Local Face Completion and the Preservation of Gate-3 Holonomy

Why Refinement Annihilates Contractible Curvature but Need Not Fill Non-Bounding Record Cycles — and Why the Last Clause Is the Winding of the Primitive Fact

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Summary for the General Reader

This paper is about whether a certain kind of "memory" can survive in the deepest layer of physics that VERSF describes — and it reaches the answer: yes, almost certainly, wherever facts exist.

Some background in plain terms. VERSF builds physics from a single starting idea: reality is made of discrete acts of distinction — moments where something becomes definite rather than remaining undecided. A settled act of this kind is called a *Fact*. The "Gate-3" question asks whether the network of settled facts can carry a particular invisible quantity — a kind of leftover twist that you only notice when you travel in a loop and come back to where you started, finding things rotated even though every local step looked fine. Mathematicians call this *holonomy*; the everyday image is currency exchange where you convert money around a loop of currencies and end up with less than you began, even though each individual trade was fair. The number that measures this twist lives in a seven-element clock arithmetic (the "$\mathbb{Z}_7$" of the title — counting 0,1,2,3,4,5,6 and then back to 0), which is why the programme is called a K=7 closure.

The worry was that this twist might not be a real feature of the world but an accounting artifact, or that even if real it would be erased as you look more finely at the substrate — "filled in" the way a small hole in a net disappears if you weave the net more tightly. This paper shows it cannot be erased. The key is a property VERSF already requires of facts: when a fact is committed, it permanently seals off a region (the "discarded region") that can never be revisited. A loop drawn around such a sealed region can never be filled in, because filling it would mean crossing into the sealed region — which would undo the fact, and facts are irreversible. So the loop, and any twist it carries, is protected forever.

That leaves one question: does such a protected loop actually carry a nonzero twist, or could it carry exactly zero? The paper argues the twist must be nonzero. The smallest possible fact is a single elementary act of distinction, and an act that changed *nothing* would not be a fact at all —

so it must register as a nonzero step on the seven-hour clock. If every fact registered zero, the entire seven-fold structure that VERSF uses elsewhere (including its celebrated calculation of the fine-structure constant, a fundamental number in physics) would be doing no work — which would be very strange. So the conclusion is that the twist is real and survives: the existence of facts forces the existence of this memory in the substrate.

One finer point remains genuinely open: whether the twist is always the *smallest* nonzero amount (one step of the clock) or could be some other nonzero amount. That depends on a single, concrete, checkable property of how a fact advances the underlying "phase" — a question for future work. But whether the twist is *present at all* — the original question — is settled here in the affirmative, resting only on facts being irreversible and the seven-fold structure being a genuine record of the substrate's state.

The technical abstract and the body of the paper make all of this precise.

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Abstract

The companion paper reduced Gate-3 occupancy to a single question about refinement dynamics. Fact Momentum is the continuum stress-energy of a U(1) connection obtained by embedding the $\mathbb{Z}_7$ closure offsets into their phase circle and taking the continuum limit; that connection decomposes into a curvature sector and a flat sector; and Gate-3 closure holonomy is exactly the flat sector. Whether Gate 3 is occupied is therefore whether the flat sector is nonempty — whether the limiting connection carries nontrivial holonomy around non-bounding record cycles, or whether refinement screens it by filling those cycles with completed faces.

This paper isolates the mechanism that decides the question and proves the structural half of it. The decision turns on the *face-completion rule*: whether refinement fills loops freely wherever they exist (globally cycle-filling, in which case every cycle eventually bounds and the flat sector is screened) or only where a completed local commitment generates a face (local-fact-generated, in which case certain cycles can remain permanently non-bounding). VERSF supplies a candidate protection mechanism already present in the fold-commitment definition: the isolation condition. A completed Fact isolates a discarded region D such that no trajectory from D rejoins the active region under any allowed local operation. A loop encircling an isolated D cannot be filled by an admissible face, because such a face would carry a trajectory from D back into the active region, violating isolation and un-committing the Fact. Isolation makes the enclosing loop a permanent puncture, not a temporary gap.

The result is a biconditional and a theorem. The biconditional: Gate-3 holonomy survives refinement if and only if face completion is local-fact-generated and isolation forbids global cycle-filling. The theorem — the Isolation-Protected Holonomy Theorem — is the forward, structural half: filling a cycle that encloses an isolated discarded region would violate the isolation condition of a completed Fact, so such cycles are permanently non-bounding and their holonomy survives every refinement. The containment that the proof requires — that refinement face-generation uses no operation outside the isolation-governed class — is not assumed but derived: an operation that could cross D would un-commit a Fact, contradicting irreversibility.

One clause remains genuinely open, and the paper marks it as such rather than assuming it. Protection guarantees that isolation-enclosing cycles *exist* and persist; it does not by itself guarantee that the admissible $\mathbb{Z}_7$ offsets *place nonzero holonomy* on such a cycle. A permanently non-bounding cycle whose offsets sum to zero is a protected hole carrying no charge. Gate 3 is occupied if and only if at least one isolation-protected cycle carries nonzero admissible holonomy, and that residual clause is a property of the offset-assignment rule, not the face rule.

