The Weak-Doublet Hessian Audit Protocol in VERSF

A Finite Pass/Fail Audit for P_W H_cl P_W — the C_3 CKM Curvature, the Weak-Commitment PMNS Kernel, and the Leakage Support Trace

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General-Reader Summary

Matter comes in three near-identical copies, called generations. They differ only in mass — the second is heavier than the first, the third heavier still. Particles can also slip from one generation into another. The surprising fact is how *unevenly* they do it: quarks barely change generation at all, while their lighter cousins the neutrinos change generation almost freely.

The Standard Model — physics' best working description of matter — handles this by writing the two mixing patterns into two tables of numbers, one for quarks and one for neutrinos. The numbers are measured in experiments and copied into the theory by hand. The model uses them with great precision, but it never explains why they take the values they do, or why quarks and neutrinos behave so differently. They sit there as brute facts.

The earlier papers in this programme made a sharper claim: those two tables are not two separate accidents. They are two views of a *single* underlying structure that the deeper theory already contains. The job, then, is not to invent a new table to fit the data. It is to look inside the structure that is already there and check whether the right pattern is built into it. A genuine explanation only counts when you actually carry out that check and watch the known numbers fall out — not when you arrange for them to.

This paper is the checklist for that check. It does not claim the full calculation has been done. Instead it pins down, in advance and exactly, what the calculation must produce if the idea is right: which specific numbers, which plus-or-minus signs, which patterns. That discipline is the value of the paper. If the calculation later returns those things, there is a real derivation. If it returns something else, you know precisely which line failed and why — the idea breaks in one identifiable place rather than failing in a vague, unrescuable way.

Two mechanisms do the heavy lifting, and both are clean. On the quark side, there is one small built-in twist between the second and third generations. On its own it does almost nothing. But combined with the one large quark mixing that is already well understood — the Cabibbo mixing between the first two generations — it automatically produces the tiny leftover mixing between the first and third generations that was otherwise unexplained. It is like turning two dials in sequence: each does its own thing, but the combination nudges a third dial you never touched. Crucially, this happens without spoiling the large mixing that the theory already gets right.

On the neutrino side, the trick is different but just as tidy. Neutrinos are only weakly "anchored" by the theory, and that weak anchoring is what makes them so light. The key point is that the same weakness which sets their tiny mass has *no* say over how they mix. Mass and mixing are governed by separate features of the structure. So neutrinos can be almost massless and still mix strongly — there is no contradiction between the two, which had always looked puzzling.

The honest bottom line is modest and precise. What used to be an open-ended mystery — "why these flavour numbers?" — has been turned into a finite, pass-or-fail audit. Either the deeper structure hands back the listed patterns, signs, and ratios, or it does not. And if it does not, the failure is local: one clearly marked step, not the whole edifice.

Contents

Abstract

The weak-doublet projection theorem reduced the residual Standard Model flavour problem to the projected closure Hamiltonian

H_W = P_W H_cl P_W ,

where **H_cl = G|_su(8)** is the inherited closure Hamiltonian and **P_W = P_Y P_R P_C** is the generation, weak-role, and mass/readout projection. Prior work fixed the admissible role decomposition

$$\Omega\_W = i\,\varepsilon\_W\, H\_W = \Omega\_even \otimes I + \Omega\_odd \otimes \tau_3 + \Omega\_mix$$

and isolated the finite list of CKM and PMNS outputs the projected block must supply.

This paper formulates the first explicit weak-doublet Hessian audit protocol, with four goals.

1. Compute the role-odd quark block and test whether it returns the inherited CKM generator **Ω_q**.
2. Compute the role-even quark block and test whether its first Cabibbo-protected common curvature is the 2↔3 channel.
3. Compute the weak-commitment neutrino block and test whether it has the scale–shape form $\mathbf{T\_\nu = D_0\, I + \varepsilon\, M\_\nu}$.
4. Compute the weak-commitment leakage projector and test the support trace $\mathbf{Tr\_support(\Pi\_leak) = (K - 2) \times 4 = 5 \times 4 = 20}$, which would make $\boldsymbol{\beta = \sqrt{3} / 20}$ a projected-Hamiltonian output rather than a benchmark.

The exact algebraic core is a single commutator identity. If the first role-even shared curvature has the form $\Omega_0(z) = z\, E_{23} - z\, E_{32}^*$, then

$$[\Omega_0(z), \Omega\_q]_{13} = -\, az\ .$$

A 2↔3 common-mode curvature is thereby converted into a 1↔3 triangle correction by the Cabibbo doorway. The convention-invariant curvature $\boldsymbol{\zeta = \lambda z}$ gives $\boldsymbol{\Delta_{13} = \tfrac{1}{2}\, a\zeta}$ in every split convention, where Δ_{13} denotes the commutator-generated *increment* on the inherited (1,3) entry. The resulting CKM curvature target is

$$\zeta\_C3 = (b / \sqrt{3}) \cdot e^{(5\pi i/6)}\ .$$

Its amplitude follows only if the projected role-even curvature is a democratic, Killing-orthonormal C_3 branch component *and* its total norm equals the inherited role-odd amplitude b (two independent premises, QF-3 and QF-3b); its phase follows if the projected Hamiltonian implements the minimal Hermitian half-transport branch. The neutrino targets are **M_ν**, $\rho\_\odot = \sqrt{3/2}$, $B/A = 6/5$, $\delta = 0 + O(\beta^2)$, $\psi = 3\pi/4$, $\beta = \sqrt{3}/20$, the octant sign σ_W, and the phase-root signs $\sigma\_\varphi q$, $\sigma\_\varphi \nu$. The paper is therefore not a completed microscopic evaluation of **H_cl**, but a finite derive-or-reject audit for that evaluation.

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0. Predictive-Content Ledger

Four grades are used throughout, and nothing is permitted to hide between them:

- **[Exact]** — true algebra once the operands are specified.
- **[Audit condition]** — a structural inequality or commutation the projection must satisfy for the sectoral reading to hold.
- **[Conditional target]** — a number that follows *if* a stated projection premise holds.

