

# The Uniqueness of the Minimal Distinction

## A No-Alternative Theorem for the Fold

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### General-reader summary

Physics always starts from *something* — a particle, a field, a string, a state vector. These starting points are usually just declared, not derived, which leaves open the question of whether some other starting point would have done equally well.

This paper asks the question directly: *what is the smallest possible thing a physical theory can be built from, and is it unique?* We lay down four conditions that any foundational starting point has to satisfy — it has to be finite, encodable, self-contained, and free of any pre-existing background. Then we go through every candidate structure and check whether it meets the conditions.

Almost everything fails. Continuous structures fail because they need infinite precision. Externally referenced structures fail because they assume the very background we are trying to do without. Structures bigger than the minimum fail because any system with three or more distinguishable states already contains a simpler yes/no question separating some of them from the rest — and that yes/no question is the more primitive thing. Structures that don't close on themselves fail because they need something outside to complete them.

This conclusion is robust enough that even if you imagine a richer substrate underneath physics, every actual measurement you can perform on it has a yes/no answer — every observation factors through a sequence of yes/no partitions, each of which is itself a Fold. So the Fold isn't a low-energy approximation that happens to look simple; it's what *operational distinction itself* has to look like. (Whether the underlying state space is itself binary, or richer at a level we cannot observe, is a separate question that this paper does not settle.)

What is left is one structure, and only one: two states, mutually relating, with no other content. We call this the Fold. It is the smallest closed act of distinction that can exist without background. Everything else physics talks about — particles, fields, spacetime — has to be built out of folds.

The Fold is therefore not assumed. It is forced.

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## Abstract

We determine the form and uniqueness of the most primitive physically admissible unit of distinction under four minimal structural conditions: finite distinguishability, finite encodability, internal closure, and absence of background structure. We prove that any ontology meeting these conditions must contain at least one irreducible distinction, that this distinction must be binary, intrinsic, and strongly connected, and that the only directed-graph structure satisfying all three properties on the minimum admissible state set is the two-cycle on two states. The argument rests on a quotient-containment lemma: any state space with  $|S| \geq 3$  distinguishable states necessarily admits a non-trivial binary partition recoverable as an admissible distinction, and therefore cannot be the minimal primitive. We further establish an *operational unavoidability* result: every admissible measurement on any finite admissible state space factors as a finite sequence of binary partitions, each of which is itself a Fold; the Fold is therefore not only the unique minimal primitive distinction but the unique operational basis of all measurable distinction. We are explicit that this is a claim about *measurement information*, not about state-space ontology — a richer pre-observable substrate is left open. By exhaustive case analysis we confirm that all alternative candidate structures — null, continuous, externally referenced, multi-element, open, or self-loop-bearing — fail at least one axiom or minimality condition. The surviving structure, which we term the **Fold**, is therefore not a postulate but a structural consequence of the axioms. We give an explicit information-theoretic characterisation (the Fold carries exactly one bit of irreducible distinction), prove uniqueness up to isomorphism, and outline the implications: the Fold is the unique minimal primitive in three senses (structural, informational, operational) and unavoidable in the operational sense. The result is conditional on the four axioms; the paper is explicit about what is forced by them, what relies on a quotient (rather than substructure) reading of containment, and what would require additional input.

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## 1. Introduction

Every physical theory rests on a primitive ontology — a stipulated unit out of which all further structure is built. Particles, fields, strings, qubits, and state vectors are all such units. They are introduced by declaration, not by derivation, and the question of whether they are *necessary* — whether some other primitive could have served — is rarely posed and almost never answered.

The aim of this paper is to pose and answer that question under minimal structural conditions. We ask:

*What is the minimal physically admissible unit of distinction, and is it unique?*

Distinguishability is plausibly the most basic structural requirement of any physical theory: without distinguishable states there is nothing for a theory to track, predict, or measure. If distinguishability is required, then any admissible ontology must contain at least one irreducible *distinction*. The question becomes: under what minimal conditions can such a distinction exist, what form must it take, and how many such forms are admissible?

We answer: under four explicit axioms (finite distinguishability, finite encodability, closure, and background-independence), the admissible primitive distinction exists, is unique up to isomorphism, and consists of a strongly connected two-cycle on a two-element state set. We call this structure the **Fold**.

The result is a *no-alternative theorem* in the strict sense: every other candidate primitive structure violates at least one axiom, and the surviving structure is unique. The result is *conditional* on the axioms; we are explicit in §11 about what the axioms commit to and what would change if any were relaxed.

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## 2. Axioms

We adopt the following four axioms as the minimal structural conditions on any physically admissible ontology.

**(A1) Finite distinguishability.** There exists at least one operationally realisable, finite distinction. Any distinction requiring infinite precision or a continuum of distinguishable values is excluded as a *primitive*; such structures may exist only as compositions or limits of finite primitives.

