The Locality Inspection

An Executable Protocol for the Three-Gate Decision over the Transport Construction

Keith Taylor VERSF Theoretical Physics Programme versf-eos.com

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General Reader Summary

The previous paper proved that a single question — is the transport construction's notion of "allowed" a purely local one? — is decided by exactly three checks, run in order, returning one of four results. It proved the procedure is correct. It did not run it.

Running it requires one thing that proof cannot supply: the actual list of rules the construction imposes. The decision is about *those* rules, and no amount of reasoning about the procedure tells you what the rules say. You have to read them off the construction and feed them in.

This paper builds the instrument that takes that list and returns the verdict. It does three things a bare procedure does not. First, it turns the vaguest of the three checks — "does any rule secretly depend on history or global structure?" — into a concrete classification with a decision tree, so that checking it is mostly reading and sorting rather than open-ended judgement — though, as the worked execution shows, a single ambiguously-worded rule can still take real argument to classify. Second, it settles a question the previous paper left as a regime choice: whether each allowed shape has a single way of being built or several, which decides whether the hardest check is a one-line computation or a heavier search. Third, for the two checks that cannot be settled by reading alone — the ones discharged by a failed hunt for a counterexample — it fixes, *in advance*, what the hunt must cover and what would count as finding something, so that "we looked and found nothing" carries real weight instead of being a place for wishful thinking to hide.

What this paper proves is that the instrument faithfully implements the procedure: fed any list of rules, it returns the result the procedure assigns — flagging a failure the moment it finds one, and reaching the "all clear" result only when a search it cannot always finish does finish, or is replaced by a proof. Appendix A then takes the step the body could not — fed the construction's actual rules, drawn from the programme's own papers, the instrument returns a **pass** on the locality-of-form question: the one rule that looked dangerous (how records flow and are conserved) is local in form after all, refuting the global-obstruction reading it was feared to carry. That is a real result, and a bounded one — it does not by itself prove the locally-allowed pieces fit together globally, a thing that can fail even for local rules — so two checks remain: can every allowed shape actually be built, and do the pieces glue? Both are identified as the work that remains.

Contents

Abstract

The Locality Decision Theorem proved that the locality of admissibility is decided by a procedure of three presuppositionally-ordered gates returning exactly one of four outputs — **L** (local), $\mathbf{N_1}$ (Gate 1 fails — conservative), $\mathbf{N_2}$ (reachability non-locality), $\mathbf{N_3}$ (assembly non-locality) — and proved the procedure correct, the verdict feeding the capstone cleanly in every case. It deferred the verdict to an inspection of the transport construction (its §8), on the ground that the verdict is a function of the construction's actual admissibility constraints, which a proof about the procedure cannot supply.

This paper builds the instrument that executes that inspection. We fix the inputs the inspection requires — the **constraint inventory** $\{\mathcal{C}_i\}$, the **realizer map** ρ, and the degeneracy datum G — and convert each gate into a concrete sub-instrument. For **Gate 1** we give a classification function class($\mathcal{C}$) ∈ {pt, germ, hist, glob} with a decision tree (Definition 3.2, Procedure 3.3), reducing the gate to sorting the inventory; we prove the classification total and the gate decided by it (Theorem 3.4). We settle the realizer regime left open in the prior paper (its §3.2 clause) with an explicit **uniqueness test** (Definition 3.6, Theorem 3.7): realizers are unique iff the realizer map is injective on L-satisfying geometries modulo the geometry-fixing equivalence, and we show Gate 1 classification already yields the data this test needs — so a single inspection stage settles both Gate 1 and the regime. For **Gates 2 and 3**, which Principle 3.7 of the prior paper requires be discharged by a *structured failed search*, we operationalize that standard into a **pre-registration**: the search space, its representativeness argument, and the falsification condition must be committed before the search (Definitions 4.1, 5.4; Principle 7.1). For **Gate 3** we give the gluing-confirmation checklist (Procedure 5.1) that licenses the cohomological form, fix the canonical object as the **realizable obstruction image** Im(G) ⊆ H^1 with Gate 3 at G

holding iff $0 \in \mathrm{Im}(G)$ (Definition 5.2), and give the obstruction-identification steps and the non-gluing fallback (Procedure 5.3, Remark 5.5).

The organizing result is the **Inspection Protocol** (Definition 6.1) and its **Faithfulness Theorem** (Theorem 6.2): executed on any inventory, the protocol halts with an N-output whenever a counterexample is found and otherwise reaches L only as a conditional terminus (a supplied exhaustive proof or a finite-space exhaustion, not a guaranteed halt); the output it returns coincides with the one the Locality Decision Procedure assigns to that inventory; moreover it returns each output with the epistemic standing Principle 3.7 prescribes — $N_1/N_2/N_3$ as [Proven] failures when reached by classification or a found counterexample, L as [Proven] only under exhaustive Gate-2/Gate-3 proofs and [Conditional] under failed searches. The protocol preserves the prior paper's output/truth distinction (Remark 6.3): an N_1 output is a [Proven] Gate-1 failure but a *conservative* locality verdict, obliging the redundancy check.

Beyond the instrument, **Appendix A executes Stage 1 against the actual transport construction**, the inventory now drawn from the VERSF corpus — the four-sector K = 7 admissibility catalogue, with the record-current conservation content supplied by the master-action and scalar-closure papers. The result: **Gate 1 passes** (conditional on four-sector inventory completeness). Every admissibility constraint classifies as pointwise or germ-local; the decisive one, closure-current conservation, is certified local *in form* at both substrate level (the pointwise continuity identity $\partial_\mu C^\mu = s_c$, forced by a locality axiom) and continuum level (the pointwise divergence identity $\nabla_\nu C^{\mu\nu} = \hat{C}^\mu$), with additivity forbidding a non-local kernel in the constitutive map. This **refutes the feared holonomy reading** of that constraint — the one the prior paper named "the likely site of the action" — so it forces neither a Gate-1 failure nor an N_3-obstruction-by-construction. It does *not*, by itself, establish that the gluing cohomology is trivial: a law local in form can still fail to glue on a topologically nontrivial base (the Maxwell case), so the general Gate-3 question (is the realizable obstruction image zero?) remains **[Open]**. Two gates therefore remain — reachability (Gate 2) and gluing-triviality (Gate 3) — and the σ-question's reduction to the clean $A \cap \Sigma$ intersection is conditional on both. The realizer regime is found **non-unique on L** (three competing constitutive realizers of one germ), rendered effectively unique only by an external physical selection. The body's faithfulness theorem stands independently of this execution; the execution supplies, at the Gate 1 stage, the partial verdict the body deferred — and Appendix A states its limits precisely.

We retain the labelling convention: [Proven], [Conditional], [Conditional-on-gluing], [Methodological], [Conjectural], and [Open — requires inventory] for results awaiting the constraint inventory not reproduced here. Appendix A, where the inventory *is* supplied, additionally uses bare **[Open]** for a gate whose constraints are in hand but whose pre-registered search has not been executed — a distinct status from [Open — requires inventory], since what is missing is the run, not the input.

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1. From Decision Procedure to Executable Inspection

The Locality Decision Theorem closed the chain

locality decision procedure → (A ∩ Σ intersection │ named global obstruction-class image) → C-vs-¬C → one residue or two → FBI-comp → RC → ℓ^2,

and proved (its Theorem 3.4) that the head of the chain is decided by three presuppositionally-ordered gates with four possible outputs. It then established (its §8) that the verdict is not a further derivation but an *inspection*: the gates are functions of the construction's admissibility constraints, and the verdict is whatever those constraints make it. A proof about the procedure cannot return the verdict, because the verdict's content is the constraint list, which the procedure does not contain.

