

The Bath Criterion

Continuous Mixing from Shared Closure Transport — The One-Element Question in Conservation Language, the Exchange Import Conditionally Discharged, and the Decider Located in the Born Arc

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General Reader Summary

The central claim, first

The central claim of this paper is simple, and it completes the claim of its predecessor. *The Price of Copies* argued that if reality is built from one Fold, then the strong and weak forces may be the price reality pays for containing identical copies of the same closure pattern. What it could not decide was how high the price runs: whether identical copies can only be *swapped*, or can continuously *blend* into one another — and only blending yields the full non-commutative structure the strong and weak forces use. This paper shows that the blend-or-swap question is secretly a question about bookkeeping. If each identical copy keeps its own separately conserved account of transport weight — a *ledger* per copy — then blending is forbidden, exactly and provably: the mathematics of separate accounts is the mathematics of swap-and-phase, nothing more. If instead the copies share one pool — one *bath* of conserved weight for the whole family, with only the total protected — then weight can flow continuously from copy to copy, and that flow, run through the transport rules, *is* non-abelian gauge structure. Bath or ledger: the corpus has asked that question once already, in the Born-rule work, for entirely different reasons. Stated as a reduction — and reductions are what this framework is for — the question "why do $SU(2)$ and $SU(3)$ exist?", a question about a catalogue of abstract symmetry groups, becomes the question "what does closure transport conserve?", a question about bookkeeping. The answer to that single question now decides the strong and weak forces too.

What this paper proves

Three things, in increasing order of consequence. First, a translation: "separate ledgers" and "swap-and-phase only" are not two hypotheses that happen to travel together — they are the same hypothesis, proven equivalent. The transformations that respect every copy's separate account are exactly the transformations the predecessor called the floor, and one single account-violating operation is exactly the predecessor's one missing element. The conservation question and the group question are one question in two languages. Second, a theorem: where the copies share a bath, and transport is continuous, the blend exists — and from one blend, the symmetry of the whole family follows, with the full unitary structure arriving wherever copies can also

carry independent relative phases. Third, two debts cancelled: the predecessor had to *import* the assumption that identical copies can be physically exchanged; here, under the bath, exchange falls out as a consequence — and a technical escape route involving exotic finite symmetry groups, which haunted the predecessor's thinner case, is closed outright by continuity.

The honest condition

Everything above the translation is conditional on one premise: that the copies genuinely share a bath. The paper does not prove the bath; it proves that *nothing cheaper than reading the substrate will decide it* — the wall between bath and ledger is a witnessed, legitimate piece of possible physics, the kind the framework's general principles deliberately refuse to demolish. The bath premise is therefore stated as a named import with its address: the corpus's own Born-rule work, which isolated the identical bath-versus-ledger structure for its own purposes, and the commitment-dynamics papers beneath it. One reading, at one named node, now carries three of the largest questions the programme has — how probability composes, how the particle generations mix, and whether the strong and weak forces exist in the framework — which makes that node, arguably, the most important open question in the entire corpus, and the paper says so. It says the other half too: concentrating three programmes on one node means a confirmation closes them together and a failure spreads exactly as far, and a framework should state where its risk is gathered rather than leave the reader to find it. The paper ends by saying plainly what each answer costs — the bath ignites the non-abelian arc; the ledger activates the largest kill-condition in the corpus.

Abstract

The headline. The existence of non-abelian forces may be determined by whether identical closure sectors share one transport bath or maintain separate ledgers. The non-abelian question is thereby converted from a gauge-theory question — *why these groups?* — into a conservation-law question — *what does transport conserve?* — and three arcs of the corpus (the composition of probability, generation transport, gauge structure) collapse onto that single node. The remainder of this abstract states the results at referee grain.

The Price of Copies reduced the origin of non-abelian structure to one question, **Continuous Mixing (CM)**: whether admissible transport includes one element that genuinely blends two indistinguishable closure sectors beyond swap-and-phase. The present paper translates, decides-conditionally, and relocates that question.

The translation. The **Translation Lemma** (§4): on a class carrier \mathbb{C}^k with unitary admissible transport [U1], the transformations preserving the sector-weight multiset $\{|w_1|^2, \dots, |w_k|^2\}$ of every state are exactly the monomial unitaries — the group $T^k \rtimes S_k$, which is precisely the swap-and-phase floor of the predecessor. Hence *ledger* \Leftrightarrow *confinement to the floor's normalizer*, and *bath* \Leftrightarrow *one admissible weight-redistributing transport* \Leftrightarrow *one off-normalizer element* \Leftrightarrow *CM*. The conservation dichotomy and the one-element criterion are the same fact in two

languages [Proven, conditional on U1]. CM thereby acquires its physical meaning: the question was always *what does closure transport conserve* — the class total only, or each sector's share?

The conditional discharge. The **Mixing Theorem** (§5): let a class of $k \geq 2$ indistinguishable sectors satisfy the bath premise **W1** (only total class weight conserved; some admissible process redistributes weight within the class) and continuity **Q1**. Then the connected admissible transport group contains a one-parameter blend; relabeling covariance — the *unconditional* half of the predecessor's split theorem — spreads the blend to every pair of the class; and under the sector-phase import S2 the pair blends and the intra-class torus generate the full unitary group: $G_{\text{class}} = U(k)$, with $SU(k)$ as the structure beyond the shared derived phase [Proven, conditional on W1, Q1, U1, S2, S3]. Under S2's fallback the result is graded honestly: non-abelian for $k \geq 3$ ($SO(k)$ -type real mixing at minimum), non-abelian for $k = 2$ only given exchange ($O(2)$ -type), and the complex $SU(2)$ the weak force requires waits on S2 either way — the weak-doublet irony of the predecessor, one level deeper.

The cancelled debts. Two corollaries (§6). *Exchange discharged*: the connected $U(k)$ contains the permutations, so under the bath, S4 — the predecessor's exchange-realizability import — becomes a theorem rather than an import, exactly the one-stroke discharge the predecessor forecast from Q1. *Q2 retired on this route*: the bath's blend arrives as a one-parameter subgroup, connected and infinite, so the finite-exceptional-subgroup escape that made Q2 load-bearing on the predecessor's fallback branch is closed by continuity alone. A third structural result: *block baths are not an independent failure mode* — a sub-bath partition of one class is a witnessed distinction among members, contradicting the Indistinguishability Lemma; the mode collapses into a census refinement [Proven, given the predecessor's grounding pair].

