The Saturation Theorem

Closure-Supported Response, the Conjugate Completion of the Free Sector, the Congruence Theorem at the Lattice's Gate, and the Conditional Discharge of CO-2

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General Reader Summary

The central claim, first

The route's previous papers answered three questions in sequence. Who owns the permanent structure of a family of identical particles? The family — the Ownership Principle. Whose whole numbers does the charge ladder force? The family's, because only the family survives transport — the Carrier Theorem. And when one member is measured, what must it show? The family's integer divided by the family's size — q⁄k, permanently, identically, by the Realization Theorem. Together those results explain why the strangest charges in physics — the thirds — appear at all. They also create, in plain view, the route's last debt: if every member of every three-member family permanently shows a third, *why has no free third ever been seen?* Every fractional charge the world contains is found locked inside a larger composite, and a theorem that forces the fraction owes an account of the lock.

This paper's answer is that there is no lock — because there was never a prisoner. The question "what prevents the fraction from escaping?" presupposes that the fraction is a thing that could escape: a self-supporting quantity riding on the member, which nature must then somehow cage. The route's own results say otherwise. The fraction is a *reading* — the family's whole number, viewed from one seat — and a reading has no existence apart from the thing it reads. Pull the member out of the family and you have not liberated a third; you have walked away from the only structure in which "a third" meant anything. The receipt was printed by the bank. Outside the bank, there is no ledger to print it from.

The bank, one last time

Recall the pooled fund: three depositors, one pound, stirred continuously. The pool owns the pound; each seat, asked, shows a third — that was the route so far. Now try to take a seat *out of the bank*. What does the detached seat hold? Not a third of a pound: the third was never the seat's property — it was the bank's standing ledger, read at the seat. Outside the bank there is no ledger, no pool, no standing entry to read; there is just a chair. The fraction did not escape, and it was not confined. It simply does not exist where its support does not exist — the way a shadow does not exist without the object that casts it, and cannot be carried away in a bag. One honesty

note belongs exactly here, because the metaphor makes that last step feel automatic and it is not: that a thing with no permanent properties cannot exist as a free permanent object is one of the paper's *named premises*, not a triviality. A world containing free, property-less quark-like objects is perfectly coherent — the paper builds that rival world in full, prices it, and identifies the premise that excludes it as one of the theorem's three load-bearing walls.

One more piece of banking completes the picture. The world's composite particles come in two families: three quarks bound together, or one quark bound with one *anti*quark. The first is a full bank — every seat occupied, the books closed, the total a whole pound; and the seats need not come from one family, because the table is the bank's, not the family's — the proton seats two members of one three-member family and one of another, and the books still close. The second is a seat paired with an anti-seat: a deposit and an exactly opposite debt, which close the books *jointly* — the loop runs forward through one and backward through the other, and the accounting completes. The previous paper had already noticed the second family and wrote it into the debt; what it could not say is *why* pairing closes the books. This paper proves that these two ways of closing them are the only admissible ways for anything to exist freely — which is exactly the composite spectrum the world shows.

What is genuinely new here, named honestly

The argument requires premises beyond what the route inherits, and the paper's discipline — learned across its predecessors — is to name every new premise in public rather than smuggle it inside the prose. There are five. That a measurement's standing output needs standing *support* — a structure whose permanent content the measurement reads — and that the support is the family's, not the member's. That a broken family cannot quietly re-close as a smaller family — family sizes are kinds of structure, not headcounts. That to exist *freely* as a permanent object is to support permanent content — a thing with no standing properties has no standing existence. And that a member paired with its anti-member genuinely closes the transport that defines response — a premise whose first two parts (that anti-families exist with opposite numbers, and that they match their families in size) the previous paper already named, and whose closing claim is the new content here. And that a family-sized table can be seated by members of *different* families of the same size — the table belongs to the geometry, not to any one family — which is what lets the proton, two seats from one three-member family and one from another, close its books at all. One further result then costs no further premise: the rule for *which* families may share a table — divide each family's whole number by the family size, and the remainders must match — is derived from the five rather than added as a sixth, because a mixed table with mismatched remainders would close its books on a fraction, and the currency mints no such note. The world's quark families pass the test, which is why the proton's books close; and a family that failed it would not be banished from existence, only from shared tables — it would dine alone — or, in exactly one arrangement the arithmetic permits, at a special table: three families whose mismatched remainders happen to add up to a whole number can share a three-seat table even though no two of them could ever pair, a table the world's current families cannot fill — which makes it a standing prediction rather than a loophole. Each premise is given an address in the programme's earlier papers where it must be confirmed, and a stated price if it fails. The theorem is exactly as strong as those five readings — and the paper says so at every joint.

The honest condition

The paper is written from the programme's standing — the bath ontology, where the corpus's independent arguments have converged — under the discipline its predecessors fixed: the formal verdict at the node W0 does not move; the two conditions on which the whole route's exposure is total are carried visibly and remain undecided; and every result is stated at a conditionality that survives either verdict. What the paper adds, if its named premises hold at their sources, is the route's empirical face, completed: fractions that must appear, must be permanent, must never be free, and must sum to whole numbers in exactly two patterns of composite — every one of which the world already shows.

Abstract

The headline. The Realization Theorem forced uniform member-grain registration at q/k and thereby created the route's last structural obligation, CO-2, named in its final form with two deliverables — (i) the balance restriction on free configurations and (ii) the sector congruence [C-RES (c)] — on pain of clause (iii)'s refutation at the free sector. This paper discharges the first deliverable, supersedes it with a stronger condition, and derives the second at its joint-support grain — the congruence of mixing classes proven, the universal form alone left open and unconsumed. The result is the **Saturation Theorem**: *admissible free standing configurations are exactly the transport-complete ones — saturated closure structures (occupancy single-class or mixed across same-capacity classes), conjugate-completed member–antimember pairs (within or across classes), and their compositions* — with the immediate **Integrality Corollary**: every admissible free configuration carries integral total response, so the world's free integral totals are recovered as theorem rather than coincidence, and fractional confinement emerges from the same closure architecture that generates the fraction. The engine is the **support/traversal distinction**: a member-grain record's *traverser* is cheap realized structure (one seat, one loop — REG-E's configuration, free or housed), but its standing output reads the standing transport structure of the class — the *support* — and the support, being class-owned and standingly undecomposable [B-IRR], is not partitioned among members and cannot accompany a detached subset. An unhoused member's loop closes over nothing standing; the comparison cancels the churn and what survives is not zero but *undefined* — the holonomy of a broken loop. The fraction is not confined. It is unsupported.

The new imports, named — the paper's engine, and its honesty. Five, each with slot and fallback — but not five of equal weight: three are load-bearing (R-SUP, FREE-R, J-COM — each failure total for the theorem or total at the table) and two are structural hygiene (NRC, C-CLO — failures graceful or cleanly local), a gradient visible in the fallbacks below and classified explicitly in §2. **R-SUP (record support)**: a record's standing output requires the standing transport structure it reads — traverser realized, support standing [slot: the charge paper's record construction, read alongside R-O for *whose* standing structure the comparison reads; *fallback:* the free-fraction rival revives — fractions independently supported, free thirds admissible — and the theorem falls whole]. **NRC (no re-closure)**: proper subsets of a closure class do not re-constitute as smaller classes — capacities are geometric kinds, not occupancy counts [slot: the

census's capacity derivation; *fallback:* exactness fails — free re-closed sub-structures with their own underived integers become admissible — while fractional confinement survives, since a re-closed carrier carries an integer, never a fraction; the degradation is graceful and priced]. **FREE-R (free standing admissibility)**: free standing existence requires standing response support — a structure supporting no standing content has no entry in the standing catalogue [slot: the admissibility conditions on standing configurations, at the Fold/G-S territory; *fallback:* the theorem demotes from confinement to *charge-indefiniteness* — free subsets admissible but with undefined standing response — with the world's total absence of free quark-like objects priced as a lean against, not a proof]. **C-CLO (conjugate closure)**: consuming the predecessor's C-RES clauses (a) and (b) under its own name — conjugate windings $-q$ [(a), Inherited at the lattice's $\mathbb{Z}$] and capacity preservation, $\bar{k} = k$ [(b), at the census's slot, carried because the cancellation arithmetic needs it] — and adding the one reading those clauses do not contain, the framework standing of the pairing itself: a member–antimember pair composes traversal with reversed traversal into a completed support, orientation reversal lifting from loops to *carriers* [slot: the charge paper's orientation-reversal structure, where the lattice already negates windings on reversed loops; *fallback:* the conjugate clause lapses, the mesonic sector is unaccounted, and the theorem's exactness is false at the particle table — a strike the paper prices rather than hides]. **J-COM (joint completion)**: completion is individuated at the closure-geometry grain, not the class-identity grain — a k-seat support is completable jointly by members of distinct same-capacity classes, each seat reading its own class's share [slot: the census's closure structure; forced by the proton, which fills no single class; *fallback:* clauses (a)/(b) revert to single-class form and the theorem is refuted at the world's table — exposure total there].

The argument. §3 installs the support/traversal distinction and reconciles it with REG-E explicitly — consuming the housed clause REG-E itself now supplies: a *housed* member completes loops against its intact class's standing structure — which is how the Realization Theorem's registrations were ever possible — while an *unhoused* member has no class structure to read. §4 proves the **Transport Completion Lemma** on the corrected ground: the support is class-owned [Ownership] and admits no standing decomposition [B-IRR], so it is owned undivided — a subset of the members is not a subset of the support, and an unhoused subset is transport-incomplete with respect to standing response. §5 closes the re-closure escape at NRC's slot. §6 consolidates the **Response Support Theorem**: standing response exists exactly on transport-complete supports. §7 names FREE-R — the bridge from "no standing response" to "no free standing existence" that the argument's most natural telling leaves silent — and §8 proves the Saturation Theorem over both new clauses — closure completion with mixed occupancy [J-COM], conjugate completion within or across classes — with the Integrality Corollary: free totals integral, the mixed clauses conditioned on the predecessor's sector congruence [C-RES (c), Inherited at its slot, Open there — the equivalence with mixed integrality Proven here, the world's compliance ($2 \equiv -1 \bmod 3$) at O1], and §9 then discharges the condition outright: the **Composite Integrality Lemma** — admissible free configurations are themselves carriers, their windings integral unconditionally by the lattice's grain-universality [L-G] — converts the congruence from a condition on integrality into the lattice's own gate on admissibility, and the **Congruence Theorem** [C-RES (c)-JS] derives it for every jointly completing pair: pair forcing through conjugates, occupancy forcing through the census's prime capacities, chain closure across mixing families. §8's Promotion Conjecture is discharged in improved form — the gate is the lattice's, no new census clause — the world's residue is entailed

by the existence of mixed hadrons — conditional on their identification as jointly completed supports, at O1 — rather than assumed, and any incongruent class is predicted segregated at pair grain — excluded from all conjugate pairing and two-class housing, with the sum-zero three-class completion the theorem's one permitted incongruent co-habitation and its named exotic. The universal congruence [C-RES (c)-U] alone remains open at the predecessor's slot, and the route no longer consumes it; the handed-forward question is assignment — why one residue family.

Audits and rivals. §10 constructs both rivals — the free-fraction world (R-SUP rejected) and the charge-indefinite world (FREE-R rejected) — prices each, and locates each decider. §11 audits every census capacity, closes with the **Minimal Composite Spectrum Theorem** — the minimal admissible free composites exactly the two hadron structures, under O1 — and boxes the observable consequences whole, ten derived lines and one standing vacancy: k = 1 (singletons self-saturate — the world's free integral charges embarrass nothing; the charge paper recovered); k = 2 (halves confined; the predecessor's null-branch freedom superseded, its free two-fold consonance inverted to exposure); k = 3 (exactly two minimal free composites — the saturated triple, occupancy possibly mixed, and the conjugate pair, possibly cross-class — against the strong sector's exactly two: baryons including the proton, now licensed rather than smuggled, and mesons including the charged pion; multiquark states as compositions; flagged [Conjectural] at O1). The deconfined collective medium is fenced as realized dynamics, outside the theorem's quantifier.

CO-2, accounted in halves; the Dichotomy, superseded; W0, re-priced. §12 splits the discharge as the truth splits: deliverable (i) — the balance restriction — moves to **[Conditionally discharged, and superseded]**, transport completion the stronger condition from which n ≡ m (mod k) falls out as arithmetic, closing at the R-SUP, NRC, FREE-R, C-CLO, and J-COM slots; deliverable (ii) — the congruence — splits at §9: the joint-support form **derived** [C-RES (c)-JS, Proven at the Composite Integrality Lemma's chain], the universal form [Open, unconsumed] handed forward as the assignment question. §12 also supersedes the predecessor's Confinement Dichotomy in public — confinement support-theoretic, not arithmetic — with the retraction, the inverted consonance, and the closed test surface stated at full width there. All carried — as every paper above CO-0 is obliged to carry them — under the two inherited total exposures, **B-IRR** and **C-ORD**, both undecided, with the composition-order audit re-recommended. §13 states the W0 chain whole, every link named: the bath now predicts appearance, value, permanence, confinement, integral free totals, the two-mode composite spectrum, and the mixing rule itself from one closure architecture — the chain consuming no residue condition, the congruence its output rather than its input — while the diagonal ledger owes every item externally. The pressure sharpens; the marker does not move.