Two facts make the inspection more than a clerical step, and both motivate building an instrument rather than running the gates ad hoc.

First, the gates are heterogeneous in kind. Gate 1 is decidable by reading and sorting the constraints; Gates 2 and 3 are discharged by argument, and the prior paper's Principle 3.7 fixed that they must be discharged by a *structured failed search*, never an unstructured proof of the clean verdict. An instrument must therefore do two different things: classify (Gate 1) and pre-register-then-search (Gates 2, 3). Conflating them is exactly the motivated-reasoning failure Principle 3.7 forbids.

Second, the prior paper left a regime open — whether each admissible geometry has a unique local realizer (its §3.2 realizer-uniqueness clause) — and the regime governs how heavy Gate 3 is: a single non-extending realizer suffices to fail it under uniqueness, whereas without uniqueness the counterexample must exhibit a geometry *all* of whose realizers fail to extend. An instrument that settles the regime before reaching Gate 3 saves the inspector from running the wrong search.

This paper builds that instrument. It inherits the procedure and its correctness wholesale; it adds the three sub-instruments, the regime determination, the pre-registration discipline, and a faithfulness theorem certifying that the assembled protocol returns the procedure's verdict — with the standing Principle 3.7 demands. The chain is unchanged; what changes is that its head is now not "examine the construction somehow" but "run this protocol on the inventory."

We state the boundary up front, as the prior paper stated its own. The constraint inventory $\{\mathcal{C}_i\}$ is transport-programme material; it is not reproduced in this paper's working context, and we do not invent it. Everything below is proven about the *protocol* — that it is total, faithful, and correctly-labelling — and is therefore independent of the inventory. The construction-specific outputs (the classification of each $\mathcal{C}_i$, the Gate 2 search result, the Gate 3 obstruction computation) are marked [Open — requires inventory] and are the work of the immediate successor execution, of which the Gate 1 stage is short.

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2. The Inputs: Constraint Inventory, Realizer Map, Degeneracy Datum

The protocol is a function of three inputs, fixed here. The degeneracy datum G and the spaces $\mathcal{D}$, $\mathscr{R}$, $\mathcal{A}$ and the involution σ are inherited from the prior papers (*The Locality Decision Theorem* §2; *One Residue or Two?* §8).

2.1 The constraint inventory

Definition 2.1 (constraint inventory). The **constraint inventory** of the transport construction is the finite family

$\{\mathcal{C}_i\}_{i \in I}$

of admissibility constraints, where each $\mathcal{C}_i$ is a predicate on transports T whose conjunction defines global admissibility:

T globally admissible $:\Leftrightarrow \bigwedge_{i \in I} \mathcal{C}_i(T)$.

The inventory is the input the inspection reads. Its enumeration — that $\{\mathcal{C}_i\}$ is complete, no constraint omitted — is itself an inspection obligation (§3), since an omitted non-local constraint would silently corrupt L. We take I finite, consistent with the construction's finite-presentation character; the totality argument of Theorem 6.2(1) uses this finiteness, so the halting guarantee is stated for finite I and is not claimed for countable I.

2.2 The data a constraint references

Each constraint references some subset of the available data at and around a point R. We name the four reference classes, which are the values of the Gate 1 classification.

Definition 2.2 (reference classes). A constraint $\mathcal{C}$ references:

pt — *pointwise* — if its predicate depends only on the pointwise degeneracy datum G(R); **germ** — *germ-level* — if it depends on the germ $\hat{G}(R) = \{ G(R') : R' \text{ in a neighbourhood of } R \}$ but on no datum outside that germ; **hist** — *history-dependent* — if it depends on the path by which the lift reached R (the transport history), i.e. on data not recoverable from the germ of G at R; **glob** — *global-selection* — if it depends on a condition on the whole carving (a property of T across the base not localizable to any germ).

The classes pt and germ are the *local* classes; hist and glob are the *non-local* classes. The candidate predicate L (prior paper, §2.3) is the conjunction of the local-class constraints:

$L(G) :\Leftrightarrow \bigwedge \{ \mathcal{C}_i : \text{class}(\mathcal{C}_i) \in \{\text{pt, germ}\} \}$ evaluated on G (germ-level constraints read on $\hat{G}$).

2.3 The realizer map

Gate 2 and Gate 3 quantify over local realizers. We name the object that relates a realizer to the geometry it realizes.

Definition 2.3 (realizer map). A **local realizer** of G is a transport T defined on a neighbourhood of R, satisfying the local-class constraints there, and realizing G at R. Write $\mathcal{R}(G)$ for the set of local realizers of G. The **realizer map**

$\rho : \mathcal{R}(G) \to \mathcal{D}$, $\rho(T)$ = (the degeneracy datum T realizes at R),

sends a realizer to its geometry; by definition $\rho(T) = G$ for $T \in \mathcal{R}(G)$. The fibers of ρ over a fixed G are the distinct realizers of that one geometry, and whether those fibers are singletons (modulo the equivalence that fixes G) is the realizer-uniqueness question of §3.6.

These three — inventory, reference classes, realizer map — are the entire input surface of the protocol. Everything the gates need is a function of them.

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3. The Gate 1 Instrument: Classification and the Realizer-Uniqueness Determination

Gate 1 is the inspection-decidable gate (prior paper, §8.A): it is settled by reading each constraint and sorting it into a reference class. We make the sorting a procedure, prove it total, and show it simultaneously yields the data that settles the realizer regime.

3.1 The classification function

Definition 3.2 (classification function). The **classification function**

$\text{class} : \{\mathcal{C}_i\} \to \{\text{pt, germ, hist, glob}\}$

assigns to each constraint the reference class of Definition 2.2. A constraint is **local** if $\text{class}(\mathcal{C}) \in \{\text{pt, germ}\}$, **non-local** otherwise.

Procedure 3.3 (the Gate 1 decision tree) [Methodological]. For each $\mathcal{C} \in \{\mathcal{C}_i\}$, evaluate in order:

(i) Does $\mathcal{C}$'s predicate reference any datum not recoverable from the germ $\hat{G}(R)$ — in particular, any feature of the lift's path to R, or any condition quantifying over the base beyond a neighbourhood of R? — If it references path data: class($\mathcal{C}$) = **hist**. Halt. — If it references a base-global condition: class($\mathcal{C}$) = **glob**. Halt. (ii) Otherwise $\mathcal{C}$ depends only on the germ. Does it depend on any datum strictly outside the single point R — i.e. on $G(R')$ for $R' \neq R$ in the neighbourhood? — If yes: class($\mathcal{C}$) = **germ**. Halt. — If no: class($\mathcal{C}$) = **pt**. Halt.

The tree is exhaustive (every constraint reaches a halt) and exclusive (the branches are mutually exclusive by construction), so class is well-defined and total.

Remark 3.3A (the enumeration obligation) [Methodological]. Procedure 3.3 classifies the constraints it is given; it cannot detect a constraint omitted from the inventory. The inspector must therefore separately certify that $\{\mathcal{C}_i\}$ is complete — that the conjunction $\bigwedge_i \mathcal{C}_i$ is exactly global admissibility, with nothing left implicit in the construction's prose. An omitted non-local constraint would make L spuriously appear to capture admissibility (a false L output) — the one way Gate 1 classification can mislead, and the reason completeness is its own check.

