The Carrier Theorem

Quantized Response and the Minimal Transport-Stable Subject — The Composition Argument for Minimality, the Decomposition-Invariance Argument for the Bath, and the Conditional Discharge of CO-0

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General Reader Summary

The central claim, first

This paper answers a question that several earlier results in the programme have been leaning on without anyone asking it out loud. When a physical quantity is quantized — when it comes only in whole-number steps, like electric charge — *what exactly is the thing being quantized?* Physics answers by habit: the particle. The electron carries the charge; the quark carries the charge; the object owns the number. But the programme's recent papers have steadily undermined the habit's foundations. The Bath Criterion showed that the conserved content of a family of identical sectors is a shared pool rather than separate accounts; the Ownership Principle showed that permanent structure belongs to families of copies rather than to any one copy. The question this paper asks is the next one in the sequence, and the last one before the route to fractional charge can open: when the charge ladder forces whole numbers, *whose* whole numbers does it force — the member's, or the family's?

The answer proposed here is a principle simple enough to state in one line: **quantization belongs to whatever survives transport.** A quantized response is read by carrying a system around a closed journey and comparing the end with the beginning — and that comparison is meaningful only for a subject that is still *there*, still the same subject, when the journey closes. Whatever does not persist through transport cannot be the thing whose response is counted. The carrier of quantization is therefore not chosen; it is determined, by the transport itself, as the smallest structure transport preserves.

And the programme's two rival pictures of transport preserve different things. Under the *ledger* — separate accounts, member by member — each member persists through every admissible journey, so each member is a carrier and the whole-number ladder applies to every member separately: every fundamental charge integral, no fractions anywhere, without extra machinery — with one piece of fine print the paper prices rather than hides: this holds for a ledger whose journeys never shuffle the accounts among themselves, and a shuffling ledger, the paper shows, begins to behave like a pool. What truly divides the two worlds is not their names but whether transport preserves *who is who*. Under the *bath* — the shared pool — transport stirs the family continuously, and no fact survives the stirring that says "this strand is still strand number two." The members do not persist *as standing subjects*; only the family does. The family becomes the carrier; the whole-number ladder applies to the family; and a family integer, read at the grain of one member, can appear as a fraction — a third, for a family of three — without any whole number ever being broken.

Why not the member? — the whole argument in a bank

Picture a fund held two ways. In the first bank, three depositors hold three separate accounts. Money may move, but every banknote stays traceable to an account: the accounts persist, so the rules of the currency — whole pounds only, in a currency that mints no coin below the pound — apply to each account separately. No account can hold a third of a pound, because each account is itself a full citizen of the currency. In the second bank, the three depositors hold one pooled fund. The pool is stirred continuously: no banknote belongs to anyone, and there is no fact of the matter, after a day's churning, as to which depositor's money is which. The currency's whole-pound rule still holds with full force — but it holds *of the pool*, because the pool is the only thing that persists. And a pool of one pound, viewed from the seat of one of three equal depositors, is a third of a pound per seat — not because the currency broke, but because the rule was never about the seats. The seats show shares; the pool owns the pound. The paper's claim is that nature's families of identical particles are pools, not accounts — and that the strange thirds of particle physics are what a whole number looks like from inside a family of three. One loose end is flagged here exactly as the body flags it: when a single quark is measured, a single seat registers the pool's number in one interaction — and how a realized seat registers a standing family quantity is a real question, named in the paper and deferred to the next obligation rather than glossed.

The honest condition

The paper is written from the programme's standing rather than from a pretended neutrality, and it says so. Over many papers, independent lines of argument — the shared-pool structure of conserved weight, the Translation Lemma, the relocation of permanent structure to families, the convergent reconstructions of the quantum weight rule — have accumulated on one side of the programme's oldest fork: the bath. This paper takes that standing seriously and develops its deepest consequence: if transport preserves only the family, quantization belongs to the family. The rival ledger picture is retained at full strength — derived, not caricatured — because it earns its keep twice: it is the comparison that makes visible what the bath uniquely supplies (a structural route from family-level whole numbers to member-level fractions, which a ledger world simply does not contain), and it is a named condition under which the paper's conclusions would have to be withdrawn. A second, deeper withdrawal condition is named rather than buried, because it is the sharpest one the paper faces: the whole analysis governs electric charge only if charge is defined on the finished, mixed transport — rather than attached to the members before the mixing exists, by the order in which the theory is built. That ordering question is decided in another paper's foundations, and this paper does not pretend otherwise; it constructs the rival ordering in full, at the rival's most favorable configuration, and finds that even there the thirds go unexplained — the rival world ends up owing exactly the explanation the separate-accounts picture owes. And the thirds themselves now do work: a world containing permanent thirds is a world the accounts picture cannot natively produce and the pool picture can, so the most familiar strange number in particle physics becomes, for the first time, a measured quantity bearing on the programme's oldest open fork — pressure, not proof, with every condition between the observation and the verdict listed in the paper. One piece of bookkeeping is kept with deliberate care. The formal verdict on the fork — the programme's node W0 — remains open on the books, because this corpus moves its markers by derivation, not by accumulated lean; everything proven here is proven at a strength that survives either verdict. What changes is the posture. The bath is no longer one of two equal guesses: it is where the programme's arguments have been pointing for years, and this paper is about what follows.

Abstract

The headline. The Ownership Principle proved that standing transport structure carried by indistinguishable members is class-owned, and named as its prior question CO-0: *what determines the carrier on which quantized response is defined?* This paper supplies the derivation CO-0 requested, accounts for its discharge status exactly — and is written from the programme's standing: the corpus's independent arcs (shared conservation, the Translation Lemma, class ownership, the convergent Born routes) weight the bath ontology, so the ledger is carried as a diagnostic comparison and refuter condition rather than as a symmetric competitor, while the formal marker at W0 does not move — [Open] it remains, with every result stated at a conditionality that survives either verdict. Against that standing, the substantive claim is the **Carrier Theorem**: the underived content of quantized response attaches at the **minimal transport-stable subject** — minimality proven by composition rather than asserted — and under bath transport, which acts irreducibly and preserves no member decomposition as standing structure, **that subject is the class: the lattice quantizes class-wise, member grain lies below the quantization grain, and the structural route from integer class response to fractional member-grain readings opens**. Under the diagnostic ledger, by contrast, member lines persist as standing subjects, quantization is member-wise, every fundamental winding is integral, and no route to fractions exists — the comparison that measures exactly what the bath supplies. The fork is established at the conditionality of its named imports; the selection between branches is W0's and remains [Open]. The remainder of this abstract states the results at referee grain.

The definition, in inherited vocabulary. §3 defines the load-bearing notion in the Ownership paper's own terms. A subject S is **transport-stable** when its identity through admissible transport is *standing* structure — an invariant fact of the transport, not a realized configuration. The definition consumes the standing/realized boundary at the Ownership paper's §10 slot and adds no exposure of its own: a subject whose persistence is merely contingent configuration is realized content, and a winding indexed to it would be standing structure hung on a non-standing peg.

Minimality, proven by composition. §4 closes the gap a referee would open first: the class is transport-stable under the ledger too, so why does ledger quantization not sit at class grain? The **Composition Argument**: where a stable subject admits an exhaustive decomposition into stable sub-subjects, the windings add — $\Sigma_i q_i = q$, by AD [Inherited — the Ownership paper's additivity

premise, the charge paper's composition law] — so the coarser subject's integer is *derived*, the composite of its parts' integers, and the lattice's underived content lives at the subjects admitting no such decomposition — a set containing the minimal subjects and, by an alignment fact verified on each branch rather than assumed in general (under L-M every stable subject is a union of member lines; under B-IRR the class has no stable sub-subjects at all), exactly them. [The general clause: Proven, conditional on AD, L-G, and the lattice at the charge paper; the exactly-minimal sharpening: verified per branch.]

The bath branch, on the corrected argument; the ledger comparison, retained. §5 (bath — the programme's case): the argument is **decomposition invariance**, not allocation flow — the correction matters and is stated as such. Flowing allocations do not destroy subjects: the corpus's own basis states are member-indexed and admissible, and the Ownership paper's realized horn is built on members that persist while their content moves. What the bath destroys is the member decomposition *as standing structure*: the class group acts irreducibly [B-IRR — slot at the Bath Criterion's transport structure], no member line is invariant under admissible transport, so member-identity-through-transport is itself realized configuration — and a member-grain winding, windings being standing classifications by the record construction itself [R-S — slot at the charge paper], would be a standing assignment indexed to a subject whose persistence is not standing. The attachment point fails before the assignment is made. The class — whose identity is standing — is the minimal transport-stable subject, and the lattice quantizes class-wise. §6 (ledger — the diagnostic comparison, retained at full derivation): admissible ledger transport is monomial and diagonal on closed loops [L-M — slot at the Bath Criterion's ledger definition], so the member lines are transport-stable, minimal absent sub-member structure [SUB — fence], and the lattice quantizes member-wise: every fundamental winding integral at member grain, no structural route to fractions, with a named refinement (loop-permuted lines coarsen the carrier to the permutation orbit — a boundary case at the slot, not a refutation). [Each branch: Proven, conditional on its named import; the asymmetry between them is one of programme standing (§1), not of proof strength.]

