

Three by Two

The Census of Closure Classes — Capacity from Hexagonal Geometry, the Factorization $6 = 3 \times 2$, and the Multiplicities of the Standard Model

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General Reader Summary

The central claim, first

The central claim of this paper is simple, and it concerns the two most famous small numbers in particle physics. The strong force organizes quarks in *threes* — the colour triplet. The weak force organizes particles in *twos* — the weak doublet. Conventionally these numbers are read off from experiment and built into the theory by hand; nothing explains why three and two rather than four and seven. This paper proposes that the numbers are not chosen but *counted* — and counted from one shape. The unique Fold's closure architecture is hexagonal, and a hexagon is secretly a multiplication: six boundary directions that decompose, exactly and uniquely, into **three axes times two orientations** — $6 = 3 \times 2$, the two interleaved triangles of the six-pointed star. Ask the hexagon how many ways its parts can be perfect, interchangeable copies of one another, and it answers twice: by threes, along its axes, and by twos, across its orientations. Those two answers are the only answers it has — and they are precisely the multiplicities the Standard Model's forces use. On this reading, the Standard Model's $SU(3) \times SU(2)$ silhouette is not a selection from an infinite catalogue. It is the factorization of hexagonal closure, read through the transport mechanism the predecessor papers derived.

What this paper does and does not prove

The paper is careful about a distinction its predecessors taught: **capacity versus occupancy**. Capacity is what the geometry permits — how many identical copies the architecture can house in a single family. The paper argues, conditionally on named geometric premises, that the capacities are exactly one, two, and three: singletons freely, pairs through the orientation structure, triples through the axis structure, and nothing larger, because a fourth member of any family would need a distinguishing channel the architecture does not contain — and a member that needs its own distinction is, by that very fact, no longer an identical copy. Occupancy is whether nature *uses* those capacities — whether the colour triplet really is the axis family and the weak doublet really is the orientation family. That identification is stated as the open question it is, with its address in the corpus's defect-and-species papers. The division of labor fixed by the predecessor holds throughout: geometry proposes the multiplicities; transport disposes their forces.

Why not four? — the whole argument in a hotel

Imagine a hotel with exactly three identical rooms. Three guests can occupy identical rooms — three perfect copies of the same accommodation, nothing distinguishing one guest's situation from another's. A fourth guest requires a different room. And the moment the room is different, that guest is no longer part of the identical-room group: their situation is distinguishable, and the group's defining feature — perfect interchangeability — excludes them by definition, not by force. The limit on the group is not imposed by a rule against fourth guests. It is imposed by running out of identical places to put them. The paper argues that closure classes behave exactly this way: the hexagon has three identical rooms along one grain (its axes) and two along the other (its orientations), and that is why nature's families of perfect copies come in threes and twos — and never, the paper predicts, in fours.

The honest condition

Every step is conditional on premises the paper names rather than hides. The deepest is indistinguishability: three axes form a *family of copies* only if no fact of the architecture singles one axis out — the geometry must be even-handed among its own directions — and likewise for the two orientations. The orientation case carries a flag the paper raises rather than buries: the two orientations are mirror twins, and the weak force famously violates mirror symmetry, so the doublet candidate sits exactly where the deepest unsolved structure of the Standard Model lives. The paper marks that entanglement as a possible clue rather than a defect, and fences it. And the census yields something a reader can hold the framework to: if the capacities really top out at three, then no fundamental particle family larger than a triplet exists — anywhere, at any energy. A single discovered quartet would refute the census outright. A counting argument that can be killed by a counting is the kind this programme exists to make.

Abstract

The headline. The Standard Model's most distinctive small numbers — the three of colour, the two of weak isospin — may be the two factors of one shape: hexagonal closure decomposes uniquely as three axes times two orientations, $6 = 3 \times 2$ (group-theoretically $C_6 \cong C_3 \times C_2$, unique by coprimality [Imported-External]) — the two factors living on two layers of the boundary structure, transport directions and triangular sectors, related by the one decomposition — and the census of closure classes the unique Fold supports is exactly the factor list. The $SU(3) \times SU(2)$ silhouette would then be the factorization of hexagonal closure, read through the Bath Criterion's transport mechanism — multiplicities counted, not chosen. The remainder of this abstract states the results at referee grain.

