The Conditional Quark Mass-Ratio Grid

RG-Invariant Targets, the Closure-Cell Suppression Unit, and the Colour-Access Problem

VERSF Theoretical Physics Programme — Quark Mass Hierarchy Series

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Summary for the General Reader

Every kind of matter particle has a mass, and those masses span an enormous range — the heaviest quark is roughly a hundred thousand times heavier than the lightest. Physics has long had no explanation for *why* the masses fall where they do; in the standard account they are simply measured and written down. This paper is part of a programme that tries to explain the **pattern** of these masses from a small number of structural rules, rather than treating each mass as a free input.

The strategy is to work with **ratios** instead of absolute masses. Asking "how heavy is the strange quark in grams?" drags in human units that mean nothing to nature. Asking "how many times heavier is the strange quark than the down quark?" is a clean, unit-free question — and that ratio is the kind of thing a structural theory can hope to predict. There are six such ratios linking the six quarks, and they are laid out as a small grid. Here it is, with every quark's mass written as a multiple of the lightest (the down quark = 1): the theory's estimate, the measured value, how close they land, and — crucially — what each cell actually *is*:

relative to down = 1	down	up	strange	charm	bottom	top
theory's estimate	1	0.46	20	235	1,040	61,500
measured	1	0.47	20	235	1,080	64,000
how close	—	consistent	exact / import	<1%	4%	4%
what it is	anchor	prediction	import (at resolution limit)	match	prediction	by-product

The estimates land within a few percent everywhere. The bottom row is what each cell has actually earned. What the labels mean:

- **anchor** — the yardstick. We measure every quark relative to the down quark, so the down quark is 1 by choice of units, not by any claim about nature. Relabel it (set the top to 1 instead) and nothing about the theory changes. It carries no information; it is just where the counting starts.

- **prediction** — a genuinely new number the theory produces from its own structure, which the measurement then confirms. The up-to-down ratio 6/13 is now of this kind: it follows from the closure framework, not from being tuned to the data.
- **import (at resolution limit)** — the operative value is borrowed: the strange-to-down ratio ≈ 20 is taken from a separate established method, so it agrees by definition. The theory's attempt to derive it instead would need a correction of about 16% to the base estimate — but that ratio is itself only measured to about ±15%, so the correction being chased is the same size as the uncertainty. There is nothing clean to derive at this resolution, so the cell stays an import; this is its settled status, not a temporary gap.
- **match** — the theory's estimate agrees with the measurement, but it rests on a flavour-law whose form is not yet settled as derived rather than tuned. Agreement, not yet prediction.
- **prediction** — a genuinely new number the theory produces from its own structure, which the measurement then confirms. Two cells are of this kind: the up-to-down ratio 6/13 (from the closure framework) and the bottom-to-strange ratio (from the three transport channels saturating, dim C = 3). The up-to-down ratio is itself measured only roughly (to about a quarter), so the construction's value is consistent with it rather than pinned against a sharp number.
- **by-product** — not an independent test: the top is just the bottom carried up by one more ratio, so its agreement adds no new evidence.

So the grid carries **two genuine predictions** — up and bottom — a match resting on an unsettled law (charm), one ratio (strange) that stays imported because its residual correction is below the resolution of both theory and data, an anchor (down), and a by-product (top). That spread — what is predicted, what is borrowed and why, what is still a calibrated match — not the row of small percentages, is what the rest of this summary unpacks.

The construction explains each ratio as the product of two effects. The first is **localisation**: each heavier generation of particle is more tightly concentrated, and that concentration multiplies the mass up by a fixed factor of about 207 per step. This single factor, derived elsewhere in the programme from geometry, already nails the muon-to-electron mass ratio to better than two parts in a thousand — the programme's best individual result. The second effect is **suppression**: a heavier particle's structure spends most of itself just holding together, leaving only a small fraction free, and that surviving fraction works out to exactly 1/12. This 1/12 is the paper's most solid piece: it is fixed by the underlying geometry *before* any actual mass is looked at, so it cannot have been quietly tuned to fit.

The honest part of the paper is about where this picture is firm and where it is not. The strange-to-down ratio is handled by a separate, better-established method that gives ≈ 20, and the simple suppression estimate (≈ 17) is openly recorded as falling 14% short rather than being patched. The harder open question concerns the bottom quark. An earlier attempt fitted its ratio using a tidy-looking colour factor of 8/3. This paper shows that the tidiness was an illusion created by comparing two masses measured at different energy scales — an apples-to-oranges error. Measured consistently, the target shifts, the 8/3 stops being special, and what the data actually call for is close to the plain number 3 (the number of "colours" a quark can carry). A reader-supplied refinement is folded in: the factor can equally be named 3/8 (a restriction) or 8/3 (its inverse). Better still, the 3 can be *derived* from the theory's own picture rather than borrowed from particle physics: when a heavy particle saturates, a colourless one keeps a single surviving "route" (the 1/12), while a coloured one keeps a route in each of its three colour channels — three routes, giving 3/12 = 1/4, which matches the corrected bottom-quark number to about four percent without ever consulting a quark mass. This even predicts a simple rule — a sector with k channels keeps k/12 — that can be tested elsewhere.

But the same picture exposes the real unsolved problem, and the paper says so plainly: it explains the *second* step of the down-quark ladder but not the *first*. The strange-to-down ratio (≈ 20) is about ten times *smaller* than its colourless counterpart, whereas counting colour channels can only ever make things *bigger*. So the first step is suppressed in the opposite direction to everything the colour picture predicts, and it is now the single largest thing the construction cannot yet account for. The likeliest fix — that coloured particles "saturate" one generation earlier than colourless ones — is named as the next thing to derive, not asserted.

The result is not a finished theory of mass. It is a disciplined map: a clear statement of what is genuinely derived (the 1/12 unit; the localisation factor — though one piece of it, a factor of 2 in the exponent, is itself still open, so even the headline muon agreement is not parameter-free), what is conditional or conjectural, and exactly which questions must be answered next. Chief among them is a two-part provenance question about the number 7 at the framework's root: whether 7 is forced (taken here as established) is one question, but whether the *particular formula* built on 7 that predicts the up-quark ratio was written down before or after seeing the data it matches is a separate, still-open question. Throughout, every claim is tagged with how well established it is, and no result is allowed to look more proven than it is.

A word on the grid's apparent success, because it is easy to overread. Laid out, the six quark ratios all land within a few percent of their measured values — but most of that agreement is borrowed rather than earned. One ratio is taken from a separate established method (it "agrees" by being an input); two more match the data but it is not settled whether the rule behind them was derived or tuned; one is just a by-product of another. That leaves **two** ratios the construction genuinely predicts — the up quark and the bottom quark — and both come qualified. The up-quark ratio is itself only measured to about a quarter, so the construction's value is consistent with experiment rather than checked against a sharp number. The bottom-quark prediction rests on a factor of 3 that is only as solid as a chain of reasoning from the number 7 down through the geometry — a chain this paper takes as given but cannot itself check. So the honest headline is: two genuine predictions, each leaning on something outside the paper's full control — a coarse measurement on one side, an unaudited foundational chain on the other. That is a real and unusual result for an independent programme, but it is carefully not oversold.

A short cautionary episode is recorded for the same reason. Attempting to turn the ratios into an actual mass for the top quark gave ≈ 289 GeV, against a true value near 165 — alarmingly wrong, and tempting to "explain" with new physics. It turned out to be the same apples-to-oranges mistake in a new place: one of the six ratios is quoted at a different energy scale than the rest, and multiplying mismatched scales together inflates the answer by precisely the (calculable) amount a quark's mass shifts between those scales. Done consistently the top comes out right to a fraction of a percent. The paper keeps the episode on the record as a *diagnosed* error, not a mystery, so no one reopens it later as a real anomaly — and as a reminder that a startling number is more often a unit slip than a discovery.

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Abstract

The charged-quark mass hierarchy is treated as a 2 × 3 grid of dimensionless ratios rather than a set of absolute masses. Each ratio is factored into a universal localisation step and a structural suppression. The localisation step is fixed by the Role-4 localisation constant $R_0 = e^{(16/3)} \approx 207.13$, inherited unchanged from the charged-lepton sector. The suppression unit $\Theta\_cell = \lambda^*/trace(L) = 2/24 = 1/12$ is fixed by the W_7 closure spectrum before any mass is consulted and carries the strongest grade in the construction.

The decisive structural move made here is to re-found every target ratio on renormalisation-group-invariant, same-charge mass ratios evaluated under a single declared convention. This separates the grid cleanly: the down-type **baseline ladder** (s/d, b/s) consists of same-charge ratios that are RG-invariant and must meet fixed physical targets, while the **flavour-split** factors (u/d, c/s, t/b) are cross-charge and inherit a conventional scale dependence that the construction does not attempt to remove.

Two consequences follow. First, the m_b/m_s target, when evaluated as a genuine RG-invariant ratio rather than a scale-mixed quotient, is ≈ 54 rather than ≈ 45; the suppression factor it implies is ≈ 0.26, for which the previously favoured value 2/9 ≈ 0.222 is disfavoured. Second, the factor that must multiply Θ_cell to reach this suppression is ≈ 3, not 8/3. This dissolves a numerical collision between the proposed colour factor 8/3 and the localisation constant κ = 8/3, and reposes the open problem as the derivation of a plain factor 3.

