The C_3 Substrate Reduction and the Minimal-Hermitian-Lift CKM Branch in VERSF

Reducing the Two Remaining Quark-Sector Bottleneck Facts — [P_C H_cl P_C, R_3] = 0 and σ_φ^q = +1 — to Substrate Admissibility

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Successor to *The First Weak-Doublet Projection Audit in VERSF*.

Summary for the General Reader

Quarks come in three families, and there are precise rules for how those families blend into one another. Physicists package those rules as a small geometric shape — the "CKM triangle." Two features of the triangle matter here: its **size** (how much blending there is between two of the families) and its **handedness** (a left- or right-leaning orientation, which is tied to the tiny imbalance between matter and antimatter in the universe). An earlier paper in this series showed that VERSF can account for the triangle *if* its deeper machinery produces these two features on its own — without anyone feeding in the measured blending values. This paper asks whether the machinery actually does that, and is careful to say how strong each answer is.

The **size** answer is clean. Before the step where particles acquire their masses, VERSF builds the three quark families even-handedly, singling none of them out. That even-handedness is a symmetry — a threefold rotational symmetry among the families. Whenever a fixed quantity is shared evenly across three equivalent directions, each direction receives a definite fraction of it; here that fraction fixes the size of the triangle exactly, with no blending data used. The one thing the calculation still takes from the wider theory is the *total* amount being shared — a single inherited number.

The **handedness** answer is more delicate, and the paper is open about it. Choosing left over right comes down to a single yes/no decision, and the natural-looking way to make that decision gives the *wrong* answer. The correct orientation appears only when the decision is referred to one particular vantage point inside the theory rather than another. There is a good reason to favour that vantage point, but turning "good reason" into "proven" is the one piece of work still outstanding. So the size is essentially settled; the handedness is narrowed down to a single clearly-stated assumption that still has to be earned.

The honest bottom line: the paper does not yet rebuild the deepest layer of the theory from scratch. What it does is reduce the whole quark-blending question to exactly two checkable claims — one symmetry of the underlying machinery (for the size) and one principle about which vantage point sets the orientation (for the handedness) — and it spells out precisely how each could fail.

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Abstract

The first weak-doublet projection audit reduced the committed-quark CKM curvature target to two remaining substrate facts. The amplitude target **$|\zeta| = b/\sqrt{3}$**, where **$\zeta = \lambda z$** is the convention-invariant common curvature, requires the pre-readout generation-completion block to act democratically on the three Killing-orthonormal C_3 branch supports. The branch target **$\sigma_\varphi^q = +1$**, selecting **$\arg \zeta = 5\pi/6$** over its antipode **$11\pi/6$**, requires the projected Hamiltonian to choose the positive orientation of the C_3 half-transport. This paper derives both from substrate admissibility, and grades each derivation against its remaining owed inputs.

For the amplitude, we prove a **C_3 equivariance theorem**. Let **R_3** be the cyclic generation operator on the pre-readout generation-completion cell. If the substrate functional **F** is C_3-invariant on this cell (**C3-INV**), the admissible background is fixed by **R_3** (**C3-BG**), and the closure restriction and generation projector **P_C** are **R_3**-equivariant (**C3-EQV**), then the Hessian **$G = \delta^2 F|_{\mathcal{M}_{\text{adm}}}$** commutes with **$R_3$**, and hence

$$[P\_C\ H\_cl\ P\_C, R_3] = 0$$

on the role-even committed-quark support. A Hermitian three-generation block commuting with R_3 is circulant, so its forward branch entries are *equal as complex numbers*,

$$(K)_{12} = (K)_{23} = (K)_{31},$$

and in particular $w_{12} = w_{23} = w_{31}$. With Killing-orthonormal branch supports, an inherited total common-mode branch norm b^2 summed over the three branches (**C3-NORM**), and norm-preserving readout on the surviving common mode (**C3-SURV**), this gives $w_{ij} = b^2/3$ and therefore $|\zeta| = b/\sqrt{3}$.

For the branch, we formulate and prove the **minimal-Hermitian-lift theorem**. Let $h = (H\_even\hat{}q)_{23}$ and $z = i\ \varepsilon\_W\ h$, with $\varepsilon\_W > 0$. The generator-side half-transport rule $z^2 \propto C \cdot O\_q$, with $C = e\hat{}(i\pi)$ and $O\_q = e\hat{}(2\pi i/3)$ (**ML-HAM**), is equivalent on the Hermitian Hamiltonian side to

$$h^2 \propto O\_q.$$

The two Hermitian square-root lifts are $h/|h| = e\hat{}(i\pi/3)$ and $e\hat{}(i4\pi/3)$, with principal phase magnitudes $\pi/3$ and $2\pi/3$. If admissibility selects the lift of minimal Hermitian generator length — the geodesic lift from the identity in the bi-invariant Killing metric (**ML-LIFT**) — and does so on the Hermitian side rather than the frame-generator side (**ML-SIDE**), then the first root is forced. Hence $\mathrm{Re}\ h > 0$, $\sigma\_\varphi\hat{}q = \mathrm{sgn}\ \mathrm{Re}\ h = +1$, and $\arg z = \pi/2 + \pi/3 = 5\pi/6$.

The result is not a completed microscopic computation of **H_cl**. It is a reduction of the two remaining CKM bottleneck facts to a short, explicit, CKM-free premise set: **pre-readout C_3 invariance** for the amplitude, and **minimal Hermitian lift on the generator side** for the branch. Both are falsifiable the moment the explicit substrate functional and projectors are supplied.

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0. Predictive-Content Ledger

Grades are four-way: **[Exact]** (proven algebra), **[Audit]** (exact given a named premise that can be checked directly), **[Conditional]** (theorem contingent on an owed substrate input), **[Owed]** (not yet supplied).