The paper locates that clause precisely on the primitive Fact — the smallest cyclic commitment structure the substrate admits, and the natural generator of the protected cycles — and reframes it as a sharp dichotomy rather than an aesthetic expectation. Either at least one primitive Fact carries nonzero $\mathbb{Z}_7$ winding, in which case Gate 3 is occupied wherever Facts exist; or the winding of every primitive Fact vanishes, in which case the offset cochain is exact on a generating set of the record network's first homology, hence globally exact, hence the $\mathbb{Z}_7$ closure charge is identically zero and the closure group is observationally inert in the transport sector.

The dichotomy is then sharpened to a conditional theorem. Defining a primitive Fact as a single fold-commitment isolating a single discarded region — consistent with the VERSF usage of minimal cyclic structure, first irreversible separation, and threshold event — the minimal cycle around the region is the boundary of that one isolation event and registers exactly one commitment's phase advance, with no sum over commitments and no cancellation. The Phase-Tick Occupancy Theorem then states: if a fold-commitment advances the closure phase by one generator of the $\mathbb{Z}_7$ phase circle ($\Delta\theta = \pm 1$), the minimal Fact cycle carries unit holonomy $Q = \pm 1 \neq 0$, and Fact existence implies Gate-3 occupancy. The arc thus closes on a single irreducible substrate input — the value of one commitment's phase advance — with no remaining fork: one generator gives occupancy with unit charge, an arbitrary residue gives occupancy unless the closure charge is inert, and there is no third branch.

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1. Where the Question Now Stands

The companion paper established the decomposition

Fact Momentum = curvature sector + flat sector, Gate 3 = flat sector,

with the arrow running substrate → continuum: the $\mathbb{Z}_7$ closure holonomy is fundamental, and Fact Momentum is the continuum stress-energy of the U(1) connection it limits to. The decomposition holds in both the flat and curved cases. What it does not settle is the *size* of the flat sector: whether there is any non-bounding-cycle holonomy to be the topological summand.

That is occupancy, and it was handed to this paper in a sharp form. Completing a face annihilates contractible curvature, not non-contractible holonomy: setting a face's holonomy to a coboundary removes curvature on the cycle bounding it but does not touch the holonomy of a cycle that bounds no completed region. So flat-sector holonomy survives refinement unless refinement adds faces that fill previously non-bounding cycles. The whole question is therefore:

Under VERSF refinement, can the face set grow to fill the non-bounding cycles that carry holonomy?

If yes, every cycle is eventually contractible, the flat sector is screened, and Gate 3 is empty. If no — if some cycles are structurally barred from ever being faced — those cycles are permanently non-bounding, and the flat sector survives. This paper identifies the structural bar already present in VERSF, proves that it protects the cycles, and isolates the one remaining clause that decides whether the protected cycles carry charge.

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2. Two Face-Completion Regimes

The face set of the record network is not fixed; refinement may add faces. Everything depends on the rule governing what may be added.

Globally cycle-filling completion. A face may be added wherever a loop exists. Under this regime refinement can subdivide and fill any cycle given enough steps, so every loop is "eventually contractible." No cycle is permanently non-bounding, the flat sector is screened in the limit, and Gate 3 is empty regardless of any other structure. This is the trivial/confined regime.

Local-fact-generated completion. A face may be added only where a completed local commitment generates one. Under this regime the face set is not free: it is constrained to faces that correspond to admissible completions. If some cycles enclose a region that admissible completion is *forbidden* to face, those cycles can never be filled, remain permanently non-bounding, and protect the flat sector. This is the topological/deconfined regime.

The two regimes are distinguished not by the curvature/holonomy distinction — which holds in both — but by whether the face set is bounded by a structural prohibition. The question is therefore whether VERSF face-completion is constrained by such a prohibition, and the answer is that it is: the isolation condition of the fold-commitment definition.

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3. The Isolation Condition as a Structural Prohibition

The fold-commitment event is defined by two conditions on a discarded region D:

- **Invariance:** the discarded region maps to itself, T(D) = D.
- **Isolation:** no trajectory from D rejoins the active region under any allowed local operation.

Isolation is the operative content for this paper. It states that, once a Fact is committed with discarded region D, the active region is the complement of D with respect to the allowed local operations: nothing in D can be brought back into the active region by any such operation. In topological terms, D is excised — the space of admissible configurations is the active region, and D is a puncture in it.

A Fact, by the minimal-fact threshold, requires nontrivial cyclic topology: irreversible fact formation needs a non-contractible loop / nontrivial first homology class, and tree-like structures cannot support permanent separation. Combining the two: a completed Fact is a non-contractible loop together with an isolated discarded region D that the loop encircles. The loop is non-contractible *because* it cannot be shrunk through D — shrinking it would carry the active region across D, which isolation forbids.

This is precisely the defect structure occupancy requires. A gap is a hole that enough admissible completions can tile over; a defect is a puncture no admissible completion can cover. Isolation makes D a defect: a loop around D cannot be filled by any admissible face, because the face would be a 2-cell spanning the loop, and such a cell contains trajectories from D into the active region — exactly what isolation forbids. The loop is therefore not merely unfilled but unfillable.

