The Second Integer

Monopole Sectors from Compact Phase — Magnetic Winding Forced, the Sector Question Located, and Both Answers Prepared

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General Reader Summary

The question

The charge paper in this programme showed why electric charge comes in exact whole-number steps: the phase of physics is a circle, and a particle's response to a circle can only wind a whole number of times. The structure-group paper then verified that the circle electromagnetism uses is the same circle the substrate derives — one dial, not two. Together they closed an old question. But closing it opened a new one, and the structure-group paper named it on its way out: a circular phase makes room for a kind of object a straight-line phase cannot accommodate — the magnetic monopole, a knot in the electromagnetic field that cannot be combed away. Conventional physics has searched for monopoles for a century without a confirmed detection, and conventional theory cannot say whether they should exist. This paper asks what the programme can say.

What this paper proves

Imagine surveying a magnetic field over the whole of a closed surface — a balloon, with no edge and no outside boundary. Patch by patch, the surveyor records how much the phase dial turns. Where two patches meet, their books must agree about the dial's *position* — but positions on a dial repeat every full turn, so the books can agree perfectly while differing by whole revolutions. Over a closed surface the discrepancies cannot hide: they add up to a total, and this paper proves that the total is forced to be an exact whole number of turns. Never half a turn, never 1.37 turns — the same circle that forced electric charge onto a ladder forces this second quantity onto a ladder of its own. That whole number is the magnetic charge of the region inside the surface: the monopole number. Electric charge is the first integer the circle forces; this is the second.

The paper also proves something about the famous consistency condition Dirac discovered in 1931 — that electric and magnetic charge, if both exist, must be quantized against each other. In conventional physics that condition is a *constraint*: the monopole is invoked to force charge into whole numbers. Here both numbers are already whole, each forced independently by the circle, so Dirac's condition is not a constraint at all. It is an identity — automatically true, the product of two integers being an integer.

One earlier finding of the programme is not disturbed by any of this but completed. The programme's work on magnetism shows that magnetic fields have no local sources — there is no such thing as a point of magnetic substance, and magnetic structure arises as circulation, never as charge density. Nothing here revisits that. The second integer is not a local source; it is a global property of a whole configuration — the kind of property a knot has, which no inspection of any single strand reveals. The paper proves the two results fit together exactly: because fields have no local sources, a monopole, if the programme permits one at all, could never be a speck of magnetic matter. It could only be a hole or a handle — a place the electromagnetic pattern cannot be smoothed flat, a feature of where the field lives rather than of what it contains. And the programme adds a demand ordinary mathematics never makes: a hole must be detectable to count as real — you cannot loop around a nothing that cannot be told apart from ordinary space — so a monopole, if one exists, must carry its own evidence of the hole it lives on.

The honest condition

What the paper does not settle is whether nature *uses* the nonzero rungs of this second ladder. Half of the question, it turns out, is already answered: the programme's earlier work on magnetism rules out monopoles as local objects — free magnetic endpoints, specks of magnetic substance — outright, so what remains open is only the global, topological half. The circle forces the ladder to exist; whether any real configuration sits above rung zero depends on how the substrate's electromagnetic sector is actually built — a question about the construction, not about the principles. The paper proves that no shortcut decides it: the programme's own general principles, which settled so much elsewhere, genuinely decide neither answer here, and the paper shows exactly why. It then states the deciding question precisely, writes down the procedure for answering it from the source papers, and prepares both outcomes in full. If monopole sectors are forbidden, the programme acquires a sharp prediction — no magnetic monopoles of the basic type — that distinguishes it from grand unified theories, which generically expect them. If they are permitted, Dirac's century-old condition is re-derived from the ground up. Either answer is a finding; the paper makes sure the finding is ready to be claimed the day the question is decided.

Abstract

The structure-group paper discharges the Gauge Identification (conditional on its import slots) and records one named successor obligation: whether the substrate construction of the Maxwell sector permits or forbids nontrivial bundle topology — monopole sectors. The present paper takes up that obligation and divides it exactly into what the inherited results force, what they provably cannot decide, and what therefore constitutes the deciding question.

Forced. Four results are established from inherited inputs alone. (i) The **Cap Lemma** (§3): for a realized 2-cap S with boundary γ, the boundary holonomy is the exponential of the cap's curvature record, $h(\gamma) = e^{i\Phi(S)}$ — a substrate Stokes relation derived potential-free, by tiling the cap with small loops whose interior edges cancel under reversibility while the realized record accumulates their phases [Proven, conditional on the structure-group paper's inputs]. (ii) The **sector quantization theorem** (§3): the total curvature record over any realized closed 2-surface

is an exact integer multiple of 2π — $\Phi(S) \in 2\pi\mathbb{Z}$ — because the two caps of any splitting must assign one fiber point to their shared boundary, forcing $e^{i\Phi_+}e^{i\Phi_-} = 1$. The integer $m(S) = \Phi(S)/2\pi$ is the magnetic sector label, coinciding with the first Chern class of the configuration [Imported-External correspondence]; it is splitting-independent, additive, deformation-invariant, and flips with orientation alongside electric charge, $m \mapsto -m$ as $q \mapsto -q$, leaving the pairing $q \cdot m$ invariant — consistent with the canonical identification's surviving bit (§4). The mechanism is the charge paper's mechanism on the dual side: electric charge is the winding of the carrier's *response* around the circle; magnetic charge is the winding of the configuration's *records* around the circle over a closed surface — the taxonomy's first and second objects each carry an integer, both forced by one compactness. (iii) The **Bounding Lemma** (§3): given the programme's source-freeness result for magnetism (B1: $\nabla \cdot B = 0$, $dF = 0$, magnetic fields as rotational entropy flow [Inherited]), the record over the boundary of any realized 3-region vanishes — the same telescoping one dimension up — so $m(S) \neq 0$ *requires* S to be non-bounding in the configuration's realized domain. Source-freeness says the curvature is closed; the sector question asks whether it is exact; a nonzero sector is closed-but-not-exact curvature, never a local source — the earlier magnetism result is completed by this paper, not contested, and the monopole is characterized in advance as necessarily topological: a handle of the emergent geometry or a core the configuration cannot extend across. (iv) **Dirac's condition, inverted** (§5): with $q \in \mathbb{Z}$ inherited and $m \in \mathbb{Z}$ established, the Dirac pairing $q \cdot m \in \mathbb{Z}$ holds identically — a consistency identity between two independently forced windings, not a constraint requiring the monopole to enforce quantization. The conventional inference *monopole ⟹ quantization* is thereby fully retired: both quantizations stand without the monopole, and the monopole's consistency is guaranteed by them.

Provably undecided. The **no-cheap-answer lemma** (§6): neither branch of the sector question follows from catalogue-level principles. The closure argument cannot open the sectors — the Conservation Lemma removes only *unwitnessed* boundaries, and a boundary between sectors is witnessed by m itself, an admissible record-level invariant. The economy argument cannot close them — the single-Fold catalogue is the fiber, all bundles over any base share the fiber $\mathbb{T}$, and triviality is a property of how configurations glue, not of the value space. Both shortcuts are foreclosed [Proven], locating the decision at construction level.