Target	Statement	Grade	Load-bearing premises
Weak-doublet object	$H\_W = P\_W\ H\_cl\ P\_W$	inherited	—
Convention-invariant curvature	$\zeta = \lambda z$	inherited (audit)	—

Target	Statement	Grade	Load-bearing premises
CKM residue	**$\Delta_{13} = \frac{1}{2} a \zeta$**	**[Exact]** once a role-even 2↔3 curvature exists	—
C_3 branch orbit	**B_{12}, B_{23}, B_{31}** Killing-orthonormal	**[Exact]**	—
C_3 covariance	**$[P_C H_{cl} P_C, R_3] = 0$**	**[Audit]**	C3-INV, C3-BG, C3-EQV
Circulant forward entries	**$(K)_{12} = (K)_{23} = (K)_{31}$**	**[Exact]** given covariance	—
Democracy weights	**$w_{12} = w_{23} = w_{31}$**	**[Exact]** given covariance	—
Amplitude	**$\lvert\zeta\rvert = b/\sqrt{3}$**	**[Conditional]**	C3-INV, C3-BG, C3-EQV, C3-NB, C3-NORM, C3-SURV, C3-ORD, C3-READOUT
Hamiltonian phase root	**$h^2 \propto O_q$**	**[Audit]**	ML-HAM
Minimal lift	**$h/\lvert h\rvert = e^{i\pi/3}$**	**[Exact]** given ML-LIFT, ML-SIDE	—
CKM branch	**$\sigma_\varphi^q = +1$, $\arg \zeta = 5\pi/6$**	**[Conditional]**	ML-HAM, ML-LIFT, ML-SIDE
Evaluated curvature	**$\zeta_{C3} = (b/\sqrt{3})e^{5\pi i/6}$, $b = 81/2000 \to -0.02025 + 0.01169i$**	**[Conditional]**	C3 + ML premises + inherited **b**
Full substrate computation	explicit **$F \to G \to H_{cl} \to P_W$**	**[Owed]**	—

The paper has two central results:

1. **Amplitude reduction.** C_3-invariant pre-readout substrate functional ⇒ **$[P_C H_{cl} P_C, R_3] = 0$** ⇒ equal branch weights ⇒ **$\lvert\zeta\rvert = b/\sqrt{3}$**.
2. **Branch reduction.** Hermitian-side C_3 square root + minimal-length lift ⇒ **$\sigma_\varphi^q = +1$** ⇒ **$\arg \zeta = 5\pi/6$**.

A consolidated statement of every named premise appears in **Appendix C**.

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1. The Bottleneck After the Projection Audit

The projection audit reached a precise boundary. The CKM curvature mechanism is algebraically exact once a role-even 2↔3 common curvature exists:

$$[\Omega_0(z), \Omega_q]_{13} = -a z,$$

and, in convention-invariant form,

$$\Delta_{13} = \tfrac{1}{2} a \zeta, \quad \zeta = \lambda z.$$

The remaining quark-sector question is not whether the commutator works — it does, exactly. The question is whether the projected closure Hamiltonian *returns the required curvature without being told the CKM answer*.

The target is

$$\zeta_{C3} = (b/\sqrt{3})\, e^{5\pi i/6}.$$

It separates into two logically independent parts:

amplitude: $|\zeta| = b/\sqrt{3}$, **branch:** $\arg \zeta = 5\pi/6$ rather than $11\pi/6$.

The projection audit reduced the amplitude to a C_3 Killing-norm-sharing condition and the branch to a single sign:

$$\sigma_\varphi^q = \mathrm{sgn}(\mathrm{Im}\, z) = \mathrm{sgn}\, \mathrm{Re}(H_{\mathrm{even}}^q)_{23}.$$

This paper asks whether those two remaining facts follow from the substrate side. The two halves are governed by *different blocks* of the projected Hamiltonian — a distinction made explicit in §2, and on which the whole construction turns.

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2. Notation and the Two-Block Distinction

Let

$$H_{cl} = G|\mathfrak{su}(8)$$

be the closure Hamiltonian: the $\mathfrak{su}(8)$-restricted Hessian of the substrate free-energy/admissibility functional **F** at the admissible manifold.

Let

$$P_W = P_Y P_R P_C$$

be the weak-doublet projection, with P_C the generation-completion projection, P_R the weak-role projection, and P_Y the mass/readout projection. These are ordered projections and need not mutually commute, so the product $P_Y P_R P_C$ is not assumed to be a single self-adjoint projector. Accordingly $H_W = P_W H_{cl} P_W$ is to be read as the ordered compression

H_W = P_Y P_R P_C H_cl P_C P_R P_Y,

which is the symmetric sandwich used throughout; the shorthand **P_W H_cl P_W** denotes this object and nothing stronger.

Two role-even quark blocks appear, and they must never be conflated.

Pre-readout block (before mass/readout selection):

K_even^q = ½ Tr_role(P_q P_R P_C H_cl P_C P_R P_q).

Post-readout block (after mass/readout selection):

H_even^q = ½ Tr_role(P_q P_Y P_R P_C H_cl P_C P_R P_Y P_q).

The distinction is the spine of the paper:

- **The amplitude lives on K_even^q.** The C_3 democracy that produces $\mathbf{b/\sqrt{3}}$ is a property of the pre-readout block, before **P_Y** selects a generation frame. It is a statement about a *magnitude* — the even split of a conserved common-mode norm — and that magnitude is C_3-protected.
- **The branch lives on H_even^q.** The visible 2↔3 curvature, and therefore its phase **arg** ζ, is read off the post-readout block. **P_Y** is the mass/readout projection and is *not* required to be C_3-equivariant: it selects a generation frame, and in doing so fixes an *orientation* without disturbing the inherited *magnitude* (premise **C3-SURV**, §4.4).

This is what allows the amplitude to be democratic while the phase is a definite selected branch: the size is read before the frame is chosen, the handedness after. There is no tension, because a magnitude and an orientation are read from two different stages of the same projection chain.

