The Closure-Charge Register and the Fold Floor

Why the Gate-3 Charge Is the $\mathbb{Z}_2$ Class — a Synthesis Resting on Fold Uniqueness and Observational Saturation

Keith Taylor

VERSF Theoretical Physics Programme

Summary for the General Reader

This paper closes the Gate-3 closure arc by answering a question the arc had reduced to a single point but deliberately left open: when the closure charge survives, *what kind of charge is it?* Earlier work in the arc carried the charge as a seven-valued object — $\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_7)$, with the seven coming from the K = 7 constraint count used as a comparison alphabet. The question this paper settles is whether a *measurable* closure charge can genuinely be seven-valued, or whether the substrate's own foundations force it to be the two-valued ($\mathbb{Z}_2$) class instead.

The answer is that it must be the two-valued class, and the reason is not a new assumption but two results already proven elsewhere in the programme. The first is the uniqueness theorem for the fold: the minimal unit of distinguishability is a two-state object whose intrinsic symmetry is $\mathbb{Z}_2$, and every distinction one can actually *measure* factors through such two-state comparisons. The second is the saturation theorem: once a theory contains the fold, the entire algebra of observable quantities is the one the fold generates, with nothing left over. A seven-valued *measurable* register is not in that algebra and cannot be added without contradicting the theorem that defines it.

The route to this conclusion is a reduction the arc had already performed. The closure charge, by a telescoping identity, lives entirely in one place: the comparison labels carried on the edges of the substrate complex. The question "is the charge seven-valued?" is therefore the question "do the substrate's edge labels carry a measurable seven-valued register?" And on that question the foundational papers are not silent — they are decisive. The fold's automorphism is $\mathbb{Z}_2$, forced by minimality and provably the *only* option; the observable algebra is the four-by-four complex matrix algebra over the fold's state space, which carries no seven-valued grading; and the finite structure that *does* appear (the $\mathbb{Z}_2$ of orientation, the $\mathbb{Z}_3$ of the colour split) leaves no structural slot for a seven-valued observable.

So the seven survives — but only as a *count*. It counts admissibility constraints (K = 7), loop channels (N_loop = 14), gauge generators (8 + 3 + 1 = 12). It never becomes a seven-position dial that an experiment could read. The measurable Gate-3 charge is the $\mathbb{Z}_2$ class, $\kappa \in H^1(\Gamma\_vac;$

$\mathbb{Z}_2$), and it is occupied by the same trapping argument that makes facts necessary in the first place.

One seam is left honestly open, and the paper marks it rather than papering over it. The uniqueness theorem is explicitly a theorem about *measurement* — about operational distinctions — and it brackets off the possibility of a richer "pre-observable" substrate. A reader who wishes to keep a $\mathbb{Z}_7$ charge alive has exactly one move: locate it in that pre-observable layer, where the uniqueness and saturation theorems do not reach. The cost of that move is that the charge stops being the observable closure holonomy the Gate-3 sector was defined to carry. The paper makes this fork explicit and assigns the burden accordingly, rather than declaring it closed.

Note on This Paper's Place in the Arc

This paper is the synthesis capstone of the Gate-3 closure arc. It does not introduce new substrate structure. It takes the reduction performed in *Uniform Readout and Global Integrability* — which terminated the occupancy question at a single containment, $A \subseteq B^1$, and located the surviving freedom entirely in the edge offsets ρ — together with the foundational results of the One-Fold strand (*The Uniqueness of the Minimal Distinction*; *The Fold Saturates Observable Physics*; *One Fold*; and the three structural-conditions audits), and shows that those foundational results decide the *register* of the charge even where they leave its *occupancy* open. The occupancy question (is $A \subseteq B^1$, given T1 availability?) remains as posed in the companion. The register question (if a charge survives, is it $\mathbb{Z}_7$ or $\mathbb{Z}_2$?) is what this paper answers.

The two questions are independent. A reader who accepts the companion's Branch-B reading (a surviving charge) and a reader who accepts Branch-A (an empty sector) are both served: this paper says only that *whatever survives in the observable sector is the $\mathbb{Z}_2$ class, not the $\mathbb{Z}_7$ class*. The $K = 7$ coefficient choice carried through the earlier arc is reinterpreted here, not as an error of computation, but as a coefficient-alphabet convention that the substrate's foundations do not support for an *observable* charge.

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Abstract

The Gate-3 occupancy question was reduced, in *Uniform Readout and Global Integrability*, to the single containment $A \subseteq B^1$ on the vacuum transport complex, with the surviving freedom — should it exist — carried entirely by the admissible offset cochains $\rho \in C^1(\Gamma\_vac; \mathbb{Z}_7)$. The earlier arc carried the coefficient group as $\mathbb{Z}_7$, the $K = 7$ constraint count repurposed as comparison alphabet. This paper addresses a question orthogonal to occupancy: the *register* of the charge. Granting a surviving charge (the T1 branch, Branch-B reading), is that charge genuinely $\mathbb{Z}_7$-valued, or is it the $\mathbb{Z}_2$ class?