That 3 is then derived internally as a participation count: under saturation a colourless closure retains one surviving anchoring route (Π = 1/12) while a coloured closure retains one route in each of three colour channels (Π = 3/12 = 1/4), with no observed mass used and with the corrected b/s target reproduced to 4%. This yields a falsifiable theorem candidate, Π = k/12 for k surviving transport channels. The same analysis exposes the first down-step m_s/m_d ≈ 20 as the dominant unexplained object: it is suppressed ≈ tenfold relative to its colourless analogue, oppositely signed to the colour enhancement, so a channel count cannot supply it; a saturation-onset rule (coloured states saturating one generation earlier) is the candidate and is gated.

The grid is presented graded cell-by-cell under the inheritance-cap principle. The down-type first step is supplied by the adjudicated primary route (Route 1, GMOR/kaon) at m_s/m_d ≈ 20, with the closure-cell prediction 17.26 retained only as a demoted cross-sector compatibility check carrying a standing 14% tension.

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1. Target Objects and Evaluation Convention

1.1 Ratios, not masses

The physical targets are dimensionless. Absolute masses in MeV or any human unit are conventional and lie outside the structural claim. The grid asserts

$m_i / m_j = \mathscr{J}_i / \mathscr{J}_j$

where $\mathscr{J}$ is the underlying VERSF structural invariant. The objects of derivation are therefore

m_μ/m_e, m_τ/m_μ (lepton ladder) m_s/m_d, m_b/m_s (down-type baseline ladder) m_u/m_d, m_c/m_s, m_t/m_b (flavour splits)

1.2 Same-charge ratios are RG-invariant; cross-charge ratios are not

This distinction is load-bearing and is enforced throughout.

The QCD anomalous dimension governing running quark mass is flavour-blind. In a ratio of two **same-charge** quark masses (both down-type, or both up-type), the running factor cancels exactly to leading order, leaving a ratio that is invariant under change of renormalisation scale. The baseline-ladder ratios m_s/m_d and m_b/m_s are of this kind. They are physical invariants and must meet fixed targets.

A ratio of two **cross-charge** masses (one up-type, one down-type) does not enjoy this cancellation through thresholds and higher order, and its numerical value depends on the scale and scheme at which it is quoted. The flavour-split ratios m_u/m_d, m_c/m_s, m_t/m_b are of this kind. Their targets are convention-dependent.

[Convention] All same-charge targets are taken as RG-invariant MS-bar ratios. All cross-charge targets are taken at the conventional scale at which the Particle Data Group quotes them (s, c at μ = 2 GeV; t, b at their respective scales). This convention is declared once and applied uniformly. A cell whose target requires a scale choice not covered by this convention is graded [Open] on that ground alone.

This convention maps exactly onto the construction's own decomposition (Section 2): the RG-invariant content lives entirely in the baseline ladder B(g); the convention-dependent content lives entirely in the flavour split χ(g). The two halves of the model are also the two halves of the renormalisation structure.

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2. The Two-Factor Decomposition

Each charged-fermion log-mass is written

$\ln m(A, g) = B(g) + A \cdot \chi(g)$

with generation index g and flavour-charge label A. The two terms are:

B(g) = down-type baseline ladder (RG-invariant content) χ(g) = ln(m_up(g) / m_down(g)) = flavour split (convention-dependent content)

Equivalently, every mass ratio is

ratio = localisation × suppression

The localisation factor is universal across sectors and carries the $e^{(16/3)}$ step. The suppression factor is sector- and step-dependent and is built from the closure-cell unit 1/12 and a structural access factor.

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3. The Localisation Constant $R_0 = e^{(16/3)}$

The Role-4 localisation law is

$$L_g = L_0 \cdot e^{(-\kappa g)}, \quad \kappa = 8/3$$

The localisation factor separating adjacent charged-lepton generations is

$$R_0 = e^{(2\kappa)} = e^{(16/3)} \approx 207.13$$

This value is structurally sourced: $\kappa = 8/3$ is fixed by CP^2 geometry independently of any mass, and supplies the strongest quantitative postdiction in the programme (Section 5).

[Open — gate G-LOC2] The factor of 2 in the exponent 2κ is carried in from the lepton construction without independent restatement here. The localisation law $L_g = L_0 e^{(-\kappa g)}$ gives an adjacent-generation localisation ratio of e^{κ}, not $e^{(2\kappa)}$. The factor $e^{(2\kappa)}$ therefore requires either mass $\propto L^2$ or a generation step $\Delta g = 2$ between adjacent flavours. Because R_0 propagates into every cell of the grid, the source of this factor is a standing dependency. It does not affect any ratio internal to the construction (R_0 cancels in step-to-step comparisons), but it gates the absolute normalisation and the structural story.

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4. The Closure-Cell Suppression Unit [Proven — structural]

The W_7 closure spectrum fixes a primitive suppression unit

$$\Theta_{cell} = \lambda^* / \mathrm{trace}(L) = 2 / 24 = 1/12 \approx 0.0833$$

with λ^* the relevant closure eigenvalue and trace(L) the closure-operator trace. This value is fixed before any particle mass is consulted, which is the condition required for it to purchase a structural claim rather than absorb a fitted number.

Interpretation. Θ_{cell} is the surviving participation fraction of a saturated closure structure:

Θ_cell = surviving channels / available channels

A completed closure must spend most of its available structure maintaining itself; only a small residual remains free for fresh irreversible commitment. The suppression is therefore not an attenuation imposed from outside but the participation budget left over after self-maintenance.

This is the model citizen of the construction. It is quarantined (fixed before masses), it is structurally sourced (read off the closure spectrum), and it is the input against which the fitted factors are disciplined. Where the construction is strong, it is strong here; where it is weak, it is borrowing this cell's credibility.

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5. The Charged-Lepton Calibration — the Clean Sector

The lepton ladder is the calibration that fixes both factors and exposes the activation asymmetry of Section 9.

First step (e → μ): localisation only.

$m\_\mu / m\_e \approx R_0 = e^{(16/3)} \approx 207.13$

Observed: 206.77. Agreement ≈ 0.17%. No suppression factor is applied. This is the programme's strongest quantitative postdiction — but it is **not** parameter-free, and the precision should not be sold as such. The fitting exponent is $2\kappa = 16/3$, not κ: $\kappa = 8/3$ alone gives $e^{(8/3)} \approx 14.4$ for e→μ, nowhere near 207. So the muon ratio fixes 2κ to ≈ 0.03% in the log, of which $\kappa = 8/3$ is claimed structural (CP^2 geometry) and the **factor of 2 is open** (gate G-LOC2). The single multiplicative factor most responsible for the 0.17% is therefore the underived one. Honest statement: the postdiction fixes 2κ tightly; one of its two factors is structural, the other is an open gate.

Second step (μ → τ): localisation × suppression.

$m\_\tau / m\_\mu \approx R_0 \times \Theta\_cell = 207.13 / 12 \approx 17.26$

Observed: 16.82. Agreement ≈ 2.6%. The third charged-lepton sector wears exactly one closure-cell unit relative to the second.

The required empirical suppression is

$\Theta\_\tau = (m\_\tau / m\_\mu) / R_0 = 16.82 / 207.13 \approx 0.0812$

against $\Theta\_cell = 0.0833$. The closure unit accounts for the tau suppression to better than 3% with no free parameter. The colourless lepton sector therefore wears the bare unit; the colour-bearing quark sectors do not (Section 8).

6. The Down-Type Baseline Ladder

The baseline ladder carries the RG-invariant content. Both of its steps are same-charge ratios and must meet fixed physical targets under the convention of Section 1.2.

6.1 First step m_s/m_d — supplied by Route 1, not by the closure route

The closure-cell construction predicts, by direct analogy to the tau:

$m_s / m_d \approx R_0 \times \Theta\_cell = 207.13 / 12 \approx 17.26$

The programme's adjudicated primary determination of this ratio is Route 1 (GMOR / kaon-ratio), which gives

$m_s / m_d \approx 20$ (RG-invariant target)

[Adjudication] The grid adopts the Route 1 value for the first down-type step. The closure-cell prediction 17.26 is **not** the grid value; it is retained as a demoted cross-sector compatibility check, on the same footing as the Routes 2 and 3 demotion elsewhere in the programme. That check currently shows

17.26 / 20 ≈ 0.863, a standing tension of ≈ 14%.

This 14% is interpreted as the signature of the colour and partial-closure corrections that distinguish a coloured quark step from the colourless tau step: the bare unit 1/12 undershoots the coloured first down-step by the same order as it will be seen to undershoot the second (Section 6.2). The tension is recorded, not absorbed. No factor is introduced to remove it, because doing so would let numerical agreement purchase a structural claim that has not been independently derived.

[Conditional — cap C-SD] The s/d cell of the grid is graded by its source, Route 1, not by the closure prediction. Its grade is whatever Route 1 carries.

6.2 Second step m_b/m_s — the scale-mixing correction

The second down-step has no competing primary route, so the construction supplies it directly. This makes the choice of target decisive, and it is here that the previous treatment was led astray by a scale-mixed quotient.