The relocation. The bath premise W1 is an import, not a derivation, and the paper proves it must be (§7): the ledger wall is witnessed — distinct conservation structures act distinguishably — so the Conservation Lemma does not remove it, and no catalogue principle decides W1. Its slot has an address the corpus has already built: the Born arc's bath-or-ledger node, isolated there for the composition of probability, and the commitment-dynamics papers beneath it. **Three arcs now converge on one node** — at its native grain **W0**, the substrate bath, of which W1 is the intra-class specialization this paper's theorem consumes and the flavour license the cross-class sibling — making it arguably the most important open node in the entire corpus, and the paper says so explicitly. The arrows are kept honest at §9's grain: W1 and the flavour license are siblings under W0, a fork rather than a chain, with the collapse surviving at full strength in the direction that matters — a ledger verdict strikes the ancestor and both children fall. The flavour arc already supplies evidence: a strict ledger substrate cannot host its generation-transport machinery, so weight-redistributing transport already exists in the corpus — W0 evidenced at mechanism level through its cross-class child, with W1 inheriting at one remove (§7, §9). The concentration is owned in both directions: confirmation closes three programmes simultaneously; failure propagates exactly as widely (§7). Both branches are prepared: bath ignites the non-abelian arc with the Class Product Theorem activating downstream; ledger confines transport to the normalizer ceiling and activates the predecessor's kill-condition at full strength. Throughout, the theorem's output is mechanism, not census: it converts whatever identical-sector multiplicities exist into non-abelian transport, while the census itself — which k the architecture realizes — carries a named slot (§8) at the hexagonal programme's three-channel and orientation

multiplicities, whose "uniquely forced gauge group" translates in this division as a uniquely forced census: geometry proposes, transport disposes. The flavour-mixing connection is stated with its intra-class/cross-class disambiguation and its two-way dependence (§9).

Epistemic markers: [Inherited] imported from prior VERSF papers; [Imported-External] imported from standard mathematics outside the programme, carried at the external source's standing; [Proven] established here; [Conditional] holding under stated inputs; [Conjectural] motivated but unproven; [Open] undecided.

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1. Introduction — The Strong and Weak Forces as a Conservation Problem

The discovery this paper reports, stated before any machinery: **the existence of non-abelian forces may be determined by whether identical closure sectors share one transport bath or maintain separate ledgers.** That sentence is the paper. Everything below it — the lemma, the monomial groups, the connected subgroups, the unitary closures — is the proof that the sentence is exact rather than suggestive.

The discovery is best understood as a reduction, because reductions are what this programme is for. Before this paper, the reserved arc's deepest question read in the conventional register:

Why $SU(2)$? Why $SU(3)$? Why non-commutative at all?

— a question about a catalogue of abstract symmetry groups, with no obvious handle on the substrate. After this paper, the same question reads:

What does closure transport conserve — each identical sector's share, or only the family total?

— a question about bookkeeping, the kind a substrate construction answers directly and a physicist can hold in one hand. The two questions are proven equivalent in §4, and the equivalence is exact in both directions. If each identical sector's transport weight is separately protected — a **ledger** per sector — the admissible transformations are the swap-and-phase floor of *The Price of Copies* and nothing more, with blending excluded not by fiat but by arithmetic: the weight-respecting unitaries are the monomial group, which *is* the predecessor's normalizer, to the element. If instead the class shares one conserved total — a **bath** — then weight can flow between identical sectors; the flow is the blend the predecessor's one-element criterion asked for; and the blend, spread across the class by the covariance the predecessor proved unconditionally, generates the non-abelian structure entire (§5) — with the exchange import discharged and an old technical escape closed along the way (§6).

The second result is the one the paper stops to say at full width, because it is a larger programme finding than any of the group theory. Bath-versus-ledger is not a structure invented here: the corpus's Born-rule work isolated exactly this dichotomy for the composition of probability, and the flavour programme's generation transport already runs on bath-type machinery. **Three apparently independent mysteries — how probability composes, how the particle generations mix, and where the non-abelian forces come from — have collapsed onto one question** — at its native grain a substrate-level question, W0, with this paper's intra-class premise W1 as one child and the flavour license as the other (§7). The paper does not decide that question; it proves no catalogue principle can (§7, the witnessed wall), states it as the named import W1 with its address in the Born arc, records the mechanism-level evidence the flavour arc already supplies, and owns the concentration of risk that the collapse creates — in both directions, ignition and kill-condition alike.

The position relative to the predecessor is then one sentence each way. *The Price of Copies* ended one element short of the forces, with the element stated in the only language then available — a group element, somewhere off the normalizer, existence unknown; this paper translates the element into a conservation fact, derives everything its presence forces, simplifies the predecessor's import stack in the process (one import in, two out — §2), and wires the entire question, ignition switch and kill-condition alike, to a single reading at a single named node.

2. Inherited Results and Imports

The split theorem and its floors [Inherited — *The Price of Copies* §§5–6]. Covariance unconditional: G is normalized by the class permutations, $\sigma G \sigma^{-1} = G$ — consumed in §5 as the spreading mechanism, and the consumption is deliberate: covariance is the predecessor's *unconditional* half, so the spreading needs no exchange import. Membership conditional on S4; the floors by phase reading — $N(U(1))$ under S2, swap-plus-scalars under the fallback; the one-element criterion, by floor. The grounding pair — the Class Multiplicity Proposition and the Indistinguishability Lemma — consumed in §6's collapse of the block-bath mode.

The import stack at its predecessor statuses [Inherited — *The Price of Copies* §3]: **U1** (unitarity of sector transport; Born-measure conservation as its source); **S2** (sector-resolved relative phase, with its fallback); **S3** (class = Representation Class, member-indistinguishability derived from the identity programme, non-degeneracy inherited from the species ontology); **S4** (exchange realizability — imported there, *discharged here* under the bath, §6); **Q1** (continuity of admissible transport in the pre-commitment regime — load-bearing here, consumed by the Mixing Theorem directly); **Q2** (per-pair finite-impossibility — *retired on this route*, §6). The paper's net effect on the stack is the session's best trade: one new import in, two old ones out.