The result is that any *measurable* closure charge is the $\mathbb{Z}_2$ class. The argument rests on two foundational results of the programme rather than on a new posit. (i) By the telescoping identity, the closure charge reduces to the edge-offset component τ; the register question is therefore exactly the question of what a *measurable* offset register can be. (ii) The Fold Uniqueness Theorem establishes that the minimal distinction has intrinsic automorphism $\mathbb{Z}_2$ — provably the unique reversible structure on one bit — and that every operationally accessible distinction factors through binary folds; a measurable seven-valued register is therefore not an admissible operational primitive. (iii) The Saturation Theorem establishes that the observable algebra of any admissible theory is the fold algebra, generated by $M_4(\mathbb{C})$ over the fold state space $\mathbb{C}^4$, which carries no $\mathbb{Z}_7$ grading; a measurable $\mathbb{Z}_7$ holonomy is excluded by the theorem's own falsifier (F-S1). Assembling these: the observable Gate-3 charge is $\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_2)$, occupied — when occupied — by the trapping/closure-completeness argument that makes facts necessary, and unifying with the $\mathbb{Z}_2$ registers of fact-trapping, fold orientation, and spin closure phase. The "seven" of $K = 7$ is a count of binary admissibility constraints throughout the corpus — of constraints, of loop channels, of gauge generators — and never a seven-valued observable register.

One seam is left explicitly open. The Uniqueness Theorem is a theorem about *operational* distinctions and brackets a possible richer pre-observable substrate. A $\mathbb{Z}_7$ charge can be preserved only by locating it in that pre-observable layer — at the cost that it ceases to be the observable closure holonomy the Gate-3 sector was defined to carry. The paper assigns the burden to that move rather than declaring the seam closed, and supplies the operational test that would decide the register directly.

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1. The Register Question

The Gate-3 arc has, throughout, pursued *occupancy*: whether the closure sector is empty ($\kappa = 0$, the containment $A \subseteq B^1$ holds) or can be occupied ($\kappa \neq 0$, some admissible offset closed but not exact). *Uniform Readout and Global Integrability* terminated that pursuit at a single residual input — whether Uniform Readout forces a global section — and stated both the Branch-A reading (empty sector) and the Branch-B reading (surviving charge) without prejudice.

This paper sets occupancy aside and asks a question the arc carried as a fixed presupposition rather than a result: the **register** of the charge. The class has been written $\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_7)$, with $\mathbb{Z}_7$ entering as the coefficient group of the offset cochains — the K = 7 constraint count repurposed as the comparison alphabet, as the companion is careful to state. The presupposition is that the coefficient group is $\mathbb{Z}_7$. The question here is whether that presupposition survives contact with the foundational theorems of the programme — specifically, whether a charge that is *measurable* (an observable closure holonomy, as the Gate-3 sector requires) can carry a seven-valued register at all.

Two clarifications fix the scope.

First, **register and occupancy are independent.** Nothing here bears on whether $A \subseteq B^1$. The claim is conditional in the same way the companion's §4–§7 are conditional on T1: *if* a charge survives in the observable sector, *then* its register is $\mathbb{Z}_2$. A reader who holds the sector empty loses nothing and gains nothing from this paper; a reader who holds it occupied learns what the occupant is.

Second, **the question is about the *observable* charge.** The Gate-3 sector was defined — and the spin-½ analogy that motivates it works — precisely because the closure holonomy is physically real and measurable: it is the committed phase recorded by an interface region after transport around a closed pathway (companion, §1, third point; and the closure-connection paper's operational reading of holonomy). The register question is therefore posed in the operational sector, the sector the foundational theorems below govern. This is the hinge on which the whole result, and its one open seam, turns.

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2. The Telescoping Reduction

The register question becomes tractable because the charge has already been localized. Any admissible edge label decomposes as

$\rho_{uv} = \chi(v) - \chi(u) + \tau_{uv}$,

where $\chi : V \to \mathbb{Z}_7$ is a local closure index (a vertex 0-cochain) and τ is a fold/interface transition cochain on the edges. The χ-part is a coboundary, $\chi(v) - \chi(u) = (d^0\chi)_{uv}$, and therefore **telescopes** around any closed loop: its loop sum vanishes identically. Hence on any cycle γ,

$\Sigma\_\gamma\ \rho \equiv \Sigma\_\gamma\ \tau \pmod 7$.

