Unified Covariant Action for Emergent Commitment Geometry in VERSF

Variational Closure, Effective Wilson Structure, Nonlocal Memory Geometry, and the Action-Level Recovery of Einstein Gravity

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General-Reader Summary

The earlier VERSF papers built the ingredients of a new picture of gravity step by step.

First came the substrate-level ideas — irreversible commitment events, finite distinguishability, and the K = 7 closure architecture. Then came the transport and memory papers, showing how information-like structures propagate through the substrate and leave behind long-lived causal memory traces. Later papers showed how a smooth Lorentzian continuum geometry could emerge from this structure, and how an effective stress–energy tensor could be built from commitment flow. Finally, the previous paper derived Einstein-compatible gravitational dynamics and showed that ordinary general relativity appears naturally as the low-memory, weak-anisotropy limit of the framework.

But one major piece was still missing.

All of these sectors — geometry, memory, transport, and source structure — had been assembled consistently, but they had not yet been shown to arise from a single variational principle. In modern physics, the deepest theories are usually written not directly as equations of motion, but as actions: compact mathematical objects whose variation generates the dynamics automatically. Einstein's equations come from varying the Einstein–Hilbert action. Maxwell's equations come from varying the electromagnetic action. Quantum field theories are built the same way.

This paper asks the VERSF version of that question:

Is there a single unified action from which the entire continuum-limit structure of VERSF gravity emerges?

The paper argues that the answer is yes.

The central result is that the previously separate sectors of the programme can now be organized into one covariant action containing:

- an Einstein-compatible geometric sector,
- a κ-field sector describing propagating commitment density,

- a nonlocal memory sector,
- an anisotropic transport-curvature sector,
- and a constrained exchange sector enforcing total conservation.

The action is not arbitrary. The paper shows that once the inherited structural rules of VERSF are imposed — finite distinguishability, irreversible commitment, CRE invariance, locality, causal propagation, tensorial closure, and parity-evenness — the allowed action structure becomes highly constrained.

One advance worth flagging is the treatment of the exchange sector. Earlier papers introduced an inter-sector tensor needed to maintain total conservation, but its origin still looked somewhat auxiliary. This paper resolves that by showing that the exchange structure can be generated variationally through a constrained action principle using a Lagrange-multiplier field. Conservation therefore becomes an Euler–Lagrange consequence of the action itself rather than something externally imposed.

The paper also clarifies the role of the anisotropic transport-curvature sector. Earlier treatments handled its couplings phenomenologically. Here, those couplings are reinterpreted as Wilson coefficients of the unique parity-even quadratic effective action compatible with the transport-curvature algebra. They become ordinary effective-field-theory coefficients rather than ad hoc corrections.

A further consequence is that Einstein gravity is recovered not only at the equation-of-motion level, but directly at the action level. In the weak-memory and weak-anisotropy limit, the full VERSF action reduces continuously to the Einstein–Hilbert action plus standard matter coupling. General relativity therefore appears as the leading continuum-limit action compatible with irreversible commitment transport.

The paper is careful about its remaining open problems. It does not yet derive the Wilson coefficients from substrate dynamics, does not yet derive the full nonlocal memory kernel from refinement theory, and does not yet derive the Einstein–Hilbert sector itself directly from the substrate. But those gaps are now sharply isolated inside a single coherent variational architecture.

Conceptually, the paper represents the variational closure of the previously derived geometry/source framework. Earlier work established ontology, geometry, memory, source structure, and Einstein-compatible dynamics. This paper unifies them into one continuum-limit field-theory framework.

Contents

Abstract

The preceding papers of the VERSF programme established:

- emergent Lorentzian continuum geometry,
- tensorial closure of the gravitational response sector,
- an effective commitment-generated stress–energy tensor,
- non-Markovian memory sourcing,
- anisotropic transport-curvature corrections,
- and Einstein-compatible dynamical field equations with continuous GR recovery.

However, the resulting framework remained partially assembled: the individual sectors were mutually compatible but had not yet been shown to descend from a single covariant variational principle.

The present paper addresses that problem.

We construct a unified covariant action

$\mathcal{S}_{\mathrm{VERSF}} = \mathcal{S}_{\mathrm{EH}} + \mathcal{S}_{\kappa} + \mathcal{S}_{\mathrm{mem}} + \mathcal{S}_{\hat{Q}} + \mathcal{S}_{\mathrm{ex}} + \mathcal{S}_{\mathrm{matter}}$,

where:

- $\mathcal{S}_{\mathrm{EH}}$ is the Einstein–Hilbert geometric sector,
- $\mathcal{S}_{\kappa}$ is the propagating commitment-density field action,
- $\mathcal{S}_{\mathrm{mem}}$ is the nonlocal retarded memory functional,
- $\mathcal{S}_{\hat{Q}}$ is the parity-even anisotropic transport-curvature action,
- $\mathcal{S}_{\mathrm{ex}}$ is a constrained exchange-sector action enforcing total conservation,
- and $\mathcal{S}_{\mathrm{matter}}$ is the effective matter sector.

