The C_3 Common-Curvature Extraction in VERSF — A Conditional Theorem for the CKM Triangle Residue

How a Shared Second-to-Third Twist Can Generate the Tiny First-to-Third Quark Mixing — Its Exact Consequence, and the Five Local Conditions Still Needed to Derive It

Keith Taylor *VERSF Theoretical Physics Programme*

General-Reader Summary

Matter comes in three "generations." The first is the lightest, the second heavier, the third heavier still. Particles in one generation can turn into — "mix with" — those in another, and among quarks this mixing is strikingly lopsided: neighbouring generations one and two mix noticeably, two and three only slightly, and one and three only very faintly.

This paper is about where that faintest mixing — between the first and third generations — comes from. The easy move would be to insert a small number by hand to match what experiments measure. That would explain nothing. The harder and more honest goal is to have the number appear on its own, forced by the structure of the theory rather than chosen to fit.

The mechanism is a kind of two-step. The mixing between generations one and two is already built in; picture it as an open doorway. The proposal is that generations two and three share a single common "twist." On its own, that twist does nothing to the first-and-third pairing. But a twist followed by a step through the doorway does not land in the same place as the doorway followed by the twist — the order matters — and that mismatch is exactly what produces a tiny first-and-third effect. Nobody dials the small number in; it emerges because two structures already present do not combine neatly.

The appeal of this is that it leaves no room to fiddle. If the deeper theory hands over a twist of the right, evenly shared ("democratic") kind, then both the size and the orientation of the resulting first-and-third effect are fixed by exact arithmetic — there is nothing left to adjust. The paper works that arithmetic out in full and checks every step on a computer.

What the paper is careful *not* to claim is that the deeper theory actually delivers such a twist. It proves an "if–then": *if* the underlying structure supplies an evenly shared two-and-three twist with one particular orientation, *then* the first-and-third mixing follows exactly, while barely disturbing the larger mixings already in place. A short list of things — five, to be precise — remains genuinely unproven, and is owed to a later, more detailed calculation.

So this is a conditional result, and it says so plainly. It states exactly what the deeper theory must supply for the small mixing to be *derived rather than fitted*, shows that everything after that point is exact, and lists the specific ways the idea could fail.

Contents

Abstract

The weak-doublet Hessian audit reduced the quark-side flavour residue to one role-even frontier object: the Cabibbo-protected common curvature inside the projected closure Hamiltonian

$$H\_W = P\_W^\dagger H\_{cl} P\_W,$$

— the symmetric ordered compression, self-adjoint by construction — with frame generator

$$\Omega\_W = i \varepsilon\_W H\_W = \Omega\_{even} \otimes I + \Omega\_{odd} \otimes \tau_3 + \Omega\_{mix}.$$

The audit identified the required role-even output as a leading 2↔3 anti-Hermitian curvature

$$\Omega_0(z) = z E_{23} - z^* E_{32},$$

with convention-invariant value $\zeta_{C3} = (b/\sqrt{3})\, e^{5\pi i/6}$.

This paper supplies the **conditional** C_3 common-curvature extraction and is explicit about its grade. It isolates the pre-readout role-even quark block K_{even}^q, states the generation-cyclic covariance test $[K_{even,pre}^q, R_3] = 0$, decomposes the support into Killing-orthonormal branch components Z_{12}, Z_{23}, Z_{31}, and passes the result through the Cabibbo-protection readout. **If** the common-mode block is a single pure democratic C_3 orbit and its inherited norm is b, the surviving branch has amplitude $|\zeta| = b/\sqrt{3}$. **If** the minimal Hermitian half-transport selects the positive quark phase root, the branch phase is $\arg \zeta = 5\pi/6$.

The exact commutator identity

$[\Omega_0(z), \Omega_q]_{13} = -\, a z$ (verified symbolically — Appendix A)

then gives, in every split convention,

$\Delta_{13} = \frac{1}{2} a\, \zeta_{C3}$ (exact to leading BCH order — verified, Appendix A).

Thus the CKM triangle residue is *generated by the Cabibbo doorway* rather than inserted directly. The paper separates exact algebra, conditional consequences, and owed microscopic returns at every step. It converts the C_3 claim into a local derive-or-reject calculation, and states plainly that the five projected-Hessian returns — C_3 covariance, single-orbit purity, the norm identification, 2↔3 readout survival, and the phase sign $\sigma_{\varphi q}$ — are **owed**, not yet computed.

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0. Epistemic Ledger

The grading discipline of the audit is retained. The purpose is to stop an algebraic consequence from masquerading as a projected-Hessian calculation, and to stop a projected-Hessian output from hiding inside a convention.

Grades. *Exact* — follows by algebra from defined objects. *Audit condition* — a consistency requirement the construction must satisfy. *Conditional* — holds if and only if a named owed return is supplied. *Owed* — a projected-Hessian output not yet computed.

Named-premise prefixes. [INH-·] inherited object; [EX-·] exact result; [AUD-·] audit condition; [SEL-·] selection rule; [LIFT-·] phase-lift rule; [COND-·] conditional result; [OWE-·] owed projected-Hessian return.

Object	Required statement	Grade
Weak-doublet block	$H_W = P_W^\dagger H_{cl} P_W$ ($P_W = P_Y P_R P_C$)	[INH-1] inherited, post-readout
Pre-readout block	$H_{pre} = P_{pre}^\dagger H_{cl} P_{pre}$ ($P_{pre} = P_R P_C$)	[INH-1′] inherited, omits readout P_Y

Object	**Required statement**	**Grade**
Weak-doublet Hermiticity	$H_W^\dagger = H_W$ (and $H_{pre}^\dagger = H_{pre}$)	[EX-0] guaranteed by construction
Frame generator	$\Omega_W = i\,\varepsilon_W H_W$, $\Omega_{pre} = i\,\varepsilon_W H_{pre}$	[EX-1] exact; anti-Hermitian via [EX-0]
Role-even block	$\Omega_{even} = \frac{1}{2}\,\mathrm{Tr}_{role}(\Omega_W) = i\,\varepsilon_W H_{even}$	[EX-2] exact Pauli projection
Quark role-even blocks	$K_{even}^q \equiv \Omega_{even}^q$ (post), $K_{even,pre}^q$ (pre) — anti-Hermitian; Hermitian parent H_{even}^q	[EX-3] exact compression once P_q defined
Anti-Hermitian branch form	branches are $A_{ij}(\eta) = \eta\, E_{ij} - \eta^*\, E_{ji}$	[AUD-1] frame consistency, secured by [EX-0]
C_3 covariance	$[K_{even,pre}^q, R_3] = 0$	[OWE-1]
Single-orbit purity	common mode is one democratic C_3 orbit, no multiplicity leakage	[OWE-2]
Equal branch weights	$w_{12} = w_{23} = w_{31}$	[EX-4'] consequence of [OWE-1] ∧ [OWE-2]
Branch basis	Z_{12}, Z_{23}, Z_{31}	[EX-4] exact once C_3 support fixed
Killing orthogonality	$\langle Z_i, Z_j\rangle_K = 0,\ i \neq j$	[EX-5] exact for distinct branches
Inherited norm identification	total common-mode norm = b	[OWE-3]
C_3 amplitude	$\lvert\zeta\rvert = b/\sqrt{3}$	[COND-1] exact given [OWE-1]∧[OWE-2]∧[OWE-3]
Cabibbo-protection rule	selection principle on the readout	[SEL-1] stated rule
Cabibbo survival	leading survivor is the 2↔3 branch	[OWE-4]
Minimal-lift rule	$\arg\zeta$ from the positive Hermitian root	[LIFT-1] premise; justification owed unless inherited theorem forces it
Phase sign	$\sigma_{\varphi q} = +1 \Rightarrow \arg\zeta = 5\pi/6$	[OWE-5]
CKM commutator	$[\Omega_0(z), \Omega_q]_{13} = -\,a\,z$	[EX-6] exact ✓
Diagonal rephasing	$b(z^*-z)$, $b(z-z^*)$ removable, no physical angle	[EX-6r] rephasing audit ✓
Convention-invariant curvature	$\zeta = \lambda\, z$	[EX-7] convention audit
Triangle residue	$\Delta_{13} = \frac{1}{2}\, a\, \zeta$	[EX-8] exact to leading BCH order ✓
Cabibbo back-reaction (leading)	$\lvert\Delta_{12}\rvert_{lead} = \frac{1}{2}\, c\, \lvert\zeta\rvert$	[EX-9] correct first BCH coefficient ✓

Object	Required statement	Grade
Cabibbo back-reaction (converged)	$|\Delta_{12}|/a = 2.11\times10^{-4}$	[EX-9'] all-orders in the inherited two-factor composition ✓
Failure mode	non-C_3, impure orbit, wrong norm, wrong survivor, wrong phase	[AUD-2] local falsification

The decisive **owed** returns of this paper are therefore exactly five:

[OWE-1] [K_even,pre^q, R_3] = 0 [OWE-2] single democratic C_3 orbit, no multiplicity leakage [OWE-3] total common-mode norm = b [OWE-4] Ω_even^(23) is the leading readout survivor [OWE-5] σ_φq selecting arg ζ = 5π/6

Everything graded [EX-·] below has been confirmed symbolically (Appendix A). Nothing graded [OWE-·] is asserted as computed.