- **[Owed output]** — a block, sign, or trace that only an actual evaluation of P_W H_cl P_W can supply.

Target	Required output	Grade
Weak-doublet object	$H\_W = P\_W\ H\_cl\ P\_W$	inherited object
Frame generator	$\Omega\_W = i\ \varepsilon\_W\ H\_W$	[Exact] once H_W defined
Role decomposition	$\Omega\_even \otimes I + \Omega\_odd \otimes \tau_3 + \Omega\_mix$	[Exact] Pauli decomposition
Role-changing gap	$\lVert\Omega\_mix\rVert \ll \lVert\Omega\_even\rVert + \lVert\Omega\_odd\rVert$	[Audit condition]
CKM role-odd split	$\Omega\_odd \to \Omega\_q$	[Owed output]
CKM common curvature	$\Omega\_even\^(23) \neq 0$, leading	[Owed output]
CKM commutator	$[\Omega_0, \Omega\_q]_{13} = -\ az$	[Exact]
Convention-invariant curvature	$\zeta = \lambda z$	convention audit
CKM triangle increment	$\Delta_{13} = ½\ a\zeta$	[Exact], leading BCH
BCH tail on (1,3)	$\lVert\text{next-order}\rVert \leq \varepsilon\_BCH$	[Owed bound]
CKM resultant (1,3)	$R_{13} = (\Omega\_q)_{13} + ½a\zeta$; $\lvert R_{13}\rvert \approx 0.00489$, arg ≈ 135.6°	decisive object; tested under deferred map
C_3 democracy	equal Killing-orthonormal branches	[Conditional target on QF-3]
C_3 amplitude	$\lvert\zeta\rvert = b / \sqrt{3}$ (needs role-even total = b)	[Conditional target on QF-3 ∧ QF-3b]
CKM phase branch	$\arg \zeta = +5\pi/6$ (least-transport root; orientation fixed by $(\Omega\_q)_{12} = +a$)	[Conditional target on QF-4]
CKM phase sign	$\sigma\_\varphi q$	[Owed output]
Neutrino weak limit	$T\_\nu = D_0\ I + \varepsilon\ M\_\nu$	[Owed output]
PMNS frame theorem	eigenframe(T_ν) = eigenframe(M_ν)	[Exact] if weak-limit form holds
Solar ratio	$\rho\_\odot = \sqrt{(3/2)}$	[Conditional target]
Pair ratio	$B/A = 6/5$	[Conditional target]
Diagonal μ–τ split	$\delta = 0 + O(\beta^2)$	[Conditional target]
Weak phase	$\psi = 3\pi/4$	[Conditional target]
Leakage support trace	$Tr\_support(\Pi\_leak) = Tr(\Pi\_R)\cdot Tr(\Pi\_Q) = 5\cdot 4 = 20$ (projector trace)	[Owed output]
Reactor leakage	$\beta = \sqrt{3} / 20$	derived only if support trace = 20
Octant sign	$\sigma\_W = \mathrm{sgn}(\lVert P\_e\ H\_W\ P\_\tau\rVert^2 - \lVert P\_e\ H\_W\ P\_\mu\rVert^2)$	[Owed output]

The decisive frontier objects are **Ω_0^(23)**, **z_C3**, **M_ν**, **σ_W**, **σ_φq**, **σ_φν**, **Π_leak**, **Tr_support(Π_leak)**, and **β = √3/20**.

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1. Named Premises

The audit is organised around explicitly named, load-bearing premises. Each falsification condition in §13 is keyed to one of them.

Weak-doublet / leakage premises (WL-).

- **WL-1 (projected-block hypothesis).** The weak-flavour operator is a projected block of the inherited closure Hessian, H_W = P_W H_cl P_W, with no observed flavour matrix used as construction data.
- **WL-2 (role gap).** ‖Ω_mix‖ ≪ ‖Ω_even‖ + ‖Ω_odd‖, so CKM and PMNS read as clean sectoral frames of one block.
- **WL-3 (charge-blind readout).** [H_W^readout, χ] = 0; the readout may distinguish closure depth, stiffness, and support, but not electric charge or gauge representation.
- **WL-4 (leakage factorisation).** Π_leak ≃ Π_R ⊗ Π_Q with dim R = K − 2 = 5 and dim Q = 4.

Quark-frame premises (QF-).

- **QF-1 (role-odd return).** The role-odd block returns the inherited generator Ω_q.
- **QF-2 (Cabibbo-protected channel).** The leading role-even common curvature lies in the 2↔3 channel. *(Not posited as a free-standing channel choice — relocated onto the audit-admissibility rules (i)–(iii) by the Cabibbo-protection lemma, §6.)*
- **QF-3 (C_3 orbit).** The projected role-even curvature is a democratic component of a Killing-orthonormal C_3 orbit.
- **QF-3b (cross-sector norm identification).** The role-even common-mode *total* norm equals the role-odd amplitude b — the (2,3) entry of Ω_q. This is the premise that ties the role-even magnitude to a role-odd number; it is logically independent of QF-3, and |ζ| = b/√3 requires *both*.
- **QF-4 (minimal Hermitian half-transport).** The projected Hamiltonian selects the minimal Hermitian phase root, fixing arg ζ = 5π/6 and the sign σ_φq.

Lepton-frame premises (LF-).

- **LF-1 (weak-commitment kernel).** The neutrino readout has the form T_ν = D_0 I + ε M_ν.
- **LF-2 (support ratios).** The solar and pair support ratios continue as ρ_⊙ = √(3/2) and B/A = 6/5.
- **LF-3 (weak-neutral diagonal).** First-order μ–τ diagonal splitting is suppressed: δ = 0 + $O(\beta^2)$.

- **LF-4 (weak phase branch).** The projected phase branch returns $\psi = 3\pi/4$ with sign $\sigma\_\varphi\nu$.