**(A2) Finite encodability.** Every distinction admits a finite specification — a finite number of states, a finite number of relations among them, and a finite description of those relations. No primitive may require an unbounded amount of information to specify.

**(A3) Internal closure.** Every state in a primitive distinction must have its relational identity determined by other states *within* the primitive. No primitive may depend on an unresolved external completion.

**(A4) Background-independence.** No pre-existing space, time, coordinate system, or external reference frame is presupposed. All structure must be intrinsic to the primitive — definable solely in terms of relations among its own states.

Each axiom is independently motivated. (A1) and (A2) are operational requirements that any *physical* primitive must satisfy if physical theories are to be finitely formulated and finitely tested. (A3) is a self-consistency requirement: a "primitive" that references something outside itself is not, in fact, primitive. (A4) is the requirement that a *foundational* ontology not presuppose the very ambient structure (space, time, frames) that it is the job of the ontology to ground.

The four axioms together pin down what we mean by a *physically admissible foundational primitive*. They do not — and are not intended to — fix every detail of physics. They fix the necessary conditions on the *most primitive layer*; the no-alternative theorem then says this layer admits exactly one structure.

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## 3. Definitions

### 3.1 Distinctions and relational systems

**Definition 1 (Relational system).** A *relational system* is an ordered pair

$$\mathcal{D} = (S, R)$$

where  $S$  is a non-empty set (the *state set*) and  $R \subseteq S \times S$  is a binary relation on  $S$  (the *distinguishability relation*). The elements of  $S$  are called *states*; the elements of  $R$  are called *relations*. A pair  $(s_i, s_j) \in R$  is read " $s_j$  is distinguishable from  $s_i$ ".

**Definition 2 (Intrinsic relational system).** A relational system  $\mathcal{D} = (S, R)$  is *intrinsic* if both  $S$  and  $R$  are defined without reference to any structure external to  $\mathcal{D}$  — that is, without reference to any ambient space, time, coordinate system, or labelling that is not itself a relation in  $R$ .

**Definition 3 (Closed relational system).** A relational system  $\mathcal{D} = (S, R)$  is *closed* if for every  $s \in S$  there exists  $s' \in S$  with  $(s, s') \in R$ , and the directed graph  $(S, R)$  is *strongly connected* — i.e., for every pair  $(s_i, s_j) \in S \times S$ , there exists a directed path in  $R$  from  $s_i$  to  $s_j$ .

**Remark.** Closure in the sense of Definition 3 is the relational implementation of axiom (A3). A non-closed system contains states whose relational identity is not determined within the system; supplying that identity would require external completion, which (A3) forbids.

### 3.2 Primitive distinctions

**Definition 4 (Distinction).** A *distinction* in a relational system  $\mathcal{D} = (S, R)$  is a non-trivial relational asymmetry — a pair  $(s_i, s_j) \in R$  with  $s_i \neq s_j$ , distinguishing  $s_j$  from  $s_i$ .

**Definition 5 (Containment of a smaller distinction).** A relational system  $\mathcal{D} = (S, R)$  with  $|S| \geq 3$  *contains a smaller distinction* if there exists a non-empty proper subset  $A \subset S$  whose indicator function

$$Q_A : S \rightarrow \{0, 1\}, Q_A(s) = 1 \text{ if } s \in A, Q_A(s) = 0 \text{ if } s \notin A,$$

separates at least one related pair — i.e., there exists  $(s_i, s_j) \in R$  with  $s_i \in A$  and  $s_j \notin A$ . The induced binary quotient  $(Q_A(S), R_{\text{quot}})$ , where  $(b, b') \in R_{\text{quot}}$  iff there exist  $s \in Q_A^{-1}(b), s' \in Q_A^{-1}(b')$  with  $(s, s') \in R$ , is a relational system on a strictly smaller state set than  $S$ , carrying  $\log_2(2) = 1$  bit of distinguishing content versus the  $\log_2|S| \geq \log_2(3)$  bits required to specify a state of  $\mathcal{D}$ .

The  $|S| \geq 3$  condition is essential: a  $|S| = 2$  system has no proper subset  $A$  whose binary quotient is on a strictly smaller state set than  $S$ , so the definition does not apply. This makes Definition 5 vacuous for the Fold itself, as required for Proposition 1 below.

**Remark — Quotient containment vs substructure containment.** Two distinct notions of one relational system "containing" another are at play in foundational arguments of this type:

- *Substructure* containment:  $\mathcal{D}'$  is an induced subgraph of  $(S, R)$  on some subset  $S' \subseteq S$ .
- *Quotient* containment:  $\mathcal{D}'$  is the image of  $\mathcal{D}$  under a state-collapsing map, in particular a binary partition  $Q_A$ .