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4. Refinement Is Contained in the Isolation-Governed Operations

The protection argument requires that refinement cannot use an operation outside the class that isolation quantifies over. Otherwise a refinement face could cross D without being an "allowed local operation," isolation would not forbid it, and protection would fail. This containment is not assumed; it is forced by irreversibility.

Lemma 4.1 — Refinement Containment *(proven, from irreversibility of fact formation)*

Refinement face-generation uses no operation outside the allowed-local-operation class relevant to commitment isolation.

Proof

Refinement is part of admissible record dynamics. Suppose, for contradiction, that refinement could generate a face using an operation outside the allowed-local-operation class — in particular, an operation carrying a trajectory from a discarded region D back into the active region. Then that operation would reverse the isolation of D, and hence reverse the commitment of the Fact whose discarded region is D. But fact formation is irreversible by definition: a committed Fact cannot be un-committed by admissible dynamics. This is a contradiction. Therefore refinement uses no operation outside the allowed-local-operation class; refinement face-generation is contained in the isolation-governed operation set. ■-free statement: any refinement operation that could cross D would un-commit a Fact, which admissible dynamics cannot do.

The containment is thus a consequence of the fact definition, not an additional hypothesis. Whatever operations refinement uses, they are a subset of the allowed local operations, because a broader set would make facts reversible.

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5. The Isolation-Protected Holonomy Theorem

Theorem 5.1 — Isolation-Protected Holonomy *(proven, modulo the offset-charge clause of Section 6)*

Let a completed Fact isolate a discarded region D, and let γ be a record cycle encircling D. Then γ is permanently non-bounding: no admissible refinement can fill γ with a face. Consequently the flat-sector holonomy supported on γ survives every refinement.

Proof

Suppose some refinement step fills γ — adds an admissible face (2-cell) σ with boundary γ. The face σ spans γ, and since γ encircles D, σ contains a path crossing D into the active region. By Lemma 4.1, σ is generated by an operation contained in the allowed-local-operation class. But the path through σ carries a trajectory from D into the active region under an allowed local operation, which violates the isolation condition of the Fact whose discarded region is D. Isolation is part of the operative definition of that completed Fact; violating it un-commits the Fact, contradicting irreversibility (Lemma 4.1). Therefore no admissible refinement can add a face with boundary γ. The cycle γ is permanently non-bounding.

Because completing a face annihilates only contractible curvature and never the holonomy of a non-bounding cycle, and because γ remains non-bounding under all refinement, the flat-sector holonomy on γ is not screened at any refinement level. It therefore survives the continuum limit. ■-free statement: an isolation-enclosing cycle can never be filled, so its holonomy persists.

Corollary 5.2 — The Biconditional

Gate-3 holonomy survives refinement if and only if face completion is local-fact-generated and isolation forbids global cycle-filling.

Proof

(Forward.) If face completion is local-fact-generated, faces are added only by completed commitments, each carrying an isolated discarded region. By Theorem 5.1, cycles encircling those regions are permanently non-bounding, so their holonomy survives. The flat sector is not screened.

(Reverse.) If face completion were globally cycle-filling — able to add a face wherever a loop exists — then in particular it could add a face across a cycle encircling an isolated D. By the proof of Theorem 5.1 that face violates isolation, so global cycle-filling is incompatible with the existence of completed Facts. Hence wherever Facts exist, completion cannot be globally cycle-filling; it must be local-fact-generated. Equivalently, if completion were genuinely global, isolation would be violable and Facts would not be irreversible, so no protected holonomy and no Facts — the flat sector is screened.

Thus survival of flat-sector holonomy and local-fact-generated (isolation-respecting) completion are equivalent. ■-free statement: protected holonomy exists exactly when completion respects isolation, which is exactly when Facts are irreversible.

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6. The One Open Clause: Charge on the Protected Cycle

Theorem 5.1 protects the *cycle*. It does not by itself place *holonomy* on the cycle. This distinction is the residual open clause, and the paper marks it rather than assuming past it.

A permanently non-bounding cycle γ carries Gate-3 holonomy only if the admissible $\mathbb{Z}_7$ offsets sum to a nonzero value around it:

$Q\_\gamma = \Sigma\_\gamma\ \rho \neq 0$ in $\mathbb{Z}_7$.

The cycle is Gate-3 occupied when $Q\_\gamma \neq 0$ and Gate-3 empty when $Q\_\gamma = 0$. Isolation guarantees γ exists and persists. It says nothing about the value of Q_γ. If the admissible offset assignment puts $Q\_\gamma = 0$ on every isolation-protected cycle, then every protected cycle is a hole carrying no charge, the flat sector is non-bounding but trivial, and Gate 3 is empty despite the protection. Protection is necessary for occupancy; it is not sufficient.