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2. The Problem After the Projection Theorem

The weak-doublet projection theorem fixes the correct object,

$H\_W = P\_W\ H\_cl\ P\_W$.

This is stronger than writing down a flavour Hamiltonian by hand. It asserts that the weak-flavour operator must be a projected block of the inherited closure Hessian. The task is not "write a Hamiltonian that gives CKM and PMNS," but "project the already-defined closure Hamiltonian and test what it forces."

The weak-doublet projection theorem also constrains the admissible shape of that block: the CKM common curvature appears in the role-even projection, the CKM leading split in the role-odd projection, and the PMNS weak-commitment kernel in the weakly anchored neutrino readout. It also isolates the support-trace computation behind $\beta = \sqrt{3}/20$.

The remaining problem is therefore not conceptual but computational. The audit must compute, or at least bound,

$P\_even\ P\_W\ H\_cl\ P\_W\ P\_even$, $P\_odd\ P\_W\ H\_cl\ P\_W\ P\_odd$, $P\_e\ P\_W\ H\_cl\ P\_W\ P\_\mu$, $P\_e\ P\_W\ H\_cl\ P\_W\ P\_\tau$, $Tr\_support(\Pi\_leak)$,

and return the listed entries, signs, and support traces **without** importing the observed CKM or PMNS matrices as construction data (WL-1).

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3. The Projected Weak-Doublet Object

Let

$H\_cl = G|\_su(8)$,

where G is the Hessian of the substrate free-energy / admissibility functional at the admissible manifold. Let

$P\_W = P\_Y\ P\_R\ P\_C$,

with P_C the generation-completion projector, P_R the weak-role projector, and P_Y the mass/readout projector. Then

$\mathbf{H\_W = P\_W\, H\_cl\, P\_W^\dagger}$, with $\mathbf{P\_W^\dagger = P\_C\, P\_R\, P\_Y}$,

and the frame generator is

$$\mathbf{\Omega\_W = i\, \varepsilon\_W\, H\_W} .$$

The dagger makes Hermiticity manifest: with H_cl Hermitian, $H\_W = P\_W\, H\_cl\, P\_W^\dagger$ is Hermitian by construction for any ordering of the factors, with no need to carry the left/right support in the reader's head. On commuting support $P\_W^\dagger = P\_W$ and the expression collapses to the symmetric shorthand $P\_W\, H\_cl\, P\_W$ used elsewhere.

Ordered compression vs. orthogonal projector. $P\_W = P\_Y\, P\_R\, P\_C$ denotes the *ordered* weak-doublet compression, and $P\_W^\dagger = P\_C\, P\_R\, P\_Y$ its reverse-ordered adjoint. Where the three factors commute on the admissible support — i.e. $[P\_Y, P\_R] = [P\_R, P\_C] = [P\_Y, P\_C] = 0$ there — P_W is itself an orthogonal projector ($P\_W^2 = P\_W = P\_W^\dagger$), $P\_W^\dagger = P\_W$, and the block statements below are projector statements. Where they do not commute, every block statement is read as an ordered compression with the stated left/right support, $H\_W = P\_W\, H\_cl\, P\_W^\dagger = P\_Y\, P\_R\, P\_C \cdot H\_cl \cdot P\_C\, P\_R\, P\_Y$, and "projected block" means the compressed block in that ordering. The audit conclusions are framed to hold in either reading; the commutation of the three factors on the admissible support is itself an **[Audit condition]**.

On the two-dimensional weak-role space the Pauli decomposition is exact:

$$\mathbf{\Omega\_W = \Omega_0 \otimes I + \Omega_1 \otimes \tau_1 + \Omega_2 \otimes \tau_2 + \Omega_3 \otimes \tau_3} .$$

Define

$$\Omega\_even = \Omega_0 ,\ \Omega\_odd = \Omega_3 ,\ \Omega\_mix = \Omega_1 \otimes \tau_1 + \Omega_2 \otimes \tau_2 ,$$

equivalently, by partial trace over weak role,

$$\Omega\_even = \tfrac{1}{2} Tr\_role(\Omega\_W) ,\ \Omega\_odd = \tfrac{1}{2} Tr\_role(\Omega\_W\, \tau_3) ,\ \Omega\_i = \tfrac{1}{2} Tr\_role(\Omega\_W\, \tau\_i) ,\ i = 1, 2 .$$

This makes the audit unambiguous: "the role-even block contains a 2↔3 curvature" means precisely that the partial-trace block Ω_even has that leading component. **[Exact]**

Anti-Hermiticity inheritance. H_W is Hermitian and ε_W is a real scalar, so $\Omega\_W = i\, \varepsilon\_W\, H\_W$ is anti-Hermitian, $\Omega\_W^\dagger = -\,\Omega\_W$. The role basis $\{I, \tau_1, \tau_2, \tau_3\}$ is Hermitian and linearly independent, so each component carries the constraint separately: from $(\Omega\_k \otimes \sigma\_k)^\dagger = \Omega\_k^\dagger \otimes \sigma\_k$ and $\Omega\_W^\dagger = -\,\Omega\_W$ one obtains

$$\Omega\_k^\dagger = -\,\Omega\_k \text{ for } k \in \{0, 1, 2, 3\} .$$

Hence **Ω_even, Ω_odd, and each of Ω_1, Ω_2 in Ω_mix are individually anti-Hermitian**. This is a genuine constraint on the audit, not a convention: the inherited generator Ω_q and the role-even curvature $\Omega_0(z) = z\,E_{23} - z^*\,E_{32}$ used below are both anti-Hermitian, consistent with this inheritance. Any candidate projected block that returns a non-anti-Hermitian Ω_even or Ω_odd is rejected before its entries are read. **[Exact]**

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4. Audit Rules

Rule 1 — No observed flavour matrix as construction input. CKM and PMNS data may enter only *after* the projected block has been computed (WL-1).

Rule 2 — Charge-blind readout. The mass/readout projection may distinguish closure depth, stiffness, and support, but must not alter electric charge or gauge representation (WL-3): $[H\_W^{readout}, \chi] = 0$.