The scale-mixed target. Forming $m_b(m_b) / m_s(2\ \text{GeV})$ — the b mass at its own scale divided by the s mass at 2 GeV — gives ≈ 44–45. Dividing by R_0:

44.7 / 207.13 ≈ 0.216 ≈ 2/9 = 0.2222

The apparent cleanliness of 2/9 is the reason it was adopted. But this quotient mixes two renormalisation scales and is not a physical invariant. It is not an admissible target under the convention of Section 1.2.

The RG-invariant target. Evaluated as a genuine same-charge invariant,

$m_b / m_s = (m_b / m_c) \times (m_c / m_s) \approx 4.58 \times 11.76 \approx 53.9$

Dividing by R_0:

$53.9 / 207.13 \approx 0.260$

The required suppression for the second down-step is therefore ≈ 0.26, not ≈ 0.216. Against this corrected target:

$2/9 = 0.2222 \rightarrow m_b/m_s = 46.0$, low by ≈ 15% $1/4 = 0.2500 \rightarrow m_b/m_s = 51.8$, low by ≈ 4%

The favourable appearance of 2/9 was an artifact of the scale-mixed quotient. Under the correct invariant target it is disfavoured at the 15% level, and no simple fraction hits the target exactly. The cell is graded [Open] on the suppression factor.

[Conditional — gate G-DS1] The second down-step suppression is supplied by the surviving-channel count of §8.6 at $\Theta_b = 3 \times (1/12) = 1/4$, giving $m_b/m_s \approx 51.78$ against the RG-invariant target ≈ 54 (4% tension); 2/9 is disfavoured. The factor 1/4 is derived, not refitted. What remains open is not this cell's value but the absolute placement of the heavy down sector, which is coupled through the saturation-onset rule to the first down-step (gate G-DS1, §8.7).

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7. The Flavour-Split Factors

The flavour splits are cross-charge and convention-dependent. They model the offset between the up-type and down-type towers, not a running mass, so their convention dependence is admissible rather than a defect — but it must be stated.

The standing flavour-split values are

$u/d = 6/13 \approx 0.4615$ $c/s \approx 11.76$ $t/b \approx 59.4$

with the construction reading $u = 0.4615\,d$, $c = 11.76\,s$, $t = 59.4\,b$.

u/d = 6/13. Note the algebraic form

$6/13 = (K - 1) / (2K - 1)$ at $K = 7$

This is a manifestation of two distinct open provenance questions, separated at the root gate (Section 11). The integer $K = 7$ is taken as established (G-UD(integer)). But the *form* $6/13 = (K-1)/(2K-1)$ is a selected expression of $K = 7$ — one of several low-complexity options — and whether it was fixed before the ratio it reproduces is a separate open audit (G-UD(form)). If the form was blind, 6/13 is structurally sourced and the up-type column rests on derived ground; if the empirical ≈ 0.47 informed the choice of form, 6/13 is a fit. The construction cannot certify this from inside the grid. The cell is graded [prediction conditional on G-UD(form)]. The measured target carries a large experimental uncertainty ($m_u/m_d \approx 0.47 \pm 20$–25%), and the construction's 0.4615 is consistent with it. (Target: $m_u/m_d \approx 0.47 \pm 20$–25%; construction 0.4615.)

c/s ≈ 11.76. The construction value coincides with the PDG common-scale ratio $m_c(2\ \text{GeV})/m_s(2\ \text{GeV}) \approx 11.76$ to better than 1%. Whether the generation-dependent flavour-splitting law derives this value or was calibrated to it determines its grade; absent an independent derivation it is graded [Conjectural].

t/b ≈ 59.4. This is a high-scale running ratio: $m_t(m_t)/m_b(m_t) \approx 163 / 2.74 \approx 59.3$, in close agreement. It is evaluated at a different conventional scale (m_t) than c/s (2 GeV); a referee will require a single scale across the cross-charge cells. The pole-mass ratio $m_t/m_b \approx 41$ differs by ≈ 45% from the running ratio, so the cell's claimed agreement is meaningful only with the running-ratio convention stated explicitly. Graded [Conjectural].

7.1 The χ-increment observation: an apparent 2:1 halving [Conjectural — convention-locked]

The three flavour splits are the susceptibility $\chi(g) = \ln(m_{up}/m_{down})$ of the response-operator decomposition $\ln m(A,g) = B(g) + A\cdot\chi(g)$. Evaluated on the standing values:

$\chi(1) = \ln(6/13) \approx -0.773$ $\chi(2) = \ln(11.76) \approx 2.465$ $\chi(3) = \ln(59.4) \approx 4.084$

The generational increments are

$\chi(2) - \chi(1) \approx 3.238$ $\chi(3) - \chi(2) \approx 1.620$

in ratio 3.238 / 1.620 ≈ 1.999 — an apparent 2:1 halving of the susceptibility increment per generation. A profile whose increments halve each step is a clean, derivable-looking form and is the strongest target in the χ sector.

It is recorded as a target, not a result, for three reasons, and is graded [Conjectural], convention-locked:

1. **It is a relation among the inputs, not a prediction.** $\chi(g)$ is defined from the same three cross-charge ratios graded [Conditional]/[Conjectural] above. If c/s and t/b were read from data, the 2:1 is a pattern the data exhibit and the construction noticed — a derivation target, not evidence the structure is correct. It is closed only by deriving $\chi(g)$ from the operator machinery without consulting masses.

2. **It is convention-locked and scale-mixed.** The clean 2:1 requires the high-scale running t/b ≈ 59; the pole ratio (~36) gives an increment ratio ≈ 2.9:1. Moreover the three points are not at one scale — u/d and c/s are conventionally 2 GeV, t/b is at m_t — so the 2:1 is assembled from points evaluated at different energies. A structural constant must not depend on the scale at which its inputs are quoted; the number moves under a single-scale evaluation.
3. **It is a single relation.** Three generations give two increments; "their ratio is 2" is one coincidence-or-not, not a replicated pattern, and cannot carry weight alone.

[Open — gate G-CHI] Derive the χ(g) profile — and specifically whether its increments genuinely halve (2:1) — from the response-operator machinery without using quark masses, under a single declared scale convention. G-CHI is the up/down-susceptibility counterpart of the down-ladder onset rule G-DS1 (Section 8.7): both ask for a per-generation profile derived from the operator structure rather than read from the spectrum. Closing G-CHI moves u/d, c/s, t/b from match/conditional to prediction, as closing G-DS1 does for s/d and b/s.

7.2 The binary fold-resolution target: ρ = 1/2 [Open — profile theorem]

Separate the χ column into two independent claims: a **profile** (the ratio of successive increments) and a **magnitude** (the size of the first increment). They are not equally supported, and conflating them obscures which is the real result.

The profile is the strong claim. On the common-scale convention (t/b ≈ 59–60, the own-scale ratio rejected — §7.1), the increment ratio is

$$\rho = \Delta\chi_2 / \Delta\chi_1 = 1.620 / 3.238 \approx 0.5003$$

Provenance caveat (read first). §7.1 establishes that the raw 2:1 is scale-mixed: the three χ points are quoted at 2 GeV, 2 GeV, and m_t. A ratio of increments built from points at different scales cannot earn a high-precision headline, and the apparent "0.5003" is an artifact of that mixed evaluation. The only legitimate statement is at a *single* scale: evaluated under one common-scale convention, ρ is **consistent with 1/2** — but the operative claim is "consistent with ½ on a common-scale evaluation," not "½ to 0.06%." The sub-percent figure is dropped; it borrowed precision from a mixed evaluation, exactly the configuration the programme's discipline warns against. What survives is a profile *consistent with* a binary-halving structure, pending a genuinely single-scale recomputation of all three points.

It is also independent of magnitude — it does not depend on the size of the first jump, on the 6/13 anchor, or on the numerical values of c/s and t/b. It is a pure profile claim.

The proposed structural source is the two-dimensional fold sector G in R ≅ C ⊕ G. Where C supplies the three-channel transport count of the down ladder, G supplies the **binary** fold-resolution space governing the up/down susceptibility. A two-way fold distinction resolved one binary layer at a time gives, naturally,

$$\Delta\chi_{g+1} = (1/2)\, \Delta\chi_g$$

so the profile is geometric with ratio 1/2. This is a stronger and more falsifiable claim than the single ratio it rests on: it predicts the *entire* tail $\Delta\chi_3 = (1/4)\Delta\chi_1$, $\Delta\chi_4 = (1/8)\Delta\chi_1$, …, a geometric sequence in dim G = 2. There is no fourth generation to test it against, but the claim is that the whole χ profile is geometric-½, not merely that two increments happen to be in ratio 2.

Status [Open — profile theorem]: this becomes a theorem only when the response-operator machinery is shown to give the G-sector contribution a binary refinement spectrum whose successive accessible increments scale by 1/2, without using the measured masses *and* under a single declared scale. The structural candidate (binary resolution in the dim-2 G sector) and consistency with ½ make this the strongest theorem target in the up/down sector — the χ counterpart of dim C = 3 in the down ladder — but its accuracy claim is only as good as the single-scale recomputation it still requires.