Definition 5 adopts the *quotient* sense. The two notions are not equivalent. Consider the directed 3-cycle on  $S = \{s_0, s_1, s_2\}$  with  $R = \{(s_0, s_1), (s_1, s_2), (s_2, s_0)\}$ : the induced subgraph on any pair

$\{s_i, s_j\}$  is a single directed edge, which is not strongly connected and therefore not a sub-Fold. So the directed 3-cycle contains no induced sub-Fold under the substructure reading. But it does contain a smaller distinction in the quotient sense: the bipartition  $A = \{s_0\}$ ,  $S \setminus A = \{s_1, s_2\}$  crosses the edge  $(s_0, s_1)$  (and the edge  $(s_2, s_0)$ ), giving an admissible binary quotient.

We adopt the quotient sense because operational admissibility (A1) requires that any distinguishing partition recoverable from  $\mathcal{D}$  be itself an admissible distinction; under that requirement, a binary partition recoverable from an  $n$ -state system is operationally prior to the  $n$ -state system, and the  $n$ -state system therefore cannot be the *minimal* primitive even though it may be perfectly well-defined as a relational system. We acknowledge in §10 that this priority is implicitly compositional: it relies on the  $n$ -state distinction being constituted by a sequence of binary distinctions, a piece of structure whose full specification is deferred to subsequent work.

**Definition 6 (Primitive distinction).** A *primitive distinction* is a relational system  $\mathcal{D} = (S, R)$  that

(i) is intrinsic (per Definition 2), (ii) is closed (per Definition 3), (iii) contains at least one distinction (per Definition 4), (iv) does not contain a smaller independently recoverable distinction (per Definition 5), (v) admits a finite specification (per axiom A2).

**Remark — Operational reading of primitiveness.** Primitiveness is understood throughout in *operational* terms: a primitive distinction is minimal under admissible operational refinement, where refinement is the recovery of a finer distinction by a physically admissible operation in the sense of (A1). The notion of "smaller" in clause (iv) is therefore not purely mathematical but operational — a distinction is "smaller" if it can be recovered by an admissible operation acting on the larger one. A reader committed to a non-operational reading of distinctions (e.g., a purely set-theoretic refinement relation) would substitute their preferred refinement notion, and the theorem would re-cast accordingly with respect to that notion. The operational reading is the one consistent with axioms (A1)–(A2) and is adopted throughout this paper.

The question of the paper is: how many primitive distinctions exist, up to isomorphism?

## 4. Foundational lemmas

### Lemma 1 (Existence of distinction)

Any primitive distinction has  $|S| \geq 2$ .

*Proof.* If  $|S| = 0$ , the system has no states and trivially contains no distinction, violating clause (iii) of Definition 6. If  $|S| = 1$  with  $S = \{s_0\}$ , then the only possible elements of  $R \subseteq S \times S$  are the self-pair  $(s_0, s_0)$ ; by Definition 4, a distinction requires  $s_i \neq s_j$ , so no distinction exists. This violates clause (iii). Hence  $|S| \geq 2$ .

## Lemma 2 (Binary minimality)

*Any primitive distinction has  $|S| = 2$ .*

*Proof.* By Lemma 1,  $|S| \geq 2$ . We show that any structure with  $|S| \geq 3$  contains a smaller distinction in the sense of Definition 5, and therefore fails clause (iv) of Definition 6.

Let  $\mathcal{D} = (S, R)$  be a relational system with  $|S| = n \geq 3$  satisfying axioms (A1)–(A4). Set-theoretically, any  $S$  with  $|S| \geq 3$  admits at least one non-empty proper subset  $A \subset S$  — i.e.,  $A$  and its complement  $S \setminus A$  are both non-empty. The indicator function

$$Q_A : S \rightarrow \{0, 1\}, Q_A(s) = 1 \Leftrightarrow s \in A,$$

is well-defined; since the directed graph  $(S, R)$  is strongly connected (axiom A3), there exists at least one related pair  $(s_i, s_j) \in R$  crossing the partition (one state in  $A$ , the other in  $S \setminus A$ ). Hence  $Q_A$  separates a related pair, and  $\mathcal{D}$  contains the smaller binary distinction induced by  $(A, S \setminus A)$  in the sense of Definition 5. Since axiom (A1) requires distinctions to be operationally realisable, any such partition that separates a related pair corresponds to a physically admissible distinction — not merely a formal one.

The contained binary distinction carries 1 bit of distinguishing content, strictly less than the  $\log_2(n)$  bits required to specify a state of  $\mathcal{D}$ . By Definition 6 clause (iv),  $\mathcal{D}$  is therefore not the minimal primitive distinction.

This argument does not require the  $n$ -state structure to *factorise* uniquely into independent two-state systems, and does not claim that the  $n$ -state structure is *reducible*. The triangle graph contains binary cuts but is not reducible to a 2-state system; what we are establishing is *minimality*, not reducibility of all higher structures. The claim used is correspondingly weaker — and stronger in the relevant sense:

*Any physically distinguishable structure with  $n \geq 3$  states contains at least one binary distinction, and therefore cannot be the minimal primitive distinction.*

If the  $n$ -state structure contains *no* recoverable binary partition, its states are not operationally distinguishable and the structure fails axiom (A1). If it does contain such a partition, the partition is prior to the  $n$ -state structure as a fact-generating unit. Either way, the  $n$ -state structure is not the minimal primitive.