Rule 3 — Role-changing gap. A clean weak-flavour frame requires $\|\Omega\_mix\| \ll \|\Omega\_even\| + \|\Omega\_odd\|$ (WL-2). If this fails, CKM and PMNS cannot be read as clean sectoral frames of one block.

Rule 4 — Separate exact algebra from projected output. The identity $[\Omega_0(z), \Omega\_q]_{13} = -\,az$ is **[Exact]** once $\Omega_0(z)$ and Ω_q are specified. The *existence* of that $\Omega_0(z)$ inside P_W H_cl P_W is **[Owed output]**.

Rule 5 — Support traces are computed, not guessed. $\beta = \sqrt{3}/20$ is not exact algebra; it follows only if $Tr\_support(\Pi\_leak) = 20$ (WL-4).

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5. Convention Audit: the Factor-of-Two Issue

Two split conventions are in circulation. In the full-Ω_0 convention ($\lambda = 1$) the triangle increment reads $\Delta_{13} = \frac{1}{2}\,az$; in the explicit half-split convention ($\lambda = \frac{1}{2}$) it reads $\Delta_{13} = \frac{1}{4}\,az$. Both are internally correct, and the apparent factor of two is an artefact of how the relative generator is normalised. The physical object must therefore be expressed in a convention-invariant variable before any numerical comparison.

Introduce a split parameter λ and write

$$V\_CKM = \exp(-\lambda\,\Omega_0 + \tfrac{1}{2}\,\Omega\_q) \cdot \exp(+\lambda\,\Omega_0 + \tfrac{1}{2}\,\Omega\_q)\ .$$

With $X = -\lambda\Omega_0 + \frac{1}{2}\Omega\_q$ and $Y = +\lambda\Omega_0 + \frac{1}{2}\Omega\_q$, the Baker–Campbell–Hausdorff expansion gives $X + Y = \Omega\_q$ and $\frac{1}{2}[X, Y] = -(\lambda/2)[\Omega_0, \Omega\_q]$, so

$$\log V\_CKM = \Omega\_q - (\lambda/2)[\Omega_0, \Omega\_q] + \cdots\ .$$

Define the triangle increment as the commutator-generated part of the (1,3) entry,

$$\Delta_{13} \equiv (\log V\_CKM)_{13} - (\Omega\_q)_{13} = -(\lambda/2)[\Omega_0, \Omega\_q]_{13} + O(\text{curvature}^2) .$$

Since $[\Omega_0(z), \Omega\_q]_{13} = -\,az$,

$$\Delta_{13} = (\lambda/2)\, a\, z .$$

Define the invariant curvature $\boldsymbol{\zeta = \lambda z}$. Then, in every split convention,

$$\Delta_{13} = \tfrac{1}{2}\, a\, \zeta .$$

The full-Ω_0 convention is $\lambda = 1$, so $\zeta = z$; the half-split convention is $\lambda = ½$, so $\zeta = z/2$, i.e. $z = 2\zeta$. Physical targets are stated for ζ, never for the convention-dependent z. *(Numerical check across $\lambda \in \{1, ½, ¼\}$, with the bare entry $(\Omega\_q)_{13}$ subtracted, confirms $\Delta_{13} = ½\, a\zeta$ to leading order, the residual being the expected $O(\text{curvature}^2)$ BCH tail.)*

Hence the CKM curvature target is

$$\zeta\_C3 = (b / \sqrt{3}) \cdot e\text{^}(5\pi i/6) .$$

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6. Quark Sector Audit: Role-Odd Split and Role-Even Curvature

The committed quark regime has $\gamma\_u \simeq \gamma\_d \simeq 1$: both weak-doublet partners are closure-committed, so the leading weak-doublet split is small. The role-odd block should return the inherited CKM generator (QF-1),

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       ⎡  0            a          c·e^(iφ) ⎤
Ω_q =  ⎢ −a            0          b        ⎥
       ⎣ −c·e^(−iφ)  −b           0        ⎦
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with

$a = 9/40$, $b = 81/2000$, $c = 243/100000$, $\varphi = 2\pi/3$.

(Ω_q is anti-Hermitian, $\Omega\_q^\dagger = -\Omega\_q$, for real a, b, c — verified.)

The role-even block should return a first common-mode curvature in the 2↔3 channel. This is not a preference selected for giving the desired repair: under the audit admissibility rules, 2↔3 is the only surviving leading channel.

Cabibbo-protection lemma. Restrict to a single leading common-mode channel j↔k drawn from {1↔2, 1↔3, 2↔3}, and impose three constraints: (i) the 1↔2 entry is already fixed by the inherited Cabibbo split, so the leading common-mode curvature may not act in the 1↔2 channel; (ii) direct 1↔3 insertion is forbidden, since it would write the target entry by hand rather than derive it; (iii) the required 1↔3 correction must arise by first nontrivial commutator transport against the inherited generator. Then the only admissible leading common-mode channel is **2↔3**.

Proof. Constraint (i) excludes 1↔2 and (ii) excludes 1↔3, leaving 2↔3 as the only surviving pure channel. That it satisfies (iii) is the exact identity below: $[\Omega_0\text{\^{}}(23), \Omega\_q]_{13} = -az \neq 0$ transports a 2↔3 common-mode curvature into the 1↔3 entry through the Cabibbo doorway $(\Omega\_q)_{12}$. The remaining excluded channels fail (iii) trivially — 1↔2 would perturb the fixed entry, and 1↔3 would bypass transport entirely. ■

QF-2 is therefore the *unique admissible channel under the stated audit exclusions* (i)–(iii), not a free-standing channel choice. The lemma does not discharge the proof debt — it relocates QF-2's content onto the more primitive rules (i) and (ii), which are themselves audit-admissibility choices (do not perturb the fixed Cabibbo entry; admit no by-hand 1↔3 insertion). That relocation onto more primitive rules is the progress; the substrate still owes the output $\Omega\_\text{even}\text{\^{}}(23)$. The role-even curvature is

