Gate 3 and Fact Momentum

Closure Holonomy as the Flat Sector of the Continuum Connection Whose Stress-Energy Is Fact Momentum

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Summary for the General Reader

Two ideas in this programme both say, in different languages, that *the past leaves a mark that the present cannot fully erase*.

The first is **Gate 3**. Picture the universe building itself out of tiny, irreversible decisions — call them committed records — laid down on a fine network. As you trace a closed loop through that network and add up the small "twists" along the way, you sometimes return to where you started without ending up back at zero. That leftover twist is a kind of memory baked into the *shape* of the loop rather than into any single point on it. It can only take one of seven values, like the hours on a seven-hour clock. The open question for Gate 3 has always been: does any such leftover twist actually *survive* in the real network, or does it always get smoothed away?

The second is **Fact Momentum**. When a commitment happens, it disturbs an underlying field, and that disturbance carries momentum and lingers — later events feel the wake of earlier ones. This is the continuum, large-scale picture: smooth fields, integrals, the world we can describe with ordinary geometry. (In this programme, time is not assumed at the bottom; it emerges. So the language throughout is "continuum" and "geometric structure," never anything that presumes time as a given.)

This paper asks the obvious question: **are these two the same thing?** The honest answer turns out to be "almost, and in a precise way." The earlier difficulty was a units mismatch — Gate 3's memory comes in seven discrete clicks, while Fact Momentum looks like a smooth, continuous quantity, and you cannot generally squeeze a smooth number down onto a seven-hour clock. The resolution is that the seven clicks were *never* really separate from a continuous circle; they are seven evenly spaced points *on* a circle (the seventh roots of unity), and once you keep the circle, the discrete and the continuous live together comfortably. This is a standard and well-tested move in physics, borrowed from how lattice models grow into smooth field theories.

With that fix, the result is a clean split. Fact Momentum has two parts: a **local** part that lives where the field is genuinely bending and storing energy, and a **global, topological** part that depends only on the large-scale shape of loops and stores no local energy — like the difference between the energy in a stretched rubber band and the fact that the band is *looped through* a ring

and cannot be removed without cutting. **Gate 3 is exactly that second, topological part.** In the special case where the field does no local bending, Fact Momentum *is* Gate 3; in the general case Fact Momentum is bigger, and Gate 3 is the protected, shape-only piece inside it.

Two honest cautions remain, and the paper is careful to flag them rather than bury them. First, the whole picture only works if the disturbing field carries a *phase* (a rotational, clock-like degree of freedom) rather than being a plain up-or-down displacement — a plain displacement would leave the topological part empty for a boring technical reason that has nothing to do with the physics. Second, even granting all this, two things are *not* settled here and are passed to a companion paper: whether the topological part is actually *present* in the real network (occupancy), and whether the surviving memory stays locked to its seven clicks or loosens into a fully continuous value (pinning). What this paper proves is the relationship; what it defers is whether the relationship has anything to act on.

The direction of the whole argument matters: the discrete seven-valued memory is the *fundamental* thing, and Fact Momentum is what it *becomes* when you zoom out. Substrate first, smooth world second.

Contents

Abstract

The Gate-3 programme asks whether the committed record network can support a surviving closure-holonomy sector: a non-integrable $\mathbb{Z}_7$ invariant of the record network that persists beyond local committed records. Independently, VERSF developed Fact Momentum and the commitment-memory field Ξ, in which irreversible commitments perturb the committed-record amplitude, those perturbations persist, and their accumulated influence conditions later

commitments. Both are history-bearing structures, and a prior result established that a surviving closure sector would be compatible with the One Fold ontology — an emergent network invariant, not primitive relational surplus — while leaving occupancy open.