The theorem and its anchor. §7 states the **Carrier Theorem** in three clauses (minimal attachment; the ledger branch; the bath branch) and runs the **k = 1 consistency check**: a singleton class collapses class carrier to member carrier, the two branches coincide, and the charge paper is recovered verbatim — the electron's −e as the one-member case. The lattice is preserved on both branches (§8): the theorem changes the carrier, never the lattice — an off-lattice fundamental charge still refutes; a member-grain reading of a class integer does not.

CO-0, accounted. §9 states the discharge exactly: CO-0 asked for a derivation that the carrier is the minimal transport-stable unit and that under the bath that unit is the class. The derivation is supplied; its premises are R-C and R-S (the record construction's persistent loop subject and the standing-status of windings — slots at the charge paper), L-G (the lattice's grain-universality — same address), AD, L-M, B-IRR, and C-ORD (the composition order — the U(1) response defined on the transport-completed carrier; the application import whose failure is §12's bypass, slot at the structure-group imports), each named with slot and fallback; the fork is therefore [Proven, conditional as stated], and **CO-0 is conditionally discharged** — it closes outright when the named slots confirm at source, and the branch selection it leaves behind is W0's question, not CO-0's. The route's order is then fixed: CO-0 (carrier) → CO-1 (existence and modality of the uniform member-grain reading, value closed at q/k by the Uniform Reading Lemma) → the thirds.

The pressure on W0. §10 prices the consequence at its full conditionality: granted the fork, a ledger substrate quantizes member-wise and has no structural route to permanent member-grain fractions, while a bath substrate makes them available; observed $e/3$ therefore bears on W0 — conditional on CO-0's slots, on the O1 identification of the quark classes [Open — at the census], and on a positive CO-1 — the W0 node's fourth converging arc and its first contact with an observed feature of the world.

Epistemic markers: [Inherited] imported from prior VERSF papers; [Imported-External] imported from standard mathematics outside the programme, carried at the external source's standing; [Proven] established here; [Conditional] holding under stated inputs; [Conjectural] motivated but unproven; [Open] undecided.

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1. Introduction — The Question the Route Was Standing On

The corpus's route to the $e/3$ generator was assembled in the Ownership paper with one premise standing on air, and that paper — to its credit — said so. The Class Ownership Theorem proves that *whatever standing structure a class carries* is class-owned; it does not prove that the response winding is carried at class grain rather than member grain. A skeptic grants the theorem entire and then makes each member its own carrier, with its own integral winding read off its own closed-loop record — and the thirds route dies before CO-1 opens, because q/k readings exist only if the class is the unit the lattice quantizes. The Ownership paper converted that silent premise into the named obligation CO-0, specified the candidate argument's shape, its proof joints, and its principal refuter, and handed it forward. This paper is the hand-off received.

The question, stated once at full width: **when a response is quantized, what is the subject of the quantization?** The conventional answer — the particle — is not wrong so much as unexamined. It presupposes that the particle is a meaningful subject of the transport that defines the response: that there is a persistent *it* whose closed-loop record the winding classifies. The charge paper's own construction makes the presupposition visible — a winding is read by transporting a carrier around a closed admissible path and comparing the returned state with the initial one, and the comparison is contentful only for a subject whose identity spans the loop [Inherited — ⟨cite: the charge paper, the record construction⟩]. Quantization is a statement about persistent response; persistent response requires a persistent subject; and *persistent* means persistent under exactly the transport whose loops define the quantity. The carrier is therefore not an input to the theory. It is an output of the transport.

That observation, once made, forks. The corpus contains two rival accounts of what admissible transport preserves, and they have been rivals since the Bath Criterion drew the line. Under the **ledger**, conserved content is member-resolved: each member's account persists through every admissible history, transport adjusts phases and configurations but never dissolves the bookkeeping, and the member is exactly the persistent subject the record construction needs. Under the **bath**, conserved content is pooled: the class carries the total, transport redistributes realization continuously among members, and — this paper will argue, on the corrected ground of decomposition invariance rather than the tempting but wrong ground of allocation flow — no member-level decomposition survives transport *as a standing structure*. The persistent subject is the class.

The consequence is the fork this paper exists to prove: **ledger transport quantizes members; bath transport quantizes classes.** The same lattice theorem, the same integers, applied to different subjects — and everything downstream of the route to thirds turns on which. Three disciplines govern the development, all inherited from the arc's hard lessons. First, **minimality must be earned**: the class is transport-stable on *both* branches, so "the carrier is the minimal stable subject" is a theorem with a missing argument until composition supplies it (§4), and the paper proves the principle rather than installing it as a definition. Second, **the bath argument must survive the corpus's own audit**: the Ownership paper's extreme cases include member-indexed basis states carrying the entire class weight, so any argument of the form "transport moves content, therefore members do not persist" is refuted by the predecessors it cites — the correct claim is sharper, and §5 states it in the standing/realized vocabulary built for exactly this purpose. Third, **the discharge must be accounted**: CO-0 was a named obligation with joints and a refuter, and a paper answering it owes an exact statement of what is now proven, at what conditionality, and what remains open (§9) — the corpus does not let obligations dissolve into prose.

One positioning statement belongs in the introduction rather than the margins, because it governs the paper's voice. The two accounts are not, at this point in the programme, epistemically symmetric. The Bath Criterion's dichotomy was drawn when they were; since then, independent arcs — the shared-conservation structure itself, the Translation Lemma's machinery, the

Ownership Principle's relocation of standing structure to classes, the convergent routes of the Born reconstruction — have accumulated on the bath side, and the corpus's working ontology has followed. This paper is therefore written from the bath's standing: the bath branch is developed as the programme's case, and the ledger is retained — fully derived, never caricatured — as the diagnostic comparison that makes the bath's yield visible and as the refuter condition under which the route would close. One discipline binds the posture: **the marker at W0 does not move.** Accumulated lean is not derivation, this corpus inflates nothing, and every theorem below is stated at a conditionality indifferent to the verdict. The asymmetry claimed is of weight, not of status — and §10, where observed fractions enter the case, draws its force from the conditional structure alone, so that the paper's posture is never fed back into its own evidence.

The paper derives no fractional value, selects no ontology, and touches no gauge structure. It identifies the subject on which any such derivation must operate, proves that the identification is forced by transport, and prices what the identification makes observable. The route to thirds acquires its foundation; the foundation acquires its slots; and the oldest fork in the programme — ledger or bath — acquires, for the first time, an empirical lean.

2. Inherited Results and Imports

The charge lattice and the record construction (R-C) [Inherited — ⟨cite: the charge paper⟩]: carrier responses classified by winding $q \in \mathbb{Z}$; the winding read off a persistent subject's closed-loop transport record; windings additive under composition of carriers (responses multiply, windings add). The lattice is consumed intact — nothing here amends it — and the record construction is consumed as the definition of what a response *is*: a comparison across a loop, predicated on a subject that spans it. **R-C is this paper's first named import**: the response's subject is the loop-persistent unit. [Slot: the charge paper's response construction, read for the persistence presupposition. *Fallback if broken:* if responses can be defined without a persistent subject, the Carrier Principle loses its ground and the carrier question reverts to [Open] with no transport-theoretic answer; every result here is exactly as strong as R-C.]

Two further readings of the same source are promoted to named imports, because each carries weight a referee would otherwise find unexamined. **R-S (standing-status of response)**: a winding is, by the record construction itself, a *standing* classification — invariant content of the carrier, holding across admissible history — and not a property of any particular realized history. A history-relative assignment of response — a configuration-level fact about which realized seat traced which realized loop — is admissible as configuration content but is not a winding, and the lattice does not bind it. R-S is the peg beneath §5's peg-failure argument, named so that the argument's own ground is examined rather than assumed. [Slot: the charge paper's response construction, read for the invariance of the classification. *Fallback if broken:* if windings can be realized rather than standing, §5 weakens from "no member-grain fundamental winding" to "no *standing* member-grain winding," with the realized analogue's lattice-status reverting to [Open] at the charge paper.] **L-G (lattice grain-universality)**: the quantization theorem binds at every transport-stable grain — any persistent loop subject's winding is integral — and not only at a privileged construction grain. L-G is what the Minimality Proposition consumes when it takes sub-subject windings to be integers; whether the charge paper proved its theorem at that generality or at one carrier construction is exactly the kind of fact only the source reading fixes.

[Slot: the charge paper's quantization theorem, read for the generality of its carrier. *Fallback if broken:* minimality of attachment survives, but "coarser integers are derived sums of integers" weakens to derived sums of responses, and the branch propositions retain their carriers with the integrality clause re-priced at the slot.]