$\Omega_0(z) = z\, E_{23} - z\, E_{32}$.*

The exact audit identity. With $\Omega_0(z)$ supported only on the (2,3)/(3,2) entries, row 1 of Ω_0 vanishes, so $(\Omega_0\, \Omega\_q)_{13} = 0$. The reverse product picks up exactly one term,

$(\Omega\_q\, \Omega_0)_{13} = (\Omega\_q)_{12} \cdot (\Omega_0)_{23} = a \cdot z$,

so

$[\Omega_0(z), \Omega\_q]_{13} = (\Omega_0\, \Omega\_q - \Omega\_q\, \Omega_0)_{13} = 0 - a\, z = -\, a\, z$. [Exact]

In invariant notation, $\Delta_{13} = ½\, a\zeta$. The role-even quark audit therefore has three outputs:

$\Omega\_\text{even}\text{\^{}}(23) \neq 0$ (leading) , $|\zeta| = b / \sqrt{3}$, $\arg \zeta = 5\pi/6$.

If the role-even block gives a different leading channel, the CKM curvature mechanism is not derived (falsifies QF-2).

The increment is not the decisive number — the resultant is. The phrase "produces the missing triangle residue" is a statement about the *increment* Δ_{13}, not about the observable. The (1,3) generator entry that the eventual comparison must match is the **resultant**: the complex sum of the inherited bare entry and the increment,

$R_{13} = (\Omega\_q)_{13} + \Delta_{13} = c \cdot e\text{\^{}}(i\varphi) + ½\, a\, \zeta$.

These do not add in magnitude. The bare entry sits at arg 120° (= φ) and the increment at arg 150° (= arg ζ) — a 30° separation — so they combine as complex numbers:

$|R_{13}| \approx 0.00489$, arg $R_{13} \approx 135.6°$.

This resultant, *after the generator → matrix-element map that this paper defers*, is what the audit ultimately tests against |V_ub| and the CP phase. For scale only: |V_ub| ≈ 0.0037, so the **resultant** generator entry is ≈ 1.32× the observable (the bare entry alone is 0.00243, ≈ 0.66×) — the right decade, the discrepancy unsurprising because exponentiation and any accompanying rotations in that deferred map have not been applied, and **no discrepancy is claimed here, since the map is exactly what is owed.** The point is to name the decisive object explicitly: the pass/fail comparison is $|R_{13}|$ and arg R_{13} against |V_ub| and the CP phase under the deferred map, **not** the increment Δ_{13} in isolation.

BCH tail. $\Delta_{13} = \frac{1}{2} a\zeta$ is **[Exact, leading BCH]**; the next BCH term scales like a·O(curvature²) at this entry and is not negligible for a precision comparison — at several-percent level it can shift $|R_{13}|$ and arg R_{13} enough to matter against |V_ub|. For any downstream precision claim the tail must be *bounded*, not merely named: the audit owes an explicit ‖next-order‖ ≤ ε_BCH estimate on the (1,3) entry, carried alongside the deferred generator → matrix-element map. **[Owed bound]**

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7. C_3 Amplitude Audit

The amplitude target is **$|\zeta| = b / \sqrt{3}$**. It rests on *two* independent premises, and conflating them is the hazard this section guards against. QF-3 supplies the C_3 democracy — three equal-norm, Killing-orthonormal branches. QF-3b supplies the cross-sector identification — that the role-even common-mode *total* norm equals the role-odd amplitude b. Only the conjunction gives the target. Let the branch components be Z_{12}, Z_{23}, Z_{31}, with

$\langle Z\_i, Z\_j\rangle\_K = 0 \ (i \neq j)$, $\|Z\_i\|\_K = |\zeta|$ (QF-3) .

If the role-even total norm equals b (QF-3b), then

$\|Z_{12} + Z_{23} + Z_{31}\|^2\_K = 3\,|\zeta|^2 = b^2 \Rightarrow$ **$|\zeta| = b / \sqrt{3}$.**

The algebra ($3|\zeta|^2 = b^2$) is elementary and is **not** where the weight sits. The weight sits in QF-3b: the equality "role-even total = role-odd b" is a genuine cross-sector identification of a role-even magnitude with a role-odd entry, and it can fail even with QF-3 fully intact. A projected block that returns a perfectly C_3-democratic, Killing-orthonormal orbit whose total norm is some $b' \neq b$ yields $|\zeta| = b'/\sqrt{3}$ and misses the target — with QF-3 untouched. The two premises must therefore be tested separately (see F6 and F6b).

Provenance of b. The quantity b = 81/2000 is not introduced here as an experimental CKM fit. It is the inherited role-odd 2↔3 closure amplitude carried over from the prior quark-frame construction — the same b that sits in the (2,3) entry of Ω_q in §6 — and it enters the audit upstream of any flavour data, consistent with Rule 1 (WL-1). The present section does not re-derive b; it tests whether the role-even common-mode branch inherits a *democratic* C_3 norm $b/\sqrt{3}$ from that same upstream amplitude. The number being matched is therefore fixed before the audit begins; what is at stake here is only whether the role-even branch shares it democratically.

The proof debt is therefore twofold. **For QF-3** (democracy), the explicit test is

$$[P\_C\ H\_cl\ P\_C, R_3] = 0$$

on the pre-readout role-even block, together with survival of the selected 2↔3 common-mode component through the mass/readout projection, and equal branch weights $w_{12} = w_{23} = w_{31}$. **For QF-3b** (norm identification), the separate test is

$$\|\Omega\_even\text{^}common\|\_K = b ,$$

i.e. the role-even common-mode total norm returned by the projection must equal the role-odd amplitude b, not merely some democratic b′. **[Conditional target on QF-3 ∧ QF-3b]**

7a. Bridge: Pre-Readout Democracy vs. Post-Readout 2↔3 Selection

A critic will immediately press the apparent tension between this section and the Cabibbo-protection lemma of §6. If the role-even orbit is C_3-democratic, with equal 1↔2, 2↔3, and 3↔1 branch weights, why is the 2↔3 branch singled out as the *leading* channel? Why are the other two branches not equally present in the CKM correction?