The relationship between the two faces a coefficient obstruction: Gate-3 holonomy is $\mathbb{Z}_7$-valued (torsion), while a naively real circulation is $\mathbb{R}$-valued (torsion-free), and no well-defined reduction $\mathbb{R} \to \mathbb{Z}_7$ exists on cohomology unless the continuum transport is already quantized. The closure architecture supplies the quantization: $\mathbb{Z}_7$ is the group of seventh roots of unity, the finite cyclic subgroup of the closure phase circle U(1). One therefore does not reduce a real circulation down to torsion; one embeds the torsion up into the circle it already inhabits,

$k \in \mathbb{Z}_7 \longmapsto \exp(2\pi i k / 7) \in U(1)$,

and takes the continuum limit into a U(1) connection rather than into $\mathbb{R}$. This is the standard lattice-gauge construction, and it makes the relationship a theorem rather than a conjecture — but a *decomposition* theorem, not an identity, and one carrying two explicit conditional premises.

The result, stated with its premises: provided the committed-record perturbation that responds to the connection is the **phase of a complex amplitude** (not a real displacement), the continuum U(1) connection splits into a curvature sector (local field energy, generated by completed-face defects) and a flat sector (pure holonomy, carried by non-bounding cycles). Fact Momentum is the full continuum stress-energy — the gauge-covariant momentum — of that amplitude. Gate-3 closure holonomy is exactly its flat sector. In the special case where refinement leaves the limiting connection flat, Fact Momentum reduces to Gate 3; in the generic case Fact Momentum is strictly richer, and Gate 3 is its protected topological summand. The arrow runs substrate → continuum throughout: the discrete $\mathbb{Z}_7$ holonomy is fundamental and Fact Momentum is what it becomes in the macroscopic limit.

Two things this paper does not prove, and explicitly defers. First, **occupancy**: whether the flat sector is nonempty — equivalently, whether VERSF refinement preserves non-bounding record cycles or fills them. Second, **pinning**: whether the holonomy that survives the continuum limit remains exactly seventh-root-valued (so that "Gate 3 = flat sector" holds exactly) or relaxes to a continuous U(1) value (so that the identity holds only approximately). Both reduce to the dynamics of face-completion under refinement and are handed to the companion paper; here they are sharpened and handed off, not decided.

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1. Scope, and What Is and Is Not Being Claimed

Two prior results frame this paper and are not re-argued here.

Compatibility (proven, modulo the monadic-primitive premise of One Fold). A surviving closure-holonomy sector does not contradict the distinction-first ontology: holonomy is an emergent dyadic invariant built from per-locus distinctions by composition around cycles, carrying no primitive relational surplus. This result is standalone and depends on nothing below.

Occupancy (open, and computable). Whether the sector survives is the question of whether the record network carries a nontrivial first $\mathbb{Z}_7$-homology class reached by admissible offsets: is there admissible ρ and a non-bounding loop γ in Γ_rec with Σ_γ ρ ≠ 0? This paper sharpens what occupancy *means* in continuum terms but does not settle it.

What this paper establishes:

- the coefficient obstruction is resolved — the $\mathbb{Z}_7 \hookrightarrow U(1)$ embedding is forced **given** that the closure offsets are genuine U(1) phases at the substrate (Section 3 makes this premise explicit);
- the relationship between Gate 3 and Fact Momentum is a *decomposition*, proven under two stated premises, with the arrow pointing substrate → continuum;
- Gate 3 is the flat/topological sector of that decomposition, in both the flat and curved cases.

What it does **not** do:

- it does not claim Gate 3 and Fact Momentum are identical (they are equal only in the flat special case);
- it does not prove the flat sector is occupied — deferred to the companion paper (the *occupancy* question);
- it does not prove the surviving holonomy stays seventh-root-valued — deferred to the same paper (the *pinning* question);
- it does not make compatibility depend on any of this.

The two conditional premises on which the decomposition rests are flagged where they enter and collected in Section 8.

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2. Two History-Bearing Structures

VERSF contains two structures expressing the same informal idea — that record formation does not erase history — in different mathematical languages and at different levels of the programme.