The paper establishes five principal results.

(i) Variational Closure Theorem (§4). Under the admissibility conditions (A1)–(A8) of §3 — local Lorentz covariance, CRE invariance, parity-evenness, finite-distinguishability compatibility, causal propagation, second-order geometric scope, tensorial closure compatibility, and total conservation compatibility — the admissible continuum-limit action is restricted to a narrow variational class.

(ii) Einstein–Hilbert Compatibility (§5). The unique admissible local geometric action at leading second-order scope is the Einstein–Hilbert action,

$\mathcal{S}_{\mathrm{EH}} = (1 / 2\kappa_{\mathrm{eff}}) \int d^{(D+1)}x \sqrt{|g|}\, (R - 2\Lambda_{\mathrm{eff}})$.

(iii) Nonlocal Memory Variational Structure (§7). The memory sector is fundamentally bilocal and retarded:

$\mathcal{S}\_\mathrm{mem} = \tfrac{1}{2} \int d^{(D+1)}x \int d^{(D+1)}x' \sqrt{|g(x)|} \sqrt{|g(x')|}\, \Xi(x)\, \mathcal{K}(x, x')\, \Xi(x')$,

with asymptotic kernel envelope

$\mathcal{K}(\tau) \sim \cos(m\tau + \phi) / \tau$.

(iv) Constrained Exchange Closure (§8). The inter-sector exchange tensor arises as the Euler–Lagrange consequence of a Lagrange-multiplier-enforced conservation constraint imposed on the source-sector divergences, rather than as an externally assembled balance tensor.

(v) Action-Level GR Recovery (§10). In the weak-memory, weak-anisotropy, slowly-varying regime, the full VERSF action reduces continuously to the Einstein–Hilbert action plus ordinary matter coupling.

The paper does not yet:

- derive the Wilson coefficients of the anisotropic sector from refinement dynamics,
- derive the full bilocal kernel from substrate transport,
- derive the Einstein–Hilbert action directly from substrate combinatorics,
- or quantize the geometry.

Its contribution is structural: it supplies the first unified variational closure of the continuum-limit VERSF gravity programme.

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1. Introduction

The preceding VERSF papers established the ontology, geometry, source structure, and dynamical equations of the framework. The remaining open question is **variational closure**.

A modern continuum field theory is not complete when it merely supplies equations of motion. The deeper organizing structure is the action principle — the compact variational object from which the equations descend.

General relativity descends from variation of

$\mathcal{S}\_\mathrm{EH} = (1 / 16\pi G) \int d^4x \sqrt{|g|}\, (R - 2\Lambda)$.

Quantum field theories descend from analogous local or effective actions.

The present paper asks:

Can the continuum-limit structure of VERSF gravity be derived from a single admissible covariant action?

The answer developed here is affirmative, subject to the inherited continuum-limit assumptions of the previous papers.

The resulting action contains:

- a local Einstein-compatible geometric sector,
- a propagating κ-field sector,
- a fundamentally nonlocal retarded memory sector,
- an anisotropic transport-curvature sector,
- and a constrained exchange sector enforcing total conservation.

The paper's central claim is that, once the inherited VERSF admissibility structure is imposed, the variational architecture becomes highly constrained. The present work therefore represents not a new modification of gravity, but the variational closure of the previously derived geometry/source framework.

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2. Inherited Structures

We inherit without re-derivation:

- emergent Lorentzian continuum geometry $g_{\mu\nu}$,
- the four-sector effective stress tensor

  $$T^{\mathrm{eff}}_{\mu\nu} = T^{(\kappa)}_{\mu\nu} + T^{(\Xi)}_{\mu\nu} + T^{(\hat{Q})}_{\mu\nu} + T^{(\mathrm{int})}_{\mu\nu},$$

- total conservation

  $$\nabla^\mu T^{\mathrm{eff}}_{\mu\nu} = 0,$$

- tensorial closure forcing the response sector into $(1,1) \oplus (0,0)$,
- the $K = 7$ fixed κ-field mass

  $$m^2 = (4 / 3)\, \xi^{-2},$$

- the worldline memory kernel

  $$\mathcal{K}(\tau) \sim \cos(m\tau + \phi) / \tau,$$

- and the inherited substrate-scale coupling structure

  $$\kappa_{\mathrm{eff}} = 8\pi\, C_\lambda\, \hbar \xi^2 / c^3.$$

These structures are treated as the input architecture of the present construction. They are not re-derived; they are organized.

Notation now uses $\mathcal{S}$ for actions and $\mathcal{K}$ for the memory kernel, consistent with the variational-layer convention; the dynamical paper's S and K denote the same objects. The substrate-scale parameter ξ retains the convention of dynamical paper §2.7.

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3. Variational Admissibility Conditions

We now impose admissibility conditions on the action itself.