The resolution is that the two statements live at different stages of the compression, and the audit must respect that ordering.

- **C_3 democracy is a pre-readout substrate statement.** It is a property of the role-even block *before* the weak-doublet / mass-readout compression P_W acts — the statement $[P\_C\ H\_cl\ P\_C, R_3] = 0$ with equal branch weights $w_{12} = w_{23} = w_{31}$. At this stage no channel is preferred; the three branches carry equal Killing norm and the total is b.
- **The Cabibbo-protected 2↔3 channel is the post-readout admissible branch.** It is the branch that *survives* the ordered weak-doublet readout under the §6 admissibility exclusions. The 1↔2 branch is suppressed in the correction because rule (i) forbids disturbing the already-fixed Cabibbo entry; the 1↔3 branch is suppressed because rule (ii) forbids by-hand 1↔3 insertion; only the 2↔3 branch reaches the (1,3) correction, and it does so *indirectly*, by commutator transport through the Cabibbo doorway rather than by sitting in the (1,3) slot itself.

There is therefore no contradiction: democracy describes how the norm b is *distributed across branches upstream*, while the 2↔3 selection describes which branch is *admissible as the leading correction downstream*. The other two branches are not absent from the substrate — they are present and equal pre-readout — they are merely inadmissible as the leading post-readout correction.

This makes the audit's burden explicit and two-staged, and the two stages must be tested separately:

1. **Pre-readout democracy.** The role-even block before compression is C_3-democratic and Killing-orthonormal, total norm b: $[P\_C\ H\_cl\ P\_C, R_3] = 0$, $w_{12} = w_{23} = w_{31}$, $\|\cdot\|\_K = b$ (QF-3 ∧ QF-3b).
2. **Readout selection without norm loss.** The ordered weak-doublet readout P_W selects the 2↔3 component as the leading admissible branch *without destroying the inherited C_3 norm* — i.e. the surviving branch still carries $\|Z_{23}\|\_K = b/\sqrt{3}$, so that the democratic distribution upstream is exactly what feeds ζ downstream.

If stage 2 fails — if the readout selects 2↔3 but in doing so rescales or contaminates the inherited norm — then $|\zeta| = b/\sqrt{3}$ fails even though both pre-readout premises hold. This is the load-bearing junction between QF-2 (channel) and QF-3/QF-3b (amplitude), and it is added to the falsification table as F6c.

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8. CKM Phase-Branch Audit

The target phase is **$\arg \zeta = 5\pi/6$** (QF-4). The selection rule is minimal Hermitian half-transport. Let the Hamiltonian-side phase be h with $h^2 \propto O\_q$, where $O\_q = e^{(2\pi i/3)}$. The square roots are

$h/|h| = e^{(i\pi/3)}$ (60°) and $h/|h| = e^{(i4\pi/3)}$ (240° ≡ −120°) .

Precise selection criterion. "Minimal Hermitian lift" is not a vague appeal to simplicity. It is the explicit rule: *of the two Hermitian-side square roots, select the one of least transport angle from the identity* — equivalently, the root with the smaller principal argument magnitude, $|\arg| \leq \pi$. Here that is $e^{(i\pi/3)}$ (transport 60°) over $e^{(i4\pi/3)} \equiv e^{(-i2\pi/3)}$ (transport 120°). Because the entire CKM CP phase and the sign σ_φq ride on this single choice, the criterion is stated as a named rule, not left to "minimal": **the projection must return the least-transport Hermitian root, and σ_φq records that it did.** Then, since the frame curvature is the i-rotation of the Hermitian side,

$\arg \zeta = \pi/2 + \pi/3 = 5\pi/6$.

On the second grammar. The complement-half-transport form $h\_q^2 = C \cdot O\_q$ with $C = e^{(i\pi)}$, $O\_q = e^{(2\pi i/3)}$ gives $h\_q^2 = e^{(5\pi i/3)}$ and root

$h\_q = e^{(5\pi i/6)}$,

but this is *the same operation rewritten, not a second derivation* — and must not be read as corroboration. The constant $C = e^{(i\pi)}$ is precisely the i-rotation of the first route, relocated across the square root: $e^{(i\pi)}$ halves to $e^{(i\pi/2)}$ under the root, landing the explicit π/2 rotation inside the radical instead of after it. Both grammars therefore execute the identical least-transport selection and reach the identical branch; the agreement is algebraic identity, carrying no independent evidential weight.

Orientation convention. The least-transport rule selects between the two Hermitian roots, but the *labelling* of which root is "least transport" depends on the orientation of the anti-Hermitian generator — and that orientation is set by the sign of ε_W in $\Omega_W = i\,\varepsilon_W H_W$. A reversed ε_W (or a reversed generator orientation) would conjugate the whole construction and flip $5\pi/6$ to its antipode. To make $\sigma_{\varphi q}$ a genuine projected output rather than a hidden sign convention, the orientation is fixed *externally*, by the inherited Cabibbo entry:

Positive generator orientation is fixed by $(\Omega_q)_{12} = +a$ with $a = 9/40 > 0$.

This is not a free choice at the phase stage — it is the same orientation already carried by the role-odd block in §6, where $(\Omega_q)_{12} = +a$ is part of the inherited generator. With that orientation fixed upstream, the least-transport Hermitian root gives **$\arg \zeta = +5\pi/6$**, and $\sigma_{\varphi q}$ records that the projection returned this branch rather than the antipodal $-\pi/6$ conjugate branch. The sign is therefore tied to an inherited, data-quarantined input (the sign of the Cabibbo entry), not posited here.

