The PFD–Standard Model Dictionary in VERSF

Persistent Fold Defects, Internal Probability Geometry, Hypercharge Selection, and the Emergence of Particle Representation Sectors

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General reader summary

The Standard Model of particle physics is the most accurate theory humans have ever written. It describes every observed particle — electrons, quarks, neutrinos, photons, and the rest — and the forces between them. But it does this by *listing* the particles and forces as ingredients. It does not explain *why* this particular list, with these particular charges, in three generations, with confinement for quarks but not for leptons, and with the weak force acting only on left-handed particles.

This paper proposes a structural explanation. The previous paper in this programme established that matter, at the substrate level, takes the form of *Persistent Fold Defects* — stable topological knots in the substrate from which space, matter, and geometry all emerge. That paper said *what kind of thing matter is*. The present paper says *which kinds of knot are which particles*.

The dictionary the paper constructs is structural rather than numerical. It does not claim to derive exact particle masses or scattering amplitudes. What it does claim is that the substrate forces the Standard Model representation structure rather than allowing it as one option among many: the joint admissibility constraints of the substrate leave only the observed Standard Model orbit classes as admissible. Confinement is not a force — it is the substrate refusing to admit isolated triality. The three generations are not posited — they are the only refinement-stable closure-depth excitations. Hypercharge is not chosen — it is the unique abelian projection compatible with anomaly cancellation. Each piece of the Standard Model lands as the only admissible solution to a substrate-level constraint.

The paper also makes explicit *what would falsify it* — specific empirical observations that would break the dictionary — and provides a dependency map showing which substrate results each piece of the dictionary depends on. The framework is therefore structurally falsifiable rather than merely interpretive.

What this paper does not yet do: it does not derive exact masses, it does not derive the mixing matrices (CKM, PMNS), it does not settle the ν_R PFD existence question (the substrate-structural question of whether a right-handed neutrino exists as a PFD class — distinct from the separate question of whether the neutrino is Majorana), and it does not close the no-alternatives

statement from an overdetermination argument to a uniqueness proof. These are the next stages of the programme.

Contents

Abstract

The VERSF and BCB programmes have progressively reconstructed the structural foundations of physical reality from information-theoretic and topological principles. Previous papers established the Fold as the minimal committed distinguishability event, irreversible fact formation from topological trapping, the emergence of gauge structure from distinguishability conservation, Lorentzian transport geometry from refinement-compatible transport, Persistent Fold Defects (PFDs) as the substrate ontology of matter, spinorial exchange topology and fermionic antisymmetry, and the emergence of Standard Model gauge sectors from internal probability geometry.

One major structural gap remained:

How do specific Standard Model particle sectors emerge from the topology and representation structure of Persistent Fold Defects?

The present paper develops the missing classification layer.

We construct a PFD–Standard Model dictionary connecting closure topology, fold orientation structure, holonomy sectors, confinement admissibility, and internal probability geometry to SU(3)_C × SU(2)_L × U(1)_Y representation sectors. The gauge group is *inherited* from the upstream gauge-necessity programme; the dictionary classifies which PFD classes carry which representations under it.

The central thesis is:

Standard Model particles are not primitive objects but the unique stable representation classes of Persistent Fold Defects on the committed distinguishability substrate.

The paper establishes nine principal structural results:

1. **PFDs as representation carriers (§4).** Particles correspond to stable closure-defect sectors in $\Sigma = (F, E)$, classified by an invariant tuple $\boldsymbol{\mathcal{J}}(D)$.
2. **Matter / gauge-boson type distinction (§5).** Matter and gauge bosons are *ontologically distinct kinds* of substrate object.
3. **Closure-confinement equivalence (§6, Theorem 6.7).** Complete closure ⟺ trivial $\mathbb{C}^3$ content ⟺ free transport; partial closure ⟺ nontrivial $\mathbb{C}^3$ content ⟺ confinement.
4. **Uniqueness of the dictionary (§7).** The assignment is *forced*, not chosen: substrate admissibility constraints simultaneously overdetermine the mapping.
5. **Isolated triality obstruction (§8, Theorem 8.2).** Substrate closure conservation forbids isolated nontrivial triality, establishing confinement as a topological admissibility obstruction.
6. **Spin-statistics from exchange holonomy (§10).** Fermionic antisymmetry emerges from exchange-path holonomy on the spinorial closure sector.
7. **Hypercharge as oriented closure-ledger invariant (§11).** U(1)_Y is the unique abelian factor compatible with anomaly cancellation under the substrate fermion content, given

anomaly cancellation as an admissibility constraint inherited from the substrate transport programme.

8. **Stabilization-depth dynamics for composites (§14).** Baryons are dynamically stabilized by TPB-constrained admissibility contraction governed by $R_{n+1} = \eta R_n$, $\eta = 3/5$.
9. **Generations as generation-depth transport sectors (§16).** The three-generation structure follows from refinement-stable closure-depth excitations with stiffness $\mathcal{D} = \mathrm{diag}(1, 2, 4)$.

The paper additionally develops *structural implications* (§19): finiteness of the spectrum, the closure-norm bridge to the hexagonal-geometry companion paper's substrate Higgs derivation (its Theorem 6), and the mass-as-closure-cost relationship. It states explicit *falsification criteria* (§20.5) and provides a *dependency map* (Appendix A).

Epistemic status. The contribution is structural and ontological. The paper does *not* claim a derivation of exact masses, mixing angles, scattering amplitudes, or the substrate Higgs mechanism. Theorems 6.7 and 8.2 are proven from inherited primitives, with the substrate $\mathbb{Z}_3$ closure-conservation law itself taken from the BCB transport programme (§2.10); the chain of conditionality from prior programmes to dictionary results is therefore explicit. Most other results are conditional on previously-established programmes and are flagged as such throughout.

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1. Introduction

One of the deepest unresolved questions in fundamental physics is

Why does matter possess the specific organisational structure observed in the Standard Model?

The Standard Model contains quarks, leptons, gauge bosons, chirality, confinement, generations, and gauge representations, but treats these largely as primitive ingredients or empirically inserted structures.

The VERSF programme approaches the problem differently. Rather than treating particles, gauge structure, and geometry as independent primitives, the framework derives them from distinguishability, irreversible commitment, topology, and closure dynamics.

Earlier papers established gauge structure from distinguishability conservation, confinement from closure incompleteness, spinorial structure from exchange topology, and matter ontology from Persistent Fold Defects. The matter paper (*Matter from Persistent Fold Closure in VERSF*) closed the substrate genealogy

Void → Fold → Fact → Persistent Closure → Matter → Geometric structure → Gravity,

and identified — in its §10 — the dictionary between PFD topological invariants and Standard Model representations as the defining open problem of the next stage.

The present paper develops that dictionary. It is the bridge between the substrate-level claim *matter is persistent fold closure* and the phenomenological claim *matter is the Standard Model spectrum*. It is the first paper in the programme in which substrate primitives are mapped directly to observed particle classes.

A key structural claim, made explicit in §7, is that the dictionary is *not free*. Each piece of the assignment is forced by substrate admissibility constraints that simultaneously overdetermine the representation structure. The observed Standard Model is therefore not a list anyone could have written differently — it is the unique admissible organisation of committed distinguishability.

The dictionary operates *downstream* of a prior gauge-necessity programme. SU(3) × SU(2) × U(1) is not derived here; it is inherited from the distinguishability-conservation and minimal-internal-symmetry results of earlier work. The present paper takes the gauge group as given and asks which PFD classes carry which representations under it.

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2. Ontological architecture inherited from prior papers

This paper does not rederive the VERSF programme. It uses the following inherited results.

2.1 The Fold

A Fold is the minimal committed distinguishability event, located at the reversible/irreversible interface. Informationally it carries one bit of committed distinguishability. Physically it possesses orientation structure, boundary polarity, and admissible transport embedding. Earlier work distinguishes

informational bit → physical fold → emergent particle closure.

Particles are stable closure defects composed of folds, not primitive material objects.

2.2 The committed distinguishability graph

Committed folds organise into

$\Sigma = (F, E)$,

where F is the set of committed folds and E is the set of admissible committed relations among them. Particles are interpreted as stable topological closure defects in Σ.

2.3 Persistent Fold Defects

A Persistent Fold Defect is a connected closure structure $D \subset \Sigma$ satisfying the four conditions of Definition 3.1 of the matter paper:

- (P1) nontrivial closure topology ($\beta_1(D) \geq 1$, with at least one admissibility-fixed homology class),
- (P2) nontrivial closure holonomy on at least one non-contractible loop,
- (P3) refinement persistence,
- (P4) positive closure stability ($\delta^2 S\_int[D] \succ 0$).

Its internal structure is encoded by an invariant tuple

$\mathcal{I}(D) = (C\_D, \beta_1(D), h\_D, \pi\_D, \chi\_D, \gamma\_D, \ell\_D, \rho\_D)$,

with:

- C_D : closure completeness (defined operationally as "the PFD's internal image lies entirely in the $\mathbb{C}^1$ sector of $\mathbb{C}^4 = \mathbb{C}^1 \oplus \mathbb{C}^3$"),
- $\beta_1(D)$: first Betti number / loop persistence,
- h_D : holonomy class,
- π_D : orientation parity,
- χ_D : chirality assignment,
- γ_D : generation depth,
- ℓ_D : ledger charge vector,
- ρ_D : confinement requirement.

The dictionary maps this tuple to Standard Model representation data.

2.4 Gauge necessity — the dictionary operates downstream

BCB distinguishability conservation forces gauge connections. If distinguishability is transported locally across the internal Fisher information manifold (FIM), a connection is required to compare internal states at neighbouring events. Gauge fields appear not as arbitrary additions but as the mathematical structure needed to preserve local distinguishability relations.

This is established in the *Distinguishability Conservation and Gauge Structure* paper and in the *Minimal Internal Symmetry Theorem* paper. Together with the per-fold Hilbert space $\mathcal{H}\_fold \cong \mathbb{C}^4$ established by *One Fold*, they constrain the gauge group to SU(3) × SU(2) × U(1) as the commutant of the $3\oplus1$-block hopping matrix on $\mathbb{C}^4$.

The present paper operates *downstream* of these results. The dictionary does not presuppose the Standard Model gauge group — it inherits it from the gauge-necessity programme and asks which PFD classes carry which representations under it. This is a critical epistemic point: the dictionary is not assuming what it is meant to explain. The hexagonal-geometry programme — comprising *The Standard Model from Hexagonal Geometry* (Taylor, VERSF Theoretical Physics Programme) together with its rigorisation companion *A Unified Derivation of Closure Geometry, Gauge Redundancy, and Mass Structure in the Hexagonal Framework* (same programme) — provides a further independent derivation of the same gauge group from closure-vertex axioms; throughout this paper, references such as "the hexagonal paper §3j" or "the hexagonal companion paper Theorem 6" point into these two documents. The explicit relationship between that programme and the present dictionary is laid out in §2.11.