The paper's four results, separated. They are distinct contributions and a reader should be able to strike one without losing sight of the others. **New Result 1 — the Support/Traversal Distinction (§3):** a record's traverser (one realized seat) is not the record's support (the class's standing transport structure); housed registration reads intact support, unhoused traversal reads nothing — the distinction that reconciles member-grain measurement with class-owned response. [Installed; load at R-SUP's slot.] **New Result 2 — the Saturation Theorem (§8):** admissible free standing configurations are exactly the transport-complete ones — saturated

closure structures with occupancy single-class or mixed [J-COM], conjugate pairs within or across classes, and their compositions — with integral totals as corollary; confinement derived from support, not imposed as a rule; elevated in §11 to the **Minimal Composite Spectrum Theorem**: the minimal admissible free composites are exactly the baryon and meson structures, everything else their compositions. [Proven, conditional as stated.] **New Result 3 — joint completion and the congruence's role (§2, §8):** the import J-COM — completion at the closure-geometry grain, the proton audit's verdict — together with the completion-theoretic role of the predecessor's sector congruence [C-RES (c), Inherited]: mixed-sector integrality ⟺ the congruence [the equivalence: Proven], with the Promotion Conjecture stated at §8 and discharged at §9. **New Result 4 — the Composite Integrality Lemma and the Congruence Theorem (§9):** admissible free configurations are themselves carriers, their windings integral unconditionally [L-G] — so the congruence of jointly completing equal-capacity classes is *derived*, not assumed: the lattice's own gate on admissibility, bought without a sixth premise, entailing the world's residue from the existence of mixed hadrons (conditional on their identification as jointly completed supports, at O1) and predicting pair-grain segregation for any incongruent class — the sum-zero triple left open as the theorem's named exotic. [The Lemma and the Theorem: Proven, conditional at their chain; C-RES (c)-U: Open, unconsumed — the assignment question, the route's handed-forward remainder.]

Epistemic markers: [Inherited] imported from prior VERSF papers; [Imported-External] imported from standard mathematics outside the programme, carried at the external source's standing; [Proven] established here; [Conditional] holding under stated inputs; [Conjectural] motivated but unproven; [Open] undecided.

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1. Introduction — The Debt, Received

The Realization Theorem ended by naming its own price. Clause (iii) forces every member of every k-member class to register q/k — permanently, identically, on every admissible loop — and a world containing admissible *free* members would therefore display standing free fractions. The world displays none: every free configuration ever registered carries class-integral total charge, and no isolated fractional object has ever been observed. The Realization paper converted that tension into the named obligation **CO-2**, stated deliberately at integral totals rather than seat-filling: *derive, from the transport and admissibility structure, that free standing configurations carry integral total readings* — occupancy saturation and conjugate neutralization the two known modes, the second read from the world's table and awaiting framework standing — and stated the stakes without softening: a route with forcing and without saturation predicts free thirds the world refuses to show, so CO-2 is load-bearing for clause (iii)'s empirical consistency, and the route must discharge it or die by it. This paper is the discharge.

The question's shape must be fixed before the answer is built, because the natural framing is the wrong one. *What prevents the fraction from escaping?* presupposes that the fraction is the kind of thing that could escape — an independently supported quantity, resident on the member, which the world must then cage. Everything the route has proven points the other way. The fraction is the class's integer in member notation [the Uniform Reading Lemma's identification clause]; the integer is the class's because only the class survives transport as a standing subject [the Carrier Theorem]; and the registration that displays the fraction reads standing content the comparison is built to isolate [the Realization Theorem]. At every level the member-grain fraction is *derived* — a reading of class-owned structure — and a derived quantity does not outlive its derivation. The right question is therefore not what confines the fraction but **what supports it**: on what structure does standing response depend for its very definition, and does that structure survive the detachment of a member? The paper's answer — standing response is closure-supported, and the support does not partition — converts confinement from a rule the world obeys into a shadow the question was casting: nothing is confined, because nothing detachable was ever there.

Two disciplines govern the development, both inherited — and the first is the one this argument most tempts. First, **every new premise is named.** The route's predecessors each turned on making a smuggled premise visible — the Carrier paper's minimality earned by composition, the Realization paper's further premises named as REG-E, REG-G, and R-O rather than derived from symmetry. The present argument needs five premises beyond its inheritance, and the corpus's rule is absolute: an analogue is a motivation, not a license, so the holonomy parallel that motivates the support principle is spent as motivation only, and the principle itself is grounded in named readings with slots and fallbacks (§2). In particular the bridge from *supports no standing response* to *cannot exist freely* — the step on which the entire theorem turns, and the step most temptingly taken for free — is named as the import it is [FREE-R], because a world of free, charge-indefinite members is consistent with everything upstream of it, and the difference between that world and ours is a premise, not a corollary. Second, **the consequences are priced, including the strange ones.** A theorem about the free sector quantifies over everything the

world sets free, so §8's conjugate clause must supply the framework standing the predecessor's second mode was named without, §11's audits must run at every census capacity, and the one residue condition on which mixed-composite integrality turns must be flagged as the open question it is rather than absorbed as a convenience.

One positioning statement, carried from the predecessors because it governs this paper's voice equally. The paper is written from the programme's standing — the bath, with the ledger retired from scope by inheritance, since the Carrier Theorem proved the ledger contains no member-grain fractions for a saturation account to govern. The discipline binding the posture is unchanged: **the marker at W0 does not move**; the two conditions at which the route's exposure is total — B-IRR at the Bath Criterion, C-ORD at the structure-group imports — appear in every conditionality statement below, undecided; and §13, where the results feed the W0 pressure, draws its force from the conditional chain alone, so that the posture is never fed into its own evidence.

The paper derives no charge values, decides no ontology, and touches no gauge interior. It identifies the structure on which standing response depends, proves that the structure does not survive detachment, converts that fact into an admissibility criterion for the free sector, and recovers — as theorem, at stated conditionality — the three facts about fractional charge the world insists on: it appears, it never appears alone, and everything free is whole.

2. Inherited Results and Imports

The charge lattice and the record construction (R-C, R-S, L-G, AD) [Inherited — ⟨cite: the charge paper⟩, at the statuses and slots the route fixed]: responses classified by winding $q \in \mathbb{Z}$; the winding read off a persistent subject's closed-loop transport record [R-C]; windings as standing classifications [R-S]; the lattice binding at every transport-stable grain [L-G]; windings additive under composition [AD], with the member-grain clause [AD-m] consumed at the Realization paper. One further structural fact of the same source is consumed here for the first time and promoted where it is spent: the lattice's **orientation-reversal structure** — a loop traversed in reverse carries the negated winding, $-q$ for q — which the conjugate-closure import [C-CLO, below] reads for its ground.

The Carrier Theorem [Inherited — at its full conditionality]: quantized response attaches at the minimal transport-stable subject; under the bath [B-IRR] that subject is the class; member grain lies below the quantization grain; member-identity-through-transport is realized, not standing — the class's standing structure admits **no standing decomposition** into member parts. That last clause, proven at the Carrier paper's §5 and consumed there for the attachment question, is consumed here for the *support* question (§4): what admits no standing parts is owned undivided. The two total exposures are carried forward visibly, as the route's discipline requires. **B-IRR** [slot: the Bath Criterion's transport structure]: if broken, the route closes at CO-0 and everything here lapses. **C-ORD** [slot: the structure-group imports]: if the U(1) response is ordered member-wise on the pre-mixing carrier, the carrier analysis is bypassed for U(1), the route closes at CO-0 on both ontologies, and everything here lapses with it. Neither is decided here; both appear in every conditionality statement below; the composition-order audit remains the route's cheapest insurance and is re-recommended.

The Ownership Principle [Inherited — at its statuses]: standing structure is class-owned; the standing/realized boundary at its §10 slot, consumed, not duplicated; the Uniform Reading Lemma with its identification clause — the uniform reading is the class's winding in member notation, indexed to no line.

The Realization Theorem [Inherited — at its conditionality: REG-E, REG-G, R-O at its scope clause, AD-m, and the carrier chain entire]: clauses (i)–(iii) — existence, uniqueness, and forced registration of the uniform reading at q/k; the registration configuration (a single realized seat traversing a closed admissible loop, free or *housed* — REG-E, whose housed clause this paper consumes directly in §3 rather than reconstructing); the grain reading (outputs are seat-grain response content composing to the class winding, not the class total read whole — REG-G); the standing-output reading (the comparison cancels the realized complement given the specification — R-O, at its scope clause); the **Class Uniformity Criterion** and the **k = 2 winding constraint**, handed to O1 as sieves and consumed by §11's audits at their stated grades; **C-RES** at its three clauses — conjugate windings, $-q$ for q [(a), Inherited at the lattice's $\mathbb{Z}$], capacity preservation, $\bar{k} = k$ [(b), at the census's slot], and the **sector congruence** — equal-capacity class windings congruent modulo capacity [(c), the predecessor's structural finding, at the census's slot, its refuter carried at that paper's §13 — inherited here under its own name, chain of custody, nothing re-derived]; the **Confinement Dichotomy** — confinement ⟺ invertible residue, null-residue sectors unconfined with free seats admissible at integral forced readings, the leptons recovered on the trivial branch — inherited *at its stated conditionality*: the dichotomy was expressly conditional on integral totals being the operative free-admissibility constraint, which is precisely the constraint this paper replaces, and §12 states the supersession in full rather than letting it happen; and **CO-2 in its final two-deliverable form** — (i) the balance restriction, free configurations balanced $n \equiv m \pmod k$ on invertible-residue sectors, and (ii) the congruence — stated, load-bearing, and handed forward. The forcing clause is what creates this paper's obligation, and the registration configuration is what this paper's Lemma must remain consistent with — a consistency §3 establishes explicitly rather than leaving to inspection, because the argument's most natural telling violates it.

The Capacity Census [Inherited — *Three by Two*, at its slots]: admissible class capacities $k \in \{1, 2, 3\}$; the O1 occupancy identifications [Open]; the G-S housing territory, where the saturation question was fenced and where two of this paper's imports take their slots.

Five new imports, named with slots and fallbacks — the paper's engine, and its honesty.

R-SUP (record support). A closed-loop record's standing output requires the existence of the standing transport structure it reads: the record's *traverser* — the realized structure that carries the comparison around the loop — is distinct from the record's *support* — the standing transport structure whose content the comparison isolates — and the output is defined exactly where the support is. Under the bath, the support of a response record is the class's standing structure [the Carrier Theorem: the class is the minimal transport-stable subject; R-O: the output is a function of the loop and the standing transport structure]. R-SUP is the reading that makes "the standing transport structure" *referential*: the comparison does not conjure standing content from the loop's geometry alone — it reads the standing content of the carrier through which the loop runs, and where no such carrier exists, the comparison completes over nothing and its output is not zero

but undefined. The import is grounded rather than bare: §3's Support Non-Creation Principle derives its shape from R-O's own functional form — a record reveals standing content and cannot create it — leaving to the slot only the referential question. [Slot: the charge paper's record construction, read alongside R-O for the dependence of the output on the *existence* of the carrier's standing structure — whether the construction defines outputs for loops run where no transport-complete carrier is present. *Fallback if broken:* standing response is loop-supported rather than carrier-supported, the free-fraction rival of §10 revives in full — detached members carry their reading with them — and the theorem falls whole; the paper is exactly as strong as R-SUP and says so.]

NRC (no re-closure). Proper subsets of a closure class do not constitute closure classes: the census's capacities $k \in \{1, 2, 3\}$ are **geometric kinds** — distinct closure structures, each with its own transport architecture — and not occupancy counts of a generic container, so two unhoused members of a broken $k = 3$ class are not, and cannot re-close as, a $k = 2$ class. [Slot: the census's capacity derivation, read for the kind/count question — whether the admissible capacities are derived as distinct closure geometries or as fillings of one structure. *Fallback if broken:* re-closure admissible; the Saturation Theorem's *exactness* fails — free re-closed sub-structures, carrying their own underived integers, join the admissible free sector — while **fractional confinement survives the strike**, since a re-closed carrier is a carrier and carries an integer, never a fraction; the degradation is graceful and is priced as such in §§5 and 15.]

FREE-R (free standing admissibility). Free standing existence requires standing response support: an admissible free standing configuration is an entry in the standing catalogue, catalogue entries are individuated by their standing content, and a structure supporting no standing response has no standing content at response grain — hence no catalogue entry, hence no free standing existence. The import extends rather than introduces the programme's admissibility criterion, and the continuity is the argument for it: throughout the corpus, standing existence has been individuated by standing transport content — the Ownership paper's catalogue is a catalogue of invariant-tuple content, the Carrier paper's transport-stable subjects are subjects *because* their identity is standing structure, and nothing in the corpus has ever admitted a standing entry individuated by anything else. FREE-R asserts no new principle of individuation; it asserts that the catalogue's one existing principle governs the free sector too. It is nonetheless named as an import — the bridge from *unsupported* to *inadmissible* — precisely because the corpus's boundary discipline makes the alternative coherent: realized structure without standing content is not a contradiction — it is the bath's own description of member allocation — so a world of free, charge-indefinite members is consistent with everything upstream, and excluding it is a premise about what the standing catalogue admits, not a theorem of what records read. [Slot: the admissibility conditions on free standing configurations, at the Fold/G-S territory — read for whether catalogue entry requires standing transport content. *Fallback if broken:* the theorem demotes from **confinement** to **charge-indefiniteness**: unhoused subsets admissible as free structure with undefined standing response; the route's empirical face then rests on the observed total absence of free quark-like objects — which the demoted theorem accommodates less sharply — priced in §10 as a lean against the fallback world, not a proof.]