3.2 Gate 1 decided by classification

Theorem 3.4 (Gate 1 reduces to classification) [Proven]. Given a complete inventory, Gate 1 holds iff class($\mathcal{C}_i$) ∈ {pt, germ} for every i; equivalently iff no constraint is classified hist or glob. The procedure's Gate 1 output is:

all local → **Gate 1 passes**; L = ⋀{local constraints} is the full local content of admissibility (prior paper, Gate 1 ⟹ (admissible ⟹ L)); some non-local → **output N_1** — a [Proven] Gate-1 failure, *conservative* on the locality verdict per the prior paper's Remark 3.3A.

Proof. Gate 1 (prior paper, Definition 3.2) is exactly the statement that every admissibility constraint references only pointwise or germ data — i.e. class($\mathcal{C}_i$) ∈ {pt, germ} for all i. Procedure 3.3 computes class totally (Procedure 3.3), so the conjunction is decidable by inspection of the classification table. If all local, L collects every constraint and the necessity direction holds (prior paper, Gate 1 clause). If some constraint is hist or glob, that constraint sits outside L; by the prior paper's Remark 3.3A this is output N_1, a [Proven] failure of Gate 1's *form* condition that does not by itself establish non-locality. ■

Remark 3.4A (germ triggers the Definition 3.5 amendment). If any local constraint classifies as germ, the prior paper's Definition 3.5 applies: the carrier becomes the germ $\hat{G}$, σ extends germ-wise, and the amendment propagates to *One Residue or Two?* Definition 8.1 as a dependency. The classification table records, per constraint, whether it is pt or germ, so the amendment is triggered exactly when the germ column is non-empty — a by-product of the same pass, requiring no separate inspection.

3.3 The realizer-uniqueness determination

The prior paper left the realizer regime as an open property of the construction (its §3.2). We give the test and show the Gate 1 pass already supplies its inputs.

Definition 3.6 (the geometry-fixing equivalence and unique realizers). Two local realizers T, T′ ∈ $\mathcal{R}$(G) are **geometry-equivalent**, T ~_G T′, if they differ only by a transformation fixing the degeneracy datum G (a relabelling of substrate data that leaves G(R), and the germ Ĝ(R) under the amendment, invariant). The construction has **unique realizers** if for every L-satisfying G the fiber $\mathcal{R}$(G) is a single ~_G-class — equivalently, if ρ is injective on L-satisfying geometries modulo ~_G.

Theorem 3.7 (the uniqueness question is posed by the local constraints) [Proven]. Suppose Gate 1 passes, so L collects every admissibility constraint. Then the construction has unique realizers iff the local constraints pin the realizer germ up to ~_G — i.e. iff, for each L-satisfying G, any two realizers of G satisfying the local constraints are geometry-equivalent. The *question* of uniqueness is therefore a function of L alone. Its *answer* need not be: deciding whether any two L-satisfying realizers of G are ~_G-related is a universally-quantified claim that may require substantive argument, and when L admits several ~_G-distinct admissible realizers of one germ (competing realizers), the regime is non-unique on L alone and is rendered unique, if at all, only by an external physical selection among them (cf. Appendix A.6). "Readable from the table" would overstate this: the classification fixes the well-posed question, not always the verdict.

Proof. Under Gate 1, a local realizer of G is a neighbourhood transport satisfying L and realizing G (Definition 2.3, with "the constraints there" = the local constraints, which under Gate 1 are all constraints). $\mathcal{R}$(G) is a single ~_G-class iff any two such transports are ~_G-related, which is precisely the condition that L, together with the requirement ρ(T) = G, determines T up to ~_G. Since L is the conjunction of the classified local constraints, the *condition* — whether L forces any two G-realizers to be ~_G-related — is a function of L and ~_G alone, both fixed by the Gate 1 output and Definition 3.6. Hence the uniqueness question is well-posed from the classification. Whether it holds is a separate, universally-quantified matter: if L pins the realizer up to ~_G the regime is unique; if L leaves several ~_G-classes admissible the regime is non-unique on L, and only an external selection among the competing realizers can restore uniqueness. ■

Corollary 3.7A (one stage settles Gate 1 and decides the regime question on L) [Proven]. A single inspection stage — running Procedure 3.3 and applying Theorem 3.7 to the resulting local constraints — settles whether Gate 1 passes and decides the realizer-regime question on L: either L pins the realizer up to ~_G (unique regime) or it admits ~_G-distinct competing realizers of a germ (non-unique regime). In the non-unique case the very multiplicity L exhibits is what an external physical selection may later collapse to one — so "L decides non-unique" and "there exist externally-selectable competing realizers" are the same finding, not alternatives. The regime, once read, sets the form of the Gate 3 search (§5) and the Gate 3 object (single class vs realizable image). This is the de-risking observation in its honest form: the cheapest stage fixes the shape of the most expensive one, and where it returns non-unique it also names the external selection that would simplify it. ■

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4. The Gate 2 Instrument: Pre-Registered Reachability Search

Gate 2 — every L-satisfying G has a locally admissible realizer — is an argued gate. Principle 3.7 of the prior paper requires it be discharged by a *structured failed search* for a counterexample (an L-satisfying G with empty $\boldsymbol{\mathcal{R}}$(G)), with the search space stated and its representativeness argued, on pain of the failed search carrying no evidential weight. We operationalize that into a pre-registration.

Definition 4.1 (Gate 2 pre-registration) [Methodological]. A Gate 2 search is **pre-registered** if, *before* searching, the inspector commits in writing to:

(a) **Search space** — a parametrization $\mathcal{P}_2$ of the L-satisfying geometries to be examined, explicitly delimited (which families of G, over what range of the substrate axes and germ data); (b) **Representativeness argument** — a stated reason $\mathcal{P}_2$ exhausts, or faithfully samples, the L-satisfying geometries, so that "no counterexample in $\mathcal{P}_2$" bears on "no counterexample in $\mathcal{D}$"; (c) **Falsification condition** — the explicit test by which a candidate G ∈ $\mathcal{P}_2$ is declared a counterexample, namely a demonstration that $\boldsymbol{\mathcal{R}}(G) = \emptyset$ (no neighbourhood transport satisfies L and realizes G).

Procedure 4.2 (executing Gate 2) [Methodological]. Search $\mathcal{P}_2$ under the falsification condition.

Found — a G ∈ $\mathcal{P}_2$ with $\boldsymbol{\mathcal{R}}(G) = \emptyset$: output $\mathbf{N_2}$, [Proven] reachability non-locality (prior paper, Theorem 3.4 N_2 cell). The witnessing G is the certificate. **Not found** — the structured search of the pre-registered $\mathcal{P}_2$ yields no counterexample: Gate 2 is **supported but not established** (Principle 3.7). The output is [Conditional] support for the gate, upgraded to [Proven] only if the search is in fact an exhaustive surjectivity proof — a proof that ρ surjects onto the L-satisfying geometries — rather than a sampled failed search.

Remark 4.3 (why pre-registration, not post-hoc rationale). The evidential asymmetry the prior paper insisted on (its Principle 3.7) is that a failed search over a *characterized* space is admissible evidence, whereas a failed σ-construction over an uncharacterized space is not. Characterization that is supplied *after* the search has seen the data is not characterization — it is curve-fitting the search space to the result. Pre-registration is the discipline that keeps the characterization honest: $\mathcal{P}_2$ and its representativeness are fixed before the search can be tempted to shrink them around a clean pass. This is the methodological content Principle 3.7 requires and the bare procedure does not enforce.