Combined with Lemma 1 (which excludes  $|S| = 0, 1$ ), this leaves  $|S| = 2$  as the unique admissible cardinality. By Theorem 1 (proved below) the resulting structure is unique up to isomorphism and equals  $\mathcal{F} \cong \mathbb{Z}_2$ .

**Remark — Operational reduction to binary distinctions**

A natural objection to Lemma 2 runs: the deeper microscopic substrate of physics may be richer than binary — continuous, ternary, or otherwise structured — so why should the *primitive* be binary?

What Lemma 2 establishes, given the operational reading of (A1)–(A2) adopted throughout, is the following: any operational measurement performed on an admissible state space reduces to a sequence of binary partition questions, each of which is itself a Fold in the sense of Definition 7. There is no admissible measurement of less granularity than 1 bit, and every measurement factors through binary partitions.

Whether the underlying state space is itself binary, or whether a richer substrate exists at the pre-observable level, is *left open* by this observation. The substantive operational claim is: under any admissible bipartition of any candidate primitive structure, the first non-trivial recoverable distinction is binary, and that binary distinction is the Fold. A 3-state system has a 3-state ontology even though every yes/no question one can ask about it is binary; what is binary is the answer space, not the state space. So the present Lemma does *not* show that all richer substrates collapse to  $\mathbb{Z}_2$  as ontologies — it shows that all operational distinctions recoverable from such substrates are binary, and that the minimal admissible primitive is the Fold.

A formal theory of how richer pre-observable substrates relate to admissible primitives would require a definition of *admissible coarse-graining* as a structure-preserving map and a fixed-point analysis of its action; that programme is outside the scope of the present paper. The conclusion of Lemma 2 — that the Fold is the unique minimal admissible primitive — does not depend on this stronger fixed-point claim. It depends only on the containment argument established in the proof above.

## Lemma 2A (Exhaustiveness of binary questions)

*Any admissible measurement on a finite admissible state space  $S$  decomposes into a finite sequence of binary partitions of  $S$ . Equivalently, every operationally accessible distinction on  $S$  factors through a finite sequence of Folds.*

*Proof.* An admissible measurement  $M : S \rightarrow O$  delivers, for each state  $s \in S$ , an outcome  $M(s)$  drawn from a finite outcome set  $O$  (axiom A2 forces  $|O| < \infty$  for any operationally realisable measurement; axiom A1 forces  $M$  to be operationally distinguishing on its image). Set  $m = \lceil \log_2 |O| \rceil$  and label the elements of  $O$  by binary strings of length  $m$ . For each  $i \in \{1, \dots, m\}$ , let  $Q_i : S \rightarrow \{0, 1\}$  be the function returning the  $i$ -th bit of the binary label of  $M(s)$ . Each  $Q_i$  is a binary partition of  $S$ . The composition  $(Q_1, Q_2, \dots, Q_m)$  recovers  $M(s)$  by yielding its complete binary label.

The partition lattice on a finite set is generated by binary partitions: any partition of  $S$  with  $k$  cells is the meet of a system of  $\lceil \log_2 k \rceil$  binary partitions sufficient to label its cells. Applied to the partition  $\Pi_M$  induced by  $M$ , this gives  $\Pi_M = Q_1 \wedge Q_2 \wedge \dots \wedge Q_m$ . Each  $Q_i$  is structurally a Fold on the quotient  $(Q_i(S), R_{\text{quot}})$  per Definition 7 and Theorem 1. Hence  $M$  factors through a finite sequence of Folds.

This lemma upgrades Lemma 2's structural claim into an operational claim: not only is the Fold the minimal primitive distinction (Lemma 2), but the Fold is the *unique operational basis* through which any finite admissible measurement is performed. We note carefully what is and is not claimed: this is a statement about the *information-theoretic factorisation* of admissible measurements, not about the underlying state ontology of the system being measured. A 3-state system retains its 3-state ontology; what factors through binary partitions is the *measurement information* extracted from it, not the system itself.

### **Lemma 2B (Minimal observable distinction)**

*Under finite distinguishability (A1), the minimal non-trivial observable distinction on any admissible state space is binary: no non-trivial observable distinction carries less than 1 bit of distinguishing content.*

*Proof.* By Lemma 1, any observable distinction requires at least 2 distinct states, hence at least  $\log_2(2) = 1$  bit of distinguishing content. There is no non-trivial finite distinction below 1 bit, since any finer refinement of a binary partition presupposes the binary partition itself. The minimal non-trivial observable distinction is therefore the Fold.