C-CLO (conjugate closure). The import consumes the predecessor's **C-RES** clauses (a) and (b) under its own name — chain of custody, nothing re-derived: clause (a), conjugate windings, $-q$

for every q [Inherited — the lattice's $\mathbb{Z}$ structure]; clause (b), the conjugate class $\boldsymbol{\mathcal{C}}$ at the **same capacity k** as $\boldsymbol{\mathcal{C}}$ [at the census's slot, carried visibly because the Integrality Corollary's cancellation arithmetic, $q_i/k + (-q_i)/k = 0$, fails without it] — and adds the one reading those clauses do not contain, the framework standing of the pairing itself: a member of $\boldsymbol{\mathcal{C}}$ paired with a member of $\boldsymbol{\mathcal{C}}$ composes forward traversal with reversed traversal into a **jointly completed support** — the pair, as a composite, transport-complete with respect to response, the loop running through one and conjugately through the other and closing. The ground is the lattice's own orientation-reversal structure — reversed loops negate windings — read as the existence of conjugate carriers rather than merely conjugate paths: the lift from loops to carriers is exactly what C-CLO asserts and C-RES did not. [Slot: the charge paper's record construction, read for conjugation — whether orientation reversal lifts from loops to carriers — together with the G-S pairing admissibility. *Fallback if broken:* the conjugate clause lapses; the theorem's free sector contains only saturated classes; the mesonic composite family is unaccounted and the exactness claim is false at the particle table — a clean strike, stated at full width in §15.]

J-COM (joint completion). Completion — **in both of the theorem's modes** — is individuated at the **geometry/capacity grain**, not the class-identity grain: a k-seat closure support is completable by members of one class or **jointly by members of distinct same-capacity classes**, and a conjugate completion may pair a member with an antimember **across distinct same-capacity classes**, the completed support in either mode the completion structure's own, each participant reading its own class's share q_i/k. The import is an audit verdict, not a rescue: the proton — two seats of the up class and one of the down — fills no single class, so *any* successful saturation theorem must license completion at the closure-geometry grain rather than the flavour-class grain; the proton does not force an adjustment to this theorem in particular, it identifies a clause every theorem of this kind was always going to need. The import also fixes how the Transport Completion Lemma's vocabulary is consumed: the Lemma's "the class's standing structure" individuates, under J-COM, the *geometry's* standing structure carrying the participating classes' shares — coinciding with the class's own when occupancy is single-class, and leaving the Lemma's conclusion untouched in every case, since detachment destroys the geometry regardless of whose members filled it. [Slot: the census's closure structure, read together with the Carrier paper's bath branch for what "the class's standing structure" individuates — whether k-seat supports are completable only by one class's members or by any same-capacity occupancy. *Fallback if broken:* clauses (a) and (b) of the Saturation Theorem revert to single-class form; the proton and the charged pion are inadmissible; the theorem is refuted at the world's table — the exposure is total there and the paper says so.]

One inherited condition, consumed at §8 and derived at §9: C-RES clause (c) — the congruence. The mixed clauses of §8 state the predecessor's sector congruence — participating equal-capacity class windings congruent mod k — as the arithmetic condition of mixed integrality, inherited at the predecessor's census slot, nothing re-derived and nothing re-slotted. §8 adds its completion-theoretic role and names the promotion candidate; §9 then discharges it in improved form: the **Composite Integrality Lemma** proves that admissible free configurations are themselves carriers with unconditionally integral windings [L-G], the congruence relocates from a condition on integrality to the lattice's own gate on admissibility, and the **Congruence Theorem** derives it for every jointly completing pair — consuming no import beyond this

section's. What remains open is the universal form alone [C-RES (c)-U, at the predecessor's slot], and the route does not consume it.

No new physical imports beyond R-SUP, NRC, FREE-R, C-CLO, and J-COM. The paper adds one distinction (§3 — support/traversal, consuming inherited material), one principle grounding it (§3 — Support Non-Creation), one lemma (§4), one exclusion (§5 — at NRC), one theorem pair (§§6, 8), one lemma–theorem pair deriving the congruence (§9 — consuming only already-slotted material, no import added), two rival constructions (§10), the audits (§11), one supersession stated in public (§12), and one split accounting (§12). The congruence is inherited [C-RES (c)] and its joint-support form derived (§9), not introduced; the discharge consumes no residue.

The imports classified by weight, so the load-bearing wall is visible. The five are not equally heavy, and a reader pricing the theorem should know which failures kill and which merely trim. **Essential — the load-bearing walls:** R-SUP (failure: the free-fraction rival wins and the theorem falls whole), FREE-R (failure: confinement demotes to charge-indefiniteness — the bridge to the free sector is gone), and J-COM (failure: the theorem is refuted at the world's table). **Structural hygiene — failures graceful or local:** NRC (failure: exactness trims to admit free integral sub-multiplets, while fractional confinement survives entire) and C-CLO (failure: clean but local — the mesonic clause lapses, the class-completion sector intact). Three walls, two fences; the theorem's real price is the three.

