Holonomy Assignment from Distinguishability

Closing the Generation Face of the Shared Phase Obligation

Keith Taylor *VERSF Theoretical Physics Programme*

General Reader Summary

The previous paper argued that the phase used in quantum theory is forced to be a continuous circle, U(1). That result answered one major question: why the menu of possible phase values is continuous rather than merely finite or arbitrary.

But it left one final question.

Even if the menu is fixed, who decides which phase belongs to which history?

This paper addresses that question.

A useful analogy is a clock face. The previous paper argued that the clock face must be a complete circle. But that does not yet tell us where the hand should point for a particular journey. The remaining question is whether the hand's position is determined by the journey itself, or whether an extra hidden choice has to be added.

The claim of this paper is that the phase assignment is not free — but the claim must be stated with care, because in its fallback form it rests on a single named premise. If two histories are distinguishably different, then the distinguishability structure must already contain the information needed to assign their relative phase. If two proposed assignments give different phases while producing no difference in any admissible distinguishability relation, then that difference is physically empty. It is an unsupported distinction, and the programme's no-pre-individuation discipline forbids it.

The result is not that every arbitrary angle is visible locally. It is that the gauge-invariant content of phase — the holonomy around closed paths — is fixed by admissible distinctions. Different mathematical descriptions may remain possible, but if they differ only by a relabelling that no admissible comparison can detect, they are not different physics.

The paper is honest about where its weight sits, and the weight is carried by two routes of different strength. The stronger route is constructive, and its idea can be said plainly. Suppose two proposed assignments disagreed about the phase around some closed history by even a tiny amount, too small for any comparison to register. Histories can be repeated: go around the loop

twice, three times, n times. Each repetition adds the disagreement again. A tiny angular error, accumulated in fixed steps, cannot skip past a quarter of the circle — at some explicit number of repetitions the disagreement lands above any fixed threshold of comparison. So a hidden phase disagreement is impossible in principle: it would always be amplified into the open by repetition. That argument uses only resources the programme has already established — that histories compose without bound, and that comparison has a fixed finite resolution — though three fine points about those resources require checking against the earlier papers. Two are settled, one of them more strongly than expected: the earlier work explicitly repeats single loops; and a comparison threshold that held steady under repetition turns out to be something the earlier paper's own central argument could not do without — if comparison degraded with length, that paper's result would fall before this one did. One point remains open: that a repeated loop can always be entered into an actual interference comparison between two histories, rather than merely existing as a path. That last point is precisely flagged, and the result's strongest form waits on it. The weaker route is a fallback that does not need these checks: once the key premise is granted — that the distinguishability structure is complete, containing every admissible comparison invariant of transport — the result follows almost by unpacking definitions. Along that route the substantive contribution is the correct statement of the problem and the relocation of the entire remaining burden onto one precisely named premise, where the programme can see it and where a refuter can attack it.

If the argument holds, the open node left by the U(1) paper closes. The phase catalogue is U(1), and the assignment of phase to admissible closed histories is determined by the distinguishability structure up to gauge — indeed uniquely determined: over a fixed distinguishability structure there is only one physical assignment, and nothing left to choose. Taken together with the previous paper, the message is simple to state: distinguishability fixes not only which phase values can exist but which phase relations can occur. Phase is real, but it carries no physical content of its own beyond what admissible comparison already contains. The Born face had already been discharged by continuous U(1); this paper discharges the generation face — conditionally, with the conditions named and route-sorted.

The honest refuters are aimed at the premises, not at the theorem, and they come in two kinds. From outside the framework, a critic can reject the operational criterion of physicality itself — insist that something can be physically real while being in principle invisible to every admissible comparison. That is a coherent philosophical position, but it is a rejection of the programme's foundations wholesale, not a counterexample within them; the paper says so plainly rather than presenting it as a constructible test. From inside the framework, the live refuters are concrete: show that no-pre-individuation fails in the transport sector; show that the support functional does real work while leaving no trace in any admissible commitment pattern; show that a repeated loop cannot in fact be entered into an interference comparison between two histories — the one refuter currently live against the stronger, constructive route; or show that the dyadic-loading paper evaluates the phase kernel between histories that do not share endpoints. One further objection has already been met partway: the corpus does in places describe distinguishability narrowly, as bare distances, and the paper absorbs this openly as a terminology amendment — folding the comparison structure its arguments always used into the definition — rather than leaving it as an open attack. If any of the live refuters can be shown, the corresponding node reopens. If none can, the assignment is fixed by distinguishability.

Abstract

The predecessor paper *Why Finite Distinguishability Forces Continuous U(1) Phase* established that the admissible phase catalogue of substrate transport is the compact continuous circle U(1), but explicitly left open whether the holonomy assignment $L \mapsto h(L)$ is determined by the distinguishability structure D. This open node blocks the dyadic-loading chain: the support functional s, built from the phase kernel $W(P, P') = e^{i(\theta(P) - \theta(P'))}$, supervenes on D only if the gauge-invariant phase assignment carries no information beyond D.

This paper addresses the assignment problem. The central claim is not that a particular phase coordinate θ(P) is absolutely fixed — such coordinates are gauge-dependent — but that closed-loop holonomies and the relative phase kernel are fixed by admissible distinguishability data up to gauge. The argument proceeds in six steps. First, the distinguishability structure D is consolidated: D is the full transport-compatible relational structure of admissible comparison, not the bare symmetric metric that some corpus passages name. The consolidation is flagged as a definitional amendment, and its consequence for the dyadic-loading chain — a restatement obligation — is recorded and graded. Second, the support functional is shown to be admissibly witnessable, so that differences in s count as admissible distinctions. Third — the constructive core — an Amplification Lemma shows that no nonzero closed-loop discrepancy can remain unwitnessed: a sub-quarter-turn discrepancy accumulates under loop iteration by fixed steps of its own angle and cannot skip the quarter-turn band, so an explicit iterate exceeds any uniform finite-resolution threshold and becomes an interference-witnessable invariant. The mechanism is inherited from the U(1) paper's amplification construction; the application to assignment discrepancies is new. Fourth, a premise-based Holonomy Dilemma stands as fallback: any discrepancy between assignments agreeing on D is either an admissible invariant, hence in D by transport-completeness, yielding contradiction; or unwitnessed, hence excluded by no-pre-individuation. Fifth, any difference in open-path phases that leaves all closed-loop holonomies invariant is gauge, not physical structure, on each path-connected sector. Sixth, the kernel W(P, P′) is identified, for co-terminal histories, as the holonomy of the closed comparison loop $P \circ P'^{-1}$, so that W and hence s supervene on D.

The result's conditionality is graded by route. The amplification route rests on premises the programme has already paid for — unbounded composition and finite-resolution witnessability — subject to three checks. Two are closed at the source: the U(1) paper's §7.4 explicitly iterates single loops, settling single-loop iteration outright; and threshold uniformity is discharged as a presupposition of that paper's own catalogue argument, whose amplification witness operates at unbounded composition lengths and whose tightness audit records that losing it would coarsen the catalogue — so degrading witnessability under composition would unseat the closure theorem before it touched this paper. The third is open: comparison realizability — that an admissible loop and its iterates enter admissible interference comparisons between co-terminal histories, rather than merely existing as paths. If it holds, the instance of transport-completeness the assignment problem needs is discharged constructively, and only the weak claim that witnessed invariants belong to D remains. The fallback route requires full Transport-Completeness as a named premise, under which the theorem is close to analytic — a relocation

of the physics into the premise, where the programme's method requires it to sit. In both routes the direction of construction is fixed so that interference-grade relations are operationally prior to the support functional, blocking circularity. The named refuters are sorted by kind. The extra-systematic refuter — a physically consequential holonomy difference with no admissible witness — is acknowledged to be incoherent within the programme's operational vocabulary and is presented honestly as a rejection of the verificationist criterion of physicality, not as a constructible counterexample; the Amplification Lemma, operating wholly inside the operational criterion, leaves this unchanged. The in-framework refuters are: exhibit a failure of no-pre-individuation in the transport sector; show that the support functional is not admissibly witnessable; show that the dyadic-loading chain evaluates the kernel off the co-terminal domain; show that comparison realizability fails; or show that the chain's validity depends on the narrow metric D — the mild form of which is already realized and graded as an amendment obligation. Absent all of these, the Shared Phase Obligation is discharged on both faces: the Born face by U(1) catalogue continuity, and the generation face by assignment supervenience — equivalently, by uniqueness: D admits only one physical holonomy assignment up to gauge per sector. Read together with the catalogue result, this is a closure theorem for distinguishability: the phase sector contains no independent physical degree of freedom beyond admissible comparison, in its value space or in its assignment.

Epistemic markers: [Inherited] imported from prior VERSF papers; [Imported-External] imported from standard mathematics outside the programme; [Proven] established here without conditional premises; [Conditional] holding under stated premises; [Open] unresolved if the named refuter can be constructed.