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5. The Gate 3 Instrument: Gluing Confirmation and Obstruction Identification

Gate 3 — every locally realizable G is globally realizable — is the gate the prior paper identified as the likely site of the action (its §8.C), because it is where global record-current consistency acts. Its instrument has three parts: confirm the gluing hypothesis (which licenses the cohomological form), fix the object, and pre-register the search in the regime Theorem 3.7 returned.

5.1 Gluing confirmation

Procedure 5.1 (gluing-confirmation checklist) [Methodological]. Proposition 3.6 of the prior paper holds only if global assembly is a gluing problem. Confirm, against the construction:

(i) **Cover** — local realizers are defined on a cover $\{U_\alpha\}$ of the base; (ii) **Overlap compatibility** — on each overlap $U_\alpha \cap U_\beta$, two local realizers agree up to admissible transition data $g_{\alpha\beta}$; (iii) **Cocycle** — the transition data satisfy the consistency (cocycle) relation on triple overlaps, so that a global realizer is exactly a consistent choice of transitions.

If (i)–(iii) hold, global assembly is a gluing problem and the cohomological form (Definition 5.2) applies. If any fails, take the non-gluing fallback (Remark 5.5).

5.2 The canonical object

We fix the canonical Gate 3 object, in the per-G form the prior review settled, so that the singular and regime-dependent phrasings coincide on one definition.

Definition 5.2 (realizable obstruction image). Under the gluing hypothesis, each local realizer $T \in \mathcal{R}(G)$ has an **obstruction class** $[T] \in H^1(\{U_\alpha\}; \mathcal{G})$, the first cohomology of the cover with coefficients in the relevant sheaf $\mathcal{G}$, where $[T] = 0$ iff T extends to a global realizer. The **realizable obstruction image** of G is

$\mathrm{Im}(G) := \{ [T] \in H^1(\{U_\alpha\}; \mathcal{G}) : T \in \mathcal{R}(G) \}$ — the image, under $T \mapsto [T]$, of G's entire set of local realizers.

Gate 3 holds at G iff $0 \in \mathrm{Im}(G)$; Gate 3 holds universally iff $0 \in \mathrm{Im}(G)$ for every locally realizable G. Under unique realizers (Theorem 3.7) Im(G) is the singleton $\{[T_G]\}$ and the condition reads $[T_G] = 0$; this is the canonical phrase used throughout — *the realizable obstruction image*, collapsing to a single class exactly when the regime is unique.

Proposition 5.2A (Gate 3 in the canonical object) [Conditional-on-gluing]. Under (i)–(iii) of Procedure 5.1, for each locally realizable G, Gate 3 at G $\Leftrightarrow$ $0 \in \mathrm{Im}(G)$; and N_3 at a witnessing G is the [Proven] statement $0 \notin \mathrm{Im}(G)$ — under uniqueness, the single non-vanishing class $[T\_G] \neq 0$.

Proof. By the standard cohomological description of gluing (prior paper, Proposition 3.6 proof sketch), a given local realizer extends iff its class is zero; G is globally realizable iff *some* realizer of G extends, i.e. iff some [T] with $T \in \mathcal{R}(G)$ is zero, i.e. iff $0 \in \mathrm{Im}(G)$. The negation $0 \notin \mathrm{Im}(G)$ is the existential failure "no realizer of G extends," which is N_3 at G; under uniqueness $\mathrm{Im}(G) = \{[T\_G]\}$ so the negation is $[T\_G] \neq 0$. ■

5.3 Obstruction identification and the regime-correct search

Procedure 5.3 (executing Gate 3 under gluing) [Methodological]. Having confirmed gluing and read the regime from Theorem 3.7:

Identify the obstruction cohomology $H^1(\{U\alpha\}; \mathcal{G})$ — the cover, the sheaf $\mathcal{G}$, and the group — from the transition data of Procedure 5.1. **Pre-register** (Definition 5.4) a search over locally realizable G for one with $0 \notin \mathrm{Im}(G)$. — *Unique-realizer regime:* the search seeks a single non-extending realizer, $[T\_G] \neq 0$. A found such G is the N_3 certificate. — *Non-unique regime:* the search must exhibit a G with *every* realizer non-extending, $0 \notin \mathrm{Im}(G)$ — the strictly heavier burden the prior paper flagged (its §8.C). A found such G is the N_3 certificate; a single non-extending realizer is *not* a counterexample here, since another realizer of the same G may extend. **Found** → output $\mathbf{N_3}$, [Proven] assembly non-locality; *name the class* (unique regime) or *name Im(G)* (non-unique regime) — it is the structure the capstone inherits (prior paper, Remark 4.3). **Not found** → Gate 3 supported but not established (Principle 3.7); [Proven] only under an exhaustive proof that $0 \in \mathrm{Im}(G)$ for every locally realizable G.

Definition 5.4 (Gate 3 pre-registration) [Methodological]. As Definition 4.1, with: search space $\mathcal{P}_3$ a delimited parametrization of locally realizable G; representativeness argument for $\mathcal{P}_3$ against the locally realizable geometries; falsification condition the regime-correct test of Procedure 5.3 (single non-vanishing class, or whole-image non-vanishing).

Remark 5.5 (non-gluing fallback) [Conditional]. If Procedure 5.1 fails — global assembly is not a gluing problem of the stated type — Gate 3 remains well-defined (global realizability of locally realizable G, prior paper §9) but loses the cohomological shortcut. The search of Procedure 5.3 reverts to a general pre-registered adversarial search (Definition 5.4 with the falsification condition stated directly in terms of global realizability rather than class vanishing). The instrument still functions; only the obstruction-class computation is unavailable, and the N_3 certificate is then "a locally realizable G shown not globally realizable" without a named class.

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6. The Protocol and Its Faithfulness

We assemble the three instruments into one protocol and prove it implements the decision procedure faithfully, with the standing Principle 3.7 demands.

Definition 6.1 (the Inspection Protocol). Given a complete inventory $\{\mathcal{C}_i\}$, execute:

Stage 1 (Gate 1 + regime). Taking inventory completeness as a precondition on the input (Remark 3.3A — the protocol classifies what it is given and cannot detect an omission), run Procedure 3.3 on every $\mathcal{C}_i$. If any constraint is hist or glob → halt, output $\mathbf{N_1}$ [Proven Gate-1 failure, conservative]. Else Gate 1 passes; form L; pose the realizer regime per Theorem 3.7, returning it when L decides it and otherwise flagging the competing realizers awaiting external selection (Corollary 3.7A); if any germ constraint, flag the Definition 3.5 amendment. **Stage 2 (Gate 2).** Pre-register (Definition 4.1) and run Procedure 4.2. If a counterexample is found → halt, output $\mathbf{N_2}$ [Proven]. Else record Gate 2 as supported, with standing [Proven] (exhaustive surjectivity) or [Conditional] (failed search). **Stage 3 (Gate 3).** Run Procedure 5.1; if gluing confirmed use Definition 5.2 / Procedure 5.3, else the fallback (Remark 5.5). Pre-register (Definition 5.4) and search in the regime from Stage 1. If a counterexample is found → halt, output $\mathbf{N_3}$ [Proven]; name the class or image. Else record Gate 3 as supported, with standing [Proven] (exhaustive vanishing proof) or [Conditional] (failed search). **Output L.** If all three stages pass: output **L**, with standing [Proven] iff Stages 2 and 3 were both discharged exhaustively, else [Conditional].