Dependency structure of the owed returns. The five are listed because each has a distinct falsification test (C3, C4, C5, C6–C8, C9), but they are **not five co-equal independent debts**. The honest map is *four-and-a-corollary*:

- *Orbit pair* — [OWE-1] covariance and [OWE-2] orbit-exhaustion (purity) are logically distinct tests (commuting with R_3 vs. occupying exactly the orbit). Jointly they force equal democratic coefficients $\eta_{12} = \eta_{23} = \eta_{31}$ as a **theorem** (§3, Appendix A.4). Thus *equal branch weights is never separately owed*; it is a corollary, not a return. The distinctness itself is an **ambient-space fact**: in a bare generation $\mathfrak{su}(3)$ the two collapse — covariance forces purity, since the C_3-commutant there *is* the democratic orbit (Appendix A.6) — and [OWE-2] becomes vacuous. They separate only under the $\mathfrak{su}(8)$ embedding, where C_3 can act with multiplicity > 1. One may equivalently bundle the pair as a single "C_3-orbit return" and quote four independent debts — a bookkeeping choice, not a physics change.
- *Entanglement* — [OWE-2] purity and [OWE-3] norm are not independent: purity fixes *which* invariant subspace carries the common mode, the norm fixes *how much* of the inherited norm lands there (§4 standing note).
- *Crux* — [OWE-4] readout survival is the **load-bearing return, more than the others combined**. Because the C_3 symmetry is democratic, the pre-readout block carries a 1↔2 common-mode curvature of *equal amplitude* to the 2↔3 one. The symmetry is therefore, by construction, blind to direction: it cannot itself prefer 2↔3. All directionality of the mechanism — the entire reason the residue lands in 1↔3 and not 1↔2 — lives in [OWE-4] (§5).

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1. Purpose and Status Claim

The weak-doublet Hessian audit left a finite list of required computations. The most economical first computation is the quark role-even common curvature, because its algebraic consequence is already fixed by [EX-6]–[EX-8].

The quark generator inherited from the role-odd block is [INH-1]:

$$\Omega_q = \begin{pmatrix} 0 & a & c\cdot e^{i\varphi} \\ -a & 0 & b \\ -c\cdot e^{-i\varphi} & -b & 0 \end{pmatrix}$$

with

$a = 9/40$, $b = 81/2000$, $c = 243/100000$, $\varphi = 2\pi/3$.

The role-even block must not be used as a fitting surface. It must be *read from* the projected closure Hessian. The target is not "choose a correction that repairs CKM." The target is:

compute the role-even quark block and test whether the first surviving common-mode curvature is the 2↔3 branch of a single democratic C_3 orbit.

If it is, the missing 1↔3 triangle residue is no longer an independent parameter — it is generated by the commutator with the Cabibbo entry.

Status claim. This paper proves the *conditional* implication

$[\text{OWE-1}] \wedge [\text{OWE-2}] \wedge [\text{OWE-3}] \wedge [\text{OWE-4}] \wedge [\text{OWE-5}] \Rightarrow \zeta_{C3} = (b/\sqrt{3})\, e^{5\pi i/6} \Rightarrow \Delta_{13} = \tfrac{1}{2}\, a\, \zeta_{C3}$,

with every step from the owed returns onward graded Exact and verified. It does **not** compute the left-hand side. The microscopic objects H_{cl}, P_C, P_R, P_Y and the resulting entries of K_{even}^q are not evaluated here. Consequently the yes/no verdict on whether the projected Hessian returns the branch is, on the evidence of this paper alone:

not yet proven — conditional success, not completed calculation.

This is the honest scope of the result, and §13 states it again once the algebra is in hand.

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2. The Role-Even Quark Block

Let $H_{cl} = G|_{\mathfrak{su}(8)}$ be the inherited closure Hessian, and let

$$P_W = P_Y P_R P_C$$

be the ordered weak-doublet compression — P_C the generation-completion projection, P_R the weak-role projection, P_Y the mass/readout projection. The weak-doublet block is defined as the **symmetric ordered compression**

H_W = P_W† H_cl P_W, [INH-1]

with P_W† = P_C† P_R† P_Y†. This definition is deliberate: it secures Hermiticity without assuming the factors commute. Since H_cl† = H_cl,

H_W† = (P_W† H_cl P_W)† = P_W† H_cl† (P_W†)† = P_W† H_cl P_W = H_W, [EX-0]

so H_W is self-adjoint by construction for **any** ordering of P_C, P_R, P_Y, whether or not they commute. When the factors are self-adjoint and commute on the admissible support, P_W is an ordinary projector, P_W† = P_W, and H_W reduces to the familiar P_W H_cl P_W. [AUD-3]: commutativity of the factors remains relevant — not for Hermiticity, which [EX-0] now guarantees, but for the clean weak-role Pauli reading (falsification C10).

Two compressions, pre- and post-readout. The mass/readout projector P_Y is the operator that selects which branches survive into the observable matrix. The C_3 orbit structure is a property of the block *before* that selection. We therefore use two distinct compressions. The **pre-readout** compression omits P_Y:

P_pre = P_R P_C, H_pre = P_pre† H_cl P_pre [INH-1′] (Hermitian by the same [EX-0] argument),

and the **post-readout** compression is the full weak-doublet block H_W = P_W† H_cl P_W defined above. The readout maps the first to the second by inserting P_Y; the whole mechanism is the implication *pre-readout democracy → post-readout 2↔3 survival*, so the two must not be conflated.

The corresponding frame generators are

Ω_pre = i ε_W H_pre, Ω_W = i ε_W H_W, [EX-1]

both anti-Hermitian, since H_pre and H_W are Hermitian [EX-0]. This is the load-bearing point that the anti-Hermitian branch form [AUD-1] rests on; it no longer depends on any commutativity assumption.

On the two-dimensional weak-role space the Pauli decomposition is exact:

Ω_W = $\Omega_0 \otimes I + \Omega_1 \otimes \tau_1 + \Omega_2 \otimes \tau_2 + \Omega_3 \otimes \tau_3$.

Define the **Hermitian-side** role-even blocks (built from the Hermitian H_pre, H_W):

H_even,pre = ½ Tr_role(H_pre), H_even = ½ Tr_role(H_W), both Hermitian, and the corresponding **anti-Hermitian frame** role-even generators by the $i\,\varepsilon_W$ rotation:

$\Omega_{\text{even,pre}} = i\,\varepsilon_W H_{\text{even,pre}} = \frac{1}{2}\,\mathrm{Tr}_{\text{role}}(\Omega_{\text{pre}})$, $\Omega_{\text{even}} = i\,\varepsilon_W H_{\text{even}} = \frac{1}{2}\,\mathrm{Tr}_{\text{role}}(\Omega_W) = \Omega_0$, [EX-2] $\Omega_{\text{odd}} = \frac{1}{2}\,\mathrm{Tr}_{\text{role}}(\Omega_W \tau_3)$, $\Omega_{\text{mix}} = \Omega_1 \otimes \tau_1 + \Omega_2 \otimes \tau_2$.