The deciding question. Sector Admissibility (SA) (§7) [Open, partially read]: whether the substrate construction admits realized configurations of nonzero total curvature record over some realized closed 2-surface — refined by the Bounding Lemma into exactly two channels: nontrivial second homology of the emergent geometry (SA-top), or admissible non-extendable loci, cores (SA-core). Read at source, the node returns a partial verdict: isolated local magnetic charges are excluded outright [Inherited — verified; citation to be typeset] — the local half of the monopole question is closed — while the Maxwell admissibility paper's own explicit caveat (global potential assumed, not derived; nontrivial bundles named as where Dirac-type monopoles would live) converges on SA exactly as this paper formalizes it. Both topological channels remain open, with a recorded mechanism-level bias toward closure but not yet a theorem — rotational flow that cannot terminate has nowhere obvious to be undefined — and one named formalization obligation, **C1 (Core Exclusion)**: rotational entropy flow is everywhere defined, no admissible cores; C1 plus a trivial-homology return at G1 resolves SA to branch F in full. The bias is mechanism-level, the marker is the marker, and the decision procedure stands: branch P (permitted) is decided by exhibiting one admissible $m \neq 0$ configuration on a live channel; branch F (forbidden) by closing both channels at source. A **distinguishability refinement** (§7) sharpens the node by the programme's oldest discipline: absence is what cannot be distinguished, so a non-bounding region is admissible only if its non-bounding character is itself witnessed (**WNB**) — the Bounding Lemma's contrapositive supplying the canonical witness, $m \neq 0$ certifying non-bounding. The refinement removes bare holes (no stockpile of unwitnessed handles awaits winding), merges the channels at the grain of admissibility (hole and witness co-arise, one package), narrows C1's burden to witnessed cores, and leaves SA's remaining physical content as a single question: what realizes the distinction m measures? Both branches are prepared (§§8–9): F yields the prediction *no magnetic monopoles of abelian-bundle (Dirac) type* — the corpus's second standalone falsifiable commitment and its first discriminator against grand unification, stated at its exact grain with the non-abelian question reserved to the multi-Fold arc; P re-derives Dirac's condition from below and hands the abundance question to the cosmology arc, noting that the conventional monopole problem does not transfer automatically into the programme's cosmology. The experimental situation (§10) — no confirmed detection across a century of searches — is consistent with F outright and with P at low abundance, so observation currently constrains but does not decide. The paper consumes only configuration-level and record-level structure and is assignment-neutral (§11); everything is conditional on the structure-group paper's imports at their slot status, recorded as inherited exposure.

Epistemic markers: [Inherited] imported from prior VERSF papers; [Imported-External] imported from standard mathematics outside the programme, carried at the external source's standing; [Proven] established here; [Conditional] holding under stated inputs; [Conjectural] motivated but unproven; [Open] undecided.

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1. Introduction — The Question the Discharge Created

A compact structure group makes mathematically available what a non-compact one cannot: nontrivial bundle topology — configurations of the electromagnetic sector that no single global phase description covers, classified by an integer, and housing, where they exist, the magnetic monopole. The structure-group paper, in reading the Maxwell connection's group as the derived compact $\mathbb{T}$, created this question for the corpus and recorded it as a named successor obligation at [Open]: *does the substrate construction of the Maxwell sector permit or forbid nontrivial bundle sectors?* It noted there that either answer would be a finding — a forbidden sector is a prediction, a permitted one re-derives Dirac's condition from below — and that the question is only available to ask in substrate terms because compactness is derived from a construction fine-grained enough to interrogate. The present paper is the taking-up of that obligation.

The paper's architecture follows from an honest division of the question into three parts of different epistemic kinds, and announcing the division up front prevents the paper from being read as either more or less than it is. The first part is **forced**: the existence and integrality of the sector label — that the total curvature record over any realized closed surface is an exact multiple of 2π — follows from the inherited results alone, by the same mechanism that forced electric charge onto its lattice, and §§3–5 prove it, together with the inversion of Dirac's condition from constraint into identity. The second part is **provably undecided**: whether nonzero sectors are admissible does *not* follow from the inherited principles, and §6 proves that too — foreclosing one tempting shortcut in each direction, so that no reader, friendly or hostile, can claim the catalogue settles what only the construction can. The third part is the **decision node** itself: §7 states the deciding question exactly, with citation slots and a procedure, in the import-slot discipline the structure-group paper established; and §§8–9 prepare both outcomes in full, so that the day the slots are read, the corpus claims its finding without further drafting.

The paper therefore does not end with an answer to the sector question, and says so plainly. What it ends with is better than a premature answer: a theorem (the second integer, forced), a retirement (Dirac's inference, inverted), a locating result (no cheap answer), a decision instrument (SA, with both branches loaded), and a partial verdict — the local half of the monopole question closed at source, the remaining distance to branch F reduced to two named obligations. The conventional literature has carried the monopole question for a century as a free speculation — neither required nor forbidden, attached to quantization by an inference this corpus has now dissolved. The present paper converts it into a determinate question about a specific construction, with the procedure for answering it written down.

2. Inherited Results and Their Reach

Five inherited results are consumed throughout, and one external correspondence; their reach — what they do and do not determine here — is fixed before the work begins.

The compact structure group [Inherited — structure-group paper, conditional on its imports M1–M3 at their slot status]. The Maxwell connection is substrate transport; its structure group is the derived compact $\mathbb{T}$; the identification is canonical up to orientation. Everything below inherits the conditionality of that paper's slots, recorded once here and in §11 rather than re-marked at every step.

The three-object taxonomy [Inherited — structure-group paper §5, step three]. The holonomy $h(\gamma) \in \mathbb{T}$ is fiber-level and circle-valued; the flux $\Phi(S) \in \mathbb{R}$ through a realized spanning surface is configuration-level and record-borne — integrated curvature, composed holonomy data; the bare circulation absent a record is a coordinate. The present paper is, in one sentence, the study of the second object on *closed* surfaces, where there is no boundary holonomy to anchor to and the record must answer to itself.

The charge lattice [Inherited — charge paper]. Carrier responses to the compact phase are classified by $q \in \mathbb{Z}$: electric charge as the winding number of the carrier's response. Consumed in §§4–5, where the second winding is set beside the first.

The holonomy composition structure [Inherited — U(1) paper, as fixed in the charge paper §§2, 4]. Holonomies compose; reversal inverts, $h(\gamma^{-1}) = h(\gamma)^{-1}$; curvature is the infinitesimal loop holonomy, the local density of loop phase [the last per M2]. These are the working parts of the Cap Lemma.

Source-freeness of magnetism (B1) [Inherited — the conventional form verified at source; the surface-level transcription below to be confirmed at the same sections; ⟨cite: magnetism paper (rotational entropy flow; rotational flow admits no isolated beginnings or endings) and Maxwell admissibility paper (no fundamental isolated U(1) magnetic charge): titles and sections to be typeset⟩]. The programme's magnetism work establishes that magnetic fields carry no local sources, stated here in the form the present paper consumes: the curvature record over every realized small closed surface vanishes — no circulation closes on itself around a point. The conventional gloss is $\nabla \cdot B = 0$, $dF = 0$, with magnetic structure arising as rotational flow, never as magnetic charge density; the import is stated record-level and surface-level so that it consumes nothing the source papers do not establish and presupposes no three-dimensional integral relation not yet derived in substrate terms. The present paper does not revisit that result; it consumes it (§3, Bounding Lemma), and the apparent tension — a monopole paper downstream of a no-magnetic-sources result — dissolves under a distinction the differential language makes exact. Source-freeness says the curvature is *closed*; the sector question asks whether it is *exact*. A nonzero sector is curvature that is closed but not exact: divergence-free everywhere it is realized, yet integrating to $2\pi m \neq 0$ over a surface that bounds no realized region. The sector label m is not a local magnetic source and never contradicts $\nabla \cdot B = 0$; it is a global topological invariant of a realized configuration, and the question of its occupancy differs categorically from the question of local magnetic charge density — which the earlier work answers, and this paper leaves answered.