The entire holonomy content of ρ — the only thing that can represent a nonzero class κ, by the survival criterion of the companion's §2 — lives in τ. The χ-part contributes nothing to any loop sum and is pure coboundary: if $\tau \equiv 0$ then $\rho = d^0\chi \in B^1$ and the sector is empty regardless of availability.

This is the reduction the arc had reached and accepted. Its consequence for the present question is immediate and exact:

The register of the closure charge is the register of τ. Asking whether κ can be $\mathbb{Z}_7$-valued is asking whether the fold/interface transition τ carries a *measurable* seven-valued register on the edges of Γ_vac.

The remainder of the paper answers that question by asking what the corpus says τ — a fold/interface object — can measurably be. The offset rule that would assign a specific $\mathbb{Z}_7$ value to an oriented adjacency is not a derived rule anywhere in the corpus; this was confirmed in the arc and is not invented here. What *is* in the corpus, and decisively, is a characterization of the fold's measurable structure. That characterization is what §§4–5 bring to bear.

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3. The Register Tally

Before invoking the theorems, it is worth recording what finite structure the corpus actually realizes when it writes down the fold and its dynamics explicitly. This is not yet an argument — it is the empirical pattern the theorems of §§4–5 will then explain as forced rather than accidental.

Across the foundational papers, the finite registers that appear are these:

- $\mathbb{Z}_2$ **— the load-bearing register.** The fold's intrinsic automorphism is $\mathbb{Z}_2$ (the swap $s_0 \leftrightarrow s_1$ of the minimal distinction; *Uniqueness*, Cor. 1). The reversible directionality label $d \in \{\pm1\}$ is $\mathbb{Z}_2$, and is *proven* the unique group structure on one-bit reversible dynamics (*One Fold*, Theorem D2: "S_2 has 2 elements; the only 2-element group is $\mathbb{Z}_2$"). The fold-interface orientation $\sigma \in \{\pm1\}$ and closure parity $\omega \in \{\pm1\}$ are each $\mathbb{Z}_2$ (*The Fold and the Record*, §4.1: $\varphi_i \in \mathbb{Z}_2 \times \mathbb{Z}_2$). The fact-trapping register — trapped/not-trapped on a cycle

— is binary (*The Topological Threshold for Fact Formation*). The Bell-sector commitment outcomes are ±1-spectrum (the synthesis paper's §9–10). Spin closure phase is $\mathbb{Z}_2$.

- **U(1) — the transport phase.** The closure transport that becomes Maxwell carries U(1) link variables and U(1) holonomy (*The Fold and the Record*, §4.2, §7); the amplitude phase of the measurement layer is U(1). This is a continuous register, not a finite cyclic one, and it is the phase of complex amplitudes — explicitly *not* a candidate home for a $\mathbb{Z}_7$ comparison register, as the companion's §1 (first point) is at pains to state.
- **$\mathbb{Z}_3$ — the colour split.** The internal decomposition $\mathbb{C}^4 = \mathbb{C}^3 \oplus \mathbb{C}^1$ is governed by $\mathbb{Z}_3$ (the centre of SU(3), via $U(3) \cong (SU(3) \times U(1))/\mathbb{Z}_3$; *The Fold and the Record*, §12.11; *One Fold*, Lemma GG2 and Appendix D.5). This is the only finite cyclic group above order two that the corpus realizes, and it is order three, from the 3 of the $3 \oplus 1$ split — itself derived as $4 - 1$.

And the status of seven, uniformly, throughout:

- **"Seven" is a count, never a register.** K = 7 is "the integer counting independent admissibility constraints" (*The Fold and the Record*, §8). N_loop = 14 = 2K is a loop-channel count. The 12 = 8 + 3 + 1 of the gauge generators is a generator count. In every appearance, seven (and its multiples and neighbours) is the cardinality of a set of binary constraints, not the order of a cyclic register that any quantity takes values in.

The pattern is unbroken across the corpus: the *registers* are $\mathbb{Z}_2$, U(1), and $\mathbb{Z}_3$; *seven* is always a count. The earlier arc's $\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_7)$ is the one place the count was promoted to a coefficient group. The next two sections show why that promotion fails for an observable charge — why the pattern is forced, not coincidental.

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4. The Fold Floor I — Uniqueness

The first foundational result is the no-alternative theorem for the fold (*The Uniqueness of the Minimal Distinction*). Under four minimal structural conditions — finite distinguishability, finite encodability, internal closure, background-independence — the unique primitive distinction is the Fold: two states, a strongly connected two-cycle, one bit, with automorphism group $\mathbb{Z}_2$. Two of its results bear directly on the register of τ.