Gate-3 closure holonomy. On the committed record network Γ_rec, an admissible offset assignment is a 1-cochain $\rho \in C^1(\Gamma\_rec; \mathbb{Z}_7)$, with $\rho_{vu} = -\rho_{uv}$. Around a loop γ the closure holonomy is

$$\mathrm{hol}(\gamma) = \Sigma\_\gamma\ \rho \in \mathbb{Z}_7.$$

If $\rho = d^0 a$ for some 0-cochain a, all holonomies vanish and the sector is exact. A loop with Σ_γ ρ ≠ 0 is closed but not exact, and Gate 3 is occupied. This is a discrete, scalar, $\mathbb{Z}_7$-valued (torsion) invariant living at the substrate / record-network level.

Fact Momentum and commitment memory. A commitment event perturbs the committed-record amplitude. The relevant perturbation is **not** a bare real displacement: a real, single-valued field has exact gradient, $\oint_\gamma \partial_i(\delta\kappa)\, dx^i = 0$ around every cycle, and therefore carries no holonomy and no flat sector at all. The perturbation that responds to the closure connection is the **phase of a complex committed-record amplitude**

$$\Psi = \sqrt{n} \cdot \exp(i\theta),$$

where n is the committed-record density and θ the transported phase — the only structure that couples gauge-covariantly to the U(1) connection $\mathcal{A}$ that the closure offsets become in the continuum (Section 4). The gauge-covariant momentum density is

$$p_i = n\,(\partial_i\theta - \mathcal{A}_i),$$

with total Fact Momentum

$$P_i = \int n\,(\partial_i\theta - \mathcal{A}_i)\, d^3x,$$

conserved jointly with the sources, $d/dt\,(P_{field}^i + P_{sources}^i) = 0$. This reduces to the real-field expression $\tilde{p}_i = \delta\dot{\kappa}\, \partial_i\delta\kappa$ only in the holonomy-free sector, where θ is globally single-valued and $\mathcal{A}$ is pure gauge; that limit is the degenerate one in which the flat sector is empty.

Premise P1 (phase-bearing amplitude). The committed-record perturbation sector that responds to the closure connection is complex — it carries a U(1) phase θ — rather than a single real displacement. This is a commitment about the field content of the committed-record sector, not a relabelling, and it must be reconciled with any part of the programme that treats the relevant κ-mode as a real field. *(conditional.)*

For several sources the phase decomposes additively, $\theta = \Sigma_a \theta_a$, and the momentum carries cross-terms,

$$p_i = n\, \Sigma_a (\partial_i\theta_a - \mathcal{A}_i^a) + n\, \Sigma_{a<b}\, (\text{cross-terms}),$$

irreducible to isolated events. The retarded response defines a memory kernel $\mathcal{M}(x,t; x',t') = G_{ret}(x-x', t-t')\, \theta(t-t')$ and an accumulated memory field

$$\Xi(x,t) = \int d^3x' \int_{-\infty}^{t} dt'\, \mathcal{M}(x,t; x',t')\, \rho_{committed}(x',t'),$$

so that two systems with identical instantaneous state $\Psi_A(x,t_0) = \Psi_B(x,t_0)$ but $\Xi_A \neq \Xi_B$ can evolve differently: future = F(state, Ξ), not F(state) alone. This is a continuous, vector-indexed, $\mathbb{R}$-valued quantity living at the emergent-geometry level, where $\int d^3x$ and ∂_i are defined.

Both structures express "history survives." Both are emergent network invariants — generated by committed records, history-dependent, network-level, non-primitive, invisible to any single record. Those shared properties establish co-membership in the class "emergent network invariant," not identity of the member; the σ-transport resemblance ($\partial x = \partial y$ but $[x] \neq [y]$ —

same boundary, different transport history) motivates seeking a relationship but does not by itself supply one. The relationship is established in Sections 4–6 by construction.

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3. The Two Obstructions, and How Each Is Discharged

For "Gate 3 is the X of Fact Momentum" to have content, two things must hold.