The quark role-even blocks are the corresponding compressions with P_q the quark-sector generation readout. To keep the two sides of the i-rotation unambiguous we name them explicitly:

pre-readout, anti-Hermitian: $K_{\text{even,pre}}^{q} \equiv \Omega_{\text{even,pre}}^{q} = P_q\, \Omega_{\text{even,pre}}\, P_q$ (built from H_{pre}, omits P_Y), **post-readout, anti-Hermitian:** $K_{\text{even}}^{q} \equiv \Omega_{\text{even}}^{q} = P_q\, \Omega_{\text{even}}\, P_q$ (built from H_W, includes P_Y), **Hermitian parents:** $H_{\text{even,pre}}^{q} = P_q\, H_{\text{even,pre}}\, P_q$, $H_{\text{even}}^{q} = P_q\, H_{\text{even}}\, P_q$, with $\Omega_{\text{even,(pre)}}^{q} = i\,\varepsilon_W H_{\text{even,(pre)}}^{q}$. [EX-3]

Throughout, **K denotes the anti-Hermitian frame block** ($K = i\,\varepsilon_W H_{\text{even}}^{q}$); the Hermitian parent is H_{even}^{q}. This matters in §8: the minimal lift of the phase is taken on the *Hermitian* side (on H_{even}^{q}) and only then rotated by i to the frame side. The distinction is load-bearing: the C_3 orbit is a *pre-readout* structure (test on $K_{\text{even,pre}}^{q}$), while the readout P_Y may select or suppress branches (test on K_{even}^{q}). The extraction therefore proceeds in two stages:

1. **C_3 orbit test before readout** — compute $K_{\text{even,pre}}^{q}$ and test cyclic covariance [OWE-1] and single-orbit purity [OWE-2].
2. **Cabibbo-protected survival after readout** — compute the selected branch in K_{even}^{q} [OWE-4].

Since Ω_{pre} and Ω_W are anti-Hermitian [EX-0]→[EX-1], every off-diagonal branch of $K_{\text{even,pre}}^{q}$ and K_{even}^{q} must take the form

$A_{ij}(\eta) = \eta\, E_{ij} - \eta^{*} E_{ji}$. [AUD-1]

A computed block that violates this falsifies the construction (condition C2, §12).

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3. The C_3 Covariance Test and Single-Orbit Purity

Let R_3 be the generation-cycle operator $1 \to 2 \to 3 \to 1$, acting on matrix units by conjugation:

$R_3 E_{12} R_3^{-1} = E_{23}$, $R_3 E_{23} R_3^{-1} = E_{31}$, $R_3 E_{31} R_3^{-1} = E_{12}$.

The extraction opens with two projected-Hessian returns. The first is covariance [OWE-1]:

$[K_{\text{even,pre}}^{q}, R_3] = 0$.

The second is single-orbit purity [OWE-2]:

the common-mode content of K_even,pre^q is one democratic off-diagonal C_3 orbit, with no leakage into other C_3-invariant components (the Cartan/diagonal invariants, or any second copy of the cyclic representation).

These are genuinely distinct, and the second is the stronger. Here is why they must be separated. Write the candidate common-mode block on the off-diagonal orbit as

$$X = A_{12}(\eta_{12}) + A_{23}(\eta_{23}) + A_{31}(\eta_{31}).$$

A direct computation (Appendix A) shows that covariance alone, $[X, R_3] = 0$, already forces

$$\eta_{12} = \eta_{23} = \eta_{31},$$

i.e. **equal weights and equal phase**. So on a pure single orbit, "democratic weights" is not an independent assumption — it is a theorem [EX-4'], a consequence of [OWE-1]. The real owed content of [OWE-2] is therefore not equality of weights but *purity*: that the projected block is this single orbit and nothing else. A block that is C_3-covariant but contaminated by a C_3-invariant diagonal piece, or that carries a second multiplicity of the cyclic representation, would pass [OWE-1] yet fail to deliver the clean amplitude. The falsifiable test is multiplicity-counting:

dim(C_3-isotypic common-mode support of K_even,pre^q) = 1, realised as the single democratic orbit.

Why purity is non-vacuous: it is an ambient-space condition. A reader picturing the block in a bare generation $\mathfrak{su}(3)$ would correctly object that purity follows from covariance and is empty. That objection is right *downstairs*: the C_3-commutant of $\mathfrak{su}(3)$ under cyclic conjugation is exactly two-real-dimensional, and both basis elements are the democratic off-diagonal orbit (Appendix A.6). There is no C_3-invariant Cartan direction in $\mathfrak{su}(3)$ — the cyclic-fixed diagonal subspace is zero — and no second copy of the cyclic representation. So in $\mathfrak{su}(3)$, covariance alone forces the block into the single democratic orbit and purity is automatic. The contamination channels named above — invariant Cartan directions and a second cyclic copy — are **ambient-space phenomena**: they exist only because the generation C_3 is embedded in the larger compressed space $\mathfrak{su}(8)$, where it can act with multiplicity greater than one. [OWE-2] is therefore a genuine, non-vacuous owed return precisely because the microscopic support is bigger than a single $\mathfrak{su}(3)$ generation triple; it asks whether the embedding has introduced the extra invariant directions that bare $\mathfrak{su}(3)$ lacks, and whether the role-even common mode nonetheless lands in exactly one regular-representation copy.

If purity fails — if the common mode is a *mixture* of invariant components — the amplitude derivation of §4 does not go through (condition C4). If covariance fails, the orbit structure is absent entirely (condition C3).

The weight diagnostic w_ij = ‖Π_ij K_even,pre^q Π_ij‖_K² remains useful as a *downstream signature* only: under [OWE-1] ∧ [OWE-2] it must return $w_{12} = w_{23} = w_{31}$ automatically, so an observed inequality flags broken purity. But it is not the real computation. The real [OWE-2] test is a **representation-multiplicity count** in the ambient space (§11.2): how many copies of the C_3

regular representation appear in the compressed generation support, and whether the role-even common mode occupies exactly one.

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4. Killing-Orthonormal Branch Decomposition

Take the anti-Hermitian branch basis $A_{ij}(\eta) = \eta E_{ij} - \eta^* E_{ji}$ and the Killing-normalised inner product

$\langle X, Y\rangle_K = -\frac{1}{2}\mathrm{Tr}(X Y)$

for anti-Hermitian generators in the chosen normalisation. Then $\|A_{ij}(\eta)\|_K^2 = \|\eta\|^2$, and distinct generation branches are orthogonal [EX-5]:

$\langle A_{12}(\eta), A_{23}(\xi)\rangle_K = \langle A_{23}(\eta), A_{31}(\xi)\rangle_K = \langle A_{31}(\eta), A_{12}(\xi)\rangle_K = 0,$

because products such as $E_{12} E_{23} = E_{13}$ have zero trace, and the trace is the only quantity entering the branch norm.

Let the pre-readout C_3 common curvature be $Z_{C3} = Z_{12} + Z_{23} + Z_{31}$. **Given** covariance [OWE-1] on a **pure** orbit [OWE-2], §3 forces $\eta_{12} = \eta_{23} = \eta_{31} = \eta$, hence

$\|Z_{12}\|_K = \|Z_{23}\|_K = \|Z_{31}\|_K = \|\zeta\|,$

and orthogonality gives the Pythagorean sum

$\|Z_{C3}\|_K^2 = \|Z_{12}\|_K^2 + \|Z_{23}\|_K^2 + \|Z_{31}\|_K^2 = 3 \|\zeta\|^2.$

If the inherited common-mode norm on this orbit is b [OWE-3], then

$3 \|\zeta\|^2 = b^2 \Rightarrow \|\zeta\| = b/\sqrt{3}$. [COND-1]

The dependency is now explicit and minimal. [COND-1] rests on **two** owed returns — purity [OWE-2], which (with covariance) supplies the equal split, and the norm identification [OWE-3], which supplies the scale. The map from {[OWE-2], [OWE-3]} to $b/\sqrt{3}$ is exact; the inputs are owed. Either failing breaks [COND-1] (conditions C4 and C5).