The Chern correspondence [Imported-External]. The integer this paper derives — total curvature record over a closed surface in units of 2π — coincides with the first Chern class of the configuration, the classical classifying invariant of circle bundles over closed surfaces. The correspondence is recorded for orientation toward the standard literature; nothing below leans on it, and the derivation is self-contained in substrate terms.

What the inherited results do *not* determine — and §6 proves they cannot — is the occupancy of the sectors. The reach of the catalogue ends at the integrality of the label; the label's realized values are the construction's to decide.

3. The Sector Label, Forced — Total Flux on Closed Surfaces Is $2\pi\mathbb{Z}$

The section's three results are a lemma, the theorem it carries, and a second lemma that converts an inherited result into the strongest forced constraint on the sector question.

The Cap Lemma. *For a realized 2-cap S — a surface with boundary γ, tiled by realized small loops — the boundary holonomy is the exponential of the cap's curvature record:*

$h(\gamma) = e^{i\Phi(S)}$, $\Phi(S) = \iint_S F$.

Proof sketch, potential-free. Tile S by small loops $\{\lambda_i\}$. By M2, each small loop carries holonomy $e^{i\varphi_i}$ with φ_i its curvature record — curvature being the local density of loop phase. Compose the small loops in tiling order: every interior edge is traversed exactly twice, in opposite directions, and reversibility cancels the two traversals, $h(e^{-1})h(e) = 1$; what survives the telescoping is the boundary, so the composite holonomy is $h(\gamma)$ — composition order and basepoint connectors drop because the group is abelian; the lemma is stated for the single-Fold $\mathbb{T}$ and does not claim the non-abelian case. On the fiber side the composite is $\Pi_i\, e^{i\varphi_i} = e^{i\Sigma\varphi_i}$; on the record side the realized composition accumulates $\Sigma\varphi_i = \Phi(S)$ as configuration-level data — this is exactly the composed-holonomy record of the taxonomy's second object, and the accumulation is licensed because the tiling is *realized*, not merely constructible. Hence $h(\gamma) = e^{i\Phi(S)}$. [Proven, conditional on inherited composition, reversibility, and M2.]

Two features of the lemma deserve flags. It is the substrate's Stokes relation, derived without a potential — no A appears, matching the potential-free form in which M2 is stated, and matching the situation the lemma will be used in, where a global A may not exist. And it is *grain-exact*: the real number $\Phi(S)$ lives in the realized record of the composition, the fiber receives only its exponential — the taxonomy enforced, not merely respected.

The Sector Quantization Theorem. *For any realized closed 2-surface S, the total curvature record is an exact integer multiple of 2π:*

$\Phi(S) \in 2\pi\mathbb{Z}$. Define $m(S) = \Phi(S)/2\pi \in \mathbb{Z}$.

Proof. Split S along any realized loop γ into two caps S_+ and S_-, each with boundary γ but with opposite induced orientations. By the Cap Lemma applied to each cap with its own orientation,

$h(\gamma) = e^{i\Phi_+}$ and $h(\gamma^{-1}) = e^{i\Phi_-}$,

where Φ_+, Φ_- are the caps' records with the closed surface's consistent global orientation. By reversibility $h(\gamma^{-1}) = h(\gamma)^{-1}$, so

$e^{i\Phi_+} \cdot e^{i\Phi_-} = h(\gamma) \cdot h(\gamma)^{-1} = 1$,

whence $\Phi(S) = \Phi_+ + \Phi_- \in 2\pi\mathbb{Z}$. [Proven, conditional as above.]

The mechanism should be stated in one plain sentence, because it is the entire physical content and it is the charge paper's mechanism re-run on the dual side. The boundary loop has exactly *one* holonomy — one point of the circle, by the compactness and exact periodicity of the derived phase — while the two caps each carry a *real* record; the single fiber point cannot remember which real representatives the records chose, and the slack between the records and the point is forced to be whole turns. On the non-compact line the argument evaporates: there $h(\gamma)$ would itself be a real number, each cap's record would equal it exactly, and $\Phi(S) = 0$ identically — no sectors, no monopoles, nothing to ask. Compactness does not merely permit the second integer; compactness is the only reason there is a second quantity to quantize.

Properties of the label, in brief. *Splitting-independence:* $\Phi(S)$ is a sum of tile records and does not reference the splitting; the theorem's split is a proof device, not part of the definition. *Additivity:* over disjoint closed surfaces, records add, so m adds. *Deformation invariance:* under admissible continuous deformation of the configuration with finite records throughout, $\Phi(S)$ varies continuously within $2\pi\mathbb{Z}$, hence not at all — integers do not drift, exactly as the charge paper says of q. *Orientation:* reversing the surface's orientation sends $\Phi(S) \mapsto -\Phi(S)$, $m \mapsto -m$. The label m(S) is therefore a sector invariant of the configuration: a second integer, with the first integer's exactness and for the first integer's reason. [All Proven, same conditionality.]

The Bounding Lemma. *If a realized closed surface S bounds — $S = \partial V$ for a realized 3-region V throughout which the configuration is realized — then $\Phi(S) = 0$ and $m(S) = 0$.*

Proof. Tile V by realized small cells. Each cell's boundary is a realized small closed surface, whose record vanishes by B1. Interior faces are shared by exactly two cells with opposite induced orientations and cancel pairwise; the telescoping — the Cap Lemma's mechanism, one dimension up — leaves only the boundary $\partial V = S$, so $\Phi(S)$ is a sum of vanishing cell records. Hence $\Phi(S) = 0$ and $m(S) = 0$. [Proven, conditional on B1 and the §3 machinery.]

The lemma converts the programme's earlier magnetism result into the strongest forced constraint on the sector question, and characterizes the monopole in advance of deciding its existence. A nonzero sector cannot live over any surface that bounds: $m(S) \neq 0$ *requires* S to be non-bounding in the configuration's realized domain. And there are exactly two ways a realized closed surface can fail to bound: the emergent geometry itself carries a nontrivial 2-cycle — a handle — or the configuration fails to extend across some locus inside S — a core that the surface encloses and no realized 3-region fills. The monopole, if the construction permits it at all, is therefore never a speck of magnetic substance — source-freeness stands untouched — but a hole or a handle: a topological feature of *where the configuration lives*, not a local property of what it contains. The conventional Dirac monopole quietly agrees, living as it does on a region with a worldline excised; the substrate derivation recovers that geometry as a necessity rather than a modeling choice. One consistency observation ties the section's three results together: for a realized small cell the Sector Quantization Theorem already forces the boundary record into $2\pi\mathbb{Z}$, so B1's additional content is exactly the exclusion of realized quantized point sources — m

$= \pm 1, \pm 2, \ldots$ at a point — which is why any core must be a non-realized locus, and why SA-core is the only escape the lemma leaves open.

4. The Second Winding — Magnetic Charge as Record Winding

What kind of object is m? The charge paper's geometric reading of q supplies the template, and the parallel is exact enough to state as a structural result rather than an analogy.