The Bath Criterion [Inherited — at its statuses]: the ledger/bath dichotomy; the ledger's member-resolved conserved accounts and monomial transport; the bath's class-level conserved total, the Translation Lemma with its basis-state proof, and the class group's action on the carrier; W0/W1 at the Born-arc slot [Open]. Two structural readings of this material are promoted to named imports here, because the fork's two branches each stand on one: **L-M (ledger monomiality)** — admissible ledger transport preserves the member-line decomposition, acting diagonally on closed loops up to phase [slot: the Bath Criterion's ledger definition; the loop-permutation boundary case is treated in §6]; and **B-IRR (bath irreducibility)** — the class group acts irreducibly on the class carrier: no proper subspace, in particular no member line, is invariant under all admissible transport [slot: the Bath Criterion's transport structure]. *Fallbacks:* stated at the branches they carry (§6, §5).

C-ORD (composition order) — the application import, named where it is consumed rather than where it is decided: *the U(1) response is defined on the transport-completed carrier — the carrier on which the class group already acts — and not ordered member-wise on the pre-mixing carrier.* The Carrier Theorem is a statement about transport-stable subjects; C-ORD is what makes the U(1) winding the theorem's business at all, since a response attached by construction order, upstream of the mixing, has its carrier fixed before transport-stability is ever consulted. It is different in kind from the other imports — they condition the fork's *proof*, C-ORD conditions its *application* — and its failure mode is correspondingly different: not a weakening but a bypass, constructed in full and priced in §12. [Slot: the structure-group paper's imports — the order in which the derived phase and the class group enter admissible transport. *Fallback if broken:* the member-wise-prior alternative of §12 activates wholesale — every theorem here stands and none of them governs U(1), the route closes at CO-0, and the §10 pressure dies with it; the exposure is total.]

The Ownership Principle [Inherited — *Class-Owned Standing Structure*, at its statuses]: the standing/realized distinction (standing = invariant-tuple content; realized = contingent configuration, with contingency rather than motion as the criterion); the Class Ownership Theorem in both clauses; the Uniform Reading Lemma (a standing member-grain reading, if it exists, is permutation-forced to q/k) with its additivity premise **AD**; the boundary-drawability exposure at its §10 slot; and CO-0 itself, with its specified joints — the candidate argument's shape, the $k = 1$ collapse, and the member-wise-prior refuter at the structure-group slot. This paper consumes the vocabulary throughout, answers the obligation in §§4–9, and inherits the boundary exposure rather than creating a new one: the definition of transport-stability (§3) is built from the standing/realized line and is exactly as drawable as that line is.

The Capacity Census [Inherited — *Three by Two*, at its slots]: admissible class capacities $k \in \{1, 2, 3\}$; the O1 occupancy identifications [Open]. Consumed in §10, where the pressure on W0 is priced — the identification of the quark classes is O1 content and the pricing says so.

No new physical imports beyond the named readings (R-C, R-S, L-G, C-ORD, AD, L-M, B-IRR, SUB). The paper adds one definition (§3 — built from inherited vocabulary, no new exposure), one proven principle (§4 — conditional on R-C, L-G, and AD), two branch propositions (§§5–6 — the bath on B-IRR and R-S, the ledger comparison on L-M), one theorem consolidating them (§7), and one fence: **SUB (no sub-member subject)** — the architecture's transport inventory contains no admissible transport-stable subject below member grain [slot: the Fold papers' carrier structure; consumed only by the ledger branch's minimality clause, §6].

3. Transport-Stable Subjects — The Definition, in Inherited Vocabulary

The paper's central notion is defined in the Ownership paper's vocabulary, so that it inherits that paper's standing — and that paper's one exposure — rather than petitioning for its own.

Transport-stable subject. A structure S within the carrier architecture is *transport-stable* when its identity through admissible transport is **standing** structure: the fact "this is S, before and after" is invariant content of the transport, holding across all admissible history, rather than a realized configuration that admissible history could alter. Transport may act *within* S — redistributing content, moving allocations, changing every realized fact about S's interior — and S remains stable so long as S itself, as a subject, is an invariant of the action.

Three consequences of drawing the definition at this line, each spent later.

First, **stability is not rigidity.** A transport-stable subject's contents may churn without limit; the Bath Criterion's conserved class total $W\_\boldsymbol{\mathcal{C}}$ is carried by a class whose member allocations flow continuously, and the class is stable throughout, because the flowing is realized and the class's identity is standing. Contingency of content does not touch stability of subject — the distinction the Ownership paper drew between what a thing *is* and what it currently *carries*, applied now to the subjects of transport themselves.

Second, **persistence-as-configuration is not stability.** A structure may happen to persist through some particular history — a member line undisturbed by a particular sequence of transports — without being transport-stable, because stability requires the persistence to be *standing*: guaranteed by the transport's invariant structure, not contingent on which history was realized. This is the joint on which the bath branch turns (§5), and it is why the definition must be drawn in the standing/realized vocabulary rather than in the looser language of "survives transport": survival in fact is configuration; survival by invariance is structure. One choice inside this clause is made explicitly rather than left to reconstruction, because it carries weight in two later sections: stability is defined across **all admissible history**, not per loop. The record construction reads windings on individual closed loops, and a skeptic may therefore propose per-loop subjects — *the comparison needs the subject to survive only this loop, and this loop happens to act diagonally*. The answer is R-S: the winding a loop reads is a standing classification, and a standing assignment cannot be indexed to a particular realized history — so its subject's identity must be standing across the whole admissible class of histories, not realized along one. Per-loop survival is persistence-as-configuration, exactly the case this clause excludes, and the per-loop skeptic's "subject" is a realized structure carrying, at most, the realized content the lattice does not bind. The choice's second consequence — that loop-level structure shapes the carrier only where it is admissibility-wide, as in §6's permutation boundary case — is spent where §10 prices the fork.

Third, **the definition consumes the boundary, and only the boundary.** Whether subject-identity is standing or realized is an instance of the catalogue/configuration separation the Ownership paper's §10 named as its single new exposure. This paper adds no second exposure: transport-stability is drawable exactly where the standing/realized line is drawable, the slot is the same slot, and the fallback is the same fallback — if the boundary admits intermediate cases at source, subjects whose identity is invariant across *all* admissible history remain cleanly stable, and intermediate subjects inherit [Open] standing pending the finer analysis. [Definition; the boundary: at the Ownership paper's §10 slot, consumed, not duplicated.]

The record construction now connects. A winding is read by comparing a subject's state across a closed admissible loop [R-C]. The comparison presupposes that "the same subject" is a fact at both ends — and *fact*, for a quantity defined by transport, must mean standing fact of the transport: a comparison across a loop whose subject-identity is merely realized would be a comparison whose own terms the transport is permitted to dissolve. Hence the **Carrier Principle**, stated now and proven minimal in §4: *quantized response attaches to transport-stable subjects, and its underived content lives at those admitting no exhaustive stable decomposition — the minimal subjects always among them, and on each of this paper's branches, only them (§4).*

4. Minimality — The Composition Argument

The principle's second clause is where a referee would press first, and the paper meets the press with the argument rather than the assertion. The objection: the class is transport-stable under the **ledger** too — monomial transport preserves the class trivially, since it preserves every member — so if stability were the whole criterion, ledger quantization would sit at class grain as comfortably as at member grain, and the fork would collapse into a free choice. The answer is that stability is not the whole criterion: stability determines where quantization *can* attach; **composition determines where its content is underived**, and the two together force minimal grain on each of this paper's branches — with the general clause held at exactly the strength the argument proves, as the proposition below states.

The Composition Argument. Let S be a transport-stable subject admitting an **exhaustive** decomposition into transport-stable sub-subjects $S_1, \ldots, S_n$ — the sub-subjects jointly covering S, each line of the decomposition itself an invariant of admissible transport. Exhaustiveness is a condition of the argument, not a nicety, and it is named where it is consumed: a *partial* stable decomposition — stable sub-subjects plus a residue that is no stable subject — supports no composition identity, since the residue's content is neither a sub-winding nor attached at any persistent grain; the argument quantifies over exhaustive decompositions only, and the proposition below says so. Each S_i, being a persistent loop subject, carries its own well-defined record and hence its own winding q_i, integral by the lattice at its grain [L-G — the grain-universality import, named in §2 precisely because this step consumes it]. By AD — the charge paper's composition law, windings additive over composed carriers [Inherited — the Ownership paper's named premise, consumed here at exactly its inherited strength] — the windings compose:

$q(S) = \Sigma_i \, q(S_i)$, each $q_i \in \mathbb{Z}$.