7.3 The magnitude question: why (3/5)·ln R_0 is *not* adopted

The remaining χ question is the size of the first increment, $\Delta\chi_1 \approx 3.238$. A tempting comparison: since $\ln R_0 = 16/3 \approx 5.333$,

$$\Delta\chi_1 / \ln R_0 \approx 3.238 / 5.333 \approx 0.607 \approx 3/5 = \dim C / \dim R$$

suggesting $\Delta\chi_1 \approx (3/5)\cdot\ln R_0 = (3/5)(16/3) = 16/5 = 3.200$. This is **explicitly not adopted**, for two reasons.

First, it degrades the fit. $\Delta\chi_1 = 16/5 = 3.200$ is 1.2% below the observed 3.238, and because the increment feeds an exponential twice, that 1.2% compounds: combined with $\rho = 1/2$ it gives c/s = (6/13)e^{16/5} ≈ 11.33 (target 11.76, −3.7%) and t/b = (6/13)e^{24/5} ≈ 56.1 (target 60, −6.5%). So committing to both clean fractions (1/2 and 3/5) makes the column **worse**, not better — the giveaway of a near-coincidence rather than a structural law. A real magnitude law should not cost the accuracy it claims to explain.

Second, and more fundamentally, it is **internally inconsistent with the profile claim**. The magnitude (3/5)·ln R_0 ties χ's scale to dim C/dim R — the *channel* sector C. But §7.2 derives χ's profile from the *fold* sector G (binary, dim 2). A quantity cannot be a G-sector object for its profile and a C-sector object for its magnitude. So the 3/5 appearing in the magnitude is either evidence that χ is not a pure G-sector quantity (it mixes C and G — which would itself need explaining) or a numerical coincidence (3.238 sits near 3.2). Either way, adopting $\Delta\chi_1 = (3/5)\ln R_0$ commits to a sector-home for χ that contradicts the one $\rho = 1/2$ requires.

Status [Open — magnitude]: the prior question is not "what fraction is $\Delta\chi_1$" but "which sector sets χ's magnitude — G (consistent with the $\rho = 1/2$ profile) or C (as the 3/5 coincidence hints)?" That sector question must be resolved before any magnitude law; until then the magnitude is open and 16/5 is recorded as a coincidence not adopted, on the same footing as the retired 2/9, 8/3, and 2/7 — clean fractions near measured numbers that cost accuracy or contradict structure when adopted.

Clean separation: profile target $\rho = 1/2$ (consistent with ½ on a single-scale evaluation, [Open theorem] pending a genuinely single-scale recomputation); magnitude open, with the C-vs-G sector home as the prior question and $(3/5)\ln R_0$ explicitly not adopted.

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8. The Colour-Access Problem: the 3/8 ↔ 8/3 Reciprocal and Why It Reposes as N_c

The coloured down-steps wear more than the bare unit. The access factor is defined by

$\Theta_{step} = \Theta_{cell} \times f_{colour} = (1/12) \times f_{colour}$

and the problem is to derive f_colour from closure structure.

8.1 The restriction/access reciprocal: 3/8 and 8/3 are one object

The colour factor admits two names that are reciprocals of one another, corresponding to whether it is stated as a restriction on realisation or as the resulting boost in effective access.

The **restriction** reading is the physically primitive one. The colour sector carries the full adjoint burden

$\dim SU(3) = 3^2 - 1 = 8$

but a realised quark state occupies only a single colour-triplet channel,

$N_c = 3$

so the realised fraction of the closure burden is

ρ_{colour} = realised triplet / adjoint burden = $3/8 \approx 0.375$ [restriction]

The **access** reading is its inverse: converting a restriction on realisation into an effective enhancement of the surviving participation budget gives

$f_{colour} = 1 / \rho_{colour} = 8/3 \approx 2.667$ [access boost]

so that

$\Theta_{b/s} = \Theta_{cell} \times 1/\rho_{colour} = (1/12) \times (8/3) = 8/36 = 2/9$

$m_b/m_s = e^{(16/3)} \times 2/9 \approx 207.13 \times 0.2222 \approx 46.03$

This is a genuine clarification and is retained as vocabulary independently of the numerical question below: 3/8 (restriction) and 8/3 (access boost) are not two factors but one ratio under two namings, and "realised triplet / adjoint burden" is a precise, falsifiable ansatz rather than a free factor. It converts the open object from "a number near 2.7" into a specific structural claim that can be checked.

What it does **not** settle is whether 2/9 is the right target value. The ansatz lands $\Theta_{b/s}$ on 2/9 and m_b/m_s on 46. The two objections below are independent of how elegantly 8/3 is derived, because a clean derivation of 8/3 is a clean derivation of 46 — and the target is elsewhere.

8.2 The 8/3 collision under the scale-mixed target

The restriction ansatz reproduces the value reached under the scale-mixed target, $\Theta_b \approx 2/9$, giving $f_{colour} = 8/3$. But 8/3 is **also** the localisation constant κ (Section 3), derived from CP^2 geometry on entirely unrelated grounds. Two unrelated derivations producing the identical rational 8/3 is either a result demanding explanation or a coincidence dressing a fit. As the construction stood, the latter reading was difficult to exclude: the second down-step needed a factor of 8/3 over the unit, and 8/3 was available elsewhere in the programme. The reciprocal ansatz of §8.1 supplies an independent route to 8/3 and so weakens — but does not remove — the coincidence concern, since it still lands on the disfavoured target (§8.3, §8.4).

8.3 The target objection: a clean 8/3 lands below the invariant target

The reciprocal ansatz is elegant but does not move the numerical dispute, because the dispute was never whether 8/3 has a tidy story — it was whether 2/9 hits the target. A derivation of 8/3 is a derivation of $m_b/m_s \approx 46$, and the RG-invariant target (Section 6.2) is ≈ 54. The ansatz therefore lands $\approx 15\%$ below the physical invariant. This is exactly the configuration the programme's own discipline covers: structural tidiness cannot purchase the claim when the target it lands on is disfavoured. The decision remains upstream of the colour story — it is the target — and the invariant target requires an enhancement $\approx 3.12 \approx N_c$, not 8/3.

8.4 The step-selectivity objection: a colour factor cannot be step-selective

A second objection bites **even if the scale-mixed target ≈ 45 is granted**, and is independent of §8.3. If the factor is genuinely colour — ρ_{colour} = triplet / adjoint = 3/8 — then it must be colour-**universal** across the down-steps, because s and b are colour-identical: both are colour triplets sitting in the same adjoint. A colour factor cannot distinguish the two steps, because their colour content is the same.

But the factor is step-selective. Applying the access boost to the **first** down-step gives

$$m_s/m_d = e^{16/3} \times (1/12) \times (8/3) = 207.13 \times 2/9 \approx 46$$

against a target of ≈ 20 — an overshoot of more than 130%. The boost must therefore land on s→b and **not** on d→s. Colour content cannot explain that selectivity, because the colour content of s and b is identical. Whatever distinguishes the two down-steps is generational / saturation depth, not colour.

The decisive tell is the colourless ladder. The lepton sector shows the **same** step-onset structure: no suppression at $e \to \mu$, one unit at $\mu \to \tau$ (Section 5). Leptons carry no triplet and no adjoint to take 3/8 of, yet they reproduce the pattern in which suppression switches on at the later step. The variable doing the work is therefore something leptons and quarks share — closure-saturation depth — with colour at most **modulating the magnitude** in the quark sector, not supplying the factor. A 3/8 read as pure colour restriction is contradicted by a colourless sector that behaves the same way.

8.5 Resolution: derive the inter-step enhancement, with N_c as modulation

Combining §8.3 and §8.4, the open object is reposed one notch further. The thing to derive is not a per-step colour access factor but the **inter-step enhancement** of the down-ladder:

$f_{step} = \Theta_{(s\to b)} / \Theta_{(d\to s)} \approx (1/4) / (1/12) = 3 \approx N_c$

i.e. why the second down-step is suppressed **less** than the first by a factor $\approx N_c$, and why a colourless ladder exhibits the same later-step onset. Under the RG-invariant target of Section 6.2 the required access factor is

$f_{colour} = \Theta_b / \Theta_{cell} = 0.260 / 0.0833 \approx 3.12 \approx 3$

so the target points at the plain colour count $N_c = 3$ entering as a **saturation-depth modulation**, not at 8/3 entering as a colour-access ratio, with $\Theta_b \approx 3 \times (1/12) = 1/4$. This is the better-posed outcome:

1. The numerical collision with $\kappa = 8/3$ (§8.2) disappears: it was an artifact of the scale-mixed target, not a structural coincidence requiring explanation.
2. The factor to be derived becomes the plainest colour quantity, $N_c = 3$, rather than the composite adjoint/multiplicity ratio 8/3.
3. The restriction reading 3/8 survives as vocabulary, but the burden it must discharge is now explicit: show why the adjoint burden discounts the **second** step and leaves the first alone, and why a **colourless** ladder shows the same step shape. Discharging both forces the colour story to become a saturation story in which N_c modulates magnitude; a pure triplet/adjoint restriction does not survive the colourless-sector test.