Electric charge, per the charge paper §7, is the winding number of the carrier's *response*: a carrier of charge q winds q times in its own phase per single circuit of the transport phase — a fiber-side winding, an integer because two circles can only mesh at whole ratios. Magnetic charge, per §3 above, is the winding number of the configuration's *records*: over a closed surface, the real-valued records wind m times around the circle their exponentials live on — a configuration-side winding, an integer because a closed surface's records must hand the fiber a single consistent point. In the taxonomy's terms: the first object (holonomy, fiber-level) carries the electric integer through the responses it classifies; the second object (record, configuration-level) carries the magnetic integer through the closed-surface accumulations it permits. One compactness, two windings — the response winding and the record winding — and the two exhaust the taxonomy's invariant-bearing objects, the third object being a coordinate and carrying nothing.

The orientation behavior seals the parallel and supplies a consistency check against the structure-group paper. Under the canonical identification's one surviving bit — conjugation $z \mapsto \bar{z}$, the charge-sign convention — the carrier winding flips, $q \mapsto -q$ [structure-group paper §6], and the record winding flips with it, $m \mapsto -m$ (records change sign with orientation, §3). The physically meaningful pairing $q \cdot m$ is therefore orientation-invariant — as it must be, since it will turn out in §5 to be the Dirac pairing, an observable consistency datum that no convention should touch. The check passes, and its passing is not trivial: a framework in which one winding flipped under conjugation and the other did not would have smuggled an orientation-sensitive observable into a convention, and the canonicality result would have been in trouble. [Proven; the check is consumed again in §5.]

One terminological note keeps the section honest about its reach. "Magnetic charge of the region enclosed by S" is the conventional gloss of m(S), and it is adopted here — but nothing in this paper constructs a monopole as an *object*: a localized configuration, with mass, size, and dynamics. The sector label classifies configurations; whether any admissible configuration realizes a nonzero label is §7's question, and what such a configuration would be like as a piece of physics is downstream of even that. The second integer is established as a classification; its occupancy and its physics are not claimed.

Why the second integer does not violate M3. One question must be asked aloud, because the structure-group paper's M3 appears to forbid exactly what this paper has built: M3 states that no Maxwell-sector construction treats total winding — the integer separating θ from $\theta + 2\pi n$ — as physical structure at any step, and m is an integer built from accumulated real records that distinguish Φ from $\Phi + 2\pi$. The reconciliation is exact, and it was prepared in advance by the structure-group paper itself. M3 forbids the lift: the promotion of a fiber holonomy e^{iθ} to a real number θ by selecting a branch of the logarithm — catalogue-level structure that no admissible invariant witnesses. The records composing m never take that step. Curvature is real-valued by kind — the local density of loop phase, integrated over realized tiles — and $\Phi(S)$ is the realized accumulation of those densities: at no point is any holonomy's logarithm taken, no branch is ever selected, and no unwitnessed distinction is introduced. The real number enters as data, not as a choice. This is precisely the realized-record structure the structure-group paper's §5 step three carved out when closing the lift loophole: a history carrying a record of its own circuit realizes an invariant — "ordinary physics, not a property of the holonomy catalogue" — and the realized-witness discipline was needed to keep the structure group at $\mathbb{T}$ despite such records, not in ignorance of them. The second integer is that anticipated record, constructed; under branch P its nonzero values would be realized; and its consistency with M3 holds by the predecessor's own argument, not by a concession made here. The same observation settles the well-definedness of m: because the tile records are curvature data rather than extracted logarithms, $\Phi(S)$ carries no per-tile 2π ambiguity for a branch choice to create, and the label is determined by the realized configuration alone. [Proven — the consistency; the well-definedness is the Cap Lemma's grain, made explicit.]

5. Dirac's Condition, Inverted

The 1931 argument runs: if a single monopole of magnetic charge g exists anywhere, then consistency of quantum mechanics — single-valuedness of any charged particle's wavefunction transported around it — requires $qg = 2\pi n$, quantizing every electric charge in the universe. The monopole is the *enforcer*: quantization is the price of its consistency, and the argument's celebrated feature is that one monopole suffices. The inference direction is *monopole ⟹ quantization*, and for ninety years it has been one of the two great reasons to suspect the monopole exists — the other being grand unification, which the charge paper already addressed.

The corpus inverts the direction, and the inversion is now an identity rather than an argument. Electric charge is quantized, $q \in \mathbb{Z}$, by the charge paper — with no monopole premise, from the compactness of the phase. Magnetic charge is quantized, $m \in \mathbb{Z}$, by §3 — with no charged-particle premise, from the same compactness. The Dirac pairing is then

$$q \cdot m \in \mathbb{Z},$$

identically: the product of two integers. (The conventional form $qg = 2\pi n$ carries the unit of charge inside g; the identity here fixes the integrality of the pairing, and the unit remains the fine-structure arc's, exactly as the charge paper's §10 divides it.) The consistency that Dirac's argument *imposed* as a condition is here *guaranteed* before any monopole is so much as permitted to exist. A carrier of charge q transported over a sector-m configuration accumulates total response winding $q{\cdot}m$ — a whole number of turns, returning the carrier exactly to its starting configuration — and it could not have been otherwise, because both factors were separately forced onto their lattices by the circle. [Proven, conditional on the charge paper and §3 at their statuses.]

The conceptual consequence deserves its own paragraph, because it completes a retirement the charge paper began. The conventional literature offers the monopole as an *explanation* of charge quantization — the inference runs from a hypothetical object to an observed regularity, and the object's non-observation has always been the explanation's open wound. The charge paper removed the regularity's need for the explanation: quantization follows from derived compactness. The present section removes the explanation's remaining logical role: Dirac's condition, the entire content of the monopole route to quantization, is an identity between two independently derived integers, with nothing left for the monopole to enforce. The monopole question survives this — §§7–9 are its new home — but it survives *as a question about what exists*, stripped of every quantization duty. The structure-group paper's §8.1 separated *why is charge quantized?* from *do monopoles exist?*; the present section finishes the separation by showing that even the consistency bridge between them is one-way: the integers guarantee Dirac, Dirac no longer argues for anything.

6. No Cheap Answer — Why Inherited Principles Decide Neither Branch

A corpus with powerful general principles faces a characteristic temptation: to let the principles decide questions that are genuinely the construction's to decide. Two shortcuts present themselves here — one opening the sectors, one closing them — and each has the surface form of arguments that succeeded elsewhere in the corpus. This section forecloses both, and the foreclosure is a result: it proves the decision node of §7 is genuinely open at catalogue level, so that neither branch can be claimed, by anyone, without reading the construction.

Shortcut one — "the closure argument opens the sectors." The U(1) paper's closure argument removed catalogue boundaries: no admissible mechanism can maintain a boundary in the phase structure, because a boundary unaccompanied by any admissible invariant change is an unwitnessed distinction, forbidden by the Conservation Lemma. The tempting transfer: excluding $m \neq 0$ while permitting $m = 0$ is a boundary in configuration space — does the Lemma not remove it, forcing all sectors open? It does not, and the reason is exact. The Lemma removes *unwitnessed* boundaries — differences in admissibility located where no admissible invariant changes. A boundary between sectors is located precisely where an admissible invariant changes: m itself, which §3 established as record-level, realized, admissible data — the very thing the taxonomy classifies as invariant-bearing. An admissibility rule that excludes configurations by their m is a rule conditioned on a witnessed quantity, exactly the kind of physical structure the Lemma permits. The closure argument owes its victories to boundaries nothing could see; the sector boundary is seen by the second integer, and the argument has no purchase. [Proven.]