2.5 Internal probability geometry

Earlier BCB papers establish that the admissible internal state spaces admit a complementary description as complex projective manifolds $\mathbb{CP}^{n-1}$ with $n \leq 3$. This presentation is *alternative* to the commutant route of §4.2 — it produces the same gauge sector structure SU(3) × SU(2) × U(1) by a different mathematical route. The canonical formulation used downstream in the dictionary, and in Appendix A's dependency map, is the commutant route via $\mathscr{H}$_fold $\cong \mathbb{C}^4$ established in *One Fold*. The $\mathbb{CP}^{n-1}$ presentation is retained here only to indicate that the two derivations agree on the substrate gauge content.

2.6 Confinement and quark partial closure

Prior confinement work interprets quarks as partial closures — structures that cannot close admissibly in isolation and therefore acquire effective mass only inside committed hadronic configurations. The confinement paper distinguishes the structural $K = 7$ closure layer from the dynamical entropic surface-tension layer and interprets quark mass as inseparable from confinement.

2.7 Spinorial and fermionic structure

The spin–statistics results of the matter paper (Theorems 7.1 and 7.2) establish that fermionic exchange arises from topological exchange holonomy via the Finkelstein–Rubinstein mechanism. Full CAR/Fock completion remains open and is deferred to the PFD second-quantisation companion paper.

2.8 Flavour mixing and generation depth

The flavour-mixing work interprets generations as closure-depth sectors with transport between them, deriving a conditional CKM hierarchy from a generation-space stiffness operator

$\mathcal{D} = \mathrm{diag}(1, 2, 4)$,

with attenuation and projection factors arising from residual closure space.

2.9 TPB dynamics and stabilization depth

The proton/baryon paper establishes that baryon stabilisation proceeds via TPB-constrained admissibility contraction in the residual closure space, governed by the iteration

R_{n+1} = η R_n, η = 3/5,

on a five-dimensional residual closure space. The iteration converges to a stable composite closure structure under finitely many steps. This result is used in §14 below. The depth-variable here — *stabilization depth* — is distinct from the single-PFD *generation depth* of §2.8; both are layered closure-depth notions but act at different layers of the substrate hierarchy.

2.10 Substrate closure-conservation law for $\mathbb{Z}_3$

The BCB transport programme establishes that admissible closure transport on Σ preserves a substrate analogue of Gauss's law for each admissibility-fixed conserved current. For the $\mathbb{Z}_3$-graded source associated with the centre $\mathbb{Z}_3 \subset SU(3)$, this takes the form

every admissibly closed loop in Σ carries vanishing total $\mathbb{Z}_3$-graded enclosed source.

This is taken as an inherited substrate result in the proof of Theorem 8.2 below.

2.11 Relation to the Standard Model from Hexagonal Geometry programme

The dictionary developed in this paper is *complementary* to, but logically *independent* from, the hexagonal-geometry programme of *The Standard Model from Hexagonal Geometry* (also within the VERSF programme). The two frameworks address different questions at different layers of description, and their results — when taken together — close more of the gap to a fully non-arbitrary Standard Model than either does alone.

2.11.1 Scope distinction

The hexagonal-geometry programme is *structural-numerical*. From the four axioms A1–A4 (uniformity, isotropy, closure, economy) plus statistical axioms S1–S3, together with the explicit Hexagonal Closure Field Model defined in its §3a, the hexagonal paper derives:

- the closure-count integer K = 7 as the unique admissible count compatible with closure-by-distinguishability + commitment,
- the numerical values of $\alpha^{-1} = 137.14$, $\sin^2\theta\_W = 3/13 = 0.2308$, $M\_H = 125.8$ GeV, $\sigma = 9\ m\_\pi^2$, and other dimensionless ratios,
- the gauge group SU(3) × SU(2) × U(1) as the unique connected continuous internal symmetry algebra compatible with closure, finite entropy density, singlet formation, and chirality (hexagonal paper §3j),
- the universality of the loop-correction factor $(2K+1)/(2K) = 15/14$ across electromagnetic, hadronic, and electroweak sectors.

The dictionary inherits K = 7 from this upstream programme as a *substrate-fixed value*, not as a uniqueness claim. Whether K = 7 is *uniquely* selected (the hexagonal paper's stronger claim) or simply the *correct* value (the weaker claim the dictionary actually depends on) is a question about the upstream programme. If the hexagonal-programme uniqueness derivation were later refined, the dictionary's representation-theoretic content would be unaffected, as long as the value K = 7 remained (§2.11.4).

The present paper is *representation-theoretic*. It takes the gauge group G_SM = SU(3) × SU(2) × U(1) as inherited from the upstream gauge-necessity programme (the hexagonal-geometry programme being one of the available upstream derivations; see §2.4), and answers:

- which PFD topological-invariant classes carry which orbit types of G_SM on 𝓗_fold,
- how the closure-completeness, orientation, chirality, generation-depth, and ledger invariants of 𝑱(D) map onto the standard SM matter content,
- under what joint admissibility constraints the dictionary is *forced* (§7).

The dictionary does not derive numerical constants. The hexagonal-geometry programme does not classify PFD matter content. The two are complementary, not redundant.

2.11.2 Structural alignment

Despite their different aims, the two frameworks are tightly aligned on substrate content. The same physical phenomena appear in both, expressed in different vocabularies:

Concept	**Hexagonal-geometry programme**	**Present dictionary**
K = 7	derived from closure vertex count (its §3); rigorised as Theorem 1 of the hexagonal companion paper (4-step proof: K ≥ 6 from boundary adjacency, K = 6 insufficient, K = 7 sufficient, K > 7 excluded via orbit-stabilizer Lemma 1)	inherited; surfaces in the Wilson-Limit normalisation (§11.3)
Gauge group SU(3) × SU(2) × U(1)	proven unique by its Theorem 3j.1	inherited as substrate input (§2.4) and as the kinematic layer of §7.1
Confinement	"quarks are level-2: 2 of 6 triangles, cannot complete a hexagon"	Theorem 8.2: isolated nontrivial triality is forbidden by closure conservation
Three generations	hexagon admits exactly 3 direction pairs (its §19)	$\gamma_D \leq 3$ from refinement persistence + $\mathcal{D} = \mathrm{diag}(1,2,4)$ stiffness (§16)
$\mathbb{C}^3$ internal structure	three triangle-pair channels (its §3j, Appendix G)	$\mathbb{C}^3$ block of $\mathbb{C}^4 = \mathbb{C}^1 \oplus \mathbb{C}^3$ (§4.1, Lemma GG2 of *One Fold*)
Colour SU(3) emergence	from unitary mixing of three triangle-pair channels (its Appendix G)	inherited; $\mathbb{C}^3$ content classifies quark-like PFDs (§8.1)
Chiral SU(2)	from orientation field on S^2 (its Appendix D)	inherited; chirality from SU(2) restriction to one orientation parity (§9.2)
Higgs scalar	closure-norm radial mode $\rho(x) = \lvert C(x) \rvert - 1$ (its Appendix E); rigorised as Theorem 6 of the hexagonal companion paper with the bound $M_H^2 \geq N_{scalar} \cdot (M_W^2 + M_Z^2)$	closure-norm bridge cited in §19.2 to the hexagonal companion paper's substrate Higgs derivation;

Concept	Hexagonal-geometry programme	Present dictionary
	and saturation at leading order via Schur complement + channel-counting Lemma 5	a PFD/dictionary-vocabulary re-derivation is open (§20.3)

The agreement is non-trivial: the hexagonal-geometry programme reaches these substrate identifications from numerical-derivation axioms (closure, isotropy, economy, statistical independence); the dictionary reaches them from representation-theoretic admissibility constraints on PFD classes. The two paths converge.

2.11.3 Translation between vocabularies

A partial translation table:

- *level-3 committed hexagonal cell ↔ substrate cell with committed fold and per-cell Hilbert space $\mathscr{H}$_fold ≅ $\mathbb{C}^4$*,
- *level-4 5-7 defect (particle) ↔ PFD with $\beta_1(D) \geq 1$* satisfying (P1)–(P4),
- *level-2 triangle (quark constituent, 2 of 6 triangles) ↔ partial PFD with C_D = partial* (Theorem 6.7),
- *triality singlet (3-quark baryon, completing 6 triangles) ↔ three-channel composite* $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \supset \mathbf{1}$ (§13.1),
- *closure-norm fluctuation ρ ↔ radial closure-norm mode* (§19.2),
- *15/14 universal loop correction* ↔ no direct counterpart, since the dictionary does not derive numerical constants.

The dictionary's confinement-via-Theorem-8.2 and the hexagonal paper's confinement-via-incomplete-closure are the *same physical claim*: an isolated quark cannot exist because its closure structure fails. The dictionary expresses this as a $\mathbb{Z}_3$-graded substrate Gauss-law statement; the hexagonal paper expresses it as a count of triangles.

The bridge between the two confinement pictures. The hexagonal-paper picture is *count-and-completion*: an isolated quark contributes 2 of the 6 boundary triangles needed for a committed hexagonal closure, leaving $6 - 2 = 4$ missing triangles. This geometric incompleteness is not in itself a Gauss-law statement. The substrate identification, in dictionary vocabulary, is that the 2 triangles a quark contributes carry a nontrivial $\mathbb{Z}_3$ triality charge $\tau(D) \in \mathbb{Z}_3$, the 6-triangle closed configuration is the minimal triality-singlet, and the 4-missing-triangles state is the geometric realisation of a substrate-level $\mathbb{Z}_3$ Gauss-law violation: an admissibility-fixed loop around the isolated quark encloses nonzero $\mathbb{Z}_3$-graded source, which Theorem 8.2 forbids. The two pictures are therefore the same: *count-and-completion at the geometric level is $\mathbb{Z}_3$ Gauss-law obstruction at the substrate level*. A full proof of this identification — explicitly identifying triangle-pair occupancy with $\mathbb{Z}_3$ triality grading — is part of the cross-programme unification work flagged in §20.3.

On the $\mathbb{C}^3$ identification. Among the rows of the §2.11.2 alignment table, the $\mathbb{C}^3$ identification (hexagonal three triangle-pair channels ↔ *One Fold* $\mathbb{C}^4 = \mathbb{C}^1 \oplus \mathbb{C}^3$) is doing more structural work than the others. The $\mathbb{C}^3$ block of $\mathbb{C}^4$ is the kinematic core of the dictionary; the three triangle-pair channels are the geometric core of the hexagonal programme's colour sector. If these are literally the same $\mathbb{C}^3$, that is a non-trivial cross-derivation consistency result for the VERSF programme as a whole — not just a notational coincidence but evidence that two independent substrate routes terminate at the same internal-Hilbert-space structure. The hexagonal paper's §3j and Appendix G derive the $\mathbb{C}^3$ from local triality mixing; the dictionary's §4.1 derives it from per-fold $\mathbb{C}^4$ commutant structure. The explicit identification theorem ("these two $\mathbb{C}^3$'s coincide as substrate carriers of SU(3) colour, with the same triality structure and the same orbit types") is open (§20.3). The other rows of the alignment table can be cleanly checked one-by-one against existing material; the $\mathbb{C}^3$ row, by contrast, is the structural keystone for whether the two programmes describe the same substrate.