Table of Contents

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1. Introduction — The Remaining Open Node

The U(1) paper closed one part of the phase problem and deliberately left another open.

It established that the admissible value space of phase is not finite, not a dense proper subgroup, and not a freely chosen continuum. The admissible phase catalogue is the compact circle U(1), forced by reversibility, unbounded composition, and no-pre-individuation.

But the catalogue is not the assignment.

A catalogue tells us what values are available. An assignment tells us which value belongs to which admissible history. The previous paper therefore ended with a precise unresolved question:

Given that $H = U(1)$, is $L \mapsto h(L)$ determined by D?

This paper answers yes, in the only form that can be physically meaningful:

$L \mapsto h(L)$ is determined by D up to gauge.

The succession is explicit: the U(1) paper's §9 names the candidate closing argument for exactly this question — no-pre-individuation applied to assignments, under which two assignments inducing identical admissible distinguishability data differ by no admissible witness and hence by no admissible structure — and records it as the successor paper's burden, with its refuter stated in that paper's §10. This paper is the execution of that named argument, refined: the Dilemma route (§6.3) is the candidate argument carried through; the Amplification route (§6.1–6.2) is the constructive strengthening the candidate did not anticipate, built from that same paper's §7.4 mechanism; and the refuter the U(1) paper stated — two assignments identical on D differing on some closed-loop holonomy — is inherited and recharacterised in §12, where it is shown to be premise-dependent and replaced by refuters aimed where the weight actually sits.

The qualification "up to gauge" is essential. A phase coordinate attached to an open path is not absolute physical structure. Only gauge-invariant phase differences and closed-loop holonomies carry physical content. If two descriptions differ by a relabelling of phase origins, they are not different assignments in the physical sense.

The real question is therefore:

Can two physically distinct holonomy assignments share the same D?

This paper argues no — and is explicit about the logical character of the argument, which arrives by two routes of different strength. The constructive route is a genuine deductive chain: it derives the witnessability of any nonzero closed-loop discrepancy from unbounded composition and finite-resolution comparison — a sub-quarter-turn discrepancy, iterated, accumulates in fixed steps and cannot skip the quarter-turn band — so a hidden assignment difference is amplified into the open rather than excluded by fiat. The fallback route is premise-based: it emerges from one premise about what D is, plus the programme's standing no-pre-individuation discipline, and once those are in place its conclusion is nearly forced by unpacking. The work of the paper is correspondingly two-layered: along the constructive route, the proof itself; along the fallback, the correct statement of the problem, the correct location of the burden, and the audit of what the conclusion does and does not license downstream.

The fallback's logic can be stated in two sentences. If two assignments differ on a closed-loop holonomy, that difference is an oriented transport invariant: either it can be witnessed by some admissible comparison — in which case, by the completeness premise, it is recorded in D, contradicting the assumption that the assignments agree on D — or it cannot be witnessed by any admissible comparison, in which case no-pre-individuation classifies it as surplus labelling, not physics. If two assignments differ only on open-path phases while agreeing on all closed loops, then on each connected sector the difference is gauge.

Thus the assignment is either recorded in D, or it is not physical. That dilemma is the core of the fallback route, and the Main Lemma of §6.3 is stated as the dilemma it actually proves; the amplification argument of §6.1 is the core of the constructive route, and it proves more — that the unwitnessed horn is empty.

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2. What the Previous Papers Established

The paper inherits four results and imports one standard mathematical fact.

2.1 Phase catalogue [Inherited]

The admissible phase catalogue is the full compact circle:

$H = U(1)$.

This is inherited from the U(1) paper.

2.2 No-pre-individuation [Inherited]

Physical structure may not contain distinctions for which no admissible comparison exists. If a proposed difference cannot be witnessed by any admissible invariant, it is surplus mathematical labelling, not physical content.

2.3 Loop structure of admissible transport [Inherited]

Admissible closed paths compose, reverse, and contain the identity. A holonomy assignment is therefore constrained to respect:

$h(L_1 \circ L_2) = h(L_1) \cdot h(L_2)$,

$h(L^{-1}) = h(L)^{-1}$,

$h(\mathrm{id}) = 1$.

Because U(1) is abelian, the holonomy of a composite of loops at a common basepoint is independent of the order and grouping of composition; no choice among loops at that base commitment introduces hidden structure into h. Comparison of holonomies across *different* basepoints additionally requires a connecting admissible path, and is therefore available only within a path-connected sector — a dependence made explicit in §7. The remark is restricted accordingly: at a common basepoint, the composition law smuggles in no preferred origin; across basepoints, nothing is claimed until connectedness is in hand.

2.4 The support functional depends on phase differences [Inherited]

The dyadic-loading support functional is built from the kernel

$$W(P, P') = e^{i(\theta(P) - \theta(P'))}.$$

Therefore s supervenes on D if and only if the gauge-invariant content of W is determined by D. The phase coordinate θ itself need not be absolute. The kernel must be.

2.5 Gauge-invariant content is closed-loop content [Imported-External]

The holonomy principle — that the gauge-invariant content of a phase assignment is exhausted by its closed-loop holonomies on each connected component — is imported from standard mathematics. The version imported must be specified, because the substrate at this layer is a discrete relational structure, not a manifold. What is imported is the lattice/graph gauge result: for U(1) link variables on a graph, two assignments are gauge-equivalent if and only if they agree on the holonomies of all closed walks (Wilson loops) within each connected component. The continuum U(1)-bundle theorem is *not* imported; it presupposes manifold structure the programme has not earned at this layer, and nothing here uses it. The lattice form requires only what the substrate supplies: a relational structure of commitments and admissible transport segments with composition and reversal.

The principle is not derived within the substrate programme and is marked accordingly. The Gauge Lemma of §7 is this imported result restated in the paper's vocabulary, not an additional fact; it carries the same [Imported-External] marker, conditional only on per-sector path-connectedness.

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3. The Assignment Problem, Stated Correctly

Let $\mathscr{L}$ be the set of admissible closed paths.

A holonomy assignment is a map

$h : \mathscr{L} \rightarrow U(1)$

satisfying the composition, reversal, and identity laws of §2.3.

The assignment problem asks whether h is fixed by D. But this must be stated carefully, because part of any phase description is pure convention.

If two phase descriptions differ by

$\theta'(P) = \theta(P) + \chi(\text{end}(P)) - \chi(\text{start}(P))$

for some function χ on commitments, then closed-loop holonomies do not change. This is gauge. It does not change physical content.

So the physical assignment problem is:

Are closed-loop holonomies determined by D?

Equivalently:

Is the gauge-invariant phase kernel determined by D?

Any formulation that asks instead whether $\theta(P)$ is absolutely determined is asking the wrong question, and a negative answer to that wrong question refutes nothing.

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4. The Distinguishability Structure D and the Transport-Completeness Premise

4.1 What D is — consolidating the definition

The programme's distinguishability structure D has, in places, been presented through its simplest face: a record of separations among commitments, metric-like and symmetric. The U(1) paper's §1 states this face in terms — there the distinguishability datum is a symmetric metric d, contrasted with the oriented holonomy d does not contain. That face is genuine but partial, and the corpus's own forcing arguments outrun it: the defining commitments of the programme — finite distinguishability, admissible comparison, closure, and irreversible record formation — characterise D as the totality of admissible comparison relations, everything that admissible operations of comparison, composition, reversal, and interference can establish about the relations among commitments and transport segments, and it is that totality which no-pre-individuation invokes when it states that admissible physical structure is exhausted by admissible distinguishability relations.

This paper consolidates the definition, and flags the move honestly:

Definitional consolidation. D is the full transport-compatible relational structure of admissible distinguishability. It includes, in addition to symmetric separations: oriented comparison relations, composition behaviour, reversal behaviour, and interference-grade relations among co-terminal histories.

This is a consolidation, not a discovery — the rich structure was already in use in the corpus's forcing arguments — but neither is it a mere restatement, since the narrow term was also in print. Because the dyadic-loading paper's supervenience requirement — $s = s(D)$ — refers to D, the consolidation carries an amendment obligation for that paper, recorded and graded in §10.2.

A bare symmetric metric cannot encode orientation, and this paper at no point claims otherwise. The claim is not

metric distance $d \Rightarrow$ phase,

which is false. The claim is

complete admissible distinguishability $D \Rightarrow$ gauge-invariant phase,

which is the content of the Main Lemma.

4.2 Interference as admissible comparison, and the direction of construction [Inherited]

The Main Lemma's proof invokes interference among co-terminal histories as one mode of admissible comparison. This is not introduced here for convenience; it is inherited from the Born-arc papers, where interference of admissible alternatives sharing preparation and registration is the operational core of the support construction. Its status as admissible comparison is therefore an import, and it is listed in the premise set rather than appearing unannounced inside a proof.