A standing referee note for the explicit microscopic paper: [OWE-2] and [OWE-3] are not obviously independent. Purity is a statement about *which* C_3-invariant subspace the block occupies; the norm identification is a statement about *how much* of the inherited role-odd norm maps onto that subspace. The microscopic compression must pin down both — the map from the role-odd norm to the role-even orbit norm is the object that fixes [OWE-3], and it should be computed alongside the multiplicity count that fixes [OWE-2].

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5. Cabibbo-Protection Selection

The role-even common curvature is not permitted to repair CKM by arbitrary insertion. The selection rule [SEL-1]:

the first common-mode curvature must preserve the already successful Cabibbo entry at leading order, must not directly install the 1↔3 correction, and must generate any 1↔3 residue only through the inherited quark generator.

Cabibbo-Protection Lemma (selection rule + owed return)

Stated premises (selection choices under [SEL-1], not derived):

1. the role-odd block has already returned Ω_q [INH-1];
2. the leading 1↔2 Cabibbo entry a is not reselected by the role-even readout;
3. a direct 1↔3 common-mode survivor is excluded as a target insertion;
4. the first 1↔3 residue must arise through the lowest nontrivial commutator with Ω_q.

Consequence of the premises. Under 1–4, the only admissible leading common-mode survivor is the 2↔3 branch

$\Omega_0(z) = z\, E_{23} - z^*\, E_{32}$.

The argument is structural: a surviving 1↔2 curvature competes with the fixed Cabibbo channel; a surviving 1↔3 curvature *is* the residue meant to be explained; only the 2↔3 curvature is neither the doorway nor the target, becoming visible in the 1↔3 channel solely through its failure to commute with the doorway.

Owed return [OWE-4]. Premises 2–4 are selection *choices* under [SEL-1]. That the *actual* readout obeys them — i.e. that the projected K_even^q in fact returns the 2↔3 branch as leading survivor — is a projected-Hessian output, not a theorem. The lemma states what *must* be returned; it does not prove the return. Explicitly,

Π_readout(Z_C3) = Z_{23} + (higher-order suppressed), i.e. Ω_even^(23) = $z\, E_{23} - z^*\, E_{32}$ leading.

If the readout returns 1↔2 first, Cabibbo protection fails (C6). If it returns 1↔3 first, the residue is inserted rather than generated (C7). If no branch survives, there is no C_3 correction (C8).

The crux of the mechanism. This return carries disproportionate weight, and the reason is worth stating plainly. Covariance [OWE-1] on a pure orbit [OWE-2] forces all three branch coefficients *equal* — so before readout there is a 1↔2 common-mode curvature of the **same amplitude** as the 2↔3 one. The C_3 symmetry is democratic precisely so that it cannot, by itself, prefer one branch over another. Consequently the entire directionality of the mechanism — why the generated residue lands in 1↔3 via the doorway, rather than competing with the Cabibbo entry in 1↔2 — rests *wholly* on [OWE-4]. Premises 2–4 above are selection choices, not theorems; the symmetry is deliberately silent on direction. Of the five owed returns, [OWE-4] is the one whose failure most directly collapses the construction, and it is the return the explicit microscopic paper should compute first.

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6. The Exact Commutator Extraction

Take Ω_q [INH-1] and the surviving role-even curvature $\Omega_0(z) = z\,E_{23} - z^*\,E_{32}$. Row 1 of Ω_0 vanishes, so $(\Omega_0\,\Omega_q)_{13} = 0$, while the reverse product has a single contribution $(\Omega_q\,\Omega_0)_{13} = (\Omega_q)_{12}\,(\Omega_0)_{23} = a\,z$. Hence the central exact identity [EX-6]:

$$[\Omega_0(z), \Omega_q]_{13} = (\Omega_0\,\Omega_q - \Omega_q\,\Omega_0)_{13} = -\,a\,z.$$

The full commutator (verified symbolically, Appendix A) is

$$[\Omega_0, \Omega_q] = \begin{pmatrix} 0 & c\cdot z^*\cdot e^{i\varphi} & -a\cdot z \\ -c\cdot z\cdot e^{-i\varphi} & b(z^*-z) & 0 \\ a\cdot z^* & 0 & b(z-z^*) \end{pmatrix}$$

The desired entry is the 1↔3 term $-a\,z$. The 1↔2 back-reaction is suppressed by c, not a, hence far smaller than the Cabibbo entry.

Diagonal rephasing audit [EX-6r]

The diagonal terms must not be waved away; they are audited. Writing $z = z_r + i\,z_i$,

$(2,2) = b(z^* - z) = -\,2\,i\,b \cdot z_i$, $(3,3) = b(z - z^*) = +\,2\,i\,b \cdot z_i$, $(1,1) = 0$.

So the diagonal block is

$\mathrm{diag}(\,0, -\,2\,i\,b \cdot \mathrm{Im}\,z, +\,2\,i\,b \cdot \mathrm{Im}\,z\,)$,

which is **traceless, purely imaginary, and diagonal** (verified, Appendix A). It therefore lies in the quark Cartan and generates a diagonal phase rotation — exactly an allowed left-quark-field rephasing. A diagonal phase rotation is unphysical in the CKM matrix: it is absorbed into the quark field redefinitions and contributes **no new physical mixing angle**. As shown below this holds at all orders, not merely leading (the removability *form* is structurally guaranteed; only the ambient availability is live — C11b). The only physical content of $[\Omega_0, \Omega_q]$ in the mixing sector is the off-diagonal $-a\,z$ (and the c-suppressed back-reaction).

Removability is load-bearing, not a convenience — and it is exact to all orders. Two facts settle the diagonal term. *First, its size.* Compared level-consistently — generator diagonal against generator increment — the diagonal entry of log V_{CKM} is $b \cdot \mathrm{Im}\,\zeta$, magnitude $b\|\mathrm{Im}\,\zeta\| = b\|\zeta\|/2 = 0.000474$, while $\|\Delta_{13}\| = \tfrac{1}{2}\,a\|\zeta\| = 0.0026306$. The ratio is exactly

$\|\text{generator diagonal}\| / \|\Delta_{13}\| = b/a = 0.18$.

(Note: a naïve comparison of the *commutator* diagonal, 2b·Im ζ = 0.000947, against the BCH-halved generator increment Δ_{13} would mismatch levels and overstate this as 0.36; the level-consistent figure is b/a = 0.18.) Either way the diagonal term is **the same order as the triangle residue — ~18% of it — not parametrically suppressed**. Its harmlessness cannot come from smallness. *Second, its removability.* Because V_CKM is unitary, L = log V_CKM is anti-Hermitian, so its diagonal is purely imaginary; and because det V_CKM = det e^{X} det e^{Y} = e^{tr Ω_q} = 1, L is traceless. A traceless imaginary diagonal is exactly a Cartan generator — a left-quark rephasing — and this holds **at every order**, not merely the leading commutator (Appendix A.5). So the *form* of the diagonal as a removable rephasing is structurally guaranteed; [EX-6r]'s argument does not weaken with higher orders.

What remains genuinely live is narrower and couples to the embedding: whether the Cartan rephasing this generates is a *bona fide field-redefinition freedom* in the ambient $\mathfrak{su}(8)$, or whether the embedding makes that Cartan direction physical. Bare $\mathfrak{su}(3)$ has no C_3-fixed Cartan direction (Appendix A.6), but the same ambient structure that makes [OWE-2] non-vacuous could in principle render this diagonal direction non-removable. The live residue of C11 is therefore not the size or the form of the diagonal — both settled — but the availability of the ambient rephasing (C11b below).

Rephasing-invariant content (precision). Because [EX-6r] is precisely the generator of the left-quark phase freedom, the *physical* output is rephasing-invariant: a left-field rephasing V → P_u† V P_d multiplies entries by phases, preserving every modulus ‖V_ij‖ and the Jarlskog invariant while shifting individual entry phases. The rephasing-invariant content of this result is therefore $\|\Delta_{13}\| = 0.0026306$ and its contribution to the unitarity triangle; the complex value −0.0022781 + 0.0013153 i quoted in §9 is the entry in *one fixed rephasing convention*, and its bare phase is convention-dependent — [EX-6r] is exactly the freedom that moves it.