The coarse subject's integer is therefore **derived**: it is the arithmetic of its parts' integers, fixed the moment the parts' windings are fixed, carrying no quantization content of its own. The proposition's exact yield is therefore: **the lattice's underived content — the quantization that is a fact about the world rather than a sum over prior facts — lives at the subjects admitting no exhaustive stable decomposition.** That set contains the minimal subjects, and the paper is careful about whether it contains more, because the corrected argument alone does not say: a non-minimal stable subject admitting only *partial* stable decompositions would carry an underived integer above minimal grain, the composition identity never firing for it. Whether such subjects exist — equivalently, whether every non-minimal stable subject decomposes exhaustively into stable sub-subjects — is an **alignment fact about the transport inventory**, not a tautology, and the paper verifies it per branch, where it is cheap, rather than importing a general fence for it. Under L-M, every stable subject is a union of member lines and is exhaustively decomposed by them, so the no-exhaustive-decomposition set is exactly the member lines: minimal grain. Under B-IRR, the class admits no stable sub-subjects at all, exhaustive or partial, so the set is exactly the class: minimal grain, trivially. On both branches the alignment holds and "exactly at minimal grain" is recovered; between the branches, the general clause stands at the strength the argument proves, and no more. Below minimal grain, on any branch, no subject persists to carry a record at all [by transport-stability, §3]. [**Minimality Proposition** — the general clause, underived content at subjects without exhaustive stable decomposition: Proven, conditional on R-C, L-G, AD, and the lattice at the charge paper; the "exactly minimal" sharpening: verified per branch, at L-M and B-IRR respectively.]

Two remarks fix the proposition's weight. First, it dissolves the opening objection exactly: under the ledger the class *is* stable and *does* carry an integer — the sum of its members' integers, the member decomposition being exhaustive by L-M, so the composition identity applies — and nothing about that integer is wrong; it is merely derived, and the fundamental lattice sits one grain down, at the members, because the members are stable and nothing stable sits below them. The fork of §§5–6 is therefore not "where can quantization live?" — it can live at every stable grain — but "where does the *underived* lattice bite?", and that question has a unique answer per ontology. Second, the proposition is symmetric in its service: it is what makes the ledger branch quantize members *rather than* the class, and equally what will make the bath branch quantize the class *rather than* nothing — under the bath the class admits no stable decomposition at all, exhaustive or partial — it is not merely a stable subject among others but the minimal one, and the lattice's underived content lands on it with nowhere finer to go.

5. The Bath Branch — Decomposition Invariance, Not Allocation Flow

The bath branch carries the route's weight, and its argument must be the right one, because the obvious one is wrong — wrong by the corpus's own results, and the paper says so before building.

The wrong argument, retired explicitly. It is tempting to argue: *under the bath, transport continuously redistributes realization among members; a quantity attached to one member cannot remain invariant, since transport may move it elsewhere; therefore the member is not transport-stable.* The corpus refutes this in one paragraph. The Translation Lemma's basis states are member-indexed configurations in which one member observably carries the entire class weight — admissible, load-bearing, and consumed by the Bath arc's central proof. The Ownership paper's realized horn is built on members that persist as loci of allocation while their content flows; its audit *passed* precisely because flowing content leaves subjects intact. Allocation flow is realized configuration, and realized configuration, by the inherited boundary, touches nothing standing. If member instability followed from content mobility, the Ownership paper's extreme cases would be inadmissible and the Bath Criterion would consume forbidden objects. They are not, and it does not. The members of a bath class *exist*, *persist as loci of realized content*, and *may carry anything, including everything, at any moment*. None of that is in dispute, and none of it decides the carrier question.

The right argument — decomposition invariance. What the carrier question turns on is not whether members exist but whether member-identity-through-transport is **standing** — whether "this line, before the loop, is that line, after it" is an invariant fact of the transport. And under the bath it is not, for a reason structural rather than dynamical: by B-IRR, the class group acts **irreducibly** on the class carrier — no proper subspace is invariant under all admissible transport, and in particular **no member line is preserved as a line**. One precision is owed and paid here, because irreducibility is being consumed at exactly its strength and not beyond: B-IRR excludes invariant *lines*, not necessarily an invariant line-*set* permuted among itself — monomial irreducible actions exist. The conclusion consumes only the former exclusion: a line-identity that survives only up to permutation is not a *standing* identity, the carrier analysis runs on identity rather than on sets, and the monomial case is the mirror of §6's permuting-ledger boundary case — the same object approached from the dichotomy's other side, where §10 names what the mirroring means for the fork's true grain. A member decomposition $\{c_1, \ldots, c\_k\}$ of the bath carrier is a *basis*: a piece of descriptive apparatus, available, useful, and realized — exactly as the basis states it generates are realized configurations. There is no transport-invariant fact that tracks "line two" around an admissible loop; the only decomposition-level facts that survive every admissible transport are the facts about the whole space. Member-identity-through-transport is configuration content, not catalogue content — persistence-as-configuration, in §3's terms, not stability.

The consequence for attachment is then immediate, and it operates *before* any winding is assigned, which is the point a looser argument would miss. A member-grain winding would be a standing assignment — invariant content, label-indexed to a member line, standing by R-S as every winding is — hung on a subject whose own identity across transport is realized rather than standing. The assignment's *peg* fails, not merely its value: there is no standing fact of the form "this line, around the loop" for the standing fact "this line's winding" to attach to. One escape is closed by import rather than by improvisation, and the import is the one §2 named for this purpose. A skeptic may offer a *history-relative* winding: a realized, configuration-level assignment of response to a realized seat along a realized history — and the seat does persist on that history, as a matter of realized fact. The assignment is admissible, and it is not a winding: by R-S, a winding is a standing classification, the lattice binds standing classifications only, and a per-history analogue is configuration content carrying no quantization. The realized analogue may exist; the fundamental integers remain the class's; and the skeptic's fallback is priced at R-S's slot, not absorbed silently. In the Ownership paper's vocabulary, run one level deeper than that paper ran it: the bath does not merely forbid members from *owning* standing structure — it withholds from member lines the standing identity that owning would presuppose. The minimal transport-stable subject — the smallest structure whose identity is an invariant of the irreducible action — is the class carrier itself.

Bath Carrier Proposition. *Under the bath ontology [B-IRR], the class is the minimal transport-stable subject, and the lattice quantizes class-wise: the fundamental winding $q \in \mathbb{Z}$ is the class's, and member grain lies below the quantization grain.* [Proven, conditional on B-IRR and §§3–4. *Fallback if B-IRR is broken:* a transport-invariant member decomposition under the bath restores member lines as stable subjects, the branch collapses onto §6's conclusions, and the thirds route closes at CO-0 — the strike is total for the route and is priced as such in §12.]

Reconciliation, recorded. The two branches' pictures of the member must be held together without contradiction, and the standing/realized line holds them. Under the bath: members exist as realized structure — loci of allocation, basis directions, the seats at the pool — and everything the corpus's proofs do with them remains licensed, because everything the proofs do with them is configuration-level. What members lack is standing identity through transport, which is precisely and only what a fundamental winding requires of its subject. And the Ownership paper's uniform reading — each member standingly at q/k, the route's eventual deliverable — is *consistent* with this, not in tension with it: the uniform reading is, by that paper's identification clause, the **class's** winding in member notation — quantifying over seats while relabeling-invariant, with "indexed to no line" licensed exactly by the Uniform Reading Lemma's permutation-forcing — presupposing no standing member identity, which is why it is the only member-grain reading that can exist and why the Lemma found no other. The skeptic's natural objection deserves its answer spelled out rather than left in apposition: *the paper has just argued that members lack standing identity — how can each member standingly carry anything?* The answer is relabeling-invariance. A relabeling-invariant predicate quantifies over seats without naming one; its standing therefore requires only the *class's* identity, which is exactly the identity the bath supplies — and "each member carries q/k" is such a predicate: true at every seat, indexed to none, standing on the class. The bath forbids windings *of* members and licenses the class's winding *read at* member grain: the route was always threading exactly this needle, and the carrier analysis is what shows the needle has an eye. One operational question is fenced where it belongs rather than left to surface on its own: a measurement is made by realized structure — a single realized quark traverses the loop — and how a realized member registers a standing class winding is CO-1's first deliverable, with the record construction's compatibility with that registration read at the R-C slot; the carrier analysis locates the winding and does not yet describe its registration. [The reconciliation: Proven at the level of the papers' statements; the registration: at CO-1 and the R-C slot.]

6. The Ledger Branch — Member Carriers, Retained as the Diagnostic Comparison

The branch is retained at full derivation, and its standing is stated before its content: the ledger is no longer carried as a symmetric competitor. The programme's independent arcs converge on the bath (§1), and the ledger's role here is twofold. **Diagnostic**: the comparison isolates exactly what the bath uniquely supplies — under the ledger, as the proposition below proves, there is no structural route from quantization to fractions, so the route is the bath's signature rather than a generic feature of class vocabulary, and the contrast is the measure of the yield. **Refutational**: the ledger is the named condition under which the route closes (§12), and a condition can be audited only if its consequences are derived rather than gestured at. Nothing below is weakened by the demotion — the proposition is proven at the same grain as its sibling, the k = 1 check consumes it, and §10's pressure argument requires it whole.

Ledger Carrier Proposition. *Under the ledger ontology [L-M], the member lines are the minimal transport-stable subjects, and the lattice quantizes member-wise: every fundamental winding is integral at member grain.* [Proven, conditional on L-M, SUB, and §§3–4.]