The minimal dependency chain, for the referee in a hurry. The paper's spine in five lines:

```
R-SUP   → an unsupported subset carries no standing response        (§4)
FREE-R  → without response support, no free standing existence      (§7)
J-COM   → completion lives at closure-geometry grain;
          mixed occupancy licensed                                  (§8)
L-G     → an admissible free composite is itself a carrier;
          its winding is integral, unconditionally — and the
          same clause gates which completions constitute
          carriers at all                                          (§9)
─────────────────────────────────────────────────────────────────────
∴ the free sector is exactly the completions; totals are integral;
  and mixed occupancy forces congruent residues — confinement,
  integrality, and the mixing rule from one chain
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3. Support and Traversal — The Distinction the Route Was Missing

The paper's engine is a distinction the Realization Theorem used without naming, and naming it is what reconciles this paper with its predecessor — a reconciliation owed in print, because the obvious form of this paper's central lemma *contradicts* the Realization Theorem, and the contradiction must be dissolved before anything is built on the lemma's corrected form.

The apparent contradiction, stated. The natural argument for transport incompleteness runs: *the response q is supported by the class; a proper subset omits part of the class; therefore the*

loops that define q cannot be completed within the subset. But the Realization Theorem's entire engine is that loops are completed by far less than the class: by REG-E, a **single realized seat** traverses a closed admissible loop — free or housed — and completes a record — that is how anything registers q⁄k at all. If completing the response's loops required class-wide traversal, member-grain registration would be impossible and the Realization Theorem would be empty. An incompleteness argument that counts *traversers* is therefore refuted by the route's own predecessor before it starts.

The distinction, installed. What the record construction requires is two things, and they are different in kind. The record's **traverser** is realized structure: the seat that carries the comparison around the loop — one member, on one history, cheap, and available to any realized configuration whatsoever [the Carrier paper's reconciliation: bath members exist, persist as loci of realized content, and may carry anything]. The record's **support** is standing structure: the standing transport content of the carrier through which the loop runs — the thing the comparison *reads*, once the realized churn is cancelled [R-O]. The Realization Theorem's own language contains the distinction in embryo — *the realized seat is the record's traverser, not its subject-matter; the ship is realized, the depth it sounds is standing* — and R-SUP is the reading that completes it: the depth must be *there* to be sounded. A record's output is a function of the loop and the standing transport structure [R-O]; R-SUP adds that the function is evaluated *at* a carrier — the comparison reads the standing content of the transport-complete structure the loop runs through, and where no such structure exists, there is nothing for the surviving content to be content *of*.

The two configurations, separated. A **housed** member — a seat within an intact, transport-complete class — traverses a loop whose support exists: the class's standing structure is present, the comparison cancels the churn against it, and what survives is the one standing entry available at seat grain, q⁄k [the Realization Theorem, recovered exactly — this is the housed registration REG-E names as its model configuration, consumed here rather than re-derived]. An **unhoused** member — detached, housed in no transport-complete structure — traverses the same loop geometry with the same realized ease, and the comparison completes over nothing: there is no class standing structure for the cancellation to isolate, and the output is not zero — zero is a winding, a standing classification the lattice binds [R-S] — but **undefined**, the response of a carrier that is not there. The programme owns the precedent exactly: generation-space holonomy on a broken loop is not a smaller phase or a zero phase — it is no phase, the quantity losing not its value but its definition. The parallel is spent here as the motivation it is; the load is carried by R-SUP at its slot, not by the analogy. [The distinction: installed; the reconciliation with REG-E: explicit — traverser cheap, support load-bearing; the precedent: motivation only.]

The Support Non-Creation Principle — why response requires support. The obvious question against R-SUP deserves an answer better than "that is how the construction reads," and the answer is already implicit in the inheritance: **a record may reveal standing content; it cannot create it.** The ground is R-O's own functional form. R-O states that a record's output is a *function of* the standing transport structure (given the specification) — and a function is evaluated at its argument: where the argument does not exist, there is no value, because there is nothing for the output to be a function *of*. The alternative — an output defined on carrier-absent loops — would be standing content manufactured by the act of measurement: a comparison

whose subtraction of realized churn from realized churn leaves a standing residue that no standing structure supplied. That is not a strange edge case of the record construction; it is a contradiction of what the construction says a record *is* — revelation of invariant content, never production of it — and it would breach the standing/realized boundary itself, realized procedure generating catalogue entries. Stated as a principle: *every standing output presupposes a standing support, because measurement reveals the invariant and the invariant must be there to reveal.* The principle does not eliminate the import — R-SUP remains the slot reading, since whether the source construction contains some output-defining convention for carrier-absent loops is a fact only the source fixes — but it converts R-SUP from a free-standing postulate into the **referential reading of a clause R-O already contains**, and it fixes exactly what a striker must produce: not a reinterpretation, but a demonstration that the charge paper's records create standing content from its absence. [The principle: stated, grounded in R-O's functional form; the residual import: R-SUP, at its slot, narrowed to the referential question.]

Corollary (Standing Content Conservation). *No standing quantity may appear in a record unless it already exists in the standing support structure.* This is the principle restated at the quantity grain, and its denial has a name worth writing down: were a record to carry a standing quantity its support does not contain, **measurement would be a source of standing structure rather than a probe of it** — realized procedure minting catalogue content, the standing/realized boundary breached not at an edge case but at the construction's core. Every standing quantity in every record on the route is therefore an *inventory item*, not a *product*: the fraction a housed seat registers exists in the class's standing support before any loop runs, and the unhoused member's record is empty not because something was destroyed but because there was never anything at that address to reveal. The corollary does not move R-SUP's marker — the import remains the slot's to confirm, since only the source fixes whether the construction contains an output convention for carrier-absent loops — but it raises the striker's price to its true level: whoever denies R-SUP must affirm that measurement creates standing structure, and own every consequence of a world in which probing the catalogue writes to it. [Corollary: stated at the principle's grounding; the import's marker: unmoved; the rival's burden: named.]

What the distinction already secures. Three boundary facts, each spent later. First, the Realization Theorem is untouched: every registration it quantifies over is a housed registration, its forcing clause governs seats whose class support is intact, and nothing in this paper revises what a housed seat must show. Second, the deep-inelastic configuration — the world's actual member-resolved probe — is a housed registration: the probed quark's class structure is intact around it, which is exactly why the probe reads a standing third; the theorem's empirical contact survives the support requirement because the contact never involved an unhoused seat. Third, the question this paper decides is thereby isolated: not what housed seats show (decided), not what realized structure can do (anything, fenced), but whether *unhoused* structure can exist freely as a standing configuration at all — the free sector, and only the free sector.

4. The Transport Completion Lemma

The lemma must be proven on the corrected ground — support, not traversal — and the corrected ground supplies a proof the naive ground could not: the support is not merely *larger* than any subset; it is **undivided**, so that no subset inherits a share of it.

The support is class-owned and admits no standing parts. The standing transport structure that supports response is the class's: the class is the minimal transport-stable subject [the Carrier Theorem], standing structure is class-owned [the Ownership Principle], and — the clause this paper consumes for the first time at its full depth — the class's standing structure admits **no standing decomposition into member parts**: by B-IRR, no member line is standing, so there is no standing fact of the form "this member's portion of the support" [the Carrier paper's §5, consumed here for ownership's granularity rather than attachment's location]. The support is therefore owned the way the Bath arc has always said class property is owned: **undivided** — not allocated among members in standing shares, because standing shares of it do not exist. A pool, not a partition.

Detachment therefore transfers nothing. Consider M, a proper subset of class $\mathcal{C}$'s members, unhoused: detached as candidate free structure, housed in no transport-complete carrier. What does M take with it? Its members' realized structure — allocation, configuration, traversal capacity — all of it, intact, because realized structure is the members' to keep [the reconciliation, §3]. And of the standing support: **nothing** — not because the support resists division the way a conserved fluid resists copying, but because there are no standing parts of it for M's members to have been carrying. A subset of the members was never a subset of the support; the support has no subsets at member grain. Unhoused, M's members traverse loops whose support does not exist where they are: the class standing structure they would read is the property of a subject [$\mathcal{C}$, or nothing] that M is not.

Transport Completion Lemma. *Let $\mathcal{C}$ be a closure class carrying standing response q, and let M be any proper subset of $\mathcal{C}$'s members, unhoused — housed in no transport-complete closure structure. Then M is transport-incomplete with respect to standing response: no loop traversed by M's members possesses the standing support a response record requires, and no standing response is defined on M.* [Proven, conditional on R-SUP at its slot, the Carrier Theorem's bath branch entire (B-IRR; C-ORD carried at total exposure), and the Ownership Principle at its statuses. *Consistency with REG-E: explicit* — the lemma governs supports, not traversers; housed registration, REG-E's own model configuration, is untouched, and the Realization Theorem's forcing clause is consumed intact.]

Corollary (mixed occupancy, under J-COM). *The Lemma's quantifier extends to any unhoused, incompletely occupied closure geometry, occupancy single-class or mixed.* Under J-COM the support is the geometry's, so an incomplete occupancy — whatever classes its members are drawn from — leaves the geometry's standing support unconstituted on exactly the ground a single-class subset does: the free **mixed diquark** — one seat each from two k = 3 classes, an incompletely occupied three-geometry, a proper subset of *no single class* — is the canonical instance, and without this corollary it would slip every single-class quantifier in the paper. [Proven, at the Lemma's conditionality plus J-COM; the gap it closes: named.]

One scope clause, fixed where a referee would press. The lemma says nothing about what becomes of the *remainder* — whether $\mathcal{C}$ minus M persists as a class with unoccupied seats, collapses, or reconfigures is occupancy dynamics at the G-S territory, fenced. The lemma needs only what it states: that M, unhoused, supports nothing standing — and that claim is indifferent to the remainder's fate. [The fence: stated; the remainder: at G-S.]

5. Re-Closure Excluded — Why a Broken Class Is Not a Smaller Class

The lemma as proven leaves one escape, and the escape must be closed by a named import rather than by the slide the naive form invites. The lemma establishes that M supports no standing response *as a fragment of* $\mathcal{C}$ — incomplete with respect to q. It does not, by itself, exclude M from constituting a **new** carrier: the census admits k = 2, so why may two unhoused members of a broken three-member class not re-close as a two-member class, transport-complete in its own right, carrying its own underived integer q′? If re-closure were admissible, the free sector would contain liberated sub-multiplets — free diquark-like objects with standing integral charge — and the Saturation Theorem's exactness would fail even though no fraction ever escaped.

The exclusion is NRC, and its content is that the question contains a category error: **capacities are kinds, not counts.** The census derives $k \in \{1, 2, 3\}$ as the admissible closure *geometries* — distinct transport architectures, each a different standing structure — not as occupancy levels of one generic container that any handful of members might fill. On that reading, a member of a k = 3 class is a k = 3-class member through and through: its transport structure is the three-seat closure's, its identity as realized structure is indexed to that architecture, and two such members are two fragments of a three-geometry — not raw material awaiting a two-geometry, which is a different kind of structure and not constructible from fragments of this one. Re-closure is not difficult; it is undefined, the way two-thirds of a triangle is not a smaller triangle.

Whether the census's derivation supports this reading is exactly the kind of fact only the source fixes, and NRC is slotted there [§2]. The fallback is priced at its true cost, which is lower than a referee might assume: if capacities are occupancy-like and re-closure admissible, the theorem's *exactness* fails — the free sector admits re-closed sub-structures — but **fractional confinement survives**, because a re-closed M is a carrier, and a carrier carries an underived integer [the Carrier Theorem]: the escape route NRC closes leads to free *integers*, never to free fractions. The route's empirical face is therefore exposed at NRC only in its exactness clause — the absence of free integral sub-multiplets — while the absence of free fractions, the obligation CO-2 actually names, holds on either reading. [The exclusion: at NRC's slot; the degradation: graceful, and priced.]

6. The Response Support Theorem

The pieces consolidate into the paper's structural core.

Response Support Theorem. *Standing response exists exactly on transport-complete supports. In particular: (a) a transport-complete closure structure — a saturated class, or a same-capacity mixed housing [J-COM] — supports its standing response, with housed member-grain registration at each seat's class share q/k [the Realization Theorem, intact at every seat]; (b) an unhoused, incompletely occupied closure geometry — occupancy single-class or mixed, covering both any proper subset of one class and any mixed partial occupancy such as the free two-class diquark [the Lemma's J-COM corollary, §4] — supports no standing response — not the fraction, not zero, not a re-closed integer — its candidate records lacking standing support [§4]*

and its candidate re-closure lacking definition [§5]; (c) what unhoused incomplete structure retains is realized structure entire, on which no standing response classification is defined and which the lattice does not bind [R-S]. [Clause (a): Inherited, intact. Clause (b): Proven, conditional on R-SUP, NRC, and the inherited chain (B-IRR; C-ORD at total exposure). Clause (c): the Carrier paper's reconciliation, extended one step at the same statuses.]

The theorem's reading, fixed against the misreading the route has met before: this is not a weakening of the Realization Theorem's forcing — it is the forcing's domain, drawn. Clause (iii) of the Realization Theorem forces what every admissible member-grain registration reads, and §3 establishes that admissible registrations are housed registrations: the forcing governs every seat the world can actually ask, and the present theorem adds that the seats the forcing does not govern — unhoused ones — cannot be asked, because there is nothing standing for the asking to read. Between them, the two theorems partition the member-grain question exhaustively: housed seats must answer q⁄k; unhoused seats cannot answer at all. What remains — and it is the route's last step — is whether unhoused seats can *exist* as free standing structure. The Response Support Theorem says they would exist responseless; it does not yet say they would not exist. That step is a premise, and it gets its own section.

7. From Unsupported to Inadmissible — FREE-R, Named

The step from clause (b) to confinement is the step the argument's most natural telling takes silently, and the route's discipline — the same discipline that made the Realization paper name REG-E, REG-G, and R-O rather than derive forcing from symmetry — requires it taken in public. The gap is real: the corpus's own boundary vocabulary makes "free realized structure supporting no standing response" a coherent description, not a contradiction — realized structure without standing content is precisely what the bath says member allocation *is*. A world containing free, charge-indefinite quark-like objects — admissible as realized configuration, present in the catalogue of happenings, absent from the catalogue of standing things — satisfies the Transport Completion Lemma, the Response Support Theorem, and every result upstream of them. That world is refuted by observation — the world shows no free quark-like objects at all, charge-definite or otherwise — but a theorem owes its conclusion to its premises, not to the observation it hopes to explain, and the premise separating the two worlds must be named.

FREE-R, stated at §2 and consumed here: *free standing existence requires standing response support.* An admissible free standing configuration is an entry in the standing catalogue; catalogue entries are individuated by standing content; a structure supporting no standing response has no standing content at response grain, hence no entry, hence no free standing existence. The premise continues the corpus rather than amending it: standing existence has been identified with standing transport content at every prior node — the Ownership catalogue, the Carrier paper's transport-stable subjects — and FREE-R is that identification applied to the one sector the predecessors never quantified over, the free one; what is new is the application, not the principle. What it may have is realized existence — transient, configuration-level, bound into the realized dynamics of larger structures — which is exactly the mode of existence the route has always granted bath members and exactly not the free-sector mode whose absence CO-2 names. The import's slot is the Fold/G-S admissibility structure — the conditions under which the corpus admits standing configurations — read for whether catalogue entry requires standing

transport content. Its fallback is the charge-indefinite world above, constructed in full as the second rival of §10 and priced there: the demoted theorem still forbids free *fractions* (nothing unhoused supports one) but no longer forbids free *unhoused structure*, and the world's total silence at the free quark sector becomes evidence the demoted theorem merely accommodates rather than predicts. [FREE-R: named, slotted, its fallback constructed; the bridge: no longer silent.]

8. The Saturation Theorem

The route's last structural result now follows, and its statement must face the particle table whole — which requires two clauses the single-class telling lacks. The conjugate clause, because the world's free composites come in two families and a theorem naming only one is refuted by the other. And the joint-completion clause [J-COM], because the world's most common free object fills no single class: the proton seats two members of the up class and one of the down, and a theorem admitting only single-class saturation excludes it — the route's own Class Uniformity Criterion forces distinct flavors to be distinct classes (they differ in standing winding), so mixed occupancy cannot be absorbed by redrawing the classes and must be licensed by the closure structure itself.

One definitional clause, fixing the theorem's quantifier. *Transport completion is carrier-constituting*: a configuration is transport-complete exactly when its completion constitutes a transport-stable subject — a carrier. Full occupancy of a closure geometry is necessary for this; §9 will show it is not sufficient: a saturated mixture of *incongruent* classes constitutes no carrier, because the subject it would constitute carries an off-lattice winding, which the lattice forbids to exist [L-G] — the candidate fails at subject-constitution, not at a filter applied after the fact. The theorem's biconditional is stated, and holds, under this reading; a reader who takes "transport-complete" as bare seat-occupancy will find §9 producing occupied-but-inadmissible mixtures and should return here.