Definition 3.1 — Admissible continuum-limit action

An action

$\mathcal{S}[g, \kappa, \Xi, \hat{Q}, \Psi]$

is **admissible** if it satisfies:

- **(A1) Local Lorentz covariance.** All sector Lagrangian densities are scalars under the local Lorentz action on the emergent continuum geometry.
- **(A2) CRE invariance.** The action is invariant under commitment refinement equivalence, inherited from the substrate (source paper §3.4).
- **(A3) Parity-evenness.** No parity-odd operators appear at leading admissible order; this is inherited from the parity structure of the underlying commitment dynamics.
- **(A4) Finite-distinguishability compatibility.** No operator structure presupposes infinite resolution on the substrate; the continuum-limit operator basis respects the finite-distinguishability cutoff of source paper §2.1.
- **(A5) Causal propagation.** All propagating sectors satisfy finite-cone causal propagation on the emergent Lorentzian geometry.
- **(A6) Second-order geometric scope.** The geometric sector contains at most two derivatives of g_μν; higher-derivative corrections are subleading.
- **(A7) Tensorial closure compatibility.** The response-sector operator content is closed under the $(1,1) \oplus (0,0)$ tensorial decomposition inherited from the dynamical paper's tensorial closure theorem.
- **(A8) Total conservation compatibility.** The full action admits a constrained completion enforcing ∇^μ T^eff_μν = 0.

Weak-field Einstein recovery is **not** included as an admissibility input. Earlier framings of this construction included a corresponding (A9) condition; we drop it here to avoid the circularity of assuming what Theorem 10.1 derives. GR recovery is recovered as a consequence in §10.

Each of (A1)–(A8) restricts the algebraic and operator content of admissible Lagrangians. The Variational Closure Theorem (§4) then shows that the joint imposition of (A1)–(A8) leaves only a narrow class of continuum-limit actions.

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4. Variational Closure Theorem

Theorem 4.1 — Variational Closure Theorem

Under (A1)–(A8), the admissible continuum-limit action is restricted to

$\mathcal{S}_{\mathrm{VERSF}} = \mathcal{S}_{\mathrm{EH}} + \mathcal{S}_{\kappa} + \mathcal{S}_{\mathrm{mem}} + \mathcal{S}_{\hat{Q}} + \mathcal{S}_{\mathrm{ex}} + \mathcal{S}_{\mathrm{matter}}$,

up to boundary terms, higher-derivative Lovelock corrections, and freedom in the numerical values of Wilson coefficients within a fixed operator basis.

Proof sketch

The theorem assembles five previously established uniqueness results, each at its own admissibility level. We summarise the contribution of each sector.

Geometric sector. Tensorial closure plus Bianchi compatibility force the geometric sector into Einstein–Hilbert form, by the Bianchi-Compatible Geometry Theorem (Theorem 4.1 of the dynamical-geometry paper). The action-level analogue is fixed in §5 below: under (A6) and (A1), the unique parity-even local scalar at most second order in derivatives of $g_{\mu\nu}$ is $\sqrt{|g|}(c_1 R + c_0)$.

κ-field sector. Under (A1), (A4), (A5), and commitment-density sourcing, the unique admissible local propagating scalar action is the Klein–Gordon form of Definition 6.1. This is Theorem 5.2 of the source-structure paper (κ-Field Uniqueness Theorem); the action-level statement is the dual of the Klein–Gordon equation that theorem fixes at the equation-of-motion level.

Memory sector. Non-Markovian memory inheritance from the κ-memory programme (source paper §6.6; dynamical paper Definition 7.2) cannot be carried by any local truncation: a bilocal retarded functional is required. The local memory-curvature tensor of the dynamical paper is the leading derivative-expanded truncation, treated as such in §7.

Anisotropic transport sector. Parity-even (A3) tensorial closure (A7) of the transport-curvature algebra forces the quadratic $\hat{Q}$-sector into the unique parity-even effective action of fixed operator content, with Wilson coefficients $(\alpha_{\hat{Q}}, \beta_{\hat{Q}}, \gamma_{\hat{Q}})$ as the only remaining freedom. This is the Wilson-coefficient EFT result of the dynamical paper §8.2 / Proposition 8.3.

Conservation sector. (A8) is enforced via the constrained-action construction of §8 below, which promotes total conservation of $T^{\mathrm{eff}}_{\mu\nu}$ — established at the source level by Theorem 8.3 of the source paper — to an Euler–Lagrange consequence of the action principle. The exchange sector $\mathcal{S}_{\mathrm{ex}}$ is the unique constrained completion compatible with this requirement.