The argument, in three steps at the definitions. **Stability:** by L-M, admissible ledger transport preserves the member-line decomposition as standing structure — the bookkeeping is member-resolved as an invariant of the transport, not as a fortunate configuration — and on closed loops the action is diagonal up to phase, so each member line spans every admissible loop as the same subject. Member-identity-through-transport is standing; the members are transport-stable. **Minimality:** no admissible transport-stable subject exists below member grain — the architecture's carrier inventory contains no sub-member persistent unit. This is the fence **SUB**, named rather than assumed [slot: the Fold papers' carrier structure, read for sub-member transport subjects. *Fallback if broken:* a stable sub-member subject would relocate the fundamental lattice one grain further down, with member integers becoming derived sums — a refinement of the branch, not a refutation of the principle]. **Attachment:** by the Minimality Proposition, the underived lattice bites at the members; every coarser subject's integer — including the class's — is the AD-composite of member integers.

The consequence the route cares about follows at once and is stated at full strength: **the ledger contains no structural route to permanent member-grain fractions.** Every member is itself a full citizen of the lattice; its winding is an underived integer; and a permanent member value of q⁄k would be an off-lattice fundamental winding — exactly the object the charge paper's theorem excludes. The claim's scope is exact and the paper marks it: not that fractional *phenomena* are impossible in a ledger world, but that they cannot arise from the quantization mechanism itself — any ledger account of observed thirds owes a separate construction, external to the lattice, which the corpus does not currently contain. [The consequence: Proven at the branch's conditionality; its empirical weight: priced in §10, not here.]

One boundary case, named at the slot rather than buried. L-M as stated takes closed-loop ledger transport to be diagonal on member lines. The Bath Criterion's ledger admits monomial transport — preservation of the line *set*, possibly with permutation — and a closed admissible loop that returns the set while permuting its lines would make the individual line non-stable across that loop, with the minimal stable subject coarsening to the **permutation orbit**: a class-like carrier arising inside the ledger. The case is a refinement with teeth — orbit carriers would import a fragment of the bath branch's conclusions into the ledger — and whether admissible ledger loops permute is a fact of the source's ledger definition, added to the L-M slot as a named condition. The branch proposition holds as stated for the diagonal reading; the permuted reading re-runs §§3–4 with orbits in place of lines and is recorded as the slot's second branch, not silently excluded. [The boundary case: stated; its resolution: at the L-M slot.]

7. The Carrier Theorem

The branches consolidate.

Carrier Theorem. *Let response winding be defined through closed-loop transport records [R-C], with windings additive over composed carriers [AD]. Then:*

(i) — attachment — the underived content of quantized response lives at the subjects admitting no exhaustive stable decomposition: coarser stable subjects admitting such decompositions carry derived integers composed by AD, no finer subject persists to carry a record, and on each branch below the no-decomposition set is exactly the minimal subjects [the Minimality Proposition, §4, with the alignment verified per branch rather than assumed in general];

(ii) — the bath branch, the programme's case — under bath transport [B-IRR], the class is the minimal transport-stable subject; the lattice quantizes class-wise, member grain lies below the quantization grain, and the only standing member-grain content available is the class winding read uniformly, at q/k where it exists [§5, with the Uniform Reading Lemma at its inherited status];

(iii) — the ledger branch, the diagnostic comparison — under ledger transport [L-M, SUB], the member lines are the minimal transport-stable subjects; the lattice quantizes member-wise, and every fundamental winding is integral at member grain — no structural route to fractions [§6].

[Proven, conditional on R-C, R-S, L-G, AD, and the branch imports (B-IRR; L-M, SUB) as stated; the theorem's application to the U(1) response additionally consumes C-ORD (§2, §12); the selection between (ii) and (iii) is not the theorem's content and is held [Open] at W0 — the paper's standing (§1) notwithstanding.]

The k = 1 consistency check, run. A singleton class collapses the fork: with one member, the class carrier and the member line are the same subject, irreducibility and diagonality coincide trivially, branches (ii) and (iii) issue the same verdict, and the theorem reproduces the charge paper verbatim — one persistent subject, one integral winding, the electron's −e recovered as the one-member case. The check is the route's anchor audit, specified in CO-0's joints, and it passes on both branches: nothing in the carrier analysis disturbs a single integral charge already derived. [Audit: passed.]

Three remarks fix the theorem's weight. First, **it is transport-driven and quantity-blind**: nothing in clauses (i)–(iii) consumes charge, gauge structure, or any particular response — the statement concerns subjects, and any quantity defined by closed-loop records inherits it. Second, **it is independent of ownership, and jointly exhaustive with it**: the Ownership Theorem governs where standing structure *belongs* once a carrier exists; the Carrier Theorem governs where the quantized carrier *is*. Neither implies the other — a referee may grant class ownership and dispute the class carrier, which is exactly the skeptic CO-0 was named against — and together they close the route's structural prerequisites: carrier located, ownership assigned, value forced, with only existence and modality (CO-1) remaining. Third, **the fork is physical, not interpretive**: the two branches differ in the subject on which the lattice's underived content sits, and §10 converts that difference into an observational lean — the divergence between ledger and bath, drawn by the Bath Criterion in the language of conserved accounts, reappears here as a difference in what the world's quantized numbers are *numbers of*.

8. The Lattice, Preserved on Both Branches

One misreading must be closed within a section of the theorem, because the route has been misread this way before: the carrier analysis is not a weakening of the charge lattice. It is the opposite — a statement about where the lattice's full, unweakened force applies.

The integer quantization theorem is consumed intact on both branches. Under the ledger, the carrier is the member; the lattice forces $q \in \mathbb{Z}$ member-wise; every fundamental charge is integral, and the class's integer is the derived sum. Under the bath, the carrier is the class; the lattice forces $q \in \mathbb{Z}$ class-wise, with exactly the same theorem at exactly the same strength; and member grain, lying below the quantization grain, hosts no fundamental winding at all — only the class integer's uniform reading, where CO-1 licenses one. In neither branch does any subject carry an off-lattice fundamental winding; in neither branch is the lattice's modality reduced; and the refutation conditions are unchanged on both: **an off-lattice fundamental charge — a winding of a carrier that is not an integer — refutes the lattice theorem today exactly as it did before this paper, while a member-grain reading of a class-level integer refutes nothing**, being the same integer in finer notation, as the Ownership paper proved at statement level. The theorem changes the carrier. It does not change the lattice. The route's compatibility with every integer quantization result in the corpus is not a hoped-for consistency to be checked later; it is structural, and this section is its record. [Proven at the level of the papers' statements.]

9. CO-0, Accounted — What Is Discharged and What Remains

The Ownership paper named CO-0 at [Conjectural] with a specified shape: *derive that the carrier of the U(1) response is the minimal transport-stable unit, and that under the bath that unit is the class* — with three joints (the record construction's persistent subject; the $k = 1$ collapse; the member-wise-prior refuter) and one priced consequence (observed $e/3$ as pressure on W0). A paper answering a named obligation owes an exact accounting, and here it is.

Discharged, conditionally. The derivation CO-0 requested is supplied: the Carrier Principle is proven minimal by composition (§4), the bath's class-carrier clause is proven by decomposition invariance (§5), and the fork is consolidated as the Carrier Theorem (§7) — [Proven, conditional on R-C, R-S, L-G, AD, L-M, B-IRR, and SUB at their named slots; applied to the U(1) response additionally under C-ORD]. The obligation's first joint is honored (R-C is the first import, §2), its second is run and passed (the k = 1 check, §7), and its third is carried forward and now analyzed rather than merely named (the member-wise-prior alternative, constructed in §12). **CO-0's status therefore moves from [Conjectural] to [Conditionally discharged]**: the structural argument exists and is proven at its stated grain; the obligation closes outright when the named slots confirm at source — the record construction's persistence presupposition and the standing-status of windings at the charge paper [R-C, R-S], the lattice's grain-universality at the same address [L-G], the composition of windings across stable decompositions at the charge paper's composition law [AD, as consumed in §4 — its *member-grain* application is a different consumption and remains CO-1's, not this paper's], monomiality with its loop-diagonal clause at the Bath Criterion's ledger [L-M], irreducibility at the Bath Criterion's transport structure [B-IRR] — and the composition order at the structure-group imports [C-ORD], named last because it is different in kind: the other slots condition the fork's *proof*, C-ORD conditions its *application* to U(1), and its failure is not a weakening but the §12 bypass, under which CO-0's claim is false of U(1) with every theorem here intact. A discharge clause that omitted it would be exactly the dissolution-into-prose this section exists to prevent, and the paper's own §12 supplies the reason it cannot be omitted. (One deliberate absence is marked so it is not mistaken for an oversight: SUB does not appear in the closure list because it conditions only the ledger branch's minimality clause — the diagnostic comparison — and not the discharge of CO-0's bath-side claim; its slot is carried at §6.) No marker inflates: the fork is [Proven, conditional]; the slots are readings; the closure is theirs.

Not CO-0's to decide, and not decided. The selection between branches — whether the world's transport is ledger or bath — was never CO-0's question and is not answered here: it is W0, at the Born-arc slot, [Open] exactly as the Bath Criterion left it. What this paper changes is W0's *exposure*: the fork now has observable consequences (§10), so W0 is no longer adjudicable only from inside the formalism. The paper's standing (§1) changes nothing here: posture is not derivation, and the marker holds.