The hierarchy produced is therefore

doorway: a • common curvature: $\zeta \sim b/\sqrt{3}$ • triangle residue: $\Delta_{13} \sim \tfrac{1}{2}\, a\, \zeta$ • back-reaction (leading): $\Delta_{12} \sim \tfrac{1}{2}\, c\, \zeta$.

A caution carried forward to §7 and §10: ½ c ζ is the *correct leading BCH coefficient* of the (1,2) entry, but it is a poor proxy for the converged back-reaction. Unlike the (1,3) channel — whose leading term is a-driven and large, so its higher-order corrections are relatively negligible — the (1,2) leading term is c-driven and small, while its higher-order corrections are a- and b-driven and hence comparable in size to the leading term itself. The converged value is ~1.7× the leading estimate (Appendix A.5).

Since $c \ll a$, the mechanism creates the 1↔3 residue without spoiling the leading Cabibbo channel.

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7. Convention-Invariant Residue

The common curvature appears as a relative weak-role split, and different conventions assign different factors of two to Ω_0. The physical statement is therefore written in a convention-invariant curvature. Introduce a split parameter λ and write

$$V_{CKM} = \exp(-\lambda\,\Omega_0 + \tfrac{1}{2}\,\Omega_q) \cdot \exp(+\lambda\,\Omega_0 + \tfrac{1}{2}\,\Omega_q).$$

This antisymmetric split is not a free convention; it is the mathematical content of the no-direct-insertion principle (§5, premise 3). Placing $-\lambda\Omega_0$ in X and $+\lambda\Omega_0$ in Y, with the symmetric $\tfrac{1}{2}\Omega_q$ in both, is exactly what makes $X + Y = \Omega_q$: Ω_0 *cancels at first order* and can enter the result only through the commutator $[X, Y]$. So the 2↔3 curvature is structurally forbidden from appearing as a direct additive entry, and the 1↔3 residue is forced to be commutator-generated — premise 3 of §5 realised algebraically rather than imposed. Moreover the construction is **exactly λ-invariant**: since $\lambda\,\Omega_0(z) = \Omega_0(\lambda z) = \Omega_0(\zeta)$, both $X = -\Omega_0(\zeta) + \tfrac{1}{2}\Omega_q$ and $Y = +\Omega_0(\zeta) + \tfrac{1}{2}\Omega_q$ depend only on ζ, not on the split parameter. V_{CKM} therefore has an unambiguous value, fixed by ζ alone — the well-posedness that licenses treating ζ, not z, as the physical object (Appendix A.5). One caveat on the word "all-orders": this value is converged with respect to the **two-factor doublet composition** $V_{CKM} = \exp(X)\exp(Y)$, which is the inherited modeling choice for how the two weak-role half-transports compose — not something this paper derives. "Exact" here means exact in that composition law; a different inherited closure (a single exponential, or a three-factor structure) would yield a different convergent value. The λ-invariance is a property of the two-factor form, and holds within it.

With $X = -\lambda\,\Omega_0 + \tfrac{1}{2}\,\Omega_q$ and $Y = +\lambda\,\Omega_0 + \tfrac{1}{2}\,\Omega_q$, one has $X + Y = \Omega_q$ [verified], and the first Baker–Campbell–Hausdorff correction is

$$\tfrac{1}{2}\,[X, Y] = -\,(\lambda/2)\,[\Omega_0, \Omega_q] \text{ [verified]},$$

so

$$\log V_{CKM} = \Omega_q - (\lambda/2)\,[\Omega_0, \Omega_q] + \mathcal{O}(\text{curvature}^2).$$

Defining the commutator-generated increment by subtracting the inherited bare entry,

$$\Delta_{13} = (\log V_{CKM})_{13} - (\Omega_q)_{13} = (\lambda/2)\,a\,z + \mathcal{O}(\text{curvature}^2),$$

and setting the convention-invariant curvature $\zeta = \lambda\,z$ [EX-7] gives, in every split convention,

$$\Delta_{13} = \tfrac{1}{2}\,a\,\zeta + \mathcal{O}(\text{curvature}^2). \text{ [EX-8, verified]}$$

The physical target is ζ, not z. The $\mathcal{O}(\text{curvature}^2)$ tail in the (1,3) channel is **discharged at all orders** by the converged computation of A.5 (C11a — leading is within 0.1% of converged), and the diagonal rephasing of §6 is removable in form at all orders (C11b live only via ambient availability). The same expansion does **not** converge as fast in the (1,2) channel: there the leading coefficient $\tfrac{1}{2}\,c\,\zeta$ is c-suppressed while the second-order term is a,b-driven, so the two are comparable and the leading estimate understates the converged back-reaction by ~1.7×

(Appendix A.5). λ-invariance gives the converged value unambiguously; it is quoted in §10 and §13.

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8. The C_3 Phase Branch

The amplitude extraction gives $\|\zeta\| = b/\sqrt{3}$ [COND-1]. The phase is a *separate* projected-Hamiltonian output [OWE-5]; it cannot be chosen after comparison with CKM data. The projected Hessian must return the branch sign $\sigma_{\varphi q}$.

The proposed structural rule is **minimal Hermitian half-transport** [LIFT-1]. The notation is kept strict by separating the two sides of the i-rotation:

- **h_q** — the *Hermitian-side* phase root;
- **g_q** — the *frame-side* branch, obtained from h_q by the i-rotation.

Let h_q satisfy $h_q^2 \propto O_q$ with $O_q = e^{2\pi i/3}$. The two Hermitian-side roots are

$h_q = e^{i\pi/3}$ or $h_q = e^{i4\pi/3}$.

The minimal Hermitian lift [LIFT-1] selects the principal root

$h_q = e^{i\pi/3}$.

The frame-side branch follows by the i-rotation $g_q = i \cdot h_q$:

$g_q = i \cdot e^{i\pi/3} = e^{i\pi/2} \cdot e^{i\pi/3} = e^{i5\pi/6}$,

so

$\arg \zeta = \arg g_q = \pi/2 + \pi/3 = 5\pi/6$.

Equivalently, the same frame-side branch is fixed by the complement-half-transport identity $g_q^2 = C \cdot O_q$ with $C = e^{i\pi}$:

$g_q^2 = e^{i\pi} \cdot e^{2\pi i/3} = e^{i5\pi/3} \Rightarrow g_q = e^{i5\pi/6}$.

These two routes are the **same result reached differently, not literally the same equation**. The first applies the i-rotation once, to the Hermitian root: $g_q = i \cdot h_q$. The second applies it at the level of the square, where the factor $C = e^{i\pi} = i^2$ is precisely that rotation squared — squaring $g_q = i \cdot h_q$ gives $g_q^2 = i^2 h_q^2 = C \cdot O_q$. So C encodes the frame rotation absorbed into the squared relation; the two paths agree because $i^2 = e^{i\pi}$, not because they assert one identity.

The phase audit is binary:

$\sigma_{\varphi q} = +1 \Rightarrow \arg \zeta = 5\pi/6$, $\sigma_{\varphi q} = -1 \Rightarrow$ antipodal branch ($\arg \zeta = 11\pi/6$), CKM phase claim fails (C9).

So the C_3 curvature target is

$$\zeta_{C3} = (b/\sqrt{3})\, e^{5\pi i/6},$$

with the exact rectangular form

$$\zeta_{C3} = -\, b/2 + i \cdot b/(2\sqrt{3}) = -\, 81/4000 + (27\sqrt{3}/4000)\, i = -0.02025 + 0.0116913\, i.$$

The phase debt is split, and that asymmetry matters. Of the five returns, the phase is the only one whose proof debt does not sit wholly in the owed column. It rests on two things: the rule [LIFT-1] that maps the Hermitian root to the frame branch by selecting the *principal* root, and the sign [OWE-5] that the projected Hessian must return. This splitting is honest only if [LIFT-1] is a forced structural principle rather than a convenient choice. If the inherited programme's Minimal-Hermitian-Lift theorem genuinely *forces* the principal root — derives it, rather than selecting it by convention — then [LIFT-1] is a justified inherited premise and the only owed quantity is the sign $\sigma_{\varphi q}$. But if "principal root" is itself a choice, then the phase carries **two** premises collapsed into one rule plus one owed sign, and the antipodal branch (C9) becomes reachable by varying *either* — flip the lift rule, or flip the sign, and the phase claim fails the same way. The microscopic paper therefore owes, for the phase specifically, **the rule's justification as well as the sign**: it must either cite the inherited theorem that forces minimal-lift or grade the lift rule itself as a second owed return. Pending that, [LIFT-1] is flagged here as a load-bearing premise, not a free convention; this section fixes the target, not the return.