(U1) The minimal distinction is $\mathbb{Z}_2$, and this is forced, not chosen. The fold's intrinsic symmetry is the order-two swap (Cor. 1: "the Fold thus carries an intrinsic $\mathbb{Z}_2$ symmetry that is not added but forced by minimality"). In the One-Fold development the same fact appears as Theorem D2, with the sharper statement that the reversible transformations on one bit form $S_2 \cong \mathbb{Z}_2$ and that *this is the only group of order two* — so the binary register is not a modelling choice that could have come out otherwise. The robustness audit (*Structural Conditions*, §8) records the failure mode of this result as the single word **impossible**: "reversible bit transforms $\neq S_2 \rightarrow$ Impossible (S_2 is unique); result is unconditional." This is the most secure register fact in the corpus.

(U2) Every operational distinction factors through binary folds. Lemma 2 establishes that any structure with three or more distinguishable states is non-primitive — it contains a recoverable binary partition and therefore is not the minimal unit. Lemma 2A upgrades this to the operational statement: every admissible measurement on a finite admissible state space factors as a finite sequence of binary partitions, each of which is itself a Fold; "there is no admissible measurement of less granularity than 1 bit, and every measurement factors through binary partitions." Lemma 2B fixes the floor: no non-trivial observable distinction carries less than one bit.

Apply (U1)–(U2) to τ. The transition τ is, by the telescoping reduction, the *measurable* holonomy-bearing component of the edge data — the thing an interface region records on transport around a loop. For τ to carry a $\mathbb{Z}_7$ register, there would have to be an admissible measurement whose answer space is irreducibly seven-valued: a single operational distinction with seven mutually exclusive outcomes that does *not* factor through binary partitions. Lemma 2A says no such measurement exists. A seven-valued register, were it operational, would be a non-binary primitive distinction — and Lemma 2 rules out exactly that. The seven-valued *count* of admissibility constraints is untouched (a count of seven binary constraints is seven Folds, entirely admissible); what is excluded is a seven-valued *answer space* for a single measurable comparison.

The conclusion at this floor:

A *measurable* $\mathbb{Z}_7$ register on the fold interface is not an admissible operational primitive. The fold's measurable comparison structure is $\mathbb{Z}_2$, forced by uniqueness; every operational distinction factors through it. Therefore τ, as a measurable register, cannot be seven-valued. [proven, given the four conditions of the Uniqueness Theorem and the operational reading of τ]

The scope of this conclusion is exactly the scope of the Uniqueness Theorem, which is explicit that it concerns *measurement information, not state-space ontology* — "a richer pre-observable substrate is left open." That bracket is the seam of §7. Within the operational sector — the sector the Gate-3 holonomy lives in — the exclusion is clean.

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5. The Fold Floor II — Saturation

The second foundational result is the Saturation Theorem (*The Fold Saturates Observable Physics*). It supplies, where the Uniqueness Theorem governs the *primitive*, a statement about the *entire observable algebra* — closing the one gap the uniqueness argument leaves, namely whether a seven-valued register could arise not as a primitive but as a composite or emergent observable.

(S1) The observable algebra is the fold algebra, with zero residual freedom. For any admissible theory T containing the fold, Obs(T) = Obs(fold) (Theorem 4.1). The single-event generating algebra is $M_4(\mathbb{C})$ — the full algebra of 4×4 complex matrices on the fold state space $\mathbb{C}^4$ — with observables the Hermitian elements and dynamics the U(4) action (Remark 4.4). The full-theory algebra is the tensor tower over these generators; every individual observable reduces to fold-type commitment outcomes.

(S2) The three escape routes are closed. Hidden sectors, emergent overlays, and dual descriptions are each shown (§5.1–5.3) to either produce fold-type commitment outcomes already in Obs(fold) or produce no admissible facts at all. Any genuinely new observable would require a new distinguishable commitment outcome, which by the necessity chain has fold structure and is therefore already counted. There is no third option and no residual observable freedom.

Apply (S1)–(S2) to τ. A measurable $\mathbb{Z}_7$ holonomy is an *observable*: an element of Obs(T). By saturation, Obs(T) = Obs(fold), whose generating algebra is $M_4(\mathbb{C})$ over $\mathbb{C}^4$. The relevant structural fact is that **$M_4(\mathbb{C})$ carries no $\mathbb{Z}_7$ grading**: the finite structure available in it is the $\mathbb{Z}_2$ of the (σ, ω) interface base and the directionality label, and — through the $3 \oplus 1$ decomposition forced by the unique void state — the $\mathbb{Z}_3$ of the colour split. There is no order-seven element forced by the algebra and no seven-valued grading for a seven-valued observable to be a component of. A measurable $\mathbb{Z}_7$ holonomy is therefore not an element of Obs(fold), and the saturation theorem says there are no observables outside Obs(fold).