Theorem 6.2 (Faithfulness) [Proven]. Executed on any complete inventory, the Inspection Protocol (Definition 6.1):

(1) **halts with an N-output on a found counterexample**, returning one of $\{N_1, N_2, N_3\}$; the L-branch is not a halting enumeration but a proof obligation — over a continuous search space $\mathcal{P}$ it terminates only given a supplied exhaustive proof or a finite-space exhaustion, so "returns L" is a conditional terminus, not a guaranteed halt; (2) **agrees** with the Locality Decision Procedure (prior paper, Theorem 3.4): the output is the output that procedure assigns to the inventory; (3) **labels** each output with the standing Principle 3.7 prescribes — N_1/N_2/N_3 [Proven] (classification or found counterexample), L [Proven] only under exhaustive Stage-2/3 discharge, [Conditional] otherwise.

Proof. (1) Stage 1 halts because class is total (Procedure 3.3) over finite I (completeness being a precondition on the input, not a step the protocol executes — §9, Remark 3.3A). On a non-local constraint it halts with N_1; otherwise it proceeds. Stage 2 halts with N_2 on a *found* counterexample; Stage 3 halts with N_3 on a *found* counterexample. On the no-counterexample branches the corresponding gate is not decided by halting: the search either exhausts a finite $\mathcal{P}$ or stands as a [Conditional] failed search, and "output L" is reached only once both Stage 2 and Stage 3 conclude on that footing. So the protocol halts with an N-output whenever a counterexample is found, and reaches L only as the conditional terminus of (possibly non-terminating) proof obligations. Exactly one output is returned because the first halting stage fixes it and no later stage runs (the staging mirrors the prior paper's first-failure structure).

(2) The Locality Decision Procedure (prior paper, Theorem 3.4) returns N_1 iff Gate 1 fails, N_2 iff Gate 1 passes and Gate 2 fails, N_3 iff Gates 1–2 pass and Gate 3 fails, L iff all pass. Stage 1 outputs N_1 iff some constraint is hist or glob, which by Theorem 3.4 (this paper) is iff Gate 1 fails. Stage 2 outputs N_2 iff a found G has $\boldsymbol{\mathcal{R}}(G) = \emptyset$, which by Procedure 4.2 is iff Gate 2 fails under Gate 1. Stage 3 outputs N_3 iff a found locally realizable G is not globally realizable ($0 \notin$

Im(G) under gluing, or the fallback condition), which by Proposition 5.2A / Remark 5.5 is iff Gate 3 fails under Gates 1–2. The protocol outputs L iff no stage halts with an N-output, iff all gates pass. Hence the protocol's output equals the procedure's on every inventory.

(3) N_1 is reached by classification — a [Proven] determination that a constraint is non-local — and is conservative per Remark 3.3A. N_2, N_3 are reached only by *found* counterexamples, which Principle 3.7 makes [Proven] failures. L is reached only by Stages 2 and 3 passing; Procedure 4.2 and Procedure 5.3 record the pass as [Proven] iff discharged by exhaustive proof, [Conditional] iff by failed search. The protocol carries these tags to the output. ■

Remark 6.3 (the output/truth distinction is preserved) [Proven]. Faithfulness inherits the prior paper's distinction (its Theorem 3.4): the protocol's outputs L, N_2, N_3 pin ground-truth locality, non-locality, non-locality; N_1 pins a [Proven] Gate-1 failure but is *conservative* on the locality verdict — the ground truth may still be locality if the offending non-local constraint is redundant on the realizable geometries (prior paper, Remark 3.3A). On an N_1 output the protocol therefore does not declare non-locality; it hands forward the redundancy check, which is itself a reachability-flavoured pre-registered search (Definition 4.1 form), per the prior paper's §8.A. The protocol is faithful to the procedure precisely in *not* over-resolving N_1.

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7. The Pre-Registration Record

The two argued gates are where motivated reasoning can enter, and Principle 3.7 of the prior paper is the discipline against it. We fix what an honest execution must place on record, so that the protocol's [Conditional] and [Proven] tags on Gates 2 and 3 are auditable rather than asserted.

Principle 7.1 (the pre-registration record) [Methodological]. An execution of the Inspection Protocol is *admissible as evidence* only if it deposits, before searching at each of Stages 2 and 3, a record containing:

— the search space ($\mathcal{P}_2$, $\mathcal{P}_3$), explicitly delimited; — its representativeness argument against the full target (L-satisfying geometries; locally realizable geometries); — the falsification condition, in the regime Stage 1 returned (for Stage 3); — the standing claimed on a clean pass: [Proven] (with the exhaustive proof attached) or [Conditional] (failed structured search).

A pass recorded without its pre-registration is, per Principle 3.7, not evidence — regardless of how clean it looks. The record is what distinguishes a structured failed search (admissible) from an unstructured one (inadmissible), and it is the artifact a referee audits.

Remark 7.1A (the asymmetry, restated for the instrument). The prior paper's asymmetry — a failed σ-construction over an uncharacterized space proves nothing, a failed search over a *characterized* space can be evidence — is enforced here structurally: the pre-registration record *is* the characterization, fixed before the search. The protocol cannot launder an unstructured search into evidence, because the admissibility tag (Theorem 6.2(3)) is conditioned on the record existing first. This is the methodological teeth the bare procedure lacked.

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8. Outputs and Hand-Forward

Each protocol output routes into the programme exactly as the prior paper's §4, Remark 4.3, and §7 prescribe. We restate the routing as the instrument delivers it, with the standing attached.

L (Locality, [Proven] or [Conditional]). Admissibility is the predicate L on $\mathcal{D}$ (germ $\hat{G}$ under the amendment). Route to the compatibility paper: the σ-question becomes the emptiness of $A \cap \Sigma$ (prior paper, Theorem 4.2), and the prior enumeration gate is reduced (its Proposition 6.1). Standing of the downstream compatibility verdict inherits the standing of the L output. **N_1 (Gate-1 failure, [Proven]; conservative).** Run the redundancy check (Remark 6.3): is each non-local constraint redundant on the realizable geometries? This is a pre-registered reachability-flavoured search (Principle 7.1). It resolves N_1 into either locality (constraint redundant — re-enter at L) or a genuine non-local verdict (constraint substantive). **N_2 (reachability non-locality, [Proven]).** The non-local datum is the unrealizable L-satisfying geometry; characterize it and hand it forward (prior paper, Remark 4.3, N_2 clause). **N_3 (assembly non-locality, [Proven]).** The non-local datum is the realizable obstruction image — named class (unique regime) or named Im(G) (non-unique regime). Hand it to the capstone as the sector residue's final gate: part-geometric (is a σ-invariant G a local realizer?), part-cohomological (does its realizable image meet zero?) — prior paper, Remark 4.3, N_3 clause. Test the §6 conjecture that this object coincides with the prior residual mode, which would consolidate two capstone gates.

