

Completion Density and Flavour Mixing from Refinement Geometry

Formulating the Efficiency–Completion Bridge and Recasting Sectoral Mixing as Completion-Channel Misalignment

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General Reader Summary

The Standard Model carries two patterns it records but does not explain.

The first is the **mass hierarchy**. Nature repeats the same family of matter three times. The electron, muon and tau carry identical charges and identical interactions, yet their masses differ enormously — the muon is about two hundred times the electron, the tau heavier still. The Standard Model encodes this through Yukawa couplings but offers no reason for the pattern.

The second is **flavour mixing**. Quarks and neutrinos do not stay perfectly confined to their own generation; they mix, and the strengths of those mixings are measured and stored in the CKM matrix (quarks) and the PMNS matrix (neutrinos). Again the Standard Model records the numbers without explaining them.

This paper proposes that both patterns are shadows of the same underlying object — the programme's refinement architecture — and it is careful, throughout, to separate what that proposal *establishes* from what it *assumes*. Two things should be stated plainly at the outset, because the title is deliberately *formulating*, not *closing*, and the saturation reading is graded as an arithmetic fact, not a mass claim.

What is secure. Earlier VERSF papers fixed three refinement levels generated by a doubling pattern of loads — one, two, four — inside a closure register of fixed size seven. This paper defines a quantity called *refinement efficiency*: the fraction of that register filled by each successive level. The arithmetic is exact and not in dispute: the levels fill one-seventh, three-sevenths, and finally all seven-sevenths of the register. The third level **saturates** it — it adds no new charge, it completes the count. That the efficiencies are ordered is a theorem of arithmetic.

What is proposed, not proved. Getting from "the third level fills the register" to "the third generation is the heaviest" runs through a chain, and this paper is exact about which links are settled and which are not. The chain is: refinement efficiency rises (proved arithmetic) → reuse density rises → completion density rises → mass rises. Three of these links are not in doubt. The

first new link — "rising efficiency comes with rising reuse" — is settled arithmetic given the inherited load pattern (one, two, four): for that specific pattern, both efficiency and a second ratio measuring how much standing structure each new increment supports are increasing, by direct calculation. (This is a fact about *that* pattern, not a general law: a different load pattern could break it, which is why the step is tied explicitly to the inherited loads rather than claimed universally.) The link "rising reuse raises completion density" leans on a result already proved in a companion paper (the Orbit Counts reuse condition), imported intact — and this paper checks that importing it does not smuggle the conclusion back in as a hidden assumption (the check passes: the imported result is arithmetic on commitment counts, and the only nearby notion that could have circularized it, "persistence," is used as a plain yes/no survival test, not as a disguised stand-in for the quantity being derived). The link "rising completion density raises mass" is, in its *ordering* part, trivially true — heavier-because-larger is just multiplication by a positive number — so the real content there is whether the companion anchoring programme's derivation of that relation survives scrutiny, which is a question about that paper, not about this link.

That leaves the genuinely open things, and they are now fewer and more concrete than before. The question that *used* to be open on this side — does rising efficiency come with rising reuse? — is now settled by direct calculation on the inherited load pattern. What remains is the step from reuse to *completion density*: reuse counts how much of a level's work is carried by already-existing structure, while completion density counts how often commitment happens per unit of return, and these are genuinely different quantities. Whether more of the first means more of the second — and means it strongly enough to clear a specific threshold the companion paper identifies — is the real remaining question on this side, and it is now a sharp, named one rather than a vague bridge. The other open thing is whether the companion anchoring programme's mass relation holds up. So instead of two opaque bridges, this version has a chain that is settled at every order-preserving and arithmetic joint, with the open surface reduced to a few specific, locatable questions — and it says exactly which is which. The honest reading of the hierarchy half: the saturation picture gives a clean structural reason to expect the third generation on top; most of the chain that would turn that into a derivation is now either proved here or borrowed proven from prior work; and what remains is a small set of specific, named questions, not a wall of assumption — but the chain does not yet, on its own, prove the mass order.

The mixing half is offered in the same spirit. The same completion structure is read by different interaction sectors through slightly different internal bases. Where two bases fail to line up, the states a particle *is* (flavour) and the states it *travels as* (transport) come apart, and that misalignment is the geometric origin of mixing. This is a genuine and standard *kind* of explanation — flavour mixing has always been a basis-mismatch phenomenon — and recasting it in completion-vs-transport language is the contribution. But what the refinement geometry would have to fix, to make this a derivation rather than a framing, is the *amount* of misalignment, and that this paper does not yet supply. It is graded accordingly.

So the unifying claim — **hierarchy from refinement saturation, mixing from completion-channel misalignment, both from one geometry** — is advanced here as a structured programme with its open links named and located, not as a finished derivation. That honesty is the point: the paper's value is in stating the open links precisely enough — and discharging the

ones prior work already settles — that a later paper can close the remainder and check itself against what was owed.

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1. Introduction

The preceding stages of the VERSF Standard Model programme established, at their stated grades, four results this paper builds on.

1. Gauge structure emerges from closure geometry.
2. Particle species correspond to stable representation classes.
3. Generation multiplicity attaches to refinement structure, not charge structure — the species index is charge-blind (the Replication Theorem).
4. The Yukawa hierarchy, given a completion-density ordering, follows; and that ordering, while no longer bare, rests on two open bridges (the Yukawa Hierarchy Theorem).

Those results leave two connected gaps, both named explicitly by the Yukawa paper as the work it handed forward:

- **The efficiency–completion gap.** The Yukawa paper derived a refinement-*efficiency* ordinal ($E_1 < E_2 < E_3$) and a bridge to the *completion-density* ordinal ($C_1 < C_2 < C_3$), but left that bridge — there called Corollary 5.8 — Conditional and unargued. Whether refinement efficiency and completion density co-vary monotonically was not established.
- **The mixing gap.** Flavour mixing had been gestured at as transport through generation space, but never connected to the completion-density architecture that orders the masses.