Exhaustiveness of the sector decomposition. The sector-by-sector arguments above fix each named sector given that the sector appears in the decomposition. What remains is to argue that no other sectors can appear at leading admissible order. The operator basis admissible under

(A1)–(A8) is exhausted by two independent classifications acting in concert. On the source side, the Admissible Source Uniqueness Theorem (source paper Theorem 3.1 / Theorem 8.2) together with (A7) tensorial closure fixes the response-sector operator basis at $(1,1) \oplus (0,0)$, exhausting the κ, Ξ, and $\hat{Q}$ sector families. On the geometric side, the Bianchi-Compatible Geometry Theorem fixes the geometric operator basis at the second-order parity-even level, exhausting the Einstein–Hilbert sector. The constrained-exchange sector is then unique by Theorem 8.2 once the source and geometric sectors are fixed: it is the unique Lagrange-multiplier completion enforcing $\nabla^\mu T^{eff}_{\mu\nu} = 0$ without modifying the field-equation content of the other sectors. (A2) CRE invariance acts as a filter on this joint classification: the inherited uniqueness results respect CRE invariance by construction, but the filter remains operative in excluding spurious operator candidates that would otherwise be admissible under (A1), (A3)–(A8) but not CRE-equivalent to the inherited substrate dynamics. No additional operator family survives the joint imposition of these classifications. This is the action-level analogue of the dynamical paper's Theorem 6.4 (admissibility uniqueness at leading order). ■

Remark on the 'up to' caveat

The Wilson-coefficient freedom is a freedom in **numerical values**, not in **operator content**. The theorem fixes the admissible operator basis of each sector; the numerical coefficients within that basis (notably $(\alpha_{\hat{Q}}, \beta_{\hat{Q}}, \gamma_{\hat{Q}})$ of the anisotropic sector and the kernel parameters of the memory sector) remain unconstrained at the continuum-limit level. Closing these values is OP3 and OP4 below; it is a closure of values within a fixed structure, not an ambiguity in the structure itself. Lovelock and boundary-term ambiguities are standard EFT subleading freedoms and are explicitly out of scope.

The theorem fixes the **sector content** of the action. The next sections fix each sector individually.

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5. Einstein–Hilbert Compatibility

Theorem 5.1 — Einstein–Hilbert uniqueness

Within the parity-even, local, second-order geometric action class, the unique admissible leading-order geometric action is

$\mathcal{S}_{EH} = (1 / 2\kappa_{eff}) \int d^{(D+1)}x \sqrt{|g|} (R - 2\Lambda_{eff})$.

Proof

Under (A6) (second-order geometric scope) and (A1) (Lorentz covariance), together with diffeomorphism invariance inherited from the Lorentzian continuum structure of the previous papers, the unique parity-even local scalar Lagrangian density at most second order in derivatives of $g_{\mu\nu}$ is

$\sqrt{|g|}\,(c_1 R + c_0)$,

up to total derivatives. This is the action-level uniqueness of the Einstein–Hilbert form among local diffeomorphism-invariant scalars at most second order in derivatives of g; it is a standard result of the diffeomorphism-invariant local-scalar classification at this order, distinct from (though parallel to) Lovelock's theorem at the equation-of-motion level. The Bianchi-Compatible Geometry Theorem of the dynamical-geometry paper supplies the corresponding equation-level statement. Variation with respect to $g_{\mu\nu}$ produces

$c_1 (R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu}) - \tfrac{1}{2} c_0\, g_{\mu\nu} = 0$,

i.e. the Einstein equations with cosmological constant. The matching condition of the GR Recovery Theorem of the dynamical paper ($\kappa_{\mathrm{eff}} = 8\pi G$ via §2.7) fixes $c_1 = 1/(2\kappa_{\mathrm{eff}})$ and $c_0 = -\Lambda_{\mathrm{eff}}/\kappa_{\mathrm{eff}}$, yielding the displayed form.

Higher-curvature Lovelock terms (Gauss–Bonnet and beyond) are excluded at leading admissible order by (A6) and inherited EFT minimality; they reappear as subleading admissible corrections beyond the scope of the present paper. ■

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6. κ-Field Sector

Definition 6.1 — κ-field action

$\mathcal{S}_\kappa = \int d^{(D+1)}x\, \sqrt{|g|}\, [\, -\tfrac{1}{2} \nabla_\mu\kappa\, \nabla^\mu\kappa - \tfrac{1}{2} m^2 \kappa^2 + \kappa\, \rho_{\mathrm{committed}}\,]$.

Variation of the linear source coupling $\kappa\rho_{\mathrm{committed}}$ against κ reproduces the inherited Klein–Gordon equation

$(\Box_g - m^2)\,\kappa = -\rho_{\mathrm{committed}}$

of source paper §2.5 and dynamical paper §2.2. Definition 6.1 is therefore the source-coupling **action form** of the κ-field sector, whose Euler–Lagrange equation coincides with the inherited field equation. The two formulations are equivalent up to boundary terms and field redefinitions.

Theorem 6.2 — κ-sector uniqueness

Under (A1), (A4), (A5), and commitment-density sourcing, the unique admissible local propagating scalar action — equivalent up to boundary terms and field redefinitions to the inherited Klein–Gordon equation of motion — is the action of Definition 6.1.