The level obstruction (resolved by direction). The VERSF chain runs substrate → $\mathbb{Z}_7$ closure → operational Hilbert geometry → emergent geometric structure. Gate-3 holonomy is a substrate / $\mathbb{Z}_7$-closure-level object; Fact Momentum is a continuum, emergent-geometry-level object (it needs $\int d^3x$, ∂_i). Coarse-graining runs fundamental → emergent, so the discrete object cannot be a coarse-graining of the continuum one without inverting the chain. The only ordering consistent with the programme is the reverse: the discrete $\mathbb{Z}_7$ holonomy is fundamental and the continuum field is its limit. This fixes the direction of any relationship and is not in dispute.

The coefficient obstruction (resolved by the closure architecture, under a stated premise). Gate-3 holonomy is $\mathbb{Z}_7$-valued; an $\mathbb{R}$-valued continuum circulation reduced "mod 7" is not loop-class-invariant unless the underlying transport carries an intrinsic $\mathbb{Z}$- or $\mathbb{Z}_7$-periodicity. The quantization needed is not an extra hypothesis if the closure offsets are genuine phases:

$\mathbb{Z}_7$ appears in VERSF as the K=7/$\mathbb{Z}_7$ closure group, the seventh roots of unity, which are the finite cyclic subgroup of a phase circle U(1). The correct continuum limit is then not $\mathbb{Z}_7 \to \mathbb{R}$ (which would lose the torsion) but $\mathbb{Z}_7 \hookrightarrow U(1) \to$ continuum U(1) connection (which carries the torsion forward as monodromy valued in the seventh roots of unity). With U(1) as the target the obstruction does not arise, because the holonomy is a U(1) Wilson-loop variable throughout and the continuum limit is the standard lattice-gauge construction.

This resolution is exactly as strong as its premise, which must be stated rather than assumed:

Premise P2 (U(1)-phase substrate). The closure offsets are genuinely valued in the seventh roots of unity *as a subgroup of an actual closure phase circle* U(1) — not merely in an abstract cyclic coefficient group $\mathbb{Z}_7$ that happens to admit an embedding into U(1) (every finite cyclic group does, and into many Lie groups besides). If the K=7/$\mathbb{Z}_7$ closure architecture establishes a genuine phase circle, the embedding $k \longmapsto \exp(2\pi i k/7)$ is forced. If it establishes only abstract $\mathbb{Z}_7$-valued offsets, then U(1) is the natural, architecture-respecting target but not the uniquely forced one, and "forced" weakens to "canonical." *(conditional — to be discharged by citation to the closure-architecture result establishing the phase-circle origin.)*

The asymmetry the level obstruction noted resolves the right way under P2: producing an $\mathbb{R}$-valued circulation *from* $\mathbb{Z}_7$/U(1) data in a continuum limit is routine; the reverse was obstructed. The favourable direction is exactly the one the level obstruction demanded. Both obstructions point the same way, and both are discharged by the same fact — the finiteness and phase-circle origin of the K=7 closure.

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4. The Coarse-Graining Construction

Let Γ_rec be a sequence of committed record networks generated by VERSF record dynamics, with lattice spacing $\varepsilon \to 0$ under admissible coarse-graining. The construction proceeds in five steps. Their epistemic status is stated honestly: **two** steps carry open or conditional obligations, not one.

Step 1 — Embed (forced under P2). Each closure offset $\rho_{uv} \in \mathbb{Z}_7$ becomes a U(1) link variable $U_{uv} = \exp(2\pi i\, \rho_{uv} / 7)$. Under Premise P2 this is the inclusion of a subgroup into the phase circle it generates, not a modelling choice. The closure holonomy around γ becomes the Wilson-loop variable

$$W(\gamma) = \prod_\gamma U_{uv} = \exp(2\pi i\, (\Sigma_\gamma \rho) / 7),$$

valued in the seventh roots of unity.