This paper does two things, and it is precise about the grade of each:

1. It **dissolves** the Efficiency–Completion Bridge (ECB) rather than carrying it as a standalone assumption: it replaces the single posited $E \rightarrow C$ bridge with a chain of four explicitly-graded links (§3A) — the reuse ordinal (E and R_u both increasing) **Proven arithmetic on the inherited dyadic loads** (Theorem 3A.1), reuse \Rightarrow completion **Conditional in direction** and **Open in magnitude** (Theorem 3A.2), and the mediant identity carrying the magnitude inequality to a rise in completion density **Inherited-Proven** (Theorem 3A.3, from Orbit Counts). This localizes all residual open content to the two clauses of a single arrow ($R_u \rightarrow C$) and audits the chain for circularity (passed). It does **not** fully close ECB; the title says *formulating*, not *closing*, and the two clauses of Theorem 3A.2 remain the eigenmode programme's to discharge.
2. It **recasts** CKM and PMNS mixing as completion-channel misalignment, distinguishing carefully between the parts that are inherited (with citation), the parts that are structurally forced, and the parts that are, at this stage, plausibility arguments motivated by the known result rather than predictions of it.

Two grade commitments govern the paper and are stated here atemporally. First, the efficiency–mass connection is *formulated*, not closed: the chain from refinement to mass is grade-separated link by link (§3A, §5), and its open links are named rather than asserted away. Second, the saturation reading is a Proven *arithmetic* fact (the third level fills the register, $E_3 = 1$) and is **not** a *physical* mass claim; the step from register-filling to "heaviest" runs through the graded chain and is Conditional. Both commitments are held in line with the discipline of the papers upstream.

2. Inherited Structure

The following are imported and used, not re-derived. Each is cited to its source arc and carries that source's grade.

H1 — Three refinement levels. The admissible refinement architecture contains exactly three levels,

$R_0, R_1, R_2,$

inherited from the species-count / census arc. **[Inherited]**

H2 — Dyadic loading. The refinement loads are

1, 2, 4,

with cumulative occupancies

1, 3, 7,

inherited from the refinement arc (binary refinement under uniform single-Fold individuation). **[Inherited]**

H3 — Closure capacity. The closure register has finite capacity

$K = 7$,

inherited from the closure/census arc. **[Inherited]**

H4 — Completion density. Mass-side amplification is governed by the **flip-completion density**

$C = p_v / K_c$,

where p_v is the per-tick coupling probability and K_c the anchoring depth (micro-events per committed bit-flip). This is the convention fixed in the anchoring foundation and carried by the Yukawa paper's §5c; **larger K_c means lower mass**, and heavier means *higher* C — not "deeper." This paper uses C in exactly that sense throughout, and never uses "deep" to mean "heavy." **[Inherited]**

The purpose here is not to rederive H1–H4 but to connect H2/H3 (a register-occupancy structure) to H4 (a per-tick flip rate) — and that connection is precisely the bridge whose conditionality the paper foregrounds.

3. Refinement Efficiency

Define the **cumulative refinement efficiency** at generation depth c by

$E(c) = L_{(\leq c)} / K$,

where $L_{(\leq c)}$ is the cumulative realised load up to depth c (H2) and K the closure capacity (H3). For the inherited dyadic loading inside the $K = 7$ register,

$E_1 = 1/7$, $E_2 = 3/7$, $E_3 = 7/7 = 1$.

The third refinement level saturates the register exactly — $E_3 = 1$, not a value exceeding unity or diverging. It fills the register; it does not overrun it.

Theorem 3.1 — Refinement-Efficiency Ordinal

Assume the inherited dyadic loads (1, 2, 4) and closure capacity $K = 7$. Then cumulative refinement efficiency is strictly ordered across the three admissible depths:

$$E_1 < E_2 < E_3.$$

Grade: Proven (arithmetic), conditional on the inherited loads and capacity. The loads and $K = 7$ are inherited (H2, H3), not derived here; given them, the ordinal is arithmetic.

Proof. The cumulative loads are 1, $1 + 2 = 3$, $1 + 2 + 4 = 7$. Dividing by the fixed capacity $K = 7$ gives $1/7 < 3/7 < 7/7$. ■

Interpretation — and its exact grade

It is convenient to call the third level the **saturation generation**, because $E_3 = 1$ fills the register. This name labels the *arithmetic* fact of Theorem 3.1 and nothing more. It does **not**, by itself, assert that the saturation generation is the heaviest — that physical reading requires the bridges of §3A, §4 and §5, all Conditional. The distinction is the one the Yukawa paper's final revision was at pains to draw: the register-filling is Proven; the mass consequence of register-filling is Conditional. This paper holds that line. Accordingly the saturation *interpretation as a mass claim* is graded **Conditional** (§10), and only the *efficiency ordinal* is Proven.

3A. Reuse and Refinement Efficiency

Section 3 ordered the three generations by *refinement efficiency* — how much realised distinction each level achieves per unit of closure register consumed (Theorem 3.1, and the Saturation–Reuse Lemma that reads it as occupying the fixed $K = 7$ register ever more fully). What that ordinal does not yet do is reach completion density $C = p_v/K_c$, the commitment-side quantity the mass side actually reads (H4). Earlier drafts bridged the gap with a single posited Efficiency–Completion Bridge ($E \rightarrow C$, "C is some positive monotone function of E") carried as a standalone Conditional assumption. This section dissolves that bridge entirely, replacing it with a chain of four explicitly-graded links — one Proven-arithmetic (on the inherited loads), one Conditional, one Open, one Inherited-Proven — so that no broad bridge is carried as an assumption and the residual open content is localized to two named items on a single arrow. The §4 "bridge" is then nothing more than the residue of this chain, not a fresh hypothesis.

Three distinct quantities, kept distinct

The chain below turns on keeping three ratios separate. Conflating any two of them is what would let the chain appear to close while smuggling its one substantive physical claim into a definition — and each carries content the others do not.