2.11.4 Logical independence

The dictionary does not depend on the hexagonal-geometry programme's specific numerical derivations. G_SM is taken as inherited (§2.4) from any of the upstream gauge-necessity papers; the hexagonal paper provides one such derivation. If the hexagonal programme's $K = 7$ uniqueness derivation were later refined or strengthened, the dictionary's representation-theoretic content would be unaffected.

A subtlety on $K = 7$ inheritance: while the dictionary's *representation-theoretic core* is independent of any specific upstream derivation, the dictionary's *numerical normalisations and stabilization-depth constants* — specifically the $K = 7$ Wilson-Limit convention in §11.3 and the $\eta = 3/5$ contraction rate from the proton/baryon paper's $K = 7$ closure architecture in §14 — inherit the value $K = 7$ from upstream papers, including the hexagonal-geometry programme via the proton/baryon paper. So "G_SM inherited from any of the upstream gauge-necessity papers" applies cleanly to the gauge group, but the $K = 7$ numerical inputs trace through a specific subset of the upstream chain. The substantive logical-independence claim is therefore: the dictionary's representation-theoretic content is independent of which upstream derivation of $K = 7$ turns out to be canonical, as long as the value is $K = 7$.

Conversely, the hexagonal-geometry programme does not depend on the PFD ontology of the matter paper or the dictionary's classification scheme. Its numerical results stand on their own substrate axioms.

This logical independence is a feature, not a bug: each paper can be audited for soundness without auditing the other.

2.11.5 Interpretive synthesis

The two papers support the following programme-level synthesis:

1. **The hexagonal-geometry programme** shows that the Standard Model's *gauge group and certain key numerical constants* (α^{-1}, $\sin^2\theta\_W$, M_H, σ, the 15/14 loop correction) are forced by closure geometry at the substrate level. It answers *why these particular constants and this gauge group* — but it does not derive every Standard Model parameter; fermion masses, mixing matrices beyond Cabibbo, the electroweak scale, and Yukawa structure are not addressed.

2. **The present dictionary** shows that the Standard Model's *matter content* (which PFD class is which particle, with which gauge representation) is forced by joint admissibility constraints on representations of the inherited gauge group. It answers *which kinds of substrate defect are which Standard Model particles*.

Both questions need answers to render the Standard Model fully non-arbitrary. The hexagonal-geometry programme without the dictionary leaves the question "which substrate excitation is the electron?" unanswered. The dictionary without the hexagonal-geometry programme leaves the question "why $\alpha^{-1} = 137$ and why this gauge group at all?" unanswered. Together they cover both faces.

What the conjunction of both papers still does not achieve. Even taken together, the two programmes do not yet deliver: exact fermion mass values, CKM/PMNS mixing angles beyond Cabibbo, the electroweak symmetry-breaking scale, Yukawa coupling structure, CP-violating phases, or running of the coupling constants. These remain genuinely open at the programme level after both papers are taken into account. The combined result is therefore "Standard Model gauge structure + matter classification + certain key constants forced from substrate", not "Standard Model fully derived from substrate." Closing the residual gap is the work of further programme papers (§20.3, §20.6).

The relationship is analogous to that between

- a *no-alternatives theorem* establishing that a particular symmetry group is the unique admissible one for a physical theory (the hexagonal paper's §3j and the present paper's §7.3 are both no-alternatives statements at different scales), and
- a *representation-classification theorem* identifying which physical objects carry which representations under that group (the present paper's central content).

Neither replaces the other; each strengthens the other's interpretive position.

2.11.6 Convergence on no-alternatives logic

Both papers culminate in no-alternatives statements, at different levels:

- The hexagonal paper's Theorem 3j.1 establishes that *SU(3) × SU(2) × U(1) is the unique gauge group* compatible with closure, entropy, singlet formation, and chirality.
- The present paper's Dictionary No-Alternatives Statement (§7.3, conjectural) extends this logic to the *matter content under that gauge group*: the observed Standard Model orbit content is the unique simultaneous solution of substrate admissibility constraints on PFD representation classes.

The two no-alternatives statements together form a layered uniqueness argument: gauge group forced by substrate geometry (hexagonal §3j, proven), matter content forced by joint admissibility on that gauge group (present §7.3, conjectural). Closing the second from conjecture to theorem is one of the major remaining targets of the programme (§20.3).

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3. The dictionary problem

The PFD matter ontology answers one question:

What is a particle?

It answers:

A particle is a stable, localized, transportable closure defect formed from committed folds in Σ.

But this does not yet answer:

Which PFD is an electron? Which is a quark? Which is a neutrino?

This is the dictionary problem.

The shape of its solution is now sharp. The matter paper (§10) reduced the dictionary to a precise classification target:

Identify which PFD topological invariants correspond to which orbit types of the SU(3)_C × SU(2)_L × U(1)_Y action on $\mathscr{H}$_fold = $\mathbb{C}^4$ at the substrate cells supporting the defect.

The dictionary therefore takes the form of a map

Φ_dict : { admissible PFD classes [D] } ⟶ { orbit types of G_SM on $\mathscr{H}$_fold }.

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4. Internal probability geometry and the PFD internal image

4.1 Substrate carrier of internal structure

At every substrate cell, the per-fold Hilbert space is

$\mathscr{H}$_fold ≅ $\mathbb{C}^4$,

with basis $|b, d\rangle$, $b \in \{0, 1\}$, $d \in \{+1, -1\}$, established in *One Fold* (Theorem T1). The classical bit b encodes distinguishability content; the direction label d encodes orientation.

Two complementary decompositions of $\mathbb{C}^4$. Both play roles in the dictionary:

(a) **Colour decomposition.** $\mathbb{C}^4 = \mathbb{C}^1 \oplus \mathbb{C}^3$, with SU(3) acting on the $\mathbb{C}^3$ block. This is Lemma GG2 of *One Fold*.

(b) **Doublet decomposition.** At each value of the orientation index $d \in \{+1, -1\}$, the two values of the distinguishability bit b span a $\mathbb{C}^2$ subspace. SU(2) acts on this $\mathbb{C}^2$ at one orientation parity (the left-handed sector, §9.2); at the other parity, SU(2) acts trivially.

The gauge group SU(3) × SU(2) × U(1) is the commutant of the $3\oplus 1$-block hopping matrix K that simultaneously preserves both decompositions. The product structure of the commutant — and in particular the appearance of SU(2) rather than an additional U(1) — follows from K's nontrivial action across the doublet $\mathbb{C}^2$ at one orientation parity, which forbids independent abelian rotation of the two doublet components and forces a non-Abelian SU(2) factor in the commutant. *Concretely*, K mixes the $|b = 0, d = +1\rangle$ and $|b = 1, d = +1\rangle$ states through an off-diagonal coupling within the doublet, so any U(1) acting independently on the two components would fail $[U, K] = 0$; only an SU(2) that rotates the doublet as a whole (and a single diagonal U(1) acting on the doublet's overall phase) commutes with K. The full canonical form of K and its commutant calculation is given in *One Fold* Appendix D.5; the diagonal U(1) survives in the commutant as $U(1)\_Y$, the hypercharge factor of §11.

4.2 The gauge group as commutant

The Standard Model gauge group emerges as the commutant of the $3\oplus 1$-block hopping matrix K acting on $\mathbb{C}^4$:

$$G\_SM = SU(3) \times SU(2) \times U(1) = \{ U \in U(\mathbb{C}^4) : [U, K] = 0 \} / (\text{overall phase})$$

(Appendix D.5 of *One Fold*). K is the unique admissible hopping matrix compatible with the per-fold Hilbert space decomposition, finite distinguishability, and the substrate-level $\mathbb{Z}_2$ direction label.

4.3 The PFD internal image and its topological invariants

Each PFD $D \subset \Sigma$ has support on a localized set of substrate cells. The internal image of D is the assignment of an internal state in $\mathscr{H}\_\text{fold}$ to each cell in supp(D), together with the admissibility conditions enforced by closure transport. We denote the internal image

$$\mathcal{I}^{\text{int}}(D) \subset \bigotimes\_\{x \in \text{supp}(D)\} \mathscr{H}\_\text{fold}(x),$$

modulo gauge equivalence. The invariant tuple $\mathcal{I}(D)$ of §2.3 is the *topological-invariant projection* of $\mathcal{I}^{\text{int}}(D)$: each entry of $\mathcal{I}(D)$ is a gauge-invariant function of the internal image (e.g. C_D is determined by whether $\mathcal{I}^{\text{int}}(D)$ lies in the $\mathbb{C}^1$ block, π_D by the d-parity of the dominant orientation, etc.).

Two PFDs with identical $\mathcal{I}(D)$ but inequivalent internal images correspond to *distinct particle states within the same gauge representation*. Two PFDs with identical internal images modulo gauge equivalence correspond to *the same particle*. The dictionary Φ_dict acts on the $\mathcal{I}(D)$-equivalence classes.

4.4 Substrate orbits

The action of G_SM on ⊗ ℋ_fold(x) decomposes into orbits. The dictionary identifies each orbit type with a particle sector. The classification of orbits is the central technical content of §§8–13.

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5. Matter vs gauge boson — the type distinction

A structural distinction must be made before the sector-by-sector classification can begin.

5.1 The ontology distinction

Ontology class	Substrate identification	Topological signature	Refinement behaviour
Matter particle	Persistent closure defect (PFD)	$\beta_1(D) \geq 1$	Refinement-stable
Gauge boson	Closure-preserving transport mode	$\beta_1 = 0$	Disperses freely

The single line that captures the distinction:

Matter corresponds to persistent topology; gauge fields correspond to admissible closure transport dynamics.

5.2 Matter particles are PFDs

A matter particle is a Persistent Fold Defect — a localized region of nontrivial closure topology, $\beta_1(D) \geq 1$, satisfying (P1)–(P4). A matter particle is a *defect* in the substrate: a knot that cannot be unwound by admissible refinement.

5.3 Gauge bosons are not PFDs

A gauge boson is a propagation mode of the gauge connection itself — a closure-preserving reconfiguration of admissible transport that carries no nontrivial closure topology of its own. Gauge bosons satisfy $\beta_1 = 0$; they are not refinement-persistent in the (P3) sense.

5.4 Why this distinction matters for the dictionary

This type distinction prevents the dictionary from being forced into representing photons or gluons as "degenerate kinds of fermion." A photon is not a PFD with trivial topology; it is a different ontological category entirely. The dictionary therefore proceeds in two stages: it classifies matter PFDs by the orbit types they carry (§§8–12), and it identifies gauge bosons as the propagation modes of the gauge connection corresponding to each gauge factor (§15).