This branch choice is not cosmetic. The projection must return the phase-branch sign **$\sigma_{\varphi q}$**, selecting $+5\pi/6$ over the antipodal branch — a decisive output, not a convention. **[Conditional target on QF-4, orientation fixed by $(\Omega_q)_{12} = +a$; sign $\sigma_{\varphi q}$ is Owed output]**

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9. Neutrino Sector Audit: Weak-Commitment Kernel

The lepton doublet is asymmetric: $\gamma_e \simeq 1$, $\gamma_\nu = \varepsilon \ll 1$. The charged lepton is closure-norm anchored; the neutrino is weakly committed. Therefore the neutrino readout should have the weak-commitment form (LF-1)

$T_\nu = D_0\, I + \varepsilon\, M_\nu$.

PMNS frame theorem. For every $\varepsilon > 0$, T_ν and M_ν share eigenvectors: if $M_\nu v = m v$ then $T_\nu v = (D_0 + \varepsilon m) v$. The mass scale is governed by ε; the mixing frame is governed by the shape of M_ν. Small neutrino masses therefore do **not** force small PMNS mixing. **[Exact, given LF-1]**

The target weak-commitment kernel is

$$M_\nu = \begin{bmatrix} r_e & A & A(1 + \beta \cdot e^{-i\psi}) \\ A & r_s + \delta & B \\ A(1 + \beta \cdot e^{i\psi}) & B & r_s - \delta \end{bmatrix}$$

up to the electron–muon / electron–tau attachment branch. The audit targets are

$\rho\_\odot = (r\_s + B - r\_e) / A = \sqrt{(3/2)}$ (LF-2) , $B / A = 6/5$ (LF-2) , $\delta = 0 + O(\beta^2)$ (LF-3) , $\psi = 3\pi/4$ (LF-4) , $\beta = \sqrt{3} / 20$ (WL-4) .

These match the target list of the weak-doublet projection theorem: weak-commitment form, solar ratio $\sqrt{(3/2)}$, pair ratio 6/5, weak-neutral diagonal suppression, phase $3\pi/4$, leakage $\sqrt{3}/20$, and octant sign σ_W.

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10. Leakage Support Trace Audit

The leakage amplitude is **$\beta = \sqrt{3} / 20$**. Numerator and denominator carry different meanings.

- The numerator **$\sqrt{3}$** is an RMS amplitude from a threefold leakage source. It is rooted because amplitudes add by norm.
- The denominator **20** is a support trace. It is linear because it counts independent support channels.

Formal definition of Tr_support. To prevent the denominator from reading as mere channel-counting, the support trace is defined operationally:

For a sharp leakage support projector Π ($\Pi^2 = \Pi = \Pi^\dagger$), **$Tr\_support(\Pi) \equiv Tr(\Pi)$**, the ordinary projector trace — equivalently rank(Π), the dimension of the active leakage support. For a tensor-factorised sharp projector $\Pi = \Pi\_R \otimes \Pi\_Q$, $Tr(\Pi) = Tr(\Pi\_R) \cdot Tr(\Pi\_Q) = \dim R \cdot \dim Q$.

This is a genuine Hilbert-space trace of an idempotent, not an enumeration. The number 20 is then *computed* as $Tr(\Pi\_R) \cdot Tr(\Pi\_Q) = 5 \cdot 4$, conditional on the factorisation being sharp.

Approximate-factorisation clause. If the projected leakage support is only approximately factorised, the audit must additionally specify a tolerance and show trace stability:

$\|\Pi\_leak - \Pi\_R \otimes \Pi\_Q\| \leq \varepsilon\_leak$, and $Tr\_support(\Pi\_leak) = 20 + O(\varepsilon\_leak)$.

Where the factorisation is inexact, Tr_support is taken as the effective trace after the leakage weights below ε_leak are thresholded out, and the audit owes a demonstration that this effective trace remains pinned at 20 across the admissible ε_leak window — i.e. that 20 is not an artefact of where the threshold is placed. **[Owed bound on ε_leak stability]**

The proposed factorisation (WL-4) is $\Pi\_leak \simeq \Pi\_R \otimes \Pi\_Q$ with

$\dim R = K - 2 = 7 - 2 = 5$, $\dim Q = 4$,

so

$Tr\_support(\Pi\_leak) = 5 \times 4 = 20$, $\beta = \sqrt{3} / Tr\_support(\Pi\_leak) = \sqrt{3} / 20$.

The load-bearing caution. The factor dim $Q = 4 = 2 \times 2$ has one secure factor and one fragile one. The weak-neutral quadrature factor of two is secure. The second factor of two is the load-bearing premise: it must be a genuine closure-sector binary — **not** the Hermitian return entry and **not** the Axiom-B-gapped weak-role complement. The closure two-cycle binary is the candidate, but its residence in the closure/holonomy sector must be verified by the projection, not assumed. The audit is binary:

$\mathrm{Tr\_support}(\Pi\_\mathrm{leak}) = 20$?

If yes, $\beta = \sqrt{3}/20$ is derived. If no, the reactor-angle value remains a benchmark.

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11. Octant and CP-Sign Audit

The atmospheric octant is not fixed by the leakage magnitude alone. The magnitude fixes $|\theta_{23} - 45°|$; the branch is fixed by which electron-attachment channel carries the leading breaking. Define

$A\_e\mu = \|P\_e\ H\_W\ P\_\mu\|^2$, $A\_e\tau = \|P\_e\ H\_W\ P\_\tau\|^2$,

and

$\sigma\_W = \mathrm{sgn}(A\_e\tau - A\_e\mu)$.

- $\sigma\_W > 0 \Rightarrow$ leading breaking is electron–tau $\Rightarrow$ upper-octant branch.
- $\sigma\_W < 0 \Rightarrow$ leading breaking is electron–muon $\Rightarrow$ lower-octant branch.
- $\sigma\_W = 0 \Rightarrow$ no first-order octant selection.

The octant is thus not a loose ambiguity but the sign of a specific projected-Hamiltonian norm difference — a decisive frontier output. **[Owed output]**

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12. Minimal Calculation Checklist

The next computation must supply the following objects without using CKM or PMNS data as input (WL-1).