Shortcut two — "the economy argument closes the sectors." The single-Fold economy argument did real work in the charge paper: the substrate supplies one independent compact phase, so a second independent electromagnetic circle is surplus structure. The tempting transfer: one phase, therefore one global phase description, therefore trivial bundle, therefore $m = 0$ only. The transfer fails at its middle step, and the failure is a category distinction. The single-Fold result is a statement about the *catalogue* — the value space, the fiber: there is one circle $\mathbb{T}$ for phases to live on. But every bundle over every base, of every Chern class, shares that same fiber:

a sector-m configuration is not a second circle, it is a way of arranging phase data valued in the *one* circle such that no single global description covers it. Triviality is a property of how configurations glue, not of what they are valued in; the economy argument constrains the value space and is silent about the gluing. One circle is exactly as compatible with $m = 7$ as with $m = 0$. [Proven.]

What the foreclosures establish. The sector question is construction-level: it turns on how the Maxwell sector's admissible configurations are actually assembled from substrate transport — globally, or patchwise with comparisons that can wind — and on whether any admissibility constraint at source bounds the total record. No catalogue principle reaches it. The source-freeness import narrows the question without deciding it: by the Bounding Lemma it closes every *local* route to $m \neq 0$, leaving only the topological routes — non-bounding surfaces, through handles or cores — and those are construction-and-geometry facts, exactly where the foreclosures locate the decision. One principle does reach the question's *grain* without deciding its answer: no-pre-individuation removes unwitnessed non-bounding structure outright (§7, the distinguishability refinement), reshaping what a sector-opening exhibition must contain — a hole co-realized with its witness — while leaving the witnessed package exactly as open, and exactly as construction-level, as the foreclosures find it. The corpus's honest position is therefore neither branch, held at [Open], with the deciding question stated exactly — which is §7's business. The foreclosure also disciplines the future: any later paper claiming either branch from general principles alone contradicts this section and must locate the error here or withdraw the claim. [Proven; the discipline is the result.]

7. The Decision Node — Sector Admissibility, with Slots and Procedure

The deciding question is stated in the import-slot form the structure-group paper established, with its verification asymmetry made explicit.

Sector Admissibility (SA) — *the decision node of this paper* [Open]

The substrate construction of the Maxwell sector admits realized configurations whose total curvature record over some realized closed 2-surface is nonzero: $m \neq 0$ sectors are admissible. By the Bounding Lemma (§3) this refines exactly: nonzero sectors require realized *non-bounding* closed 2-surfaces, through at least one of two channels — **(SA-top)** the emergent geometry carries nontrivial 2-cycles (nonzero second homology) on which admissible configurations wind, or **(SA-core)** admissible configurations may fail to extend across loci — cores — so that a surface enclosing a core bounds no realized region. (Negation of SA: both channels closed — every realized closed surface bounds a realized region of the configuration, and by the Bounding Lemma every sector label vanishes.)

Slot G1 (homology of realized domains). The emergent geometry's supply of closed 2-surfaces *and* their bounding status: whether realized domains carry nontrivial second homology. Trivial homology closes channel SA-top. [Inherited — ⟨cite: geometry arc, emergent 4-manifold structure; the homological reading is the substantive part of this slot⟩]

Slot G2 (configuration assembly and extendability). The Maxwell-sector papers' construction of admissible configurations: whether phase data is assembled globally — which closes both channels at once — or patchwise; and whether admissibility requires every configuration to extend across every locus of its domain. Universal extendability closes channel SA-core; admissible cores open it to the exhibition step. *Post-reading status (per the partial reading below):* read at source — assembly global-by-assumption, non-closing; extendability unconstructed, formalized as C1; the slot's residue is the citation typesetting. [⟨cite: Maxwell-sector paper, configuration-assembly construction; admissibility conditions on configuration domains⟩]

The decision procedure, in order. (i) Read G2's assembly form: if global, and the globality is established as a property of every admissible configuration rather than a feature of the configurations so far constructed, SA is decided negative — branch F — with the globality's reason recorded as the finding's ground; an incidental globality decides nothing and routes to (ii). This is the section's existence/universality asymmetry applied to its own first step: F needs globality universal. (ii) If patchwise, read G1's homology: nontrivial 2-cycles leave SA-top live; trivial homology closes it. (iii) Read G2's extendability: universal extendability closes SA-core; admissible cores leave it live. (iv) If both channels are closed, SA is decided negative — branch F — with the two closings jointly as the finding. (v) If either channel is live, exhibit one admissible $m \neq 0$ configuration explicitly — the minimal candidate being the $m = 1$ configuration on the smallest realized non-bounding surface (an SA-top cycle) or around the minimal admissible core (SA-core) — which decides branch P.

The verification asymmetry, flagged. Branch P is decided by an existence proof: one admissible configuration, exhibited on a live channel. Branch F is decided by universality proofs: both channels closed at source. Existence is checkable; universality must be derived — so P is the easier verdict to *establish* and F the easier to *refute* (one exhibited configuration overturns it), and the asymmetry now runs per channel: F requires the geometry arc and the Maxwell arc to *each* deliver a closure. A verifier should expect the F-reading to take longer and to be the one a hostile referee probes hardest; the asymmetry is structural, not a comment on which answer is likelier. No likelihood is assessed from principles: §6 establishes that the corpus's principles do not lean, and the paper records bias only where its sources supply it — the partial reading below records a mechanism-level bias toward closure, not yet a theorem, that the markers do not anticipate. [The node is [Open], partially read; the procedure and the partial reading are the deliverables.]

The partial reading at source. The slots, read against the magnetism and Maxwell admissibility papers, return a three-part verdict, and recording it exactly is the difference between a bias and a claim.

Decided at source — no isolated local magnetic charges. The magnetism paper excludes free magnetic endpoints outright: magnetic structure is rotational entropy flow, and rotational flow admits no isolated beginnings or endings — the poles of a magnet are directions of flow, not separable charges. The Maxwell admissibility paper concurs: no fundamental isolated U(1) magnetic charge exists in the construction. The *local* half of the monopole question is therefore closed [Inherited — verified at source; citation to be typeset] — and it is exactly what B1 and the

Bounding Lemma require the sources to say, since a local magnetic charge is a nonvanishing divergence record, forbidden at the mechanism level.

Anticipated at source — the topological carve-out. The Maxwell admissibility paper's exclusion carries an explicit caveat: it assumes a globally valid potential — trivial topology — and names nontrivial topology, U(1) bundles, as precisely where Dirac-type monopoles would live if anywhere. The convergence deserves its record: the source arc carved out, in its own terms, exactly the question this paper formalizes — SA is the source's own caveat, given channels, slots, and a procedure. Two consequences follow. The construction is global *by assumption*, not by derivation, so the procedure's step (i) does not close the channels: the source confines its constructions to the trivial sector without legislating against the others. And the structure-group paper's insistence on stating M2 potential-free is retroactively vindicated as necessary care rather than caution: the source's global-potential assumption is a working restriction, and importing it silently would have begged the present question.