Matter is what is committed and persists. Gauge bosons are how committed structure communicates.

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6. PFD invariants and their representation images

This section provides the component-wise map from each entry of $\mathcal{I}$(D) to its representation-theoretic image.

6.1 Closure completeness C_D

We *define*:

C_D = complete ⇔ $\mathcal{I}^{\mathrm{int}}$(D) lies entirely in the $\mathbb{C}^1$ block of $\mathbb{C}^4 = \mathbb{C}^1 \oplus \mathbb{C}^3$, C_D = partial ⇔ $\mathcal{I}^{\mathrm{int}}$(D) has nontrivial component in the $\mathbb{C}^3$ block.

Under SU(3)_C:

C_D = complete ⟹ colour singlet ⟹ lepton-like. C_D = partial ⟹ colour triplet ⟹ quark-like.

The complete/partial distinction is the substrate-level origin of the lepton/quark divide. Partial closures cannot close admissibly in isolation (Theorem 8.2).

6.2 First Betti number β_1(D) and holonomy h_D

The pair (β_1(D), h_D) classifies the closure-transport structure of D. By the persistence theorem (Theorem 4.1 of the matter paper), $\beta_1(D) \geq 1$ is the topological signature of matter. The holonomy class h_D is an element of the relevant gauge group acting on the orientation and ledger fibres.

6.3 Orientation parity π_D

The orientation parity $\pi\_D \in \{+1, -1\}$ is the substrate image of the $\mathbb{Z}_2$ direction label d. It is the ancestor of:

- the $\mathbb{Z}_2$ that lifts to $\pi_1(SO(3)) = \mathbb{Z}_2$ under dimensional emergence (matter paper Theorem 7.1),
- the orientation sector that distinguishes left- and right-handed Weyl components (§9),
- the sign of the closure-loop holonomy in the U(1)_Y sector (§11).

One substrate $\mathbb{Z}_2$ does all three jobs; *One Fold* establishes consistency.

6.4 Chirality assignment χ_D

The chirality assignment $\chi_D \in \{L, R, \text{none}\}$ records the action of $SU(2)_L$ on D. The substrate origin of chirality is the restriction of SU(2) transport to one Weyl sector (§9).

6.5 Generation depth γ_D

The generation depth $\gamma_D \in \{1, 2, 3\}$ records the admissible radial excitation sector of D. The bound $\gamma_D \leq 3$ follows from refinement persistence (§16). *(Conditional on the flavour-mixing programme.)*

6.6 Ledger charge ℓ_D

The ledger charge ℓ_D is the oriented closure-circulation invariant. It is the ancestor of hypercharge Y (§11).

6.7 The confinement–closure equivalence

Theorem 6.7 — Confinement–Closure Equivalence *(proven, conditional on the confinement programme and Theorem 8.2)*

For every admissible PFD D, the following three conditions are equivalent:

$C_D = \text{complete} \Leftrightarrow \text{trivial } \mathbb{C}^3 \text{ content in } \boldsymbol{\mathcal{I}}^{\mathrm{int}}(D) \Leftrightarrow \rho_D = \text{free}.$

Equivalently:

$C_D = \text{partial} \Leftrightarrow \text{nontrivial } \mathbb{C}^3 \text{ content} \Leftrightarrow \rho_D = \text{confined}.$

Proof

The first equivalence ($C_D = \text{complete} \Leftrightarrow \text{trivial } \mathbb{C}^3$ content) is the *definition* of C_D given in §6.1, not a substantive claim.

The second equivalence:

($\Leftarrow$) Trivial $\mathbb{C}^3$ content implies trivial triality charge $\tau(D) = 0$, hence no obstruction from Theorem 8.2 to free admissible transport, hence $\rho_D = \text{free}$.

($\Rightarrow$) Free admissible transport — $\rho_D = \text{free}$ — implies admissibility under isolation. Theorem 8.2 then forces $\tau(D) = 0$, which is incompatible with nontrivial $\mathbb{C}^3$ content (since nontrivial $\mathbb{C}^3$ content places D in a faithful representation of SU(3) carrying nontrivial centre charge under $\mathbb{Z}_3 \subset SU(3)$). Therefore the $\mathbb{C}^3$ content of $\boldsymbol{\mathcal{I}}^{\mathrm{int}}(D)$ must be trivial. ▢

Remark 6.7.1

The substrate-level content of the theorem is that *confinement is admissibility failure of isolated partial closure*, not a separate force law. §7 then shows that this and the rest of the dictionary are jointly forced.

7. Uniqueness constraints on admissible particle sectors

The dictionary is *not a free symbolic assignment* between topological classes and observed particles. The mapping is *forced* — overdetermined — by simultaneous substrate admissibility constraints.

7.1 The simultaneous admissibility constraints

The constraints fall into four distinct *layers*, which the substrate imposes simultaneously:

Kinematic constraints (per-cell Hilbert space):

- $\mathbb{C}^4 = \mathbb{C}^1 \oplus \mathbb{C}^3$ decomposition (Lemma GG2 of *One Fold*),
- the doublet decomposition at fixed orientation parity (§4.1),
- gauge group G_SM = SU(3) × SU(2) × U(1) as commutant — *inherited from the gauge-necessity programme, not re-derived here* (§2.4).

Topological constraints (substrate closure topology):

- closure admissibility (P1)–(P4) of the matter paper,
- triality conservation: isolated nontrivial triality is forbidden (Theorem 8.2),
- $\beta_1(D) \geq 1$ with admissibility-fixed homology classes.

Quantum-consistency constraints (on the U(1) sector):

- anomaly-free transport: $\Sigma\_f\ Y\_f = \Sigma\_f\ Y\_f^3 = \Sigma\_f\ T(R\_f)\ Y\_f = 0$ (§11.2).

Refinement-dynamical constraints:

- (P3) refinement persistence (no PFD with $\gamma\_D \geq 4$ survives),
- generation-depth bound $\gamma\_D \leq 3$.

No single layer alone forces the dictionary. *Their joint imposition does*: the kinematic layer fixes the Hilbert space and gauge group, the topological layer fixes which PFD types exist and how they organize into colour singlets and triplets, the quantum-consistency layer fixes hypercharge, and the refinement-dynamical layer bounds generations. Their intersection is, conjecturally, the observed Standard Model orbit content.

7.2 Structural requirement to forced consequence

Within the *fixed* substrate kinematic layer (G_SM inherited), the remaining constraints map directly to dictionary content:

Structural requirement	**Forced consequence**
Complete isolated admissibility	colour-singlet sector (lepton-like)
Partial admissibility only	confined SU(3) sector (quark-like)
Spinorial holonomy	fermionic exchange
One-sided SU(2) transport	chirality (L vs R asymmetry)
Triality conservation	three-channel baryonic closure
Refinement persistence	finite generation depth (γ_D ≤ 3)
Abelian anomaly freedom	unique U(1)_Y projection

Each row is not a choice. Each row is a *forced equivalence* given the fixed gauge group inherited from §2.4. The gauge group itself is *not* included in this table — it is upstream of the dictionary.

7.3 The Dictionary No-Alternatives Statement

Dictionary No-Alternatives Statement (conjectural / programme target): *The observed Standard Model orbit content is the unique simultaneous solution of the substrate admissibility constraints (§7.1) on PFD representation classes, given G_SM as inherited from the gauge-necessity programme.*

This is in the same epistemic category as the gauge-group uniqueness theorem of *One Fold* (which fixes G_SM upstream of this paper) and the *Uniqueness of the Minimal Distinction* paper. Those papers established that the gauge group and the minimal distinction are forced. The hexagonal-geometry programme's Theorem 3j.1 provides a further independent derivation of gauge-group uniqueness from closure-vertex axioms (see §2.11.6 for the layered-uniqueness picture). The present paper extends the no-alternatives logic from the *gauge group* to the *matter content carried under it*.

7.4 Epistemic positioning

A full *proof* of the Dictionary No-Alternatives Statement — closing it from conjecture to theorem — requires:

- substrate-level *derivation* (rather than imposition) of anomaly cancellation as an admissibility constraint,
- substrate-level *derivation* of γ_D ≤ 3,
- substrate-level *uniqueness proof* for the canonical ledger direction ω_Y,
- a substrate-level *matching theorem* establishing that the admissible PFD classes Φ_dict assigns coincide with the standard SM orbit content of G_SM on ⊗ ℋ_fold(x) (the classification of irreps of compact Lie groups is standard mathematics; the substrate-specific content is the matching between admissible PFD classes and the resulting orbit content, not the orbit classification itself).

These are explicit programme targets (§20.3). The argument of §7.1–§7.2 is *forcing-by-overdetermination* rather than *forcing-by-uniqueness-proof*. The former is already substantial; the latter is the natural target of the next stage of work.

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8. The colour sector — quarks as partial closure

8.1 Quark sector definition

A **quark-like PFD** is an admissible PFD with C_D = partial. By Theorem 6.7, every quark-like PFD has ρ_D = confined.

The internal image carries nontrivial $\mathbb{C}^3$ content. Under SU(3)_C, it transforms in the fundamental **3** or its conjugate $\bar{\mathbf{3}}$, determined by orientation parity π_D:

π_D = +1 ⟹ D ∈ **3** of SU(3)_C (quark), π_D = −1 ⟹ D ∈ $\bar{\mathbf{3}}$ of SU(3)_C (antiquark).

8.2 The isolated triality obstruction

Theorem 8.2 — Isolated Triality Obstruction *(proven, conditional on the substrate closure-conservation law of §2.10)*

Let D be a PFD carrying nontrivial $\mathbb{C}^3$ triality charge τ(D) ∈ $\mathbb{Z}_3$. If D admits isolated admissible transport, then the substrate closure-conservation law requires

τ(D) = 0 in $\mathbb{Z}_3$.

Therefore any isolated PFD with nontrivial triality violates admissibility.

Consequently:

- isolated quark-like sectors are *forbidden*,
- only triality-neutral composites admit persistent closure.

Proof

The $\mathbb{C}^3$ sector of $\mathbb{C}^4$ carries the centre $\mathbb{Z}_3 \subset$ SU(3) as a triality grading. By §2.10, admissible closure transport on Σ preserves a substrate analogue of Gauss's law for the $\mathbb{Z}_3$-graded source: every admissibly closed loop in Σ carries vanishing total $\mathbb{Z}_3$-graded enclosed source.

For an isolated PFD D, consider any admissibility-fixed closure loop γ encircling D. By additivity of the enclosed $\mathbb{Z}_3$-graded source over the region bounded by γ, the total enclosed $\mathbb{Z}_3$ charge equals τ(D) (with the substrate-conserved source contained entirely within supp(D) for an isolated PFD). The closure-conservation law of §2.10 then requires τ(D) = 0.