The theorem supplies its own falsifier for exactly this situation. F-S1: "an admissible observable not in Obs(fold)" would refute Theorem 4.1. A measurable $\mathbb{Z}_7$ closure charge is precisely such an observable — accessible to a finite observer through a repeatable protocol (it is a recorded holonomy), not determinable by the fold structure (it has no home in $M_4(\mathbb{C})$), yet a stable observable of T. Its existence would not merely sit awkwardly with the corpus; it would *refute the saturation theorem by the theorem's own stated criterion*.

The conclusion at this floor:

A *measurable* $\mathbb{Z}_7$ holonomy is not in the observable algebra Obs(fold) = $M_4(\mathbb{C})$-generated, which carries no $\mathbb{Z}_7$ grading; and saturation forbids observables outside Obs(fold). A surviving $\mathbb{Z}_7$ observable charge would refute the Saturation Theorem by its falsifier F-S1. [proven, given the Saturation Theorem and the operational reading of τ]

Where the Uniqueness Theorem blocked a seven-valued *primitive*, Saturation blocks a seven-valued *composite or emergent* observable as well. Between them the two floors close the operational sector completely: no seven-valued measurable register, primitive or composite.

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6. The Register Verdict

Assembling the reduction and the two floors:

1. **(Telescoping, §2)** The closure charge reduces entirely to the edge-offset component τ; the register of κ is the register of τ. [proven, the arc's accepted identity]

2. **(Operational reading, §1)** The Gate-3 charge is, by definition, a measurable closure holonomy — an element of the observable sector. [definitional, the sector's defining property]
3. **(Fold Floor I, §4)** A measurable seven-valued register is not an admissible operational primitive: the fold's measurable structure is $\mathbb{Z}_2$ (forced, "impossible" to be otherwise), and every operational distinction factors through binary folds. [proven, given the Uniqueness Theorem]
4. **(Fold Floor II, §5)** A measurable $\mathbb{Z}_7$ holonomy is not in Obs(fold) = $M_4(\mathbb{C})$-generated, which has no $\mathbb{Z}_7$ grading; its existence would refute Saturation by F-S1. [proven, given the Saturation Theorem]

Therefore the observable Gate-3 charge cannot be the $\mathbb{Z}_7$ class. The register that *is* available — the $\mathbb{Z}_2$ of the fold automorphism, of orientation, of fact-trapping, of spin closure phase — is what the charge must be:

The observable Gate-3 closure charge is the class $\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_2)$. When occupied (the T1 branch, Branch-B reading of the companion), it is occupied by the same trapping/closure-completeness argument that makes facts necessary — a fact *is* a non-recombinable trap on a cycle (*Topological Threshold*) — and it unifies with the binary registers the corpus realizes throughout. [proven, conditional on the occupancy branch and on the operational reading; the register conclusion is independent of which occupancy branch holds]

The reinterpretation of the earlier coefficient choice is now precise. $\mathbb{Z}_7$ was the K = 7 constraint count used as a comparison alphabet — a coefficient-group convention. The convention is not *wrong* as a piece of cochain bookkeeping on a fixed complex; what the two floors establish is that it cannot be the register of an *observable* charge, because the observable sector admits no seven-valued register. The genuine observable object is the $\mathbb{Z}_2$ class. The seven remains, in its proper role, a count of binary constraints.

A note on what is *not* claimed. This does not show $H^1(\Gamma\_vac; \mathbb{Z}_7)$ is an empty or ill-defined group — it is a perfectly good cohomology group on the fixed complex. Nor does it compute occupancy. It shows that the *measurable charge the Gate-3 sector carries* is valued in $\mathbb{Z}_2$, not $\mathbb{Z}_7$ — that the physically realized holonomy, if any, is the two-valued class.

6.5 A Candidate Offset Rule and the Choice It Requires

The telescoping reduction of §2 fixes the *form* of any offset rule but leaves the *primitive interface value* unspecified. It is worth writing down the cleanest candidate rule explicitly, both because it sharpens exactly what a $\mathbb{Z}_7$ occupancy would demand and because its status — what is algebra, what is posit — can then be read off cleanly.

The candidate rule (proposed, not derived). Assign to each oriented edge

$\rho_{uv} = j(v) - j(u) + \omega_{uv} \pmod 7$, where $j(u) \in \mathbb{Z}_7$ is the local closure-index of site u, and ω_{uv} is the interface twist introduced by the fold/closure constraint on the edge ⟨u,v⟩, with $\rho_{vu} = -\rho_{uv}$. Completed closure faces require Σ_∂f ρ = 0 (mod 7), so $d^1\rho = 0$ and $\rho \in Z^1$. This is exactly the decomposition of §2 with the identification $\omega_{uv} \equiv \tau_{uv}$: the closure-index term j(v) − j(u) is the coboundary part $d^0 j$ (the χ-part of §2), and the interface twist ω is the transition part τ. The telescoping is then immediate —

Q_γ = Σ_γ ρ = Σ_γ ω (mod 7),

since the j-terms cancel around any closed loop. If ω ≡ 0 everywhere, $\rho = d^0 j \in B^1$ and the sector is empty; occupancy requires ω ≠ 0 on a surviving non-bounding cycle.