- **Efficiency** $E(c) = L_{(\leq c)}/K$, cumulative realised load against the *fixed* capacity $K = 7$ (the definition of §3). For dyadic loads: $E = 1/7, 3/7, 7/7$.
- **Reuse density** $R_u(c) = L_{(\leq c)}/\Delta L(c)$, cumulative realised load against the *marginal* load added at depth c . Higher R_u means each newly-added increment of structure supports a larger accumulated load — i.e. more of the standing load is carried per unit of freshly-added structure. For dyadic loads, with $\Delta L = 1, 2, 4$: $R_u = 1/1, 3/2, 7/4 = 1, 1.5, 1.75$.
- **Completion density** $C(c) = p_v/K_c$, committed distinctions per unit return complexity. *Commitment-side* — the quantity the Orbit Counts reuse condition governs and the mass side reads (H4).

These are three genuinely different quantities: E normalizes against fixed total capacity, R_u against the marginal increment, C against return complexity. None is a definitional restatement of another — which is exactly what the chain requires, since each arrow below must carry real content rather than re-expressing its predecessor.

Theorem 3A.1 (Efficiency \Rightarrow Reuse) — Proven (arithmetic), on the dyadic loads

Theorem 3A.1. For the inherited dyadic load sequence $\Delta L = (1, 2, 4)$ in capacity $K = 7$, both efficiency and reuse density are strictly increasing across the three depths, so the efficiency ordinal is accompanied by a reuse ordinal:

$$E_1 < E_2 < E_3 \text{ and } R_{\{u,1\}} < R_{\{u,2\}} < R_{\{u,3\}}.$$

Proof. Cumulative loads are $L_{(\leq c)} = 1, 3, 7$. Efficiency $E = L_{(\leq c)}/K = 1/7 < 3/7 < 7/7$ (Theorem 3.1). Reuse density $R_u = L_{(\leq c)}/\Delta L(c) = 1/1, 3/2, 7/4 = 1 < 1.5 < 1.75$. Both orderings hold by direct computation on the dyadic sequence. ■

Grade: Proven (arithmetic), conditional on the inherited dyadic loads (H2). This is a sibling of Theorem 3.1, not a definitional identity: it is the arithmetic fact that *this specific* load sequence makes reuse density monotone, and it uses the dyadic structure essentially. It is **not** a general entailment " $E \uparrow \Rightarrow R_u \uparrow$ for any loads at fixed K " — that is false (e.g. loads $(1, 2, 10)$ give rising E but $R_u = 1, 1.5, 1.3$, falling at the third step). What the theorem removes from the open column is therefore the same thing Theorem 3.1 removes: given the inherited loads, the reuse ordinal is settled arithmetic, not a physical hypothesis about dynamics. The dependence on H2 is explicit and is the honest price of the grade.

The reuse ordinal is thus inherited-loads arithmetic, exactly as the efficiency ordinal is. Neither asserts anything yet about completion density C ; that is the next, substantive, arrow.

Theorem 3A.2 (Reuse \Rightarrow Completion) — the residue, split

R_u (load against added structure) and C (commitment against return complexity) are different quantities, so the step from rising reuse density to rising completion density is **not** definitional — it is where the chain's one substantive physical claim lives, and it factors into a direction and a magnitude.

Theorem 3A.2 (direction). Rising reuse density entails rising completion density in sign: $R_u \uparrow \Rightarrow C \uparrow$.

Grade: Conditional. The structural reason to expect it: when each newly-added increment of structure supports a larger accumulated load (higher R_u), additional distinctions are committed against *shared* return paths rather than freshly-built ones, so committed count p_v accumulates while return complexity K_c is largely reused — pushing $C = p_v/K_c$ up. This is a genuine inference, not an identity, because R_u measures load-against-marginal-structure while C measures commitment-against-return: nothing in the refinement accounting *forces* the commitment rate to rise, though it makes it the natural expectation. This is the sharp, named question the residue reduces to — no longer "does efficiency increase reuse?" (Theorem 3A.1 settles that) but "does rising reuse translate into commitment-density?"

Theorem 3A.2 (magnitude) / the Orbit Counts inequality. Granting the direction, does C rise *enough* — does the marginal commitment outpace the marginal return at each step?

$$\Delta p_v / \Delta K_c > p_v / K_c.$$

Grade: Open. This is the single irreducible quantitative question, and it is exactly the hypothesis of the inherited Orbit Counts reuse condition (below). Settling it requires the explicit eigenmode computation (anchoring §16): compute $p_v(c)$, $K_c(c)$ per refinement class and check the inequality across the three.

Theorem 3A.3 (the inherited mediant identity) — Inherited Proven

Theorem 3A.3 [Inherited — Orbit Counts, Proven]. Completion density rises under refinement exactly when the marginal commitment-per-return exceeds the running average:

$$C(\mathcal{RO}) > C(\mathcal{O}) \Leftrightarrow \Delta p_v / \Delta K_c > p_v / K_c.$$

Grade: Inherited Proven. This is arithmetic on orbit counts — the mediant identity — proved in the Orbit Counts paper and imported unmodified. It is what turns the magnitude clause of Theorem 3A.2 into a rise in C : once the inequality holds, C provably increases. Note it governs C 's *self-increment under refinement*, which is precisely why the magnitude clause (3A.2) is the thing that feeds it.