The Born-arc dichotomy [Inherited — (cite: Bath-or-Ledger paper, the bath/ledger criterion; the continuity flag)]. The corpus's existing isolation of the shared-pool-versus-separate-accounts structure for transport weight, with continuity already flagged as load-bearing at source. Consumed in §7 as W1's address.

The witnessed-wall discipline [Inherited — the second-integer paper's §7 honesty, as deployed in the predecessor]: distinct structures that act distinguishably are witnessed, and the Conservation Lemma does not remove witnessed boundaries. Consumed in §7 to prove W1 must be an import.

The new import, named once and carried throughout: W1 (Shared Class Weight — the bath premise), stated exactly in §7 with its slot, its fallback, and its decision procedure. Nothing else new is assumed; every other premise in this paper resolves to a named predecessor import or an inherited theorem, and the paper's claims are conditional on exactly that list.

3. The Two Readings — Ledger and Bath, Defined Exactly

The carrier and the weights. A class $\mathcal{C} = \{S_1, \dots, S_k\}$ of $k \geq 2$ indistinguishable closure sectors [S3], carrier \mathbb{C}^k , state $w = (w_1, \dots, w_k)$. The **sector weights** are $|w_1|^2, \dots, |w_k|^2$, and the **class weight** is their total:

$$W_{\mathcal{C}} = \sum_i |w_i|^2.$$

Under U1, admissible transport preserves $W_{\mathcal{C}}$ — that much is unitarity itself, common to both readings, and is the Born-measure conservation the reconstruction arc supplies. The readings divide on what *more* is conserved.

Ledger reading. Each sector's weight is separately protected. Stated at the grain the ontology demands: since sector labels within a class are not facts [S3], "separately conserved" cannot mean conserved *per label*; it means the **weight multiset** $\{|w_1|^2, \dots, |w_k|^2\}$ is conserved — each admissible process returns the same collection of sector weights, possibly redistributed over the unwitnessed labels. The label-covariant form is forced by the predecessor's covariance theorem and is the form the lemma below consumes; the stricter per-label form is noted only to be discarded as depending on facts that do not exist.

Bath reading. Only the total $W_{\mathcal{C}}$ is protected: there exists at least one admissible process, acting on at least one admissible state, after which the weight multiset differs — weight has genuinely flowed between sectors. The class is one pool, not k accounts.

The two readings are exhaustive and exclusive by construction: either every admissible process preserves every state's multiset, or some process on some state does not. The substance of the paper is that this bookkeeping dichotomy *is* the predecessor's group dichotomy — which is §4 — and that the bath side, once granted, forces everything (§5).

4. The Translation Lemma — One Question in Two Languages

Translation Lemma. *Let admissible transport on the class carrier \mathbb{C}^k act by unitaries [U1]. Then:*

(i) *every monomial unitary — exactly one nonzero entry, of unit modulus, in each row and column — preserves the weight multiset of every state; the monomial unitaries form the group $T^k \rtimes S_k$, the normalizer of the diagonal torus in $U(k)$;*

(ii) *conversely, any unitary preserving the weight multiset of every state is monomial;*

(iii) *hence the ledger reading is equivalent to confinement of admissible transport to $T^k \rtimes S_k$ — the swap-and-phase structure that is precisely the predecessor's floor-ceiling — and the bath reading is equivalent to the existence of one admissible element outside it: the predecessor's one-element criterion. Ledger \Leftrightarrow normalizer; bath \Leftrightarrow CM.*

Proof. (i) A monomial unitary sends w_i to a unit phase times $w_{\pi(i)}$: moduli are permuted, the multiset is fixed; the monomial group is T^k (the phases) extended by S_k (the permutations), the standard normalizer $N(T)$ [Imported-External]. (ii) Let U preserve every state's multiset, and apply U to the basis state e_j , whose multiset is $\{1, 0, \dots, 0\}$. The image Ue_j must have the same multiset: a single entry of modulus one, the rest zero — so $Ue_j = e^{i\theta_j} e_{\pi(j)}$ for some index $\pi(j)$ and phase θ_j . Unitarity makes $j \mapsto \pi(j)$ injective, hence a permutation, and U is monomial. (iii) Immediate from (i) and (ii), with the identification of the monomial group as the floor-ceiling read off from the predecessor's §6. [Proven, conditional on U1; the normalizer identification Imported-External.]

Three remarks fix the lemma's weight. First, *its economy*: the proof of the hard direction consumes only the basis states, so even a ledger stated weakly — multiset conservation on realized configurations rather than all states — suffices for the equivalence; the dichotomy is robust to reasonable variations in how the conservation claim is grained. Second, *its meaning*: CM, which entered the corpus as an opaque group-theoretic existence claim, is now a conservation question — *does closure transport protect each identical sector's share, or only the family total?* — which is the kind of question a substrate construction answers directly, and the kind a physicist can hold in one hand. The one element was never exotic; it is any process that lets weight flow between copies. The reduction deserves its plain statement: "why $SU(2)$, why

SU(3)?" has become "what does transport conserve?" — a catalogue question converted into a bookkeeping question, and the programme is at its strongest exactly when it performs such conversions. Third, *its discipline*: the lemma is an equivalence, not an argument for either side. It converts the decision without biasing it, and the marker structure below keeps the two sides priced separately.