Step 2 — Limit (open: occupancy and pinning). Take $\varepsilon \to 0$. U(1) link variables admit a continuum limit to a U(1) connection $\mathcal{A}$ by the standard lattice-gauge construction. This step carries the two deferred obligations. *Occupancy*: whether the nontrivial Wilson loops survive the limit or are screened by the proliferation of completed faces under refinement (Section 7). *Pinning*: whether what survives stays seventh-root-valued or relaxes to a continuous U(1) value (Section 7). The construction below is valid whether or not the flat sector survives and whatever its value group; what these questions decide is the size of that sector and the exactness of the final identity.

Step 3 — Complexify and source (conditional on P1; carries a claim). The coarse-grained transport induced by the limiting connection acts on the committed-record amplitude $\Psi = \sqrt{n} \cdot \exp(i\theta)$. This step is **not** merely definitional: it asserts that the responding field is phase-bearing (Premise P1). A real, single-valued $\delta\kappa$ would source no holonomy; the flat sector would be empty independent of occupancy. The substantive content here is that the connection has a phase to act on.

Step 4 — Stress-energy (computed). The gauge-covariant momentum density of Ψ in the hydrodynamic (phase) regime is computed, not posited:

$$p_i = \mathrm{Im}(\Psi^* D_i \Psi) = n\, (\partial_i\theta - \mathcal{A}_i), \text{ where } D_i = \partial_i - i\mathcal{A}_i,$$

with total $P_i = \int n\, (\partial_i\theta - \mathcal{A}_i)\, d^3x$. The gauge-invariant circulation around a loop γ follows directly:

$$\oint_\gamma (p_i / n)\, dx^i = \oint_\gamma \partial_i\theta\, dx^i - \oint_\gamma \mathcal{A}_i\, dx^i = 2\pi w(\gamma) - \Phi(\gamma),$$

where $w(\gamma) \in \mathbb{Z}$ is the winding of θ around γ (forced to be integer by single-valuedness of Ψ) and $\Phi(\gamma) = \oint_\gamma \mathcal{A}$ is the connection's period — the holonomy. This is, by definition, Fact

Momentum, and its circulation is exactly the connection holonomy modulo the integer winding. This is the form that carries a flat sector; the real-field expression does not.

Step 5 — Identification. The chain

$\mathbb{Z}_7$ closure offsets → U(1) link variables → continuum U(1) connection → phase of Ψ → Fact Momentum

relates the substrate holonomy to the continuum stress-energy, every arrow pointing toward the continuum. Steps 1, 5 are forced/definitional under P2. Step 3 carries Premise P1. Step 2 carries the two open dynamical obligations (occupancy, pinning). Step 4 is a computation. So the construction has **one conditional premise (P1, Step 3) and two open questions (Step 2)** beyond the standing premise P2 — not a single open step.

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5. The Decomposition Theorem

The continuum object produced by Section 4 is a U(1) connection acting on a phase-bearing amplitude, and such a connection decomposes canonically. This decomposition is the result.

Theorem 1 — Curvature/Holonomy Decomposition of Fact Momentum ***(proven, conditional on Premises P1 and P2 and the construction of Section 4)***

Let $\mathcal{A}$ be the continuum U(1) connection obtained as the $\varepsilon \to 0$ limit of the embedded $\mathbb{Z}_7$ closure offsets on Γ_rec, acting on the phase-bearing committed-record amplitude $\Psi = \sqrt{n}\exp(i\theta)$. Then the continuum stress-energy of Ψ — Fact Momentum — decomposes into two sectors:

Fact Momentum = curvature contribution + flat/topological contribution,

where

1. the **curvature sector** is the local field energy generated by completed-face defects, supported on contractible cycles, carrying the genuine local committed-record dynamics, with bulk density set by $F = d\mathcal{A}$;
2. the **flat sector** is pure holonomy, supported on non-bounding cycles, with no bulk field energy — its contribution to P_i is a *global* winding quantity concentrated at defect cores, not a local density.