The route's order, fixed. With CO-0 conditionally discharged, the route to thirds stands in its final structural order: **CO-0** locates the carrier (this paper — the class, on the bath branch); **CO-1** determines existence and modality of the uniform member-grain reading (open at the realization-dynamics slots, its value closed in advance at q/k by the Uniform Reading Lemma, its consistency guaranteed in advance by the identification clause); and the housing-family saturation question — why free configurations show integral totals — remains CO-1's second deliverable, in the census's G-S territory, untouched here. Nothing in this paper advances CO-1; the carrier analysis is its foundation, not its discharge. [The accounting: stated; CO-1: [Conjectural], at its paper.]

10. The Pressure on W0

The theorem's consequence for the programme's deepest open node is priced here at its exact conditionality — neither inflated into a verdict nor deflated into a remark.

Granted the fork, the two ontologies now differ in an observable's *availability*. A **ledger** substrate quantizes member-wise: every fundamental winding integral at member grain, with no structural route to permanent fractions — observed thirds would stand as external inputs awaiting a mechanism the corpus does not contain (§6). A **bath** substrate quantizes class-wise: member grain lies below the quantization grain, and permanent member-grain fractions become structurally available as uniform readings of class integers — observed thirds would stand as candidate consequences of the transport structure itself (§5, pending CO-1). The world manifestly contains permanent thirds. The observation therefore **bears on W0**: it is evidence the two branches accommodate asymmetrically, the first time the ledger/bath fork has faced data.

The conditionality is stated in full, because the pressure is exactly as strong as its chain. The lean of observed $e/3$ toward the bath consumes: **the fork itself** at §9's full conditionality (C-ORD included); **the O1 identification** of the quark species with a $k = 3$ class [Open — at the census's PFD slot], without which the thirds are not yet attached to any class whose carrier is in question; and **a positive CO-1**, without which the bath's availability of fractions remains availability rather than account. The pressure is therefore conditional three deep — and genuine at that depth: the thirds do not prove the bath and do not refute the ledger, but they shift the burden of explanation onto the ledger branch, which must either supply the external mechanism it owes or concede the lean.

One of the fork's own conditions is different in kind and is pointed at here rather than left inside the list, because a reader auditing the pressure from this section alone would count three conditions and miss it: **C-ORD**. Under §12's member-wise-prior ordering the pressure does not degrade — it dies wholesale, on both ontologies, since even a bath world under that ordering quantizes its U(1) member-wise. The bypass is constructed and priced in §12; the audit of this section's claim must include it. This is the W0 node's **fourth converging arc** — beside the Born-rule chains the corpus has assembled from the geometric, informational, and eliminative directions — and its first external data point: the prior three arcs argue toward W0 from inside the framework's structure, and this one, alone so far, from an observed number. It is also the first member of the very convergence from which the paper's own standing (§1) is drawn — and for exactly that reason the pressure is run on the conditional fork and never on the standing: feeding the posture back into its own evidence would be circular, so the branches are compared here as live alternatives, precisely as §12's refuters require, and the lean earns its place by derivation alone. [The pressure: Conditional, three deep, as priced; its discharge: at the named slots and obligations, not here.]

A second dependence is priced visibly rather than folded into "the fork's slots," because its failure collapses the asymmetry entirely rather than weakening it: **the lean survives only on L-M's diagonal clause.** A permuting ledger — admissible closed loops permuting member lines, §6's named boundary case — generates orbit carriers inside the ledger, orbit integers readable at member grain as fractions, and therefore precisely the fractional availability this section prices as bath-borne. On the permuted reading the two ontologies converge in what they make available, the observational asymmetry vanishes, and the burden-shift dissolves. The permuted reading is thus the ledger branch's cheapest escape from the pressure — cheaper than the external mechanism the diagonal ledger owes, since it costs one clause of one slot — and the paper says so plainly: the lean on W0 is conditional on the *diagonal* reading of the ledger specifically, decided where L-M's loop clause is decided, and a referee auditing the pressure should look there first. [The dependence: stated; the clause: at the L-M slot.]

The two visible dependences are, in fact, one observation, and it is promoted here from half-concession to statement: **the operative fork for the carrier question is not ledger-versus-bath as labelled, but identity-preserving versus identity-dissolving transport** — diagonal action against everything else. A permuting ledger and a monomial bath are the same object approached from the dichotomy's two sides (§5's precision; §6's boundary case): both preserve the member-line *set* while dissolving line *identity*, both therefore deliver class-grain or orbit-grain carriers, and both sit on the fraction-available side of the asymmetry. The lean of observed e⁄3 is, stated at its sharpest grain, a lean toward **identity-dissolving transport** — and the bath is the corpus's name for the ontology that dissolves identity natively, by B-IRR, rather than as a boundary case of its rival. The relabelled fork sharpens the result rather than weakening it, and it locates the L-M diagonal clause exactly: that clause is the membrane between the fork's two true sides — and it is *through* that membrane that the pressure reaches W0 at all: observed thirds bear directly on the relabelled divide, and on W0 via the bridge that the ledger, as the Bath Criterion draws it, is the diagonal side. The chain is explicit rather than reconstructible — thirds → identity-dissolving transport → W0, the last step crossing at the diagonal clause — which is why the audit instruction above points there. [The relabelling: stated at the level of the two branch propositions; the dichotomy's labels at the Bath Criterion: untouched.]

11. Position in the Programme

Inherited exposure. R-C, R-S, L-G, and the lattice at the charge paper; AD at the charge paper's composition law (§4's consumption), with its member-grain application remaining at CO-1's slot; L-M (with its loop-diagonal clause) and B-IRR at the Bath Criterion; SUB at the Fold papers' carrier inventory; the standing/realized boundary at the Ownership paper's §10 slot, consumed once and not duplicated; the census and O1 at their statuses; W0 [Open] at the Born-arc slot. No new physical imports beyond the named readings; no new exposure; one definition, one proposition, two branch propositions, one theorem, one conditional discharge.

Assignment-neutrality. The inputs are transport-structural and definitional throughout — persistence presuppositions, decomposition invariance, composition of windings — and none consumes which path carries which holonomy. The independence chain from Transport-Completeness extends unbroken through the carrier analysis. [Proven, given the stated grain.]

The diagram, extended:

```
Charge paper [lattice q ∈ ℤ; record construction R-C; standing-status R-S;
        grain-universality L-G; composition law → AD]
Bath Criterion [ledger: L-M | bath: B-IRR; W0 at the Born-arc slot, Open]
Ownership paper [standing/realized; Class Ownership; Uniform Reading Lemma;
CO-0 named]
        |
```

```
        ├── transport-stable subject: identity-through-transport as
        │        standing structure                                  ← this
paper, §3
        │        [boundary: consumed at the Ownership §10 slot]
        │
        ├── Minimality Proposition: coarser integers compose by AD;
        │        the underived lattice bites at minimal grain          ← this
paper, §4
        │
        ├── CARRIER THEOREM                                            ← this
paper, §7
        │        (ii) bath [B-IRR] ⟹ class carrier — the programme's case:
        │             lattice class-wise; member grain below
        │             quantization grain; uniform reading q⁄k the
        │             only standing member-grain content
        │        (iii) ledger [L-M, SUB] ⟹ member carriers — diagnostic:
        │             fundamental windings integral member-wise;
        │             no structural route to fractions
        │        k = 1 check: charge paper recovered verbatim ✓
        │
        ├── CO-0: conditionally discharged — closes at the
        │        R-C / R-S / L-G / AD / L-M / B-IRR slots, applied
        │        to U(1) under C-ORD [structure-group imports —
        │        §12's bypass condition]; selection = W0's
        │
        ├── pressure on W0: observed e⁄3 leans bath, conditional on
        │        the fork (C-ORD included — §12's bypass) + O1 + CO-1
        │        — the node's fourth arc, first external data point
        │
        └── CO-1 [Conjectural, downstream]: existence and modality
                 of the uniform reading; value closed at q⁄k;
                 housing-family saturation its second deliverable
```

The division of labor, extended by one line. Geometry proposes; transport disposes; ownership allocates; **the carrier locates** — the census supplies the multiplicities, the bath supplies the mechanism, the principle governs how what they build is held, and the theorem fixes the subject on which the holding is quantized. The four statements consume disjoint imports and meet only at the assembly, exactly as the predecessors' division provided.

12. What Would Refute or Decide This

Against R-C (§2, §3). Show the charge paper's response construction defines a winding without presupposing a loop-persistent subject. Effect: the Carrier Principle loses its ground entirely; the carrier question reverts to [Open] with no transport-theoretic answer, and every result here lapses with it. The paper is exactly as strong as the persistence presupposition, and says so.

Against R-S (§2, §3, §5). Show the charge paper's windings admissible as realized, history-relative content — a per-history response the lattice binds. Effect: the per-loop skeptic of §3 and the history-relative skeptic of §5 both revive; the bath branch weakens from "no member-grain fundamental winding" to "no standing member-grain winding," with the realized analogue's quantization status [Open] at the charge paper; the route narrows but does not close, since the standing thirds it seeks remain class content either way.