Let $\mathbf{R_3}$ be the cyclic generation operator,

$$\mathbf{R_3 : 1 \to 2 \to 3 \to 1,}$$

acting on branch supports as

$$\mathbf{B_{12} \to B_{23} \to B_{31} \to B_{12}.}$$

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3. The C_3 Branch Supports

On the three-generation readout space, define the Hermitian branch quadratures

$$\mathbf{S_{ij} = (E_{ij} + E_{ji})/\sqrt{2},\ A_{ij} = i(E_{ij} - E_{ji})/\sqrt{2}.}$$

The branch support is

$$\mathbf{B_{ij} = span\{S_{ij}, A_{ij}\}.}$$

Under the trace/Killing inner product $\boldsymbol{\langle X, Y\rangle_K \propto \mathrm{Tr}(XY)}$, distinct branches are orthogonal,

$$\mathbf{B_{12} \perp B_{23} \perp B_{31},}$$

and the three branches carry equal norm after normalisation. The branch-support geometry required for C_3 norm sharing is therefore already present in the three-generation algebra — it is structural, not assumed. **[Exact]**

For a pre-readout Hermitian block $\mathbf{K_{even}^q}$, define branch weights

$$\mathbf{w_{ij} = \|\Pi_{B_{ij}} K_{even}^q\|^2_K.}$$

In a fixed normalisation $\mathbf{w_{ij} \propto |(K_{even}^q)_{ij}|^2}$; the proportionality factor is conventional, so every absolute-norm statement below uses the single Killing normalisation that also fixes the inherited norm **b**.

4. First Bottleneck — Proving $[P_C H_{cl} P_C, R_3] = 0$

4.1 Named premises

The covariance follows if the pre-readout generation-completion sector is genuinely C_3-symmetric. The premises are CKM-free: they say only that, before mass/readout, the substrate prefers no generation branch over another.

- **C3-INV — C_3-invariant substrate functional:**

  $\mathbf{F[R_3 x] = F[x]}$ on the pre-readout generation-completion cell.

- **C3-BG — C_3-symmetric admissible background:**

  $R_3 x = x$** at the background where the Hessian is taken.

- **C3-EQV — equivariant restriction and projector:** the restriction $\mathbf{G \mapsto H_{cl} = G|\mathfrak{su}(8)}$ preserves the C_3 action (R_3 is realised as a unitary commuting with the closure embedding, so conjugation by $\mathbf{R_3}$ preserves $\mathbf{\mathfrak{su}(8)}$), and

  $$\mathbf{P_C R_3 = R_3 P_C.}$$

Moreover the weak-role projector **P_R**, the quark-sector projector **P_q**, and the role trace **Tr_role** are *generation-blind* — each commutes with **R_3**, since **R_3** acts only on generation labels: **P_R R_3 = R_3 P_R, P_q R_3 = R_3 P_q, [Tr_role, R_3] = 0.** This is what carries the substrate commutator **[P_C H_cl P_C, R_3] = 0** cleanly through the role reduction into the quark block **K_even^q**, so the circulant conclusion holds there and not merely on the unrestricted closure block.

4.2 Hessian equivariance theorem

Theorem 1 — C_3 Hessian Equivariance. Under **C3-INV** and **C3-BG**, the Hessian $G = \delta^2F|_{x}*$ commutes with **R_3**:

[G, R_3] = 0.

If in addition **C3-EQV** holds, then

[P_C H_cl P_C, R_3] = 0.

Proof. Differentiate **C3-INV** twice and evaluate at **$x*$**. Because $R_3\, x = x**$ (**C3-BG**), the chain rule gives

$R_3^\dagger$ G R_3 = G, equivalently G R_3 = R_3 G,

so **[G, R_3] = 0**. By **C3-EQV**, **R_3** acts on the closure sector as a unitary preserving **$\mathfrak{su}(8)$** (it permutes generation labels and adds no trace component), so the restriction **H_cl = G|$\mathfrak{su}(8)$** inherits **[H_cl, R_3] = 0**. Finally, using **P_C R_3 = R_3 P_C**,

P_C H_cl P_C R_3 = P_C H_cl R_3 P_C = P_C R_3 H_cl P_C = R_3 P_C H_cl P_C,

hence **[P_C H_cl P_C, R_3] = 0**. ■

4.3 Consequence — branch democracy

A Hermitian three-generation block **K** with **R_3 K $R_3^\dagger$ = K** is circulant. Writing the cyclic generator as the shift **1→2→3→1**, the circulant form is

K = circ(d, h, $\bar{h}$),

so every *forward* link carries the **same complex number** and every *reverse* link its conjugate:

$(K)_{12} = (K)_{23} = (K)_{31} = h$, $(K)_{21} = (K)_{32} = (K)_{13} = \bar{h}$.

This is stronger than equal magnitudes: C_3 covariance equalises the forward branch entries *as complex numbers*. In particular

$|K_{12}| = |K_{23}| = |K_{31}|$, $w_{12} = w_{23} = w_{31}$. [Exact]

4.4 From democracy to amplitude

Further premises close the amplitude:

- **C3-NB — no non-equivariant leakage:** after restriction to the role-even committed-quark support, no transport or record term introduces a non-C_3-equivariant off-diagonal contribution before **P_Y** acts. (This is *not* a restatement of the commutator. Covariance already forces the surviving off-diagonal block to be circulant, hence the weights equal — §4.3; what C3-NB adds is that no further term, equivariant-looking after the closure step but non-equivariant after the role reduction, leaks in to spoil that circulant block. It is a no-contamination condition, owed to the explicit transport and record penalties.)
- **C3-NORM — inherited norm:** the total role-even common-mode branch norm, summed over the three C_3 branches and carried into **K_even^q**, is $\mathbf{b^2}$:

  $\mathbf{w_{12} + w_{23} + w_{31} = b^2}$**.** The sum runs over all three branches; the visible curvature branch carries one share, $\mathbf{b^2/3}$. The **2↔3** identification of that visible share belongs to **C3-READOUT**, not here — it is precisely what makes the $\mathbf{\sqrt{3}}$ mean "one share out of three." (Read instead as **b** being a norm already assigned to the visible 2↔3 branch, the sharing would give a different amplitude; the $\mathbf{b/\sqrt{3}}$ result requires $\mathbf{b^2}$ to be the all-branches total.)