What is algebra here, and what is choice. Everything up to and including Q_γ = Σ_γ ω is sound and is just §2 restated in this notation; it adds no commitment. The entire substantive content sits in one further step: assigning the *primitive* interface twist a fixed nonzero value in $\mathbb{Z}_7$ — for instance $\omega_{uv} = +1$ for a positively oriented primitive fold crossing, $\omega_{vu} = -1$ for the reverse, so that a primitive isolated cycle carries Q_γ = ±1 (mod 7). This assignment is **not derived from the K = 7 geometry**; it is a posited rule, and is offered as such. The reduction does not supply it, and — this is the point — the corpus's foundations do not merely fail to supply it; they bear on it.

Where the posit meets the floors. The assignment $\omega_{uv} \in \mathbb{Z}_7$, read as a *measurable* residue the fold interface carries, is precisely the object §§4–5 exclude from the observable sector. To say that an interface measurably transports a seven-valued residue is to assert a seven-valued operational register on the fold — which Fold Uniqueness (§4) forbids as an operational primitive (every measurable distinction factors through binary folds) and which Saturation (§5) forbids as any observable (no $\mathbb{Z}_7$ grading in Obs(fold); F-S1 triggered). Writing the realized values as ω ∈ {+1, −1} does not evade this: those are two values *embedded in* $\mathbb{Z}_7$, and the rule's force depends on ω being genuinely $\mathbb{Z}_7$-valued, so that Q_γ = ±1 (mod 7) is a seven-position holonomy rather than a sign. The moment ω is a sign — a two-valued orientation residue — it is the $\mathbb{Z}_2$ interface parity ω ∈ {±1} of *The Fold and the Record* §4.1, which is $\mathbb{Z}_2$, not $\mathbb{Z}_7$, and Q_γ is the $\mathbb{Z}_2$ class, not a $\mathbb{Z}_7$ one.

The fork the rule makes concrete. The candidate rule is therefore the sharpest available statement of *what a* $\mathbb{Z}_7$ *occupancy requires*, and it resolves into the §7 fork with no slack:

- **$\mathbb{Z}_7$ reading of ω** — the primitive twist is an irreducibly seven-valued residue the interface carries. This is the pre-observable relocation of §7: it can be posited as substrate ontology, but it is not in any committed record (Saturation), and so it is not the observable Gate-3 holonomy. The rule then defines a pre-observable structure, not the measurable charge the sector was about.
- **$\mathbb{Z}_2$ image of ω** — the primitive twist is an orientation sign, ω ∈ {±1}, its $\mathbb{Z}_2$ image under $\mathbb{Z}_7 \to \mathbb{Z}_2$ being the only operationally admissible content. This is the observable reading; it gives Q_γ ∈ $\mathbb{Z}_2$, recovering the verdict of §6, and it coincides with the fold's derived orientation/parity register.

So the rule is welcome and clarifying, and it is recorded here in full. What it does not do — and what the §4–§5 floors prevent it from doing — is occupy the *observable* Gate-3 sector with a $\mathbb{Z}_7$

charge by fiat. It makes the price explicit instead: a $\mathbb{Z}_7$ occupancy needs the primitive interface twist to be a measurable seven-valued residue, and that residue is the one object the framework's own uniqueness and saturation theorems exclude from the observable algebra. The rule is the clean statement of the cost, not its payment.

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7. The Open Seam

Intellectual honesty requires marking the one place a $\mathbb{Z}_7$ charge could still be preserved, and stating exactly what it costs.

Both floors are explicitly *operational*. The Uniqueness Theorem states, in its own scope section, that it concerns "measurement information, not state-space ontology," and that "a richer pre-observable substrate is left open" — "whether the underlying state space is itself binary, or whether a richer substrate exists at the pre-observable level, is left open by this observation." The Saturation Theorem governs Obs(T) — the algebra of quantities "determinable by a finite observer through a repeatable protocol." Both, by construction, fall silent on a layer of the substrate that no measurement reaches.

This is the seam, and it admits exactly one move to preserve a $\mathbb{Z}_7$ charge:

The pre-observable relocation. Locate the $\mathbb{Z}_7$ register not in the observable holonomy but in the pre-observable substrate the Uniqueness Theorem brackets off. There, neither floor reaches: a seven-valued structure could in principle exist as pre-observable ontology without being an operational primitive (Uniqueness does not forbid it) and without being an element of Obs(fold) (Saturation does not govern it).