Gate-3 closure holonomy is exactly the flat sector:

Gate 3 = flat/topological contribution.

Proof

By Hodge decomposition on the record manifold, the connection 1-form splits as

$\mathcal{A} = d\alpha + \delta\beta + h$,

with h harmonic ($dh = 0$, $\delta h = 0$). The curvature $F = d\mathcal{A} = d(\delta\beta)$ sees only the co-exact part; the exact part $d\alpha$ is pure gauge and the harmonic part h is closed and co-closed, so $F = 0$ there. Curvature is therefore a property of contractible cycles: the holonomy around a small loop equals the enclosed curvature flux, $\oint_{\partial S} \mathcal{A} = \int_S F$, so where $F \neq 0$ the loop bounds a region S carrying that flux. Holonomy around a non-bounding loop bounds no such S and is not determined by any enclosed curvature; it is the period $\oint_\gamma h$ of the harmonic part — the flat/topological content. This is the standard de Rham / Hodge separation: the closed-modulo-exact (harmonic) sector is precisely the non-bounding-cycle holonomy.

The momentum $p_i = n(\partial_i\theta - \mathcal{A}_i)$ is built from $\mathcal{A}$ and inherits the split through its circulation, $\oint_\gamma (p_i/n)\, dx^i = 2\pi w(\gamma) - \oint_\gamma \mathcal{A}$:

- around contractible γ, the gauge-invariant circulation reduces to $-\int_S F$, the curvature sector;
- around non-bounding γ, it reduces to the harmonic period $2\pi w(\gamma) - \oint_\gamma h$, the flat sector — finite even where F vanishes identically along γ.

Gate-3 holonomy, by definition, is $\Sigma_\gamma \rho$ on cycles γ of Γ_{rec} — the period of the offset cochain over a cycle, which under the embedding of Step 1 is the period of $\mathcal{A}$ over that cycle, i.e. the harmonic period. It is nonzero exactly when the cycle is non-bounding (closed but not exact). Hence Gate-3 holonomy is precisely the flat sector of the decomposition and contains no curvature contribution.

Equivalently stated: the flat sector and Gate-3 holonomy are the same object — the harmonic-period content of the limiting connection.

Corollary 1.1 — Two Cases

- **Flat limit.** If refinement leaves the limiting connection flat (curvature sector vanishes), Fact Momentum has no content but holonomy, and Fact Momentum = Gate 3. The two coincide necessarily, because a flat U(1) connection carrying only its harmonic periods has no degrees of freedom other than its holonomy.
- **Curved limit.** If refinement injects curvature through completed faces, Fact Momentum is strictly richer than Gate 3, and Gate 3 is its flat/topological summand. This is the generic case, and the cleaner unification: Gate 3 is not equal to Fact Momentum but is its protected topological part, the remainder being local committed-record dynamics.

In both cases the relationship holds and the arrow runs substrate → continuum. The decomposition does not depend on which case obtains; only the size of the flat sector does. (The *exactness* with which the flat-sector value coincides with the seventh-root-valued Gate-3 holonomy is the separate pinning question of Section 7.)

6. Why This Is a Decomposition and Not an Identity

The honest relationship is containment-as-a-summand, not equality, except in the flat special case. This matters for three reasons.

First, equality would require the curvature sector to be empty, which is the non-generic case; asserting it in general would re-import an unproven identity of the kind the arc has had to retract before.

Second, the decomposition survives both outcomes of the occupancy question, whereas an identity claim would be false in the curved case. A result that holds in both cases is stronger than one that holds only in a special case not yet established to obtain.

Third, the decomposition correctly locates *where* the open questions live. Fact Momentum exists as a continuum quantity regardless of occupancy; what is open is whether its flat sector is nonempty — whether there is any holonomy to be the topological summand — and whether that holonomy is exactly seventh-root-valued. Theorem 1 hands both off cleanly rather than burying either in an equality.