8.6 The surviving-channel derivation: 3 as a participation count, not a group factor

The saturation reading supplies a concrete internal route to the factor 3 that uses no observed quark mass. It begins from the participation interpretation of the closure unit:

Θ_{cell} = surviving anchoring participation / total closure participation = 1/12

When a closure saturates, its participation collapses to the surviving routes. For a **colourless** state there is one surviving anchoring route, so

$\Pi\_lepton = N\_survive / N\_anchor = 1/12, N\_survive = 1, N\_anchor = 12$

A **coloured** state is not a single closure realisation. It carries three admissible colour realisations {r, g, b}. The structural question is whether saturation acts on each colour channel separately or on the colour-neutral object as a whole. If closure maintenance acts on the colour-neutral object, each colour channel still contributes one surviving anchoring route, so

$N\_survive = 3, N\_anchor = 12, \Pi\_quark = 3/12 = 1/4$

The factor 3 is then not a group-theory invariant but a **participation count**:

$\Pi\_quark = 3 \times \Pi\_lepton = 3 \times (1/12) = 1/4$

This is structurally attractive for three reasons. It does not assume colour = 3 as an input; it assumes only that saturation leaves one surviving route per independent colour sector and that a coloured state possesses three such sectors, and 3 emerges. It uses no quark mass. And it lands on exactly the value the corrected RG-invariant b/s target wants: $\Theta\_b = 1/4$ gives $m\_b/m\_s = R_0/4 \approx 51.78$ against ≈ 54, a 4% tension of the same order as the muon and tau postdictions.

It also generalises into a falsifiable theorem candidate. If

$\Pi = N\_survive / 12$ with $N\_survive$ = number of surviving transport channels

then leptons (1 channel) give 1/12, quarks (3 channels) give 1/4, and any future sector with k transport channels predicts $\Pi = k/12$. This is gradeable content beyond the single b/s data point, and ties the suppression directly to the three-channel transport architecture the colour-transport programme independently favours.

[Conditional — gate G-CA] The down-ladder enhancement factor 3 now has a candidate derivation — the surviving-channel count — rather than an imported number. It is graded [Conditional], not [Open], on the strength of that derivation, conditional on two structural premises (one surviving route per colour sector; three independent colour sectors per coloured state) and on the saturation-onset rule of §8.7. The reciprocal vocabulary (3/8 restriction, 8/3 access) is retained but superseded as the operative reading: the operative reading is $\Pi = k/12$. The residual 4% between the channel value 1/4 and the target 0.261 is recorded, not absorbed.

8.7 What the channel count does not yet explain: the first down-step

The surviving-channel count derives the factor 3 and fixes the **second** down-step, but it sharpens rather than removes the step-selectivity problem, and the sharpening is quantitative. Stripping each step to its participation ratio $\Pi(g+1)/\Pi(g)$ = (observed ratio)/R_0:

Step	Observed ratio	÷ R_0 = participation ratio	Channel count predicts
e → μ	206.77	0.998 ≈ 1	1 (k = 1)
μ → τ	16.82	0.081 ≈ 1/12	1/12 (k = 1)
d → s	≈ 20	0.097 ≈ 1/10	3 × 1 = 3 — miss
s → b	≈ 54	0.261 ≈ 1/4	3 × 1/12 = 1/4 — hit

The second down-step is a clean hit. The first down-step misses in the **opposite direction** and by a large factor. The channel count states that a coloured step retains three times the routes of its colourless analogue — an **enhancement**. At s→b this is exactly what is seen (≈ 54 against the lepton ≈ 17). At d→s the quark is **suppressed** roughly tenfold relative to the lepton (≈ 20 against ≈ 207), and a count of surviving channels can only add routes; it cannot manufacture a suppression. The naive channel prediction at the first step is $m_s/m_d \approx 3 \times 207 \approx 621$ (or, with no first-step saturation, ≈ 207); the observed value is ≈ 20, a miss of 10–30×.

This is forced by colour-identity: s and b are both colour triplets carrying three channels, so a colour-channel count must act identically at both down-steps. It acts correctly at one and oppositely-wrongly at the other. The factor 3 is therefore secure for s→b, but the ≈ 1/10 suppression at d→s is now the dominant unexplained object in the down ladder, and it runs against the colour enhancement rather than with it.

The candidate reconciliation is that coloured states saturate one generation earlier than colourless ones: the strange is first-saturated with channels not yet open (one surviving route, ≈ 1/12, recovering $m_s/m_d \approx 17$), while the bottom has the three channels open ($\Pi = 3/12 = 1/4$). This is coherent but reintroduces the selectivity one level down — why one surviving route at the strange and three at the bottom, when both are coloured — so the theorem to be proved is not "three channels exist" but "channels open progressively with saturation depth." Until that rule exists, the first down-step is gated.

[Open — gate G-DS1] The first down-step suppression $\Pi(s)/\Pi(d) \approx 1/10$ is underived and is oppositely signed to the colour-channel enhancement. The surviving-channel count cannot produce it. A saturation-onset rule (coloured states saturating one generation earlier, with channels opening progressively) is the candidate but is itself underived. G-DS1 is coupled to G-CA: any onset rule that fixes d→s also moves the s→b prediction, so the two cells must be solved together, not separately.

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9. The Activation Rule and the Lepton/Quark Offset

The construction contains an unexplained asymmetry in **which** step first carries suppression, sharper than a generic "derive the activation rule" statement admits.

Lepton ladder: e → μ bare localisation (no unit) μ → τ localisation × (1/12)

Down ladder: d → s localisation × (1/12) × f_colour, already suppressed at step one s → b localisation × (1/12) × f_colour, suppressed again

The lepton ladder is unsuppressed at its first step and acquires one unit at its second. The down-quark ladder is suppressed from its first step. The activation rule must reproduce this offset: the coloured ladder begins wearing the closure unit one step earlier than the colourless ladder.

[Open — gate G-ACT] The activation rule determining which sectors and which steps wear the suppression unit is underived. Any candidate rule must satisfy the constraint above: colourless ladder unsuppressed at step 1, coloured ladder suppressed at step 1. The surviving-channel/saturation-onset picture of §8.6–§8.7 is now the leading candidate mechanism: if channels open progressively with saturation depth and coloured states saturate one generation earlier than colourless ones, the same rule fixes both the step offset here and the first down-step (G-DS1). The open objects G-ACT, G-CA, and G-DS1 are plausibly a single object — one rule for participation as a function of saturation depth and channel count.

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10. The Conditional Grid, Graded Cell-by-Cell

The grid is built from the structural construction and compared against the convention-correct targets of Section 1.2. Normalising d = 1 and adopting the Route 1 primary for s/d:

Cell	Construction	Value	Target (convention-correct)	Tension	Grade	Gate
u/d	6/13	0.4615	≈ 0.47 (±20–25%)	consistent	[prediction]	G-UD(form)
s/d	Route 1 (primary)	≈ 20	≈ 20 (GMOR/kaon, invariant)	—	[import, at resolution limit]	G-DS1
c/s	flavour-split law	11.76	≈ 11.76 (PDG, 2 GeV)	< 1%	[match]	—
b/s	$R_0 \cdot (1/12) \cdot \dim C$, $\dim C = 3$	51.8	≈ 54 (invariant)	≈ 4%	[prediction]	G-CA
t/b	flavour-split law	59.4	≈ 59 (running, m_t)	≈ 0–1%	[match]	—

Absolute placements (d = 1). Using $b/s = R_0 \cdot (1/12) \cdot \dim C = 51.8$ and the imported s/d = 20:

Constructed: u : d : s : c : b : t = 0.4615 : 1 : 20 : 235.2 : 1036 : 61,540 Target: u : d : s : c : b : t ≈ 0.47 : 1 : 20 : 235.2 : 1078 : 64,000

with c = 20 × 11.76, b = 20 × 51.8, t = 1036 × 59.4. Every cell lands within a few percent. The two down-steps are different kinds of object (G-DS1): b/s is a channel-saturation quantity (n = dim C = 3, denominator 3) and is a clean prediction; s/d's first-step correction is a residual quantity (denominator 5 = dim R) that the channel structure cannot supply and that sits at the data/theory noise floor, so s/d stays imported. The earlier {2/5, 1} accessibility pairing and its s/d candidate (20.7) are retired: the 2/5 was the non-adjacent kernel fraction, not licensed for the adjacent first step.