Contrapositively, if $\tau(D) \neq 0$, then no admissibility-fixed loop encircling D in isolation can satisfy the closure-conservation law, contradicting the assumption that D is admissibly transported in isolation. Therefore isolated admissible transport requires $\tau(D) = 0$. ▢

Corollary 8.2.1 — Confinement as Admissibility Obstruction

Confinement of quark-like PFDs is *not* a dynamical binding postulate; it is a topological admissibility obstruction. The continuum description of confinement — flux tubes, string tension, the QCD coupling running into the infrared — is the *emergent dynamical realisation* of this substrate-level obstruction.

8.3 Triality singlets

The substrate admits two minimal triality singlets:

- the three-channel singlet $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \supset \mathbf{1}$ (baryonic, §13),
- the quark–antiquark singlet $\mathbf{3} \otimes \bar{\mathbf{3}} \supset \mathbf{1}$ (mesonic).

8.4 Fractional ledger charge

Quark-like PFDs carry fractional ledger charge by a one-step argument from triality. Among the two minimal triality singlets of §8.3, only the all-quark baryonic singlet $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \supset \mathbf{1}$ carries pure-quark ledger circulation; the mesonic singlet $\mathbf{3} \otimes \bar{\mathbf{3}} \supset \mathbf{1}$ is triality-neutral by quark–antiquark cancellation rather than by accumulation, and therefore does not constrain the quark ledger projection. The minimal admissibly closed circulation in pure-quark ledger content therefore involves three triality steps (the baryonic singlet). Each triality element accordingly carries 1/3 of the minimal admissibly closed ledger circulation. The factor of 1/3 in quark hypercharge is the substrate-level image of this triality projection.

Given this projection, anomaly cancellation (§11.2) then fixes the specific Y values across the SU(2) doublet/singlet structure:

$Y(q_L) = +1/3$, $Y(u_R) = +4/3$, $Y(d_R) = -2/3$.

The factor 1/3 is the substrate-level *consequence* of triality projection; the specific values are the *consequence* of anomaly cancellation under that projection. Neither is independently posited.

8.5 Quark mass — confinement-inseparable

$m_q \approx m_q^{(\text{intrinsic})} + m_q^{(\text{confinement})}$,

with $m_q^{(\text{confinement})}$ dominating for the light quarks and $m_q^{(\text{intrinsic})}$ becoming significant only for the heavy quarks. *(Conditional on the confinement programme.)*

8.6 Confinement reinterpreted

Confinement is not a force law. It is admissibility failure of isolated partial closure.

The dynamical flux-tube picture of QCD is the *emergent* description of how this substrate-level admissibility failure manifests in the continuum.

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9. The weak sector — chirality from SU(2) restriction

9.1 SU(2) action on the orientation sector

SU(2) acts on the $\mathbb{C}^2$ spinorial sector that arises from the substrate $\mathbb{Z}_2$ direction label after dimensional emergence. Distinct from the colour $\mathbb{C}^3$ sector, the SU(2) sector carries the chirality structure of the dictionary.

9.2 Chirality from orientation parity selection

The defining feature of the weak sector is that SU(2) acts faithfully on only one orientation parity:

SU(2) acts faithfully on π_D = +1 (the χ_D = L sector), SU(2) acts trivially on π_D = −1 (the χ_D = R sector).

The substrate-level reason: BC1-compatible distinguishability transport forbids simultaneous independent SU(2) transport currents on both Weyl sectors. The *One Fold* construction selects one orientation as the carrier of non-Abelian SU(2) holonomy; the other becomes an SU(2) singlet by construction.

This is the substrate origin of chirality. The Standard Model does not need to *posit* that SU(2) is chiral; the substrate forces it.

9.3 The L/R labelling — empirical, not structural

The *structural* content is that exactly one orientation parity carries faithful SU(2) action. *Which* of the two parities is labelled "L" rather than "R" is an empirical orientation identification. The substrate produces the asymmetry; the convention "L" attaches to the orientation that matches observed weak interactions.

9.4 Doublet structure

PFDs in the χ_D = L sector carry SU(2) doublet structure:

(ν_L, e_L) ↔ doublet of leptonic PFDs with χ_D = L, (u_L, d_L) ↔ doublet of quark-like PFDs with χ_D = L.

9.5 Singlet structure

PFDs in the $\chi_D = R$ sector carry no SU(2) action:

e_R, u_R, d_R ↔ SU(2) singlets with $\chi_D = R$.

Whether a ν_R PFD exists with $\chi_D = R$ is the substrate-level form of the right-handed neutrino question. *(Open; see §12.2.1.)*

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10. Spin-statistics in the dictionary

The matter paper's Theorems 7.1 and 7.2 establish the spin-statistics structure at the substrate level. This section applies those results to the PFD classes of the dictionary.

10.1 Spinorial transport on the dictionary's matter PFDs

A matter PFD D carries spinorial transport when its admissibility-fixed homology class $[\gamma]$ generates orientation holonomy acting faithfully on the $\mathbb{Z}_2$ direction label after dimensional emergence (matter paper Theorem 7.1). For such PFDs,

$$U_{2\pi}(D) = -\mathbb{1}$$

on the spinorial sector, and the transport representation is spinorial.

All matter PFDs identified by this dictionary — leptons, quarks, baryon constituents — are spinorial: PFDs with admissibility-fixed homology and faithful $\mathbb{Z}_2$ orientation transport are exactly those that satisfy (P1)–(P4) on the orientation sector.

10.2 Fermionic exchange from exchange-path holonomy

For two spinorial PFDs of the same closure type, exchange in the emergent 4-manifold is implemented by braiding their world-tubes within a fixed spatial slice. The Finkelstein–Rubinstein construction supplies the operative homotopy: the exchange path is homotopic, in emergent 3-space, to a path in which one PFD remains fixed while the other undergoes a 2π rotation of its orientation frame.

The exchange-sign sector is $\mathbb{Z}_2$-valued, $U_{\text{exch}} \in \{\pm\mathbb{1}\}$, so homotopy invariance is automatic in this sector. The exchange path and the 2π-rotation path carry the same sign, and matter paper Theorem 7.2 then gives

$$U_{\text{exch}} = U_{2\pi} = -\mathbb{1}$$

on the spinorial sector.

FR for extended PFDs. The Finkelstein–Rubinstein construction is canonically stated for point particles in $\mathbb{R}^3$. For *extended* PFDs, framed-cobordism subtleties enter. We invoke the construction here under the localization assumption: at scales large compared to the PFD localization length L_D — the spatial extent of the closure defect's support, equivalently the closure-Hessian correlation length of matter paper §6.3 — each PFD is effectively point-like in the emergent 3-space and the FR argument applies on the centre-of-mass world-line. For any standard-model fermion L_D is at most the Compton-wavelength scale of the corresponding matter sector, which is far below the de Broglie wavelength relevant to exchange in any physical scattering regime; the localization assumption is therefore not in tension with experimentally accessible kinematics. A full extended-framed-cobordism treatment is deferred to the PFD second-quantisation companion paper.

10.3 Spin-statistics is forced, not imposed

The dictionary therefore inherits the substrate spin-statistics result:

Fermionic antisymmetry emerges from exchange-path holonomy on the spinorial closure sector, not from an axiom of quantum field theory imposed by hand.

The conventional spin-statistics theorem of relativistic quantum field theory appears at the substrate level as a *consequence* of the spinorial transport structure of orientation-faithful PFDs.

10.4 Bosonic composites and exchange of whole composites

The exchange operation for composite PFDs requires unpacking. Exchanging *whole composites* X ↔ X' resolves into pair-exchanges of their constituents: if $X = D_1 \otimes \ldots \otimes D_n$ and $X' = D_1' \otimes \ldots \otimes D_n'$ are composites of the same type, exchanging X ↔ X' is implemented by n simultaneous pair-exchanges $D_i \leftrightarrow D_i'$ for i = 1, ..., n. Each pair-exchange contributes a factor of $U\_exch = -\mathbb{1}$ on the joint spinorial sector by §10.2.

Mesons ($\mathcal{M} = D\_q \otimes D\_\bar{q}$, n = 2) therefore carry composite exchange sign $(-\mathbb{1})^2 = +\mathbb{1}$: they are bosonic.

Baryons ($\mathcal{B} = D_1 \otimes D_2 \otimes D_3$, n = 3) carry composite exchange sign $(-\mathbb{1})^3 = -\mathbb{1}$: they are fermionic.

This recovers the standard meson-vs-baryon statistics distinction at the substrate level.

10.5 CAR algebra and Fock structure

A full reconstruction of the canonical anticommutation relation (CAR) algebra and Fock structure on the space of multi-PFD configurations requires the PFD second-quantisation companion paper. The spin-statistics result of §10.2–§10.3 is the substrate-level prerequisite for that construction.

11. The hypercharge sector — U(1)_Y as oriented closure-ledger invariant

11.1 The ledger structure

Every admissible PFD carries an oriented closure-circulation invariant ℓ_D taking values in a free abelian group $\mathscr{L}$. The ledger captures the net oriented circulation of distinguishability around the admissibility-fixed closure of D.

11.2 Anomaly cancellation as Abelian uniqueness condition

The substrate admissibility constraints impose that a U(1) gauge field admits a consistent quantum theory only if its charge assignments satisfy

$\Sigma_f Y_f = 0$, $\Sigma_f Y_f^3 = 0$, $\Sigma_f T(R_f) Y_f = 0$ (for each non-Abelian factor R_f),

summed over each generation. These are the substrate-level consistency conditions for admissible transport in the presence of both Abelian and non-Abelian sectors.

The hypercharge direction $\omega_Y \in \mathscr{L}$ is uniquely determined (up to overall sign and rescaling) by the requirement that the standard fermion content of one generation satisfies these conditions simultaneously. This is **Abelian uniqueness**: of all directions in $\mathscr{L}$ on which U(1) could act, only ω_Y is consistent with admissible transport.

11.3 Hypercharge assignments

$Y(D) = \langle \ell_D, \omega_Y \rangle$

(with overall normalisation conventional; the K = 7 Wilson Limit referenced in earlier drafts and in upstream papers only fixed this overall convention and adds no derivable content beyond what anomaly cancellation already supplies). The standard-generation assignments:

PFD class	Y
(ν_L, e_L)	−1
e_R	−2
(u_L, d_L)	+1/3
u_R	+4/3
d_R	−2/3

(Y values shown in the standard $Q = T_3 + Y/2$ convention; see §11.5.)

These values are *jointly forced*: the triality projection of §8.4 fixes the fractional structure (the 1/3 for quark-like PFDs), and the anomaly conditions of §11.2 fix the specific Y values across the SU(2) doublet/singlet structure.

11.4 Why U(1) and not a larger abelian group

Larger abelian factors fail anomaly cancellation under the substrate fermion content; smaller ones (discrete subgroups) fail to support continuous closure transport. U(1)_Y is therefore *forced*, not selected.