Proof

The result inherits from Theorem 5.2 of the source-structure paper (κ-Field Uniqueness Theorem), which fixes the Klein–Gordon structure as the only admissible propagating commitment-density equation compatible with finite distinguishability (A4), Lorentz covariance (A1), causal propagation (A5), and commitment-density sourcing. The action-level statement is the variational dual of that equation-level result: any local propagating scalar action whose Euler–Lagrange equation is the Klein–Gordon equation with linear source coupling is equivalent to Definition 6.1 up to boundary terms and field redefinitions. Metric variation yields $T^{(\kappa)}_{\mu\nu}$. No additional local propagating scalar sector is admissible at leading order. ■

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7. Nonlocal Memory Sector

Definition 7.1 — Nonlocal retarded memory action

$\mathcal{S}_{mem} = \frac{1}{2} \int d^{(D+1)}x \int d^{(D+1)}x' \sqrt{|g(x)|} \sqrt{|g(x')|}\, \Xi(x)\, \mathcal{K}(x, x')\, \Xi(x')$.

The kernel $\mathcal{K}(x, x')$ is retarded, causal, and parity-even, and supported on x' in the causal past of x.

Theorem 7.2 — Memory-kernel asymptotic structure

The retarded kernel satisfies

$\mathcal{K}(\tau) \sim \cos(m\tau + \phi) / \tau$

in the worldline asymptotic regime.

Proof

The result is inherited from the κ-memory programme of the VERSF source-structure paper (§6.6) and is unpacked in the dynamical paper §7.5 (with the substrate-scale conventions of dynamical paper §2.4). The four substrate-level ingredients that fix the displayed envelope are:

- the κ-field of source paper §2.5, with K = 7-fixed mass $m^2 = (4/3)\xi^{-2}$;
- the CCC coherence scale ξ inherited from the Lorentzian Completion paper;
- the Gaussian causal-coherence projection operator from substrate transport onto the worldline;
- the retarded Green-function reduction hierarchy whose Mellin asymptotic analysis produces the oscillatory algebraic envelope.

Their joint action fixes the $\cos(m\tau + \phi)/\tau$ structure as the universal worldline asymptotic form. ■

Proposition 7.3 — Local effective limit

In the weak-memory derivative-expansion regime — the (L1)–(L3) regime of source paper Proposition 6.5, carried through the dynamical paper's Proposition 7.4 —

$$\mathcal{S}_{\text{mem}} \to \int d^{(D+1)}x \sqrt{|g|}\, [\, a_0\, \Xi^2 + a_1\, \nabla_\mu \Xi\, \nabla^\mu \Xi + \cdots \,],$$

where the $\cdots$ denote the standard higher-order tail of the inherited derivative expansion. The local memory-curvature tensor is the leading effective truncation of the fundamentally bilocal action.

The memory sector is therefore elevated from an appended correction to a first-principles retarded bilocal functional whose asymptotic structure inherits directly from the κ-memory programme.

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8. Constrained Exchange Sector

Definition 8.1 — Exchange action

Introduce a Lagrange-multiplier covector field

A_{ex}^{ν}.

Define

$$\mathcal{S}_{\text{ex}} = \int d^{(D+1)}x \sqrt{|g|}\, A_{\text{ex}}^{\nu}\, \nabla^\mu \left(T^{(\kappa)}_{\mu\nu} + T^{(\Xi)}_{\mu\nu} + T^{(\hat{Q})}_{\mu\nu} + T^{(\text{int})}_{\mu\nu} \right).$$

Theorem 8.2 — Variational conservation closure

(a) Variation with respect to A_{ex}^{ν} yields

$$\nabla^\mu T^{\text{eff}}_{\mu\nu} = 0.$$

Total conservation therefore becomes an Euler–Lagrange consequence rather than an externally imposed condition.

(b) Variation with respect to $g_{\mu\nu}$, κ, Ξ produces additional terms proportional to A_{ex}^{ν}. These vanish on the multiplier's own field equation ($A_{\text{ex}}^{\nu} = 0$ in the non-degenerate sector), so the field equations reduce to those of $\mathcal{S}_{\text{VERSF}} - \mathcal{S}_{\text{ex}}$, recovering Definition 6.1 of the dynamical-geometry paper. ■

Parts (a) and (b) together ensure that $\mathcal{S}_{\text{ex}}$ modifies the field content only through the conservation constraint, with no additional contribution to the gravitational or matter sector equations of motion.

Structural significance

This resolves one of the major remaining weaknesses of the earlier dynamics paper. The inter-sector exchange tensor is no longer an externally assembled balance object; it is the constrained completion required by variational consistency.

Crucially, A_ex^ν is auxiliary — it carries no propagating degree of freedom. Its sole role is to enforce the divergence constraint at the action level, just as constraint multipliers do in standard gauge theory. The substrate-derivation target inside 𝒮_ex is T^(int)_μν itself, not the multiplier A_ex^ν, which remains a book-keeping device at all levels of the construction.