One further statement is required to block a circle. If "interference-grade relations in D" were defined as whatever relations determine s, then the supervenience claim "W supervenes on D" would collapse toward "W supervenes on W" — a tautology dressed as a theorem. The direction of construction is therefore fixed explicitly:

Direction-of-Construction Premise. Interference-grade relations are operationally prior to the support functional. They are the relative commitment statistics among co-terminal admissible alternatives — which alternatives reinforce, cancel, or shift one another's commitment patterns — inherited from the Born arc as primitive admissible comparison data. The support functional s is *constructed from* these relations, never the reverse. At no point is membership in D defined by reference to s.

With the direction fixed, the architecture is acyclic: D is characterised by admissible operations (comparison, composition, reversal, interference-grade statistics); W is identified as closed-loop content within that structure; s is built from W. Each layer is defined before the next refers to it. This premise appears in the condition lists of §11 and §13.

4.3 The Transport-Completeness Premise [Conditional]

Premise (Transport-Completeness of D). D contains all admissible transport-comparison invariants. If an invariant difference can be detected by admissible comparison, composition, reversal, or interference, then it belongs to D.

This premise is the hinge of the paper, and the paper does not pretend otherwise. Without it, D could be defined too narrowly — for example, as only a symmetric distance metric — and would then trivially fail to contain oriented holonomy information. With it, holonomy cannot float freely outside D: a physically effective holonomy difference is an admissible distinction; if it is admissible, it is in D; if it is not in D, it has no admissible witness and is not physical.

The premise is named, isolated, and carried through every downstream marker. Everything proven along the fallback route is conditional on it; the amplification route (§6.1–6.2) requires only its weak form.

4.4 Why Transport-Completeness Is Not Ad Hoc

A natural concern is that Transport-Completeness has been chosen to make the theorem true. The answer is that the premise's content is narrower than the concern assumes, and its source is methodological rather than physical.

Transport-Completeness does not assert that D secretly contains a phase variable. It asserts only that D contains every admissible comparison invariant. That assertion follows from the operational role assigned to D throughout the framework. Finite distinguishability does not classify physical structure by ontology but by admissible comparison; the purpose of D is precisely to record the total relational content accessible to admissible operations. A transport invariant capable of changing admissible comparison outcomes therefore cannot consistently lie outside D — if it did, D would cease to be a complete distinguishability structure and would become a partial bookkeeping device, contradicting its defining role.

Transport-Completeness is therefore not an additional physical hypothesis about phase. It is a completeness requirement on the notion of distinguishability itself. The premise may still be rejected — but its rejection requires replacing the programme's notion of admissible distinguishability wholesale, not identifying a missing transport variable within it. This is the same structural fact that makes Refuter 1 (§12) extra-systematic rather than constructible: the exit from the premise is an exit from the framework's criterion of physicality, not a counterexample inside it. The amplification route (§6.1) makes the point concrete for the holonomy instance: there, witnessability is derived rather than presumed, and the premise's role shrinks to the near-trivial claim that witnessed invariants are recorded.

Status. This subsection reduces the arbitrariness of the premise; it does not discharge it. The argument shows that Transport-Completeness is forced *by the framework's own characterisation* *of D*, which relocates the question to whether that characterisation is correct — a question the programme answers by its verificationist foundations, not within this paper. The premise therefore remains listed as undischarged in §13.

[Conditional — a methodological defence, marked as such; the premise's entry in the Not-Established ledger is unchanged.]

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5. Witnessability of the Support Functional

Before the Main Lemma can do its work, one loop must be closed. The refutation conditions in §12 treat "changes the support functional s" as a physical difference. That treatment must be earned, not assumed. If s were a purely theoretical construct with no admissible witness of its own, then a holonomy difference that changed s but nothing else would fall into the "unwitnessed" horn of the dilemma and be classified as non-physical — while the generation chain simultaneously treated s as physically loaded. That would be an inconsistency in the architecture.

The loop closes as follows.

Claim (s-witnessability). Differences in the support functional are admissible distinctions.

Argument. The support functional is not an idle label. Within the dyadic-loading programme, s governs which candidate-menu structures are loaded and therefore which commitment patterns are admissible downstream. Generation structure — the very thing the chain is built to derive — consists of differences in committed records. Committed records are the paradigm case of admissibly witnessed structure: they are precisely what admissible comparison compares. Therefore any difference in s that has downstream effect manifests in differences among admissible commitment patterns, and is thereby admissibly witnessed. Conversely, a difference in s with no downstream effect on any admissible commitment pattern is, by no-pre-individuation, no difference at all — and then the generation chain loses nothing by ignoring it.

So either an s-difference is witnessed, or it is empty. In both cases the architecture is consistent: "changes s" entails "admissibly distinguishable or physically null," and the refuter conditions of §12 may legitimately use s-differences as markers of physical difference.

[Conditional — on no-pre-individuation and on the dyadic-loading paper's characterisation of how s governs commitment structure.]

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6. The Main Lemma — Amplification and the Holonomy Dilemma

This section establishes the central result in two layers. The first layer, the Amplification Lemma, is constructive: it shows that no nonzero closed-loop discrepancy can remain below the witnessability threshold under all admissible compositions, using only machinery the programme has already paid for. The second layer, the Holonomy Dilemma, is the premise-based fallback: it secures the conclusion from Transport-Completeness alone, and remains in force even if any of the Amplification Lemma's three named checks fails.

6.1 The Amplification Lemma

The Dilemma's second horn — an unwitnessed nonzero discrepancy — can be attacked directly rather than excluded by premise. The attack uses three inherited resources: unbounded composition (load-bearing in the U(1) paper's catalogue-continuity argument), finite distinguishability as a fixed witnessability resolution, and interference among co-terminal histories as admissible comparison (§4.2). The amplification mechanism itself is not new to this paper: it is the construction of the U(1) paper's §7.4 — iterate a loop to drive a sub-resolution phase difference above threshold — there applied to differences between realized holonomies, here applied to discrepancies between candidate assignments. The mechanism is [Inherited]; its application to the assignment problem is what this section adds.

Lemma (Amplification). Let $\Delta = e^{i\alpha} \in U(1)$ with $\Delta \neq 1$ be the discrepancy of two holonomy assignments on an admissible closed path L. Suppose: (A) unbounded composition licenses iteration of a single admissible loop, so that L^n is admissible for every $n \in \mathbb{N}$; (B) witnessability has a uniform resolution threshold ε, independent of loop length, with $\varepsilon < \pi/2$; and (C) every admissible closed path is realizable as the comparison loop of some pair of co-terminal admissible histories, so that its holonomy enters an admissible interference comparison. Then there exists an explicit n such that the discrepancy on the admissible loop L^n exceeds the threshold and is interference-witnessable.

Proof. By the composition law, $\Delta(L^n) = \Delta(L)^n = e^{in\alpha}$. Write d(·) for angular distance from 1. By reversal symmetry, take $\alpha \in (0, \pi]$: if the discrepancy angle exceeds π, pass to L^{-1} — admissible by §2.3 — whose discrepancy is the inverse, with angle $2\pi - \alpha \in (0, \pi)$.

If $\alpha \geq \pi/2$, then n = 1 already gives $d(\Delta) = \alpha \geq \pi/2$.

Otherwise $\alpha < \pi/2$. Let $n^* = \lceil \pi/(2\alpha) \rceil$. Then $(n^* - 1)\alpha < \pi/2 \leq n\alpha$, *and* $n\alpha < \pi/2 + \alpha < \pi$. The accumulated angle therefore lands in $[\pi/2, \pi)$ without wrapping:

$$d(\Delta^{n^*}) = n^*\alpha \in [\pi/2, \pi).$$

In either case there is an explicit witness index n with $d(\Delta^n) \geq \pi/2 > \varepsilon$. The discrepancy on L^n is above threshold; by (C), L^n enters an admissible interference comparison among co-terminal histories, on which the two assignments predict admissibly distinguishable relative commitment statistics (§4.2). The discrepancy is thus an admissible transport-comparison invariant.

[Conditional — on checks (A), (B), and (C), statuses below, and on interference as admissible comparison (§4.2). The arithmetic core — that a sub-quarter-turn step sequence cannot skip the band $[\pi/2, \pi)$ — is elementary and unconditional; no density theorem and no external mathematics is used.]

Remark on the uniformity of the bound. The bound $d(\Delta^n) \geq \pi/2$ is uniform across all nonzero discrepancies, however small α may be: a sub-quarter-turn discrepancy accumulates by fixed steps of α and cannot skip the quarter-turn band. This makes the demand on finite distinguishability very weak: only quarter-turn phase discrepancies need be witnessable. The Born arc presupposes interference contrast resolvable far below a quarter turn — without it, the support construction would have no operational content at all — so condition (B)'s magnitude clause, $\varepsilon < \pi/2$, is inherited de facto. What (B) adds beyond the Born arc is the *uniformity* clause: the threshold must not degrade with loop length.