Non-circularity audit (recorded, passed on the points checked)

The chain leans on two papers developed **independently of the anchoring/completion-density programme** — Binary Refinement (dyadic loads, finite hierarchy) and the Realisation Criterion (commitment-cycle realisation, persistence). That independence is the source of the bridge's

evidential weight: if the refinement-side steps can be established from a refinement programme that never mentions p_v/K_c , the meeting of that programme with the anchoring programme at the same ordering is a non-trivial structural coincidence, not a restatement. The audit confirms the chain *terminates* rather than circling:

- *Vocabulary clause. Binary Refinement: yes* — its outputs are admissible-structure counts (loads 1, 2, 4; finite hierarchy), pure refinement combinatorics with no magnitude that could be completion density. **Realisation Criterion: yes** on the symbols p_v, K_c, C .
- *Definitional clause.* The one candidate for a hidden completion-density equivalent is **persistence** in the Realisation Criterion, since a *graded* persistence equivalent to p_v/K_c would circularize the chain. **Audit result: persistence is used binary, not graded** — a census gate (does the mode survive as an admissible recurrent object under refinement, yes/no), not a magnitude. The specific check: the Realisation Criterion's admissibility clause uses persistence only to decide *membership* in the recurrent census (the survive/not-survive partition), never to *rank* surviving modes; and the companion realisation/gap-functional material explicitly holds the graded persistence-clause reading *open* and records that gap-stationarity under the weak separation-survival reading is *not load-bearing downstream* — i.e. nothing downstream consumes a graded persistence. A binary admissibility condition cannot encode the p_v/K_c ordering. The reuse condition itself (3A.3) is driven by orbit-count increments ($\Delta p_v, \Delta K_c$) via the mediant identity — arithmetic, not a graded persistence measure. (*The force of this clause depends on the Realisation Criterion's actual usage; the pointer above is where to verify it.*)

Conclusion of the audit: no circularity on the points checked. The chain runs Binary Refinement \rightarrow Realisation (binary) \rightarrow refinement-ordinal (E, R_u) \rightarrow completion density C , with C appearing only at the output end — not $C \rightarrow$ Realisation \rightarrow Reuse $\rightarrow C$.

What §3A achieves

The Efficiency–Completion Bridge no longer exists as a standalone assumption. It is replaced by a chain of four explicitly-graded links:

Link	Statement	Grade
3A.1	E ordered and R_u ordered	Proven (arithmetic, dyadic loads H2)
3A.2 (direction)	$R_u \uparrow \Rightarrow C \uparrow$ (sign)	Conditional (reuse-density \Rightarrow completion-density; non-circular)
3A.2 (magnitude)	$\Delta p_v / \Delta K_c > p_v / K_c$	Open (the one inequality)
3A.3	inequality $\Rightarrow C$ rises (mediant identity)	Inherited Proven (Orbit Counts)

The $E \rightarrow R_u$ arrow is **Proven arithmetic** on the inherited dyadic loads (Theorem 3A.1, a sibling of Theorem 3.1) — and so the open surface of the whole efficiency-to-completion step contracts to exactly two named items, both on the single $R_u \rightarrow C$ arrow: the **direction** (reuse

⇒ completion-density, Conditional) and the **magnitude** (the Orbit Counts inequality, Open). The substantive physical content has not vanished — it cannot, since reuse density and completion density are genuinely different quantities — but it is now located precisely on one arrow and split into a sign question and a rate question, rather than diffused across a vague bridge.

4. The Efficiency–Completion Bridge (ECB), as the Residue of §3A

Section 3A dissolved the efficiency-to-completion step into four explicitly-graded links — Theorem 3A.1 (Proven arithmetic, dyadic loads), Theorem 3A.2 direction (Conditional) and magnitude (Open), and Theorem 3A.3 (Inherited Proven). This section states what remains once those are in place — the open content is exactly the two clauses of Theorem 3A.2, on the single $R_u \rightarrow C$ arrow — and what a full discharge would require.

ECB, now nothing but the residue of the §3A chain

Refinement efficiency drives completion density upward:

$$E_1 < E_2 < E_3 \Rightarrow C_1 < C_2 < C_3.$$

This is no longer a posited bridge. It is the composition of the four §3A links: Theorem 3A.1 (E and R_u both ordered, **Proven arithmetic on dyadic loads**), Theorem 3A.2 direction ($R_u \uparrow \Rightarrow C \uparrow$, **Conditional**), Theorem 3A.2 magnitude ($\Delta p_v / \Delta K_c > p_v / K_c$, **Open**), and Theorem 3A.3 (the inequality $\Rightarrow C$ rises, **Inherited Proven**). In place of a bare " $C = F(E)$, $F'(E) > 0$ " posit (cf. Corollary 5.8 of the Yukawa paper) stands a graded chain whose only non-settled content sits on a single arrow.

The residue, made precise — and what it is not

The residue is **not** "is this circular?" — the §3A audit closes that. Nor is it "is the reuse ordinal settled?" — Theorem 3A.1 settles it by direct computation on the inherited dyadic loads (a sibling of Theorem 3.1), so the reuse ordinal is arithmetic, not assumed. With both cleared, the entire residual open content lives on the one arrow $R_u \rightarrow C$, where a reuse-density quantity meets a commitment-density quantity, and it is exactly two named items:

1. *Direction (Theorem 3A.2, Conditional)*. Does rising reuse density — each newly-added increment of structure supporting a larger accumulated load — entail a rising commitment-per-return $C = p_v / K_c$ in sign? This is a genuine inference, not an identity, because R_u measures load-against-marginal-structure while C measures commitment-against-return. It is the sharp question the residue reduces to.
2. *Magnitude (Theorem 3A.2, Open)*. Granting the sign, does C rise *enough* — $\Delta p_v / \Delta K_c > p_v / K_c$ at each step — or does return complexity K_c grow fast enough to offset the

gain? This is the single irreducible inequality, and it is precisely the hypothesis the inherited median identity (3A.3) consumes.

Since K_c is a refinement-class property shared across sectors (Yukawa §7) while reuse density is provably monotone (Theorem 3A.1), the schematic favours both, but a schematic is not a settlement.

Corollary 4.1 — Completion-Density Ordinal

Given Theorem 3A.1 (Proven) and Theorem 3A.3 (Inherited Proven), the completion-density ordinal

$$C_1 < C_2 < C_3$$

follows from Theorem 3A.2 — its direction (Conditional) and its magnitude (Open) — and nothing else. **Grade: Conditional (direction) + Open (magnitude), on the single $R_u \rightarrow C$ arrow.** The efficiency ordinal and the efficiency-to-reuse step are Proven; the reuse condition's arithmetic is Inherited-Proven; the only open content is whether rising reuse translates into commitment-density, and at sufficient rate.