Independence — why the biconditional is not a definition. With completion read as carrier-constituting, a referee should ask whether the theorem has quietly become a tautology — admissible iff complete, with "complete" defined to track admissibility. It has not, and the two sides are characterized by independent machinery. *Completion* decomposes into two independently checkable components: **occupancy** — every seat of an admissible closure geometry filled — a census-grade geometric fact, verifiable by counting seats without ever consulting the lattice; and **constitution** — the occupancy yielding a transport-stable subject — which is L-G's filter, operating on the candidate's winding alone. The two can come apart, and in §9's incongruent case they do: the lattice's gate excludes a fully occupied geometry, so occupancy-without-constitution is a live category — exactly what a definitional reading would make impossible. *Admissibility*, meanwhile, is characterized at the standing catalogue [FREE-R, at the Fold/G-S conditions] and consults neither component. The theorem's content is therefore a genuine coincidence-claim between independently characterized sides — the catalogue's free entries are exactly the constituted completions — and it could have failed in either direction: a catalogue admitting responseless entries (FREE-R's rival world, §10) breaks one direction; unconstituted occupancies admitted as standing entries would break the other. The biconditional

is carried by the named imports, not unrolled from a definition. [The independence: stated; the paper's softest joint: closed.]

Saturation Theorem. *Under the bath ontology, with the Carrier and Realization theorems at their full conditionality and R-SUP, NRC, FREE-R, C-CLO, and J-COM at their named slots: admissible free standing configurations are exactly the transport-complete ones —*

(a) — closure completion — saturated closure structures: every seat of a k-seat closure geometry occupied, by members of one class or jointly by members of distinct same-capacity classes [J-COM], the geometry's standing support intact, each seat carrying its class's share q_i/k, the configuration carrying total $(\Sigma_i\, q_i)/k$ — equal to the class winding $q \in \mathbb{Z}$ when occupancy is single-class;

(b) — conjugate completion — member–antimember pairs, within or across same-capacity classes: a member of class $\mathcal{C}$ paired with an antimember of class $\mathcal{C}'$ (the case $\mathcal{C}' = \mathcal{C}$ included), the forward and reversed traversals composing into a jointly completed support [C-CLO; J-COM for the cross-class case], the pair carrying $(q - q')/k$;

(c) — composition — finite compositions of (a) and (b), windings summing by AD;

and nothing else: unhoused, incompletely occupied closure geometries — occupancy single-class or mixed — are inadmissible as free standing configurations — unsupported by the Response Support Theorem at its full quantifier [clause (b), with the Lemma's J-COM corollary], unrescued by re-closure [NRC], and excluded from the standing catalogue by FREE-R. [Proven, conditional as stated; the exposure at B-IRR and C-ORD total, as everywhere on the route.]

Integrality Corollary. *Every admissible free standing configuration carries integral total response.* For single-class completion: the k seats at q/k compose by AD-m to $k\cdot(q/k) = q \in \mathbb{Z}$. For mixed closure completion [J-COM]: the total is $(\Sigma_i\, q_i)/k$, integral **exactly when the participating windings are congruent mod k** [C-RES (c)] — all $q_i \equiv r$ gives $\Sigma_i\, q_i \equiv k\cdot r \equiv 0 \pmod{k}$. For conjugate completion within one class: $q/k + (-q)/k = 0 \in \mathbb{Z}$ — the arithmetic consuming C-RES's clause (b), the conjugate class at the same capacity k. For cross-class conjugate completion: $(q - q')/k$, integral **exactly when $q \equiv q' \pmod{k}$** [C-RES (c)]. For compositions: sums of integers. [Proven, conditional as the theorem. The mixed clauses state an arithmetic equivalence; their residue condition is discharged one section ahead — §9 proves integrality unconditional over the admissible sector, the congruence relocating from condition on integrality to gate on admissibility.]

The congruence — the predecessor's finding, given its completion-theoretic role, with a promotion candidate. The residue condition the mixed clauses consume is not this paper's discovery: it is the predecessor's **C-RES clause (c)** — equal-capacity class windings congruent modulo capacity — named there as a structural finding, slotted at the census, its refuter carried in that paper's §13, and inherited here under its own name: chain of custody, nothing re-derived, nothing re-slotted. What this paper adds is the condition's *role*. **The equivalence** [Proven, conditional as the corollary]: the mixed free sector is integral if and only if C-RES (c) holds across the participating classes — and on the O1 identification of the quark species with k = 3

classes ($q_{up} = +2$, $q_{down} = -1$, with $2 \equiv -1 \bmod 3$), the entire mixed composite sector turns on that one shared residue: the meson $u\cdot\bar{d}$ at $(2-(-1))/3 = +1$, the proton $u\cdot u\cdot d$ at $(2+2-1)/3 = +1$ — every hadron integral not by a hadron-by-hadron coincidence but by one standing fact. Saturation and conjugation alone do not explain the proton: the proton forced the joint-completion import and consumes the congruence, and the paper says so plainly rather than absorbing either as a convenience. **And the Promotion Conjecture** — the repair that opened the mixed sector points at where the residue may live. Once joint completion is a named import, the natural question is *which* cross-class completions the closure structure admits, and the structurally cleanest candidate answer is: exactly the congruent ones. If J-COM's admissibility condition is the congruence itself — joint completion defined precisely for same-capacity classes congruent mod k — then mixed integrality ceases to be conditioned and is **derived from the admissibility of the completion**: the corollary's mixed clauses lose their asterisk, integrality is exact across the whole free sector, and C-RES (c) is promoted from an exposed assignment fact to the completion structure's own admissibility clause. The conjecture is taken up one section ahead and discharged in a form **stronger than it proposes**: §9 shows the gate is not a new clause of the census's closure structure at all — it is the lattice's own grain-universality [L-G], already in the inheritance — an incongruent mixture would be a transport-stable carrier with a non-integral winding, which the lattice forbids at every stable grain, so it is inadmissible with nothing new asked of any slot. Mixed integrality is thereby derived, the asterisk dissolves, and what survives open is the universal form of the residue and the assignment question behind it. [C-RES (c): Inherited at the predecessor's slot; the equivalence: Proven; the promotion: discharged at §9; the residual: §9's split.]

One fence, drawn before it is needed. The theorem quantifies over *free standing configurations* — asymptotic, isolated, standing. It says nothing about collective realized media: a deconfined plasma of class members is realized dynamics — members as loci of flowing allocation within a larger interacting structure — not a set of free standing configurations, and its description belongs to the realized-dynamics territory every paper on this route has fenced. The theorem forbids a free standing third; it is silent on what realized matter does in a fireball, and the silence is scope, not embarrassment. And the exclusion is by kind, not degree: fm-scale, finite-time quasi-free behaviour inside an interacting medium is not a *small or brief* free configuration — the quantifier runs over standing catalogue entries, asymptotic, isolated, individuated by standing content, and a transient locus of realized allocation within a larger interacting structure is not a candidate entry at any size or duration. [The fence: stated; the exclusion: by kind.]

9. The Congruence Theorem — Mixed Completion at the Lattice's Gate

§8 left the mixed clauses conditioned on the predecessor's congruence and named the Promotion Conjecture as its candidate resolution. This section discharges both at once, and in a form stronger than the conjecture proposed: the congruence is **derived**, the gate is the lattice's own, and — the fact worth stating before the mathematics — the section consumes **no new import**: every premise below is already named, slotted, and carried in §2 — including, essentially and not decoratively, the census's capacity bound, which clause (ii) of the theorem consumes as arithmetic. The congruence of mixing classes is bought from material the route already owns.

No circularity — what is and is not assumed. A hostile reading would ask whether integrality is here derived from integrality — the congruence smuggled in through §8's corollary, whose mixed clauses are conditioned on it. It is not, and the dependence runs one way only. Nothing below assumes the mixed total is integral. What is assumed is that an admissible free configuration is a **transport-stable carrier** — standing by FREE-R, transport-complete by the Saturation Theorem — and integrality then *follows*, from L-G, which binds every transport-stable grain and knows nothing of the mixture's arithmetic. The congruence is therefore never assumed as a property of mixed arithmetic; it is **forced as the condition under which a candidate mixture can qualify as a carrier at all**. §8's corollary, with its conditional mixed clauses, is consumed nowhere in this section: the Lemma re-derives integrality from the lattice directly, and the corollary's conditionality is discharged *by* the result, not used in its proof. [The dependence: one-way; the corollary: this section's output, never its input.]

Composite Integrality Lemma. *Every admissible free standing configuration is a carrier, and its total response is an integral winding.* The argument, at the definitions. An admissible free standing configuration is standing — a catalogue entry, individuated by standing content [FREE-R] — and transport-complete [the Saturation Theorem]; its identity through admissible transport is therefore standing structure, which makes it a **transport-stable subject** in the Carrier paper's exact sense, hence a persistent loop subject, hence a carrier. By **L-G** — the lattice's grain-universality, binding at *every* transport-stable grain, not at a privileged construction grain — its winding is integral, unconditionally: no admissible free configuration of any composition carries an off-lattice total. And the composite's winding is identified with the AD-m composition of its seats' readings — the consumption the Integrality Corollary already performs at $k\cdot(q/k) = q$, extended to mixed geometries under J-COM's clause that each seat reads its own class's share. [Proven, conditional on FREE-R, the Saturation Theorem at its conditionality, L-G at the Carrier paper's slot, AD-m at geometry grain, and J-COM. No new import.]

The gate, relocated — the Promotion Conjecture discharged in improved form. The Lemma inverts §8's bookkeeping. Integrality is **unconditional over the admissible sector**; what the residue governs is admissibility itself. A candidate mixture of incongruent equal-capacity classes would be, if admissible, a transport-stable carrier whose winding $(\Sigma_i\, q_i)/k$ is not an integer — exactly the object the lattice forbids at every stable grain [L-G]. It is not a strange-but-tolerated configuration; it is not admissible at all — and, under §8's carrier-constituting reading of completion, it is not transport-complete at all: there is no transport-stable subject for the saturated incongruent occupancy to constitute, so the Saturation Theorem's biconditional survives verbatim, the gate operating at subject-constitution rather than as a filter bolted on behind the theorem. The conjecture had located the gate at J-COM's slot, as a possible new admissibility clause of the census's closure structure; the derivation needs no new clause anywhere — **the gate was in the inheritance the whole time, at L-G** — and the conjecture moves from [Conjectural] to **[Discharged — in improved form]**: mixed integrality derived, the corollary's asterisk dissolved, and the census's closure structure asked for nothing beyond what J-COM already requested.

Congruence Theorem (C-RES (c)-JS — joint-support congruence). *Let $\mathcal{C}$ and $\mathcal{D}$ be closure classes of capacity k, with windings q_$\mathcal{C}$ and q_$\mathcal{D}$. Then:*

(i) — pair forcing — if a cross-class conjugate pair of $\mathcal{C}$ and $\mathcal{D}$ is admissible, then $q_{\mathcal{C}} \equiv q_{\mathcal{D}} \pmod{k}$: the pair's total $(q_{\mathcal{C}} - q_{\mathcal{D}})/k$ is an integral winding by the Lemma, and its integrality is the congruence, directly;

(ii) — occupancy forcing — if a two-class closure completion is admissible — a seats of $\mathcal{C}$ and $k - a$ of $\mathcal{D}$, with $0 < a < k$ — then $q_{\mathcal{C}} \equiv q_{\mathcal{D}} \pmod{k}$: the total $(a \cdot q_{\mathcal{C}} + (k - a) \cdot q_{\mathcal{D}})/k$ is integral by the Lemma, so $a \cdot (q_{\mathcal{C}} - q_{\mathcal{D}}) \equiv 0 \pmod{k}$; and a is a unit modulo k at every admissible capacity — $k \in \{1, 2, 3\}$, prime or trivial, so every $0 < a < k$ is invertible [Imported-External — elementary modular arithmetic: every nonzero residue modulo a prime is a unit] — whence $q_{\mathcal{C}} \equiv q_{\mathcal{D}} \pmod{k}$;

(iii) — chain closure — congruence propagates along joint admissibility: any classes connected by a chain of clause-(i) or clause-(ii) co-occurrences share one residue mod k. The one configuration forcing less is the three-distinct-class completion at $k = 3$, which forces $q_1 + q_2 + q_3 \equiv 0 \pmod{3}$ alone; pairwise congruence there is recovered through any pair link — any two-class or conjugate co-occurrence — in the same mixing family, which the world's table supplies abundantly wherever such mixtures occur at all [empirical, at O1 — as every table-facing claim in this clause].

[Proven, conditional on the Composite Integrality Lemma at its chain and — for clause (ii) — on the census's capacity result $k \in \{1, 2, 3\}$, consumed as an essential structural premise: the occupancy-forcing step depends on every proper occupancy being a unit, which holds *because* the admissible capacities are prime or trivial, and without the census restriction the clause weakens from pairwise to subgroup-wise selection. The theorem is about completion *plus the census*, not a generic theorem of completion, and says so rather than letting a referee discover the dependence.]

The census's part — a premise, not a flourish — shown by the counterfactual. Clause (ii) holds because every admissible capacity is prime or trivial — the census's *one, two, three, never four* doing arithmetic duty as a consumed structural input, stated in the theorem's own conditionality above rather than left implicit. At a hypothetical $k = 4$, $a = 2$ is a zero divisor [Imported-External — composite moduli admit zero divisors]: classes whose windings differ by 2 could jointly complete at half-occupancy with no congruence forced, and the selection rule would weaken from pairwise to subgroup-wise. The theorem's pairwise sharpness is the census's gift, and one more place the capacity bound earns its keep.

The replacement argument, retired to a remark. A natural alternative proof swaps one member of $\mathcal{C}$ for one member of $\mathcal{D}$ in a completed configuration and reads the change $\Delta = (q_{\mathcal{D}} - q_{\mathcal{C}})/k$ against preserved integrality. The argument is sound under one premise the theorem above never needs: that the swapped configuration is admissible whenever the original is — *occupancy exchangeability* — which is exactly what the selection rule governs, and so cannot be assumed in its own proof. The single-configuration derivation replaces it; the remark records the alternative with its premise named rather than leaving the temptation unexamined. [Remark; the exchange premise: named, unconsumed.]

What the theorem buys, stated at full strength. Three things. First, **the world's residue stops being an input.** Under C-RES (c)-JS, the observed compliance — $2 \equiv -1 \pmod 3$ — is *entailed by the existence of mixed hadrons*, at one stated conditional: **the proton's identification as a jointly completed support** — clause (a)'s mixed case, at O1. The chain is the full one, not a slogan: the proton exists; identified as a jointly completed support, it is admissible free standing structure; admissible free structure is a carrier [the Lemma]; carriers obey the lattice [L-G]; therefore the up and down classes are congruent. Granted the identification, the route assumes no residue and consumes none. Second, **a segregation prediction — at pair grain, with one named exception that is itself a prediction.** An equal-capacity class incongruent with its fellows is not forbidden — it is **segregated at pair grain**: admissible in its own pure and own-family composites, and excluded by clauses (i) and (ii) from every conjugate pairing and every two-class housing with the incongruent rest. Exactly one co-habitation is left open by the theorem's own arithmetic, and the paper names it rather than overclaiming past it: the three-distinct-class completion at $k = 3$, where clause (iii) forces only the sum, so a triple of *mutually incongruent* classes whose residues sum to zero mod 3 — the family $\{0, 1, 2\}$ — is lattice-admissible, co-habiting one free composite with no pair congruent. The exception is not a leak; it is the theorem's most distinctive **prediction-in-waiting**: the world's current table, every flavour in one residue family, has no occupants for it — and if assignment ever yields a third residue family, the theorem states in advance exactly which mixtures it may join: sum-zero triples only, no pairs, no two-class housings. The prediction cuts both ways and §15 carries it at its corrected grain: a confirmed incongruent *pair-type or two-class* composite refutes the theorem at the Lemma's chain; a sum-zero incongruent triple does not — it instantiates clause (iii); and incongruent classes found in segregation refute nothing, answering part of the assignment question. Third, **the proton's arc closes.** The proton's audit identified J-COM (§8); J-COM exposed the congruence; and the congruence is the lattice refusing to mint a fractional carrier — the diagnostic case become the keystone, the theorem's strongest confirmation rather than its awkward edge.

The split, recorded under custody. The predecessor's C-RES (c) is the universal form, and it divides. **C-RES (c)-JS** — equal-capacity classes that jointly complete share a residue mod k — [Proven here, at the Lemma's conditionality]. **C-RES (c)-U** — all equal-capacity classes share a residue, whether or not they mix — [Open, at the predecessor's slot], and, decisively, **no longer consumed by the route**: the route needs only that existing mixtures are integral, which the selection guarantees, so the universal form demotes from the route's most exposed coincidence to a pure assignment curiosity. The Congruence Theorem, named in §8's earlier accounting as the route's next paper, is hereby absorbed; what is handed forward is the narrower **assignment question** — why the world's flavour classes all lie in one jointly admissible residue family, equivalently why every flavour chose to mix — at the predecessor's C-RES (c) slot. A smaller and better problem. [The split: stated; JS: Proven; U: Open, unconsumed; the successor: re-aimed at assignment.]

10. The Two Rivals — Constructed and Priced

The route's discipline, fixed at the member-wise-prior alternative and held since: a result is only as credible as its best rivals are explicit, and a rival is constructed in full, priced exactly, and sent

to a named slot for judgment. The Saturation Theorem has two, one per named bridge, and they fail differently.

The free-fraction rival [against R-SUP]. Suppose record outputs are loop-supported rather than carrier-supported: the standing content a comparison isolates is a property of the loop and the geometry, conjured wherever the loop closes, with no carrier required. Then a detached member carries its reading with it — the fraction is an independently supported standing quantity, resident on the seat, surviving the class's destruction — and free thirds are admissible standing configurations. The rival's world is clean, and its debt is total: it must explain why independently supported, permanently registering fractional objects never once appear free, anywhere, at any energy — an absence it can address only with an external confinement mechanism of exactly the kind this route exists to make unnecessary. The decider is R-SUP's slot at the record construction: whether the construction defines outputs for loops run where no transport-complete carrier is present. The comparison's honest state: the theorem predicts the absence the rival owes; the rival, if its slot reading prevailed, would simply win, inheriting its debt. [Constructed; decider: R-SUP at the record construction; the rival's obligation: total at the free sector.]