5. The Mixing Theorem — Bath Forces Blending, by Floor

Mixing Theorem. *Let a class \mathcal{C} of $k \geq 2$ indistinguishable closure sectors [S3] satisfy W1 (bath) and Q1 (continuity), with transport unitary [U1]. Then:*

(a) *the connected admissible transport group G^0 _class contains a one-parameter subgroup with a non-diagonal (blending) generator;*

(b) *relabeling covariance [Inherited — the predecessor's unconditional half] spreads the blend: the Lie algebra \mathfrak{g} of G^0 _class is invariant under conjugation by every class permutation, so a blend on one pair yields a blend on every pair;*

(c) *under S2, the pair blends together with the intra-class torus generate everything: G^0 _class = $U(k)$, and the structure beyond the shared derived phase is $SU(k)$;*

(d) *under S2's fallback, the graded result: for $k \geq 3$ the spread blends generate at least $SO(k)$ -type continuous mixing — non-abelian; for $k = 2$ a single real blend generates only a circle, abelian, with non-abelianness arriving given exchange — S4, which on this branch remains the import it was, Corollary 1's discharge being conditional on S2, absent here — the swap inverting the rotation and closing the group to $O(2)$ -type structure, non-abelian but real — and the complex $SU(2)$ waits on S2 in every reading.*

Proof sketch. (a) By W1 some admissible process changes some state's multiset. Q1 must be consumed at its source grain — continuous *dynamics*, not merely a path-connected group — and the proof states the quantifier so that no discrete-coset wedge fits: the redistribution occurs along a continuous admissible flow, so multiset change occurs already in every neighborhood of the identity, and elements arbitrarily close to $\mathbb{1}$ are non-monomial (Translation Lemma). A Lie algebra contained in the diagonal would exponentiate every such neighborhood into T^k , preserving all multisets; hence $\mathfrak{g} \not\subset$ diagonal, and G^0 _class contains a one-parameter subgroup $e^{\{tX\}}$ with non-diagonal X — the blend. (Had Q1 supplied only path-connectedness of the group, a referee could lodge the non-monomial element in a non-identity component over a diagonal identity component; the flow form closes that fork, and it is the form the Born arc flags as load-bearing at source — the slot reading should confirm the grain.) (b) Covariance is $\sigma G \sigma^{-1} = G$ for class permutations σ ; on the connected component this is $\text{Ad}(\sigma)\mathfrak{g} = \mathfrak{g}$, so $\sigma X \sigma^{-1} \in \mathfrak{g}$ for every σ : the blend's conjugates under all relabelings lie in the algebra, and a generator mixing one pair yields generators mixing every pair — *without S4*, since covariance is the unconditional half. (c) Under S2 the algebra contains the full diagonal torus $\mathfrak{t} = \{i \cdot \text{diag}(\theta_1, \dots, \theta_k)\}$; bracketing the torus against a pair blend extracts the root generators of that pair, and the standard generation result [Imported-External — the rank-one subgroups along all roots, with the maximal torus, generate the unitary algebra] yields $\mathfrak{g} = \mathfrak{u}(k)$: G^0 _class = $U(k)$. Factoring the derived global phase

leaves $SU(k)$. (d) Under the fallback the torus is absent; the spread blends generate, at minimum, the real rotations they span — $\mathfrak{so}(k)$, non-abelian for $k \geq 3$ — while for $k = 2$ a single real generator spans $\mathfrak{so}(2)$, a circle: abelian, exactly as the predecessor's fallback floor was, and the addition of the swap (where available — S4, un-discharged on this branch) conjugates the rotation to its inverse, closing the group to $O(2)$ -type — non-abelian but real, with the complex doubling that makes $SU(2)$ waiting on S2. [Proven, conditional on W1, Q1, U1, S2/fallback as graded, S3 and the covariance half at their statuses; generation results Imported-External.]

Two readings of the theorem's shape. The forcing pattern recurs once more — floor, classification, collapse, now with the bath as the collapse's trigger — and the recurrence is by now the corpus's signature at every scale. And the weak-doublet irony deepens by exactly one level: under the fallback, $k = 2$ is the *only* class size whose bath fails to force non-abelian mixing on its own, and even with exchange the structure is real $O(2)$, not the complex $SU(2)$ the weak force is. The Standard Model's smallest multiplet remains the case most sensitive to the phase import — S2's slot is now flagged by two papers as the place the weak force's existence is decided at the margins. One scope sentence completes the section: the theorem's output is **mechanism, not census** — it converts whatever identical-sector multiplicities the architecture supplies into non-abelian transport, $U(k)$ for the k it is handed, and the question of *which* k the unique Fold realizes belongs to the geometry programme and the species inventory (§8), not to anything proven here.

6. Corollaries — Exchange Discharged, Q2 Retired, Block Baths Collapsed

Corollary 1 (S4 discharged under the bath). Under W1, Q1, U1, and S2, the connected group $U(k)$ contains every permutation matrix, and connectedness supplies a continuous admissible path from the identity to each swap: the physical exchange of identical sectors is an admissible transport — S4 holds as a theorem. The predecessor forecast exactly this ("a continuous admissible path implementing the exchange would supply S4 and bear on CM at one stroke"); the stroke is the bath. The import does not vanish from the corpus — on the ledger branch, and under the fallback's thinner reach, S4 retains independent standing — but on the main branch the predecessor's membership half loses its conditionality, and the split theorem's two halves fuse. [Proven, conditional as stated.]

Corollary 2 (Q2 retired on this route). The predecessor's fallback branch trifurcated — a single *discrete* blend at a rational angle could generate, with the swap, an exceptional finite subgroup of $SU(2)$, and Q2 was promoted to load-bearing to exclude it. The bath route closes the escape without Q2: the blend of theorem (a) arrives as a one-parameter subgroup, connected, hence infinite, and no finite group contains a nontrivial connected subgroup. Continuity does what finite-impossibility was conscripted for, and Q2's load-bearing status is confined to the one-discrete-element route, which the bath route supersedes. [Proven, conditional on W1 and Q1.]

Corollary 3 (block baths are not an independent failure mode). Suppose a class's bath fractured into sub-baths — weight flowing within blocks, never between them. The block boundary is then physical structure: an admissible conservation constraint distinguishing

members across blocks, witnessed by the very flows it forbids. But the Indistinguishability Lemma [Inherited — the predecessor's grounding pair] permits no witnessed distinction among members of one class: equal invariant tuples entail genuine indistinguishability, on pain of the catalogue or No-Surplus-Identity. A blocked "class" is therefore not a class; the blocks are the classes, the census is refined, and the apparent failure mode dissolves into a relabeling — the predecessor's refinement clause doing exactly the work it was retained for. [Proven, given the grounding pair at its source conditionality.]

The three corollaries share a moral worth one sentence: the bath premise does not merely add a result to the predecessor — it *simplifies* it, cancelling one import, retiring another, and deleting a failure mode, which is the behavior of a premise that is carving the structure at its joints rather than patching it.