10.1 What the grid does and does not demonstrate

The grid lands within a few percent at every cell, and that agreement is real, but the cell-by-cell provenance must be read before the agreement is credited, because the cells differ sharply in what they have earned. Within the K = 7 closure framework, sorting the six cells:

- **u/d = 6/13 is a prediction, conditional on the form provenance.** The form (K−1)/(2K−1) is a *selected* algebraic expression of K = 7, not a forced dimension count; establishing the integer (G-UD(integer)) does not establish that this form was fixed before the ratio it reproduces (G-UD(form), open). So it is a prediction conditional on the form being blind. The measured target $m_u/m_d \approx 0.47$ carries a large experimental uncertainty (≈ ±20–25%), and the construction's 0.4615 is consistent with it.
- **b/s ≈ 51.8 is a prediction, conditional on the dim C = 3 descent — but carries one read-off attachment.** It is the channel sector saturating: n(s→b) = dim C = 3, giving b/s = $R_0 \cdot (1/12) \cdot 3 \approx 51.8$ against ≈ 54 (4%). The *value* 3 is forced (dim C), so this is **not** a two-parameter step-assignment fit — the {2/5, 1} pairing was retired. But the claim is not assignment-free: that the three channels saturate at the *second* down-step (s→b) rather than the first is read off, not derived — §11's assignment-rule ledger books this as a *partial* assignment (forced value, read-off attachment). So b/s is a one-forced-number prediction carrying one open attachment, and it has two conditions, not zero: the dim C = 3 descent (the K = 7 → C_3 → channel-count chain this paper cannot referee from inside), and the step-attachment. It is a clean prediction in value, given those.
- **s/d ≈ 20 is an import.** The operative value is Route 1's 20. The theory's own attempt to derive it stalls (G-DS1): the first-step correction is ≈ 16% on a ratio known only to ≈ ±15%, i.e. at the noise floor, so there is likely nothing clean to derive at this resolution. A heuristic "denominator" argument (the correction looks like a /5 = dim R fraction, not a /3 = dim C one, so it can't come from the channel structure) is *suggestive but untested* — it has so far been invoked only to explain this one cell, so it is a plausible reason s/d may be underivable, not an established principle. Either way, s/d stays an import.
- **c/s and t/b are matches resting on the flavour-split law.** They agree with PDG, but whether the law *derives* 11.76 and 59.4 or was calibrated to them is unresolved independently of K = 7. Agreement, not yet prediction.
- **t is a by-product.** It is b × 59.4 and inherits b's tension and t/b's open law-form; it tests nothing the b and t/b cells did not already test.

So within the framework the grid carries **two predictions, both qualified** — u/d (conditional on G-UD(form); its measured target carries ≈ ±20–25% uncertainty) and b/s (channel saturation, dim C = 3, conditional on the foundational descent) — a match resting on an unsettled law (c/s, t/b), an imported s/d (its residual correction sits at the data/theory noise floor, ≈16% on a ±15% number, so likely nothing clean to derive), an anchor (d), and a by-product (t). The honest map: what is predicted-but-qualified (u/d, b/s), what is a calibrated match (c/s, t/b), what is imported at the resolution limit (s/d), and what is inherited (t). That spread, not the uniform row of small percentages, is the real content — and §11's assignment-rule ledger shows the residual fitting freedom lives in the open assignment rows beneath even the predictions.

This is not a deficiency to be hidden; it is the map of where the next work pays off — and one place where the work has reached its terminus. s/d will not convert to a prediction by a channel rule: its correction has denominator dim R = 5, not dim C = 3, so it is a residual quantity the channel structure cannot supply, and it is in any case at the noise floor (G-DS1). The live frontier is therefore the χ column (G-CHI), not the down ladder, which is now resolved into "b/s predicts, s/d imports, for a structural reason."

Inheritance caps. By the inheritance-cap principle a product cell carries the grade of its weakest input:

- The **up-type column** (u, c, t) is capped at the weakest of {u/d [prediction conditional on G-UD(form)], c/s [match], t/b [match]} → the absolute up-type masses are capped at [match] by the unsettled flavour-law form. Numerical proximity in c and t does not lift this cap.
- The **down-type ladder**: b/s is a [prediction] (channel saturation, value dim C = 3 forced, with the step-attachment read off — a partial assignment, §11 ledger); s/d is an [import at resolution limit]. The two behave as different kinds of quantity (b/s a channel quantity, s/d a residual one whose correction sits at the noise floor) and do not share a single rule (G-DS1). The absolute b placement is as firm as b/s; the absolute scale of the down sector inherits the s/d import.
- Only Θ_cell = 1/12 [Proven structural] and $R_0 = e^{(16/3)}$ [structurally sourced, given K = 7] stand above the cell grades.

The grid must not be read as uniformly derived. Its proven structural objects are the suppression unit 1/12 and (given K = 7) the localisation factor R_0; the predictions u/d and b/s rest on those plus the K = 7 framework; the rest is match, import, or inherited, and is graded as such.

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11. Status Ledger and Gates

Structurally sourced / proven

- Θ_cell = λ^*/trace(L) = 2/24 = 1/12 [Proven — structural; fixed before masses]
- $R_0 = e^{(16/3)} \approx 207.13$ [Structurally sourced; $\kappa = 8/3$ from CP^2 geometry] — subject to gate G-LOC2
- $m_\mu/m_e \approx 207.13$, postdiction to 0.17% [strongest quantitative result — but fixes 2κ, of which the factor 2 is open (G-LOC2); not parameter-free]
- $m_\tau/m_\mu \approx R_0/12$, postdiction to 2.6% [bare unit, colourless sector]

Conditional

- u/d = 6/13 = (K−1)/(2K−1)|_{K=7} [prediction conditional on G-UD(form): the integer K=7 is established, but the *selected form* (K−1)/(2K−1) is not yet audited as blind]
- s/d ≈ 20 [Conditional; sourced from Route 1, primary]
- f = 3 down-ladder enhancement [grounded; the factor is dim C = 3 in R ≅ C ⊕ G, a forced dimension count given K = 7 (G-UD(integer)), not a selected form — so it is bedrock with the integer, distinct from 6/13 which depends on G-UD(form)]

Conjectural

- c/s ≈ 11.76, t/b ≈ 59.4 [flavour-split law; derivation vs calibration unresolved]
- χ-increment 2:1 halving [Conjectural; §7.1–7.2 — consistent with ½ on a common-scale evaluation, but the raw 2:1 is scale-mixed (points at 2 GeV, 2 GeV, m_t), so no sub-percent precision is claimed and a genuinely single-scale recomputation of all three χ points is still pending (G-CHI). Remains a relation among the inputs until χ(g) is derived from the C ⊕ G fold sector G]
- up-type absolute masses [capped Conjectural by inheritance]

Open gates

- **G-UD(integer) — taken as established.** Is K = 7 forced independently of any mass or flavour target? The chain K = 7 → (6-boundary + 1-hub) wheel W_6 → cyclic C_3 (120° rotations giving two 3-orbits of the hexagonal boundary) → channel decomposition → dim C = 3, dim G = 2 is clean given K = 7; C_3 is forced by the hexagonal boundary, not chosen to yield a 3. K = 7 is taken here as independently established (six convergence routes, two rigid). This certifies the **integer** 7 and the dimension counts dim C = 3, dim G = 2 that follow from it — i.e. the factor 3 (G-CA), η = 3/5, χ = 2/5, the C ⊕ G foundation.
- **G-UD(form) — open, and distinct from the integer.** Establishing K = 7 does **not** establish the functional forms that consume it. At K = 7 the up/down split 6/13 = (K−1)/(2K−1) is one of many low-complexity expressions — 3/7 = (K−1)/2K, (K−1)/(2K−1), and others — and the one landing on ≈ 0.46 was *selected*. So u/d's grade rests on a provenance claim the integer audit does not deliver: **was the form (K−1)/(2K−1) fixed before the ratio it reproduces, or chosen to hit 0.46?** This is a separate audit from G-UD(integer) and the entire up-type column rests on it. Until it is discharged, 6/13 is "prediction *conditional on the form being blind*," not a clean prediction. The same form-provenance question applies to any cell whose value comes from a *chosen functional expression* of K rather than a forced dimension count. (Dimension counts like dim C = 3 are covered by G-UD(integer); selected algebraic forms like 6/13 are covered by G-UD(form).)
- **G-CA** — Closed at the level of the *number*, and now grounded rather than merely plausible. The factor 3 is the transport-sector dimension dim C = 3 in the flavour-mixing paper's admissibility decomposition R ≅ C ⊕ G (dim C = 3, dim G = 2, dim R = 5), derived from the K = 7 closure architecture without using masses. The three channels are the admissible transport directions u_1, u_2, u_3. This supersedes the earlier surviving-channel premises (§8.6) by supplying their structural source: triplicity is transport admissibility, not eigenvalue multiplicity. Residual precondition: G-UD(integer) — dim

C = 3 is a forced dimension count given K = 7, so the factor 3 is bedrock with the integer (unlike 6/13, which is a *selected form* and depends on G-UD(form)). The factor 3 is covered by the integer audit, not the form audit.

- **G-DS1** — Status after a full derivation push: **the two down-steps are different *kinds* of quantity and are not unifiable by a single channel rule.** The ladder has two steps. Their participation relative to the colourless 1/12 unit:
  - **b/s saturates the channel sector C: n(s→b) = dim C = 3**, a clean fraction of dim C (denominator 3). b/s = $R_0 \cdot (1/12) \cdot 3 \approx 51.8$ vs ≈ 54 (4%). This is a pure *channel* (/3) quantity, derived from dim C (G-CA), and it closes.
  - **s/d requires a fraction of the full residual R, not of C.** The first-step excess over the bare unit is n(d→s) ≈ 1.16–1.2, i.e. ≈ dim C · (2/5). The decisive fact: 2/5 has denominator **5 = dim R**, not 3 = dim C. Any "k of the 3 channels active" mechanism gives a /3 fraction (e.g. 2 of 3 → 2/3 → n = 2 → m_s/m_d ≈ 34, wrong); only a fraction of the full residual space R gives /5. So the first-step correction cannot come from the adjacent channel structure C at all — it is a residual (/5) quantity.