11.5 Electric charge

$Q = T_3 + Y/2$,

with T_3 the third component of weak isospin. Q is a *derived* quantity, not a primitive.

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12. Lepton classes

12.1 Charged leptons

A **charged lepton PFD** has:

- C_D = complete (singlet under SU(3)_C),
- nontrivial ledger charge ℓ_D ≠ 0,
- π_D ∈ {+1, −1} (both orientations realised),
- γ_D ∈ {1, 2, 3}.

The three generations are (e, μ, τ). Within each generation, SU(2) doublet structure (§9.4) produces a left-handed (ν, ℓ^-)_L pair and the SU(2) singlet ℓ^-_R.

Mass hierarchies follow 𝒟 = diag(1, 2, 4) (§16). *(Conditional / schematic.)*

12.2 Neutrinos

A **neutrino PFD** has:

- C_D = complete,
- ℓ_D = 0 (no ledger charge — electromagnetically neutral),
- χ_D = L only, or χ_D ∈ {L, R} (if ν_R exists as a PFD class),
- π_D = +1 (orientation-trivial),
- γ_D ∈ {1, 2, 3}.

Open question 12.2.1 — Two distinct neutrino questions

Two questions about the neutrino sector are conflated in casual discussion but are distinct at the substrate level. The dictionary distinguishes only the first directly:

Question A — Does ν_R exist as a PFD class? This is the substrate-structural question of whether the χ_D = R orientation parity admits a complete, ℓ_D = 0 PFD on the neutrino orbit type. The dictionary developed here is compatible with both answers. If the answer is yes, the neutrino sector admits a Dirac-type mass; if no, no Dirac-type mass is admissible.

Question B — Is the neutrino Majorana? This is the question of whether the neutrino is its own antiparticle. It is logically *independent* of Question A: type-I seesaw scenarios have ν_R present (Question A: yes) but the neutrino mass is Majorana (Question B: yes); pure Dirac scenarios have ν_R present (Question A: yes) and Majorana mass absent (Question B: no); a no-ν_R scenario answers Question A negatively and forces Majorana mass via a different mechanism if neutrinos have mass at all.

Empirically, neutrinoless double beta decay (0νββ) probes Question B (Majorana mass), not Question A directly. Observation of 0νββ would establish a Majorana mass component but would not by itself resolve the Question A PFD existence question — that requires direct substrate-level structural inference about which orientation parities are realized.

The substrate-level mechanism by which the neutrino acquires mass — and therefore the prediction about 0νββ — depends on the closure-norm and substrate Higgs structure (§19.2), not on the present dictionary alone. The dictionary fixes Question A as a substrate-structural open problem; Question B is downstream of both Question A and the closure-norm mechanism.

12.3 Neutrino mass

The vanishing of ℓ_D removes the U(1) contribution to the mass expression. The dictionary predicts neutrino mass is *smaller* than charged lepton mass within the same generation, by reduction of the contributing mass channels (matter paper §6.5). A *parametric* scaling argument requires the closure-cost coefficients of §19.3 and the substrate Higgs programme; the present paper does not derive it. *(Schematic.)*

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13. Baryonic and mesonic composite closure

13.1 Three-channel $\mathbb{C}^3$ singlet

A **baryon PFD** is a composite of three quark-like PFDs whose joint $\mathbb{C}^3$ content forms an SU(3)_C singlet:

$\boldsymbol{\mathcal{B}} = D_1 \otimes D_2 \otimes D_3$, with $[\boldsymbol{\mathcal{B}}] \in \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \supset \mathbf{1}$.

The "three" comes from the dimension of the $\mathbb{C}^3$ sector of $\mathbb{C}^4$.

13.2 Closure completeness of baryons

A baryon is a *complete* PFD in the composite sense. The triality charges sum to zero modulo 3 (Theorem 8.2), restoring closure conservation.

13.3 Mesons

A **meson PFD** is a quark-antiquark composite:

$\mathcal{M} = D_q \otimes D_{\bar{q}}$, with $[\mathcal{M}] \in \mathbf{3} \otimes \bar{\mathbf{3}} \supset \mathbf{1}$.

Mesons are bosonic by §10.4.

13.4 Exotic composites

The substrate admits in principle:

tetraquark : $\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \supset \mathbf{1}$, pentaquark : $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \supset \mathbf{1}$, glueball : pure colour-singlet bound state of gauge-mode propagation.

Whether these are *stable* PFDs is open. *(Conditional.)*

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14. Stabilization-depth dynamics and composite closure stabilization

Baryons are forced by triality conservation to be three-channel composites (§13). But triality conservation alone does not specify *how* the composite stabilizes. The dynamical stabilization mechanism is supplied by the *stabilization-depth dynamics* of the upstream proton/baryon paper.

The depth-variable in §14 — *stabilization depth* — is *distinct from* the *generation depth* γ_D of §16. Both are layered closure-depth notions but act at different layers of the substrate hierarchy. The §14 mechanism governs composite admissibility contraction; the §16 mechanism governs single-PFD refinement-stable closure excitations.

14.1 TPB-constrained admissibility contraction

The proton/baryon paper establishes that baryon stabilization proceeds via an iterative admissibility-filtering process in the residual closure space. Each TPB tick contracts the residual space by a fixed factor:

$R_{n+1} = \eta R_n$, $\eta = 3/5$,

acting on the five-dimensional residual closure space. The iteration converges in finitely many steps to a stable composite closure structure. The variable controlling the iteration is *stabilization depth*: the number of contraction steps the composite has undergone.

14.2 Iterative admissibility filtering

The mechanism is:

- at each TPB tick, admissibility constraints are re-imposed on the composite,
- inadmissible configurations contract out at the rate $\eta = 3/5$,
- the iteration is iterative admissibility filtering, not phenomenological fitting,
- the fixed point of the iteration is the stable composite.

The contraction ratio $\eta = 3/5$ and the residual-space dimension are both *derived* in the upstream proton paper from substrate primitives — finite distinguishability, TPB locality, and the $K = 7$ closure architecture — and are not adjustable parameters.

14.3 Consequence for the dictionary

Baryons are dynamically stabilized closure structures whose confinement depth emerges from TPB-constrained admissibility contraction, not from phenomenological fitting.

This is the substrate-level form of the statement that QCD does not require external parameter inputs to produce bound hadrons: the stabilization-depth dynamics produces them by iterative admissibility, and the parameter values η and R_0 are forced by substrate primitives.

14.4 Relation to generation depth

The stabilization-depth dynamics of §14.1 is conceptually distinct from the generation-depth structure of §16 (which is about radial excitation sectors of *individual* PFDs). The dynamical stabilization of §14 is about *composite* baryon admissibility contraction; the generation structure of §16 is about *single-PFD* refinement-stable closure sectors. The unified treatment of both layers is a programme target. *(Conditional on the proton/baryon paper.)*

15. Gauge boson sectors

The type distinction of §5 already established that gauge bosons are not PFDs. This section identifies the propagation-mode content of each gauge factor.

15.1 Photon

The **photon** is the closure-preserving propagation mode that carries the unbroken U(1)_EM transport on the emergent geometric structure after electroweak symmetry breaking. Zero ledger charge, zero SU(3) content, exact admissibility under refinement flow. *(Conditional on the substrate Higgs programme: the identification of U(1)_EM as the unbroken combination of U(1)_Y and the diagonal SU(2)_L generator depends on the electroweak-breaking mechanism.)*

15.2 W and Z bosons

The **W± and Z bosons** are propagation modes of the broken SU(2)_L × U(1)_Y combinations. Their masses arise from the substrate Higgs mechanism via the closure-condensate structure of the vacuum (§19.2). *(Conditional on the substrate Higgs programme.)*

15.3 Gluons

The **gluons** are propagation modes of SU(3)_C transport carrying colour-octet structure (**8** of SU(3)). They are themselves charged under colour — the substrate origin of QCD's non-Abelian self-interaction. Gluons are confined by the same admissibility law (Theorem 8.2) that confines quarks.

15.4 Why no graviton in the dictionary

Gravity is not a propagation mode of a gauge connection in VERSF. It is the dynamics of the emergent metric g_μν = (1/λ★) Φ_μν, with Φ_μν the fundamental commitment field sourced by PFDs. The substrate does not contain a "graviton PFD"; the dictionary developed here is silent on the spin-2 quantum question.

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16. Generations and generation-depth transport geometry

16.1 Three generations from generation depth

The substrate carries exactly three admissible generation-depth excitation sectors, indexed by γ_D ∈ {1, 2, 3}. The bound γ_D ≤ 3 is the substrate-level constraint that PFDs with γ_D ≥ 4 fail (P3): the higher generation-depth excitations are not refinement-stable. *(Conditional on the flavour-mixing programme.)*

16.2 Generation-space stiffness operator

The flavour-mixing programme derives the generation-space stiffness operator

$$\mathcal{D} = \mathrm{diag}(1, 2, 4),$$

acting on the three-dimensional generation space. The eigenvalues set the generation-depth stiffness contrasts between generations.

16.3 Generation structure within each particle class

Each particle class admits three generation copies:

charged leptons: e, μ, τ ↔ γ_D = 1, 2, 3, neutrinos: ν_e, ν_μ, ν_τ, up-type quarks: u, c, t, down-type quarks: d, s, b.

16.4 Transport geometry and the CKM/PMNS matrices

Generation-depth eigenstates do not in general coincide with mass eigenstates or with SU(2)_L doublet eigenstates. The off-diagonal projection between these two bases is the substrate-level origin of the CKM (quark) and PMNS (lepton) mixing matrices.

The flavour-mixing programme derives a conditional CKM hierarchy from a transport-attenuation factor associated with each cross-generation projection. The transport-attenuation operator acts as a generation-depth attenuation between adjacent generations, with the structure forced by the stiffness contrasts of 𝒟.

Generations correspond to transport-separated generation-depth sectors within the persistent admissibility geometry.

The substrate origin of the CKM hierarchy is therefore not a list of free Yukawa couplings but a geometric attenuation in the residual closure space. *(Conditional on the flavour-mixing programme.)*

16.5 Mass hierarchies

The mass of a PFD scales with its generation-depth eigenvalue under 𝒟:

$m(\gamma=1) < m(\gamma=2) < m(\gamma=3)$.

Quantitative mass predictions require the matter paper §6.5 bridge constants. *(Schematic / conditional.)*

16.6 Why exactly three generations

Three is the number of admissible generation-depth excitations on the substrate that satisfy (P3) refinement persistence.

The bound is forced by 𝒟 having exactly three persistent eigenvalues (1, 2, 4). *(Conditional on the flavour-mixing programme.)*

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17. Matter–geometry coupling

The matter paper established that PFDs source the committed record field ρ, which sources the fundamental commitment field $\Phi_{\mu\nu}$, which sources Einstein curvature through the quadratic-in-Φ stress-energy $T^{(\Phi)}_{\mu\nu}$.

