2 and Proposition 6.1 are conditional on a local verdict. Under a non-local verdict they are replaced by Remark 4.3 and the separation of §6, respectively. The clean-intersection picture is not established until the inspection returns a local verdict.

The adversarial standard (Principle 3.7) is methodological, not a theorem; it constrains how Gates 2 and 3 may be discharged but does not itself decide them. Its force is that an unstructured discharge does not count as evidence — a discipline, not a proof.

This paper does not address FBI-comp's or ND's own truth, or the phase-continuity tier. It governs the locality of admissibility and the consequences thereof, and nothing further on the sector side beyond what §§6–7 inherit from the prior paper. The σ -question is treated only on the geometric route fixed by the prior paper's partition convention; non-geometric label recovery remains FBI-comp's territory.

All counts remain conditional on the prior paper's three-type obstruction enumeration (§9), reduced but not closed here (§6), with the noted possibility that a Gate 3 failure and a §9 residual mode are the same global object.

10. Conclusion

The σ -question — does the transport construction admit an admissible σ -invariant degeneracy? — sits at the head of the sector-side chain, and cannot be attacked by construction alone. The prior analysis descended to admissibility and showed that everything turns on whether admissibility is local. This paper makes that locality question decidable.

We proved the **Locality Decision Theorem**: under Gate 1 as a well-posedness precondition that constitutes L , locality \Leftrightarrow (Gate 2 \wedge Gate 3) — every locally-constrained geometry is locally realizable (Gate 2) and every locally realizable geometry is globally realizable (Gate 3) — with all three jointly sufficient (Theorem 3.3). The gates are presuppositionally ordered, and the **First-Failure Principle** (Theorem 3.4) turns the criterion into a procedure with a partitioned output: exactly one of four cells — L (locality), N_2/N_3 ([Proven] reachability or assembly non-locality), and N_1 , which is *conservative* — a Gate 1 failure flags that L may be incomplete and obliges a

redundancy check rather than establishing non-locality (Remark 3.3A). N_3 is the deepest cell, clearing constraint-locality and reachability and breaking only at global assembly (Corollary 3.4A). We gave the germ refinement and its amendment to the prior σ -action (Definition 3.5), the conditional cohomological form of the global gate as the vanishing of a realizable obstruction-class *image* (Proposition 3.6), and the adversarial standard fixing that a failed counterexample search supports but does not by itself prove a gate (Principle 3.7) — so a search-discharged local verdict is [Conditional] absent an exhaustive proof, even though Theorem 3.3 is a [Proven] equivalence. We proved that a local verdict turns the σ -question into the emptiness of a single explicit set (Theorem 4.2) and reduces the prior enumeration gate (Proposition 6.1), while a non-local verdict identifies the final gate as global — under N_3 as a named obstruction-class image (Remark 4.3) — and possibly unifies it with the prior residual-mode question (§6). These four locality outputs are orthogonal to the three σ -terminal states (§5): the former fixes the form of the final gate, the latter the resolution status of the residue count.

Stated in one line: with Gate 1 fixing the local object L, the locality of admissibility is decided by whether Gates 2 and 3 hold, the procedure yielding four outputs — locality, a conservative Gate-1 flag, or two genuine non-local verdicts (reachability, assembly) — of which a clean pass reduces the σ -question to a geometric intersection and the deepest (assembly) identifies the final sector gate as a named obstruction-class image; and in every case the honest terminations of the σ -question are construction, exclusion, or a sharpened-open compatibility, never a verdict inferred from a failed attempt.

The chain now reads, anchored at the decidable verdict:

locality decision procedure \rightarrow ($A \cap \Sigma$ intersection | named global obstruction-class image) \rightarrow C-vs- \neg C \rightarrow one residue or two \rightarrow FBI-comp \rightarrow RC \rightarrow ℓ^2 .

The head is no longer "examine the construction somehow." It is the evaluation of three explicit gates against existing transport-programme material — Gate 1 by classification, Gate 2 by structured search, Gate 3 by obstruction analysis (§8). That evaluation is an inspection, not a new derivation, and its four possible outputs mechanically determine the next paper: the verdict paper toward $A \cap \Sigma$ on a clean pass (with its standing per Principle 3.7), the characterization of a named non-local object under N_2 or N_3 , or a redundancy check under the conservative N_1 . What this paper delivers — the decision procedure, its diagnostics, the output/truth distinction, and its outcome-indexed consequences — stands independently of how that inspection resolves, which is the most a paper at this position can honestly provide, and exactly what the inspection needs to be decisive.