1. **Quark role-odd block.** $\Omega\_\mathrm{odd} = \frac{1}{2}\,\mathrm{Tr\_role}(\Omega\_W\ \tau_3)$. Test: $\Omega\_\mathrm{odd} \to \Omega\_q$.
2. **Role-even common curvature.** $\Omega\_\mathrm{even} = \frac{1}{2}\,\mathrm{Tr\_role}(\Omega\_W)$. Test that the first nontrivial component is $\Omega_0\hat{}(23) = z\ E_{23} - z^*\ E_{32}$; convert to $\zeta = \lambda z$; test $|\zeta| = b/\sqrt{3}$ and $\arg \zeta = 5\pi/6$. Then form the **resultant** $R_{13} = (\Omega\_q)_{13} + \frac{1}{2}a\zeta$ and record $|R_{13}|$ and $\arg R_{13}$ as the objects for the deferred V_ub comparison; bound the next-order BCH tail.

3. **C_3 covariance and norm.** Compute the pre-readout role-even generation block K_even^q; test [K_even^q, R_3] = 0 (QF-3); verify equal branch weights $w_{12} = w_{23} = w_{31}$ (QF-3); and verify the total-norm identification ‖Ω_even^common‖_K = b (QF-3b), distinct from democracy.
4. **Neutrino weak-commitment block.** Compute T_ν = P_ν H_W P_ν; test T_ν = D_0 I + ε M_ν; extract ρ_⊙, B/A, δ, ψ, β.
5. **Leakage support trace.** Compute Π_leak; test Π_leak ≃ Π_R ⊗ Π_Q with dim R = 5, dim Q = 4, Tr_support(Π_leak) = 20.
6. **Discrete signs.** Compute σ_φq, σ_φν, and σ_W = sgn(‖P_e H_W P_τ‖² − ‖P_e H_W P_μ‖²).

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13. Falsification Conditions

The audit fails **locally** — and diagnosably — under any of the following, each keyed to a named premise.

#	Condition	Premise broken
F1	P_W H_cl P_W cannot be defined without fitting flavour data	WL-1
F2	The mass/readout projection violates charge-blindness	WL-3
F3	Ω_mix is not gapped; weak-role frames do not separate	WL-2
F4	Ω_odd does not return the inherited CKM hierarchy	QF-1
F5	The leading role-even common curvature is not 2↔3	QF-2
F6	The role-even orbit is not C_3-democratic / not Killing-orthonormal (branch weights unequal)	QF-3
F6b	The role-even common-mode total norm is some b′ ≠ b, so |ζ| ≠ b/√3 even with the orbit democratic	QF-3b
F6c	The ordered readout selects 2↔3 but rescales or contaminates the inherited C_3 norm, so the surviving branch does not carry b/√3 (junction between QF-2 and QF-3/QF-3b)	§7a bridge
F7	The phase branch is not 5π/6, or the selection is not the least-transport Hermitian root, or the generator orientation is not fixed by $(Ω\_q)_{12}$ = +a, or σ_φq selects the antipodal CP branch	QF-4
F7b	The resultant $R_{13} = (Ω\_q)_{13}$ + ½aζ fails to match |V_ub| and the CP phase under the deferred generator → matrix-element map (with the BCH tail bounded)	§6 resultant
F8	The neutrino block is not of the form T_ν = D_0 I + ε M_ν	LF-1
F9	The solar ratio does not return ρ_⊙ = √(3/2)	LF-2
F10	The pair ratio does not return B/A = 6/5	LF-2
F11	First-order diagonal μ–τ splitting appears	LF-3
F12	The leakage support trace does not return 20	WL-4

#	Condition	Premise broken
F13	The second binary in dim $Q = 2 \times 2$ is not closure-sector-resident, or collapses into quadrature or role-changing structure	WL-4
F14	σ_W selects the atmospheric branch opposite to stable experimental data	§11 octant
F15	CKM and PMNS require unrelated projection rules, breaking the unified weak-doublet claim	WL-1

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14. Conclusion

This paper does not claim to have evaluated the microscopic substrate Hessian. It does something narrower and more useful: it turns the next Standard Model flavour step into a finite projected-Hamiltonian audit.

The correct object is $\mathbf{H_W = P_W H_{cl} P_W}$, with the exact role decomposition $\mathbf{\Omega_W = \Omega_{even} \otimes I + \Omega_{odd} \otimes \tau_3 + \Omega_{mix}}$.

The quark-sector test is whether the role-odd block returns Ω_q and the role-even block returns a Cabibbo-protected 2↔3 common curvature whose convention-invariant value is

$$\zeta_{C3} = (b / \sqrt{3}) \cdot e^{(5\pi i/6)} .$$

The exact commutator $[\Omega_0, \Omega_q]_{13} = - az$ then produces the triangle increment $\Delta_{13} = ½ a\zeta$, and the decisive comparison is the resultant $R_{13} = (\Omega_q)_{13} + ½a\zeta$ — magnitude and CP phase — against $|V_{ub}|$ under the deferred generator → matrix-element map.

The neutrino-sector test is whether the weak-commitment block returns $\mathbf{T_\nu = D_0 I + \varepsilon M_\nu}$, so that the PMNS frame is set by M_ν, independent of the tiny neutrino mass scale.

The leakage test is whether the projected weak-commitment attachment block gives $\mathbf{Tr_{support}(\Pi_{leak}) = (K - 2) \times 4 = 20}$, making $\mathbf{\beta = \sqrt{3/20}}$ a derived support-trace amplitude.

The octant test is whether $\mathbf{\sigma_W = sgn(\|P_e H_W P_\tau\|^2 - \|P_e H_W P_\mu\|^2)}$ selects the observed branch.

The audit is now well-posed and the burden is local. There is no room left for vague flavour language. Either the projected Hessian returns these blocks, signs, ratios, support traces, and the resultant (1,3) entry — or it does not, at one identifiable line. The remaining work is the evaluation of P_W H_cl P_W against this checklist, together with the deferred generator → matrix-element map and a bound on the BCH tail.