Remark on graceful degradation. The explicit witness index $n^* = \lceil \pi/(2\alpha) \rceil$ converts a partial failure of (B) into a bounded loss rather than a total one. If threshold uniformity holds only out to loops of length ℓ, the Lemma still excludes every discrepancy of angle α with $\lceil \pi/(2\alpha) \rceil \cdot |L| \leq \ell$ — that is, every discrepancy above a floor set by the length scale at which witnessability degrades. A failure of uniformity therefore does not surrender the constructive route wholesale; it bounds the size of any surviving hidden discrepancy by the degradation scale, and only discrepancies below that floor revert to the fallback route. The exclusion above the floor still runs through the witness step and therefore carries the same dependence on (C) as the Lemma itself; the bound on the index is unconditional arithmetic, the exclusion is not.

Three checks against source papers, with current status.

(A) *Single-loop iteration.* **Verified.** The U(1) paper's §7.4 explicitly iterates a single admissible loop — "iterating it n times is admissible (unbounded composition, §3) with holonomy $n\delta$" — so the licence is not merely available but already load-bearing at the source. [Inherited — U(1) paper §7.4, §3.]

(B) *Threshold uniformity.* **Discharged — a presupposition of the inherited catalogue result.** The tie-back runs deeper than verified usage, and deserves its own statement:

Stability of Witnessability Under Composition. Admissible composition preserves the operational status of admissible comparison: the witnessability threshold does not degrade with composition length. This is not a new principle of the present paper; it is a presupposition of the U(1) paper's own catalogue argument. That paper's §7.4 amplification witness must operate at iteration counts $n \approx \lceil \varepsilon_{min}/\delta \rceil$ for arbitrarily small realized differences δ — that is, at unbounded composition lengths — and its window condition $\varepsilon_{min} < n\delta < 2\pi - \varepsilon_{min}$ applies a fixed threshold at every such count. Its §10 tightness audit states the dependence outright: lose the witness where the required iteration count grows too large, and "the asymmetry collapses, and the conservation argument would coarsen the catalogue as readily as close it." If distinguishability resolution degraded arbitrarily under repeated composition, arbitrarily long admissible histories would cease to support meaningful comparison, and the catalogue theorem itself would become unstable. The present paper therefore requires no new principle for (B) — only that admissible composition preserve what the catalogue argument already requires it to preserve. A challenger who denies uniformity refutes the U(1) closure argument before reaching this one. [Inherited — as a presupposition of the U(1) paper's §7.4 and closure theorem; the residual recommendation is that the principle be stated explicitly at the source, as a recorded presupposition, not a new premise. The graceful-degradation remark above stands as belt-and-braces: even under a hypothetical partial failure, the loss is a bounded floor, not the route.]

(C) *Comparison realizability.* **Open — flagged for verification.** The witness step requires that an admissible closed path enter an interference comparison: that L^n (indeed L itself, already at n = 1) is realizable as $\tilde{P} \circ \tilde{P}'^{-1}$ for some pair of co-terminal admissible histories. The U(1) paper's §2.3-analogue closes closed paths under composition, which with (A) makes L^n an admissible *loop*; but an admissible loop is not automatically an admissible *comparison*. A sufficient and perhaps easier condition is general composition — that open paths compose admissibly with closed loops, so that $\tilde{P} = L^n \circ P'$ is admissible for an admissible history P′ into the basepoint. Whether the Born arc's definition of the comparison-loop set discharges (C) by construction — if $\mathscr{L}$ is *defined* as the set of comparison loops, (C) holds for L by definition and reduces, for L^n, to closure of the comparison set under iteration — must be read at the source, not presumed. Note that the Dilemma route is untouched by this gap: there the witness is hypothetical and the dilemma cases on its existence; the amplification route asserts a witness and must supply one. [Conditional — audit item (e), §10.2; Refuter 5c, §12.]

All three checks are carried in the audit register (§10.2) and given refuters (§12, Refuters 5a–5c).

6.2 Consequence — the unwitnessed horn is empty for nonzero discrepancies

Under (A), (B), and (C), no nonzero closed-loop discrepancy can be unwitnessed: every $\Delta(L) \neq 1$ amplifies to an above-threshold, interference-witnessable invariant on some admissible comparison built from L^n, which by Transport-Completeness — indeed, by the much weaker claim that *witnessed* invariants are in D — belongs to D. Two assignments agreeing on D therefore cannot differ on any closed-loop holonomy at all:

$h(L) = h'(L)$ for every admissible closed path L.

This upgrades the result's epistemic standing. The specific instance of Transport-Completeness that the assignment problem needs — that closed-loop holonomy discrepancies cannot hide outside D — is discharged constructively, from premises the programme has already paid for elsewhere. Transport-Completeness remains a named premise only for its general form (all admissible invariants whatsoever are in D), and for the fallback route below.

[Conditional — on checks (A), (B), and (C); no premise beyond inherited machinery and weak completeness is used.]

6.3 The Holonomy Dilemma — premise-based fallback

The Dilemma stands in its honest logical form, independent of amplification. What it establishes is a dilemma — every closed-loop discrepancy is either recorded in D or physically empty — not a bare equality. It secures the supervenience conclusion from Transport-Completeness alone, and is the operative route if any of checks (A), (B), or (C) fails.

Lemma (Holonomy Dilemma)

Let h and h′ be two holonomy assignments on the same admissible transport structure, both valued in U(1) and both respecting composition, reversal, and identity. Suppose h and h′ agree on all distinguishability data in D. Then for every admissible closed path L with $h(L) \neq h'(L)$, exactly one of the following holds:

(i) the discrepancy $\Delta(L) = h'(L) \cdot h(L)^{-1}$ is an admissible transport-comparison invariant — in which case, by Transport-Completeness, Δ(L) belongs to D, contradicting the assumption that h and h′ agree on D; or

(ii) Δ(L) has no admissible witness — in which case, by no-pre-individuation, the discrepancy is not admissible physical structure.

Consequently, no physically distinct closed-loop holonomy assignment can vary independently of D. More sharply: under the agreement hypothesis, case (i) is empty by construction — an admissible invariant discrepancy would already contradict agreement on D — so every actual discrepancy between assignments agreeing on D falls under case (ii) and is unwitnessed, hence non-physical. The agreement hypothesis converts the dilemma into a verdict: all residual variation is surplus labelling.

Proof

Suppose there exists an admissible closed path L with $h(L) \neq h'(L)$. Define

$\Delta(L) = h'(L) \cdot h(L)^{-1}$.

Since $h(L), h'(L) \in U(1)$, we have $\Delta(L) \in U(1)$, and by assumption $\Delta(L) \neq 1$.

Δ(L) is a closed-loop phase difference. Closed-loop phase differences are gauge-invariant: no relabelling of phase origins of the form $\theta \mapsto \theta + \chi(\text{end}) - \chi(\text{start})$ can alter the holonomy of a closed path [Imported-External, §2.5]. So Δ(L) cannot be removed by convention. It is either a feature of the physics or nothing at all.

Case (i): Δ(L) has an admissible witness. Some admissible comparison — interference among co-terminal histories, composition, reversal, or transport response — detects Δ(L). Then Δ(L) is an admissible transport-comparison invariant. By Transport-Completeness, every such invariant belongs to D. But h and h′ were assumed to agree on D. Contradiction.

Case (ii): Δ(L) has no admissible witness. Then h and h′ differ in a way that no admissible comparison can detect. The difference is a distinction with no admissible witness. By no-pre-individuation, such a distinction is not admissible physical structure. The difference is not physical.

The two cases are exhaustive and exclusive: a discrepancy either has at least one admissible witness or has none. In the first case the agreement hypothesis is contradicted; in the second the discrepancy is surplus labelling. There is no third case in which the discrepancy is both physical and absent from D.

[Conditional — on Transport-Completeness of D (§4.3), no-pre-individuation (§2.2), interference as admissible comparison (§4.2), and the imported lattice holonomy principle (§2.5).]

Remark on logical character

The two routes differ in character, and the difference should be stated plainly. The Dilemma route is close to analytic: as its concluding clause records, case (i) is empty under the agreement hypothesis, and the route unpacks what the completeness premise and no-pre-individuation jointly mean for closed-loop phase. The Amplification route is not analytic: it derives the emptiness of the unwitnessed horn from unbounded composition and finite-resolution witnessability, doing substantive work with inherited machinery rather than postulating the conclusion's precondition. The paper's standing is therefore graded by the three checks. Check (A) is verified at the source; check (B) is discharged as a presupposition of the inherited catalogue theorem itself (the Stability of Witnessability statement, §6.1); so the constructive route's standing turns entirely on check (C), comparison realizability, which awaits its source reading. If (C) holds, the assignment result is a substantive theorem whose conditionality rests on premises already paid for, and the near-analyticity concession applies only to the fallback. If (C) fails, the fallback carries the result alone, the physics lives in the completeness premise, and the relocation of weight into that named premise — exposed to refutation rather than hidden in an argument — is the method working as intended. §9 develops this point; §12 aims the refuters accordingly.