For the dictionary developed here:

PFDs (this paper) → $\rho(x, t) \rightarrow \Phi_{\mu\nu} \rightarrow T^{(\Phi)}_{\mu\nu} \rightarrow G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T^{(\Phi)}_{\mu\nu}$.

Each particle class contributes to ρ in proportion to its closure-density structure (matter paper §6.1). Gauge bosons contribute only via their kinetic content in the propagation sector. The dictionary closes the substrate genealogy at both ends: matter classes are identified at the representation level (§§8–14), and matter classes source emergent geometry at the gravitational level (this section).

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18. The full dictionary tables

Table A — Particle-class summary

Sector	PFD type	Closure status	Representation
Electron	Minimal complete closure	Stable isolated	SU(3)-singlet, SU(2) doublet/singlet by χ
Neutrino	Weak-sector closure, $\ell_D = 0$	Stable isolated	SU(3)-singlet, SU(2) doublet
Charged lepton (μ, τ)	Higher-γ charged lepton	Stable isolated	as electron, with $\gamma_D = 2, 3$
Quark	Partial closure	Requires confinement	SU(3)-triplet
Proton/Baryon	Composite three-channel closure	Colour singlet	SU(3)-singlet composite
Meson	Composite quark–antiquark	Colour singlet	SU(3)-singlet composite
Photon	Closure-preserving mode	Non-persistent (not a PFD)	$U(1)_{EM}$ transport
W±, Z	Closure-preserving mode	Non-persistent, massive	$SU(2)_L \times U(1)_Y$ transport
Gluon	Closure-preserving mode	Non-persistent, confined	$SU(3)_C$ transport

Table B — Full invariant-tuple projection (one generation)

Settled rows (standard SM matter content):

PFD class	C_D	π_D	χ_D	ℓ_D / N_Y	(SU(3), SU(2))	Y	Particle
L_L	C	+1	L	⟨−1⟩	(**1**, **2**)	−1	(ν_L, e_L)
e_R	C	−1	R	⟨−2⟩	(**1**, **1**)	−2	e_R
Q_L	P	+1	L	⟨+1/3⟩	(**3**, **2**)	+1/3	(u_L, d_L)
u_R	P	−1	R	⟨+4/3⟩	(**3**, **1**)	+4/3	u_R
d_R	P	−1	R	⟨−2/3⟩	(**3**, **1**)	−2/3	d_R

Open row (PFD existence not yet settled — see §12.2.1):

PFD class	C_D	π_D	χ_D	ℓ_D / N_Y	(SU(3), SU(2))	Y	Particle
ν_R	*C*	*−1*	*R*	*⟨0⟩*	*(1, 1)*	*0*	*ν_R (open)*

(C = complete, P = partial; ⟨·⟩ denotes the ledger projection onto ω_Y.)

For γ_D = 2 and γ_D = 3, replace (ν_e, e, u, d) with (ν_μ, μ, c, s) and (ν_τ, τ, t, b). The two tables together exhaust the standard one-generation matter content of the Standard Model, modulo the open ν_R PFD existence question (§12.2.1, Question A).

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19. Structural implications

19.1 Why the particle spectrum terminates

The dictionary predicts a *finite* low-energy particle spectrum. Finiteness, not just discreteness, follows from explicit substrate-level bounds on each component of 𝓙(D):

- **C_D ∈ {complete, partial}** — 2 values,
- **π_D ∈ {+1, −1}** — 2 values,
- **χ_D ∈ {L, R, none}** — 3 values,
- **γ_D ∈ {1, 2, 3}** — 3 values (bound by refinement persistence, §16.6),
- **ledger projection ⟨ℓ_D, ω_Y⟩** — constrained by anomaly cancellation to a finite set per generation (§11.2),
- **C_D = partial sub-classification under SU(3)** — finite (the triplet **3** and antitriplet **3̄**, distinguished by π_D, already counted above).

The conjunction of these component-wise finite bounds gives a finite total number of admissible PFD orbit types. *Discreteness* of the orbit space (e.g. integer-labelled winding sectors) would not by itself give finiteness; the additional finite bounds on each invariant component do.

The substrate admits a finite number of stable representation classes — and therefore a finite low-energy particle spectrum — rather than an unbounded particle zoo.

The Standard Model is *small* because the substrate forces it to be small. This contrasts sharply with many beyond-Standard-Model frameworks in which representation proliferation is structurally unconstrained.

19.2 Closure norm and representation stability

The dictionary classifies which PFD classes are *admissible*. The *energetic stiffness* of those classes under closure-norm perturbation is classified by the substrate Higgs derivation developed in the hexagonal-geometry companion paper (*A Unified Derivation of Closure Geometry, Gauge Redundancy, and Mass Structure in the Hexagonal Framework*, Theorem 6 and §5):

- **representation structure** (this paper) determines *which closure sectors may exist*,
- **closure-norm stiffness** (hexagonal companion paper, Theorem 6) determines *the energetic persistence and effective mass scale of those sectors*, with the bound $M_H^2 \geq N_{scalar} \cdot (M_W^2 + M_Z^2)$ and saturation at leading order via the Schur-complement argument on the closure stiffness matrix.

Gauge representation, confinement, and mass generation are three faces of the same closure machinery:

closure topology → representation (this paper), closure conservation → confinement (Theorem 8.2), closure-norm stiffness → mass (hexagonal companion paper, Theorem 6).

The hexagonal companion paper's derivation works in hexagonal-tiling vocabulary (the closure-norm scalar $\rho(x) = |C(x)| - 1$, with channel-counting normalisation Lemma 5 giving $K_{RR} = [(2K+1)/(2K)] \cdot \mathrm{Tr}(K_{AA})$). A direct re-derivation in PFD/dictionary vocabulary — establishing the radial mode of $\mathcal{J}^{int}(D)$ and the corresponding stiffness eigenvalue — would establish the same result within the dictionary's substrate language. This is one of the open cross-programme identification problems flagged in §20.3.

The Standard Model particle spectrum is the stable representation manifold of persistent closure on the substrate.

19.3 Mass as closure cost

The closure-norm bridge admits a schematic restatement at the energy level:

$m_D \sim E_{closure}(D)$,

where $E_{closure}(D)$ is the energetic cost of maintaining persistent closure against admissible perturbation. Four contributions enter (matter paper §6.5): commitment-density loading, closure-Hessian stiffness, confinement/localization cost, and persistent distinguishability content.

In dictionary terms:

- **stabilization cost** — the cost of holding the PFD topology against admissible perturbation,
- **confinement layering** — for partial closures, the additional cost imposed by composite stabilization (§14),
- **closure persistence** — the cost of being refinement-stable,
- **distinguishability maintenance** — the Landauer-type cost of holding admissibility-fixed folds.

Mass corresponds to the energetic cost of being a stable persistent closure rather than dispersing into the reversible substrate sector.

The m ~ E_closure form is not a derivation of any specific mass; it is the relationship through which the closure architecture *produces* mass at all.

20. Epistemic status and open problems

20.1 What this paper has done

This paper has constructed the representation-level map

Φ_dict : { admissible PFD classes } ⟶ { orbit types of G_SM on $\mathscr{H}$_fold }

between PFDs and Standard Model matter sectors. The construction is forced in the sense that each component is required either by substrate primitives or by admissibility consistency. §7 made the joint forcing — the no-alternatives logic — explicit.

The dictionary identifies the substrate signatures of:

- the matter / gauge-boson type distinction,
- the lepton/quark distinction (closure completeness),
- chirality (orientation parity restriction in SU(2)),
- colour ($\mathbb{C}^3$ sector content),
- hypercharge (oriented ledger projection + anomaly cancellation),
- generations (generation-depth excitations),
- spin-statistics (substrate $\mathbb{Z}_2$ direction label, via §10),
- confinement (substrate triality conservation, Theorem 8.2),
- baryon stabilization-depth dynamics (§14),
- finiteness of the spectrum (§19.1),
- forcing of the dictionary (§7),
- energetic cost structure of mass (§19.3).

20.2 What this paper has not done

The dictionary is representation-level. It does not deliver:

- exact masses for any particle,
- mixing angles for either CKM or PMNS,
- scattering amplitudes,
- a substrate derivation of the electroweak symmetry-breaking scale,
- the resolution of the ν_R PFD existence question (§12.2.1, Question A),
- the resolution of the Majorana mass question (§12.2.1, Question B),
- a full *proof* (rather than conjecture) of the Dictionary No-Alternatives Statement.

20.3 Open problems specific to the dictionary

- **Proof of the Dictionary No-Alternatives Statement** (§7.3). Requires: substrate-level derivation of anomaly cancellation; derivation of γ_D ≤ 3; uniqueness proof for ω_Y; matching theorem between admissible PFD classes and SM orbits under G_SM.
- **Exact quark mass intrinsic vs confinement split** (§8.5).
- **PMNS structure** (§16.4).
- **ν_R PFD existence** (§9.5, §12.2.1, Question A) — distinct from the Majorana question.
- **Majorana mass mechanism for the neutrino** (§12.2.1, Question B) — depends on the closure-norm and Higgs structure.
- **Exotic stable composites** (§13.4).
- **The L↔R empirical identification** (§9.3).
- **Closure-depth layering unification** (§14.4) — unified treatment of stabilization depth and generation depth.

Cross-programme unification with the hexagonal-geometry framework (§2.11). The hexagonal-geometry programme (and especially its rigorisation companion *A Unified Derivation of Closure Geometry, Gauge Redundancy, and Mass Structure in the Hexagonal Framework*) provides numerical-derivation theorems whose substrate identifications with dictionary objects remain to be proven:

- **$\mathbb{C}^3$ identification.** The hexagonal-programme $\mathbb{C}^3$ (three triangle-pair channels, its §3j and Appendix G) and the dictionary's $\mathbb{C}^3$ (the $\mathbb{C}^3$ block of $\mathbb{C}^4 = \mathbb{C}^1 \oplus \mathbb{C}^3$ from *One Fold* Lemma GG2, used in §4.1) are *claimed* to be the same $\mathbb{C}^3$ but not yet proven equivalent. A direct mapping between triangle-pair channels and *One Fold* internal-image basis vectors would close this.
- **15/14 appearance in the dictionary.** The hexagonal programme makes (2K+1)/(2K) = 15/14 a universal loop-correction factor across all hexagonal-vacuum-mediated processes (its §5, Theorem 6, Lemma 5). The dictionary does not currently carry this factor anywhere; the natural place would be the closure-cost scaffold of §19.3. Either the dictionary's E_closure(D) inherits 15/14 from the same channel-counting structure (in which case §19.3 should reflect this), or 15/14 is not universal in the way the hexagonal programme claims.
- **Level-4 defect ↔ PFD identification.** The hexagonal programme classifies matter as "level-4 5-7 defects" in the four-level hierarchy (its §7, Theorem 7). The dictionary classifies matter as PFDs with $\beta_1(D) \geq 1$ satisfying (P1)–(P4). These are *almost certainly*

the same objects in different vocabularies, but the identification is not proven. A theorem stating "every level-4 5-7 defect is a PFD with the corresponding $\mathcal{J}$(D), and conversely" would establish the equivalence.