Open at source, with a mechanism-level bias — the two channels. SA-top is untouched: neither source paper reads the second homology of realized domains, and the channel waits on the geometry arc (G1). SA-core is unconstructed: no admissible singular core — no locus where the rotational flow is undefined — appears anywhere in the magnetism arc, and the arc's mechanism points against one: flow that cannot terminate is flow with nowhere obvious to be undefined. The sources therefore supply a mechanism-level bias toward closure — the existing construction naturally points one way — but not yet a theorem. The mechanism-level exclusion of endpoints is not a theorem that every admissible configuration is everywhere defined, and the gap between them is exactly one formalization:

C1 (Core Exclusion) — *named obligation, handed to the magnetism arc* [Open]

Every admissible configuration of the Maxwell sector is everywhere defined: rotational entropy flow admits no non-extendable loci — no admissible cores. If proven, SA-core closes; and with G1 returning trivial homology, SA resolves to branch F in full.

The burden posture is recorded with the obligation, in the corpus's standard form. No admissible core is constructed at source, so the case for opening SA-core does not currently exist: whoever would open the channel must build the core, and whoever would close it must prove C1. Until one of them does, the channel is open, the bias is mechanism-level, and the marker is the marker. The bottom line of the reading, stated once: the magnetism arc may well be SA's decider — but only when its everywhere-definedness is a theorem rather than a mood.

The distinguishability refinement — witnessed non-bounding. One further sharpening is forced by the programme's own oldest discipline, and it changes what kind of question SA is. Ordinary topology may posit a hole and stop: the hole is a feature of the mathematics whether anything detects it or not. This corpus cannot stop there. Absence, in this programme, is what cannot be distinguished — and a distinction located where no admissible invariant differs is not admissible structure. One cannot loop around a nothing that cannot be told apart from ordinary continuum. The principle, in the node's terms:

Witnessed Non-Bounding (WNB). A non-bounding region is admissible only if its non-bounding character is itself witnessed by some realized admissible invariant. [Proven, as an application of no-pre-individuation at its source status; agreement with the geometry arc's distinguishability-built construction is expected, and is confirmed or refuted at G1.]

The refinement does not close the channels, and the reason is the Bounding Lemma's own contrapositive: $m(S) \neq 0$ entails that S bounds no realized region, so a nonzero record winding is itself a witness of non-bounding — the canonical one. A realized $m \neq 0$ configuration is a self-certifying hole: the winding does not merely live on the hole, it circumscribes it into admissibility. The grain is exact and worth one sentence: the core itself is never realized (§3), so absence is never witnessed directly — it is circumscribed; what is witnessed is always the configuration around the nothing, never the nothing.

What the refinement does is three things. First, it removes the bare hole: topology unaccompanied by any witness is unwitnessed absence, removed at principle level — there is no stockpile of empty handles waiting for configurations to wind on them. Second, it merges the channels at the grain of admissibility: SA-top and SA-core differ in which structure supplies the hole, but under WNB both reduce to a single admissibility form — the hole and its witness must co-arise, one package — and the procedure's exhibition step is revealed as not merely sufficient but essentially the only route: exhibiting the witnessed configuration *is* exhibiting the hole, with no independent geometric fact to verify first. G1's question is re-grained accordingly: not whether abstract second homology exists, but whether the geometry arc's distinguishability-built structure admits witnessed non-bounding — a question that arc is exactly positioned to answer, since structure built from distinguishability has no unwitnessed features to remove in the first place. Third, it narrows C1's burden: unwitnessed cores are removed by WNB without the magnetism arc's help, so the everywhere-definedness theorem need exclude only the witnessed package — prove that rotational entropy flow cannot co-realize a core together with the winding that would certify it.

The refinement raises the bar for branch P — a mathematical hole is no longer enough; a physical mechanism must realize the package — and in doing so it strengthens the recorded mechanism-level bias without moving the marker: continuity, closure, no free boundaries, and flow that cannot terminate are now joined by a principle that forbids looping around undistinguished nothing. The bar is higher; the question is open; and its remaining physical content is one sentence: *what realizes the distinction that m measures?* That question — not the existence of holes — is what the geometry and core investigations decide. [The refinement and the channel merger: Proven, given WNB; the bar-raising recorded as bias, the marker unchanged.]

8. Branch F — If the Sectors Are Forbidden

Prepared, not claimed; everything in this section is [Conditional on SA resolving negative].

The prediction, in two levels. The first level is no longer conditional. Isolated local magnetic charges — free magnetic endpoints, magnetic charge density anywhere — are excluded at source [Inherited — verified; citation to be typeset; §7]: that much of "no monopoles" the corpus owns outright, branch or no branch. Branch F's content is the second level: no magnetic monopoles of abelian-bundle (Dirac) type exist — no nonzero sector through either topological channel, at any abundance, anywhere. Under the partial reading, the branch's remaining distance is exactly two named closures, neither yet delivered: C1 (core exclusion, the magnetism arc's formalization obligation) and a trivial-homology return at G1 (the geometry arc's reading). This second level is the corpus's second standalone falsifiable commitment, beside the charge paper's e/3 lattice — and unlike the lattice, which the Standard Model shares, it is a *discriminator*: grand unified theories generically produce monopoles, as topological defects of the embedding of U(1) in a broken simple group, at abundances large enough that their non-observation is itself a celebrated problem. Under branch F the corpus and grand unification disagree about an observable. A single confirmed monopole refutes branch-F; continued null searches corroborate it weakly and continuously. A programme becomes scientific in proportion to what can kill it; branch F is a clean kill-condition.

The exact grain of the prediction. The forbidden objects are abelian-bundle monopoles — nonzero m of the substrate Maxwell sector. The conventional literature's other monopole channel, the 't Hooft–Polyakov construction, requires the electromagnetic U(1) to sit inside a spontaneously broken non-abelian group; the corpus has no grand-unified embedding, so that channel does not arise as conventionally constructed — but whether the multi-Fold arc's non-abelian sectors generate any analogous configuration is that arc's question, reserved to it, and the prediction is stated at its honest grain: *no monopoles of the abelian-bundle type; the non-abelian sectors to be audited at their construction.* A reader should not quote this branch as "VERSF predicts no monopoles, full stop" until the multi-Fold reservation is discharged.

Structural consequences. Triviality means a global phase description exists for every admissible configuration — a global potential A, retroactively licensing the conventional global-A practice as a fact about this construction rather than an assumption smuggled into it (and the structure-group paper's care in stating M2 potential-free is then revealed as necessary while the question was open, even though the stronger fact ultimately holds — the right order of operations, vindicated twice). The abelian sector's topological vacuum structure trivializes with it. And the finding's *ground* matters as much as the finding: per the procedure, branch F arrives with a reason — the global assembly, or the named constraint — and that reason is a new piece of substrate physics in its own right, likely with consequences beyond this question.

9. Branch P — If the Sectors Are Permitted

Prepared, not claimed; everything in this section is [Conditional on SA resolving positive].

Dirac from below. The condition $qg = 2\pi n$, imposed in 1931 as a consistency demand on a hypothetical object, becomes a theorem of the substrate: occupied sectors carry integer m, carriers carry integer q, and the pairing is integral identically (§5). The third and last conventional route to charge quantization — after compactness-by-assumption (absorbed by the charge paper) and anomaly constraints (positioned as complementary there) — is thereby absorbed as a corollary: every classical explanation of the lattice stands as a consequence of the one derived structure. That absorption is branch P's principal theoretical yield, and it is already proven; the branch merely activates it — a small improvement on the structure-group paper's forecast, which assigned the Dirac derivation to the permitted branch; the derivation in fact lands before the branch is chosen, and only its application to occupied sectors waits on P.