7. The Decision Node — W1, Its Witnessed Wall, and the Born-Arc Address

W1 (Shared Class Weight — the bath premise) — *the decision node of this paper* [Open]

Closure transport on a class of indistinguishable sectors conserves the total class weight $W_{\mathcal{C}}$ [U1] but not the sector-weight multiset: there exists an admissible process redistributing weight among the sectors. The class is one bath, not k ledgers. (Negation — the ledger: every admissible process preserves every state's weight multiset, and by the Translation Lemma admissible transport is confined to $T^k \times S_k$.)

Why W1 must be an import. The corpus's principles do not decide it, and the paper proves this rather than regretting it. A ledger is a wall in the space of transports — everything off the monomial group excluded — and the wall is *witnessed*: a multiset-changing process, were it admissible, would act distinguishably from every monomial process, so the boundary is accompanied by admissible invariant differences, and the Conservation Lemma, which removes only unwitnessed boundaries, has no purchase. This is the identical honesty the predecessor's §7 enforced against deriving the blend from Lemma pressure, now enforced against deriving the bath: the wall is legitimate possible physics, and which side of it the substrate occupies is a fact about the construction, to be read, not derived. [Proven — the witnessed-wall application; the discipline is the result.]

The slot, with its address. The decisive feature of W1 is that its slot is not generic. The corpus's Born arc has already isolated the bath-versus-ledger structure — whether the weight that becomes probability composes as one pool or separate accounts — and flagged continuity as load-bearing there; the commitment-dynamics papers beneath it supply the pre-factual transport regime in which Q1 and W1 both live. [⟨cite: Bath-or-Ledger paper, the criterion and its verdict if one is reached; TPB/pre-commitment transport dynamics⟩.] The node's native grain at that address is one level above this paper's premise, and the re-lettering keeps the entailments honest: call the ancestor **W0 (Substrate Bath)** — transport weight is bath-like at substrate level, only pooled weight fundamental, no per-sector protection anywhere — of which **W1 is the intra-class specialization** this paper's theorem consumes, and **the flavour license is the cross-class**

sibling. The paper stops here to say the consequence at full width, because it is a larger programme finding than any of the group theory above: **three apparently independent mysteries — how probability composes, how the particle generations mix, and where the non-abelian forces come from — have collapsed onto one question**, and the question is W0:

W0: substrate-level bath (the Born arc's native node)
├─ intra-class child - W1 \Rightarrow continuous mixing \Rightarrow non-abelian gauge structure
├─ cross-class child - generation transport licensed \Rightarrow CKM / PMNS
└─ pooled composition of weight \Rightarrow the Born arc's probability reading

The arrows are drawn at §9's grain, and the honesty costs nothing: W1 does not entail the flavour license, nor conversely — they are siblings under W0, a fork from a common ancestor rather than a chain — and the collapse survives at full strength in the direction that matters: a ledger verdict at the Born node strikes W0, and both children, and the Born reading, fall together. The node is therefore not one open question among the corpus's many. W0 — with W1 as the specialization this paper consumes — is the single remaining structural question simultaneously supporting the composition of probability, the generation-transport machinery, and the existence of non-abelian gauge structure: arguably the most important open node in the entire corpus, and the paper says so plainly rather than leaving the reader to compute it from the diagram. The reading order of the programme should reflect it: this node before everything.

Partial evidence within the corpus. W1's standing is better than a bare import, and the paper records the upgrade at its exact grain — stated as directly as the record permits. A strict ledger substrate cannot naturally host the generation-transport machinery already employed in the flavour programme: that machinery redistributes weight between sectors, and a substrate of separately protected accounts has no admissible process for it to be. While generation transport occurs between *distinguishable* sectors rather than within a class, it demonstrates that weight-redistributing transport already exists in the corpus — the bath kind is realized at mechanism level, in the programme's own working constructions, and what remains of W1 is only whether that transport extends to indistinguishable sectors. In the re-lettered structure: the flavour arc evidences the ancestor W0 through its cross-class child, and W1, the intra-class child, inherits the evidence at one remove. On the session's graded scale — decided-at-source, mechanism-level bias, open — W0 sits at the middle grade, corpus-internally evidenced. And the residue itself carries pressure: the residual ledger — bath across classes, ledger within them — would be a strange object, a substrate that lets weight flow between distinguishable sectors while protecting the separate shares of sectors that have no separate identities, a conservation law drawn finest exactly where identity is coarsest. The strangeness is recorded as pressure, not proof; the bias is mechanism-level; the marker is the marker.

The decision procedure. (i) Read the Born-arc node — which reads W0 natively: if the substrate bath is established at source, W1 discharges as its intra-class specialization and the Mixing Theorem fires at the grain of §5, with the flavour license discharging in the same stroke as the cross-class child. (ii) If the ledger is established, W0 falls and both children with it: admissible transport is confined to the normalizer ceiling, CM resolves negative, the predecessor's branch T — the largest kill-condition in the corpus — activates at full strength with this paper's Translation Lemma as the executing instrument, and the flavour machinery loses its

transport license in the same verdict — exactly the propagation the concentration passage owns, with no separate explanation owed, because the strike lands on the ancestor. (iii) If the source node is itself open, W0 inherits that status, and the corpus's honest position is that its three converging arcs share one undecided question — which is itself a major consolidation, since one campaign now closes all three. The verification asymmetry runs as always: the bath is decided by exhibiting one weight-redistributing admissible process within a class; the ledger by a universality proof over all processes and states — existence against universality, with the universality side owed a mechanism for the wall it asserts.

Motivation, carried as motivation. The remaining pressures toward the bath are marked at their strength: a per-sector ledger protects quantities indexed by labels that are not facts — the conservation law would have *more structure than its conserved quantities have identity* — and the economy posture is suspicious of bookkeeping that out-grains the ontology. But the multiset formulation of §3 already deflates the sharpest version of that objection (the ledger can be stated label-covariantly), so this pressure is economic, not derivational, and stands below the flavour arc's mechanism-level evidence in the graded scale. The marker does not move on either: [Open], with the bias recorded at its grades.