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9. Unified Field Equations

Variation of 𝒮_VERSF with respect to g_μν yields

G_μν + Λ_eff g_μν + M^(Ξ)_μν + Q^(Q̂)_μν + I^(int)_μν = κ_eff T^(κ)_μν,

where:

- M^(Ξ)_μν is the memory-curvature contribution from 𝒮_mem,
- Q^(Q̂)_μν is the anisotropic transport-curvature contribution from 𝒮_Q̂,
- I^(int)_μν is the geometric-side contribution from 𝒮_ex, equal to T^(int)_μν of §2 transposed to the curvature side of the field equation. The two notations denote the same object on opposite sides of the equality; I^(int)_μν is used in §9 to emphasize its geometric appearance as a curvature-side correction, T^(int)_μν elsewhere to emphasize its source-side role as the inter-sector exchange tensor.

This reproduces the dynamical field equation of the previous paper, now as a strict Euler–Lagrange consequence of a single variational principle rather than a multi-sector assembly.

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10. Action-Level GR Recovery

Theorem 10.1 — Action-level Einstein recovery

In the regime (R1)–(R3) of the dynamical-geometry paper (Theorem 10.2) — weak memory, weak anisotropy, slowly varying commitment density — and under the boundary-data assumptions (B1)–(B2) of the same theorem, the full action reduces continuously to

𝒮_EH + 𝒮_matter.

Proof

Under (R1)–(R3):

- $\mathbf{M^{(\Xi)}_{\mu\nu} \to 0}$ by the weak-memory derivative-expansion regime of (R1) (Proposition 7.3 above; source paper Proposition 6.5).
- $\mathbf{Q^{(\hat{Q})}_{\mu\nu} \to 0}$ by the weak-anisotropy regime of (R2).
- $\mathbf{T^{(int)}_{\mu\nu} \to 0}$ by the defining-relation argument of the dynamical paper's Theorem 10.2: the inter-sector exchange tensor inherits vanishing directly from the source-side divergences once the boundary-data assumptions (B1)–(B2) hold. The divergence-to-tensor implication $\nabla^\mu T^{(int)}_{\mu\nu} \to 0 \Rightarrow T^{(int)}_{\mu\nu} \to 0$ is **not** automatic; it requires the boundary-data conditions and is established as in the dynamical paper.

The corresponding sectors of $\mathcal{S}_{VERSF}$ therefore vanish continuously, with $H^k(K)$ continuity established by the Sobolev bound of the dynamical paper's Theorem 10.3 for any compact K. The remaining action is

$$\mathcal{S}_{EH} + \mathcal{S}_{matter},$$

whose Euler–Lagrange equations reduce to Einstein gravity. ■

Weak-field Einstein recovery is therefore a derived consequence of the variational closure, not an admissibility input — as anticipated in the remark following Definition 3.1. Moreover, the recovery operates at the level of the variational principle itself, not merely at the level of field equations: the Einstein–Hilbert action emerges as the continuum-limit attractor of the full VERSF variational structure.

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11. Structural Interpretation

The unified action organizes the programme into one continuum-limit field theory.

The architecture becomes:

commitment ontology → κ propagation → memory accumulation → effective stress-energy → Einstein-compatible geometry → variational closure.

The Einstein tensor therefore appears not as a primitive geometric object, but as the unique admissible local curvature structure compatible with:

- conserved irreversible commitment transport (Admissible Source Uniqueness Theorem of the source-structure paper, Theorem 3.1 / Theorem 8.2),
- Lorentzian causal completion (Lorentzian Completion paper of the VERSF programme; cf. CCC framework as inherited there),
- and variational closure (Theorem 4.1 above).

In standard field theory, geometry and matter are primitive ingredients and actions are postulated. In VERSF, irreversible commitment structure is primary, and geometry, memory, source structure, and the admissible action itself emerge from the closure constraints governing conserved commitment transport.

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12. Open Problems

The paper does not yet derive:

- the closure-normalisation factor C_λ,
- the Wilson coefficients of the anisotropic sector,
- the full bilocal kernel 𝒦(x, x′),
- the Einstein–Hilbert action directly from substrate combinatorics,
- Standard-Model matter emergence,
- quantum fluctuations of the geometry.

The remaining problems are now sharply isolated.

OP1 — Substrate derivation of the unified action

OP-numbering in this paper continues the dynamical-paper numbering, with each problem reframed at the variational level rather than the equation-of-motion level.

Can 𝒮_VERSF be derived directly from refinement dynamics on the void substrate? The present paper establishes the existence and uniqueness of the action under continuum-limit admissibility, but does not yet derive it from the underlying combinatorial commitment dynamics. Closing OP1 would mean showing that the entire variational structure — sectors, signs, and relative weightings — emerges as the refinement-stable closure of irreversible commitment transport, with no continuum-level input.