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7. Gauge Freedom, Open Paths, and Connectedness

The Lemma concerns closed loops. But the support kernel is often written using open-path phases:

$W(P, P') = e^{i(\theta(P) - \theta(P'))}$.

Does the argument fix $\theta(P)$ itself?

No. And it should not. Absolute open-path phase is not physical. Only phase differences around closed comparison loops are physical.

Suppose two assignments θ and θ' differ on open paths but agree on every closed-loop holonomy. The natural claim is that they differ by gauge. That claim is true, but it requires a hypothesis that must be stated rather than assumed.

Gauge Lemma

Let the admissible comparison structure be path-connected — every commitment is joined to every other by some admissible path. If two phase assignments agree on all closed-loop holonomies, then they differ by a gauge transformation

$$\theta'(P) = \theta(P) + \chi(\mathrm{end}(P)) - \chi(\mathrm{start}(P))$$

for some function χ on commitments, and hence define the same admissible physical structure.

[Imported-External (§2.5, lattice form) — conditional only on per-sector path-connectedness. This Lemma is the §2.5 import restated in the paper's vocabulary, not an independent result; it is stated here so the connectedness hypothesis is visible at the point of use.]

Remark on disconnected sectors

If the admissible comparison structure decomposes into multiple path-connected sectors, the Gauge Lemma holds on each sector separately. Between sectors, no admissible comparison loop exists, so relative phase between sectors is not gauge — it is undefined. By no-pre-individuation this is harmless: a quantity that no admissible comparison can probe carries no admissible content, and inter-sector phase is exactly such a quantity. The supervenience results of this paper are therefore stated per sector, which is the only place they have content.

Corollary (Sector closure of the generation chain)

Co-terminal histories share endpoints and are therefore automatically same-sector; hence W on its domain (§8.1) is a within-sector quantity. [Proven — immediate from §7 and the §8.1 domain restriction.]

Consequently, *if* the dyadic-loading construction evaluates W only on co-terminal pairs, the chain never consumes inter-sector phase and the per-sector restriction of the Gauge Lemma costs it nothing. [Conditional — §10.2 audit, item (b): this second clause holds exactly insofar as the kernel's co-terminal domain covers the chain's actual usage, which is checked by reading and reversible.]

Consequence

This is why the assignment result must be stated as:

The holonomy assignment is D-determined up to gauge, on each connected sector.

Not:

$\theta(P)$ is absolutely determined.

The latter is false. The former is the physically meaningful claim.

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8. The Kernel Is Determined by D

The support kernel is

$W(P, P') = e^{i(\theta(P) - \theta(P'))}$.

A condition on its domain must be made explicit, because without it the kernel is not even gauge-invariant.

8.1 The co-terminality condition

The identity that converts the kernel into a holonomy,

$\theta(P) - \theta(P') = \theta(P \circ P'^{-1})$,

holds only when P and P′ share both endpoints: start(P) = start(P′) and end(P) = end(P′). Only then is $P \circ P'^{-1}$ a closed loop. For histories that do not share endpoints, $\theta(P) - \theta(P')$ shifts under gauge and carries no invariant content.

This restriction is not a technical inconvenience; it is physically exact. The histories compared by the support construction are interfering alternatives, and interfering alternatives share preparation and registration — they begin at a common commitment and end at a common commitment. The comparison loop closes at commitments, which is precisely where the programme's record structure lives. The kernel's domain is therefore the set of co-terminal admissible history pairs, and on that domain it is a closed-loop quantity.

One caution: the claim that the support construction compares only co-terminal alternatives is a claim about the dyadic-loading paper's construction. It carries the same epistemic status as the §10.2 audit — checked by reading, reversible if a closer reading contradicts it. If dyadic loading ever evaluates W on non-co-terminal pairs, the supervenience result proven here does not cover the kernel's full domain of use, and the discharge would fail silently. The point is therefore added to the §10.2 audit checklist and named as a refuter (§12, Refuter 4b).

8.2 Supervenience of the kernel

For co-terminal P, P′:

$W(P, P') = h(P \circ P'^{-1})$,

and $P \circ P'^{-1}$ is an admissible closed comparison loop. Its holonomy cannot vary independently of D, by either route: along the amplification route (§6.2), assignments agreeing on D coincide on it outright; along the fallback route (Holonomy Dilemma, §6.3), any variation is either recorded in D or physically empty. By the Gauge Lemma, the residual descriptive freedom on each connected sector is gauge and does not touch W.

Therefore W is determined by D on its domain. Since the support functional s is built from W, and since s-differences are admissible distinctions or null (§5):

s = s(D).

This is the desired supervenience result.

[Conditional — route-graded: via §6.2 under checks (A)–(C) and weak completeness, or via §6.3 under full Transport-Completeness; in either route also on no-pre-individuation, interference as admissible comparison and the Direction-of-Construction Premise (§4.2), the imported lattice holonomy principle (§2.5), s-witnessability (§5), and per-sector connectedness where the Gauge Lemma is invoked (§7).]

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9. Where the Weight Sits — Why This Is Not Circular and Not Free

A critic might object:

You have solved the assignment problem only by putting phase into D.

That objection deserves a careful answer, because it is half right, and the half that is right is openly embraced.

9.1 What would be illegitimate

It would be illegitimate to define D as a bare symmetric metric, discover that holonomy does not supervene on it, and then quietly enlarge D until the theorem comes out true. That manoeuvre would make the result empty and would also corrupt the dyadic-loading chain, which refers to D under its original characterisation.

9.2 What is actually done

The paper does three things instead, all in the open.

First, it anchors the characterisation of D in the programme's defining commitments (§4.1) and identifies the move honestly as a consolidation: the rich structure — admissible comparison, not bare metric — was always *in use* in the corpus's forcing arguments, while the narrow metric term was sometimes in print; §4.1 folds the operative structure into the definition. Second, it flags the consolidation as definitional and audits the dyadic-loading chain against it (§10.2), recording the resulting amendment obligation rather than substituting silently. Third, it states the logical consequence with the grading it deserves: along the fallback route, under Transport-Completeness alone, the supervenience theorem is close to analytic and the substantive question becomes the truth of the premise; but along the amplification route (§6.1–6.2), the instance of completeness the assignment problem needs is derived constructively from unbounded composition and finite-resolution witnessability, so that — with check (A) verified and check (B) discharged at the source, and check (C) outstanding — the result is a substantive theorem resting on premises the programme has already paid for, not a postulate restated.

This relocation is the method of the programme, not a violation of it. A claim that hides inside a proof cannot be attacked; a claim isolated as a named premise can. The paper's contribution is exactly this isolation, together with the structural clarifications that make it possible: the assignment problem is a closed-loop problem; open-path freedom is gauge; the kernel is a comparison-loop holonomy on co-terminal pairs; the support functional is witnessable.

9.3 The forced choice

With the premise named, the situation is a genuine dichotomy:

1. holonomy differences are physically distinguishable, and therefore part of D; or
2. they are not physically distinguishable, and therefore not physical.

There is no third option in which holonomy is physical but absent from all admissible distinguishability data — unless Transport-Completeness fails. And for the closed-loop case specifically, the amplification route closes even that exit: a nonzero holonomy discrepancy cannot remain below the witnessability threshold under all compositions, so "indistinguishable but physical" is not merely excluded by premise but ruled out constructively, given the three checks of §6.1 — two closed, one open. That is why the refuters of §12 are aimed at the premises, the discipline, and the open check — not at the theorem.

9.4 Why Assignment Freedom Would Violate No-Pre-Individuation

The relationship between the assignment result and no-pre-individuation can be turned around, and the reversed direction is the deeper statement. It is not merely that no-pre-individuation helps prove assignment supervenience. It is that assignment freedom would itself violate no-pre-individuation.

Suppose holonomy assignment were not determined by D. Then there would exist two admissible configurations sharing the same distinguishability structure while differing in holonomy assignment. Either the difference changes admissible comparison outcomes or it does not.

If it changes admissible comparison outcomes, then the two configurations do not in fact share the same D — the supposition contradicts itself, since D records exactly the admissible comparison content.

If it does not change admissible comparison outcomes, then the difference is an individuating label attached to physical structure without support from any admissible distinction. That is precisely the structure no-pre-individuation excludes: individuation in excess of distinguishability.

Assignment freedom is therefore not merely unnecessary, an idle wheel the formalism could carry. It is incompatible with the programme's central discipline. A framework that admitted free holonomy assignment over fixed D would have readmitted pre-individuated structure through the transport sector, and the discipline would be broken not at its periphery but at a load-bearing joint.