What the branch does not deliver. A monopole as a *particle* — mass, size, stability, production — is not derived by sector admissibility; $m \neq 0$ classifies configurations, and the physics of localized realizations is downstream work requiring the sector's dynamics. One characterization, however, is already forced by the Bounding Lemma and should govern that downstream work: under branch P the monopole is necessarily topological — it lives on a handle of the emergent geometry or around a core the configuration cannot extend across, and never as a local density of magnetic substance, which source-freeness forbids unconditionally. The monopole is not a thing in the field; it is a place the field cannot be. Under the SA-core channel in particular, the "particle" question becomes the physics of admissible non-extendable loci — what a core is, what sustains it, what it costs — which is a sharper and stranger question than the conventional one, and is named here as the branch's principal successor obligation. The abundance question — how many, made when — belongs to the cosmology arc, with one transfer warning: the conventional "monopole problem" (GUT-scale production overclosing the universe, historically part of inflation's motivation) is a problem of grand-unified phase transitions in conventional cosmology, and transfers to the programme's cosmology neither automatically nor obviously; the corpus's cosmological arc differs at its foundations, and the abundance computation must be done in its terms, not imported with conventional assumptions attached.

The empirical posture under P. Null searches then constrain abundance, not existence — fully consistent with P at low abundance, so branch P is not in tension with the experimental record. It is, however, *less* falsifiable than F in the near term, and the asymmetry should be owned: P's distinctive successes are theoretical (the Dirac derivation) unless and until a detection occurs, at which point P is spectacularly confirmed and F is dead. The corpus does not get to choose; the construction decides.

10. The Experimental Situation

Recorded briefly, as standing context rather than as evidence for either branch. A century of searches — induction experiments, accelerator searches, cosmic-ray and ice-detector programs, and indirect astrophysical bounds from the survival of galactic magnetic fields — has produced no confirmed detection of a magnetic monopole. The null record is consistent with branch F outright, and with branch P at abundances below current sensitivity; it therefore constrains without deciding, and nothing in this paper leans on it. What the record does establish is the stakes of branch F's prediction: a forbidden-sector finding converts every future null result from an unexplained absence into a corroboration, and converts any future detection from a discovery into a refutation — of this branch, of the construction reading that grounded it, and upstream into the conjunction the structure-group paper consolidated. The corpus would be committed, which is the point. [Standing context; no marker.]

11. Position in the Programme

Inherited exposure, stated once. Every result above is conditional on the structure-group paper's imports M1–M3 at their slot status — the discharge chain's pending verifications flow through to here — and on the U(1) arc's foundations at source. The exposure is the corpus's, shared, and not re-audited.

Assignment-neutrality. The inputs consumed are: composition and reversibility (catalogue-level, §3); M2's curvature-as-infinitesimal-holonomy (kind-level, §3); the taxonomy (grain-level, §§3–4); the charge lattice (catalogue-level at its source, §5); and the foreclosure arguments (§6), which quantify over principles, not paths. The sector label m is configuration-level and record-borne: it classifies realized configurations without ever consuming which path carries which holonomy — two rival assignments sharing the catalogue would induce identical sector structure on identical configurations. The chain therefore remains assignment-neutral: a failure of Transport-Completeness in the Assignment Paper leaves every result here untouched, extending the independence the charge paper proved and the structure-group paper preserved. [Proven, given the stated grain of the inputs.]

The programme diagram, extended at this paper's branch:

```
Dense + closed ⇒ admissible phase structure = 𝕋
Reversible (ℂ-realized), single-Fold ⇒ exactly U(1)
        │
        ├── U(1) ⇒ Born obligation discharged
        │
        ├── U(1) + GI ⇒ charge lattice (q ∈ ℤ)
        │        └── GI discharged [Proven, conditional on M1–M3 — structure-
group paper]
        │                 │
        │                 ├── closed-surface records ⇒ m ∈ ℤ         ← this
paper
        │                 │       ⇒ Dirac pairing q·m ∈ ℤ, identically
        │                 │
        │                 └── Sector Admissibility (SA)
        │                        [Open, partially read — local half decided
at source;
        │                         SA-top open (G1: homology), SA-core open
(C1: core exclusion)]
        │                        ├── F: no abelian-bundle monopoles
[prediction]
        │                        └── P: Dirac's condition from below
[theorem activated]
        │
        └── U(1) + assignment L ↦ h(L) D-determined up to gauge
                [Conditional — Assignment Paper, on Transport-Completeness of
D]
                ⇒ support supervenience ⇒ dyadic loading
```

What does not advance here. The unit of charge (fine-structure arc), the occupancy pattern of the electric lattice (multi-Fold arc), the assignment question (Assignment Paper), and the cosmological abundance question under branch P (cosmology arc) — all reserved exactly as divided at their sources.

12. What Would Refute or Decide This

The section divides into refuters of the forced results and deciders of the open node — different kinds, named separately.

Against the Cap Lemma and the quantization theorem (§3). Exhibit a realized closed surface whose total curvature record is *not* in $2\pi\mathbb{Z}$. Effect: since the theorem consumes only composition, reversibility, M2, and the compactness of the fiber, an off-lattice record strikes upstream — at the structure-group paper's reading or the U(1) arc's compactness — and is an inconsistency alarm of the same class as the audit's M3 fallback, not a local repair. Standing: the derivation is short and potential-free; the place to press is its inputs, which are the corpus's load-bearing arcs.

Against the Bounding Lemma (§3). Exhibit a realized bounding surface — $S = \partial V$, the configuration realized throughout V — with $m(S) \neq 0$. Effect: by the lemma's own proof this requires a nonvanishing record over some realized small closed surface — a cell boundary in V — striking B1, the programme's source-freeness result, at its source: a local magnetic source, contradicting the magnetism arc directly. The refuter is therefore another upstream alarm, and the lemma is exactly as secure as $\nabla \cdot B = 0$. Standing: the telescoping is the Cap Lemma's, one dimension up; the inputs are the place to press.

Against the no-cheap-answer lemma (§6). Produce a catalogue-level argument that decides SA — opening the sectors despite m being a witnessed invariant, or closing them despite triviality being a gluing property. Effect: §7's node collapses into a theorem and this paper's division of labor is redrawn. Standing: the foreclosures are each one category distinction; a successful challenge must dissolve the distinction itself, and should be welcomed if it can — a derived answer is better than a decided one.

Deciders of SA (§7). After the partial reading, the live deciders are two: C1 — the everywhere-definedness theorem, closing SA-core — and G1's homological return, closing or opening SA-top; one exhibited admissible $m \neq 0$ configuration on either channel decides P at any time. The local question is decided at source and is no longer a decider. These are the named obligations this paper hands forward, with the verification asymmetry as flagged.

Empirically. A confirmed magnetic monopole detection decides SA = P over the heads of the slots, refutes branch F if it has been claimed, and — if the measured Dirac pairing were ever *non*-integral, $q \cdot m \notin \mathbb{Z}$ — would strike the entire phase programme at once, since §5 makes the integrality an identity of the two derived lattices. The pairing's integrality is therefore this paper's contribution to the corpus's consolidated empirical lever: one more observable that cannot fail without everything failing together.

Inherited exposure. As §11: the structure-group paper's slots, and the U(1) arc's foundations, at their statuses.