Remark 8.1 (the immediate execution). Corollary 3.7A makes the order of execution clear: **run Stage 1 first and alone.** It is inspection-decidable (no search, no pre-registration), it returns Gate 1 and the realizer regime in one pass, and its output already forks the work — N_1 to a redundancy check, or a Gate 1 pass that fixes the regime and hence the form of the Stage 3 search before any of the heavier Stage 2/3 labour is committed. A short Stage-1-only note against the constraint inventory is therefore the natural immediate successor to this paper, de-risking Stages 2 and 3 by telling the inspector which searches they will be running. We flag this as the recommended next execution; it requires only the inventory of §2.1.

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9. Limitations

The body of this paper does not itself return the verdict; it proves the protocol total, faithful, and correctly-labelling (Theorem 6.2), and builds the three sub-instruments (Procedures 3.3, 4.2/5.3; Definitions 4.1, 5.2, 5.4) and the pre-registration discipline (Principle 7.1). Appendix A then executes Stage 1 against the corpus inventory and returns **Gate 1 PASS** (the local-form branch), establishing that the conservation constraint is local in form and refuting its feared holonomy reading — but explicitly *not* closing the general Gate-3 gluing-triviality question, which a law local in form can still fail (the Maxwell case). What remains unexecuted is therefore two gates: Gate 2 (reachability) and Gate 3 (is Im(G) = 0?), both [Open], both requiring pre-registered searches, with Gate 2 the immediate next gate. The realizer regime is found non-unique on L, fixed only by an external selection (Appendix A.6).

The completeness of the inventory (Remark 3.3A) is an inspection obligation the protocol cannot discharge internally: it classifies what it is given and cannot detect an omitted constraint. A complete inventory is a precondition for Theorem 6.2's faithfulness; an incomplete one can yield a false L.

Proposition 5.2A and Procedure 5.3's cohomological form are conditional on the gluing hypothesis (Procedure 5.1). If gluing fails, Gate 3 reverts to the general adversarial search (Remark 5.5) and the N_3 certificate carries no named class — only "locally realizable but not globally realizable." The instrument still functions; the obstruction shortcut does not.

The realizer-uniqueness determination (Theorem 3.7) is decidable from the local constraints *only once Gate 1 passes*; on an N_1 output the regime is not yet defined, since L is not yet the full local content. The regime question is therefore downstream of Gate 1, not independent of it.

Principle 7.1 is methodological, not a theorem; it constrains what counts as an admissible execution but does not itself search. Its force is that the protocol's [Proven]/[Conditional] tags on Gates 2 and 3 are conditioned on the pre-registration record existing, so an unstructured discharge cannot acquire a [Proven] tag.

This paper does not address FBI-comp's or ND's truth, the phase-continuity tier, or the σ-terminal-state resolution (prior paper, §5); it builds only the instrument that returns the locality output, which is one coordinate of the programme state (prior paper, §5 orthogonality). The downstream σ-resolution remains construction, exclusion, or sharpened-open, as before.

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10. Conclusion

The prior paper proved the locality of admissibility is decided by three presuppositionally-ordered gates with four outputs, and deferred the verdict to an inspection of the construction's constraints. This paper builds the instrument that runs that inspection.

We fixed the inputs — constraint inventory, reference classes, realizer map (§2) — and converted each gate into a sub-instrument: Gate 1 into a total classification with a decision tree and an enumeration obligation (Procedure 3.3, Theorem 3.4), carrying as a by-product the posing of the realizer-uniqueness question that the prior paper left open (Theorem 3.7, Corollary 3.7A); Gates 2 and 3 into pre-registered structured searches enforcing Principle 3.7's evidential standard (Definitions 4.1, 5.4; Principle 7.1), with Gate 3's cohomological form fixed on the canonical realizable obstruction image Im(G) and its gluing-confirmation and non-gluing fallback supplied

(Procedures 5.1, 5.3; Definition 5.2; Remark 5.5). We assembled these into the Inspection Protocol (Definition 6.1) and proved it faithful (Theorem 6.2): on any complete inventory it halts with an N-output whenever a counterexample is found and otherwise reaches L only as a conditional terminus; the output it returns agrees with the decision procedure, and each output carries the standing Principle 3.7 demands — preserving the output/truth distinction so that N_1 is handed forward conservatively, not over-resolved (Remark 6.3).

Stated in one line: **the three-gate decision procedure is implemented by an executable protocol whose Gate 1 stage is pure classification — settling the gate and *posing* the realizer regime in one inspection-decidable pass — and whose Gate 2 and Gate 3 stages are pre-registered structured searches whose clean passes count as evidence only against a characterization fixed before the search; the protocol is proven faithful to the procedure and correctly-labelling, halting with an N-output on a found counterexample and reaching L only as the conditional terminus of a supplied proof or a finite-space exhaustion.**

What a protocol cannot contain is the inventory — and Appendix A supplies it, from the programme's own papers, and runs Stage 1. The result is **Gate 1 PASS**: every admissibility constraint is pointwise or germ-local, and the one that carried the risk — closure-current conservation — is certified local *in form* at both substrate and continuum levels, with additivity forbidding a non-local kernel in the constitutive map. This refutes the feared holonomy reading of that constraint, so it forces neither a Gate-1 failure nor an N_3-by-construction; it does not, however, close the general Gate-3 gluing-triviality question, which concerns the cocycle on overlaps and the topology of the base and which a law local in form can still fail (the Maxwell case). The construction is on the local-form branch with N_1 excluded; two gates remain open — Gate 2 (reachability) and Gate 3 (is $\mathrm{Im}(G) = 0$?) — and the σ-question's reduction to the clean $A \cap \Sigma$ intersection is conditional on both. The realizer regime is non-unique on L, fixed only by the external M2 selection (Appendix A.6). The head of the chain is therefore no longer "examine the construction somehow," nor even "feed the inventory to this protocol," but the two concrete, scoped searches that the executed Gate 1 has isolated, together with the one external selection they depend on.

Inspection Protocol (this paper) ← constraint inventory ⟶ **[Gate 1: PASS, local-form branch; holonomy reading of $\mathcal{C}$_circ refuted]** ⟶ { L ┃ N_2 ┃ N_3 } decided by Gate 2 (reachability) and Gate 3 (is $\mathrm{Im}(G) = 0$?) ⟶ ($A \cap \Sigma$ intersection, *if both clean*) → C-vs-¬C → one residue or two → FBI-comp → RC → ℓ^2.

Appendix A. Stage-1 Execution Against the Transport Construction

This appendix runs Stage 1 of the Inspection Protocol (Definition 6.1) against the actual admissibility constraints of the VERSF transport construction. It is the inspection the body deferred, executed. The faithfulness theorem (Theorem 6.2) is independent of what follows; the appendix adds the verdict for *this* construction at the Gate 1 stage, with its standing labelled exactly.

Sources. The admissible-motion space $\mathcal{A}$, refinement base $\mathscr{R}$, projection $\pi : \mathcal{A} \to \mathscr{R}$, the involution σ, and the geometric-route restriction are from *Operationally Invisible Sector Boundaries* and *One Residue or Two?* §8. The conservation sector's constraint content is from *The Leading-Order Unique Record Current in the VERSF Master Action* (continuum; constraints K1–K6) and *The Unique Scalar Closure of the VERSF Record Current* (substrate; Theorem 1, axioms A1–A4). The four-sector decomposition is the K = 7 admissibility catalogue.

A.1 The inventory

The transport construction's admissibility catalogue closes on four constraint sectors, which enter the leading-order admissible transport functional

$A_{cl} = A_{inc} + A_{hub} + A_{circ} + A_{comp}$,

giving the working inventory

$\{\mathcal{C}_i\} = \{ \mathcal{C}_{inc}, \mathcal{C}_{hub}, \mathcal{C}_{circ}, \mathcal{C}_{comp} \}$.