- **C3-SURV — norm-preserving readout: P_Y** selects the visible common mode without rescaling its norm, so the magnitude established pre-readout survives into **H_even^q**. (This is the premise that lets a pre-readout magnitude set a post-readout amplitude; it is named here so it cannot hide.)
- **C3-ORD — readout order: P_Y** acts *after* the C_3 supports are formed, so there is a pre-readout object on which the $\sqrt{3}$ sharing can act.
- **C3-READOUT — visible branch identification:** the mass/readout projection **P_Y** selects the surviving CKM curvature branch as the visible 2↔3 common mode. (This is where the 2↔3 label is fixed; the upstream sharing in C3-NORM is branch-blind.)

Given **C3-NB** and **C3-NORM**,

$\mathbf{w_{12} = w_{23} = w_{31} = b^2/3,}$

and given **C3-SURV** and **C3-ORD** the visible branch amplitude is

$\mathbf{|\zeta| = b/\sqrt{3}.}$ **[Conditional]**

This is the amplitude theorem: $\mathbf{b/\sqrt{3}}$ is the consequence of one substrate symmetry (C3-INV/BG/EQV) plus inheritance, ordering, and no-contamination bookkeeping (C3-NB/NORM/SURV/ORD/READOUT), with no CKM input.

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5. A Minimal Substrate Cell Realising the C_3 Premise

Theorem 1 proves the reduction *if* **F** is C_3-invariant. The following minimal, non-circular cell shows that the premise is natural rather than imposed.

Let

$$\mathcal{H}_{\mathrm{min}} = \ell^2(\mathcal{G}_3) \otimes \mathbb{C}^2_{\mathrm{role}} \otimes \mathbb{C}^8_{\mathrm{closure}},$$

with $\mathcal{G}_3 = \{1,2,3\}$ the generation triangle, $\mathbb{C}^2_{\mathrm{role}}$ the weak role, $\mathbb{C}^8_{\mathrm{closure}}$ the closure fibre. Use the *inherited* substrate functional restricted to this cell:

$$F_{\mathrm{CKM,min}}[\rho] = \Sigma_i V_{\mathrm{sub}}(\rho_i) + \alpha_C A_C^\Delta + \alpha_T A_T^\Delta + \alpha_R A_R^\Delta,$$

with V_{sub} the saturation potential, A_C^Δ the closure-consistency penalty on the generation triangle, A_T^Δ the transport-consistency penalty, A_R^Δ the record-composition penalty — all inherited, none fitted. No CKM matrix, no ζ_{C3}, and no empirical phase enters.

For small role-even branch transports $U_{ij} = \exp(i\varepsilon X_{ij})$, $X_{ij} \in B_{ij}$, closure around the triangle gives

$$U_{12} U_{23} U_{31} - \mathbb{1} = i\varepsilon(X_{12} + X_{23} + X_{31}) + O(\varepsilon^2),$$

so the quadratic closure penalty is

$$A_C^{(2)} \propto \|X_{12} + X_{23} + X_{31}\|^2_K = \|X_{12}\|^2_K + \|X_{23}\|^2_K + \|X_{31}\|^2_K,$$

the second equality by Killing-orthogonality (§3). The closure penalty is therefore **manifestly democratic** on the C_3 branch orbit. **[Exact]**

The transport and record penalties must be checked in the explicit model. Covariance under the generation cycle already does the heavy lifting: a Hermitian block commuting with R_3 is circulant, so the three forward branch weights are equal as a matter of form (§4.3) — the abelian isotype freedom (distinct eigenvalues on the characters $1, \omega, \omega^2$) changes the spectrum of the block, not the equality of the forward magnitudes. So branch-weight equality is *not* the owed condition. What is owed is **C3-NB** in its no-leakage form: that no transport or record term, on the role-even committed-quark support, contributes a non-C_3-equivariant off-diagonal piece before P_Y that would add to the circulant block. That is a genuine extra condition on A_T and A_R, not a corollary of covariance, and it is the residual content the closure penalty alone does not settle.

So the minimal cell does not discharge the full Hessian automatically. It discharges the closure part exactly and reduces the rest to the single sharp test $[P_C H_{\mathrm{cl}} P_C, R_3] = 0$.

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6. What Would Falsify the Amplitude Derivation

The amplitude **$|\zeta| = b/\sqrt{3}$** fails *locally* — at one diagnosable place — if any of the following holds:

1. **C3-INV fails:** the explicit pre-readout **F** is not C_3-invariant on the generation cell.
2. **C3-BG fails:** the admissible background **x^*** spontaneously breaks C_3 before readout.
3. **C3-EQV fails:** **P_C** is not C_3-equivariant, or the **$\mathfrak{su}(8)$** restriction does not preserve the C_3 action.
4. **C3-NB fails:** **A_T** or **A_R** contributes a non-C_3-equivariant off-diagonal term on the committed-quark support before **P_Y**, leaking into and spoiling the circulant block.
5. **C3-NORM fails:** the inherited role-even common-mode norm is not **b^2**.
6. **C3-SURV fails:** **P_Y** rescales the surviving common-mode norm.
7. **C3-ORD fails:** **P_Y** acts before the C_3 supports form, leaving the $\sqrt{3}$ sharing nothing to act on.

If none holds, the amplitude is derived.

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7. Second Bottleneck — The Minimal Hermitian Lift for the CKM Branch

The amplitude sets the *size* of the triangle repair; the branch sets its *handedness*.

Let

$$h = (H_{even}^{q})_{23},\ z = i\,\varepsilon_W\, h,\ \varepsilon_W > 0.$$

The generator-side half-transport grammar (**ML-HAM**) states

$$z^2 \propto C \cdot O_q,\ C = e^{i\pi},\ O_q = e^{2\pi i/3}.$$

Since **$z = i\,\varepsilon_W\, h$**, we have **$z^2 = -\varepsilon_W^2 h^2$**, and the reversal **$C = -1$** is exactly the **$i^2$** of the Hamiltonian-to-generator conversion. The two cancel, leaving the clean Hamiltonian-side rule

$h^2 \propto O_q$. [Audit on ML-HAM]

Its two square-root lifts are

$$h/|h| = e^{i\pi/3} \text{ and } h/|h| = e^{i4\pi/3} = e^{-i2\pi/3},$$

with principal phase magnitudes **$\pi/3$** and **$2\pi/3$** respectively.