Together, Lemmas 2A and 2B establish that the Fold is both *sufficient* (every measurement factors through binary partitions) and *minimal* (no observable distinction is finer than binary) for the operational basis of measurement. Combined with Lemma 2's structural minimality, the three lemmas establish that the Fold is the unique minimal primitive in *all three* senses: structural ( $|S| = 2$ ), informational (1 bit), and operational (binary measurement basis).

### **Lemma 3 (Intrinsic necessity)**

Any primitive distinction is intrinsic.

*Proof.* Direct from axiom (A4). A relational system that is not intrinsic refers to structure external to itself (an ambient space, coordinate system, or labelling); axiom (A4) forbids such reference at the foundational level. Hence any foundational primitive must be intrinsic.

### **Lemma 4 (Closure necessity)**

Any primitive distinction is strongly connected.

*Proof.* This is a direct unpacking of axiom (A3) under Definition 3, which formalises closure as strong connectivity. We state it as a separate lemma to expose its role in subsequent proofs. If the directed graph  $(S, R)$  were not strongly connected, there would exist states  $s_i, s_j$  with no directed path connecting them; the relational system would then decompose into mutually independent components, each of which fails (A3)'s requirement that no relational obligation be left unresolved within the system. Hence  $(S, R)$  must be strongly connected.

## 5. Exhaustive elimination of alternatives

We now show that every candidate primitive structure other than the Fold violates at least one axiom or definitional clause.

**Case 1: Null structure ( $|\mathcal{S}| = 0$  or  $|\mathcal{S}| = 1$ ).** By Lemma 1, no distinction exists. Violates clause (iii) of Definition 6 and axiom (A1). *Rejected.*

**Case 2: Continuous structure ( $|\mathcal{S}|$  uncountable, with continuous distinguishability).** A continuous primitive requires that distinct states be distinguishable to arbitrary precision. By axiom (A1), only finite distinguishability is admitted at the primitive level. Any operationally realisable observer accesses only a finite quotient of the continuum; the finite quotient — not the underlying continuum — is the operationally primitive object. Furthermore, axiom (A2) requires finite encodability, which a continuum of states cannot satisfy. *Rejected.*

A continuum may emerge as a limit of compositions of finite primitives, but it cannot itself be primitive.

**Case 3: Externally referenced structure.** A relational system whose distinctions are defined by reference to external coordinates, frames, or labels violates axiom (A4). *Rejected.*

**Case 4: Multi-element primitive ( $|\mathcal{S}| \geq 3$ ).** By Lemma 2, any  $|\mathcal{S}| \geq 3$  system contains at least one binary distinction (the indicator function  $Q_A$  on some non-empty proper subset  $A \subset \mathcal{S}$ ) and therefore violates clause (iv) of Definition 6. *Rejected as primitive*; such systems may carry composite physical content but cannot themselves be the minimal admissible unit.

**Case 5: Open relational structure.** A system with non-strongly-connected  $(\mathcal{S}, \mathcal{R})$  contains states whose relational identity is undetermined within the system. By Lemma 4, this violates axiom (A3). *Rejected.*

**Case 6: Self-loop-bearing structure on  $|\mathcal{S}| = 2$ .** A two-state system with self-loops — i.e.,  $\mathcal{R}$  containing  $(s_0, s_0)$  or  $(s_1, s_1)$  in addition to inter-state edges — adds non-distinguishing relations to the underlying Fold. By Definition 4, a distinction requires  $s_i \neq s_j$ ; a self-loop  $(s_i, s_i)$  is therefore not a distinction. Adding such relations strictly increases  $|\mathcal{R}|$  without contributing distinguishing content, so the loop-free two-cycle  $\mathcal{F}$  carries the same distinguishing content with strictly less structure.

This is an argument from *parsimony* — that a minimal primitive should contain no non-distinguishing relations — and is structurally distinct from Lemma 2's *quotient containment* argument. We adopt parsimony as an extension of the minimality reading: from minimal  $|\mathcal{S}|$  (established by Lemma 2) to minimal  $|\mathcal{R}|$  consistent with carrying the required distinguishing content. This extension is natural but not strictly forced by (A1)–(A4); it could equivalently be stated as an implicit further axiom (A5) of relational minimality, but we treat it as the canonical reading of "minimal primitive" throughout rather than as a separate commitment. Under that reading, the loop-bearing system is not the minimal primitive; the loop-free Fold is. *Rejected as a primitive distinct from  $\mathcal{F}$ .*

**Case 7: Two-state systems with incomplete relation  $R$  on  $|S| = 2$ .** On  $|S| = 2$  there are exactly four possible directed-graph structures excluding self-loops:

$$R = \emptyset, R = \{(s_0, s_1)\}, R = \{(s_1, s_0)\}, R = \{(s_0, s_1), (s_1, s_0)\}.$$

We eliminate the first three:

- $R = \emptyset$ . No distinction exists; violates clause (iii) and axiom (A1). *Rejected.*
- $R = \{(s_0, s_1)\}$ . The state  $s_1$  has no outgoing relation; its identity is not relationally determined within the system. The directed graph is not strongly connected. Violates Lemma 4 and axiom (A3). *Rejected.*
- $R = \{(s_1, s_0)\}$ . Symmetric to the previous case;  $s_0$  has no outgoing relation. *Rejected.*

The only remaining candidate is  $R = \{(s_0, s_1), (s_1, s_0)\}$ .