Against L-G (§4). Show the quantization theorem bound to one privileged construction grain rather than to every stable subject. Effect: the Minimality Proposition's "derived sums of integers" weakens to derived sums of responses; minimal attachment survives, the branch carriers survive, and the integrality clause re-prices at the slot — a weakening, not a collapse, as §2's fallback states.

Against the Minimality Proposition (§4). The strikeable joint above the lattice is AD: exhibit stable sub-subjects whose windings do not compose to the coarse subject's winding. Effect: coarse integers cease to be derived, underived quantization content can sit at multiple grains simultaneously, and the fork's exclusivity fails — both branches would weaken from "the lattice bites here" to "the lattice bites here among other places." The strike lands at AD's slot, where CO-1's reading confirms it.

Against the bath branch (§5) — the route's load-bearing wall. Exhibit a transport-invariant member decomposition under the bath: a partition of the class carrier into member lines preserved as standing structure by all admissible transport, B-IRR failing at source. Effect: member lines regain standing identity, the ledger branch's conclusions return inside the bath, the class-carrier clause falls — and **the thirds route closes at CO-0**, before CO-1 opens. This is the fork's total strike, it is decided at the Bath Criterion's transport-structure slot, and the paper prices it at full cost rather than softening it.

Against the ledger branch (§6). Two joints. *L-M's diagonal clause:* exhibit admissible closed ledger loops that permute member lines — effect: the minimal stable subject coarsens to the permutation orbit, an orbit carrier inside the ledger, re-running the analysis with orbits for lines; a refinement at the slot, not a refutation. *SUB:* exhibit an admissible transport-stable subject below member grain — effect: the fundamental lattice relocates one grain down and member integers become derived; again refinement, priced in §6's fallback.

The member-wise-prior alternative, constructed — the principal refuter, analyzed. CO-0's specification named this refuter; naming is no longer enough, because it is the strongest rival the route faces, and a route is only as credible as its best rival is explicit. So the alternative world is built in full rather than gestured at. Suppose the source construction orders the structures thus: the U(1) response is defined **first**, on the pre-mixing carrier — each member line a carrier by construction, the derived phase coupling to member lines before the class group enters the admissible transport — and the non-abelian mixing is introduced **afterward**, acting on members whose windings are already fixed:

```
abelian layer first: member lines as U(1) carriers
        ↓
integer member windings, fixed by the lattice at member grain
        ↓
class group enters: non-abelian mixing among already-quantized members
        ↓
class structure as bookkeeping over fixed member integers
```

In this world the carrier question is settled by construction order before transport-stability is ever consulted: the U(1) response's subject is the member line *at the layer where the response is defined*, and at that layer the member line is standing by fiat — the mixing that would dissolve it has not yet entered the transport group. B-IRR is not refuted; it is **bypassed**: irreducibility governs the completed transport, but the winding was attached upstream of completion, and §5's peg-failure argument never gets to run, because the peg was driven before the ground became unstable.

The consequences, priced exactly. The thirds route closes at CO-0 — member windings integral by the lattice, q/k readings nonexistent because the class never carries the underived integer — and it closes **on both ontologies**: even a bath world, under this ordering, quantizes its U(1) member-wise, so the §10 pressure on W0 dies with the route, and observed thirds revert to unexplained inputs everywhere. This is what makes the alternative the *principal* refuter rather than one among several: it is the unique rival that kills the route without striking any theorem in this paper — the Minimality Proposition, both branch propositions, and the Carrier Theorem all survive intact, conditionalized on an ordering the source did not choose. A refuter that leaves the mathematics standing and removes only its application is the hardest kind to argue against from inside, and the paper does not try: it locates the decision instead.

The divergence point, located exactly. The two worlds differ in one structural fact: **whether the U(1) response exists as structure on the pre-mixing carrier, or is defined only on the transport-completed carrier on which the class action already acts.** That is a fact about how the abelian and non-abelian structures compose in the source construction — the order in which the derived phase and the class group enter the admissible transport — and its address is the structure-group paper's imports, where that composition is built. The fact is carried in this paper as the named import **C-ORD** (§2), so that the discharge accounting (§9) and the pressure chain (§10) consume it explicitly rather than by reference to this subsection. Nothing in this paper, the Ownership paper, or the Bath Criterion decides it; the slot is genuinely open, and the route's exposure there is total and stated as such.

And the alternative's one apparent debt is priced honestly rather than inflated, because the cheap answer exists and the paper supplies it before a skeptic does. The question: how do the member-wise windings remain *readable* once mixing is introduced, given that the completed transport no longer preserves the lines that carried them? The cheap answer: take the commuting case — the abelian phase central with respect to the mixing. The case is marked for what it is: the alternative's *cheapest construction*, supplied here deliberately as the rival's best configuration because it carries the lightest readability debt — the non-commuting version pays more, as below — so the repricing runs against the alternative at its strongest, not at a configuration chosen for it. Under commutation, the member windings are constant on transport orbits, collapse to a single class-uniform value, and are trivially readable as a class scalar. The readability obligation is therefore light, not unpaid — *and the repricing serves the route better than the debt did.* Under commutation, the member-wise-prior world's entire readable content is one uniform integer per member: it reproduces the diagonal ledger's phenomenology exactly, and with it the diagonal ledger's debt — an external mechanism owed for every observed third, since uniform member integers contain no fractions. **The alternative does not escape §10's burden; it relocates it.** The only version of the alternative with different phenomenology — distinct member windings surviving the mixing — is the only version on which the readability obligation genuinely bites, and that version owes the non-commuting structure and the persistence account at once. Construction order, if the source chose it, would still win the carrier question — but it would win into the diagonal ledger's empirical position, not out of the route's problem. [The alternative: constructed; its decider: C-ORD at the structure-group imports; the route's exposure there: total for the carrier claim; the burden of thirds: relocated, not escaped — the commutation repricing.]

Against the reconciliation (§5). Exhibit a corpus result that consumes standing member-identity-through-transport under the bath — a proof, anywhere in the arcs, requiring "the same member line" as an invariant fact across an admissible bath loop. Effect: either the result's transport was ledger-typed (reclassification at the Bath Criterion's dichotomy) or the decomposition-invariance argument is refuted in place. Standing: the audit of §5 retired the allocation-flow argument precisely because the corpus's member-indexed proofs are configuration-level; the refuter's burden is a *standing* consumption, and none is known.

Deciders. R-C, R-S, and L-G at the charge paper; AD at the composition law (§4's consumption) and, separately, at CO-1's member-grain reading; L-M (both clauses) and B-IRR at the Bath Criterion; SUB at the Fold inventory; the composition order [C-ORD] — the member-wise-prior alternative's decider — at the structure-group imports; the branch selection at W0; the pressure's remaining conditions at O1 and CO-1, with its diagonal-clause dependence at L-M.

Empirically. The theorem adds one lever and prices it (§10): observed permanent member-grain fractions lean against the ledger branch, conditional on the fork, O1, and CO-1 — pressure, not proof, at exactly that depth. A confirmed off-lattice fundamental winding — non-integral *at the carrier* — refutes the lattice at the charge paper, unchanged by anything here. No other direct empirical face exists at this paper's grain.

13. What the Paper Establishes

Established (conditionally, with conditions named):

- The transport-stable subject, defined in the Ownership paper's vocabulary: subject-identity-through-transport as standing structure, with stability distinguished from rigidity (contents may churn) and from persistence-as-configuration (survival in fact is not survival by invariance), and the all-history reading chosen explicitly over the per-loop reading — a standing classification cannot be indexed to a realized history [R-S], so per-loop subjects carry at most realized content the lattice does not bind — the boundary consumed at the Ownership §10 slot, no new exposure created (§3). [Definition; the choice: stated; the boundary: at its inherited slot.]
- The Carrier Principle with minimality **proven by composition**: stable sub-subjects' windings compose by AD and are integral by L-G — the grain-universality of the lattice named as the import this step consumes rather than assumed as the charge paper's generality — with exhaustiveness named as a condition of the composition (a partial stable decomposition supports no composition identity; both branch applications unaffected, the ledger's decomposition exhaustive by L-M and the bath admitting none)

— so coarser integers are derived, no finer subject persists, and the underived lattice bites at the subjects admitting no exhaustive stable decomposition — exactly the minimal grain on each branch, by an alignment fact verified there (L-M: every stable subject a union of member lines; B-IRR: no stable sub-subjects at all) rather than assumed in general — dissolving the both-branches-stable objection rather than evading it (§4). [The Minimality Proposition, general clause: Proven, conditional on R-C, L-G, AD, and the lattice; the exactly-minimal sharpening: verified per branch.]