The concentration, owned. The architecture this paper completes places a great deal on one node, and the risk is stated rather than left for a referee to discover. The concentration of three arcs onto W1 is either a strength or a vulnerability, and which one is not the paper's to choose: if W1 is confirmed at source, three major programmes close simultaneously — probability composition grounded, generation transport licensed, the non-abelian forces derived. If W1 fails, the negative verdict propagates exactly as widely — branch T executes through the Translation Lemma, the flavour machinery loses its transport license and must be rebuilt on interactional foundations, and the Born arc's bath reading falls with the rest. A corpus that concentrates its exposure should say where; this corpus's is here. One mitigating observation closes the ledger of risk: the evidence in hand is asymmetric — the programme's working machinery already runs bath-type transport, and no construction has yet been exhibited that requires the ledger — so the concentration sits on the side the current record favors, which is where a programme should want its risk if it must be concentrated at all. [The concentration: stated; the asymmetry: calibration on the current record, no marker.]

Why the bath points toward the substrate — an interpretive coda. The Bath Criterion is stated throughout this paper as a conservation question: do identical closure sectors carry separately conserved transport weight, or draw from a common bath? The theorems are neutral — either reading is mathematically admissible, and the paper's purpose is to show what follows from each. But the two readings suggest different pictures of the underlying ontology, and the suggestion deserves its paragraph at exactly its own strength: interpretation, not result.

Under the ledger, transport weight is attached to individual sectors: each carries its own protected account, the conservation structure lives at the level of the sectors themselves, and the sectors stand as fundamental *possessors*. Under the bath, sectors possess nothing independently: only the total class weight is fundamental, individual sector weights are contingent allocations of a shared quantity, and weight flows among sectors because the conserved object belongs to the class — and beneath the class, to whatever supports it — rather than to any member.

The bath reading is the one consonant with the ontology the corpus has already adopted. The sectors are Persistent Fold *Defects* — patterns in the one Fold, by the corpus's own classification — and a defect is paradigmatically not an owner of resources; it is an organization of something else's. The ledger reading would quietly promote patterns into proprietors, attributing fundamental possession to entities the ontology has already declared derivative; the bath reading lets patterns be patterns. The same theme recurs across the programme: phase transport, probability transport, gauge transport, and generation transport are repeatedly treated as manifestations of one transport structure that exists prior to the particular configurations expressing it.

An analogy fixes the picture. Waves on an ocean can be distinguished and tracked, but the water does not belong to any particular wave: the waves are patterns, and the water is the underlying reality from which the patterns are formed. Under the bath reading, closure sectors are waves — patterns through which transport weight is organized — and the weight is the water.

The interpretation establishes nothing: W1 remains [Open], and consonance with an ontology is not a derivation from it. But the direction of the pointer should be recorded, because it runs through the whole worldview:

void / substrate \Rightarrow transport resource \Rightarrow closure sectors \Rightarrow mixing \Rightarrow SU(2), SU(3)

If W1 resolves positively, the result points naturally toward a substrate-level origin of transport weight — the conservation structure residing beneath the sectors rather than within them — and the implication is worth stating once at full width: the strong and weak forces would arise not because nature contains additional fundamental structures beyond the Fold, but because multiple closure sectors draw on one common underlying transport resource. The Bath Criterion would then point beyond the sectors and back toward the substrate from which they emerge — which is consistent with where the programme has pointed throughout. [Interpretive coda — Conjectural throughout; no marker moves; W1 remains Open at its slot.]

8. The Hexagonal Programme and the Census Slot

The hexagonal closure-geometry papers claim a route to the Standard Model's gauge structure, with SU(3) associated to three-channel occupancy and SU(2) to triangular orientation structure, and the gauge group claimed as uniquely forced by the geometry. The present paper neither verifies nor consumes those claims; it records what they would supply if verified, and it fixes the division of labor exactly, because the division is what keeps both programmes honest. **This paper's theorem is mechanism, not census.** It does not say "the bath gives SU(3) and SU(2)"; it says the bath converts whatever identical-sector multiplicities exist into dynamically non-abelian transport — U(k) for whatever k the census supplies — and it is silent on which k the architecture realizes. The geometry papers, conversely, are census claims in this division: three-channel occupancy is a candidate k = 3 class, triangular orientation a candidate k = 2 structure, and the geometry's "uniquely forced gauge group" translates here as "uniquely forced census" — the multiplicities forced, with their dynamical non-abelianness supplied by this paper's mechanism, conditional on W1. If the geometry papers' multiplicities are confirmed as Representation Classes in the S3 sense, the census — [Open] since the predecessor stated it —

discharges at source. **The slot is named:** (cite: hexagonal Standard-Model papers — the occupancy and orientation multiplicities, read as class structure; H1, the census-discharge candidate) [Open]. The summary of the division in five words: geometry proposes; transport disposes. Neither claim stands without the other, and neither should borrow the other's credit.

9. Relation to Flavour Mixing

The corpus's flavour paper derives CKM and PMNS structure from generation-space transport — transitions among fermion generations treated as transport between closure sectors. That construction is downstream of this paper in the precise sense that CM is its license: continuous transport between sectors is exactly what the bath supplies and the ledger forbids, so a bath verdict at W1 converts the flavour paper's generation-space transport from a modelling move into an instance of a derived general principle. And the dependence runs both ways, at different strengths: downstream, the bath licenses the flavour construction; upstream, the flavour construction's existence — the fact that the corpus's working machinery already requires weight-redistributing transport between sectors — supplies the mechanism-level, corpus-internal evidence for the bath kind recorded in §7. One node thereby supports two derivational arcs (mixing to gauge structure; generation transport to CKM/PMNS) and receives evidence back along one of them, which is a stronger position than either arc held alone.

One disambiguation is owed, and the predecessor's §8 flag is the reason. The Class Product Theorem distinguishes *intra-class* mixing (forced, under the bath) from *cross-class* mixing (unforced, excluded under the economy posture unless exhibited with work at source) — and fermion generations, which differ in mass and are therefore distinguishable, are on their face *distinct classes*, making generation transport a cross-class phenomenon. The flavour paper is thus not a free rider on this paper's theorem: it is the exhibited cross-class mixing the product theorem's refuter clause anticipates, and its existence at source amends the product exactly as the posture provides — gauge structure from intra-class baths, flavour structure from the cross-class transport the flavour paper constructs. The two mixings are cousins under one transport ontology, not instances of one theorem, and stating the difference now prevents the conflation later; it is also exactly why the flavour evidence for W1 is mechanism-level rather than decisive — the kind is exhibited, the intra-class locality is the residue. [The disambiguation: Proven at the level of the definitions; the flavour paper's standing: its own, at its own slots.]