6.1 Occupancy needs only one charged cycle, and the primitive Fact is the natural carrier

Occupancy requires nonzero *total* charge on *some* protected cycle — not on all of them. The natural place to look is the primitive Fact, for two reasons. First, by the minimal-fact threshold, irreversible fact formation requires nontrivial cyclic topology: a non-contractible loop, since tree-like structures cannot support permanent separation. So a primitive Fact is precisely a minimal non-contractible cycle together with an isolated discarded region — the smallest cyclic commitment structure the substrate admits, and by Theorem 5.1 a protected one. Second, Fact-generated cycles are the *only* source of non-contractible loops in the record network; every protected cycle is generated by the isolation regions of completed Facts. The primitive Facts are therefore the generators of the protected part of H_1, and the charge on any protected cycle is a $\mathbb{Z}_7$-combination of the windings of the primitive Facts that generate it.

A caveat follows immediately and is worth stating: occupancy does not strictly require any *primitive* Fact to be charged. Charge could be supported on a *composite* cycle — a sum of Fact-cycles — even if individual primitive windings interfere. But the primitive Facts are the generators, so if every primitive winding vanishes, every composite winding vanishes too. The primitive Fact is therefore the correct object: charged primitives give occupancy directly, and uncharged primitives force every protected cycle to be uncharged. The question reduces to the winding of a primitive Fact.

6.2 The dichotomy: charged Facts, or an inert $\mathbb{Z}_7$

The decisive observation is what *uniform zero winding* would mean — and it is far stronger than "the cycle happens to be uncharged." Suppose $Q\_\gamma = 0$ for every primitive Fact. Then the offset cochain ρ is exact on every Fact-generated cycle. Since Fact-generated cycles generate the non-contractible part of the record network's first homology (§6.1), ρ has vanishing period on a generating set of H_1, hence vanishing period on all of H_1, hence ρ is globally exact: $\rho = d^0 a$ for some 0-cochain a. That is $A \subseteq B^1$ identically, $\kappa = 0$ by construction, on every record network the dynamics produce.

This is not "the closure architecture contributes nothing beyond the cycle." It is the much sharper statement that the $\mathbb{Z}_7$ closure *charge is identically zero everywhere*, so the closure group $\mathbb{Z}_7$ could be replaced by the trivial group with no change to any record-network transport observable. The 7 would be a label with no referent in the transport sector — observationally inert.

Proposition 6.1 — Zero Winding Implies Inertness *(proven)*

If every primitive Fact has zero winding ($Q\_\gamma = 0$ for every minimal Fact cycle), then the $\mathbb{Z}_7$ closure charge is globally inert: ρ is exact on every record network the dynamics produce, $A \subseteq B^1$ identically, and no record-network transport observable depends on the closure group being $\mathbb{Z}_7$ rather than trivial.

Proof. As above: Fact-generated cycles generate the non-contractible part of H_1 (§6.1); vanishing winding on all of them is vanishing period on a generating set, hence on all of H_1; hence $\rho = d^0 a$ globally. Every closure holonomy $Q\_\gamma = \Sigma\_\gamma\ \rho$ then vanishes for every cycle γ, so no transport observable distinguishes the $\mathbb{Z}_7$ structure from the trivial group. ■-free statement: uniform zero winding makes the closure charge vanish everywhere, which is exactly inertness of $\mathbb{Z}_7$ in the transport sector.

Consequence

The zero-winding branch is not logically impossible, but it is highly constrained, because the rest of the programme treats the 7 as physically operative:

- the K=7 closure architecture is built on $\mathbb{Z}_7$ being the closure group;
- the hexagonal-closure derivation of the fine-structure constant uses the 7-fold structure as an operative quantity;
- the transport programme assigns $\mathbb{Z}_7$-valued offsets as carriers of closure content.

For the closure charge to be globally inert, all of these would have to treat as physically operative a structure that contributes nothing to any transport observable — a strong internal tension, though not a formal contradiction. The dichotomy is therefore:

Either at least one primitive Fact carries nonzero $\mathbb{Z}_7$ winding — and Gate 3 is occupied wherever Facts exist — **or** the $\mathbb{Z}_7$ closure charge is globally inert (Proposition 6.1), in tension with the operative role the 7 plays elsewhere in the programme.

This replaces the aesthetic expectation "the smallest commitment should carry the smallest charge" with a genuine constraint: zero winding is not merely unmotivated, it implies inertness of the whole $\mathbb{Z}_7$ structure. It does not *prove* Facts are charged — the inert branch is logically available — but it raises the price of the empty outcome from "a quiet null result" to "the closure group does no work anywhere."

6.2A The Tick → Fold → Fact chain and generator minimality

The phase-tick statement is not an isolated hypothesis bolted onto the theorem; it is grounded in the VERSF construction chain for a primitive Fact:

Ticks → One Fold = One Bit = One Fact.

Ticks accumulate on the substrate until closure is achieved. The first completed closure event is the Fold: the minimal irreversible commitment, simultaneously the first bit and the first Fact. This is the primitive construction, and it constrains the winding through a minimality principle.

Generator Minimality Principle. A primitive object corresponds to a *generator* of the relevant algebra, not to a composite element. A primitive Fold is, by construction, the first and indecomposable closure transition; it is therefore not a sum or multiple of smaller transitions. In the closure algebra it corresponds to a generating element, not a composite.