Why s/d does not close. The only structural object affording the right denominator (2/5 = dim ker M / dim R) is the non-adjacent matching operator M (rank 1, ker 2), which by the corpus's own adjacent/non-adjacent separation does not act on the adjacent first step. The candidate routes through the *adjacent* structure all fail on the denominator: the C_3 split of C is 3 = 1 ⊕ 2 (k=0 trivial ⊕ k=±1 pair) and the hub/rim split are both /3, giving n = 2 (m_s/m_d ≈ 34), not n ≈ 1.16 (m_s/m_d ≈ 20). Moreover the target itself — a ≈ 16% bump on the bare unit 17.26 — sits at the noise floor: s/d is known only to ≈ ±15% (17–22 in the literature), the bare-unit value is 14% low, and the correction being chased is the same size as the data uncertainty. Deriving a second-significant-figure fraction from a number pinned to ±15% is fitting noise.

Terminus: s/d remains an **import**. The load-bearing reason is the noise floor: the first-step correction is ≈ 16% on a ratio known to ≈ ±15%, so there is likely no clean structural quantity to derive at this resolution. (The denominator distinction below is a candidate explanation for *why* it resists the channel structure, but it is untested — see the heuristic note.) b/s remains a derived channel-saturation prediction (≈ 4%), conditional on the dim C = 3 descent. The earlier {2/5, 1} pairing is retired: its 2/5 was the non-adjacent kernel fraction, illegitimately imported to the adjacent step.

Heuristic banked for any future attempt — the denominator test (untested). The b/s correction is a fraction of dim C = 3; the s/d correction looks like a fraction of dim R = 5. This *suggests* the two are different kinds of quantity and that s/d's cannot come from the channel structure. But this distinction has so far been invoked **only** to explain the one cell that does not derive — it has made no independent /5-vs-/3 prediction that has been checked elsewhere. So it is recorded as a *plausible but untested reason s/d may be underivable*, not as an established structural principle. For it to earn that status it must predict a /5-vs-/3 outcome somewhere away from s/d and be confirmed. Until then s/d is import because its correction sits at the data/theory noise floor (≈16% on a ±15%

number), which is the load-bearing reason; the denominator distinction is a candidate explanation, not a proof.

- **G-CHI** — Derive the $\chi(g)$ up/down-susceptibility profile. Split into two unequal claims (§7.2–7.3). **Profile (strong target):** $\rho = \Delta\chi_2/\Delta\chi_1$ is consistent with 1/2 on a common-scale evaluation, with a structural candidate — binary fold resolution in the dim-2 G sector of $R \cong C \oplus G$, predicting the full geometric tail $\Delta\chi_{g+1} = \frac{1}{2}\Delta\chi_g$. The raw 2:1 is scale-mixed (points at 2 GeV, 2 GeV, m_t), so no sub-percent precision is claimed; the operative statement is "consistent with ½ pending a genuinely single-scale recomputation of all three χ points." This is the χ counterpart of dim C = 3 [Open — profile theorem]. **Magnitude (open, with a contradiction to resolve first):** the first increment $\Delta\chi_1 \approx 3.238$; the candidate $(3/5)\cdot\ln R_0 = 16/5 = 3.2$ is *not adopted* — 1.2% low, degrades the column to −6.5% at t/b if combined with $\rho = \frac{1}{2}$, and ties χ's magnitude to the C sector while its profile is G-sector, an internal contradiction. The prior question is which sector sets χ's magnitude. 16/5 is recorded as a coincidence-not-adopted alongside 2/9, 8/3, 2/7. First concrete sub-step: recompute all three χ points at one scale and report ρ there, once.
- **G-BS** — Subsumed: the second down-step value is fixed by the channel count at $\Theta_b = 1/4$ (4% tension). What remains is the absolute scale, gated by G-DS1.
- **G-ACT** — Derive the activation rule reproducing the lepton/quark step offset (Section 9). The channel/saturation-onset picture of §8.7 is now the leading candidate mechanism; G-ACT, G-CA, and G-DS1 are plausibly one object — a single rule for how transport channels open with saturation depth.
- **G-LOC2** — Restate the factor 2 in $R_0 = e^{(2\kappa)}$: mass $\propto L^2$ or $\Delta g = 2$.
- **G-ABS** — Absolute masses (overall scale). Out of scope here; the grid is ratios only. The grid is internally a 2 GeV object: absolute-mass work requires every chained ratio to share a scale, and the t/b step is the one cell that breaks this (it is an m_t-scale ratio). Chaining across scales produces the spurious $m_t \approx 289$ GeV diagnosed and closed in Appendix B; that artifact is recorded so it is not reopened as a real anomaly.

Assignment-rule ledger (the meta-count). The per-derivation ledgers police free choices *within* each derivation, but the framework also acquires *assignment and activation rules* — choices about which structural ingredient attaches to which cell or step — and each is a degree of freedom that the per-cell view does not see. The programme's discipline (free structural choices strictly fewer than independent targets reproduced) must be audited at this level too. Every such rule, with its provenance grade:

Rule	What it assigns	Provenance	Grade
Localisation 2κ (G-LOC2)	the factor 2 in $R_0 = e^{(2\kappa)}$	mass $\propto L^2$ or $\Delta g = 2$ — neither derived	**open**
Functional form (K−1)/(2K−1) (G-UD form)	the up/down split value 6/13	selected K=7 expression; blindness unaudited	**open**
Flavour-split law form	c/s = 11.76, t/b = 59.4	derive-vs-calibrate unresolved	**open**

Rule	What it assigns	Provenance	Grade
Channel saturation at s→b (G-DS1)	n = dim C = 3 to the second down-step	the *value* 3 is forced (dim C); the *attachment* to the saturating step is read off, not derived	partial
Activation / step-offset (G-ACT)	which step first wears the suppression unit	candidate (saturation-onset), underived	**open**
χ sector-home (G-CHI)	whether χ's magnitude is G- or C-sector	unresolved (the 3/5 contradiction)	**open**
Denominator heuristic (/3 vs /5)	classifies s/d as residual-type	invoked only on the failing cell; untested	**open**

This count is the honest location of the residual fitting freedom. K = 7 plus this set of assignment rules is where the flexibility lives, and several rows are open. The dimension-count facts (dim C = 3, dim G = 2) and the suppression unit 1/12 are *not* on this list — they are forced, not assigned. But the rows above are choices, and a fair reading of the whole apparatus must weigh them against the targets reproduced, not treat each derivation's internal ledger as the whole accounting. The grid's genuine predictions (u/d, b/s) survive this scrutiny only to the extent their assignment rows resolve: u/d on G-UD(form), b/s on the dim C = 3 descent.

Investigated and parked: the spectral +2 bridge (do not re-run). A route to unify B(g) and $\chi(g)$ through the W_7 spectrum was investigated and is parked, recorded here so it is not re-derived from scratch. The idea: the doubled eigenvalue $\lambda = 2$ (multiplicity 2, the E_1/D_6 rim-hexagon pair) could supply a +2 channel jump, gated by a fold-residue driver $r_g = 2^{-(g-1)}$ crossing a spectral threshold, so that one structure drives both profiles. It does not connect, for three reasons established in sequence: (i) a single residue driver with $r_{g+1} = r_g/2$ *decelerates*, and cannot produce an *accelerating* channel count without tuned thresholds (monotonicity mismatch); (ii) the repaired version (B reads accumulated resolution S_g, χ reads instantaneous residue r_g) gave the right shape but required an activation threshold $\theta \in [1/4, 1/2)$, and the natural candidate $\theta = \lambda^*/\lambda_{max} = 2/7$ fails a generalisation test — written as a rule across the spectrum {2,4,5,7} it opens the primitive mode *last*, contradicting its foundational status; (iii) decisively, the current corpus sources colour from **bath / transport participation, not the W_7 eigenspace**, and explicitly warns against conflating the D_6 action on the E_1 eigenspace with the D_7 transport/generation structure. The factor 3 is therefore not spectral: it is dim C = 3 in the flavour-mixing decomposition $R \cong C \oplus G$ (G-CA). The doubled eigenvalues remain a true numerical feature of the Laplacian but are an **unused coincidence**, not the colour mechanism. Consequence: under the current architecture B(g) and $\chi(g)$ share the response-operator *framework* (the $\ln m = B + A\chi$ form, §2) but not a single spectral *driver* — the unification is nominal, not connected. Connecting it would require a deliberate paper-level relocation of colour into an eigenspace, against the present D_6/D_7 separation; that is a decision to be made explicitly, not a gap to be filled. The live route to a genuine shared driver is instead the $C \oplus G$ decomposition itself (G-DS1, G-CHI): if η = dim C/dim R and the fold sector dim G are two projections of the one $R = C \oplus G$ structure, that — not the spectrum — is where one operator emitting both readouts would come from.

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12. Highest-Leverage Next Actions

The quark sector currently contains one genuinely new few-percent prediction — b/s, from the derived factor 3 — inside a grid whose other agreements are imported (s/d), inherited (t), or unaudited (u/d, c/s, t/b) (§10.1). The next actions are therefore ordered by how much derived content each converts from import or assumption into prediction, not by how much it improves an already-good-looking fit.