The Bath Criterion fixed a division of labor — geometry proposes the multiplicities, transport disposes their forces — and named the census as its slot H1: which class sizes k does the unique Fold architecture realize? The present paper is the proposing paper, and it divides the census into the two questions the predecessors' discipline requires: **capacity** (which class sizes the architecture *can* house) and **occupancy** (which it *does*).

Capacity, conditional. Four imports are named and carried. **HG** [Inherited — the hexagonal closure papers]: the architecture's six boundary directions with their reversal structure, and the triangular orientation sector. **X3 (axis indistinguishability)**: the three axes — six rays identified in opposite pairs under reversal [Inherited disposal] — are mutually indistinguishable absent realized configuration differences: the geometry is even-handed among its axes, so axis-labels are unwitnessed. **X2 (orientation indistinguishability)**: likewise the two interleaved orientations — with the **chirality flag** raised at the import itself, since the orientations are parity twins and the weak force is chiral (§10). **G-A (Geometric Anchoring)**: members of one class are realized through distinct indistinguishable channels of the architecture, so class size is bounded by channel count — the premise that converts geometric multiplicity into a class ceiling rather than a coincidence. **XC (inventory exhaustiveness)**: the architecture's indistinguishable-channel structures are exactly the axis triple and the orientation pair — the six rays reduce to axes under reversal, and no further multiplicity hides in the inventory [slot at the geometry papers' full structural inventory]. Under these, the **Capacity Census Theorem** (§7): admissible class capacities are exactly $k \in \{1, 2, 3\}$ — singletons unconstrained (consistent with the Class Multiplicity Proposition, which admits them), triples housed by the axis triple, pairs by the orientation pair *or by any two axes* (partial-family housing is admissible at every premise; a saturation import G-S is named, not assumed, with the pairing of capacities to factors thereby assigned to occupancy), and $k \geq 4$ inadmissible, since a fourth member of any class would require a distinguishing channel the architecture lacks, and a member requiring its own distinction is by the Indistinguishability Lemma no member of the class [Proven, conditional on HG, X3, X2, G-A, XC, and the predecessor stack]. One structural condition is named where a referee would find it: the half-turn that quotients rays into axes *exchanges* the interleaved triangles, so the orientation pair exists only because it lives on the sector layer the transport quotient does not touch — the layer separation is stated, disposed, and added to the X2 slot (§4).

The factorization. The paper's centerpiece (§5): the two nontrivial capacities are not two accidents in one geometry but **one factorization** — the six-fold closure boundary is the product of its axis triple and its orientation pair, $6 = 3 \times 2$, the two factors of a single structure (the interleaved triangles), with the group-theoretic backbone $C_6 \cong C_3 \times C_2$ unique by coprimality. The **Factorization Uniqueness Proposition** sharpens the claim past arithmetic: indistinguishable families are symmetry orbits, the boundary's orbits come only in sizes one, two, and three, and every rival partition (4+2, 5+1, 3+1+1+1) contains members no symmetry relates — 3 and 2 are the *only* factors compatible with indistinguishable-family structure. The Standard Model numbers 3 and 2 are thereby not explained independently: they are the two factors of a single geometric decomposition, and the silhouette $SU(3) \times SU(2)$, assembled through the Bath Criterion's mechanism, is the unique factorization of hexagonal closure [Conditional on the full chain, stated at that grain; the factorization reading itself carries a slot at the source geometry].

Occupancy, open. Whether the colour triplet *is* the axis class and the weak doublet *is* the orientation class is the occupancy question **O1** [Open], slotted at the Persistent-Fold-Defect and species-inventory papers; the candidate identifications are stated as candidates (§8).

Disambiguations and the prediction (§9): the census counts sector classes — fundamental multiplets, the carrier — while adjoint multiplicities come free from the group structure (dim $SU(k) = k^2 - 1$: eight gluons, three weak bosons — not sectors, and not census items); and the