This case enumeration is exhaustive: every two-state system without self-loops is one of these four, and only the last survives.

## 6. The Fold

**Definition 7 (Fold).** A *Fold* is a relational system

$$\mathcal{F} = (S, R), S = \{s_0, s_1\}, R = \{(s_0, s_1), (s_1, s_0)\},$$

that is intrinsic (Definition 2) and strongly connected (Definition 3).

The Fold is the minimal closed binary distinction: two states, each distinguished from and distinguishing the other, with no further structure.

**Proposition 1 (The Fold is a primitive distinction).**  $\mathcal{F}$  satisfies all five clauses of Definition 6.

*Proof.* (i) Intrinsic by stipulation; the states  $s_0$  and  $s_1$  are defined solely by their relational asymmetry, with no external reference. (ii) Closed: every state has an outgoing relation, and the directed graph is strongly connected ( $s_0 \rightarrow s_1 \rightarrow s_0$ ). (iii) Contains the distinction  $(s_0, s_1) \in R$  with  $s_0 \neq s_1$ . (iv) Definition 5 of "contains a smaller distinction" applies only to systems with  $|S| \geq 3$ ; the Fold has  $|S| = 2$ , so clause (iv) is satisfied vacuously, and Lemma 1 establishes that no relational system below  $|S| = 2$  supports a distinction at all. (v) Finite:  $|S| = 2$ ,  $|R| = 2$ , both bounded.

## 7. Uniqueness theorem

**Theorem 1 (Uniqueness of the minimal distinction).** *Under axioms (A1)–(A4), every primitive distinction is isomorphic to  $\mathcal{F}$ .*

*Proof.* Let  $\mathcal{D} = (S, R)$  be a primitive distinction.

By Lemma 2,  $|S| = 2$ . Write  $S = \{s_0, s_1\}$ .

By Lemma 3,  $\mathcal{D}$  is intrinsic.

By Lemma 4, the directed graph  $(S, R)$  is strongly connected.

By Case 6 of §5,  $R$  contains no self-loops (any such loop would render  $\mathcal{D}$  non-minimal).

By Case 7 of §5, the only two-state directed graph without self-loops that is strongly connected is

$$R = \{(s_0, s_1), (s_1, s_0)\}.$$

$$\text{Hence } \mathcal{D} = (\{s_0, s_1\}, \{(s_0, s_1), (s_1, s_0)\}) = \mathcal{F}.$$

For uniqueness up to isomorphism, let  $\mathcal{D} = (S, R)$  and  $\mathcal{D}' = (S', R')$  both be primitive distinctions. By the above,  $S = \{s_0, s_1\}$  and  $S' = \{s'_0, s'_1\}$ , with  $R$  and  $R'$  both equal to the two-cycle. The bijection  $\varphi : S \rightarrow S'$  defined by  $\varphi(s_0) = s'_0$ ,  $\varphi(s_1) = s'_1$  preserves the relation:  $(s_i, s_j) \in R \Leftrightarrow (\varphi(s_i), \varphi(s_j)) \in R'$ . Hence  $\mathcal{D} \cong \mathcal{D}'$ .

Therefore every primitive distinction is isomorphic to  $\mathcal{F}$ .

**Corollary (Operational unavoidability of the Fold).** *In any admissible physical theory whose observables satisfy (A1)–(A2) — finite distinguishability and finite encodability — every measurable distinction factors through a finite sequence of binary partitions, each of which is a Fold (up to isomorphism). The Fold is therefore not only the unique minimal primitive distinction (Theorem 1) but the unique operational basis of all measurable distinction.*

*Proof.* Direct from Lemma 2A (every admissible measurement factors through a finite sequence of binary partitions) combined with Theorem 1 (every binary partition realised as a primitive distinction is isomorphic to  $\mathcal{F}$ ). The result is operational: it concerns the factorisation of measurement information through binary partitions, not the ontology of the system being measured, which may carry richer non-binary structure at the pre-observable level. What is established is that all *measurable* structure factors through Folds; no claim is made that all *underlying* structure does so.

**Corollary 1 (Uniqueness of automorphism).** *The automorphism group of  $\mathcal{F}$  is the cyclic group of order 2, generated by the swap  $s_0 \leftrightarrow s_1$ .*

*Proof.* The only non-identity bijection on a two-element set is the swap. The swap preserves  $R$ :  $(s_0, s_1) \leftrightarrow (s_1, s_0)$ , and  $\{(s_0, s_1), (s_1, s_0)\}$  is invariant under this exchange. Hence  $\text{Aut}(\mathcal{F}) \cong \mathbb{Z}_2$ .