[Conditional — on no-pre-individuation and on D's defining role as the record of admissible comparison content (§4.1, §4.4). The argument is the Dilemma of §6.3 read in reverse: rather than using the discipline to derive the result, it exhibits the result's negation as a violation of the discipline.]

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10. Relationship to the Born, Charge, and Dyadic-Loading Papers

10.1 The chain

The Born paper needed the existence of a continuous compact phase catalogue. The U(1) paper supplied H = U(1). The charge paper needed only this catalogue result, plus the gauge-identification premise, to derive the charge lattice.

The generation chain needs more. It needs the support functional s to introduce no information beyond D. Since s is built from the phase kernel W, this requires the kernel to be D-determined. This paper supplies that step:

$D \Rightarrow h(L) \Rightarrow W(P, P') \Rightarrow s$.

The programme sequence becomes:

finite distinguishability ⇒ U(1) catalogue ⇒ D-determined holonomy ⇒ D-determined support ⇒ dyadic loading.

The Born face was discharged by the catalogue. The generation face is discharged by the assignment.

[Conditional — route-graded: via §6.2 under checks (A)–(C) and weak completeness, or via §6.3 under full Transport-Completeness.]

10.2 Audit of the dyadic-loading chain against the consolidated D

Because §4.1 consolidates the characterisation of D, the dyadic-loading paper's requirement s = s(D) must be re-checked against the consolidated definition. The audit question is: did the chain at any point rely on D being narrower than the full admissible comparison structure — for example, on D being only a symmetric metric?

The reading is mixed, and the audit records it as mixed. On one side, the U(1) paper's §1 — the passage that sets up the dyadic-loading reduction — states the narrow characterisation outright: the distinguishability datum there is a symmetric metric d, explicitly contrasted with the oriented holonomy that d does not contain. "D" in at least one load-bearing corpus passage means the metric datum. On the other side, the forcing arguments of that same paper, and the supervenience requirement's own purpose, run through the rich structure: no-pre-individuation is stated there as "admissible physical structure is exhausted by admissible distinguishability relations," and the requirement that s add no information beyond admissible distinguishability presupposes that admissible distinguishability includes interference-grade comparison — under the narrow reading the requirement would be unsatisfiable for any interference-built functional and the chain dead on arrival by its own standard. The rich structure was always *in use*; the narrow term was sometimes *in print*.

The resolution is therefore that §4.1 performs a definitional consolidation, not a restatement: it folds the comparison and interference structure — operative throughout the forcing arguments — into the definition of D explicitly, and the corpus's narrow usages become superseded terminology rather than contradicted claims. The consequence is an amendment obligation, tracked openly: the dyadic-loading paper's supervenience clause must be restated against the consolidated D at its next review round. This realizes Refuter 4 in its mild form — an amendment changing terminology and bookkeeping, not a defeat of the discharge — and the discharge claimed in this paper transfers under the consolidated definition.

A second audit item concerns the domain of the kernel. The supervenience result of §8 covers W only on co-terminal pairs, where the kernel is a closed-loop quantity. §8.1 argues that the dyadic-loading construction uses W only on such pairs, since the alternatives it compares share preparation and registration. That reading must be verified against the dyadic-loading paper's actual construction: every evaluation of W in that paper must be confirmed to occur on a co-terminal pair. If any evaluation occurs off the co-terminal domain, the quantity evaluated there is gauge-dependent, the supervenience result does not cover it, and the discharge claimed here fails for that portion of the chain.

Audit checklist, with current statuses.

(a) *Breadth of D in the dyadic-loading chain.* **Re-graded: the consolidation branch is live.** The U(1) paper's own §1, setting up the reduction, states the narrow characterisation in terms — "the distinguishability datum is a symmetric metric d, while θ is oriented and carries a holonomy that d does not contain." The corpus thus uses "D" for the narrow metric datum in at least one load-bearing passage, while the forcing arguments of that same paper run through no-pre-individuation stated broadly ("admissible physical structure is exhausted by admissible distinguishability relations"), which is the rich structure. The resolution is that §4.1's move is a genuine *definitional consolidation* — folding the comparison and interference structure, always in use in the forcing arguments, into the definition of D explicitly — rather than a restatement of standing usage. Consequence: the dyadic-loading paper's supervenience clause s = s(D) must be *restated* against the consolidated D at its next review round, as an amendment tracked openly, not merely re-read. This is the mild form of Refuter 4: realized as an amendment obligation, not as a defeat of the discharge.

(b) *Co-terminal domain of W in the dyadic-loading construction.* **Open.** Every evaluation of W in that paper must be confirmed to occur on a co-terminal pair.

(c) *Single-loop iteration (check A).* **Resolved.** The U(1) paper's §7.4 iterates a single admissible loop explicitly and load-bearingly. [Inherited — U(1) §7.4, §3.]

(d) *Threshold uniformity (check B).* **Resolved — discharged as a presupposition.** The U(1) paper's §7.4 applies a fixed ε_{min} at unbounded iteration count, and its §10 tightness audit records that losing the amplification witness at long lengths would collapse the asymmetry and coarsen the catalogue. Uniformity is therefore a presupposition of the inherited catalogue theorem, not a new premise of this paper (Stability of Witnessability, §6.1). Residual: record the presupposition explicitly at the source.

(e) *Comparison realizability (check C).* **Open.** Confirm, against the U(1) and Born-arc papers' statements of composition and the support construction's loop generation, either that every admissible closed path is realizable as a comparison loop of co-terminal histories, or that open-with-closed composition is admissible so that $L^n \circ P'$ realizes the comparison. If the Born arc defines its loop set as comparison loops, (C) discharges by definition for L and reduces, for L^n, to closure of the comparison set under iteration — to be read, not presumed.

[Items (b) and (e) open and flagged; item (a) re-graded to an amendment obligation; items (c) and (d) resolved as recorded.]

Owed edits arising from this audit. Two edits to other papers are owed and tracked here as first-class obligations, alongside the open items. To the dyadic-loading paper: restate the supervenience clause $s = s(D)$ against the consolidated D (item (a)). To the U(1) paper: state the Stability of Witnessability presupposition explicitly among its §3 inputs (item (d) residual) — the principle its §7.4 already consumes at unbounded iteration counts. Neither edit changes a result; both bring print into line with use.

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11. The Main Theorem

Theorem — Holonomy Assignment Supervenience

Let D be the complete admissible distinguishability structure of substrate transport, satisfying Transport-Completeness (§4.3). Let $H = U(1)$ be the phase catalogue inherited from the compact-phase result. Let $h : \mathscr{L} \to U(1)$ be a holonomy assignment on admissible closed paths, respecting composition, reversal, and identity.

Then, on each path-connected sector of the admissible comparison structure:

(a) the gauge-invariant content of h is determined by D. Two routes secure this clause, with distinct conditions. *Amplification route* (§6.1–6.2): any two assignments h, h′ over the same D are identical on closed-loop holonomies outright, since every nonzero discrepancy amplifies, at an explicit witness index, to an above-threshold witnessable invariant; this route requires checks (A), (B), and (C) — of which (A) is verified at the source and (B) is discharged as a presupposition of the catalogue theorem (§6.1) — and only the weak completeness claim that witnessed invariants are in D. *Fallback route* (§6.3): any two assignments over the same D are either identical on closed-loop holonomies or differ only by physically empty surplus structure; this route requires full Transport-Completeness and survives failure of any check;

(b) open-path phase differences not affecting closed loops are gauge (Gauge Lemma, §7);

(c) for co-terminal admissible histories P, P′, the phase kernel

$$W(P, P') = e^{i(\theta(P) - \theta(P'))} = h(P \circ P'^{-1})$$

supervenes on D (§8); and

(d) the support functional s, built from W and itself admissibly witnessable or null in its differences (§5), introduces no information beyond D:

$$s = s(D).$$

[Conditional — shared conditions: (1) no-pre-individuation (§2.2); (2) interference as admissible comparison (§4.2); (3) the Direction-of-Construction Premise (§4.2); (4) the imported lattice holonomy principle (§2.5); (5) per-sector path-connectedness where the Gauge Lemma is invoked (§7); (6) s-witnessability (§5); (7) the §10.2 audit of the dyadic-loading chain, items (a) — as amendment obligation — and (b). Route-specific conditions: the amplification route additionally requires checks (A) single-loop iteration [verified at source], (B) uniform threshold [discharged as a presupposition of the catalogue theorem], and (C) comparison realizability [open] (§6.1; audit items (c), (d), (e)), and only weak completeness (witnessed invariants belong to D); the fallback route additionally requires full Transport-Completeness of D (§4.3). No part of this theorem is claimed unconditionally.]

Corollary — Uniqueness of Physical Assignment

Let h and h′ be holonomy assignments over the same distinguishability structure D, under the conditions of the Theorem along either route.

If h and h′ differ physically, then they differ on some admissible transport invariant — along the amplification route, because every nonzero closed-loop discrepancy amplifies to a witnessable one; along the fallback route, because an unwitnessed difference is not physical. In either case the difference belongs to D, contradicting sameness of D.