13. What the Paper Establishes

Established (conditionally, with the conditions named):

- The Cap Lemma: boundary holonomy as the exponential of the cap's curvature record, $h(\gamma) = e^{i\Phi(S)}$ — the substrate's Stokes relation, derived potential-free by tiled composition with interior-edge cancellation under reversibility, at the taxonomy's exact grain (§3). [Proven, conditional on inherited composition, reversibility, and M2.]
- The Sector Quantization Theorem: $\Phi(S) \in 2\pi\mathbb{Z}$ on every realized closed 2-surface; the label $m(S) = \Phi(S)/2\pi$ splitting-independent, additive, deformation-invariant, orientation-odd — the second integer, forced by the same compactness that forced the first, and coinciding with the first Chern class [Imported-External correspondence] (§3). [Proven, same conditionality.]
- The Bounding Lemma: given source-freeness (B1 [Inherited]), the record over the boundary of any realized 3-region vanishes — cell-boundary records zero by B1, interior faces canceling pairwise — so nonzero sectors require non-bounding surfaces, through handles or cores; the earlier magnetism result completed rather than contested (closed versus exact), and the monopole characterized in advance as necessarily topological, never a local source (§3). [Proven, conditional on B1 and the §3 machinery.]
- The two-winding structure: electric charge as response winding (fiber-side), magnetic charge as record winding (configuration-side) — the taxonomy's two invariant-bearing objects each carrying an integer; conjugation flips both, $q \mapsto -q$ and $m \mapsto -m$, leaving the pairing invariant, in consistency with the canonical identification's surviving bit (§4). [Proven.]
- Dirac's condition, inverted: $q \cdot m \in \mathbb{Z}$ identically, the product of two independently forced integers — the monopole's quantization-enforcing role fully retired, the consistency bridge one-way: the integers guarantee Dirac, Dirac argues for nothing (§5). [Proven, conditional on the charge paper and §3.]
- The no-cheap-answer lemma: the closure argument cannot open the sectors (the boundary is witnessed by m); the economy argument cannot close them (the catalogue is the fiber, triviality is gluing) — the decision is construction-level, and any future catalogue-level claim of either branch must answer this section (§6). [Proven.]
- The decision node SA, refined by the Bounding Lemma into two exact channels — SA-top (nontrivial second homology) and SA-core (admissible non-extendable loci) — stated with slots, a five-step procedure, and the verification asymmetry flagged per channel (§7). [Open, partially read; the instrument and the reading are the deliverables.]
- The partial reading at source: isolated local magnetic charges excluded outright — the local half of the monopole question closed [Inherited — verified; citation to be typeset]; the Maxwell admissibility paper's explicit caveat (global potential assumed; nontrivial bundles named as the Dirac residence) recorded as an independent convergence on SA; both topological channels open with a mechanism-level bias toward closure recorded as bias, not yet a theorem and not a marker move; the structure-group paper's potential-free statement of M2 retroactively vindicated as necessary (§7).
- The named obligation C1 (Core Exclusion): rotational entropy flow everywhere defined, no admissible cores — handed to the magnetism arc, with the burden posture recorded: opening SA-core requires a constructed core, closing it requires C1; the burden narrowed by WNB to witnessed cores (§7). [Open.]

- The distinguishability refinement (WNB): a non-bounding region is admissible only if its non-bounding character is itself witnessed — an application of no-pre-individuation that removes bare holes, merges the channels at the grain of admissibility (hole and witness co-arise, one package), narrows C1's burden to co-realized core-and-winding packages, and re-grains G1 from abstract homology to witnessed non-bounding; the Bounding Lemma's contrapositive identified as the canonical witness, $m \neq 0$ certifying non-bounding, with the grain note that absence is never witnessed directly, only circumscribed (§7). [Proven, as an application of no-pre-individuation at source status; the channel merger Proven given WNB; the bar-raising recorded as bias, marker unchanged.]
- Both branches prepared: F with the no-abelian-bundle-monopole prediction at its exact grain, the GUT discriminator, and the structural consequences of triviality (§8); P with the Dirac derivation activated, the forced topological characterization of any monopole — a handle or a core, never magnetic substance — governing the downstream particle question, and the abundance question reserved to the cosmology arc with its transfer warning (§9). [Each Conditional on SA's resolution.]
- Assignment-neutrality of the entire chain, extended through this paper (§11). [Proven, given the stated grain of the inputs.]

Not established (open, out of scope, or pending):

- Sector Admissibility's remaining content — the two topological channels, awaiting C1 (the magnetism arc's everywhere-definedness theorem) and G1 (the geometry arc's homological reading); the local half is decided at source and no longer pending (§7);
- the monopole as a particle — mass, size, stability, production — under any resolution of SA; downstream of the sector dynamics;
- the cosmological abundance under branch P — the cosmology arc's, with the transfer warning of §9;
- the non-abelian analogue of the sector question — the multi-Fold arc's, reserved in §8's grain statement;
- the verification of B1 against the magnetism papers, in the surface-level form stated in §2 — a citation slot of this paper, handed forward alongside G1 and C1, G2 having been read into them; until fixed, the Bounding Lemma and the channel refinement of SA stand conditional on B1 as stated;
- all inherited exposure: the structure-group paper's M1–M3 at slot status, and the U(1) arc's foundations at source (§11).

The honest summary: the paper proves everything about monopole sectors that the corpus's inherited results can prove — the second integer exists and is exact, the Dirac condition is an identity, and the two tempting shortcuts to an answer both fail — and converts the one thing they cannot prove into a decision node with slots, a procedure, an asymmetry warning, and both findings pre-written. The question that entered the corpus as a century-old speculation leaves this paper as a determinate reading assignment.

14. Conclusion

The circle that forced electric charge onto its ladder was never going to stop at one integer. Over a closed surface — a balloon with no edge for the bookkeeping to escape through — the phase's real-valued records must answer to the single fiber point on any loop that splits the surface, and the slack they are permitted is exactly whole turns. The second integer is forced the way the first was: not by a new law, but by the absence of anywhere else for the remainder to go. Electric charge is how many times a carrier's dial turns per turn of the phase; magnetic charge is how many full turns the surveyor's ledger is off by when the survey closes on itself. Two windings, one circle.

What the circle forces, it forces completely — and what it does not force, this paper has declined to pretend it does. Dirac's condition, the old bridge from a hypothetical monopole to the observed lattice, is dismantled into an identity: two integers, multiplied, integer — nothing enforced, nothing owed. And the question of whether nature occupies the second ladder's upper rungs is proven to be undecidable from the catalogue, by its own principles fairly applied: the Conservation Lemma cannot open sectors whose boundary its own taxonomy witnesses; the economy of one circle cannot close configurations that all live on that one circle differently. The answer is in the construction, where this paper's procedure now points — one reading of how the Maxwell sector assembles its configurations, with a prediction waiting on one branch and a century-old condition's derivation waiting on the other.

Either way, the corpus is committed in advance, which is the posture the whole programme exists to earn. If the sectors are forbidden, the world contains no magnetic monopoles of the basic type, and every empty detector from here forward says so a little more firmly — while every grand-unified theory continues to expect otherwise. If the sectors are permitted, the most elegant consistency condition in twentieth-century physics becomes a theorem of finite distinguishability. And the question carries the programme's signature one level deeper than topology can follow: a hole, here, is not a piece of mathematics but an absence, and absence must be distinguishable to be physical — so the hole and the winding that witnesses it stand or fall as one package. The question is not whether the universe contains holes for fields to wind on. It is whether the substrate can realize a winding that certifies its own hole. The monopole spent a century as quantization's hypothetical enforcer. It ends this paper as something better: a sharp question, asked of a construction that is finally fine-grained enough to answer it.