7.1 Division of labour — which premise supplies the value

It is worth being explicit about what each premise contributes, because the value and the sign come from different places. **ML-HAM alone** fixes the *value* modulo π: from $z^2 \propto C \cdot O_q$ one reads $\arg z = \tfrac{1}{2} \arg(C \cdot O_q) = 5\pi/6 \pmod{\pi}$, so

$$\arg z \in \{5\pi/6, -\pi/6\},$$

with both candidates already set by $O_q = e^{2\pi i/3}$ together with the **i**-rotation. The minimal-lift machinery (**ML-LIFT** + **ML-SIDE**) does **not** produce the number $5\pi/6$; it only resolves the two-fold square-root sign, selecting $5\pi/6$ over $-\pi/6$. The handedness story is therefore:

ML-HAM gives the value; minimal lift picks the sign.

The entire numerical value $5\pi/6$ is inherited through O_q. Two consequences follow. First, a reader should not credit the minimal-lift principle with producing $5\pi/6$ — it is a binary sign selector acting on a value fixed upstream. Second, **ML-HAM**'s **[Audit]** grade is only as strong as the predecessor's derivation of $O_q = e^{2\pi i/3}$ and $C = e^{i\pi}$: **ML-HAM** carries the substantive half-transport grammar, not merely the $i^2 = -1$ bookkeeping that cancels **C** against the **i** in $z = i\,\varepsilon_W h$. If the upstream values of O_q or **C** are wrong, the value $5\pi/6$ is wrong regardless of the lift.

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8. The Minimal-Hermitian-Lift Theorem

8.1 The lift principle, made precise

The earlier audit phrased the selection as "the shortest reconfiguration path." We sharpen it to a statable, checkable geodesic principle.

ML-LIFT — minimal Hermitian generator length. Among admissible one-parameter Hermitian families $H(t)$, $t \in [0,1]$, that realise the half-transport of a sector orientation ($H(0)$ the identity-sector orientation, $H(1)^2$ the target O_q), H_{cl} realises the family of minimal Killing length $L[H] = \int_0^1 \|\dot{H}(t)\|_K \, dt.$ In the bi-invariant Killing metric this length is the geodesic distance, and the minimiser is the square-root lift of smallest principal phase magnitude.

ML-SIDE — anchoring side. The minimisation is performed on the Hermitian admissibility generator H_{cl}, not on the anti-Hermitian frame coefficient $z = i\,\varepsilon_W h$.

ML-LIFT fixes *which* lift; **ML-SIDE** fixes *where the lift is measured*. Both are load-bearing, and §8.3 shows they are not interchangeable.

8.2 The theorem

Theorem 2 — Minimal Hermitian Lift Selects the CKM Branch. Assume **ML-HAM** (so $h^2/|h|^2 = O_q = e^{2\pi i/3}$), **ML-LIFT**, and **ML-SIDE**. Then

$$h/|h| = e^{(i\pi/3)},$$

so

$$\mathrm{Re}\, h > 0,\ \sigma_\varphi^q = \mathrm{sgn}\, \mathrm{Re}\, h = +1,\ \arg z = \arg(i\, h) = \pi/2 + \pi/3 = 5\pi/6.$$

Proof. The square roots of $O_q = e^{(2\pi i/3)}$ are $e^{(i\pi/3)}$ and $e^{(i4\pi/3)} = e^{(-i2\pi/3)}$, with principal phase magnitudes $\pi/3$ and $2\pi/3$. By **ML-LIFT** the geodesic lift has minimal magnitude, selecting $e^{(i\pi/3)}$, so $\mathrm{Re}\, h > 0$. Since $z = i\, \varepsilon_W\, h$ with $\varepsilon_W > 0$,

$$\sigma_\varphi^q = \mathrm{sgn}(\mathrm{Im}\, z) = \mathrm{sgn}\, \mathrm{Re}\, h = +1,$$

and $\arg z = \pi/2 + \pi/3 = 5\pi/6$. By **ML-SIDE** this is the admissible selection. ■

8.3 Why the side is physics, not convention

Multiplication by **i** is a rotation of the phase circle, and on its own it preserves distances — if the reference point is rotated with it, no ordering changes. So "the **i**-rotation breaks the magnitude ordering" is *not* the right way to put it. The real question is sharper: **which side carries the neutral identity anchor** against which admissibility length is measured? The minimal-lift rule selects the square root closest to that anchor, and the two sides place the anchor differently.

- If the neutral anchor sits on the Hermitian **h** side, the shorter square-root lift is $e^{(i\pi/3)}$, giving $\arg z = 5\pi/6$.
- If the neutral anchor sits instead on the frame-generator **z** side, the shorter root is $e^{(-i\pi/6)}$, the antipode.

Concretely, on the **z** side the roots of

$$z^2 \propto C \cdot O_q = e^{(i5\pi/3)}$$

are $e^{(i5\pi/6)}$ and $e^{(i11\pi/6)} = e^{(-i\pi/6)}$, and the one nearer the **z**-side anchor is $-\pi/6$. So the branch choice is not a harmless convention — it depends on where the admissibility metric is anchored:

anchor on h (Hermitian generator): minimal lift → **5π/6** ✓ (physical) **anchor on z (frame transport):** minimal lift → **−π/6** ✗ (antipode)

Hence Theorem 2 does not say merely "take the principal branch." It says the admissibility metric is anchored on the self-adjoint generator H_{cl}, *before* the anti-Hermitian **i**-rotation. That is the one claim that must be justified from VERSF admissibility rather than from CKM data, and it is named **ML-SIDE** precisely so the burden is visible. The burden runs in a definite direction: because **z** generates the physical frame transport, the **z**-side anchor is the *default* least-action choice, so **ML-SIDE** does not break a symmetric tie — it must actively defeat the default, which selects the antipode (see §10.2).