The substrate-derivation target inside 𝒮_ex is T^(int)_μν, not the Lagrange multiplier A_ex^ν; A_ex^ν remains a book-keeping device at every level of the construction and is not a candidate for microscopic promotion. OP1 is therefore the substrate derivation of the **physical content** of 𝒮_VERSF — the geometric, κ, memory, anisotropic, and inter-sector exchange tensors — not of the multiplier scaffolding that enforces conservation.

OP2 — Derivation of the Einstein–Hilbert sector from refinement transport

The present paper inherits

𝒮_EH = (1 / 2κ_eff) ∫ d^(D+1)x √|g| (R − 2Λ_eff)

as the unique admissible local geometric action compatible with the Bianchi-Compatible Geometry Theorem and the inherited Lorentzian continuum structure. What remains open is the substrate derivation of this action from refinement-stable commitment transport itself.

The key unresolved question is therefore:

Can the Einstein–Hilbert integrand $\sqrt{|g|}\, R$ be obtained directly from refinement transport curvature, closure-consistent parallel transport, and commitment-density flow, without assuming differential geometry a priori?

A successful derivation would close the gap between emergent continuum geometry and the continuum-limit gravitational action. This is likely the deepest remaining geometric closure problem in the VERSF programme.

OP3 — Substrate derivation of the full bilocal memory kernel

The present paper establishes:

- the existence of a nonlocal retarded memory action,
- its derivative-expanded local limit,
- and its worldline asymptotic envelope $\mathcal{K}(\tau) \sim \cos(m\tau + \phi) / \tau$.

However, the full bilocal kernel $\mathcal{K}(x, x')$ is not yet derived from refinement dynamics. The inherited κ-memory programme fixes the oscillatory structure, the algebraic decay, and the coherence-scale frequency, but not the complete geometry-dependent bilocal kernel.

The remaining open problem is therefore to derive the full causal kernel $\mathcal{K}(x, x')$ directly from refinement transport, CCC projection, and irreversible commitment propagation. This would complete the nonlocal memory sector at the microscopic level.

OP4 — Substrate derivation of anisotropic Wilson coefficients

The anisotropic transport-curvature sector now possesses a fully specified effective action, a variational origin, and a Wilson-coefficient interpretation. The remaining problem is no longer "why these terms?" but "why these coefficients?"

Specifically, derive

$(\alpha_{\hat{Q}}, \beta_{\hat{Q}}, \gamma_{\hat{Q}})$

from refinement transport dynamics, $K = 7$ closure counting, RG flow, or universality arguments. This is now a standard Wilsonian closure problem: the action structure is fixed, but the UV completion determining the coefficients remains open.

OP5 — Independent dynamics of the anisotropic transport sector

The present construction treats $\hat{Q}_{\mu\nu}$ as an effective source-sector contribution to the geometry. However, it remains unresolved whether $\hat{Q}_{\mu\nu}$ is purely constrained, or whether it possesses independent propagating degrees of freedom.

The open question is therefore:

Does the transport-curvature sector support independent propagating modes beyond those already encoded in $g_{\mu\nu}$, κ, and Ξ?

If so, the framework may contain genuinely new geometric excitations beyond ordinary Einstein gravity.

OP6 — Substrate derivation of the closure-normalisation factor

The present paper inherits the structural form

$$\kappa_{\text{eff}} = 8\pi\, C_\lambda\, \hbar\xi^2 / c^3.$$

The dimensional structure and scaling are therefore fixed. What remains open is the derivation of C_λ itself.

Specifically, derive the closure-normalisation factor from fold energetics, $K = 7$ binary suppression, refinement transport closure, and the hexagonal interface structure. Closing this problem would constitute a first-principles derivation of Newton's constant within the VERSF framework.

OP7 — Matter emergence

The present paper does not derive ordinary Standard-Model matter. Matter enters only through the effective recovery limit

$$T^{(\kappa)}_{\mu\nu} \to T^{(\text{matter})}_{\mu\nu}.$$

The major remaining open question is therefore:

How do stable matter sectors emerge from irreversible commitment dynamics?

Possible routes include stable transport-localized excitations, topological commitment defects, persistent closure modes, or refinement-stable cohomological structures. This remains one of the deepest unresolved problems in the entire programme.

OP8 — Quantum fluctuations of the geometry

The present treatment is entirely classical/effective. Quantum fluctuations of

$$g_{\mu\nu},\ \kappa,\ \Xi,\ \hat{Q}_{\mu\nu}$$

remain untreated. The major unresolved question is therefore:

What is the quantum completion of commitment geometry?

In particular: how commitment discreteness relates to quantum fluctuations, whether memory sectors regularize UV structure, and whether the nonlocal kernel modifies standard gravitational quantization all remain open.

13. Conclusion

The previous VERSF papers established:

- irreversible commitment ontology,
- emergent continuum structure,
- refinement-stable transport geometry,
- Lorentzian causal completion,
- non-Markovian memory,
- effective stress–energy sourcing,
- and Einstein-compatible dynamics.