The cost of this move is exact and, I think, decisive for the Gate-3 sector specifically. The Gate-3 charge was *defined* as an observable closure holonomy — a committed, observer-independent, measurable phase recorded on transport around a cycle. That is the whole content of "occupancy," and it is what makes the spin-½ analogy work: the analogy holds only because the holonomy is physically real and measurable (neutron interferometry measures it). A $\mathbb{Z}_7$ register relocated to the pre-observable substrate is, by the relocation, *no longer the observable holonomy the sector was defined to carry*. It becomes a different object — a pre-observable structure that does not show up in any committed record — and the Gate-3 occupancy question, which is a question about a measurable charge, is not answered in its favour by positing it.

So the fork is genuine but asymmetric:

- **The $\mathbb{Z}_2$ reading** keeps the charge observable (consistent with the sector's definition) and pays no foundational cost: it rests on results the corpus marks as its most secure (§8).
- **The $\mathbb{Z}_7$ reading** can be preserved only by making the charge *un*observable (relocating it below the measurement floor), which forfeits the sector's defining property; or else by *overturning* the Uniqueness or Saturation theorems in the observable sector — both of which are load-bearing for the programme's entire uniqueness claim (Uniqueness grounds

dim $\mathscr{H}$_fold = 4 and the whole One-Fold derivation; Saturation grounds "all admissible physics is VERSF").

The burden therefore sits, explicitly, on the $\mathbb{Z}_7$ reading: it must either exhibit a pre-observable mechanism *and* accept that the resulting object is not the observable Gate-3 holonomy, or it must refute a Category I theorem. Neither is supplied by the corpus as it stands. This is a burden assignment under the operational premise, not a proof that no pre-observable $\mathbb{Z}_7$ structure exists — the premise itself (that the Gate-3 charge is the observable holonomy) is the thing the §9 test checks.

I state the seam this plainly because the arc's whole method has been to refuse to round a "burden shifted" up to a "question closed." The register verdict is as close to closed as the operational sector allows; the pre-observable layer is genuinely outside its reach, and the honest statement is that the verdict holds *for the observable charge*, which is the only charge the Gate-3 sector was about.

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8. Security Grading

The third structural-conditions audit (*Structural Sufficiency*) and its predecessor (*Structural Conditions*) tier the One-Fold results into Category I (proven from the core axioms A1–A5 plus quantum mechanics, robust against failure of any conditional axiom), Category II (conditional structural derivations), and Category III (physical identifications). It is worth recording where the register verdict's load-bearing inputs sit, because it determines how secure the verdict is.

The two facts that exclude a measurable $\mathbb{Z}_7$ register are both **Category I**:

- **Binary directionality (Theorem D2 — the $\mathbb{Z}_2$ register).** Category I. The audit's robustness table records its failure mode as *impossible* ("S_2 is unique; result is unconditional"). This is the register fact that (U1) rests on.
- **dim $\mathscr{H}$_fold = 4 (Theorem T1 — the $M_4(\mathbb{C})$ algebra).** Category I, failing "only if A5 (one-bit minimality) is wrong." This is what (S1) rests on: the observable algebra is generated on a four-dimensional fold space.

The facts the verdict does *not* need are exactly the corpus's softer results:

- The $3 \oplus 1$ split (hence $\mathbb{Z}_3$ colour) is Category II, conditional on V1.
- The gauge group SU(3) × SU(2) × U(1) is Category II.
- The α and Λ numerical identifications are Category III.

None of these is load-bearing for the register verdict. The verdict uses $\mathbb{Z}_3$ only as a *witness* in the tally (showing the one finite cyclic group above $\mathbb{Z}_2$ that the corpus realizes is order three, not seven) — not as a premise. Even if every Category II and III result failed, the binary directionality (D2) and the four-dimensionality (T1) would stand, and with them the exclusion of a measurable seven-valued register.

The consequence is worth stating sharply:

The exclusion of a measurable $\mathbb{Z}_7$ Gate-3 charge rests entirely on the Category I tier — the part of the framework the audit marks unconditional and, for D2, *impossible* to revise. The $\mathbb{Z}_2$ verdict is therefore strictly more secure than the gauge group, the fine-structure constant, or the cosmological constant, all of which are conditional or interpretive. A reader who accepts those softer results has *a fortiori* accepted the harder ones the register verdict needs.

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9. The Test, Restated for the Register

The companion supplied an operational test for occupancy (does Uniform Readout consult a path-independent ledger?). The register verdict supplies its own, sharper test, and it is the one that would decide the open seam of §7 directly.

The register question is operationally decidable because it is a question about what a committed readout can take as a value. Concretely:

Register test. Inspect the committed-record readout rule. Does any single committed comparison outcome — one edge's recorded offset, read by an admissible local protocol — admit *seven mutually exclusive values that do not factor through binary partitions*? Or does every committed comparison reduce, on analysis, to a finite sequence of binary (Fold) outcomes?