The two structural frontiers are symmetric and both have the same shape — *derive a per-generation profile from the operator machinery without consulting masses*. One governs the baseline B(g) (the down ladder: s/d, b/s); the other governs the susceptibility $\chi(g)$ (the flavour splits: u/d, c/s, t/b). Closing either converts a full row of the grid from import/match into prediction; closing both would leave only the anchor and the audited inputs unexplained. The candidate single source for *both* is the flavour-mixing decomposition $R \cong C \oplus G$ (dim C = 3, dim G = 2): if B reads the channel sector C and χ reads the fold sector G, the two profiles are projections of one structure rather than two engines. That, not the W_7 spectrum, is the live route to a genuine shared driver (see the parked-bridge note in Section 11).

1. **Audit the K = 7 convergence routes (integer) AND the functional forms (form) — separately.** Two distinct provenance questions, collapsed in earlier drafts. **G-UD(integer):** are the six convergence routes independent (disjoint premises, built blind)? This certifies the integer 7 and the forced dimension counts dim C = 3, dim G = 2 (hence the factor 3, η, χ, the $C \oplus G$ foundation). **G-UD(form):** was each *selected algebraic form* that consumes K — chiefly $6/13 = (K-1)/(2K-1)$ — fixed before the ratio it reproduces, rather than chosen from the many low-complexity K=7 expressions because it lands on 0.46? The integer audit does **not** deliver the form; the entire up-type column rests on the form audit, and it is currently undischarged. Both are literature traces, not new physics. They settle different cells: the integer settles b/s and the dimension-count cells; the form settles u/d and the up-type column.
2. **Derive the running baseline profile $\eta(g)$ — the B(g) frontier (G-DS1 + G-CA + G-ACT).** The factor-3 ceiling is settled (dim C = 3). The binding open object is now the *running* channel-survival profile: the down ladder carries not a clean integer $1 \to 3$ but **$1.16 \to 3.12$ channel-equivalents** (Π relative to the 1/12 unit). Derive a generation-dependent $\eta(g)$ from $R \cong C \oplus G$ that hits 1.16 and 3.12 with no per-step Q-factors. Sharp failure test: if η is generation-independent (flat 3/5) it cannot produce the run, and the baseline needs another driver — a clean negative. Success derives s/d instead of importing it from Route 1, collapses the grid's two down-steps into one engine, and dissolves the 14% s/d tension into η's running.
3. **Derive the susceptibility profile $\chi(g)$ — the up/down frontier (G-CHI).** The symmetric counterpart of action 2, and the other projection of $R \cong C \oplus G$ (the fold sector G). Derive $\chi(g)$ from the response-operator machinery without using masses, under a single scale convention, and test the halving (the 2:1 of §7.1, which survives at $\rho \approx 0.503$ on the paper's common-scale convention). Closing it converts u/d, c/s, t/b from match/conditional into prediction. The deeper prize: if the same $C \oplus G$ structure yields both $\eta(g)$ (action 2) and $\chi(g)$, the unification is real rather than nominal — that cross-link, not either profile alone, is the theorem.

4. **Test Π = k/12 outside the down ladder.** The channel count predicts the suppression of *any* sector from its surviving-channel count. Identify a second testable sector (colourless k = 1 elsewhere, or a sector with a different k) to falsify or confirm the theorem candidate beyond the single b/s point on which it was built.

Appendix A — Numerical Audit

Quantity	Construction	Value	Reference	Tension
$R_0 = e^{16/3}$	exp(5.3333)	207.13	—	—
Θ_cell	2/24	0.08333	—	—
Θ_τ (required)	16.82/207.13	0.0812	vs 0.0833	2.6%
m_μ/m_e	R_0	207.13	206.77	0.17%
m_τ/m_μ	$R_0/12$	17.26	16.82	2.6%
m_s/m_d (closure)	$R_0/12$	17.26	≈ 20	14%
m_b/m_s (2/9)	$R_0 \cdot 2/9$	46.0	≈ 53.9	15%
m_b/m_s (1/4)	$R_0 \cdot 1/4$	51.8	≈ 53.9	4%
f_colour (required)	0.260/0.0833	3.12	vs N_c = 3	4%
f (channel count)	3 × (1/12)	1/4 = 0.250	target 0.261	4%
Π(s)/Π(d)	20/207.13	0.097 ≈ 1/10	channel predicts ≠	unexplained
Π(b)/Π(s)	54/207.13	0.261 ≈ 1/4	3 × 1/12 = 0.250	4%
m_u/m_d	6/13	0.4615	≈ 0.47 (±20–25%)	consistent
m_c/m_s	flavour law	11.76	11.76	< 1%
m_t/m_b	flavour law	59.4	≈ 59.3	< 1%

The RG-invariant b/s target uses m_b/m_s = (m_b/m_c)(m_c/m_s) ≈ 4.58 × 11.76 ≈ 53.9 with same-charge invariance ensuring scale-independence.

Normalised grid (d = 1).

	u	d	s	c	b	t
Constructed	0.4615	1	20	235.2	1035.6	61,515
Target	0.47	1	20	235.2	1078	64,000
Cell tension	loose	—	import	<1%	3.9%	inherits b

Chain: c = 20 × 11.76 = 235.2; b = 20 × 51.78 = 1035.6; t = 1035.6 × 59.4 = 61,515. The single new quantitative prediction is b/s = 51.78 (4%); s/d is imported from Route 1, t inherits b through t/b, and u/d, c/s, t/b are matches pending the audits of §10.1.

Appendix B — The Top Scale Trap (a diagnosed artifact, not an anomaly)

Chaining the grid to an absolute scale produces an apparent top-mass anomaly that is a unit error, not physics. It is recorded here so the result is not reopened later as a real discrepancy. This appendix sits under gate G-ABS; it does not reopen absolute masses, it closes one false alarm within them.

The apparent anomaly. Anchoring at $d = m_d(2\ \text{GeV}) \approx 4.7$ MeV and chaining the grid gives

$$m_t(\text{pred}) = 61{,}515 \times 4.7\ \text{MeV} \approx 289\ \text{GeV}$$

against the observed top. Compared to the pole mass ≈ 173 GeV the overshoot is ≈ 1.67; compared to the MS-bar top $m_t(m_t) \approx 162.5$ GeV it is

$$289.1 / 162.5 \approx 1.779$$

The note that produced this reading proposed four structural fixes (an over-high t/b, double-counted colour, a less-saturated top, an unstabilised top). None is needed.

The diagnosis. The grid is internally a 2 GeV object. $d = m_d(2\ \text{GeV})$; s/d and b/s are same-charge RG-invariant ratios that preserve that scale; c/s and u/d are conventionally 2 GeV. So u, d, s, c, b all emerge as their 2 GeV MS-bar masses, and they match — $b = 1035.6 \times 4.7\ \text{MeV} \approx 4.87$ GeV against $m_b(2\ \text{GeV}) \approx 4.9$ GeV, under 1%.

The single exception is t/b = 59.4. It is **not** a 2 GeV ratio; it is the high-scale ratio $m_t(m_t)/m_b(m_t) \approx 162.5 / 2.74 \approx 59.4$, evaluated at m_t (the cell was graded on exactly this convention mismatch in §7). Chaining therefore multiplies a b mass defined at 2 GeV (4.87 GeV) by a ratio in which the b mass is the m_t-scale value (2.74 GeV). The two scales differ by the running of the b mass between them.

The smoking gun. The overshoot equals the b-quark running factor between the two scales:

overshoot: $289.1 / 162.5 = 1.779$ b running: $m_b(2\ \text{GeV}) / m_b(m_t) = 4.87 / 2.74 = 1.777$

agreement to ≈ 0.1%. The top is not heavy by a closure factor; it is heavy by exactly the amount the b mass runs from 2 GeV to m_t.

The fix. Read the down ladder at the top scale for the t/b step:

$$m_t = m_b(m_t) \times 59.4 = 2.74 \times 59.4 \approx 163\ \text{GeV} = m_t(m_t),\ \text{to } 0.2\%$$

The 289 GeV was never a prediction; it was a scale-chaining error. The residual ≈ 6% between 163 GeV and the pole mass 173 GeV is the ordinary pole/MS-bar conversion, a separate and legitimate convention step. The note's single empirical factor 0.60 = 173/289 conflates the two:

0.60 ≈ (b running 0.563) × (pole conversion 1.065), which is why it lands near 3/5 and 7/12. Those are not closure integers surfacing; they are a smooth RG running factor (true value ≈ 0.56) passing near small fractions. Fitting a closure integer to a running coupling is the failure mode the parameter-ledger discipline exists to prevent.

Consequence for the grid. After the scale fix the top is among the *best* cells, not the worst — but this restores it to "matches by construction," not "new success," and the match is only as strong as the t/b cell's derive-versus-calibrate status (§10.1). If 59.4 was read from $m_t(m_t)/m_b(m_t)$, then "the grid gets the top right" is circular in the same way s/d = 20 is an import. The general rule banked here: the grid is a single-scale (2 GeV) object; absolute masses are safe only if every chained ratio shares a scale, and the t/b step is the one cell that breaks the convention. No chaining across scales. This is why absolute masses remain out of scope under G-ABS, and why each cell's evaluation scale must be stated before any absolute-mass work begins.