Therefore physically distinct assignments cannot share the same D. Equivalently:

D determines a unique physical holonomy assignment, up to gauge, on each connected sector.

Logically, this Corollary is the contrapositive of the Theorem's clause (a) and adds no deductive content. It is stated separately because it expresses the result in its strongest form: not merely that holonomy supervenes on D, but that D admits only one physical assignment. The phase sector offers no menu of physically distinct holonomy choices over a fixed distinguishability structure; there is nothing left to choose.

[Conditional — inherits exactly the Theorem's conditions, route for route.]

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12. What Would Refute This

The refuters must be aimed where the weight actually sits. Under Transport-Completeness, it is impossible by construction for two assignments to agree on all of D while differing on an admissible holonomy invariant — the premise itself rules that out — and along the constructive route the same demand is ruled out by amplification rather than premise: the discrepancy is driven above threshold and witnessed. A "refuter" demanding such a pair would therefore be demanding something the paper's premises or its construction make contradictory, and its non-existence would confirm nothing. The honest refuters attack the premises, the discipline, and the open check, not the theorem.

Refuter 1 — against Transport-Completeness, via rejection of the operational criterion

The natural-sounding target — a closed-loop holonomy difference that is physically consequential yet has no admissible witness — must be characterised honestly, because within the programme's own vocabulary it is not merely unconstructed but incoherent. §5 defines physical consequence operationally: a difference matters precisely when it manifests in admissible commitment patterns, and commitment patterns are paradigm witnessed structure. Jointly with no-pre-individuation, this makes "consequential but unwitnessed" a contradiction in terms, not an open empirical possibility. No counterexample of this shape can be exhibited *from inside the framework*, and the paper does not pretend that its absence is evidence.

Refuter 1 is therefore a philosophical exit door, not an empirical-style search, and it is stated as such: reject the operational criterion of physicality itself. Maintain that there is a fact about closed-loop phase that does real physical work while being in principle invisible to every admissible comparison — that "physical" outruns "operationally distinguishable." This is a rejection of the programme's verificationist foundation wholesale, not a local objection to this paper. It is a coherent position; the programme simply does not hold it, and holds the contrary openly. A reader who takes this door does not refute the theorem — they decline the premise that gives "physical" its meaning here, and the disagreement moves upstream of everything the programme asserts.

Stating this explicitly forecloses a hostile reading on which the paper claims refutability while offering an unfalsifiable refuter. The refuter is not unfalsifiable; it is extra-systematic. The Amplification Lemma changes nothing here: it operates entirely inside the operational criterion, deriving witnessability from composition and resolution, so a critic who takes the extra-systematic door is untouched by it and a critic who stays inside the framework must attack the open check instead (Refuter 5c). The falsifiable targets are Refuters 2–5c below, which attack the framework on its own terms.

Refuter 2 — against no-pre-individuation in the transport sector

Exhibit admissible physical structure in transport that no admissible comparison can witness — a distinction that does real work while being invisible to every admissible operation. This would break the discipline on which case (ii) of the Holonomy Dilemma relies, and with it the dilemma's exhaustiveness.

Refuter 3 — against s-witnessability

Show that the support functional has differences that are physically loaded in the generation chain yet manifest in no admissible commitment pattern — that s does real work while being operationally silent. This would break §5, and "changes s" could no longer serve as a marker of physical difference; the architecture connecting the Dilemma to dyadic loading would be severed.

Refuter 4 — against the §10.2 audit, item (a)

Show, by close reading of the corpus, that the supervenience requirement $s = s(D)$ was stated for a strictly narrower D than the consolidated one — and that the chain's validity depends on that narrowness. **Standing: realized in its mild form.** The narrow usage exists — the U(1) paper's §1 states D as a symmetric metric — and §10.2 accordingly grades §4.1 as a definitional consolidation carrying an amendment obligation for the dyadic-loading paper, rather than a restatement. What remains open is the refuter's strong form: showing that the chain's validity *depends* on the narrowness, so that the consolidation changes content rather than terminology. The strong form is what would block the discharge from transferring; nothing read so far supports it, and the dyadic-loading restatement is the channel through which it would surface if real.

Refuter 4b — against the §10.2 audit, item (b)

Show that the dyadic-loading construction evaluates the kernel W outside the co-terminal domain — that somewhere in the chain, a phase comparison is taken between histories that do not share both endpoints. Off that domain, $\theta(P) - \theta(P')$ is gauge-dependent and the supervenience result of §8 does not cover it. The discharge would fail for exactly that portion of the chain, and the failure would be silent unless this refuter is checked.

Refuter 5a — against check (A), single-loop iteration

Standing: closed at the source. The U(1) paper's §7.4 iterates a single admissible loop explicitly and load-bearingly; the licence is established there, not awaited. To revive this refuter one must overturn that source passage itself, which would simultaneously unseat the U(1) paper's amplification asymmetry and with it the closure argument's Case 1 — a far larger casualty than this paper's constructive route.

Refuter 5b — against check (B), threshold uniformity

Show that the witnessability resolution degrades with loop length — that the threshold for distinguishing closed-loop phase grows with composition. **Standing: closed at the source, at the price of the catalogue.** Uniformity is a presupposition of the U(1) paper's own closure argument (Stability of Witnessability, §6.1): its §7.4 witness operates at unbounded iteration counts, and its §10 tightness audit states that losing the witness collapses the amplification asymmetry and lets the conservation argument coarsen the catalogue. This refuter therefore cannot strike the present paper alone — succeeding here means refuting the catalogue theorem first, and with it both client chains. And even then the explicit witness index bounds the local damage: by the graceful-degradation remark (§6.1), degradation beyond a length scale ℓ excludes only discrepancies below the floor that ℓ sets. The refuter further requires not that long loops are noisier in practice, but that admissible comparison is *in principle* threshold-degraded with length — a structural claim about the substrate irreconcilable with the inherited results as they stand.

Refuter 5c — against check (C), comparison realizability

Show that admissible closed paths — L itself, or its iterates L^n — cannot in general be realized as comparison loops of co-terminal admissible histories: that the loop set and the comparison set come apart, with some loops admissible as transport but unenterable into any admissible interference comparison. The Amplification Lemma's witness step would then assert a comparison that does not exist, the constructive route would fail for exactly the unrealizable loops, and the result would fall back to the Dilemma route for them. This is, at present, the one live refuter against the constructive route, and audit item (e) is its verification channel.

What does not count

It is not enough to show that different gauge choices give different open-path phase labels. That is gauge freedom, conceded in §7. It is not enough to show that a bare symmetric metric fails to encode phase. The paper does not claim that a bare metric is sufficient, and says so in §4.1 and §9. It is not enough to observe that the fallback route is nearly analytic under its premise. The paper says that itself, in §6 and §9; and the amplification route exists precisely because the near-analytic route is not the only one. And it is not enough to exhibit the corpus's narrow metric usages of "D" — §10.2 records them and grades the consolidation as an amendment obligation already.

If any of Refuters 2–5c can be constructed, the corresponding node reopens and is marked [Open] — with the grading that 5a and 5b are closed at the source and cannot strike this paper without first unseating the catalogue theorem; 5c, if successful, demotes the result from constructive to premise-based without unseating it; 2, 3, or 4b strike load-bearing structure; and 4 has already been realized in its mild amendment form (§10.2). Refuter 1 stands apart: it cannot be constructed within the framework, only adopted as a rejection of the framework's criterion of physicality, and its adoption relocates the disagreement upstream of the programme entirely. Within the framework, if Refuters 2–5c all fail, the assignment is forced — constructively.