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9. The CKM Triangle Residue

Substituting the C_3 branch into the exact residue [EX-8]:

$$\Delta_{13} = \tfrac{1}{2}\, a\, \zeta_{C3} = \tfrac{1}{2}\, a\, (b/\sqrt{3})\, e^{5\pi i/6},$$

with exact value

$$\Delta_{13} = -\, a b/4 + i \cdot a b/(4\sqrt{3}) = -\, 729/320000 + (243\sqrt{3}/320000)\, i = -0.0022781 + 0.0013153\, i,$$
$$\|\Delta_{13}\| = 0.0026306.$$

The rephasing-invariant physical content is $\|\Delta_{13}\| = 0.0026306$; the displayed complex value is the entry in one fixed rephasing convention, its bare phase being convention-dependent (the freedom generated by [EX-6r], §6). Physical phase information lives in rephasing-invariant combinations — the unitarity-triangle angles and the Jarlskog invariant — not in the isolated entry phase.

The inherited bare 1↔3 entry is $(\Omega_q)_{13} = c\, e^{i\varphi}$ with $\varphi = 2\pi/3$, so the leading logarithmic 1↔3 entry is

$$(\log V_{CKM})_{13} = c\, e^{2\pi i/3} + \tfrac{1}{2}\, a\, \zeta_{C3} + \mathcal{O}(\text{curvature}^2),$$

i.e. in rectangular form

$$(\log V_{CKM})_{13} = (-c/2 - a\, b/4) + i\, (\sqrt{3}\, c/2 + a\, b/(4\sqrt{3})) + \mathcal{O}(\text{curvature}^2).$$

This is the triangle residue in clean form. The correction is **not** a new fitted 1↔3 parameter; it is the product of the Cabibbo doorway a and the common curvature ζ_{C3}. Its grade is Conditional: it inherits the conditional status of ζ_{C3} through [COND-1] and [OWE-5], while the map from ζ_{C3} to Δ_{13} is Exact.

Δ_{13} is not the observable $|V_{ub}|$. This caution prevents a natural misreading. The quantity $\Delta_{13} = \tfrac{1}{2}\, a\, \zeta_{C3}$ is the commutator-generated *logarithmic* residue — an entry of $\log V_{CKM}$, not of V_{CKM} itself. It is not, by itself, the experimental CKM matrix element $|V_{ub}|$, and $\|\Delta_{13}\| = 0.0026306$ is **not** a prediction that $|V_{ub}| = 0.0026$. The observable comparison requires combining the inherited direct term $c\, e^{i\varphi}$ with this residue (giving the full logarithmic entry above) and then exponentiating the complete two-factor CKM product to recover the matrix elements. The present paper therefore derives only the conditional residue *target*; the matrix-level comparison of $|V_{ub}|$, $|V_{td}|$, and the Jarlskog invariant J against experiment belongs to the exact CKM audit, where the exponentiation and rephasing-invariant combinations (§6) are carried out in full.

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10. Numerical Ledger

Numbers are a scale audit only; they are never used to construct the projected block. All values follow from the exact rationals.

Quantity	**Expression**	**Value**
a	$9/40$	0.225
b	$81/2000$	0.0405
c	$243/100000$	0.00243
φ	$2\pi/3$	$120°$
$\|\zeta_{C3}\|$	$b/\sqrt{3} = 27\sqrt{3}/2000$	0.0233826859
$\arg \zeta_{C3}$	$5\pi/6$	$150°$
ζ_{C3}	$-81/4000 + (27\sqrt{3}/4000)\, i$	$-0.02025 + 0.0116913\, i$
$\|\Delta_{13}\|$	$\tfrac{1}{2}\, a\, b/\sqrt{3}$	0.0026306
Δ_{13}	$-729/320000 + (243\sqrt{3}/320000)\, i$	$-0.0022781 + 0.0013153\, i$
$\|\Delta_{12}\|$ (leading $\tfrac{1}{2} c \|\zeta\|$)	$\tfrac{1}{2}\, c\, b/\sqrt{3}$	0.0000284
$\|\Delta_{12}\|$ (converged, two-factor)	λ-invariant value	0.0000474
$\|\Delta_{12}\|/a$ (leading)	$c\, b/(2\sqrt{3}\, a)$	1.26×10^{-4}
$\|\Delta_{12}\|/a$ (converged)	two-factor, all-orders	2.11×10^{-4}

The hierarchy is the point:

$\|\Delta_{12}\| / a \approx 2.1 \times 10^{-4}$ (converged; the leading ½c‖ζ‖ estimate, 1.26×10^{-4}, understates this ~1.7×).

The C_3 correction is visible in the 1↔3 triangle channel while leaving the leading Cabibbo entry essentially untouched at this order.

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11. What Must Be Computed Explicitly

The extraction is local. The microscopic projected-Hessian calculation must return the **pre-readout** role-even block — built from the readout-free compression P_pre = P_R P_C, *not* from the full P_W:

K_even,pre^q = P_q (½ Tr_role(i ε_W H_pre)) P_q, H_pre = P_pre† H_cl P_pre,

and separately the post-readout block K_even^q = P_q(½ Tr_role(i ε_W H_W))P_q with H_W = P_W† H_cl P_W, P_W = P_Y P_R P_C. The four orbit/readout tests below split across these two: 11.1–11.3 are pre-readout (on K_even,pre^q), 11.4 is the readout step (pre → post, the action of P_Y), and 11.5 is the phase.

11.1 C_3 covariance [OWE-1]. Test [K_even,pre^q, R_3] = 0. Nonzero at leading order ⇒ no C_3 orbit (C3).

11.2 Single-orbit purity [OWE-2]. This is an **ambient-space** test, not a $\mathfrak{su}(3)$ test — in bare $\mathfrak{su}(3)$ covariance already forces purity (Appendix A.6), so the content lives entirely in the $\mathfrak{su}(8)$ embedding. Decompose the compressed generation support into C_3-isotypic components and count the multiplicity of the regular representation: confirm that the role-even common mode occupies **exactly one** democratic regular-rep copy, with no C_3-invariant Cartan direction (which $\mathfrak{su}(3)$ lacks but the embedding may supply) and no second cyclic copy. The weight equality $w_{12} = w_{23} = w_{31}$ is only the automatic downstream signature; an inequality flags multiplicity leakage (C4), but the primary computation is the branching multiplicity itself.

11.3 Norm identification [OWE-3]. Verify that the total inherited common-mode norm on the orbit is exactly b — i.e. that the role-odd-to-role-even norm map delivers b. A different norm rescales ‖ζ‖ and breaks [COND-1] (C5).

11.4 Cabibbo-protected survival [OWE-4]. Apply P_Y and determine the first surviving common-mode component of K_even^q. Test K_even^q = z E_{23} − z* E_{32} + (higher-order suppressed). Leading 1↔2 ⇒ protection fails (C6); leading 1↔3 ⇒ residue inserted (C7); none ⇒ no correction (C8).

11.5 Phase-root sign [OWE-5]. Compute σ_φq from the projected Hermitian half-transport [LIFT-1] and test that it selects arg $\zeta = 5\pi/6$. Antipodal ⇒ phase claim fails even if the amplitude is correct (C9).

Until 11.1–11.5 are computed, the result stands at [COND-1] et seq. — exact downstream, owed upstream.

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12. Local Falsification Conditions

A falsifier table should list the live ways the construction can fail. Two earlier items have since been discharged by the converged computation of Appendix A; they are separated out below so the live set stands clean.

12.1 Live falsification conditions

The extraction fails locally under any of the following.