These constrain the admissibility of transport, hence which degeneracy germs Ĝ are realized by an admissible transport — the object the σ-question quantifies over.

A.2 Classification (Procedure 3.3)

Constraint	Meaning	Data referenced	Class
$\mathcal{C}_{inc}$	Closure-incidence: local closure compatibility of the transport cell structure	local cell / edge incidence data	pt / germ
$\mathcal{C}_{hub}$	Hub anchoring: compatibility of closure response with the K = 7 hub structure	local hub-neighbourhood data	germ
$\mathcal{C}_{circ}$	Closure-current conservation: admissible record-current conservation	germ data at a point (see A.3)	germ
$\mathcal{C}_{comp}$	Closure-competition: admissible balancing of competing closure responses	local competing-sector data	germ

The three constraints $\mathcal{C}_{inc}$, $\mathcal{C}_{hub}$, $\mathcal{C}_{comp}$ are germ-local on inspection: each references only cell- or neighbourhood-level data, with no transport-history or base-global content. The fourth, $\mathcal{C}_{circ}$, is the decisive one and is treated in A.3 — because the phrase "circulation" admits a non-local reading that the bare classification cannot settle.

Exhaustiveness for σ-coupled constraints. Since σ is central to the programme, we confirm the four reference classes cover the σ-coupling case rather than leaving it "by construction." A constraint coupling the local datum at R to its σ-image at R is germ-local (σ acts on the substrate axes at R, within the germ). A constraint coupling R to a *distant* σ-image point is non-localizable to any germ and lands in **glob** (→ N_1). No σ-coupling escapes the four classes; in the present inventory all σ-structure is the local axis-swap of Definition 8.1, hence germ.

A.3 The decisive constraint: closure-current conservation is local

$\mathcal{C}$_circ is the constraint *The Locality Decision Theorem* §8.C named as "the likely site of the action" — the candidate for an assembly-level (N_3) obstruction. Its classification is not settled by the word "circulation," which admits two readings: a local cell-balance (germ) or a prescribed holonomy around the base 1-cycles of $\mathscr{R}$ (global → N_3). The source material settles it as **local**, at two levels and on three independent grounds.

Substrate level. *The Unique Scalar Closure of the VERSF Record Current*, Theorem 1: under axioms (A1) finite distinguishability, (A2) irreversibility, (A3) additivity, (A4) local coupling, the record current C^μ = (ρ_c, J_c) satisfies the pointwise continuity identity

∂_μ C^μ(x) = s_c(x)

at every point. This is a local cell-balance — net count change = boundary flux + interior production, via the divergence theorem — not a cycle integral. Its data-reference is the germ (first derivatives at a point), so $\mathcal{C}$_circ classifies as germ.

The locality is **axiomatic, not incidental**. Axiom (A4) local coupling states that changes in ρ_c in one region affect changes elsewhere only through transport across shared boundaries — an explicit prohibition of action-at-a-distance. Axiom (A3) additivity actively *forbids the non-local kernel*: Theorem 2's restriction to first-degree-in-current identities is justified precisely because a higher-degree term "would couple the value of the count current to itself in a way inconsistent with the additivity of fact counts — the bookkeeping identity for disjoint regions R_1, R_2 would acquire a cross-term." A base-cycle holonomy condition *is* such a cross-region coupling; additivity excludes it. The constitutive-map discussion (that paper, §9.2) reinforces this a third time: maps "introducing nonlocality (integral kernels relating ρ_c to Φ over an extended region)" are flagged as the non-minimal, non-selected option — within strictly local single-field closures the map is forced.

Continuum level. *The Leading-Order Unique Record Current*, constraint (K1): ∇_ν C^{μν} = Ĉ^μ at every point — the tensor analogue of the same pointwise differential identity — and constraint (K4) additivity, which "restricts admissible functionals to those that depend locally on Φ at each point: non-local kernels in the functional definition of C^{μν}[Φ] are excluded." The continuum and substrate statements agree, and both forbid the non-local kernel.

Conclusion (and its precise limit). $\mathcal{C}$_circ is germ-local, certified at both substrate (Theorem 1, axiom A4) and continuum (K1, K4) levels, with the **glob-class / holonomy reading excluded — not merely thought unlikely** — by the additivity axiom that prohibits the cross-region coupling. This is a Gate-1 result: it establishes that $\mathcal{C}$_circ is *local in form*, so it causes no Gate-1 failure and is not an N_3-obstruction-*by-construction* (it does not secretly encode a prescribed base-cycle holonomy). It does **not** establish that the gluing cohomology is trivial. A pointwise conservation law, local in form and additive in sources, is the textbook case of something locally satisfiable everywhere that can still fail to glue on a topologically nontrivial base — Maxwell's equations are local and additive yet carry H^1/H^2 obstructions to a global potential, and that obstruction lives in the transition cocycle on overlaps (Procedure 5.1 (ii)–(iii)), not in the constitutive kernel that additivity controls. Ruling out the non-local kernel therefore *refutes the feared holonomy reading*; it does not, by itself, close the general Gate-3 question of whether Im(G) = 0. That question is separate and is handed forward (A.5).

A.4 Gate 1 verdict

Every extracted admissibility constraint classifies as pt or germ; no constraint references transport history or base-global selection data. Gate 1 PASSES.

The candidate local predicate is

$$L(\hat{G}) \equiv \mathcal{C}\_inc(\hat{G}) \wedge \mathcal{C}\_hub(\hat{G}) \wedge \mathcal{C}\_circ(\hat{G}) \wedge \mathcal{C}\_comp(\hat{G}),$$

read at germ level. Because $\mathcal{C}$_hub, $\mathcal{C}$_circ, $\mathcal{C}$_comp are germ-level (each referencing a neighbourhood, and $\mathcal{C}$_circ first derivatives), the **germ amendment of Definition 3.5 is triggered**: the carrier is the germ $\hat{G}$, not the pointwise datum G(R). This propagates to *One Residue or Two?* Definition 8.1 as that paper's amendment dependency.

Standing. Gate 1 PASS is certified for the conservation sector — the one carrying the live risk — at both substrate and continuum levels. It is conditional in one remaining respect: inventory completeness (Remark 3.3A). The PASS rests on the four-sector catalogue { inc, hub, circ, comp } being the complete admissibility set, which traces to the K = 7 catalogue's "closes on four sectors" closure claim. That claim is a citable result of the K = 7 papers, not an open search, but it is not independently re-verified here. The label is therefore:

Gate 1: PASS [certified for the conservation sector; conditional on four-sector inventory completeness].

A.5 What the verdict resolves, and what it does not

By the First-Failure Principle (*The Locality Decision Theorem*, its Theorem 3.4 — not this paper's Theorem 3.4, which is "Gate 1 reduces to classification"), a Gate 1 PASS places the construction on the local-form branch { L, N_2, N_3 } and excludes the conservative output N_1. Within that branch we claim only what the Gate-1 argument earns.

— **What is established [Proven, given four-sector completeness].** The feared glob-class reading of $\mathcal{C}$_circ — that record-current consistency secretly encodes a prescribed holonomy around the base 1-cycles of $\mathscr{R}$ — is *refuted*. $\mathcal{C}$_circ is germ-local (A.3), so it neither causes a

Gate-1 failure nor stands as an N_3-obstruction-*by-construction*. The one route by which the conservation sector could have forced non-locality *at the level of constraint form* is closed.