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13. What This Paper Establishes

Established, conditionally

- The assignment problem concerns gauge-invariant closed-loop holonomies, not absolute open-path phase coordinates. [Proven, given Imported-External §2.5 — the structural clarification rests on the lattice holonomy principle]
- For any nonzero closed-loop discrepancy $\Delta = e^{i\alpha}$, an explicit iterate Δ^{n^*} with $n^* = \lceil \pi/(2\alpha) \rceil$ (after reversal-normalising α into $(0, \pi]$) lies at angular distance in $[\pi/2, \pi]$ from 1; hence no nonzero discrepancy stays below a uniform sub-$\pi/2$ witnessability threshold under loop iteration. [Proven — Amplification Lemma core, §6.1; the arithmetic is elementary and unconditional, with no external mathematics imported; the physical application is conditional on checks (A), (B), (C)]
- Under checks (A), (B), and (C) — (A) verified, (B) discharged as a presupposition of the catalogue theorem, (C) open — the unwitnessed horn of the Dilemma is empty for nonzero discrepancies, and assignments agreeing on D coincide on all closed-loop holonomies outright, requiring only weak completeness. [Conditional — §6.2, on audit item (e); items (c) and (d) resolved]
- Threshold uniformity is not a new premise but a presupposition of the inherited catalogue result: degrading witnessability under composition would collapse the U(1) paper's amplification asymmetry and destabilise the catalogue theorem itself. [Inherited — Stability of Witnessability, §6.1, from the U(1) paper's §7.4 and §10]
- Under a partial failure of threshold uniformity beyond a length scale ℓ, the witness index for a discrepancy of angle α exceeds ℓ only when α lies below an explicit floor set by ℓ. [Proven — graceful-degradation remark, §6.1; arithmetic of the explicit index]
- Consequently, under such partial failure, discrepancies above that floor remain excluded by the constructive route. [Conditional — on check (C), since exclusion runs through the witness step; (A) resolved, and the partial-uniformity hypothesis supplies what remains of (B) up to ℓ]
- Independently of amplification, a closed-loop holonomy difference between assignments agreeing on D is unwitnessed — the witnessed case being empty under the agreement hypothesis — and hence excluded by no-pre-individuation. [Conditional — Holonomy Dilemma, §6.3, on full Transport-Completeness]
- Open-path phase differences leaving all closed-loop holonomies invariant are gauge, on each path-connected sector. [Imported-External §2.5, conditional on per-sector connectedness — Gauge Lemma, §7]
- Inter-sector relative phase is undefined rather than gauge, and carries no admissible content. [Conditional — §7]
- Co-terminal histories are automatically same-sector; W on its domain is a within-sector quantity. [Proven — Sector-closure Corollary, first clause, §7]

- The generation chain never consumes inter-sector phase. [Conditional — Sector-closure Corollary, second clause, §7; holds exactly insofar as §10.2 audit item (b) holds]
- For co-terminal histories, the phase kernel W(P, P′) is the holonomy of the closed comparison loop $P \circ P'^{-1}$. [Proven — identity, given the domain restriction]
- W is D-determined on its domain; therefore s supervenes on D. [Conditional — §8, route-graded: via §6.2 under the checks and weak completeness, or via §6.3 under full Transport-Completeness]
- Differences in s are admissibly witnessed or physically null. [Conditional — §5]
- Physically distinct holonomy assignments cannot share the same D; D determines a unique physical assignment up to gauge on each connected sector. [Conditional — Uniqueness Corollary, §11; contrapositive of the Theorem, inheriting its conditions route for route]
- Assignment freedom over fixed D would itself violate no-pre-individuation: an undetermined assignment is individuation in excess of distinguishability. [Conditional — §9.4; the Dilemma read in reverse]
- Transport-Completeness is a completeness requirement on the notion of distinguishability, not an additional physical hypothesis about phase; its rejection requires replacing the framework's criterion of physicality. [Conditional — §4.4; a methodological defence that reduces arbitrariness without discharging the premise]
- Given the Direction-of-Construction Premise, the architecture D → W → s contains no definitional cycle. [Conditional — §4.2; the premise itself is undischarged and listed below]
- The dyadic-loading chain's supervenience requirement transfers under the consolidated D as an amendment obligation; its kernel evaluations being co-terminal remains to be verified. [Conditional — §10.2 audit: item (a) re-graded to amendment, item (b) open]

Not established

- The compact U(1) catalogue itself; inherited from the previous paper. [Inherited]
- The truth of no-pre-individuation; inherited from the programme. [Inherited]
- The truth of Transport-Completeness of D in its general form. The specific instance the assignment problem needs — closed-loop holonomy discrepancies cannot hide outside D — is discharged constructively by the Amplification Lemma, conditional on checks (A), (B), and (C) — the first two verified at the U(1) paper's §7.4, the third open; the general form remains a named, undischarged premise and carries the fallback route alone. §4.4 argues the premise is methodological rather than ad hoc, which reduces its arbitrariness without discharging it. [Conditional]
- Check (A): that the U(1) paper's unbounded-composition premise licenses single-loop iteration. Verified at the source — the U(1) paper's §7.4 iterates a single loop explicitly. [Inherited — audit item (c), resolved]
- Check (B): that finite distinguishability supplies a loop-length-uniform resolution threshold below $\pi/2$. Discharged: the magnitude clause is inherited de facto from the Born arc, and uniformity is a presupposition of the U(1) paper's own catalogue argument (Stability of Witnessability, §6.1) — denying it unseats the closure theorem before it reaches this paper. Residual: record the presupposition explicitly at the source. [Inherited — audit item (d), resolved]

- Check (C): that every admissible closed path — including iterates L^n — is realizable as the comparison loop of co-terminal admissible histories, or that open-with-closed composition is admissible. The one open condition of the constructive route. [Conditional — audit item (e), flagged; Refuter 5c]
- The Direction-of-Construction Premise — that interference-grade relations are prior to s; named here, anchored to the Born arc, undischarged within this paper. [Conditional]
- The lattice holonomy principle that gauge-invariant content is closed-walk content; imported from standard mathematics. [Imported-External]
- The detailed derivation of particle generation counts; downstream of support supervenience. [Open]
- Non-abelian gauge structure; out of scope.
- The Standard Model embedding of charges; out of scope.

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14. Conclusion

The U(1) paper showed that the phase catalogue is forced to be a compact continuous circle. It deliberately left open whether the assignment of phase to histories is also forced. This paper addresses that open node, and is explicit about how it does so.

The conclusion is that the physically meaningful assignment is fixed by D, provided D is understood as complete admissible distinguishability rather than a bare metric — and the fixing is secured by two routes of different strength. The constructive route shows that a nonzero closed-loop discrepancy cannot hide: a sub-quarter-turn discrepancy accumulates under loop iteration by fixed steps of its own angle and cannot skip the quarter-turn band, so an explicit iterate exceeds any uniform witnessability threshold and becomes an interference-witnessable invariant, recorded in D. The fallback route, independent of the checks the constructive route carries, secures the same conclusion as a dilemma under full Transport-Completeness: a holonomy difference is either detectable by admissible comparison, and then in D, or undetectable by every admissible comparison, and then not physical structure at all.

Open-path phase labels may still vary, but only by gauge, sector by sector. They do not change closed-loop holonomies and therefore do not change physical content. The support kernel W(P, P′), defined on co-terminal histories, is the holonomy of the closed comparison loop between them. Since closed-loop holonomy cannot vary independently of D, the kernel is D-determined, and the support functional — whose differences are themselves witnessed or null — introduces no information beyond D.

The paper does not overstate what this proves, and grades it honestly. Of the constructive route's three checks, two are closed at the source — single-loop iteration is verified outright at the U(1) paper's §7.4, and threshold uniformity is discharged as a presupposition of that paper's own catalogue argument, which could not survive its failure — so the route's standing turns on one open condition, comparison realizability: that admissible loops enter admissible interference comparisons. If that check holds, the assignment result is a substantive theorem resting on premises the programme has already paid for — unbounded composition and finite-resolution witnessability — and the completeness premise is needed only in its weak form. If it fails, the fallback carries the result alone, the theorem is close to analytic, and the physics lives in the named premise, exposed where the method requires. The imports are marked and specified in their lattice form; the definition of D is consolidated rather than clarified, with the corpus's narrow metric usages recorded and the resulting amendment obligation for the dyadic-loading paper graded openly; the construction D → W → s is fixed as acyclic; and the audit register spans two papers and five items, three resolved. The refuters are sorted by kind: one extra-systematic door, which rejects the operational criterion of physicality and with it the programme's foundations, stated honestly as such; in-framework targets that strike load-bearing structure; one realized already in its mild amendment form; two closed at the source, strikable only through the catalogue theorem itself; and one live against the constructive route, which if successful demotes the result to premise-based without unseating it.

If none of those refuters can be constructed, the Shared Phase Obligation closes on both faces. The Born face is discharged by the continuous U(1) catalogue. The generation face is discharged by holonomy assignment supervenience.

The result can be viewed as a closure theorem for distinguishability, and this is its deepest reading. The previous paper showed that finite distinguishability determines the admissible phase catalogue — which phase values may exist. The present paper shows that, once transport-complete distinguishability is granted — or, along the amplification route, once witnessability under unbounded composition is taken at its inherited strength — distinguishability determines also which phase relations may occur. Between the two papers, the phase sector contains no independent physical degree of freedom beyond admissible comparison itself: not in its value space, which is forced to U(1), and not in its assignment, which is forced by D up to gauge. Phase survives — it is real, structural, and load-bearing for everything downstream — but phase assignment does not float free of distinguishability. The phase sector has no physical content beyond distinguishability, and that, rather than the supervenience formula, is what the paper establishes, under its named conditions. The two-paper sequence is then summarised in three sentences: the phase catalogue paper removed freedom in the choice of phase space; the present paper removes freedom in the assignment of phase within that space; together they leave no independent phase structure beyond admissible distinguishability itself.

In plain terms: once the circle is forced, the hand is not free to point anywhere without reason. Its position around a closed history is either written into admissible distinguishability, or it is not physics.