The two floors predict the second. By Lemma 2A every admissible measurement factors through binary partitions; by saturation every committed outcome is fold-type. So the prediction is sharp: **no committed comparison outcome will be found to carry an irreducible seven-valued answer space.** Finding one would do two things at once — it would carry the $\mathbb{Z}_7$ register into the observable sector (vindicating the earlier coefficient choice as more than bookkeeping) and it would refute the Saturation Theorem by F-S1. Finding none — the predicted outcome — confirms the $\mathbb{Z}_2$ reading and discharges the seam: the pre-observable relocation of §7 would then be the *only* surviving home for a $\mathbb{Z}_7$ structure, and that home, by construction, produces no committed comparison to find.

This is the register analogue of the companion's "the entire arc now rests on one checkable property of one principle." Here: the entire register question rests on one checkable property of the readout rule — whether a committed comparison can be irreducibly seven-valued — and the floors predict it cannot.

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10. Status and Conclusion

The Gate-3 closure arc reached, in *Uniform Readout and Global Integrability*, a clean terminus for *occupancy*: one well-posed property of one axiom. This paper reaches the corresponding terminus for *register*, and the register terminus is more nearly closed than the occupancy one, because it rests on theorems already proven rather than on an axiom whose quantifier is still to be fixed.

What is established.

- The closure charge reduces entirely to the edge-offset component τ (telescoping); the register of κ is the register of τ. [proven]
- A measurable $\mathbb{Z}_7$ register is excluded as an operational primitive by the Fold Uniqueness Theorem (every operational distinction factors through $\mathbb{Z}_2$ folds; the $\mathbb{Z}_2$ register is forced, "impossible" to be otherwise). [proven, given Uniqueness and the operational reading]
- A measurable $\mathbb{Z}_7$ holonomy is excluded as any observable by the Saturation Theorem (Obs(fold) = $M_4(\mathbb{C})$-generated, no $\mathbb{Z}_7$ grading; F-S1 would be triggered). [proven, given Saturation and the operational reading]
- Therefore the observable Gate-3 charge is $\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_2)$, occupied — when occupied — by the trapping argument that makes facts necessary, and unifying with the corpus's $\mathbb{Z}_2$ registers. [proven, conditional only on the operational reading; independent of the occupancy branch]
- The exclusion rests on the Category I tier (D2, T1), the framework's most secure results — strictly more secure than the gauge, α, and Λ results it does not need. [established by the audit tiering]

What remains open.

- **Occupancy itself** — whether $A \subseteq B^1$ on the T1 branch — is untouched here; it remains the companion's single-axiom question. The register verdict holds on either branch.
- **The pre-observable seam** — a $\mathbb{Z}_7$ structure relocated below the measurement floor is not reached by either theorem. The verdict is that *the observable charge* is $\mathbb{Z}_2$; a pre-observable $\mathbb{Z}_7$ ontology is neither established nor refuted, and is, by construction, not the Gate-3 holonomy. The burden of exhibiting it — and of accepting that it is a different object — sits on the $\mathbb{Z}_7$ reading.
- **The register test** (§9) is well-posed and predicts no irreducibly seven-valued committed comparison; carrying it out against the precise readout axiom would discharge the seam directly.

The arc's coefficient choice — $\mathbb{Z}_7$, the K = 7 count as comparison alphabet — is reinterpreted, not erased. Seven is, and remains, the count of binary admissibility constraints, of loop channels, of gauge generators. What it is not, for any *observable* charge, is the order of a register. The substrate's own foundations — the uniqueness of the minimal distinction and the saturation of the observable algebra — permit only $\mathbb{Z}_2$ there, and the corpus has, in those two theorems, the reason the register tally was forced all along rather than accidental.

The measurable closure charge of Gate-3 is the two-valued class. The seven is a count. The fold floor holds.

References (programme)

The results invoked here are established in companion VERSF programme works, catalogued at the programme website. The load-bearing inputs are: the telescoping reduction and the occupancy pipeline (*Uniform Readout and Global Integrability*; *Contextual Distinguishability and Coboundary Confinement*; the closure-connection paper); the Fold Uniqueness Theorem (*The Uniqueness of the Minimal Distinction*); the Saturation Theorem (*The Fold Saturates Observable Physics*); the explicit fold dynamics and the $\mathbb{Z}_2$/T1/$3\oplus 1$ derivations (*One Fold*; *The Fold and the Record*); the fact-trapping necessity of cycles (*The Topological Threshold for Fact Formation*); and the Category-tiering and risk-concentration audits (*Structural Conditions, Physical Identification, and Robustness*; *Structural Sufficiency, Risk Concentration, and Representation Selection*).