7. The Two Deferred Questions, Sharpened and Handed Off

Theorem 1 makes both deferred questions precise statements about the limiting connection rather than about the meaning of any principle.

7.1 Occupancy — is the flat sector nonempty?

Does the limiting U(1) connection carry nontrivial holonomy around non-bounding record cycles, or is that holonomy screened as $\varepsilon \to 0$?

This is the survival half of Step 2. It is a question about the *dynamics of face-completion under refinement*, governed by a distinction that is standard in the gauge picture and exact here:

Completing a face annihilates contractible curvature, not non-contractible holonomy.

Completing a face sets the holonomy on that face to a coboundary — it removes curvature on the contractible cycle bounding it — but it does not touch the holonomy of a cycle that bounds no completed region. So holonomy survives refinement *unless* refinement adds faces that fill the previously non-bounding cycles. Whether it does is determined by the record-formation rule:

- if face-completion is **globally cycle-filling** — refinement may add a face wherever a loop exists — then every loop eventually becomes bounding, the flat sector is screened, and Gate 3 is empty (the curved/confined case with empty topological summand);
- if face-completion is **local-fact-generated** — faces are added only where a completed local commitment generates one, and certain non-bounding cycles enclose a defect that no completed-fact face can tile — then those cycles remain permanently non-bounding, the flat sector survives, and Gate 3 is occupied.

The biconditional — *Gate-3 holonomy survives refinement iff face-completion is local-fact-generated rather than globally cycle-filling, with at least one protected defect cycle* — is the subject of the companion paper. This paper does not prove it; it establishes that occupancy reduces to it.

7.2 Pinning — does the surviving holonomy stay seventh-root-valued?

A question logically independent of occupancy concerns the *value group* of whatever survives. The embedding of Step 1 pins finite-ε Wilson loops to the seventh roots of unity, but the strict ε → 0 limit flows to a fixed point, and two fixed points are compatible with a surviving flat sector:

- a $\mathbb{Z}_7$ **topological phase** (BF / Dijkgraaf–Witten type): the holonomy remains exactly seventh-root-valued, so "Gate 3 = flat sector" holds *exactly* — but propagating curvature is absent, so the curvature sector and with it the "generic curved case" is empty;
- a **deconfined U(1) phase**: curvature propagates, but Wilson loops take *continuous* U(1) values, so the seventh-root pinning is washed out and Gate-3's $\mathbb{Z}_7$ holonomy coincides with the flat sector only up to the relaxation of the pinning.

Exact seventh-root pinning *together with* a nontrivial curvature sector is a finite-ε feature whose survival to the continuum is not established by the embedding alone. Call this the **pinning question**. It is distinct from occupancy: occupancy asks whether the flat sector is nonempty; pinning asks whether the surviving holonomy is exactly $\mathbb{Z}_7$-valued or relaxes to continuous U(1). The identity "Gate 3 = flat sector" is exact under pinning and approximate without it; the cleanest unification (a nonempty *and* exactly-pinned flat sector inside a nontrivial curvature sector) requires the favourable answer to *both* questions.

Resolving which continuum fixed point VERSF record dynamics realize requires the same face-completion analysis as occupancy and is handed off alongside it. Pinning is, plausibly, the question of whether the local-fact-generated face-completion rule confines the U(1) gauge field down to its $\mathbb{Z}_7$ subgroup — the same rule that decides occupancy may decide pinning — but that is a conjecture, not a result of this paper.

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8. What Is Established, What Is Conditional, and What Is Handed Off

Established here (conditional on the stated premises).

- The coefficient obstruction is resolved under Premise P2: the $\mathbb{Z}_7 \hookrightarrow U(1)$ embedding is forced by the closure architecture's phase-circle origin, and the continuum limit into a U(1) connection carries the torsion forward as pinned holonomy (Section 3, Step 1).
- The coarse-graining construction relating substrate holonomy to continuum stress-energy is sound; its obligations are itemised honestly — one conditional premise (P1) and two open dynamical questions (Section 4).
- Theorem 1: under P1 and P2, Fact Momentum decomposes into a curvature sector and a flat sector, and Gate-3 holonomy is exactly the flat sector. This holds in both the flat and curved cases (Section 5).
- The relationship is a decomposition, not an identity; equality holds only in the flat special case (Section 6).
- Direction: the discrete $\mathbb{Z}_7$ holonomy is fundamental; Fact Momentum is its continuum limit. The arrow runs substrate → continuum throughout.