— **What is NOT established.** It does **not** follow that the realizable obstruction image Im(G) (Definition 5.2) is trivial, nor that the N_3 outcome is excluded. As A.3's limit makes explicit, a constraint local in form can still carry a gluing obstruction in the transition cocycle on overlaps; refuting the holonomy *reading* of 𝒞_circ does not touch that cocycle. The genuine Gate-3 question — is Im(G) = 0 on the relevant cover of ℛ? — is a statement about the base topology and the cover (Procedure 5.1), not about the constitutive kernel, and it is **[Open]**. It is handed forward alongside Gate 2, not closed here. (Were ℛ topologically trivial at the relevant scale, so that the relevant cohomology vanishes, N_3 would indeed be excluded — but that is a Gate-3 argument about the base, which this Stage-1 run does not make.)

— **The σ-question's routing, stated conditionally.** With N_1 excluded, the σ-question is on the local-form branch; *if* Gate 2 and the Gate-3 gluing question both come back clean, it reduces to the compatibility intersection A ∩ Σ (prior paper, Theorem 4.2) and the capstone inherits a geometric test rather than a cohomology class. Whether it does is exactly the content of the two open gates.

— **The two open gates.** Gate 2 (reachability): does every germ Ĝ satisfying L have a locally admissible realizer? Gate 3 (assembly): is Im(G) = 0? Stage 1 executes neither; both require pre-registered structured searches (Definitions 4.1, 5.4; Principle 7.1), not clean-verdict proofs. Gate 2 is the immediate next gate; Gate 3's gluing-triviality question is the one A.3 declined to close.

The honest one-line state: **Gate 1 passes (certified on the decisive conservation sector); the holonomy reading of record-current consistency is refuted, so 𝒞_circ forces neither a Gate-1 failure nor an N_3-by-construction; but the general Gate-3 gluing-triviality question (Im(G) = 0?) remains open alongside the Gate-2 reachability search, so the σ-question's reduction to A ∩ Σ is conditional on both.**

A.6 The realizer regime: non-unique on L, pending external selection

Stage 1 also poses the realizer-regime question (Corollary 3.7A), and the corpus supplies a sharp answer that corrects the optimistic reading. *The Unique Scalar Closure* exhibits three admissible constitutive maps for the record current C^μ — linear (M1), bilinear / energy-current (M2), entropy-current (M3) — each a bona fide Noether current of an exact symmetry, all admissible under the local axioms, and explicitly identified as *different conserved quantities attached to the same bookkeeping object* C^μ (they "conserve different things and predict different physics"). In the inspection's terms these are **competing realizers of one germ**: a single degeneracy germ Ĝ admits three ~_G-distinct realizations, so the fiber **ℛ**(Ĝ) carries at least three ~_G-classes.

The consequence for the regime turns on one interpretive step, which we make explicit because everything downstream hinges on it: **the constitutive map is free given the germ.** M1, M2, M3 are structure layered on top of Ĝ, not determined by it — the same degeneracy datum admits all three realizations, which is what makes them *competing realizers of one germ* rather than realizers of three different germs. (Were Ĝ to fix the map, they would not be same-germ realizers and the multiplicity would not bear on the regime at all.) The corpus supports this directly: the three maps are different conserved currents attached to the *same* bookkeeping object C^μ, and their ~_G-distinctness then follows, since genuinely different conserved physics cannot be a G-fixing relabelling. Granting that step, the regime verdict is the honest one, and it corrects any impression that Stage 1 returns uniqueness on its own:

— **On L alone, the regime is non-unique.** L admits M1, M2, M3 as ~_G-distinct realizers of the same germ; by Definition 3.6 that is precisely the non-unique regime. This is what Theorem 3.7 (as corrected) returns: the uniqueness question is posed by L, and L *decides it negatively*.

— **Uniqueness, if it holds, is external.** The physical selection of M2 is made not in *The Unique Scalar Closure* but in the BCB Lagrangian companion (its §9.5.4), on physical-interpretation grounds (positivity, source structure, second-law reproduction). That selection is external input — exactly the kind Theorem 3.7 now states is *not* contained in L. Importing it collapses the three classes to one ("effective uniqueness"), but only post-selection.

— **What this hands Gate 3.** Absent the imported M2 selection, the Gate-3 search must run in the *non-unique* regime: a counterexample requires a germ *all* of whose realizers fail to extend (0 ∉ Im(Ĝ) for the whole class), the strictly heavier burden of Procedure 5.3. With the M2 selection imported and documented as a Gate-3 precondition, the search collapses to the single selected realizer. The inspector should do one or the other explicitly — run the heavier non-unique search, or import and record the selection — not silently assume uniqueness.

This corrects the earlier framing: a single Stage-1 pass does *not* settle the regime from L. It returns non-unique on L and names the external selection (M2, BCB §9.5.4) that would render it effectively unique.

A.7 Residual obligations

Two, both citation-bounded rather than open searches:

(i) **Inventory completeness** (A.4): the exhaustiveness of the four-sector catalogue, traced to the K = 7 "closes on four sectors" claim. This is the one thread that could reopen Gate 1; it is a citation check against the K = 7 papers, not a search.

(ii) **The (K6) external coefficient closure**: continuum admissibility inherits a coefficient relation f(a, b) = 0 imported from the gravity-from-record-density paper. This constrains the constants in the local functional, not the data-class of any constraint, so it does not disturb Gate 1's locality verdict; it is recorded for completeness.

Neither reopens the conservation-sector locality established in A.3.

A.8 Summary

Stage 1, executed against the actual construction, returns **Gate 1 PASS** on the local-form branch (conditional on four-sector inventory completeness). Its substantive content, stated at the strength the argument earns: the decisive constraint — closure-current conservation — is local *in form* at both substrate (Theorem 1, axiom A4) and continuum (K1, K4) levels, so the feared holonomy reading is refuted and $\mathcal{C}$_circ forces neither a Gate-1 failure nor an N_3-obstruction-by-construction. What this does *not* settle — and A.3 / A.5 are explicit about it — is the general Gate-3 gluing-triviality question (is Im(G) = 0?), which concerns the cocycle on overlaps and the topology of $\mathscr{R}$, not the constitutive kernel, and which remains **[Open]** alongside Gate 2. The realizer regime is **non-unique on L** (three competing constitutive realizers of one germ), rendered effectively unique only by importing the external M2 selection (BCB §9.5.4).

So the inspection's remaining content is two pre-registered searches, not one — Gate 2 (reachability) and Gate 3 (gluing-triviality) — with the realizer regime to be either run heavy (non-unique) or fixed by the imported selection. The instrument did its job: it predicted that record-current consistency was the site of the action, and the analysis localized exactly what that constraint does and does not foreclose — it forecloses the *holonomy reading* by axiom, and leaves the *gluing question* open for an honest Gate-3 search. The distinction the whole verdict rests on is the one A.3's Maxwell limit fixes: the Gate-1 locality of $\mathcal{C}$_circ is a statement about the constitutive kernel (which additivity controls), while Gate-3 triviality of Im(G) is a statement about the transition cocycle on overlaps (which it does not). Conflating the two — reading kernel-locality as cocycle-triviality — is precisely the inference the Maxwell case forbids, and keeping them apart is what holds the Gate-1 result to its proper, bounded strength.