#	Condition	Broken claim
C1	K_even,pre^q cannot be defined without importing CKM data	no-fit projection
C2	K_even,pre^q not anti-Hermitian after i ε_W rotation	frame consistency [AUD-1]/[EX-0]
C3	[K_even,pre^q, R_3] ≠ 0 at leading order	C_3 covariance [OWE-1]
C4	common mode is an impure mixture (multiplicity leakage)	single-orbit purity [OWE-2]
C5	total common-mode norm ≠ b	norm identification [OWE-3]
C6	leading readout survivor is 1↔2	Cabibbo protection [OWE-4]
C7	leading readout survivor is 1↔3	no direct triangle insertion [OWE-4]
C8	no 2↔3 survivor appears	C_3 mechanism absent [OWE-4]
C9	σ_φq selects the antipodal phase branch — OR [LIFT-1] fails to force the principal root	phase branch [OWE-5] + lift-rule justification [LIFT-1] (either route reaches the antipode)
C10	Ω_mix not gapped relative to Ω_even, Ω_odd	clean weak-role reading [AUD-3]
C11b	ambient Cartan rephasing of the diagonal term is not an available field redefinition	diagonal removability [EX-6r]; form guaranteed all-orders, availability couples to $\mathfrak{su}(8)$ Cartan (A.6)

Each points to a specific block, commutator, multiplicity, norm, sign, gap, or field-redefinition freedom — not a vague objection. Of these, C6–C8 (the [OWE-4] crux) are the most load-bearing: the democratic symmetry is direction-blind, so the entire directionality of the mechanism rests there.

12.2 Already discharged — not standing conditions

These were live in earlier drafts but are settled by the all-orders computation; they cannot fail the construction and are recorded only for completeness.

#	Former condition	Resolution
C11a	(1,3) BCH tail same order as Δ_{13}	**discharged**: the converged (1,3) entry (A.5), including all diagonal feed-through, sits within 0.1% of the leading $\frac{1}{2}a\zeta$ estimate
C12	(1,2) back-reaction quoted at leading $\frac{1}{2}c\lVert\zeta\rVert$ rather than converged	**handled**: the converged $\lVert\Delta_{12}\rVert/a = 2.11\times10^{-4}$ is now the quoted figure (§10, §13); the leading estimate is labelled as such

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13. Status: Conditional Theorem, Not Completed Calculation

This section states the result's grade without softening, because the distinction is the whole point of the audit discipline.

What is proven (Exact, verified — Appendix A). $H_W = P_W^\dagger H_{cl} P_W$ is Hermitian by construction, so Ω_W is anti-Hermitian unconditionally. Given the surviving 2↔3 curvature $\Omega_0(z)$, the commutator $[\Omega_0, \Omega_q]_{13} = -a\,z$ holds exactly; the diagonal terms are removable Cartan rephasings carrying no physical angle, and that rephasing *form* is exact to all orders (L anti-Hermitian and traceless); the BCH residue $\Delta_{13} = \frac{1}{2}\,a\,\zeta$ holds at leading order and the (1,3) channel converges to within 0.1% of it (C11a discharged); covariance on a pure orbit forces equal η so $3\lVert\zeta\rVert^2 = b^2$ yields $\lVert\zeta\rVert = b/\sqrt{3}$; and the assembled numbers are exact rationals. The map

$$\{\text{covariance, pure orbit, norm } b,\ 2\leftrightarrow3 \text{ survivor, phase root } +1\} \longmapsto \{\zeta_{C3}, \Delta_{13}\}$$

is rigorous and reproducible.

What is owed (not computed here). The left-hand inputs of that map — the five returns [OWE-1]…[OWE-5] — are projected-Hessian outputs that this paper does not evaluate. The microscopic objects H_{cl}, P_C, P_R, P_Y are not constructed, and the entries of K_{even}^q are not computed from them. Two of the debts deserve their own flags. The phase debt is *split*: it is one premise ([LIFT-1], the minimal-lift rule) plus one owed sign ([OWE-5]), and is honest only if the inherited programme forces minimal-lift as a principle; otherwise the rule's justification is itself owed and the antipode (C9) is reachable by varying either (§8). And [OWE-4] is the *crux*: because the C_3 symmetry is democratic, all directionality of the mechanism lives in the readout survival (§5). Therefore the claim "the projected Hessian returns the C_3-democratic, Cabibbo-protected 2↔3 branch" is, on the evidence here,

not yet proven.

Honest verdict. The present state is *conditional success*: if the Hessian returns a single pure democratic orbit of norm b with $\sigma_{\varphi q} = +1$ and 2↔3 survival, then $\zeta_{C3} = -0.02025 +$

0.0116913 i and $\Delta_{13} = -0.0022781 + 0.0013153$ i, $\|\Delta_{13}\| = 0.0026306$, with the Cabibbo channel protected at the converged $\|\Delta_{12}\|/a \approx 2.1\times10^{-4}$ (the leading $\frac{1}{2}c\|\zeta\|$ estimate, 1.26×10^{-4}, understates this ~1.7× because the (1,2) BCH tail is a,b-driven and non-negligible — see C12). The physics conclusion is untouched: 2.1×10^{-4} is still deeply protective. The conditional is exact; its antecedent is owed. This paper is accordingly titled and graded as a *conditional extraction theorem*, and the next paper must either compute [OWE-1]…[OWE-5] explicitly from the microscopic compression or remain at this grade. It must not present the conditional as a completed Hessian calculation.

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14. Relation to the Neutrino and Leakage Papers

This paper treats only the quark C_3 common-curvature extraction. It does not compute the weak-commitment neutrino kernel, the leakage support trace, the reactor leakage amplitude, or the atmospheric octant sign. Those remain separate projected-Hessian outputs:

$T\_\nu = D_0 I + \varepsilon M\_\nu$, $\mathrm{Tr\_support}(\Pi\_\mathrm{leak}) = 20$, $\beta = \sqrt{3}/20$, $\sigma\_W = \mathrm{sgn}(\|P\_e H\_W P\_\tau\|^2 - \|P\_e H\_W P\_\mu\|^2)$.

Separating the quark C_3 extraction is methodological: the CKM triangle residue has a compact algebraic core and a small number of owed returns, and should be settled before the larger leakage and PMNS kernel calculation. The programme sequence is:

1. weak-doublet Hessian audit;
2. C_3 role-even quark curvature extraction (this paper — conditional);
3. leakage support trace and β extraction;
4. PMNS weak-commitment kernel extraction;
5. unified CKM/PMNS flavour ledger.

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15. Conclusion

The quark C_3 problem is now a local projected-Hessian extraction with a sharply bounded proof debt. The exact objects are the pre- and post-readout role-even quark blocks

$K\_\mathrm{even,pre}\hat{}q = P\_q(\frac{1}{2} \mathrm{Tr\_role}(i\, \varepsilon\_W H\_\mathrm{pre}))P\_q$, $H\_\mathrm{pre} = P\_\mathrm{pre}^\dagger H\_\mathrm{cl} P\_\mathrm{pre}$, $P\_\mathrm{pre} = P\_R P\_C$, $K\_\mathrm{even}\hat{}q = P\_q(\frac{1}{2} \mathrm{Tr\_role}(i\, \varepsilon\_W H\_W))P\_q$, $H\_W = P\_W^\dagger H\_\mathrm{cl} P\_W$, $P\_W = P\_Y P\_R P\_C$,

self-adjoint compressions secured, anti-Hermitian frames secured. The required returns are five: pre-readout C_3 covariance $[K\_\mathrm{even,pre}\hat{}q, R_3] = 0$; single-orbit purity (one democratic C_3 orbit, no multiplicity leakage), from which equal weights follow as a theorem; the norm identification fixing the total at b, which then forces $\|\zeta\| = b/\sqrt{3}$; Cabibbo-protected survival of $\Omega\_\mathrm{even}\hat{}(23) = z$

E_{23} − z* E_{32} through the readout (pre → post, the action of P_Y); and the phase sign σ_φq selecting arg ζ = 5π/6. Together these would give ζ_C3 = (b/√3) e^{5πi/6}, after which the triangle residue follows by exact algebra:

[Ω_0(z), Ω_q]$_{13}$ = − a z, Δ_{13} = ½ a ζ_C3.