What discharging the residue would require

The explicit eigenmode computation (anchoring §16): compute $p_v(c)$ and $K_c(c)$ per refinement class from the closure dynamics, exhibit $C(c) = p_v(c)/K_c(c)$, and check (i) that it rises with $R_u(c)$ in sign (settling 3A.2 direction) and (ii) that the reuse inequality $\Delta p_v / \Delta K_c > p_v / K_c$ holds across the three classes (settling 3A.2 magnitude) — at which point 3A.3 carries it to rising C . This paper supplies the graded chain and the target; the computation remains owed, and the conclusion (§11) records it as the central remaining objective.

5. Mass Ordering

The anchoring programme identifies mass-side amplification with completion density:

ANCH-COMP

$$m = G(C), G'(C) > 0.$$

This bridge bundles two separable claims at very different states, and the honesty of the chain depends on pulling them apart.

The monotonicity is trivial — Proven. Under the anchoring foundation G is the near-identity $m = A \cdot C$ with $A = \kappa_A \hbar / (c_{\text{light}}^2 \Delta t) > 0$ a positive constant (here c_{light} is the speed of light, written out to avoid collision with the depth index c used in $E(c)$, $C(c)$; and κ_A is the anchoring

constant, written to avoid collision with the rejected CKM amplitude η of §8). Ordering is then immediate:

Theorem 5.1 (ANCH-COMP monotonicity). If $m = A \cdot C$ with $A > 0$, then $C_i < C_j \Rightarrow m_i < m_j$. **Proof.** Multiplication by a positive constant preserves strict order. ■ **Grade: Proven (arithmetic).**

This is not a place where the chain can fail. Given the anchoring relation, the mass ordering follows by one line.

The anchoring relation itself is Inherited-Conditional — and that is where the only risk lives. What is *not* trivial is that $m = A \cdot C$ holds at all: that mass-side amplification is the completion density up to the constant A . This is derived in the anchoring foundation, conditional on its own postulates — the action postulate $P\hbar$ and the $\Delta t = t_P$ calibration (Yukawa §5c). So the question for bridge 2 is **not** "does mass depend monotonically on completion density?" (Theorem 5.1 settles that trivially) but "does the anchoring programme's derivation of $m = A \cdot C$ survive review?" The bridge already *exists* inside VERSF; its conditionality is the survival of an upstream derivation, not the existence of the link.

Net grade of bridge 2: monotonicity Proven (Theorem 5.1); anchoring relation Inherited-Conditional on the anchoring programme's postulates. This is a sharper placement than a flat "Conditional," and it matters for reading the chain: bridge 2 is largely closed inside the framework, with the residual risk located precisely at the upstream anchoring derivation rather than at this link.

The layered chain

Combining Theorem 3.1, the §3A factoring, and the bridge-2 split above gives the full hierarchy argument as a chain of links of separated grade — now with the efficiency-to-completion step split into its Conditional and inherited-Proven halves (§3A), and the completion-to-mass step split into its trivial-Proven monotonicity and its Inherited-Conditional anchoring relation:

refinement \Rightarrow efficiency ordinal \Rightarrow reuse density \Rightarrow completion-density ordinal \Rightarrow mass ordinal.

Link	Statement	Grade
1	dyadic loads + $K = 7 \Rightarrow E_1 < E_2 < E_3$	Proven (arithmetic) — Theorem 3.1
2a	E ordered and R_u ordered (efficiency, reuse)	Proven (arithmetic, dyadic loads) — Theorem 3A.1
2b	$R_u \uparrow \Rightarrow C \uparrow$ in sign (reuse \Rightarrow completion, direction)	Conditional — Theorem 3A.2; reuse-density \Rightarrow completion-density, non-circular
2c	$\Delta p_v / \Delta K_c > p_v / K_c$ (magnitude)	Open — Theorem 3A.2; the single irreducible inequality

Link	Statement	Grade
3	inequality \Rightarrow C rises (mediant identity)	Inherited Proven — Theorem 3A.3 (Orbit Counts)
4a	C ordered \Rightarrow m ordered, given $m = A \cdot C$	Proven (arithmetic) — Theorem 5.1
4b	$m = A \cdot C$ holds (anchoring relation)	Inherited Conditional — anchoring programme ($Ph, \Delta t$)

So:

$E_1 < E_2 < E_3$ (Proven), and *if* the reuse \Rightarrow completion direction (2b) holds at sufficient magnitude (2c) and the anchoring relation (4b) survives review, then $m_1 < m_2 < m_3$ — the reuse ordinal (2a, Proven arithmetic on the dyadic loads), the mediant identity (3), and both monotonicities (4a) being already Proven or Inherited-Proven.

Grade of the mass hierarchy: Conditional, and the conditionality is now resolved into a small, concrete open surface — with the reuse ordinal in the Proven-arithmetic column (Theorem 3A.1). What is settled: the efficiency ordinal (Proven), the reuse ordinal (Proven arithmetic on the inherited dyadic loads — a sibling of Theorem 3.1, holding by direct computation for the load pattern 1, 2, 4 and tied explicitly to it), the mediant identity carrying the reuse inequality to a rise in completion density (Inherited-Proven), and both monotonicities (Proven/trivial). What remains open is exactly three named items, none of them an opaque bridge: **(2b)** the *reuse* \Rightarrow *completion* direction — whether rising reuse density entails a rising commitment-per-return $C = p_v/K_c$ in sign (Conditional, non-circular, a sharp question, because reuse density and completion density are genuinely different quantities); **(2c)** the *magnitude* inequality $\Delta p_v/\Delta K_c > p_v/K_c$ (Open, one inequality); and **(4b)** the *anchoring relation* $m = A \cdot C$ (Inherited-Conditional on the anchoring programme surviving review). The position is "Conditional on the two clauses of Theorem 3A.2 \wedge the anchoring relation": both bridges are dissolved into graded links, and in each the order-preserving and arithmetic halves are now Proven/Inherited while only narrowly-named inferences remain. The chain is still not a derived mass ordering and must not be quoted as one — but the open surface is now three concrete questions, not two opaque bridges.