The present paper closes the next structural layer — the variational architecture.

The central result is the existence of a unified covariant action

$$\mathcal{S}_{\mathrm{VERSF}} = \mathcal{S}_{\mathrm{EH}} + \mathcal{S}_{\kappa} + \mathcal{S}_{\mathrm{mem}} + \mathcal{S}_{\hat{Q}} + \mathcal{S}_{\mathrm{ex}} + \mathcal{S}_{\mathrm{matter}},$$

whose Euler–Lagrange equations reproduce the continuum-limit field equations derived in the preceding papers.

The construction establishes that geometry, source structure, memory response, anisotropic transport, and conservation can all be organized within a single variational framework.

The paper also resolves several major structural issues of the earlier dynamics programme.

First, the inter-sector exchange tensor is no longer an externally assembled balance object. Through the constrained exchange action, total conservation becomes an Euler–Lagrange consequence of the variational principle itself.

Second, the anisotropic transport-curvature sector is no longer phenomenological. Its couplings become Wilson coefficients of the unique parity-even quadratic effective action compatible with the transport-curvature algebra.

Third, the nonlocal memory sector is elevated from an appended correction to a first-principles retarded bilocal functional whose asymptotic structure inherits directly from the κ-memory programme.

The paper further proves that ordinary Einstein gravity is recovered continuously at the action level: in the weak-memory, weak-anisotropy, slow-variation regime, the full action reduces to

$S_{\mathrm{EH}} + S_{\mathrm{matter}}$.

General relativity therefore appears not as a competing theory, but as the leading continuum-limit variational structure compatible with conserved irreversible commitment transport.

The broader conceptual shift of the paper is this. In standard field theory, geometry and matter are primitive ingredients, and actions are postulated. In VERSF, irreversible commitment structure is primary, and geometry, memory, source structure, and even the admissible action itself emerge from the closure constraints governing conserved commitment transport.

The programme has therefore advanced through the sequence:

commitment ontology → continuum emergence → transport geometry → Lorentzian completion → effective stress-energy → Einstein-compatible dynamics → variational closure.

The remaining open problems are no longer foundational in character. They are concentrated in substrate derivations, Wilson-coefficient closure, matter emergence, and quantum completion.

The framework has therefore transitioned from a collection of emergent-geometry constructions into a unified continuum-limit field-theory architecture built from irreversible commitment dynamics.

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14. References to Inherited VERSF Papers

The present paper carries inline citations to several earlier VERSF papers. For a stand-alone read, each is identified below by title and the specific results invoked.

- **Source-Structure Paper** — *Effective Source Structure and Admissibility Closure in VERSF.* Establishes the four-sector effective stress tensor $T^{\mathrm{eff}}_{\mu\nu}$, the κ-field of §2.5 with K = 7-fixed mass, the Admissible Source Uniqueness Theorem (Theorem 3.1 / Theorem 8.2), the total-conservation result (Theorem 8.3), the κ-Field Uniqueness Theorem (Theorem 5.2), the κ-memory programme (§6.6), the (L1)–(L3) regime of Proposition 6.5, and the CRE invariance framework of §3.4.
- **Dynamical-Geometry Paper** — *Bianchi-Compatible Geometry and Einstein-Compatible Dynamics in VERSF.* Establishes the Bianchi-Compatible Geometry Theorem (Theorem 4.1), the tensorial closure structure (response sector decomposition into $(1,1) \oplus (0,0)$), the admissibility uniqueness theorem at leading order (Theorem 6.4), the anisotropic transport-curvature Wilson-coefficient EFT result (§8.2 / Proposition 8.3), the GR Recovery Theorem with (R1)–(R3) regime conditions and (B1)–(B2) boundary-data assumptions (Theorem 10.2), the $H^k(K)$ Sobolev continuity bound (Theorem 10.3), the local memory-curvature tensor (Definition 7.2), the inheritance of the κ-memory worldline kernel (§7.5), and the substrate-scale conventions (§2.4, §2.7).
- **Lorentzian Completion Paper** — *Lorentzian Causal Completion of Emergent Commitment Geometry in VERSF.* Establishes the emergent Lorentzian continuum

geometry $g_{\mu\nu}$, the CCC framework, the coherence scale ξ, and the causal-propagation structure inherited as (A5) admissibility.

- **κ-Memory Programme.** A multi-paper line of development unpacked across source-structure paper §6.6 and dynamical-geometry paper §7.5, supplying the four substrate-level ingredients of the worldline memory kernel: the κ-field, the CCC coherence scale, the Gaussian causal-coherence projection operator, and the retarded Green-function reduction hierarchy.

Numerical results and structural conventions inherited from these papers — including $m^2 = (4/3)\xi^{-2}$, $\kappa_{\text{eff}} = 8\pi C_\lambda \hbar\xi^2/c^3$, and the $K = 7$ closure architecture — are treated throughout the present paper as established inputs.