The programme no longer asks vaguely whether a small 1↔3 CKM entry can be accommodated. It asks one concrete, falsifiable question:

does the projected Hessian return a single pure democratic C_3 orbit, of norm b, with the 2↔3 branch surviving the Cabibbo-protected readout and phase sign σ_φq = +1?

If yes, the CKM triangle residue is derived. If no, the C_3 common-curvature mechanism fails locally — at a named live condition (C1–C10, C11b; §12.1). Until that microscopic return is computed, the result stands exactly where it is honest to stand: a conditional extraction theorem whose downstream algebra is exact and whose upstream antecedent is owed.

Appendix A. Symbolic Verification

The exact claims [EX-6], [EX-6r], [EX-8] and the orbit-purity argument of §3 were checked with a computer-algebra system. The verification establishes four facts.

A.1 Commutator and the −a z identity. With

Ω_q = [[0, a, c·e^{iφ}], [−a, 0, b], [−c·e^{−iφ}, −b, 0]], Ω_0(z) = z E_{23} − z* E_{32},

expansion of [Ω_0, Ω_q] returns

```
[[      0,          c z* e^{iφ},     -a z     ],
 [ -c z e^{-iφ},    b(z*-z),          0       ],
 [     a z*,           0,          b(z-z*)    ]]
```

and in particular the (1,3) entry is exactly −a z. ✓

A.2 Diagonal rephasing [EX-6r]. Writing z = z_r + i z_i, the diagonal of the commutator is

(1,1) = 0, (2,2) = b(z*−z) = −2 i b z_i, (3,3) = b(z−z*) = +2 i b z_i,

a traceless, purely imaginary diagonal — a quark-Cartan rephasing generator. It induces a diagonal phase rotation only and contributes no physical mixing angle, removable by allowed left-quark-field rephasing at this order. ✓

A.3 BCH residue [EX-8]. With $X = -\lambda\,\Omega_0 + \tfrac{1}{2}\,\Omega\_q$ and $Y = +\lambda\,\Omega_0 + \tfrac{1}{2}\,\Omega\_q$: $X + Y = \Omega\_q$, and $\tfrac{1}{2}[X,Y] = -(\lambda/2)[\Omega_0, \Omega\_q]$. Hence $(\log V\_CKM)_{13} - (\Omega\_q)_{13} = (\lambda/2)\,a\,z$, and with $\zeta = \lambda\,z$, $\Delta_{13} = \tfrac{1}{2}\,a\,\zeta$. ✓

A.4 Orbit purity / equal weights (§3). For $X = A_{12}(\eta_{12}) + A_{23}(\eta_{23}) + A_{31}(\eta_{31})$ with $A\_ij(\eta) = \eta\ E\_ij - \eta^*\ E\_ji$, solving $[X, R_3] = 0$ (R_3 the cyclic permutation $1\to2\to3\to1$) returns the unique solution $\eta_{12} = \eta_{23} = \eta_{31}$. Thus C_3 covariance on the single off-diagonal orbit forces equal weights and equal phase; "democratic weights" is a consequence of [OWE-1] ∧ [OWE-2], not an independent owed return. ✓

A.5 Converged λ-invariant residues (two-factor composition). Because $\lambda\,\Omega_0(z) = \Omega_0(\lambda z) = \Omega_0(\zeta)$, the operators $X = -\Omega_0(\zeta) + \tfrac{1}{2}\Omega\_q$ and $Y = +\Omega_0(\zeta) + \tfrac{1}{2}\Omega\_q$ depend only on ζ; V_CKM = exp(X)exp(Y) is exactly λ-invariant and unitary, with converged L = log V_CKM anti-Hermitian. "Converged" is with respect to this inherited two-factor doublet composition; it is not a claim about a composition law this paper derives. Reading the off-diagonal increments at the C_3 value ζ_C3:

- (1,3): $\Delta_{13} = L_{13} - (\Omega\_q)_{13}$ has $\|\Delta_{13}\| = 0.0026332$, versus the leading $\tfrac{1}{2}a\|\zeta\| = 0.0026306$ — ratio 1.001, so the leading estimate is converged (including all diagonal feed-through) and C11a is discharged in this channel.
- (1,2): $\Delta_{12} = L_{12} - (\Omega\_q)_{12} = -4.518\times10^{-5} + 1.431\times10^{-5}\,i$, $\|\Delta_{12}\| = 4.739\times10^{-5}$, versus the leading $\tfrac{1}{2}c\|\zeta\| = 2.841\times10^{-5}$ — ratio 1.668. Hence $\|\Delta_{12}\|/a = 2.11\times10^{-4}$ (converged), not 1.26×10^{-4} (leading). The second-order BCH (1,2) entry is 2.05×10^{-5}, fully 0.72× the leading (1,2) term: the tail is a,b-driven and the same order as the c-driven leading term, confirming C12. ✓
- diagonal: L is anti-Hermitian (V_CKM unitary) and traceless (det V_CKM = 1), so its diagonal is purely imaginary and sums to zero — a traceless imaginary Cartan generator, i.e. a removable left-quark rephasing, **at all orders**. The converged diagonal entry $\|L_{22}\| = 4.755\times10^{-4}$ matches the leading $b\|\mathrm{Im}\,\zeta\| = 4.735\times10^{-4}$ to 0.4%. So the rephasing *form* of [EX-6r] is exact, not leading-order; what is live (C11b) is only whether the ambient $\mathfrak{su}(8)$ makes that Cartan direction a genuine field-redefinition freedom. ✓

A.6 The C_3-commutant of $\mathfrak{su}(3)$ (§3, [OWE-2]). With R_3 the cyclic permutation, the linear map $X \mapsto R_3\,X\,R_3^{-1} - X$ on the eight-dimensional real $\mathfrak{su}(3)$ has a kernel of dimension exactly **2**. Both kernel elements are purely off-diagonal (zero Cartan content) and realise the democratic orbit — the real and imaginary parts of $A_{12}(\eta) + A_{23}(\eta) + A_{31}(\eta)$. There is no C_3-invariant Cartan direction and no second cyclic copy in $\mathfrak{su}(3)$. Hence covariance alone forces the democratic orbit downstairs, and purity [OWE-2] is vacuous in bare $\mathfrak{su}(3)$; it is non-vacuous only under the $\mathfrak{su}(8)$ embedding, which can supply invariant Cartan directions and regular-representation multiplicity that $\mathfrak{su}(3)$ lacks. ✓

A.7 Diagonal term magnitude and removability (§6, [EX-6r]). Comparing level-consistently — generator against generator — the diagonal entry of log V_CKM is $b\cdot\mathrm{Im}\,\zeta$, magnitude $b\|\mathrm{Im}\,\zeta\| = b\|\zeta\|/2 = 0.000474$ (converged 0.000475), while $\|\Delta_{13}\| = a\,b/(2\sqrt{3}) = 0.0026306$. The exact ratio is $\|\text{generator diagonal}\|/\|\Delta_{13}\| = b/a = 0.18$ — same order, not suppressed. (Comparing the commutator-level diagonal $2b \cdot \mathrm{Im}\,\zeta = 0.000947$ against the BCH-halved Δ_{13} mismatches levels and would read 0.36; the consistent figure is 0.18.) Removability is exact to all orders: L is anti-Hermitian and traceless, so its diagonal is an imaginary traceless Cartan generator at every order; harmlessness is structural, not a matter of smallness. ✓

Numerical closure. $a = 0.225$, $b = 0.0405$, $c = 0.00243$; $\|\zeta\_C3\| = b/\sqrt{3} = 0.0233826859$; $\zeta\_C3 = -81/4000 + (27\sqrt{3}/4000)\,i$; $\Delta_{13} = -729/320000 + (243\sqrt{3}/320000)\,i$, $\|\Delta_{13}\| = 0.0026306$ (leading) / 0.0026332 (converged); $\|\Delta_{12}\|/a = 1.26\times10^{-4}$ (leading) / 2.11×10^{-4} (converged); generator-diagonal/$\|\Delta_{13}\| = b/a = 0.18$ (removable, all-orders); dim C_3-commutant $\mathfrak{su}(3) = 2$. ✓