- The Bath Carrier Proposition, on the corrected argument: **decomposition invariance, not allocation flow** — the allocation-flow argument retired explicitly against the corpus's own basis states and realized horn; member-identity-through-transport shown realized rather than standing under B-IRR's irreducible action; the attachment peg failing before any assignment, with the peg's own ground examined rather than assumed (windings standing by R-S) and the history-relative skeptic disposed at the import — a realized per-history assignment admissible but not a winding, the lattice binding standing classifications only; the class as the minimal stable subject and the lattice class-wise (§5). [Proven, conditional on B-IRR and R-S; the fallback total for the route and priced as such.]
- The Ledger Carrier Proposition, retained as the diagnostic comparison and refuter condition rather than as a symmetric competitor (§6's demotion, stated before its content): member lines stable by L-M's diagonal clause, minimal by the SUB fence, the lattice member-wise, every fundamental winding integral — with the consequence at full strength (no structural ledger route to permanent member-grain fractions; any ledger account of thirds owes an external mechanism) and the loop-permutation boundary case named at the slot as a refinement with orbit carriers, not buried (§6). [Proven, conditional on L-M and SUB.]
- The reconciliation: bath members persist as realized structure (loci of allocation, basis directions) with everything the corpus's proofs do with them licensed at configuration level; what they lack is standing transport-identity, which is precisely and only what a fundamental winding requires; the uniform reading q/k — quantifying over seats while relabeling-invariant, "indexed to no line" licensed by the Uniform Reading Lemma's permutation-forcing — as exactly the member-grain content the bath licenses; and the registration question fenced (how a realized member registers a standing class winding — CO-1's first deliverable, with R-C's compatibility read at its slot) (§5). [Proven at the level of the papers' statements; the registration: at CO-1 and the R-C slot.]
- The Carrier Theorem in three clauses — minimal attachment; the ledger branch; the bath branch — with the $k = 1$ consistency check run and passed: branches coincide on singletons, the charge paper recovered verbatim, the electron's $-e$ as the one-member case (§7). [Proven, conditional as stated; the branch selection expressly excluded from the theorem's content.]
- The lattice preserved on both branches: the integer theorem consumed intact at the carrier on either ontology, refutation conditions unchanged — off-lattice fundamental windings still kill, member-grain readings of class integers still do not — the carrier changed, the lattice untouched (§8). [Proven at the level of the papers' statements.]
- CO-0 accounted: the requested derivation supplied, the joints honored (R-C first import; $k = 1$ run; the member-wise-prior refuter not merely carried but constructed and analyzed), and the status moved from [Conjectural] to **conditionally discharged** —

closing outright at the R-C, R-S, L-G, AD, L-M, and B-IRR slots with the application to U(1) under C-ORD — the §12 bypass condition carried in the discharge clause itself, not left to the refuters — and with the branch selection assigned to W0 where it always lived, and the route's order fixed: CO-0 → CO-1 → thirds (§9). [The accounting: stated; no marker inflated.]

- The pressure on W0, priced three deep and with its sharpest dependence visible: the fork makes permanent member-grain fractions structurally unavailable to the diagonal ledger and available to the bath, so observed $e/3$ leans — conditional on the fork's slots, the O1 identification, and a positive CO-1 — with the diagonal-clause dependence priced openly: a permuting ledger generates orbit carriers and member-grain fractional readings, collapsing the asymmetry, and is the ledger's cheapest escape from the burden-shift, decided at the L-M slot — the bypass pointed at from inside the pricing (under C-ORD's failure the pressure dies wholesale on both ontologies rather than degrading, §10, §12) — and the fork's true grain promoted from half-concession to statement: identity-preserving versus identity-dissolving transport, the permuting ledger and the monomial bath the same object from the dichotomy's two sides, the lean at its sharpest a lean toward identity dissolution, with the bath as the ontology that dissolves natively and W0 receiving the pressure through the diagonal-clause membrane — thirds → identity-dissolving transport → W0 (§5, §10) — the node's fourth converging arc and first external data point, burden-shifting rather than verdict-rendering (§10). [Conditional, as priced; the diagonal dependence and the bypass: stated.]
- The member-wise-prior alternative, constructed rather than named: the rival world built in full (abelian layer first, member lines as U(1) carriers by construction order, mixing introduced over fixed integers), its mechanism identified (B-IRR bypassed, not refuted — the peg driven before the ground destabilizes), its consequences priced exactly (the route closes at CO-0 on *both* ontologies, the W0 pressure dies, observed thirds revert to inputs everywhere — the unique rival that kills the application while leaving every theorem standing), its divergence point located and promoted to the named import C-ORD (whether the U(1) response exists on the pre-mixing carrier — the composition order at the structure-group imports, consumed by §§7, 9, and 10 explicitly), and its apparent readability debt repriced honestly via commutation — the commuting case taken explicitly as the rival's cheapest construction, under which the member windings collapse to a class-uniform scalar, the alternative reproduces the diagonal ledger's phenomenology and inherits its external-mechanism debt for thirds — the §10 burden relocated rather than escaped, with the repricing run against the alternative at its strongest (§12). [The alternative: constructed; the decider: at the structure-group imports; the route's exposure there: total.]
- The repositioning, kept disciplined: the paper written from the programme's standing — the bath weighted by the corpus's convergent arcs, the ledger retained at full derivation as the diagnostic comparison and refuter condition (§1, §6) — with the marker discipline stated and held: W0 remains [Open], no upstream conditionality inflates, the §10 pressure runs on the conditional fork rather than on the standing so that the posture never feeds its own evidence, and every theorem holds at a strength indifferent to the verdict (§1, §10). [Posture: stated; no marker moves.]

- The division of labor extended — geometry proposes, transport disposes, ownership allocates, **the carrier locates** — and assignment-neutrality preserved (§11). [Proven, given the stated grain.]

Not established (open, out of scope, or pending):

- the branch selection — ledger or bath — W0, [Open] at the Born-arc slot, its exposure changed but its status untouched, and unmoved by the paper's standing, which is posture rather than derivation;
- the slot readings that close CO-0 outright — R-C's persistence presupposition, R-S's standing-status of windings, L-G's grain-universality, AD at the composition law, L-M's diagonal clause, B-IRR, and C-ORD's composition order — each at its named source;
- SUB — the sub-member inventory fence — at the Fold papers' carrier structure;
- CO-1 in its entirety — existence and modality of the uniform reading, the realization dynamics, the registration question, housing-family saturation — at its paper, untouched here;
- O1 — the identification of the quark species with the k = 3 class — at the census's PFD slot, consumed by §10's pricing only as the open condition it is;
- C-ORD's verdict — whether the U(1) response exists on the pre-mixing carrier — the member-wise-prior alternative's decider, at the structure-group imports, where the route's exposure is total;
- hypercharge, the mixing, breaking, and the arc's fenced interior — exactly as the predecessors fenced them.

The honest summary: the paper takes the question the route to thirds was standing on — *whose integers does the lattice force?* — and answers it with a theorem instead of a habit: quantization attaches at the minimal transport-stable subject, because coarser integers compose and finer subjects do not exist; the ledger's transport preserves member lines, so a ledger world quantizes members and owes an external mechanism for every fraction it contains; the bath's transport preserves no member decomposition as standing structure, so a bath world quantizes classes, and a class integer read at the grain of one of three members is a third with no whole number broken. The fork is proven at its slots, the discharge is accounted without inflation, the lattice survives untouched on both branches — and for the first time, the oldest open question in the programme's foundations faces a number the world has already measured.

14. Conclusion

Every quantized quantity in physics is an answer to a comparison: take the system around a closed journey, and ask what came back. The question this paper asks is the one hiding inside that procedure — *what* came back? — and its answer is that the procedure itself decides. A comparison across a loop is contentful only for a subject the loop preserves, and so the carrier of quantization was never a free choice among objects: it is the smallest structure the transport keeps, because everything larger inherits its integers by addition and everything smaller does not survive to be asked.

The earlier papers built, argument by argument, the case that the bath is the right picture of transport: conserved weight pools rather than partitions; permanent structure belongs to families; the reconstructions of the weight rule converge. The present paper shows what follows when that conclusion is taken seriously. If transport preserves only the class as standing structure, then quantization attaches to the class — the carrier changes, the integer lattice does not, and the route to fractional member-grain structure opens, not as an exotic possibility but as the natural reading of a whole number held in common. The ledger stays on the books at full derivation, as the comparison that measures the yield and the condition that would withdraw it: a ledger world is integers all the way down, its observed thirds an unpaid external debt, and the contrast is exactly what the bath supplies. The marker at the fork does not move on lean — the books stay honest, and the world's thirds are priced as pressure, not proof, at their stated depth. But the theorem's importance is not that it introduces a new fork. It is that it identifies one of the deepest consequences of a fork the programme has spent years resolving — and finds, waiting at that consequence, a number the world has already measured.

The route to the strangest charges in physics is now structurally complete in its prerequisites and honest about its remainder. The census counted the copies: one, two, three, never four. The ownership principle assigned the property rights: classes own, members carry, uniform carriage the title read at member grain. The carrier theorem locates the quantization: on the survivor, which under the bath is the family. What remains is one obligation — whether the family's integer is in fact read at the seats, forced, admissible, or excluded — and one verdict, at the node where the programme's deepest question has waited through every arc. If both fall the route's way, then $e/3$ was never a broken whole number. It was a whole number held in common, seen from one seat of three — and the fraction, all along, was the signature of the pool.