9. Combined CKM C_3 Reduction Theorem

Theorem 3 — C_3 Reduction and Minimal-Hermitian-Lift CKM Curvature. Assume:

1. **C3-INV** — the pre-readout substrate functional is C_3-invariant on the generation cell.
2. **C3-BG** — the admissible background is C_3-symmetric before readout.
3. **C3-EQV** — the closure restriction and $\mathbf{P_C}$ are $\mathbf{R_3}$-equivariant.
4. **C3-NB** — no non-C_3-equivariant leakage term on the committed-quark support before $\mathbf{P_Y}$.
5. **C3-NORM** — the inherited role-even common-mode branch norm, summed over the three branches, is $\mathbf{b^2}$.
6. **C3-SURV / C3-ORD** — readout is norm-preserving on, and ordered after, the surviving common mode.
7. **C3-READOUT** — the visible readout selects the CKM curvature branch as the 2↔3 common mode.
8. **ML-HAM** — the 2↔3 Hamiltonian entry satisfies $\mathbf{h^2 \propto O_q}$.
9. **ML-LIFT / ML-SIDE** — $\mathbf{H_{cl}}$ selects the minimal Hermitian square-root lift, on the Hermitian side.

Then the convention-invariant CKM curvature is

$$\zeta_{C3} = (b/\sqrt{3})\, e^{(5\pi i/6)},$$

and the leading triangle residue is

$$\Delta_{13} = \tfrac{1}{2}\, a\, \zeta_{C3}.$$

Proof. By Theorem 1, premises 1–3 give $\mathbf{[P_C H_{cl} P_C, R_3] = 0}$, so (premise 4) the pre-readout branch weights are equal. By premise 5 their sum is $\mathbf{b^2}$, so each is $\mathbf{b^2/3}$ and the branch amplitude is $\mathbf{b/\sqrt{3}}$; premises 6 carry that magnitude through readout (premise 7 identifies the visible mode as 2↔3). By Theorem 2, premises 8–9 select the positive Hermitian lift, giving $\mathbf{\arg\zeta = 5\pi/6}$. Combining magnitude and phase yields $\mathbf{\zeta_{C3} = (b/\sqrt{3})\, e^{(5\pi i/6)}}$, and the residue follows from the exact BCH identity $\mathbf{\Delta_{13} = \tfrac{1}{2}\, a\, \zeta}$. ■

9.1 Worked Numerics — Evaluation at the Inherited Norm

Theorem 3 fixes $\mathbf{\zeta_{C3} = (b/\sqrt{3})\, e^{(5\pi i/6)}}$ as a function of a single inherited input $\mathbf{b}$, where $\mathbf{b^2}$ is the *total* role-even common-mode branch norm across the three pre-readout C_3 branches (C3-NORM) — not a norm already assigned to the visible 2↔3 branch. This is the consistency the $\mathbf{\sqrt{3}}$ depends on: if $\mathbf{b}$ were already the visible 2↔3 norm, dividing by $\mathbf{\sqrt{3}}$ would be unjustified; it is justified precisely because the visible branch carries one share, $\mathbf{b^2/3}$, of the all-branches total $\mathbf{b^2}$. With the value carried in from the upstream norm budget,

$$b = 81/2000 = 0.0405, \quad \textbf{[inherited]}$$

the curvature evaluates as below. The map $b \mapsto \zeta_{C3}$ is **[Exact]** algebra; the *object* it produces carries the grade of its weakest input and is therefore **[Conditional]** on the C3 and ML premises together with the inherited **b**. No CKM datum enters: under Rule 1 the comparison of these numbers to measured mixing is quarantined to a separate section.

Magnitude.

$$|\zeta_{C3}| = b/\sqrt{3} = 27\sqrt{3}/2000 \approx 0.0233827.$$

(The rationalisation uses $(81/2000)/\sqrt{3} = 27\sqrt{3}/2000$; note the factor $81/3 = 27$, not $81\sqrt{3}/2000$.)

Phase decomposition. Since $e^{5\pi i/6} = -\sqrt{3}/2 + i/2$, the parts separate cleanly:

$$\mathrm{Re}\,\zeta_{C3} = (b/\sqrt{3})(-\sqrt{3}/2) = -b/2 = -81/4000 = -0.02025,\quad \mathrm{Im}\,\zeta_{C3} = (b/\sqrt{3})(1/2) = b/(2\sqrt{3}) = 27\sqrt{3}/4000 = 81\sqrt{3}/12000 \approx 0.0116913.$$

The real residue is a flat rational multiple of the inherited norm — $\mathrm{Re}\,\zeta_{C3} = -b/2$, with no surd — and all of the $\sqrt{3}$ from the threefold sharing sits in the imaginary part. Assembling,

$$\zeta_{C3} \approx -0.02025 + 0.01169\, i.$$ **[Conditional]**

Internal consistency. The components reproduce the target phase and magnitude exactly:

$$\arg \zeta_{C3} = 180° - \arctan\left((1/2)/(\sqrt{3}/2) \right) = 180° - 30° = 150° = 5\pi/6,\quad |\mathrm{Re}\,\zeta_{C3} + i\,\mathrm{Im}\,\zeta_{C3}| = b/\sqrt{3},\quad \mathrm{Im}\,\zeta_{C3}/|\mathrm{Re}\,\zeta_{C3}| = 1/\sqrt{3} = \tan 30°.$$

Leading triangle residue. Through the exact BCH identity $\Delta_{13} = \tfrac{1}{2} a \zeta$,

$$\Delta_{13} = \tfrac{1}{2} a\, \zeta_{C3} = (a\, b/2\sqrt{3})\, e^{5\pi i/6} = a\cdot(27\sqrt{3}/4000)\, e^{5\pi i/6},$$

so Δ_{13} inherits the same phase $5\pi/6$ and scales linearly in the structural coefficient **a** and the inherited norm **b**.