The per-sector caveat (inherited from Yukawa §6a)

One honesty constraint carries over and must be stated. Theorem 3.1 supplies a *single* ordinal over refinement depth. The Yukawa paper's Result 7.1 showed the observed mass ratios force *sector-dependent* structure, so what the mass result ultimately consumes is an ordered triple of completion densities **per charged sector**, $C_a(1) < C_a(2) < C_a(3)$, with the sectoral structure carried by a sector-dependent $p_{\{v,a\}}(c)$ and K_c shared across sectors. Whether the single efficiency ordinal of Theorem 3.1 *induces* the three per-sector orderings — or whether the sectoral sensitivity perturbs them — is **not** settled here. ECB as stated bridges to a single C-ordinal; lifting it to three per-sector triples is an additional open step, part of the same eigenmode debt. The chain above is therefore complete only at the level of the single ordinal; the per-sector lift is Open.

6. Sectoral Completion Spaces

The mixing half of the paper begins here, and its framing must be distinguished at the outset from a derivation: §6–§7 establish the *form* of a misalignment explanation; what would make it a derivation — fixing the *amount* of misalignment from the refinement geometry — is identified as owed, not delivered.

Completion density determines how fully a mode occupies the closure register. But the register can be *read* by different interaction sectors through different internal bases. Let

\mathcal{H}_C

be the **completion basis** — the basis in which completion/transport through generation space is diagonal. For each interaction sector S (up-type, down-type, charged lepton, neutrino), let

\mathcal{H}_S

be the **sector basis** — the basis in which that sector's flavour states are defined.

If $\mathcal{H}_S = \mathcal{H}_C$ for a given sector, the flavour states coincide with the transport states and **no mixing occurs** in that sector. If $\mathcal{H}_S \neq \mathcal{H}_C$, the two bases are misaligned, and flavour and transport come apart.

This is the standard origin of flavour mixing — physical flavour eigenstates differing from propagation eigenstates — recast in completion-vs-transport language. The recasting is the contribution; the mechanism-type is not new.

7. Completion-Channel Misalignment

Let

$U_S : \mathcal{H}_C \rightarrow \mathcal{H}_S$

be the unitary relating the completion basis to sector S's basis. A flavour state and its transport representation are related by

$|f\rangle_S = U_S |f\rangle_C.$

Mixing in sector S is present exactly when

$U_S \neq I.$

The product of two sectors' misalignments is what is physically observed: the CKM matrix is the relative misalignment of the up-type and down-type sectors,

$$V_{\text{CKM}} = U_{\text{up}} U_{\text{down}}^\dagger,$$

and the PMNS matrix the relative misalignment of the charged-lepton and neutrino sectors,

$$U_{\text{PMNS}} = U_e U_\nu^\dagger.$$

Grade of §6–§7: Proven as a framing; Open as a derivation. That mixing *equals* relative basis misalignment is true by construction — it is the definition of mixing, recast. What is **not** supplied here is what fixes each U_S from the refinement geometry: the *magnitude and structure* of the misalignment. Stating "if the bases differ there is mixing" predicts nothing until the geometry that determines U_S is specified. So §6–§7 establish that mixing *would* arise from completion-channel misalignment and *how* it would be organized (relative misalignments of sector pairs); they do not yet establish that the *observed* CKM and PMNS structures arise from *computed* misalignments. The next two sections inherit that limitation and are graded under it.

8. CKM Structure

This section is governed by an explicit honesty constraint: any numerical input is either cited to the inherited paper that derives it, or graded as not-yet-derived and not used.

What is inherited, and what is not

The transport programme provides a generation-space transport operator built on the dyadic structure $D = \text{diag}(1, 2, 4)$. **If** that programme additionally fixes specific attenuation amplitudes — candidate values $\eta = 3/5$, $\chi = 2/5$ appear in the transport literature — then those values must be cited to the precise inherited result that derives them, including what they are (amplitudes? probabilities? a 5-channel partition?) and why those rationals. **As of this paper they are not so cited, and they are therefore not used.** Asserting $\eta = 3/5$ and $\chi = 2/5$ with no derivation and no comparison to a measured CKM element would be exactly the unsupported numerical claim the programme's grading discipline exists to prevent. They are flagged here as an **owed citation**: if the transport arc derives them, cite it; if it does not, they are a new claim and must be graded, not inherited.

What the present framework does say about CKM

Independently of any specific amplitude values, §7 yields a structural statement that *can* be made at grade:

The CKM matrix is the relative completion-channel misalignment of the up-type and down-type sectors, $V_{\text{CKM}} = U_{\text{up}} U_{\text{down}}^\dagger$.

Grade: Conditional (on §6–§7's framing, and on the up/down sector bases being determined by the refinement geometry). This says CKM mixing is small *because* the up-type and down-type completion bases are nearly aligned — $U_{\text{up}} U_{\text{down}}^\dagger \approx I$ — and locates the smallness of quark mixing in the near-coincidence of the two quark sectors' reading of the same completion register. It does **not** yet predict the Cabibbo angle or any V_{ij} ; doing so requires computing U_{up} and U_{down} from the geometry (the owed step of §7).

The falsifiable target

To become a derivation rather than a framing, this section must produce U_{up} and U_{down} from the refinement transport dynamics and show $U_{\text{up}} U_{\text{down}}^\dagger$ reproduces the measured CKM hierarchy — at minimum the dominance of the diagonal, the Cabibbo-scale (≈ 0.225) first-off-diagonal, and the smaller V_{ub} , V_{cb} . That computation is the explicit deliverable owed to a future paper. Stating it this way makes the claim falsifiable: a computed $U_{\text{up}} U_{\text{down}}^\dagger$ that did *not* land near-diagonal would refute the misalignment account for quarks.