What this principle delivers, stated exactly. In $\mathbb{Z}_7$ — a cyclic group of prime order — *every* nonzero element generates the group, so "the primitive Fold corresponds to a generator" yields

$\Delta\theta \in \{1, 2, 3, 4, 5, 6\}$, i.e. $\Delta\theta \neq 0 \pmod 7$.

The principle therefore gives **nonzero** winding, not unit winding. And nonzero winding is already enough for occupancy: by Proposition 6.1 and the §6.2 dichotomy, a nonzero primitive winding places the programme on the occupied branch. So generator minimality alone, applied to a prime-order closure group, already implies:

If a primitive Fold corresponds to a nonzero generator of $\mathbb{Z}_7$, then primitive Facts carry nonzero winding, and Gate 3 is occupied wherever Facts exist.

This is a genuine strengthening of §6.2: it removes the inert branch on structural grounds (a primitive transition that is the zero element would not be a closure transition at all — it would be no commitment), leaving occupancy rather than the bare dichotomy.

Proposition 6.2 — Nontriviality of Primitive Commitment *(proven, given faithful phase)*

A primitive closure transition does not correspond to the identity element of the closure group; hence $\Delta\theta \neq 0$.

Premises.

1. A commitment is, by definition, an irreversible *change* of closure state: an event that left the closure state unchanged would not be a commitment but its absence.
2. The identity element of the closure group is the unique element that leaves the closure state unchanged.
3. *(Faithful phase.)* The $\mathbb{Z}_7$ closure phase is a faithful record of the closure state: distinct closure states have distinct phases, so the identity phase corresponds only to the unchanged state. In the K=7/$\mathbb{Z}_7$ architecture this is the statement that the closure group *is* $\mathbb{Z}_7$ — the phase is the closure state, not a projection of a larger state space.

Proof. By premise 1 a primitive commitment changes the closure state. By premise 2 the only group element leaving the state unchanged is the identity, so the commitment's element is not the identity. By premise 3 the phase faithfully tracks the closure state, so a nonidentity change of state is a nonidentity phase advance: $\Delta\theta \neq 0 \pmod 7$. ■-free statement: a commitment changes the closure state by definition, the identity changes nothing, and since the phase faithfully records the state, the commitment's phase advance is not the identity.

The role of premise 3 is the hinge and is stated rather than assumed silently: premises 1 and 2 give "the commitment changes the closure *state*," and only faithfulness upgrades this to "the commitment changes the *phase*." If the closure state had components the $\mathbb{Z}_7$ phase does not record, a commitment could change the state while leaving $\Delta\theta = 0$, and the proposition would fail at exactly this step. Faithfulness is the claim that the closure state is the $\mathbb{Z}_7$ phase — which is what the K=7/$\mathbb{Z}_7$ closure architecture asserts throughout.

With Proposition 6.2, occupancy no longer rests on the informal "a zero-element transition is no commitment" but on a stated proposition: $\Delta\theta \neq 0$ follows from the definition of commitment plus faithfulness of the phase. The occupancy theorem is then close to unavoidable — its only remaining escape is denial of faithfulness (premise 3), i.e. the existence of closure-state structure invisible to the $\mathbb{Z}_7$ phase, which would itself sit uneasily with the K=7/$\mathbb{Z}_7$ architecture.

What generator minimality and Proposition 6.2 do *not* deliver is the specific value ±1. Because every nonzero residue generates $\mathbb{Z}_7$, "generator" and "nonidentity" both give only $\Delta\theta \neq 0$, not $\Delta\theta = \pm 1$. Obtaining the unit value requires the stronger statement that the primitive Fold is the *unit step* of the phase circle — the elementary advance $\exp(2\pi i/7)$ — which is the content of §6.3. The distinction is kept deliberately: generator minimality and Proposition 6.2 give nonzero (hence occupancy); the unit-step reading gives ±1 (hence unit charge).

6.3 The phase-circle route to unit winding

Section 6.2A established, via generator minimality, that a primitive Fold corresponds to a nonzero element of $\mathbb{Z}_7$, hence $\Delta\theta \neq 0$, hence occupancy (the inert branch is removed because a zero-element transition is no commitment at all). What remains is the *value*: whether $\Delta\theta$ is the unit step ±1 or some other nonzero residue. Occupancy does not need this; unit charge does.

The companion paper established that $\mathbb{Z}_7$ embeds in the closure phase circle U(1) as the seventh roots of unity, with the embedding forced by the closure architecture. The Tick → Fold chain of §6.2A supplies the missing specificity: a tick is the elementary advance of the closure phase, and the primitive Fold is the *first* completed closure — one tick, one elementary step around the circle. The elementary advance is $\exp(2\pi i/7)$, the generating root, not an arbitrary one. Under this reading the primitive Fold advances the phase by exactly one unit:

$\Delta\theta = \pm 1 \pmod 7$, the sign fixed by orientation.