The charge-indefinite rival [against FREE-R]. Suppose the standing catalogue admits entries without standing response content: free unhoused members are admissible standing configurations whose response is undefined — not fractional, not zero, not anything. This rival concedes every theorem in this paper up to §6 and disputes only the bridge: in its world, breaking a class liberates not a third but a *responseless object* — free quark-like structure with no standing charge to measure. Its debt is narrower than the first rival's and is stated at its exact width: the world contains no free quark-like objects of any description — none charge-fractional, none charge-indefinite — and the rival accommodates that silence (responseless objects might evade response-keyed detection) less sharply than the theorem predicts it (no such objects exist to detect). The pricing is a lean, not a proof, and the paper marks it as exactly that: detection physics is realized-dynamics territory, the rival's evasion story is not incoherent, and the decider is FREE-R's slot at the admissibility structure — whether the standing catalogue individuates by standing content. [Constructed; decider: FREE-R at the Fold/G-S admissibility conditions; the lean: priced, not inflated.]

The asymmetry, recorded for symmetry's sake. Both rivals leave the route's upstream intact — the Carrier and Realization theorems survive either — and both, like every rival on this route, would win at their slot rather than lose by argument. What the theorem holds over both is the same one thing: it converts the free sector's entire observed pattern — no free fractions, no free sub-multiplets, no free responseless objects, integral totals always, two composite families exactly — into consequences of one support structure, while each rival buys back one piece of that pattern at the price of owing the rest. [The comparison: symmetric in method; its verdicts: at the slots.]

11. The Audits — k = 1, k = 2, k = 3, and the Composite Sector

A theorem quantified over the free sector must be audited against everything the world sets free, at every census capacity, and the audits below run the theorem's two completion modes against the table — with every identification conditioned on O1 explicitly, because the route does not let its empirical contacts borrow certainty from its structure.

k = 1 — the anchor. A singleton class is its own saturation: one seat, occupied, is class completion entire, and clause (a) admits it freely at its winding $q/1 = q \in \mathbb{Z}$. The world's free integral charges — the electron at −e foremost — are therefore not exceptions the theorem survives but instances it predicts: every free elementary standing object is a saturated k = 1 class, free *because* saturated, integral because $q/1 = q$. Conjugate completion at k = 1 admits the particle–antiparticle composite at total response 0 — the positronium-like configuration, admissible and integral. The charge paper is recovered verbatim, the route's standing anchor, passed. [Audit: passed.]

k = 2 — the prediction, carried at its conditionality. The Realization Theorem forces standing halves at the two-member class's housed seats; the Saturation Theorem adds that the halves are never free: admissible free configurations of the k = 2 class are the saturated pair (single-class total $2\cdot(q/2) = q \in \mathbb{Z}$; mixed pairs at $(q_1 + q_2)/2$, the constituents congruent by the Congruence Theorem) and the conjugate pair (total 0 within a class, $(q - q')/2$ across classes, the congruence forced by §9's pair clause). Which physical structure is the k = 2 class is O1 content, [Open], and every empirical sentence in this audit is conditional on it; the audit's deliverable is the constraint itself — *whatever* the k = 2 class is, its halves are housed and its free composites are whole — stated in advance of the identification, where a prediction belongs. One predecessor claim lands here and is superseded rather than silently dropped: the Confinement Dichotomy admitted free k = 2 seats on the null-residue branch, at integral forced readings; under this theorem's support constraint that freedom is retracted — §12 states the supersession in full — and the predecessor's free two-fold consonance inverts into this paper's exposure: a confirmed $k \geq 2$ class with free members, at any reading, now refutes rather than instantiates. [Audit: run; verdict: at O1; the supersession: at §12.]

k = 3 — the target, and the spectrum. The theorem admits exactly **two minimal free composites** of a three-member class: the saturated triple [clause (a)] and the conjugate pair [clause (b)] — and, conditional on O1's identification of the quark species with k = 3 classes — an identification the predecessor's Class Uniformity Criterion now sieves as well as licenses — the world's strong sector is built of exactly two: **baryons and mesons**. The correspondence runs to the integers, and it is licensed, not smuggled: mixed occupancy is clause (a)'s J-COM case — the saturated triple at $(q + q + q)/3 = q$ for a single class, or $(q_1 + q_2 + q_3)/3$ for joint occupancy, integral unconditionally by the Composite Integrality Lemma with the constituents congruent by the Congruence Theorem (§9), the proton u·u·d a jointly saturated three-geometry at $(2+2-1)/3 = +1$, and the saturated geometry of *antimembers* — the antibaryon, total −q — a clause-(a) instance via C-RES (b), the conjugate classes carrying the same capacity, §12's balance arithmetic counting it silently already; and the cross-class conjugate pair is clause (b)'s J-COM case — $(q - q')/3$, the congruence forced by §9's pair clause, the charged pion $u\cdot\bar{d}$ at $(2-(-1))/3 = +1$. And the composite clause (c) admits exactly the exotica the world has since found and nothing it has not: the tetraquark as a composition of two conjugate pairs; the pentaquark as a saturated triple composed with a conjugate pair — multiquark states as compositions of

completions, never as liberated fragments — and the exclusion clause names its mixed instance: the free u·d diquark, one seat each from two classes, an incompletely occupied three-geometry, is excluded on exactly the ground the single-class pair is [the Lemma's J-COM corollary], a proper subset of no class and a completion of nothing. The consonance is striking and the paper prices it at its exact modality: the *structure* — two minimal modes, compositions thereof, integral totals throughout — is the theorem's, [Proven, conditional as stated]; the *identification* — that the strong sector's classes are these classes — is O1's, [Conjectural]; and the residue that makes the mixed integers come out whole is *derived* — C-RES (c)-JS, §9's theorem, [Proven at the Lemma's chain], with only the universal form [Open] at the predecessor's slot, unconsumed. Two open markers and one derivation, none inflated, and between them the strangest sector of the particle table read off one closure architecture. [Audit: run; the chain: as marked.]

Minimal Composite Spectrum Theorem. The audits have been saying this in pieces; it deserves its name and one statement. *Given the census ($k \in \{1, 2, 3\}$) and transport completion, the minimal admissible free composites of the confined sector are exactly two: the k-seat saturated completion (occupancy single-class or mixed) and the conjugate pair (within or across classes) — under O1, the baryon and the meson — and every other admissible free composite is a composition of these.* The proof is the Saturation Theorem's clause structure read for minimality: admissibility is exhausted by clauses (a), (b), and (c); clause (c)'s entries are compositions by construction, so the minimal entries are (a) and (b) themselves; and nothing smaller is admissible — the single unhoused member and every incomplete occupancy, single-class or mixed, are excluded at the Lemma and its J-COM corollary [§4], with re-closure barred at NRC [§5]. At $k = 3$ the minimal spectrum is therefore the three-seat completion and the conjugate pair, full stop: the world's two hadron families are not two entries the framework accommodates but the *only two minimal entries it permits*. [Proven, conditional as the Saturation Theorem; the naming at O1, Conjectural — the structural content carries either way.]

Observable consequences of saturation, boxed. One table, so the pattern is seen whole — every line conditional on O1 and the five slot readings, every line derived, none imposed:

OBSERVABLE CONSEQUENCES OF SATURATION

1. Free integral particles	k = 1 entries, windings integral [L-G]
2. No free fractional charge	confinement derived from support (§8)
3. Baryons	saturated k = 3 completion, single-class or mixed [J-COM]
4. Antibaryons	the conjugate saturated geometry [C-RES (b)]
5. Mesons	conjugate pairs, within or across classes [C-CLO; J-COM]
6. Integral totals, every composite	the Composite Integrality Lemma (§9)
7. The mixing rule	congruent residues only, pair grain — derived, not assumed (§9)
8. No free diquarks	incomplete occupancy, single-class or mixed, unsupported (§4)
9. No isolated quarks	the unhoused member, transport-incomplete (§8)
10. Multiquarks as compositions	tetraquark, pentaquark — compounds of completions, never fragments

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∴ the observed strong-sector pattern, entire, from one closure
  architecture — plus one vacancy the table can fill: the sum-zero
  incongruent triple, predicted and unoccupied (§9)
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A referee should read the box as the paper's empirical face in miniature: not a confinement result with corollaries, but the whole observed pattern of the strong sector — what exists free, what does not, in what combinations, with what totals — as the output of one architecture, with one named prediction standing open. [The box: every line cross-referenced; the conditionality: stated once, governing all.]

12. CO-2, Accounted — What Is Discharged and What Remains

The Realization paper, in its final form, named CO-2 with **two deliverables**: (i) the **balance restriction** — free configurations balanced, $n \equiv m \pmod k$, on invertible-residue sectors — and (ii) the **congruence** [C-RES (c)]; with a territory (G-S), an anchor ($k = 1$), and load-bearing stakes (clause (iii)'s empirical consistency at the free sector). The accounting is split, because the two deliverables fare differently, and the route's custody rule forbids declaring whole what closes in halves:

Deliverable (i) — discharged, conditionally, and superseded. The derivation is supplied, and it delivers more than the deliverable asked: the Saturation Theorem derives **transport completion** as the free-admissibility condition — strictly stronger than the balance restriction, which falls out of it as a consequence: a free configuration composed of a saturated geometries and b conjugate pairs carries $n = a \cdot k + b$ members against $m = b$ antimembers, so $n - m = a \cdot k \equiv 0 \pmod k$ — the balance restriction recovered on the invertible branch as arithmetic of the completion spectrum, not as a separate condition. Its shape answers the principal question exactly: $k \geq 2$ differs from $k = 1$ because at $k = 1$ the member *is* the class — traverser and support coincide, detachment is meaningless — while at $k \geq 2$ the support is geometry-owned and standingly undivided [B-IRR, via the Carrier paper's §5; J-COM for the individuation], therefore unavailable to any proper subset [the Transport Completion Lemma], with re-closure excluded as a category error [NRC] and unsupported free standing existence excluded at the catalogue [FREE-R]. The two modes are unified in the same stroke: occupancy completes the support by full housing, conjugation completes it jointly by reversal [C-CLO] — both transport completion, the G-S unification supplied by the support structure itself. **Deliverable (i) moves from [Open] to [Conditionally discharged — and superseded by the stronger condition]**, closing outright when the five named slots confirm at source: R-SUP at the record construction, NRC at the census's capacity derivation, FREE-R at the Fold/G-S admissibility conditions, C-CLO at the conjugation structure with C-RES (b) at the census, and J-COM at the census's closure structure.

Deliverable (ii) — split by §9, at the deliverable's own grain. The congruence divides where the derivation reaches. Its **joint-support form** — the congruence of equal-capacity classes that actually jointly complete — is **derived**: C-RES (c)-JS, the Congruence Theorem, [Conditionally discharged at §9], the gate the lattice's own [L-G], with the consequence that the route consumes no residue condition anywhere — the corollary's mixed clauses hold over the admissible sector

unconditionally, the congruence their output. Its **universal form** — congruence across all equal-capacity classes, mixing or not — is **not discharged and not consumed**: [Open] at the predecessor's slot, handed forward as the **assignment question**: why the world's flavour classes all lie in one jointly admissible residue family, equivalently why every flavour chose to mix. A discharge accounting that declared CO-2 closed whole would still be dissolution-into-prose — the universal form remains open — but the route's books no longer carry it as debt: nothing in the free sector's account awaits it.

The Confinement Dichotomy, superseded — done, not happened. The predecessor's signature finding — confinement ⟺ invertible residue, null-residue sectors unconfined, their free seats admissible at integral forced readings, the leptons recovered on the trivial branch and a null-residue k = 2 sector predicted free-roaming — is overturned by this paper's theorem, and the overturning is recorded here as the supersession it is rather than left for a reader to discover by collision. Three clauses. *(a) The retraction, and its legitimacy.* The dichotomy was expressly conditional on **integral totals** being the operative free-admissibility constraint; this paper replaces that constraint with the strictly stronger **transport completion**, under which an unhoused seat of a null-residue class is exactly as unsupported as any other — its reading would have been integral, but FREE-R never asks the value. The null-branch freedom was an artifact of the weaker constraint and is retracted at the stronger one: **confinement is support-theoretic, not arithmetic**, and the residue now governs only the integrality of totals, never the freedom of seats. The dichotomy's integrality content survives entire; its confinement clause is superseded. *(b) The consonance, inverted.* The predecessor's null-branch consonance — the world's free two-fold-structured particles read as an unconfined k = 2 sector — is retracted with the branch, and it inverts into this paper's exposure: any confirmed $k \geq 2$ class with free members, at any reading, integral included, refutes the Saturation Theorem directly — a configuration the predecessor's account would have accommodated. §15 carries it as the sharpened kill condition it is. *(c) The screen, closed.* The predecessor's prospective unscreened test surface — free null-residue k = 2 seats as the one place R-O's slot could be discriminated without the saturation screen — closes under this theorem: no seats are free at $k \geq 2$, so on this paper's success the Realization paper's screen analysis is **total at $k \geq 2$ rather than branch-dependent**, and the stability lean priced there re-prices accordingly — narrower in venue, unchanged in kind. The k = 1 case is untouched throughout: singleton traverser and support coincide, this paper's account and the dichotomy's trivial branch agree, and the world's free integral elementary charges sit identically in both. [The supersession: stated, with its legitimacy, its inverted consonance, and its screen interaction; nothing silent.]

Carried, undecided, exposure total. B-IRR at the Bath Criterion; C-ORD at the structure-group imports — under whose failure the route closes at CO-0 and this paper's results, intact as mathematics, lose their application exactly as the predecessors' did. Both appear in every conditionality statement above; neither is decided here; and the paper repeats the standing recommendation in its own ledger: **the composition-order audit remains the route's cheapest insurance**, and four papers now stand above what it protects.

The route's order, completed — structurally. CO-0 (carrier — conditionally discharged at the Carrier paper) → CO-1 (realization — first deliverable conditionally discharged at the Realization paper) → CO-2 (saturation — conditionally discharged here). No further structural

obligation is created — the Saturation Theorem's own price is paid in named imports rather than in a handed-forward node — and the target the route had named for itself — **the Congruence Theorem**, listed in earlier accountings as the next paper — is **absorbed at §9**: the joint-support congruence is derived, the mixed sector's integrality awaits nothing, and what the route hands forward instead is the narrower **assignment question** [C-RES (c)-U, Open at the predecessor's slot, unconsumed]: why the world's flavour classes share one residue family. With that recorded, the route from the census to the confined thirds is, for the first time, **derivation-complete at its stated conditionality** — every link a theorem, every theorem's premises slotted, nothing owed but the slot readings themselves, the identifications [O1], and the verdict [W0]; the residue is the route's output, not its debt. [The accounting: stated, split where the truth is split; no marker inflated.]

13. The Pressure on W0, Re-Priced — The Chain Stated Whole

The route's pressure on the programme's deepest open node has been priced incrementally — three conditions at the Carrier paper, the third link anchored at the Realization paper — and with the route structurally complete the chain is owed once, whole, every link named.

The chain. Observed fractional charge leans toward the bath conditional on: the **fork** [R-C, R-S, L-G, AD, L-M at its diagonal clause, B-IRR — the Carrier paper's slots], applied to U(1) under **C-ORD** [the bypass condition, exposure total]; the **identifications** [O1 — the quark species as k = 3 classes, at the census's PFD slot, Open]; **realization** [REG-E, REG-G, R-O, AD-m — the Realization paper's slots]; and now **saturation** [R-SUP, NRC, FREE-R, C-CLO, J-COM — this paper's slots] — with the mixed-composite integers conditioning nothing: the Congruence Theorem (§9) derives the selection, so the chain consumes no residue; the congruence is the route's output, not its input. Many named conditions, each with an address, none hidden — and the claim purchased at that price has changed in kind across the route. At the Carrier paper, the bath *made fractions available*. At the Realization paper, the bath *predicted them, permanent*. At this paper, the bath predicts the free sector entire: **fractions forced at housed seats, absent free, integral in total, composited in exactly two minimal modes, mixed only under the lattice's own congruence gate** — appearance, value, permanence, confinement, integrality, spectrum, and the mixing rule, from one closure architecture. The diagonal ledger's position, unchanged in structure, is now priced against the full pattern: it owes an external mechanism not for one anomaly but for each item on the list, and the permuting-ledger escape [the L-M diagonal clause, still the first place a referee should look] is narrower than it was: the Realization paper's transfer clause closed it for forcing — orbit carriers register $q/(\text{orbit size})$ at every seat — so the escapee arrives owing neither availability nor permanence. Whether it arrives owing confinement too turns on the orbit analogue of NRC — whether orbit sizes are kinds or counts, a fact about the permuting transport rather than the census — which this paper locates at the same L-M slot and does not decide; the saturation clause is therefore claimed for the bath branch alone, and the escape retains exactly the composite spectrum as its one open item pending that reading.

The firewall, held. Every sentence above is conditional and says so; the lean is exactly as strong as the named slots and no stronger; the posture of §1 appears nowhere in the chain; and the

marker at W0 does not move — [Open] it remains, at the Born-arc slot, awaiting derivation, not accumulation. What has changed is what the verdict now decides: the fork's branches no longer differ in one observable's availability but in whether the free sector's whole observed pattern is structure or coincidence. [The pressure: Conditional, at the chain as stated; the marker: unmoved.]

14. Position in the Programme

Inherited exposure. The Carrier Theorem's bath branch entire and the Realization Theorem at its conditionality — R-C, R-S, L-G, AD, AD-m, REG-E, REG-G, R-O at their slots; B-IRR and C-ORD at total exposure, carried; the Ownership Principle and the standing/realized boundary at its §10 slot, consumed, not duplicated; the census's capacities, the G-S territory, and O1 at their statuses; **C-RES** at its three clauses — (a) Inherited, (b) at the predecessor's census slot, (c) split at §9: the joint-support form derived here, the universal form [Open] there and unconsumed; the **Confinement Dichotomy** at its stated conditionality, superseded in §12; W0 [Open], its marker unmoved.

New exposure. R-SUP at the record construction; NRC at the census's capacity derivation; FREE-R at the Fold/G-S admissibility conditions; C-CLO at the conjugation structure; J-COM at the census's closure structure — the theorem is exactly as strong as these five readings, and §15 prices each failure. C-RES (c) is inherited, not new — and §9 derives its joint-support form from already-slotted material, **adding no import**: the Composite Integrality Lemma and the Congruence Theorem consume only L-G, AD-m, J-COM, FREE-R, the Saturation chain, and — essentially, at clause (ii) — the census's capacity bound. The universal form alone is [Open] at the predecessor's slot, unconsumed. No exposure is created elsewhere: the support/traversal distinction consumes inherited machinery; the audits consume the census; the rivals are constructions, not premises.

The diagram, extended:

```
Charge paper [lattice q ∈ ℤ; R-C; R-S; L-G; AD; orientation
        reversal → C-CLO's ground; R-SUP slotted at the record
        construction beside R-O]
Census [k ∈ {1,2,3}; NRC slotted at the capacity derivation;
        J-COM slotted at the closure structure; C-RES (b),(c) at
        the census slot; G-S territory → FREE-R's slot; O1 Open]
Ownership [class-owned standing structure; Uniform Reading Lemma]
Carrier Theorem [class carrier under the bath; no standing
        member decomposition (B-IRR) → the undivided support]
Realization Theorem [housed registration forced at q⁄k;
        C-RES named — (c) the sector congruence, Open there;
        Confinement Dichotomy — superseded at §12 below;
        CO-2 named in two deliverables, load-bearing]
        │
        ├── support/traversal distinction: traverser realized
        │       and cheap; support standing and class-owned;
        │       REG-E reconciled explicitly; Support Non-Creation
        │       Principle — records reveal, never create —
```

grounding R-SUP in R-O's functional form ← this paper, §3

Transport Completion Lemma: the support is owned undivided [B-IRR]; unhoused subsets inherit none of it; no standing response defined ← this paper, §4

re-closure excluded: capacities are kinds, not counts [NRC]; fallback graceful — re-closure admits free integers, never free fractions ← this paper, §5

J-COM: completion at the closure-geometry grain — k-seat supports jointly completable by distinct same-capacity classes; the proton audit's verdict; fallback total at the table ← this paper, §2, §8

Response Support Theorem: standing response exactly on transport-complete supports; housed forcing intact; unhoused subsets responseless ← this paper, §6

FREE-R named: unsupported ⟹ uncatalogued ⟹ no free standing existence — the bridge, no longer silent ← this paper, §7

SATURATION THEOREM ← this paper, §8

- (a) closure completion: saturated geometries, occupancy single-class or mixed [J-COM], total $(\Sigma q_i)/k$ — = q when single-class
- (b) conjugate completion: member–antimember pairs, within or across classes [C-CLO; J-COM], total $(q - q')/k$
- (c) compositions; nothing else

Integrality Corollary: free totals $\in \mathbb{Z}$; mixed clauses under C-RES (c) [Inherited, Open at the predecessor's slot]; equivalence Proven; compliance at O1 ($2 \equiv -1 \bmod 3$)

promotion: stated here, discharged at §9 — the gate the lattice's [L-G], not a census clause

CONGRUENCE THEOREM [C-RES (c)-JS] ← this paper, §9

Composite Integrality Lemma: admissible free configurations are themselves carriers — windings integral unconditionally [L-G, AD-m, J-COM, FREE-R]; no new import

congruence = admissibility gate: (i) pair forcing; (ii) occupancy forcing — the census's capacity bound consumed as an essential premise; (iii) chain closure across mixing families

world's residue entailed by mixed hadrons —