Conditional premises (to be discharged).

- **P1 (phase-bearing amplitude).** The committed-record perturbation sector that responds to the connection is complex. Required for the flat sector to be nonzero; a real single-valued field carries no holonomy. Must be reconciled with any treatment of the relevant mode as real elsewhere in the programme. *(conditional.)*
- **P2 (U(1)-phase substrate).** The closure offsets are genuine seventh-root-of-unity phases of an actual closure phase circle, not abstract cyclic coefficients. Required for "forced" rather than merely "canonical." *(conditional — discharge by citation to the closure-architecture result.)*

Handed off (open).

- **Occupancy.** Whether the flat sector is nonempty — i.e. whether Gate 3 is occupied. Equivalent, by Theorem 1 and Section 7.1, to whether VERSF face-completion preserves non-bounding record cycles. *(open.)*
- **Pinning.** Whether the surviving holonomy stays exactly seventh-root-valued or relaxes to continuous U(1) — i.e. whether "Gate 3 = flat sector" is exact or approximate. *(open.)*

Standalone and unaffected.

- Compatibility of a surviving sector with One Fold, which depends on none of the above.

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9. Conclusion

The relationship between Gate 3 and Fact Momentum is a theorem, because the obstruction that blocked it dissolves under the closure architecture. The block was the mismatch between $\mathbb{Z}_7$ torsion and a real-valued circulation; the resolution is that $\mathbb{Z}_7$ is the seventh-roots-of-unity subgroup of the closure phase circle, so the continuum limit is taken into a U(1) connection — carrying the torsion as pinned holonomy — rather than into $\mathbb{R}$, which would lose it. The same finiteness of the K=7 closure that gives the discrete coefficient group is what forces the embedding and makes the limit well-posed, *provided* the offsets are genuine phases (Premise P2).

What the limit yields is a U(1) connection acting on a phase-bearing committed-record amplitude (Premise P1), whose gauge-covariant stress-energy is Fact Momentum and which decomposes into curvature and flat sectors. Gate-3 closure holonomy is the flat sector — the harmonic, non-bounding-cycle content with no bulk field energy, contributing to the momentum as a global winding concentrated at defect cores. The phase content is essential: without it the flat sector is empty by construction, and the decomposition collapses to its curvature part regardless of how the dynamics resolve. In the flat case Fact Momentum reduces to Gate 3; in the generic curved case Fact Momentum is richer and Gate 3 is its protected topological summand. The arrow runs substrate → continuum: closure holonomy is fundamental, Fact Momentum is what it becomes macroscopically.

The decomposition is proven under its two premises. The two things it does not settle — whether the flat sector is nonempty, and whether the surviving holonomy stays seventh-root-valued — are occupancy and pinning, and both now have sharp form: occupancy holds iff face-completion under refinement preserves non-bounding record cycles, and pinning holds iff the continuum fixed point retains the $\mathbb{Z}_7$ value group. Both are questions about the record-formation rule, not about any word. Both are handed to the companion paper.

Gate-3 holonomy is the flat sector of the continuum U(1) connection whose stress-energy is Fact Momentum — exactly, if the surviving holonomy stays seventh-root-valued; up to that relaxation otherwise. Fact Momentum is curvature plus that flat sector; in the flat limit it is the flat sector alone. The flat sector is nonzero only because the committed-record amplitude carries a phase. Whether the flat sector is occupied, and whether it stays $\mathbb{Z}_7$-valued, are decided by face-completion dynamics, not by this paper.