This is a stronger statement than generator minimality: it asserts not merely that the Fold is a generator (every nonzero residue is), but that it is the *unit* generating step of the phase circle. Its justification is the Tick → Fold identification — one tick is one elementary phase advance — rather than generic primitiveness.

If instead a commitment could advance the phase by an arbitrary nonzero residue — if a primitive Fold were a generator but not the unit step — then $\Delta\theta \in \{1,\ldots,6\}$ unconstrained in value, Q_γ would be nonzero but not necessarily ± 1, and occupancy would still hold (by §6.2A) without unit charge. The two readings and their consequences:

- **Unit step** (one tick = $\exp(2\pi i/7)$) $\Rightarrow \Delta\theta = \pm 1 \Rightarrow Q_\gamma = \pm 1 \Rightarrow$ Gate 3 occupied with unit charge wherever Facts exist.
- **Nonzero but not unit** $\Rightarrow \Delta\theta \in \{1,\ldots,6\} \Rightarrow Q_\gamma \neq 0 \Rightarrow$ Gate 3 occupied (by generator minimality), charge value unfixed.

In both readings Gate 3 is occupied. They differ only in whether the charge is the generator ±1 or another nonzero residue. The pivotal substrate question for the *value* is therefore:

Is a primitive Fold the unit step of the $\mathbb{Z}_7$ phase circle (one tick = $\exp(2\pi i/7)$), or merely some nonzero generator?

and the pivotal question for *occupancy itself* is already answered by generator minimality: a primitive commitment is a nonzero closure transition, so $Q_\gamma \neq 0$.

Conjecture 6.2 (Minimal Fact Winding) *(open — value only; occupancy itself is no longer conjectural)*

Every primitive isolated Fact carries unit $\mathbb{Z}_7$ winding: $Q_\gamma = \pm 1 \pmod 7$ for the minimal cycle enclosing its isolated discarded region, the sign fixed by orientation.

The status of this conjecture has shifted with §6.2A. Occupancy — that primitive Facts carry *nonzero* winding — is no longer conjectural: it follows from generator minimality, since a primitive closure transition is a nonzero element of $\mathbb{Z}_7$ (a zero-element transition is no commitment). What remains conjectural is only the *value*: that the winding is the unit generator ±1 rather than some other nonzero residue. That stronger claim rests on the unit-step reading of §6.3 (one tick = $\exp(2\pi i/7)$), not on generator minimality alone. So the conjecture is now a statement about the precise charge, on top of an established nonzero occupancy.

The occupancy condition, in final form

Gate 3 is occupied if and only if at least one isolation-protected cycle carries nonzero admissible $\mathbb{Z}_7$ holonomy — equivalently, if and only if the $\mathbb{Z}_7$ closure charge is not globally inert (Proposition 6.1). By generator minimality (§6.2A), a primitive closure transition is nonzero, so this condition is met wherever primitive Facts exist.

This is the whole of what remains. It is no longer a question about face-completion (settled: local-fact-generated, by isolation and irreversibility), nor about whether protected cycles exist (settled: they do, wherever Facts do), nor about the meaning of any principle. It is a question about the *offset-assignment rule* — specifically, the winding a fold-commitment deposits on its minimal cycle — and it is evaluable on a cycle already known to exist and to persist.

6.4 The Phase-Tick Occupancy Theorem

The conjecture and dichotomy above leave a residual slip that must be closed before any theorem is claimed: the phase-tick hypothesis is a statement about a *single commitment event* ($\Delta\theta = \pm1$), whereas occupancy is a statement about the *sum around a cycle* ($Q_\gamma = \Sigma_\gamma \rho$). These are different objects. The hypothesis "each commitment advances the phase by one generator" does not by itself give "the cycle sums to one generator" — a loop threading several commitments would give $Q_\gamma = \Sigma_i \Delta\theta_i$, which can be any residue in $\mathbb{Z}_7$, including zero by cancellation. Bridging the two requires knowing that the minimal Fact cycle registers exactly one commitment, and that is a question about what a primitive Fact *is*.

Definition 6.3 (Primitive Fact). A primitive Fact is a single fold-commitment isolating a single discarded region D. This is stated as a definition, consistent with the VERSF usage throughout — minimal cyclic structure, first irreversible separation, isolated discarded region D, threshold event — all of which describe a single isolation event rather than a composite of several commitments. It is adopted here explicitly rather than derived from prior text, because the prior language fixes minimality but does not, by itself, forbid a composite; making it a definition closes that gap honestly.

Under Definition 6.3 the minimal cycle γ_D is not an arbitrary transport loop. It is the boundary of the single isolation event that created D, so it registers exactly one commitment's phase advance and no other:

$Q_{\gamma_D} = \Delta\theta_{commitment}$.

Theorem 6.5 — Phase-Tick Occupancy *(occupancy proven via generator minimality; unit value conditional on the unit-step reading)*

Suppose:

1. **(definitional)** a primitive Fact is a single fold-commitment isolating a single discarded region D (Definition 6.3); and
2. **(established, §6.2A and Proposition 6.2)** a primitive fold-commitment is a nonzero closure transition — it is not the identity element, because a commitment changes the

closure state and (faithful phase) the $\mathbb{Z}_7$ phase records that change — so it advances the phase by a nonzero element of $\mathbb{Z}_7$, $\Delta\theta \neq 0$.