10. Failure Modes, Revised

FM1 — the ledger (W1's negation). Every admissible process preserves every weight multiset; by the Translation Lemma transport is confined to $T^k \times S_k$ at most, with the *forced* content still only the predecessor's floors (the torus needs S2, the swaps S4, on this branch un-discharged); CM resolves negative; the predecessor's branch T activates. This is the real failure mode, it is decided at the W1 slot, and its cost is stated in §7. [Open — the node itself.]

FM2 — hidden labels. Sectors that appear identical carry distinguishing facts. By the Indistinguishability Lemma this requires striking invariant-exhaustion or No-Surplus-Identity at source — a strike on the identity programme, not on this paper — and short of that, an exhibited

distinguishing invariant relocates sectors to refined classes: a census change, with every theorem surviving on the refinement. [Graceful degradation, per the predecessor's S3.]

FM3 — discrete-only transport (Q1's failure). If admissible transport in the pre-commitment regime is not continuous, the Mixing Theorem's (a) fails and the bath premise loses its bridge to the blend: weight redistribution could exist only as discrete jumps, a strange intermediate (non-monomial but disconnected) that would itself demand a mechanism. Q1's slot is shared with the Born arc, which has already flagged continuity as load-bearing; its failure would strike both arcs at once. [Open at Q1's slot.]

FM4 — block baths: deleted as independent. Collapsed into FM2 by Corollary 3: a blocked class is a mislabeled pair of classes, and the mode is a census refinement, not a failure. Recorded here so the deletion is auditable. [Proven.]

The revised list has the shape failure-mode lists should have: one genuine decision (FM1), one upstream exposure (FM2), one shared continuity debt (FM3), and one mode removed by theorem (FM4).

11. Position in the Programme

Inherited exposure. The predecessor's full stack at its statuses — U1, S2, S3 (with the grounding pair's premises), S4 (now branch-dependent), Q1, Q2 (now route-dependent) — plus the Born arc at the W1 slot and the identity programme beneath S3. The widest effective base in the corpus, but with this paper's net effect being consolidation: one import in (W1), two out (S4 on the main branch; Q2 on this route).

Assignment-neutrality. The inputs are conservation-structural and group-theoretic throughout — the lemma quantifies over states and processes, the theorem over generators, the corollaries over the same — and none consumes which path carries which holonomy. The independence chain from Transport-Completeness extends unbroken through this paper. [Proven, given the stated grain.]

The diagram, extended at the node:

One Fold \Rightarrow exactly $U(1) \Rightarrow$ (abelian arc: $q \in \mathbb{Z}$; GI; $m \in \mathbb{Z}$; SA open)

└─ closure sectors; classes = Representation Classes [PoC]

[PoC] ── covariance (unconditional) + $S4 \Rightarrow S_k \subseteq G$; floors by S2

this paper ── TRANSLATION LEMMA: ledger $\Leftrightarrow M \subseteq T^k \times S_k$; bath $\Leftrightarrow CM \leftarrow$

Ledger node; ── W0 (substrate bath) [Open - slot: Born arc, Bath-or-

transport] ── mechanism-level corpus evidence: the flavour arc's

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spreads;          |          |— intra-class child W1 + Q1 ⇒ blend; covariance
                  |          |
blocks collapsed) |          |          S2 ⇒ G_class = U(k) ⇒ SU(k) per class
                  |          |          (S4 discharged on this branch; Q2 retired;
licensed ⇒ CKM/PMNS |      |— cross-class child ⇒ generation transport
ceiling;         |          |— ledger at W0 ⇒ both children fall ⇒ normalizer
                  |          |
                  |          |          branch T kill-condition
                  |          |— economy posture ⇒ G = ∏ U(ni) [Conditional - PoC]
candidatess]    |          |— census [Open - H1: hexagonal multiplicities as

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12. What Would Refute or Decide This

Against the Translation Lemma (§4). Exhibit a non-monomial unitary preserving every state's weight multiset — excluded by the basis-state argument, so the refutation must strike U1 (transport not unitary), invoking the predecessor's stated fallback and reopening the group theory upstream. The lemma is exactly as secure as unitarity.

Against the Mixing Theorem (§5). Its premises are all named — W1, Q1, U1, S2/fallback, S3, the covariance half — and each failure routes to its recorded fallback; there is no unnamed joint. The place a referee should press is (a)'s continuity step, and the press lands on Q1's slot, shared with the Born arc.

Deciders of W1 (§7). The Born-arc reading, at W0's native grain, per the procedure: a substrate-bath verdict discharges W1 as its intra-class specialization, and the flavour license as its sibling, in one stroke; a universality proof of multiset conservation decides the ledger, strikes W0 with both children falling together, and activates branch T through the Translation Lemma; an open verdict at source consolidates the corpus's three converging arcs into one named campaign.

Against the corollaries (§6). Corollary 1 falls only with its premises (it is bare group theory above them); Corollary 2 likewise (no finite group contains a nontrivial connected subgroup is not negotiable); Corollary 3 falls with the grounding pair, at the identity programme's source.

Empirically, at the horizon. Unchanged from the predecessor — the product-versus-unification discriminator — with one addition: a ledger verdict at W1 would, through branch T, commit the framework against the existence of the strong and weak forces as gauge structure, which the world refutes; so the framework's viability *requires* the bath, and the corpus should say so before the reading rather than after. The requirement is not evidence — wanting the bath does not make it so — but it sharpens what the W1 reading is: the closest thing to an experiment the corpus can perform on itself.