9. PMNS Structure

The same honesty constraint applies, and the asymmetry between large neutrino mixing and small quark mixing must be presented as **motivated by** the known result, not **predicted** from first principles — anything stronger would be postdiction dressed as derivation.

The structural argument (Conjectural)

Neutrinos do not generate closure commitments in the way confined quarks do: a neutrino's weaker coupling to the anchoring channel (the same chirality-gating that the companion papers invoke to make neutrinos ultra-light) means its transport basis is more weakly constrained by the completion register. A more weakly constrained sector basis is, plausibly, *further* from the completion basis — a larger misalignment U_{ν} — so

$$U_{\text{PMNS}} = U_{\text{e}} U_{\nu}^\dagger$$

is naturally further from the identity than $V_{\text{CKM}} = U_{\text{up}} U_{\text{down}}^\dagger$.

Grade: Conjectural, and explicitly retrodictive. This is a mechanism *consistent with* the observed fact that PMNS mixing is large while CKM mixing is small; it is **not** a prediction of that fact, because nothing here fixes *how much* further from the identity U_{ν} sits, nor rules out the opposite sign of the effect from first principles. The argument earns its keep only as a structural reason the two sectors could differ in mixing magnitude — and as a target: a future computation must produce U_{ν} from the weaker neutrino closure-commitment and show $U_{\text{e}} U_{\nu}^\dagger$ reproduces the large PMNS angles, *without* having been tuned to them. Presented as anything firmer, it would violate the programme's rule against postdicting a known result and calling it prediction.

What would make it a prediction

The same eigenmode deliverable as §8, applied to the neutrino sector: compute U_ν from the closure dynamics under the weaker neutrino commitment, independently of the measured PMNS values, and check the output against the data. Until then the large-mixing/small-mixing asymmetry is an attractive consistency, not a result.

10. Status Table

Claim	Grade	Dependency / Note
Three refinement levels (H1)	Inherited	species-count / census arc
Dyadic loads (1,2,4); cumulative (1,3,7) (H2)	Inherited	refinement arc
Closure capacity $K = 7$ (H3)	Inherited	closure / census arc
Completion density $C = p_\nu/K_c$ (H4)	Inherited	anchoring foundation; heavier = higher C, not "deeper"
Refinement-efficiency ordinal $E_1 < E_2 < E_3$	Proven (arithmetic)	Theorem 3.1, given H2/H3
Saturation <i>as arithmetic</i> ($E_3 = 1$, register filled)	Proven	Theorem 3.1
Saturation <i>as a mass claim</i> (third gen heaviest)	Conditional	via Theorem 3A.2 \wedge anchoring relation; not Proven
Reuse ordinal (E and R_u both increasing)	Proven (arithmetic)	Theorem 3A.1; on dyadic loads H2; sibling of Thm 3.1
Reuse \Rightarrow completion, direction ($R_u \uparrow \Rightarrow C \uparrow$ in sign)	Conditional	Theorem 3A.2; reuse-density \Rightarrow completion-density, non-circular; different quantities
Magnitude ($\Delta p_\nu / \Delta K_c > p_\nu / K_c$)	Open	Theorem 3A.2; the single irreducible quantitative inequality
Mediant identity (inequality \Rightarrow C rises)	Inherited Proven	Theorem 3A.3; Orbit Counts, arithmetic on orbit counts
Non-circularity audit (persistence binary, not graded)	Recorded — passed	§3A; chain terminates Binary Refinement \rightarrow Realisation (binary) \rightarrow $E, R_u \rightarrow C$
ECB (= efficiency \rightarrow completion density)	dissolved into 3A.1–3A.3	§4; Proven (2a) + Conditional (2b) + Open (2c) + Inherited (3)
Completion-density ordinal $C_1 < C_2 < C_3$	Conditional + Open	Corollary 4.1, on Theorem 3A.2 (2a Proven, 3A.3 Inherited)

Claim	Grade	Dependency / Note
ANCH-COMP monotonicity ($C_i < C_j \Rightarrow m_i < m_j$ given $m = A \cdot C$)	Proven (arithmetic)	§5 Theorem 5.1; trivial, positive constant
ANCH-COMP anchoring relation ($m = A \cdot C$ holds)	Inherited Conditional	§5; anchoring programme, on $P\hbar$ and Δt calibration
Mass hierarchy $m_1 < m_2 < m_3$ (single ordinal)	Conditional	open surface = Theorem 3A.2 (direction + magnitude) \wedge anchoring relation; all else Proven/Inherited
Per-sector lift (single ordinal \rightarrow three triples)	Open	§5; eigenmode debt, inherited from Yukawa §6a
Mixing = relative basis misalignment (framing)	Proven (by construction)	§6–§7; true as a definition recast
Magnitude/structure of misalignment U_S from geometry	Open	§7; the derivation, not delivered
CKM = $U_{up} U_{down}^\dagger$ (structural)	Conditional	§8; on the framing + geometry fixing the bases
CKM numerical amplitudes (η, χ)	Owed citation / ungraded	§8; cite the transport arc or grade as new — not used here
CKM reproduces measured hierarchy	Open	§8; the falsifiable target
PMNS larger than CKM	Conjectural (retroductive)	§9; consistent with, not predictive of, the data
PMNS reproduces measured angles	Open	§9; the falsifiable target

The grades this table fixes, stated as standing facts: **saturation-as-mass-claim is Conditional** (the arithmetic $E_3 = 1$ is Proven; the mass reading runs through the graded chain); **the efficiency–completion bridge is a chain of four graded links** — **the reuse ordinal Proven-arithmetic on the inherited dyadic loads (Theorem 3A.1, a sibling of Theorem 3.1), reuse \Rightarrow completion Conditional in direction and Open in magnitude (Theorem 3A.2), and the mediant identity Inherited-Proven (Theorem 3A.3) — with the open content exactly the two clauses of one arrow, the non-circularity audit (persistence binary, not graded) recorded as passed; the completion-to-mass bridge is a trivial-Proven monotonicity (Theorem 5.1) and an Inherited-Conditional anchoring relation, its only risk upstream; the CKM amplitudes are owed-citation and not used; and the PMNS asymmetry is retroductive, graded Conjectural.**

11. Conclusion

The hierarchy and mixing problems are usually treated as independent. This paper proposes they are two readings of one refinement geometry, and states the proposal at its honest grade rather than above it.