Grade. The arithmetic is exact. Its endpoints are owed exactly where Theorem 3's premises are owed: the amplitude through the C3 covariance premises and norm bookkeeping, the phase through **ML-HAM** and the minimal-lift pair (**ML-LIFT**, **ML-SIDE**), and the absolute scale through the inherited **b**. The numbers above are the conditional output of the established structure, not a microscopic evaluation of **H_cl**.

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10. Status of the Two Remaining Substrate Facts

10.1 Amplitude fact

$$[P_C H_{cl} P_C, R_3] = 0$$

is reduced to ordinary Hessian equivariance: it follows from C_3-invariance of **F** at a C_3-symmetric background with equivariant projectors. This is a genuine derivation *if* the explicit substrate functional has that symmetry — directly checkable by verifying **F[R₃x] = F[x]** on the pre-readout cell. **[Audit]**

10.2 Branch fact

σ_φ^q = +1

is reduced to **ML-LIFT** and **ML-SIDE**, and of the two it is **ML-SIDE** that carries the weight. It is not a neutral tie-breaker but an override of the physically default reading. The generation frame is transported by the anti-Hermitian **z = i ε_W h** — that is the actual generator of the unitary that rotates the frame. If "minimal reconfiguration length" is read as the length of the path the system physically traverses, that is a **z**-side quantity, and it selects **−π⁄6**, the antipode. So the most naive least-action reading favours the *wrong* branch.

ML-SIDE therefore asserts that admissibility length is measured on the self-adjoint generator **H_cl**, not on the transport it generates. That is a claim against a real competitor, not a choice among equals — and the slogan "**H_cl** is the natural home of a length functional" assumes precisely the point in dispute. The honest statement is that the **z**-side reading is the default and **ML-SIDE** must *defeat* it, not merely be preferred over a vacuum. The Hamiltonian-admissibility result supplies one half of what is needed — it forces a self-adjoint generator to *exist* — but not the operative half, that the root-selecting minimisation is taken in **H_cl**'s metric *before* the **i**-rotation rather than in the metric of the transport **z** generates. Until that is derived from the substrate action it remains owed. **[Conditional]**

Amplitude: derived from substrate C_3 symmetry, if explicit **F** is pre-readout C_3-invariant.
Branch: derived from minimal Hermitian lift, with **ML-SIDE** as the remaining load-bearing owed principle.

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11. Falsification Conditions

Amplitude theorem fails if: C3-INV, C3-BG, or C3-EQV fails (no covariance, or P_R/P_q/role-trace not generation-blind); C3-NB fails (a non-equivariant term leaks into the quark block before **P_Y**); C3-NORM fails (all-branches total ≠ **b^2**); C3-SURV fails (readout rescales the common mode); C3-ORD fails (readout precedes support formation); or C3-READOUT misidentifies the visible branch.

Branch theorem fails if: ML-HAM fails (**$h^2 \propto O_q$** does not hold); ML-SIDE fails (minimal lift acts on **z**, not **h** — which forces the **−π⁄6** antipode by §8.3); ML-LIFT fails (**H_cl** does not realise the geodesic lift); the projected Hamiltonian returns **Re h < 0** (antipodal branch); or it returns real or zero **h** (CP handedness collapses).

The full CKM C_3 target fails if *either* theorem fails. Each failure is local and named, so any negative result points at one premise rather than the whole construction.

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12. Calculation Protocol for the Next Technical Note

The next note must not start from CKM data. It should run, in order:

1. Write the explicit minimal functional **F_CKM,min** on **$\mathcal{H}$_min = $\ell^2(\mathcal{G}_3) \otimes \mathbb{C}^2$_role $\otimes$ $\mathbb{C}^8$_closure**.
2. Verify **F[R_3x] = F[x]** before **P_Y** (tests C3-INV).
3. Verify *$R_3\, x = x$*** (tests C3-BG).
4. Compute *$G = \delta^2 F|$_{x}**.
5. Check **[G, R_3] = 0**.
6. Restrict to **H_cl = G|$\mathfrak{su}(8)$** and verify **[P_C H_cl P_C, R_3] = 0** (tests C3-EQV).
7. Extract **K_even^q**; compute **w_{12}, w_{23}, w_{31}**.
8. Verify **$w_{12} = w_{23} = w_{31}$** (tests covariance) and **$\Sigma\ w = b^2$** in the inherited Killing normalisation (tests C3-NORM); confirm no non-equivariant off-diagonal term contaminates the block (tests C3-NB).
9. Apply **P_Y**; check the visible branch is 2↔3 and that the common-mode norm is preserved (tests C3-READOUT, C3-SURV).
10. Extract **h = (H_even^q)$_{23}$**; test **$h^2/|h|^2$ = e^(2πi⁄3)** (tests ML-HAM).
11. Check whether **H_cl** realises the minimal Hermitian lift **arg h = $\pi/3$** on the generator side (tests ML-LIFT, ML-SIDE).
12. Only then compare the resulting CKM matrix with data.

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13. Conclusion

The projection audit named the two remaining quark-sector bottlenecks: the amplitude condition **$|\zeta| = b/\sqrt{3}$** and the phase-branch condition **σ_φ^q = +1**. This paper derives both from substrate admissibility, and is explicit about how much each derivation still owes.

For the amplitude, the result is clean. C_3-invariance of the pre-readout functional at a C_3-symmetric background, with equivariant restriction and projector, forces the Hessian to commute with the generation-cycle operator:

[P_C H_cl P_C, R_3] = 0.

That covariance makes the pre-readout block circulant, equalising the three forward branch entries as complex numbers; the inherited total norm **b^2** then splits as **$b^2/3$** per branch, and norm-preserving readout carries the amplitude **$b/\sqrt{3}$** into the visible 2↔3 mode. The amplitude is no longer a loose target — it is the consequence of one explicit substrate symmetry plus named inheritance bookkeeping.

For the branch, the result is promising but properly conditional. On the Hermitian side the C_3 square-root relation is **$h^2 \propto O_q$**; the minimal-length lift is **$h/|h| = e^{(i\pi/3)}$**, and the **i**-rotation to the frame generator carries it to **$\arg \zeta = 5\pi/6$**. This selects **$\sigma_\varphi^q = +1$** with no CKM input — *provided* the minimal-length selection truly belongs to **H_{cl}** and is anchored on the Hermitian side. The same rule on the frame side selects the antipode, so the side is the physics; that is the one load-bearing principle still owed.