This automorphism is the *intrinsic dual symmetry* of the Fold: there is no canonical labelling of  $s_0$  versus  $s_1$ , since the structure is invariant under their exchange. The Fold thus carries an intrinsic  $\mathbb{Z}_2$  symmetry that is not added but forced by minimality.

## 8. Information-theoretic characterisation

The Fold's state space has cardinality 2, so specifying its state requires  $\log_2(2) = 1$  bit. Combined with Lemma 1 ( $|\mathcal{S}| \geq 2$ , hence at least 1 bit) and Lemma 2 ( $|\mathcal{S}| = 2$ , hence at most 1 bit), the information content of any primitive distinction is exactly 1 bit. The Fold is therefore the unique 1-bit primitive: smaller information content (0 bits) admits no distinction; larger information content ( $>1$  bit) necessarily contains a smaller binary partition (in the quotient sense of Definition 5) and is therefore not the minimal primitive.

This gives a complementary characterisation: the Fold is the unique minimal admissible distinction in both *structural* terms (a 2-cycle on two states) and *information-theoretic* terms (a 1-bit primitive). The two characterisations coincide because the structural minimum and the information-theoretic minimum are the same: there is no admissible primitive below the 1-bit floor, and no admissible primitive distinction carrying exactly 1 bit of distinguishing content other than the Fold (up to isomorphism, by Theorem 1).

## 9. Corollaries and physical implications

**Corollary 2 (No smaller ontic unit).** No relational system satisfying (A1)–(A4) is structurally smaller than  $\mathcal{F}$ .

*Proof.* Direct from Lemma 1 (no distinction below  $|\mathcal{S}| = 2$ ) and the information-theoretic characterisation of §8 (no admissible primitive below 1 bit).

**Corollary 3 (Compositional ontology).** Every relational system satisfying (A1)–(A4) with  $|\mathcal{S}| \geq 3$  contains binary distinctions and is therefore not the minimal primitive distinction. Whether all such higher-cardinality structures are best understood as *compositions* of folds — and what the appropriate composition operation is — requires structural input beyond the present paper.

*Proof.* By Lemma 2, any  $|\mathcal{S}| \geq 3$  admissible system contains at least one binary partition  $Q_A : \mathcal{S} \rightarrow \{0, 1\}$ , hence contains a smaller distinction (Definition 5) and fails clause (iv) of Definition 6, which precludes minimality. The corollary does not assert that such systems are *reducible* to folds; it asserts only that they fail minimality. The compositional reading — that the system is *built from* folds via a specific operation — is the natural strengthening of this result, but

establishing it requires specifying the composition operation, which is the subject of subsequent work in the corpus. The present paper establishes the weaker negative claim (no  $|S| \geq 3$  structure is the minimal primitive), which is sufficient for the uniqueness theorem.

**Corollary 4 (Intrinsic dual structure).** Every fold defines a canonical pair of mutually distinguished states with intrinsic  $\mathbb{Z}_2$  exchange symmetry.

*Proof.* From Definition 7 and Corollary 1.

**Conjectural Corollary 5 (Closure-origin of conservation).** Conservation laws in any composition of folds arise from the closure conditions (A3) imposed on the fold-network.

*Proof sketch.* Closure (A3) requires that no relational obligation propagate outside the system. In a composition of folds, this constrains the joint distinguishability flow between subnetworks. Such constraints are formally identical to conserved-current statements in continuous limits. A full derivation of specific conservation laws (energy, momentum, charge) requires the composition operation explicit, which is outside the scope of the present paper; we identify the *structural origin* of conservation here, not its specific form.

## Implications for physical primitives

Under the operational reading of (A1)–(A2) adopted throughout, several conventional foundational primitives violate the axioms at the primitive level:

- **Continuous fields.** Require uncountable distinguishability; violate (A1) under the operational reading as primitives. *Admissible only as emergent limits of fold-composition.*
- **Particles.** Require spatial embedding; violate (A4) as primitives. *Admissible only as fold-composition structures within emergent space.*
- **Strings.** Require background spacetime geometry; violate (A4). *Admissible only as emergent.*
- **State vectors in Hilbert space.** Require continuous superposition; violate (A1) and (A2) under the operational reading as primitives. *Admissible only as effective descriptions of fold-network states.*

Each of these is a legitimate *emergent* description of fold compositions; none is admissible as the foundational primitive itself under the present axioms. A reader committed to a kinematical reading of state space — on which a continuum of states is admissible at the primitive level — would consistently reject (A1)–(A2) and therefore stand outside the scope of the theorem; this is consistent with the conditional framing of the result in §10.

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## 10. Honest scope of the result

We are explicit about what the theorem does and does not establish.

**What is forced by the axioms.** Given (A1)–(A4) and Definition 6, the Fold is the unique primitive distinction up to isomorphism. The argument is exhaustive: every alternative candidate has been eliminated by explicit case analysis, not by appeal to plausibility.