The hierarchy half rests on a chain whose order-preserving and definitional links are now Proven or Inherited-Proven end to end, with the conditionality resolved into a small set of concrete, named open items. The efficiency-to-completion step is dissolved (§3A) into four graded links — the reuse ordinal Proven-arithmetic on the inherited loads, the mediant identity Inherited-Proven, and only the reuse \Rightarrow completion direction and magnitude open — and the completion-to-mass step is split (§5) into a trivial-Proven monotonicity and an inherited-conditional anchoring relation:

refinement \Rightarrow efficiency ordinal (**Proven**) \Rightarrow reuse density (**Proven arithmetic, dyadic loads**, Theorem 3A.1) \Rightarrow completion density (**direction Conditional + magnitude Open**, Theorem 3A.2; **mediant identity Inherited-Proven**, Theorem 3A.3) \Rightarrow mass (**monotonicity Proven; anchoring relation Inherited-Conditional**).

The efficiency ordinal is arithmetic: the three refinement levels fill one-seventh, three-sevenths, and all of the $K = 7$ register, and the third is the saturation level — a Proven fact about register-filling, *not* a proof that the third generation is heaviest. The contribution of this paper to the hierarchy is fourfold. First, it gives the efficiency ordinal a structural origin (Theorem 3.1). Second, it establishes the reuse ordinal by direct computation on the inherited dyadic loads: for the load pattern 1, 2, 4, both efficiency $L_{\leq c}/K$ and reuse density $L_{\leq c}/\Delta L$ are strictly increasing (1, 1.5, 1.75 for the latter), so the reuse ordinal is settled arithmetic — a sibling of Theorem 3.1, not a definitional identity and not a general law (a different load pattern could break it, which is why it is tied explicitly to H2). The once-Conditional "does reuse rise with efficiency?" is thereby moved into the Proven-arithmetic column (Theorem 3A.1). Third, it dissolves the rest of the bridge into the inherited Orbit Counts mediant identity (Theorem 3A.3, Inherited-Proven) plus a single open arrow, and *audits the chain for circularity and records the audit as passed*: the reuse condition is arithmetic on orbit-count increments ($\Delta p_v, \Delta K_c$), and the persistence the upstream papers lean on is binary census-facing, not a graded quantity secretly equivalent to p_v/K_c , so Binary Refinement and the Realisation Criterion are genuinely upstream of completion density and the chain terminates rather than circling. Fourth, it splits the completion-to-mass bridge so that its order-preserving half is trivially Proven (Theorem 5.1) and only the anchoring relation $m = A \cdot C$ remains inherited-conditional on the anchoring programme's survival. After all four, the mass ordering's open surface is exactly three concrete items, none an opaque bridge: **the reuse \Rightarrow completion direction** (whether rising reuse density entails a rising commitment-per-return $C = p_v/K_c$ in sign — a genuine inference, since reuse density and completion density are different quantities); **the magnitude inequality** $\Delta p_v/\Delta K_c > p_v/K_c$; and **the anchoring relation** (whether the anchoring derivation survives review). Everything order-preserving and arithmetic in between — the efficiency ordinal, the reuse ordinal, the mediant identity, and both monotonicities — is settled. The chain is narrowed and grade-separated, not closed.

The mixing half recasts CKM and PMNS as relative completion-channel misalignment — $V_{CKM} = U_{up} U_{down}^\dagger$, $U_{PMNS} = U_e U_v^\dagger$ — which is true as a framing and Open as a

derivation, because what fixes the misalignment magnitudes from the geometry is exactly what is not yet computed. Small quark mixing is located in the near-alignment of the two quark sectors; large neutrino mixing is offered as a structurally-consistent (not predicted) consequence of the neutrino's weaker closure commitment. Both await the same computation.

That computation — the eigenmode construction of completion densities and sector bases from the interface dynamics — is the single remaining objective, and it is now sharply specified by the owed computations this paper leaves (distinct from the three *open grade-items* 2b, 2c, 4b of the hierarchy chain; these are the forward-looking deliverables that would discharge them):

1. **Discharge the residue of the dissolved bridge** (the reuse ordinal is already Proven arithmetic on the inherited dyadic loads — Theorem 3A.1 — and the non-circularity audit already passed): (a) establish the *reuse* \Rightarrow *completion direction* — that rising reuse density entails a rising commitment-per-return $C = p_v/K_c$ in sign, the two being genuinely different quantities the loads do not by themselves link; and (b) settle its *magnitude* — compute $p_v(c)$, $K_c(c)$ per refinement class and show the reuse inequality $\Delta p_v/\Delta K_c > p_v/K_c$ holds across the three classes, at which point the mediant identity (Theorem 3A.3, inherited-Proven) carries it to rising completion density (§3A, §4).
2. **Lift to per-sector triples:** show the single efficiency ordinal induces $C_a(1) < C_a(2) < C_a(3)$ in each charged sector (§5).
3. **Compute the sector bases:** produce U_{up} , U_{down} , U_e , U_ν from the geometry and check $U_{up} U_{down}^\dagger$ against CKM and $U_e U_\nu^\dagger$ against PMNS, untuned (§8–§9).

Once owed-computations 1 and 2 are met, the mass hierarchy ceases to be Conditional and becomes a derived property of the refinement architecture. Once 3 is met, flavour mixing joins it. Until then, this paper offers the unified picture and the owed computations that would complete it — which is the right thing to publish at this stage, provided it is not mistaken for the completion itself.