The frontier is therefore split cleanly into two local, falsifiable tasks: verify C_3-invariance of the explicit pre-readout functional (amplitude), and prove that **H_{cl}** implements minimal Hermitian lift on the generator side (branch). If both hold, the CKM C_3 curvature

$$\zeta_{C3} = (b/\sqrt{3})\, e^{(5\pi i/6)}$$

is derived rather than reconstructed. If either fails, the failure is local, named, and diagnostic.

Appendix A — One-Line Proofs (Numerically Verified)

All four steps below were confirmed numerically: a generic Hermitian 3×3 block, averaged over the C_3 conjugation, commutes exactly with **R_3** and is circulant with equal off-diagonal magnitudes; the **$b^2/3$** split reproduces **$b/\sqrt{3}$**; the two roots of **$e^{(2\pi i/3)}$** sit at **$+\pi/3$** and **$-2\pi/3$**; and the minimal-lift selection gives **$\arg z = 5\pi/6$**, with the z-side analogue collapsing to **$-\pi/6$**.

A.1 Hessian equivariance

If **$F[R_3x] = F[x]$** and $R_3x = x**$, then at **x^*** the second differential satisfies **$R_3^\dagger\, G\, R_3 = G$**, hence **$[G, R_3] = 0$**.

A.2 Branch equality from circulant form

If **$R_3\, K\, R_3^\dagger = K$** for a Hermitian 3×3 generation block, then **K** is circulant:

$$K = \begin{pmatrix} d & h & h^{-} \\ h^{-} & d & h \\ h & h^{-} & d \end{pmatrix}$$

so the forward entries are equal as complex numbers, **$(K)_{12} = (K)_{23} = (K)_{31} = h$**, and in particular **$|K_{12}| = |K_{23}| = |K_{31}|$**.

A.3 Root-normalised amplitude

If **$w_{12} = w_{23} = w_{31}$** and **$\Sigma w = b^2$**, then **$w_{ij} = b^2/3$** and **$\sqrt{w} = b/\sqrt{3}$**.

A.4 Minimal Hermitian lift

If **$h^2/|h|^2 = e^{(2\pi i/3)}$**, the roots are **$e^{(i\pi/3)}$** and **$e^{(-i2\pi/3)}$**, with phase magnitudes **$\pi/3$** and **$2\pi/3$**. Minimal lift selects **$\pi/3$**, hence **$\arg z = \pi/2 + \pi/3 = 5\pi/6$**. The same rule on the **z** side selects **$-\pi/6$**, the antipode.

Appendix B — Status Labels

Claim	Grade
C_3 branch orbit exists	**[Exact]** algebra
C_3 branch orbit is Killing-orthonormal	**[Exact]** algebra
Hessian commutes with R_3 if F is C_3-invariant at symmetric background	**[Exact]** symmetry theorem
$[P\_C\ H\_cl\ P\_C, R_3] = 0$	**[Audit]** — follows if restriction and P_C are equivariant
Circulant forces equal forward entries $(K)_{12} = (K)_{23} = (K)_{31}$	**[Exact]**
$w_{12} = w_{23} = w_{31}$	**[Exact]** given covariance of K_even^q
$\lvert\zeta\rvert = b/\sqrt{3}$	**[Conditional]** — equal weights + inherited norm + norm-preserving readout
$h^2 \propto O\_q$	**[Audit]** on ML-HAM (Hamiltonian-side phase grammar)
minimal Hermitian lift on generator side	**[Owed]** admissibility principle (ML-LIFT, ML-SIDE)
$\sigma\_\varphi\text{\^{}}q = +1$	**[Conditional]** — ML-HAM + minimal Hermitian lift
full H_cl numerical evaluation	**[Owed]**

Appendix C — Consolidated Named Premises

Amplitude (C_3) premises

- **C3-INV** — substrate functional is C_3-invariant on the pre-readout generation cell: **$F[R_3x] = F[x]$**.
- **C3-BG** — admissible background is C_3-fixed: *$R_3x = x$***.
- **C3-EQV** — closure restriction preserves the C_3 action and **$P\_C\ R_3 = R_3\ P\_C$**; the role projectors **P_R**, **P_q** and the role trace are generation-blind (each commutes with **R_3**), carrying the commutator into **K_even^q**.
- **C3-NB** — no non-C_3-equivariant leakage: after the role reduction, no transport or record term adds a non-equivariant off-diagonal contribution to the committed-quark block

before **P_Y**. (Not a restatement of covariance — covariance already gives equal weights; this rules out contamination of the circulant block.)

- **C3-NORM** — inherited total role-even common-mode branch norm, summed over the three C_3 branches, is **b^2** (the visible 2↔3 curvature branch carries **$b^2/3$**).
- **C3-SURV** — **P_Y** preserves the norm of the common mode it selects.
- **C3-ORD** — **P_Y** acts after the C_3 supports are formed.
- **C3-READOUT** — the visible readout selects the CKM curvature branch as the 2↔3 common mode.

Branch (minimal-lift) premises

- **ML-HAM** — Hamiltonian-side half-transport grammar: **h^2 ∝ O_q**, equivalently **z^2 ∝ C·O_q** with **C = e^(iπ), O_q = e^(2πi/3)**.
- **ML-LIFT** — **H_cl** realises the square-root lift of minimal Killing length (geodesic lift from the identity).
- **ML-SIDE** — the minimal-length selection is anchored on the Hermitian generator **H_cl**, before the anti-Hermitian **i**-rotation to **z**.

The amplitude theorem rests on **C3-INV/BG/EQV** (covariance) plus **C3-NB/NORM/SURV/ORD/READOUT** (inheritance and readout bookkeeping). The branch theorem rests on **ML-HAM** plus **ML-LIFT/ML-SIDE**. Every premise is CKM-free and individually falsifiable once the explicit substrate functional and projectors are supplied.