Then for the minimal cycle γ_D enclosing D,

$Q\_\{\gamma\_D\} = \Sigma\_\{\gamma\_D\}\ \rho = \Delta\theta \neq 0 \pmod 7$,

so by the Isolation-Protected Holonomy Theorem (5.1) the protected cycle carries nonzero admissible holonomy that survives every refinement, and

Fact existence ⇒ Gate-3 occupancy.

If, in addition, **(3, unit-step reading of §6.3)** the primitive Fold is the unit step of the $\mathbb{Z}_7$ phase circle, one tick = $\exp(2\pi i/7)$, then $\Delta\theta = \pm 1$ and

$Q\_\{\gamma\_D\} = \pm 1 \pmod 7$,

so the protected cycle carries *unit* charge — Conjecture 6.2 holds as a theorem.

Proof

By Definition 6.3, the primitive Fact is one commitment isolating one region D, and γ_D is the boundary of that isolation event. The accumulated offset around γ_D is therefore the phase advanced by that single commitment: $Q\_\{\gamma\_D\} = \Delta\theta$. There is no sum over independent commitments and hence no cancellation — the cancellation case is excluded by the singleness in Definition 6.3, not assumed away. By input 2 (generator minimality, §6.2A, and Proposition 6.2), $\Delta\theta$ is a nonzero element of $\mathbb{Z}_7$ — the commitment is not the identity and the phase faithfully records the state change — so $Q\_\{\gamma\_D\} \neq 0$. By Theorem 5.1, γ_D is permanently non-bounding and its holonomy is never screened. Hence Gate 3 is occupied wherever a primitive Fact exists. Under the further input 3, $\Delta\theta$ is the unit generator, so $Q\_\{\gamma\_D\} = \pm 1$. ■-free statement: a primitive Fact's boundary cycle registers exactly its one commitment's nonzero phase advance, on a cycle that can never be filled; if that advance is the unit step, the charge is ±1.

Status of the three inputs

- Input 1 (Definition 6.3) is definitional and matches consistent VERSF usage; under it the sum-over-cycle/single-event slip is closed, and the cancellation case ($Q\_\{\gamma\_D\} = 0$ by interfering ticks) does not arise. If a future reading of the substrate admitted composite primitive Facts, input 1 would become a genuine hypothesis and cancellation would be live; the theorem flags this explicitly.
- Input 2 (generator minimality §6.2A, Proposition 6.2) is established, not assumed: a primitive closure transition is not the identity element (a commitment changes the closure state, and the $\mathbb{Z}_7$ phase faithfully records it), so $\Delta\theta \neq 0$ and $Q\_\{\gamma\_D\} \neq 0$ — hence occupancy. Its one residual premise is faithfulness of the phase (premise 3 of Proposition 6.2); the inert branch (Proposition 6.1) is the failure of faithfulness in the extreme,

requiring the $\mathbb{Z}_7$ closure charge to vanish everywhere, in tension with the operative role of the 7.

- Input 3 (unit-step reading, §6.3) is the one statement still genuinely open, and it bears only on the *value* of the charge, not on occupancy. It asserts the primitive Fold is the unit step of the phase circle (one tick = $\exp(2\pi i/7)$), giving $\Delta\theta = \pm 1$. If accepted, the charge is the unit generator and Conjecture 6.2 is a theorem. If a primitive Fold is some other nonzero generator, the charge is a different nonzero residue but Gate 3 is still occupied. There is no branch in which occupancy fails except the inert branch ruled implausible by input 2.

The arc therefore rests, for *occupancy*, on inputs 1 and 2 — one definitional and one established — and for the *unit value* on input 3 alone. Occupancy is no longer the open question; the precise charge is.

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7. What Is Proven and What Remains

Proven.

- Lemma 4.1: refinement face-generation is contained in the isolation-governed operation class — derived from irreversibility, not assumed.
- Theorem 5.1: a cycle encircling an isolated discarded region is permanently non-bounding; its holonomy survives every refinement.
- Corollary 5.2: Gate-3 holonomy survives refinement iff face completion is local-fact-generated, which by isolation and irreversibility is the regime VERSF is in wherever Facts exist.
- Therefore: the protection mechanism is operative. Isolation-protected non-bounding cycles exist and persist into the continuum limit. The flat sector is non-bounding wherever Facts exist.
- Proposition 6.1: if every primitive Fact had zero winding, the $\mathbb{Z}_7$ closure charge would be globally inert — exact everywhere, the 7 doing no transport work.
- §6.2A (generator minimality) and Proposition 6.2 (nontriviality of primitive commitment): a primitive fold-commitment is not the identity element of the closure group — it changes the closure state, and the $\mathbb{Z}_7$ phase faithfully records that change — so it advances the phase by a nonzero element of $\mathbb{Z}_7$, $\Delta\theta \neq 0$. With Definition 6.3, the minimal Fact cycle registers exactly that one nonzero advance, so $Q_{\gamma_D} \neq 0$.
- Theorem 6.5 (Phase-Tick Occupancy): combining the above, Fact existence ⇒ Gate-3 occupancy, modulo only the inert branch of Proposition 6.1, which is in tension with the operative role of the 7 elsewhere. Occupancy is therefore established, not conjectural, save for that implausible branch.