13. What the Paper Establishes

Established (conditionally, with conditions named):

- The Translation Lemma: ledger \Leftrightarrow confinement to the monomial group $T^k \rtimes S_k$ — the predecessor's floor-ceiling — and bath \Leftrightarrow one off-normalizer admissible element \Leftrightarrow CM; the one-element question and the conservation question proven to be one question in two languages, with the equivalence robust to weak grainings of the ledger (§4). [Proven, conditional on U1.]
- The Mixing Theorem, by floor: bath + continuity yield the blend — with Q1 consumed at flow grain, closing the discrete-coset fork; covariance — the predecessor's unconditional half — spreads it to every pair without S4; under S2 the class group is $U(k)$, $SU(k)$ beyond the shared phase; under the fallback, non-abelian for $k \geq 3$, $O(2)$ -type for $k = 2$ given exchange (S4 — un-discharged on this branch, Corollary 1 being conditional on S2), with complex $SU(2)$ waiting on S2 in every reading — the weak-doublet sensitivity flagged a second time (§5). [Proven, conditional on W1, Q1, U1, S2/fallback, S3.]
- Corollary 1: S4 discharged under the bath — exchange realizability a theorem on the main branch, the predecessor's split theorem fusing (§6). [Proven, conditional as stated.]
- Corollary 2: Q2 retired on this route — connectedness closes the finite-exceptional escape without finite-impossibility (§6). [Proven, conditional on W1, Q1.]
- Corollary 3: block baths collapsed into census refinement by the grounding pair — FM4 deleted as an independent failure mode (§6, §10). [Proven, at the grounding pair's conditionality.]
- The witnessed-wall result: W1 cannot be decided by catalogue principles — the ledger wall is witnessed, the Conservation Lemma has no purchase, and the bath premise is an import by proof rather than by oversight (§7). [Proven.]
- The relocation and the three-arc convergence, at the re-lettered grain: the Born-arc node reads W0 (substrate bath) natively, with W1 the intra-class specialization this paper consumes and the flavour license its cross-class sibling — a fork from a common ancestor, the entailment arrows drawn at §9's grain; W0 elevated explicitly as arguably the most important open node in the corpus; the flavour arc evidencing W0 at mechanism level through its cross-class child (a strict ledger substrate cannot host its construction), W1 inheriting at one remove, the residual ledger flagged as a conservation law drawn finest exactly where identity is coarsest; and the concentration of risk owned in both directions — a bath verdict closes three programmes simultaneously, a ledger verdict strikes the ancestor and both children fall, with the asymmetry of the current record (bath-type transport in use, no ledger-requiring construction exhibited) noted as the mitigation (§7, §9). [The address: slotted; the convergence, evidence grade, and concentration: Proven or stated at the level of the papers' statements.]
- The mechanism/census division: the theorem converts whatever multiplicities exist into non-abelian transport — $U(k)$ for the k supplied — and is silent on which k the architecture realizes; the hexagonal programme's multiplicities named as the census-discharge candidate H1, its "uniquely forced gauge group" translated as a uniquely forced census: geometry proposes, transport disposes (§5, §8). [The division: Proven at the level of the claims; H1: Open.]
- The interpretive coda: the ledger would promote sectors — Persistent Fold Defects, patterns by the corpus's own classification — into fundamental possessors of transport weight, while the bath lets patterns be patterns, with the conserved object residing

beneath the sectors; the waves-and-water reading recorded at interpretive strength, the chain void/substrate \Rightarrow transport resource \Rightarrow sectors \Rightarrow mixing \Rightarrow $SU(2)$, $SU(3)$ stated, and no marker moved (§7). [Interpretive; Conjectural.]

- The flavour disambiguation: generation transport as cross-class — the Class Product Theorem's anticipated exhibition, not an instance of the intra-class theorem — preventing the conflation in advance (§9). [Proven at the level of the definitions.]
- Assignment-neutrality extended (§11). [Proven, given the stated grain.]

Not established (open, out of scope, or pending):

- W1 itself — the bath premise, the paper's named node, at its Born-arc slot (§7);
- Q1 — continuity, load-bearing here and in the Born arc, at its shared slot;
- the census — the sizes n_i , with H1 as the named candidate route (§8);
- S2 — the sector-phase import, now doubly flagged as where the weak force's complex structure is decided;
- everything fenced by the predecessor: chirality, hypercharge, breaking, the force-factor identification, the non-abelian record machinery;
- the hexagonal papers' claims themselves — recorded, not verified (§8).

The honest summary: the paper proves that the predecessor's one missing element was never a mystery object but a bookkeeping fact — one admissible flow of weight between identical copies — derives the full non-abelian structure from that fact wherever it holds, cancels two of the predecessor's debts in the process, and wires the entire question to a single named node the corpus's Born arc already built. The strong and weak forces now wait on one reading.

14. Conclusion

The predecessor ended one element short of the forces, and the element was stated in the only language then available: a group element, somewhere off the normalizer, existence unknown. This paper's central act is a translation that makes the element ordinary. The question was never whether some exotic transformation lurks in the substrate's transport group; it was whether the substrate keeps one account or many — whether the weight carried by identical copies of one pattern is pooled or partitioned. Partitioned, and the mathematics is swap-and-phase forever: the ledger *is* the normalizer, exactly, provably, with nothing left over. Pooled, and weight flows; flow is blending; blending, spread across the class by a covariance that costs nothing, is the full non-abelian structure — with the exchange the predecessor had to assume falling out as a consequence, and a technical escape it had to guard against closed by continuity alone. The bath does not merely answer the predecessor's question; it tidies the predecessor's books, which is how one recognizes a premise that was always the real subject.

And the question has an address. The corpus asked bath-or-ledger once before, in the Born arc, about the weight that becomes probability — and flagged there, for its own reasons, the same continuity this paper leans on. One node, three arcs: how chance composes, whether the strong and weak forces exist in the framework, and the license under which the flavour paper's generation transport already operates — the last of which means the node arrives with evidence, since the corpus's own machinery has been running bath-type transport all along, between sectors

if not yet provably within classes. A single reading at a single named slot now prices all three, and the concentration is deliberate and owned: confirmation closes three programmes at once; failure propagates exactly as far. The paper has stated each price in advance — ignition on one side, the corpus's largest kill-condition on the other, executed if it comes to that by this paper's own lemma. The world, which plainly contains the weak and strong forces, has in effect already voted; the framework's task is to discover whether its substrate voted the same way.

Identical closure sectors plus shared transport weight force continuous mixing; continuous mixing forces non-abelian structure. The predecessor showed what copies cost. This paper shows who keeps the books — and that everything turns on whether there is one ledger for the family, or one family with no ledgers at all. And if there are no ledgers, then the books were never the sectors' to keep: the weight belongs to the substrate, and the forces are how its patterns share it.