```
│          conditional on their identification as jointly
│          completed supports [at O1];
│          pair-grain segregation for incongruent classes;
│          sum-zero triple the named exotic;
│          C-RES (c)-U open, unconsumed → the assignment
│          question
│         k = 1: charge paper recovered ✓
│         k = 3: two minimal free modes — baryons and
│             mesons [Conjectural, at O1]; exotica as
│             compositions
│
├── rivals: free-fraction [decided at R-SUP];
│         charge-indefinite [decided at FREE-R]
│
├── CO-2: split discharge — deliverable (i), the balance
│         restriction, discharged and superseded (transport
│         completion the stronger condition; n − m = a·k);
│         deliverable (ii) split at §9: joint-support form
│         derived [C-RES (c)-JS]; universal form open,
│         unconsumed — the assignment question handed
│         forward; (i) closes at the R-SUP / NRC / FREE-R /
│         C-CLO / J-COM slots
│
├── Confinement Dichotomy superseded — confinement
│         support-theoretic, not arithmetic; residue governs
│         integrality only; null-branch consonance inverted
│         to exposure; R-O's unscreened surface closed —
│         the screen total at k ≥ 2;
│         B-IRR, C-ORD carried at total exposure
│
└── pressure on W0: the chain stated whole — the bath
          predicting appearance, value, permanence,
          confinement, integrality, spectrum; marker
          unmoved [Open]
```

The division of labor, completed. Geometry proposes; transport disposes; ownership allocates; the carrier locates; realization registers; **saturation houses** — the census counts the seats, the carrier fixes whose integer the lattice forces, the registration fixes what each housed seat must answer, and the support structure fixes what may stand alone: only that which closes. Six statements, disjoint imports, meeting only at the assembly — the route's architecture, now load-bearing end to end.

15. What Would Refute or Decide This

Against R-SUP (§3, §4) — the theorem's load-bearing wall. Show the record construction defines standing outputs for loops run where no transport-complete carrier is present — response as loop-supported rather than carrier-supported. Effect: the free-fraction rival wins; detached members carry their readings; the Transport Completion Lemma, the Response Support Theorem, and the Saturation Theorem fall whole, and confinement reverts to an unexplained input. Decided at the charge paper's record construction, read beside R-O — and the Support Non-Creation Principle (§3) fixes the striker's burden exactly: not a reinterpretation of support, but a demonstration that the construction's records *create* standing content on carrier-absent

loops, against R-O's own functional form. The paper is exactly as strong as R-SUP and has said so throughout.

Against NRC (§5). Show the census derives capacities as occupancy counts admitting re-closure — fragments of a k-geometry reconstituting as a smaller geometry. Effect: exactness fails; free re-closed sub-multiplets with underived integral windings join the admissible sector; **fractional confinement survives**, since re-closure delivers integers, never fractions. Empirical face: confirmed free sub-multiplet states with integral standing charge would evidence the fallback; their absence leans for NRC. A graceful strike, priced at its true width.

Against FREE-R (§7). Show the Fold/G-S admissibility conditions admit standing catalogue entries without standing response content. Effect: the theorem demotes from confinement to charge-indefiniteness — free unhoused structure admissible, responseless; the absence of free fractions still follows (nothing unhoused supports one), but the absence of free quark-like objects *simpliciter* becomes accommodation rather than prediction. Empirical face: any confirmed free quark-like object with indeterminate or unmeasurable standing charge would evidence the fallback; the world's silence leans against it, priced as a lean.

Against J-COM (§2, §8) — the table's wall. Show the census's closure structure individuates completion at class identity — k-seat supports completable only by one class's members. Effect: clauses (a) and (b) revert to single-class form; the proton and the charged pion are inadmissible; the theorem is refuted by the world's most common free object. The exposure is total at the table and the paper has said so. Decided at the census's closure structure, read with the Carrier paper's individuation of "the class's standing structure." Conversely, J-COM's confirmation completes the Composite Integrality Lemma's chain (§9): the slot that licenses the proton is one of the three joints on which the derived congruence stands.

Against C-CLO (§8). Show orientation reversal fails to lift from loops to carriers — no conjugate classes, or pairs failing to compose a completed support. Effect: the conjugate clause lapses; the admissible free sector contains only saturated classes; the mesonic family is unaccounted and the theorem's exactness is false at the particle table. The cleanest strike against the theorem as stated, decided at the conjugation structure — and conversely, the mesonic sector's existence is, conditional on O1, standing evidence at exactly this slot. A narrower strike at C-RES's clause (b) alone — conjugate classes at unequal capacities — leaves joint completion standing but breaks the cancellation arithmetic: conjugate-pair totals become generically non-integral, a consequence the world's integral mesons already lean against; decided at the census, priced as a refinement.

Against the Composite Integrality Lemma and the Congruence Theorem (§9). Three joints, every one inherited. *L-G*: show the lattice bound at a privileged construction grain rather than at every transport-stable subject — effect: composite windings lose unconditional integrality, the gate falls, the congruence reverts to the conditional arithmetic of §8's corollary with C-RES (c) re-consumed by the route; the Carrier paper's L-G fallback, propagated one theorem further. *AD-m at geometry grain*: show seat readings non-compositional over mixed geometries — effect: the Lemma's identification fails; same reversion. *J-COM*: already the table's wall; its failure takes the mixed sector whole, theorem included. Empirically, the theorem cuts clean in both directions

— at the corrected grain: **a confirmed incongruent pair-type or two-class composite refutes it directly** at the Lemma's chain [clauses (i), (ii)]; a **sum-zero three-distinct-class completion does not** — it instantiates clause (iii), the theorem's one permitted incongruent co-habitation and its named exotic; while the segregation prediction at pair grain — incongruent classes excluded from all conjugate pairing and two-class housing — gives the theorem a standing test, and discovery of incongruent classes *in segregation* refutes nothing, answering part of the assignment question instead.

Against C-RES (c)-U (§9's residual). The universal form is [Open] at the predecessor's slot — its refuter carried at that paper's §13 — and is no longer consumed by the route: exhibiting equal-capacity classes with incongruent windings refutes nothing here, the theorem predicting their segregation rather than their absence, and bears only on the assignment question's shape. The world's compliance ($2 \equiv -1 \bmod 3$) is, after §9, entailed for the classes that mix — conditional on the mixed hadrons' identification as jointly completed supports, at O1; what remains coincidental, pending assignment, is only that *all* the world's flavours chose one mixing family.

Against the Lemma's scope (§4). Exhibit a corpus result requiring standing response on an unhoused, incompletely occupied geometry — proper subset or mixed partial occupancy — anywhere in the arcs. Effect: refutation in place, or reclassification of the consuming result's configuration as housed. Standing: none known; the Realization Theorem's registrations are housed by construction, and §3's reconciliation is exactly the audit of the nearest candidate.

Empirically, consolidated. Confirmed free fractional standing charge: refutes the theorem at R-SUP or FREE-R — the route's sharpest kill condition, unchanged since CO-2 was named. **Any confirmed $k \geq 2$ class with free members — at any reading, integral included**: refutes the Saturation Theorem directly — the kill condition *sharpened* by §12's supersession, since the predecessor's Confinement Dichotomy would have accommodated exactly this on the null-residue branch, and this theorem stakes that accommodation away. Confirmed free integral sub-multiplets: evidence against NRC's exactness clause. Confirmed free responseless quark-like objects: evidence for FREE-R's fallback world. Non-integral free composites, or any confirmed incongruent *pair-type or two-class* composite: against the Composite Integrality Lemma's chain (§9) — L-G, AD-m at geometry grain, J-COM. A confirmed sum-zero incongruent triple: not a refutation but the confirmation of clause (iii)'s named exotic. The segregation prediction at pair grain, standing: incongruent classes excluded from all conjugate pairing and two-class housing. Continued universal integrality and confinement: the pattern the theorem predicts, at the chain of §13. The R-O discrimination, re-priced: the predecessor's unscreened test surface (free null-residue $k = 2$ seats) closes on this paper's success, so the stability lean for R-O runs through housed registration only — narrower in venue, unchanged in kind. Deconfined collective media: outside the quantifier, fenced at realized dynamics (§8).

Deciders. R-SUP and C-CLO at the charge paper (record construction; conjugation); NRC and J-COM at the census — the capacity derivation and the closure structure; C-RES (b) at the census, C-RES (c)-U at the predecessor's slot — the assignment question; L-G at the Carrier paper's slot — the Congruence Theorem's gate; AD-m at geometry grain; FREE-R at the Fold/G-S admissibility conditions; O1 at the census's PFD slot; the inherited totals — B-IRR at the Bath Criterion, C-ORD at the structure-group imports; the branch selection at W0.

16. What the Paper Establishes

Established (conditionally, with conditions named):

- The support/traversal distinction: a record's traverser is realized and cheap, its support standing and class-owned; R-O's "standing transport structure" made referential by R-SUP; the apparent contradiction with REG-E dissolved explicitly — housed registration, REG-E's own model configuration, completes against intact class support, which is how the Realization Theorem's registrations were ever possible — and the holonomy precedent spent as motivation only, the load carried at the slot; and the **Support Non-Creation Principle** — a record reveals standing content and cannot create it, by R-O's own functional form — grounding R-SUP as the referential reading of an inherited clause rather than a bare postulate, the slot narrowed to the referential question (§3). [The distinction: installed; the principle: stated; the reconciliation: explicit.]
- The Transport Completion Lemma on the corrected ground: the support class-owned [Ownership] and standingly undivided [B-IRR via the Carrier paper's §5 — no standing member decomposition, hence no standing shares], so that detachment transfers realized structure entire and standing support none; unhoused, incompletely occupied geometries — single-class or mixed, the free two-class diquark the corollary's named instance — transport-incomplete with respect to standing response, their loops closing over nothing standing; the remainder's fate fenced at G-S (§4). [Proven, conditional on R-SUP and the inherited chain.]
- Re-closure excluded as a category error at a named slot: capacities as geometric kinds, not occupancy counts [NRC]; two fragments of a three-geometry not a two-geometry; the fallback priced at its true, graceful width — re-closure would admit free integers, never free fractions, so confinement survives the strike and only exactness is exposed (§5). [The exclusion: at NRC's slot; the degradation: priced.]
- The Response Support Theorem: standing response exactly on transport-complete supports — housed forcing intact [clause (a), the Realization Theorem untouched], unhoused subsets responseless rather than fraction-bearing [clause (b)], realized structure retained entire and unbound by the lattice [clause (c)]; the two theorems partitioning the member-grain question exhaustively: housed seats must answer q/k, unhoused seats cannot be asked (§6). [Proven, conditional as stated.]
- FREE-R named — the bridge from unsupported to inadmissible taken in public rather than silently, and grounded as continuity rather than novelty: standing existence individuated by standing transport content at every prior node of the corpus, FREE-R the same principle applied to the one sector the predecessors never quantified over; the charge-indefinite world nonetheless coherent by the boundary vocabulary, its exclusion therefore still a premise, slotted at the admissibility conditions, its fallback constructed as the second rival (§7). [Named, slotted, continuous; the bridge: visible.]
- The Saturation Theorem in three clauses — closure completion with occupancy single-class or mixed [J-COM], conjugate completion within or across classes [C-CLO; J-COM], composition — the quantifier fixed by the carrier-constituting clause, completion and admissibility independently characterized so the gate operates at subject-constitution rather than by definition (§8) — with exactness (nothing else free), and the Integrality Corollary: free totals integral, single-class $k\cdot(q/k) = q$, mixed occupancy $(\Sigma q_i)/k$ and cross-

class pairs $(q - q')/k$ under the congruence (§8). [Proven, conditional on R-SUP, NRC, FREE-R, C-CLO, J-COM, and the inherited chain; the mixed clauses' residue condition discharged at §9, integrality unconditional over the admissible sector.]

- J-COM named — completion individuated at the closure-geometry grain, k-seat supports jointly completable by distinct same-capacity classes, each seat reading its class's share — the proton audit's verdict rather than a rescue: the world's most common free object fills no single class under the route's own Class Uniformity Criterion, so any successful saturation theorem must license geometry-grain completion; the Transport Completion Lemma's vocabulary fixed under it (the geometry's standing structure carrying the classes' shares, the Lemma's conclusion untouched); fallback total at the table and stated as such (§2, §8). [Named, slotted at the census's closure structure.]
- The congruence inherited under custody and then derived at its joint-support grain: C-RES (c) — the predecessor's structural finding — consumed at §8 as the arithmetic equivalence (mixed-sector integrality ⟺ the congruence) and discharged at §9, where the **Composite Integrality Lemma** proves admissible free configurations to be carriers with unconditionally integral windings [L-G, AD-m at geometry grain, J-COM, FREE-R — no new import] and the **Congruence Theorem** [C-RES (c)-JS] derives the congruence of every jointly completing pair: pair forcing, occupancy forcing through the census's prime capacities [the unit and zero-divisor facts Imported-External at elementary modular arithmetic] (the $k = 4$ counterfactual stated — a zero divisor would weaken the rule to subgroup-wise; *never four* does arithmetic duty), and chain closure across mixing families; the replacement argument retired to a remark with its exchange premise named; the Promotion Conjecture discharged in improved form — the gate the lattice's, not a census clause; the world's residue ($2 \equiv -1 \bmod 3$) entailed by the existence of mixed hadrons, conditional on their identification as jointly completed supports at O1; pair-grain segregation predicted for any incongruent class, with the sum-zero three-distinct-class completion named as the permitted exception and prediction-in-waiting; C-RES (c)-U alone [Open] at the predecessor's slot, unconsumed — the assignment question, the route's handed-forward remainder, the paper's most Standard-Model-facing chapter (§8, §9). [The Lemma and Theorem: Proven, conditional at their chain; the split: stated.]
- The deconfinement fence: the theorem's quantifier running over free standing configurations only; collective realized media outside it by kind, not degree — no size or duration makes a transient realized locus a catalogue entry — at the realized-dynamics territory (§8). [Scope: stated.]
- Both rivals constructed and sent to their slots: the free-fraction world (loop-supported response; decided at R-SUP; debt total at the free sector) and the charge-indefinite world (catalogue without content; decided at FREE-R; the world's silence a lean, priced as such) — with the comparison's method symmetric and its verdicts located, not argued past (§10). [Constructed; deciders: named.]
- The Minimal Composite Spectrum Theorem: the minimal admissible free composites exactly the k-seat saturated completion and the conjugate pair — under O1, the baryon and the meson — everything else their compositions, nothing smaller admissible; with the observable consequences boxed, ten derived lines and the sum-zero vacancy (§11). [Proven, conditional as the Saturation Theorem; the naming at O1.]
- The audits at every capacity: $k = 1$ passed — singletons self-saturating, the world's free integral charges predicted rather than excused, the charge paper recovered; $k = 2$ run at

O1's conditionality — halves housed, free composites whole, the predecessor's null-branch freedom superseded at this audit and its consonance inverted; $k = 3$ — exactly two minimal free modes against exactly two observed composite families, the proton licensed as clause (a)'s mixed case [J-COM] and the charged pion as clause (b)'s cross-class case, exotica as compositions, the markers each at its own claim (§11). [Audits: run; verdicts: as marked.]

- CO-2 accounted in halves, as the truth divides: deliverable (i) — the balance restriction — [Conditionally discharged, and superseded]: transport completion the strictly stronger condition, the balance recovered as completion-spectrum arithmetic (the derivation at §12), closing at the R-SUP, NRC, FREE-R, C-CLO, and J-COM slots; deliverable (ii) — the congruence — split at §9: the joint-support form derived [C-RES (c)-JS, the route consuming no residue anywhere], the universal form [Open, unconsumed] handed forward as the assignment question; the principal question (why $k \geq 2$ differs) answered in the discharge's own shape — at $k = 1$ traverser and support coincide, at $k \geq 2$ the support is undivided and untransferable; B-IRR and C-ORD carried at total exposure with the composition-order audit re-recommended; the route derivation-complete at its stated conditionality, the residue its output rather than its debt, no new obligation handed forward (§12). [The accounting: split where the truth is split; no marker inflated.]
- The Confinement Dichotomy superseded in public — done, not happened: confinement support-theoretic, not arithmetic, with the retraction's legitimacy, the inverted consonance (now the sharpest $k \geq 2$ kill condition), and the closed R-O test surface all stated at full width in §12, their proper home. [The supersession: at §12.]
- The W0 chain stated whole — fork, C-ORD, O1, realization, saturation (five slots), and no residue condition: the congruence derived at §9, the chain's output rather than its input — with the claim's kind tracked across the route: availability, then prediction-with-permanence, now the free sector entire (appearance, value, permanence, confinement, integrality, spectrum, and the mixing rule) from one closure architecture; the firewall held, the lean exactly as strong as the slots, the marker unmoved (§13). [Conditional, as priced.]
- The division of labor completed — geometry proposes, transport disposes, ownership allocates, the carrier locates, realization registers, **saturation houses** (§14). [Proven, given the stated grain.]

Not established (open, out of scope, or pending):

- R-SUP, C-CLO at the charge paper's record construction and conjugation structure; NRC and J-COM at the census — the capacity derivation and the closure structure respectively; FREE-R at the Fold/G-S admissibility conditions — the paper's engine, awaiting its five source readings; the theorem is exactly as strong as they are;
- C-RES (c)-U — the universal congruence — [Open] at the predecessor's census slot, unconsumed by the route: the assignment question — why the world's flavour classes all lie in one jointly admissible residue family — the route's sole handed-forward remainder beyond its slot readings;
- O1 — the identifications — at the census's PFD slot; every empirical contact in §§11 and 13 is conditioned on it;

- the remainder problem — the fate of a class with unoccupied seats — at the G-S occupancy dynamics, fenced (§4);
- the realized dynamics of collective media — the deconfinement interior — fenced (§8);
- C-ORD at the structure-group imports and B-IRR at the Bath Criterion — the route's total exposures, undecided, carried;
- the branch selection at W0, [Open], its marker unmoved;
- hypercharge, the mixing, breaking, and the arc's fenced interior — exactly as the predecessors fenced them.

The honest summary: the paper takes the route's last debt — *if every seat must show its fraction, why is no fraction ever found alone?* — and dissolves the question's own premise with a theorem whose five new readings are named instead of smuggled: a measurement's standing output needs standing support, and the support is the geometry's, owned undivided because the bath admits no standing shares of it; a broken class is not a smaller class, because capacities are kinds; what supports nothing standing has no standing existence to be free with; a member paired with its antimember closes transport jointly, which is the world's second composite family read off the lattice's own orientation reversal; and a closure geometry seats whoever fits, one class's members or several's, which is what admits the proton — and, one section later, derives the congruence the proton was thought to consume: the lattice itself gates the mixing, refusing to mint a fractional carrier. If the five readings hold at their sources, the free sector is exactly the closure-complete sector — fractions housed, totals whole, mixtures congruent by the lattice's own gate, two composite modes and their compositions, nothing else — and confinement was never a wall around the fraction. It was the absence of anywhere else for the fraction to be.

17. Conclusion

The route asked, paper by paper, the questions a pooled fund forces. Who owns the pound? The pool. Whose whole numbers does the currency mint? The pool's, because only the pool survives the stirring. What does a seat show, when asked? The pool's pound at the seat's share — a third, printed permanently, by a receipt built to ignore the churn. And the last question, the one the world's silence had been asking all along: *can a seat be taken out of the bank?* The answer is that the question mistakes what a seat is. The third was never in the chair; it was in the books, and the books are the bank's — kept undivided, because the stirring that made the pool the owner left no line in the ledger that any one seat could tear out and carry away. Take the chair outside and it holds nothing — not a third, not zero, nothing the currency recognizes — because the ledger that gave the question meaning stayed behind. And the one other way to close the books is the one the world also shows: a deposit walked out arm in arm with an exactly opposite debt, the account closing jointly — the meson, in the ledger's own handwriting.

That is the theorem, and it is paid for in public: five named readings, each with an address and a stated price of failure; two rivals built whole and sent to the same addresses for judgment; one predecessor finding inherited under custody and then derived at its joint-support grain — the residue on which the wholeness of every mixed composite turns, shown to be the lattice's own gate on admissibility, so that — granted the proton's identification as a jointly completed support — the world's quark charges share their congruence *because* the proton exists, rather than the proton existing because the charges happened to comply — with only the assignment question

handed forward: why every flavour chose the same residue family; and one predecessor claim superseded in the open rather than by collision. What it buys, at that price, is the end of the route's structural debts. The census counted the seats. Ownership assigned the books. The carrier located the minting. Realization fixed the receipt. And saturation closes the bank's doors at night: nothing leaves but what the books can close on — full tables, or matched and opposite pairs — which is why everything the world sets free is whole, and why the thirds, forced at every housed seat, have never once been found on the street.

The strangest numbers in physics entered this programme as an anomaly, became a capacity, then a title, then a notation, then an obligation. They leave it, at the route's stated conditionality, as a closed account: the fraction exists because the class exists; it is permanent because the receipt reads the books and not the churn; and it is never free because it was never a thing — it was a view of a thing, and views do not survive the demolition of what they are views of. What remains is not structure but verdicts: five readings at their sources, the identifications, one assignment question at the predecessor's slot, one node. The books are kept; the markers hold; and the route's remaining debts, for the first time since the census opened it, are not unnamed gaps but identified verdicts at named slots.