Closing these three identifications would unify the hexagonal-programme's numerical derivations with the dictionary's representation-theoretic classification at the substrate level. Such a unification is the natural target of a future dictionary-side companion paper paralleling the hexagonal companion paper structurally.

20.4 Epistemic colour-coding

- **Forced by substrate primitives** (proven): G_SM (inherited, §2.4); matter / gauge-boson type distinction; lepton/quark/baryon distinction; triality conservation (Theorem 8.2, conditional on §2.10); closure-confinement equivalence (Theorem 6.7); three orientation sectors; γ_D ≤ 3.
- **Conditional on prior programmes**: chirality assignment (§9.2, *One Fold*); anomaly cancellation route to Abelian uniqueness (§11.2); generations as generation-depth (§16, flavour-mixing programme); baryon stabilization-depth dynamics (§14, proton/baryon paper); meson exchange structure (§13.3, matter paper Theorem 7.2); substrate $\mathbb{Z}_3$ closure-conservation law (§2.10, BCB transport programme).
- **Conjectural at programme level**: Dictionary No-Alternatives Statement (§7.3); joint substrate finiteness theorem (§19.1, given component bounds).
- **Schematic / not derived**: explicit mass values; explicit CKM and PMNS entries; intrinsic/confinement mass split; substrate Higgs mechanism details; closure-cost coefficients of §19.3; parametric scaling of neutrino mass.
- **Empirical**: L vs R labelling (§9.3).

20.5 Falsification criteria

A framework that does not specify what would falsify it is not structurally testable. The dictionary would *fail* under any of the following observations:

1. **Stable isolated colour-triplet states propagating at distances large compared to the confinement scale.** Theorem 8.2 forbids this. (Asymptotic freedom permits quasi-free quark behaviour at short distances, which is *not* a falsifier; the substrate claim is about stable isolated propagation at distance scales above the confinement scale.) A genuine observation of stable free colour-triplet states would refute the substrate triality-conservation law.
2. **Observation of more than three refinement-stable generations.** A fourth-generation set with the standard charge assignments would refute §16.6 and γ_D ≤ 3. (Heavy unstable particles that do not fit the generation pattern would not be falsifiers; the claim is about refinement-stable generation copies of the standard fermion content.)
3. **Detection of anomaly-inconsistent hypercharge sectors.** Observation of a fundamental fermion with hypercharge incompatible with anomaly cancellation under the substrate fermion content would refute §11.2.

4. **Stable particle sectors requiring closure classes outside the admissible PFD invariant structure.** A stable matter sector that does not fit 𝕴(D) would refute the representation map.
5. **Detection of an extra fundamental U(1) factor irreducibly required by observed phenomenology.** §11.4 forces U(1)_Y as the unique abelian factor; an independent additional U(1) at the fundamental level would refute Abelian-uniqueness.
6. **Specific lattice-QCD predictions about the substrate-to-continuum bridge.** If precision lattice-QCD measurements of the confinement scale, glueball spectrum, or pure-gauge string tension turn out to be inconsistent with the substrate triality-conservation interpretation of confinement (specifically, if the continuum confinement mechanism turns out to be irreducibly *non-topological*), the substrate-to-continuum bridge for confinement would be broken. A precise falsification target requires the substrate confinement programme to produce a lattice-discriminable prediction, which is a programme target rather than a present statement.

(Parity violation is *retrodicted* by §9.2, not forward-predicted: the framework correctly retrodicts that SU(2) acts on one orientation parity only, in agreement with observed weak interactions. An empirical scenario in which SU(2) acted symmetrically on both parities would have refuted the framework had this been observed; it has not been.)

The framework is structurally falsifiable, not merely interpretive.

20.6 Where the next paper should go

- **Dictionary-side companion to the hexagonal companion paper**: a rigorisation pass in PFD/dictionary vocabulary paralleling *A Unified Derivation of Closure Geometry, Gauge Redundancy, and Mass Structure in the Hexagonal Framework*, closing the three cross-programme identification problems of §20.3 ($\mathbb{C}^3$ identification, 15/14 universality within the dictionary, level-4 defect ↔ PFD identification).
- **PFD second-quantisation paper**: full CAR/Fock structure (§10.5), extended-PFD FR (§10.2).
- **ν_R PFD existence companion paper**: substrate-level resolution of §12.2.1 Question A.
- **Proof of the Dictionary No-Alternatives Statement** (highest-stakes target): closing §7.3 from conjecture to theorem.

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21. Conclusion

The VERSF and BCB programmes have progressively reconstructed gauge structure, geometry, gravity, and matter ontology from distinguishability, closure, and topology. The matter paper closed the substrate genealogy

Void → Fold → Fact → Persistent Closure → Matter → Geometric structure → Gravity, and identified the dictionary between PFD invariants and Standard Model representations as the defining open problem of the next stage.

The present paper has developed that dictionary.

The central result is structural:

Standard Model particle sectors are the unique stable representation classes of Persistent Fold Defects on the committed distinguishability substrate.

Matter particles are stable closure defects. Gauge bosons are closure-preserving propagation modes. Leptons are complete non-colour PFDs. Quarks are partial colour-carrying PFDs that fail isolated admissibility (Theorem 8.2) and require composite stabilisation. Baryons are three-channel $\mathbb{C}^3$ singlets, dynamically stabilized via $R_{n+1} = (3/5) R_n$ stabilization-depth iteration; mesons are quark–antiquark singlets. Hypercharge is the oriented closure-ledger projection selected by anomaly cancellation. Weak chirality arises from the substrate restricting SU(2) transport to one orientation parity. Generations are transport-separated generation-depth sectors with stiffness $\mathcal{D} = \mathrm{diag}(1, 2, 4)$ and bound $\gamma_D \leq 3$. Spin-statistics is forced by exchange holonomy. Confinement is admissibility failure of isolated partial closure. Mass corresponds to the energetic cost of being a persistent stable closure rather than dispersing.

The dictionary is *not free*. §7 made the no-alternatives logic explicit. §19.1 made the consequent finiteness explicit. §19.2 connected the dictionary to the hexagonal-geometry companion paper's substrate Higgs derivation. §19.3 made the mass-energy relationship explicit. §20.5 stated falsification criteria. Appendix A maps the dependency structure.

The resulting genealogy is complete from end to end:

Void → Fold → Persistent Closure → Representation Structure → Matter → Geometric structure → Gravity.

The deepest implication of the dictionary is not merely that particles may be represented as Persistent Fold Defects. It is that the Standard Model ceases to appear as a disconnected catalogue of particles and charges.

Colour, confinement, chirality, generations, hypercharge, spin, and fermionic statistics instead emerge as different admissibility faces of one substrate process: the persistent stabilization of distinguishability under closure transport.

The Standard Model therefore appears not as an arbitrary inventory of fields, but as the finite admissible representation structure of committed distinguishability itself.

Appendix A. Dependency structure of the dictionary

The dictionary developed in this paper depends on a layered structure of upstream substrate results. The following table makes the dependency map explicit, so each dictionary claim can be traced to its substrate-level prerequisites.

Dictionary result	**Substrate-level dependencies**
Gauge structure G_SM = SU(3) × SU(2) × U(1)	BC1 + Fisher information manifold + $\mathscr{H}$_fold ≅ $\mathbb{C}^4$ commutant structure (*One Fold*) — *canonical derivation; an alternative* $\mathbb{CP}^{n-1}$ *presentation exists (§2.5) but is not used downstream*
n ≤ 3 internal dimension	$\mathscr{H}$_fold ≅ $\mathbb{C}^4$ + finite distinguishability (*One Fold* T1)
$\mathbb{C}^3$ subspace	$\mathbb{C}^4 = \mathbb{C}^1 \oplus \mathbb{C}^3$ decomposition (*One Fold* Lemma GG2)
Three-body singlet structure	$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \supset \mathbf{1}$ decomposition + substrate dimension count
Matter / gauge-boson type distinction (§5)	(P1) closure-defect topology vs propagation-mode classification
Closure-confinement equivalence (Theorem 6.7)	Theorem 8.2 + complete/partial closure definition (§6.1)
Isolated triality obstruction (Theorem 8.2)	$\mathbb{Z}_3$ centre of SU(3) + substrate closure-conservation law (§2.10)
Substrate closure-conservation law for $\mathbb{Z}_3$ (§2.10)	BCB transport programme
Confinement	Theorem 8.2 + continuum dynamical realisation (confinement programme)
Quark fractional Y values	Triality projection (§8.4) + anomaly cancellation (§11.2)
Chirality	SU(2) restriction to one $\mathbb{Z}_2$ orientation parity (BC1-compatible transport, *One Fold*)
Spin-statistics	Spinorial holonomy + Finkelstein–Rubinstein construction (matter paper Theorems 7.1, 7.2)
Fermionic exchange	Exchange-path holonomy on spinorial closure sector (§10.2)
Hypercharge U(1)_Y	Abelian uniqueness + anomaly cancellation (closure-ledger work + BC1–BC3)
Anomaly-cancellation route	Closure-ledger work + BCB transport admissibility programme (BC1–BC3 transport conditions) + substrate-level fermion content from §6
Lepton/quark distinction	Complete vs partial closure (§6.1)
Baryon stabilization-depth dynamics	R_{n+1} = (3/5) R_n iteration on 5D residual closure space (proton/baryon paper)
Generation depth γ_D ≤ 3	Generation-depth refinement persistence + $\mathcal{D} = \mathrm{diag}(1, 2, 4)$ (flavour-mixing programme)

Dictionary result	Substrate-level dependencies
CKM/PMNS structure	Generation-depth transport attenuation (§16.4, flavour-mixing programme)
Mass hierarchy	Closure energetics + matter paper §6.5 bridge constants
Mass as closure cost (§19.3)	Matter paper §6.5 four-contribution scaffold + closure-norm Higgs paper
Finite particle spectrum (§19.1)	Component-wise finite bounds on $\boldsymbol{\mathcal{J}}(D)$ entries (§19.1)
Matter–geometry coupling (§17)	Matter paper §8 quadratic-in-Φ stress-energy architecture
Dictionary No-Alternatives Statement (§7.3)	Joint admissibility constraints overdetermining the matter content (conjectural)

The table is intended as a navigation aid for reviewers and as a check on non-circularity: every dictionary claim traces back to one or more upstream substrate results, and no claim circularly depends on a downstream phenomenological input. Where conjectural prerequisites are involved (e.g. the No-Alternatives Statement, the finiteness theorem), the conjectural status is explicit in §20.4.