**What is not forced by the theorem.** The axioms themselves are not derived. They are minimal *structural conditions* on what we mean by a foundational physical primitive. Each axiom is independently motivated (operational realisability, finite specification, self-consistency, foundational background-independence), but the theorem says only that *given* these conditions, the Fold is unique. A theory operating outside the scope of one or more axioms — for instance, one that takes a continuum as primitive, or that admits external background structure — falls outside the theorem's scope and is not addressed by it.

**What requires further input.** The composition operation by which folds combine into higher structures is referenced in Corollaries 3–5 but not specified here. Lemma 2 does not formally depend on a composition operation — it establishes only that  $|S| \geq 3$  structures *contain* smaller binary distinctions in the quotient sense, not that they decompose uniquely into them. There is, however, a *tacit* compositional commitment in Lemma 2's argument: the priority of the binary partition over the n-state structure relies on the n-state distinction being constituted by some sequence of binary distinctions that the quotient recovers. Making this constitution explicit, and specifying the composition rules under which it holds, requires additional structural input (relational composition operations, closure constraints on networks of folds, admissibility conditions on composite states) that is the subject of the broader corpus. Case 6 of §5 also relies on a parsimony reading of relational minimality (minimal  $|R|$  in addition to minimal  $|S|$ ), which we treat as the canonical reading of "minimal primitive" rather than as an additional axiom; a reader who declines this reading would treat the loop-bearing two-state system as a distinct admissible primitive. The present theorem establishes that *whatever* the correct composition operation is, it acts on a unique minimal primitive — the Fold — and that the choice of quotient containment over substructure containment in Definition 5 is consistent with that compositional reading.

**What the theorem rules out.** It rules out, conditional on the axioms, the possibility of a *different* foundational primitive: there is no admissible 3-element primitive, no admissible continuous primitive, no admissible externally referenced primitive, and no admissible primitive smaller than the Fold. Any theory that claims a different foundational primitive must either reject one of (A1)–(A4) or must be redescrivable (under the theorem) as a composition of folds.

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## 11. Conclusion

We have shown that the requirement of finite, encodable, closed, background-independent distinction admits exactly one primitive structure: the Fold. The argument proceeded by:

1. fixing four minimal axioms,
2. defining primitiveness in relational-system terms,

3. proving the foundational lemmas (existence of distinction, binary minimality, exhaustiveness of binary questions, minimal observable distinction, intrinsic necessity, closure necessity),
4. exhaustively eliminating every alternative candidate, and
5. establishing uniqueness up to isomorphism, with an automorphism group  $\mathbb{Z}_2$  and information content of exactly 1 bit.

The Fold is therefore not a postulate of the theory. It is what survives when every alternative is eliminated. Combined with the operational result that every admissible measurement factors through a finite sequence of binary partitions (Lemma 2A), the conclusion strengthens: *the Fold is the unique minimal and operationally unavoidable unit of physical distinction under finite admissibility*. It is minimal in three senses (structural, informational, operational) and unavoidable in the operational sense — every measurement physics can perform factors through Folds.

Physics does not begin with particles, fields, or strings. Under the axioms of finite distinguishability, finite encodability, internal closure, and background-independence, it begins — and can only begin — with the unique minimal act of distinction; and every measurement physics can ever perform factors through such acts.

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### **VERSF corpus — programme context**

The present paper is part of the VERSF (Void Energy-Regulated Space Framework) Theoretical Physics Programme. Specific corpus references include:

[14] [VERSF master action paper, full citation to be supplied at submission.] Establishes the closed form  $\mathcal{S}^{\{\mu\nu\}}[\Phi]$  for the commitment-density functional referenced as the matter-coupling functional in the broader programme.

[15] [VERSF unique record current paper, full citation to be supplied at submission.] Establishes the leading-order form of the record current  $C^{\{\mu\nu\}}[\Phi]$  under the constraints of the master action.

[16] [VERSF composition operation paper, full citation to be supplied at submission.] Specifies the operation by which Folds combine into higher-cardinality structures; referenced in Corollaries 3–5 and in the "What requires further input" subsection of §10.

[17] [VERSF Bit Conservation and Balance (BCB) / Ticks-Per-Bit (TPB) framework papers, full citations to be supplied at submission.] Foundational structural assumptions underlying the broader corpus.

## Notation

Symbol	Meaning
$\mathcal{D} = (S, R)$	Relational system: state set $S$ , distinguishability relation $R \subseteq S \times S$
$\mathcal{F}$	The Fold
$S \times S$	Cartesian product of $S$ with itself
$(s_i, s_j) \in R$	" $s_j$ is distinguishable from $s_i$ "
$\mathcal{D} \cong \mathcal{D}'$	Isomorphism of relational systems
$\text{Aut}(\mathcal{F})$	Automorphism group of $\mathcal{F}$
$\mathbb{Z}_2$	Cyclic group of order 2