Open — value only.

- Whether the primitive Fold is the *unit step* of the $\mathbb{Z}_7$ phase circle (one tick = $\exp(2\pi i/7)$, $\Delta\theta = \pm 1$) or merely some other nonzero generator. This bears only on the *value* of the

charge, not on occupancy: unit-step gives $Q_{\gamma_D} = \pm 1$ and Conjecture 6.2 as a theorem; any other nonzero generator gives a different nonzero charge with Gate 3 still occupied. *(open — a concrete property of the fold-commitment dynamics, affecting the charge value, not whether the sector is occupied.)*
- The inert branch of Proposition 6.1 — that the $\mathbb{Z}_7$ closure charge vanishes everywhere — remains logically available but is in tension with the K=7 closure architecture, the α derivation, and the transport programme, all of which treat the 7 as operative.

Inherited and unaffected.

- The decomposition Fact Momentum = curvature + flat sector, Gate 3 = flat sector (companion paper).
- Compatibility of a surviving sector with One Fold (earlier paper), independent of all of the above.

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8. Conclusion

The occupancy of Gate 3 has been reduced, across the arc, from a question about the meaning of principles to a question about refinement dynamics to — now — a single clause about the offset-assignment rule. The reduction was achieved by identifying the protection mechanism that VERSF already contains.

A completed Fact isolates a discarded region D: no trajectory from D rejoins the active region under any allowed local operation. Refinement, being admissible record dynamics, cannot use any operation outside that class without un-committing a Fact, so refinement is contained in the isolation-governed operations (Lemma 4.1). A cycle encircling an isolated D therefore cannot be filled by any admissible refinement face — such a face would carry a trajectory from D into the active region, violating isolation — so the cycle is permanently non-bounding and its holonomy survives the continuum limit (Theorem 5.1). Filling such cycles is incompatible with the irreversibility of fact formation, which is precisely why face completion is local-fact-generated rather than globally cycle-filling (Corollary 5.2). The phrase that captures the mechanism:

Face completion annihilates contractible curvature, not non-contractible holonomy — and isolation is what makes the encircling cycle non-contractible for good.

That charge question now has a sharp form rather than a vague one, and it splits cleanly into occupancy and value. The natural carrier is the primitive Fact — the minimal protected cycle and the generator of the protected part of the record network's first homology. Generator minimality (§6.2A) settles occupancy: a primitive closure transition is a nonzero element of $\mathbb{Z}_7$, since a zero-element transition is no commitment, so under Definition 6.3 the minimal Fact cycle registers exactly that one nonzero advance and $Q_{\gamma_D} \neq 0$. The only escape is the inert branch (Proposition 6.1) — the $\mathbb{Z}_7$ closure charge vanishing everywhere — which would make the 7 do no transport work, in tension with the K=7 architecture, the α derivation, and the transport programme. So:

Gate 3 is occupied wherever Facts exist, unless the $\mathbb{Z}_7$ closure charge is globally inert — and inertness is implausible given the operative role of the 7 throughout the programme.

Occupancy is thus established, not conjectural, save for that one implausible branch. What remains genuinely open is only the *value* of the charge. If the primitive Fold is the unit step of the phase circle — one tick of the $\mathbb{Z}_7$ closure clock, $\exp(2\pi i/7)$ — then $Q_{\gamma_D} = \pm 1$ and the charge is the unit generator (Conjecture 6.2 as a theorem). If a primitive Fold is some other nonzero generator, the charge is a different nonzero residue, and Gate 3 is occupied just the same. The arc therefore closes on a single physical statement, which now fixes the charge *value* rather than occupancy itself:

A fold-commitment advances the closure phase by one generator of $\mathbb{Z}_7$; if that generator is the unit step, $\Delta\theta = \pm 1$.

The Tick → Fold = One Bit = One Fact chain supports the unit-step reading: a tick is the elementary phase advance, and the primitive Fold is the first completed closure — one tick, one elementary step. Under it the entire programme — compatibility, protection, persistence, decomposition, occupancy, and unit charge — is settled.

Gate 3 is occupied iff at least one isolation-protected cycle carries nonzero admissible $\mathbb{Z}_7$ holonomy — established by generator minimality wherever Facts exist, modulo only the implausible inert branch; the charge is the unit generator ±1 iff the primitive Fold is the unit step of the closure phase circle.

That is the terminus. The occupancy of Gate 3 — the question this arc began with — is no longer a question about topology, ontology, principles, or readings, and no longer even an open conjecture: it follows from a primitive commitment being a nonzero closure transition, on a cycle known to exist and known to persist. What is left is one quantity: whether that nonzero transition is the unit tick of the $\mathbb{Z}_7$ closure clock. Determine that tick, and not only occupancy but the charge value is fixed, and the arc is complete.