capacity ceiling is claimed as a ladder of standalone falsifiable commitments, the arc's third kill-condition — Predictions A–D: **no fundamental quartet, quintet, or sextet exists, and every fundamental multiplet ever observed must be singleton, doublet, triplet, or composite** — one discovered violation, at any energy, refutes the census outright. The chirality entanglement at X2 is developed, fenced, and partly inverted (§10): either orientation-indistinguishability holds at the unbroken level and its breaking is a named electroweak/chirality candidate [Conjectural — successor obligation X2-B] — in which case the census *places* the weak multiplicity on the parity-borne channel family (axes-rotation-strong-clean against orientations-reflection-weak-violating), anticipation rather than accident, conditional on the source's exchange operation being reflection-implemented (in the bare hexagon both rotation and reflection exchange the orientations; the condition is added to the X2 slot) — or the orientations are distinguishable at source and the cost is re-priced at the amended theorem's grain: the doublet survives via two-axis housing, while the factorization's second tenant and the correspondence are lost; both branches stated, neither claimed. The general **Distinguishability Ceiling** is extracted as exportable corpus vocabulary (§6): indistinguishable multiplicity is bounded by indistinguishable structure. The **Multiplicity–Mechanism Principle** (§11) states the joint result of this paper and its predecessor: $3 + \text{bath} \Rightarrow \text{SU}(3)$, $2 + \text{bath} \Rightarrow \text{SU}(2)$ — the gauge structures as the transport realizations of the multiplicities, one derivation in two papers.

Epistemic markers: [Inherited] imported from prior VERSF papers; [Imported-External] imported from standard mathematics outside the programme, carried at the external source's standing; [Proven] established here; [Conditional] holding under stated inputs; [Conjectural] motivated but unproven; [Open] undecided.

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1. Introduction — The Multiplicities as a Factorization Problem

The discovery this paper proposes, stated before any machinery: **the three of colour and the two of weak isospin may be the two factors of hexagonal closure — $6 = 3 \times 2$ — counted from the unique Fold's geometry rather than chosen from a catalogue.** That sentence is the paper. Everything below it is the apparatus that states the count's premises exactly, proves the ceiling they imply, and separates what is counted (capacity) from what is conjectured (occupancy).

The question arrives with its place in the arc already fixed. *The Price of Copies* proved what identical closure sectors force; *The Bath Criterion* translated the remaining decision into conservation language and fixed a division of labor in five words — *geometry proposes; transport disposes* — naming the census as the slot H1: the transport mechanism yields $U(k)$ for whatever k the architecture supplies, and is silent on which k that is. The present paper is the proposing paper. Its question is not *why are there non-abelian forces?* — the predecessors answered that conditionally — but *why these multiplicities?* Why doublets and triplets; why not quartets, septets, or families of ten?

The conventional register has no answer: $SU(3)$ and $SU(2)$ are selected because they fit, and the numbers 3 and 2 enter as the dimensions of representations chosen to match observation. The register this paper proposes is counting. The unique Fold's closure architecture is hexagonal [Inherited], and a hexagon is a multiplication: six boundary directions that decompose as three axes (opposite rays identified under reversal) times two orientations (the interleaved triangles) — exactly, uniquely, with the group-theoretic backbone $C_6 \cong C_3 \times C_2$ admitting no other factorization. If the axes are genuine copies of one another — no fact of the architecture singling one out — they are the channels of a three-member class; if the orientations are genuine copies, a two-member class; and if class members must be housed in distinct channels, nothing larger than three can ever assemble. The multiplicities of the Standard Model would then be the *only* multiplicities the architecture can count to, and the $SU(3) \times SU(2)$ silhouette — assembled by the predecessors' mechanism from these classes — would be the factorization of one shape.

Three disciplines govern the paper, inherited from the arc's hard-won habits. First, the **capacity/occupancy split**: geometry can at most bound what classes *can* exist; whether the colour triplet *is* the axis class is a fact about the defect inventory, held [Open] at its own slot, and the paper never lets the bound borrow the identification's credit. Second, the **named-import discipline**: every step from "the geometry contains a 3-structure" to "the architecture supports a $k = 3$ class" runs through premises stated as imports with slots — above all the indistinguishability of the channels themselves, which is the bridge a hasty reading would cross silently. Third, the **flag-don't-bury rule**: the orientation pair is a parity pair, the weak force is chiral, and the collision between those two facts is raised at the import that creates it, developed in its own section, and fenced — because it is either the census's deepest problem or the corpus's first handle on chirality, and pretending it is neither would be the one dishonesty a referee could not forgive.

2. Inherited Results and Imports

Fold Uniqueness and Saturation [Inherited — (cite: Fold papers)]: one Fold; its architecture exhausts observable structure. The census is therefore a census of *its* structures — there is nowhere else to count.

The hexagonal closure architecture (HG) [Inherited — (cite: hexagonal closure papers: the six boundary directions; the reversal structure on rays; the triangular orientation sector)]: the geometric substrate of every count below. The paper consumes HG's structural inventory and nothing about its dynamics.

The class machinery [Inherited — *The Price of Copies*]: classes as Representation Classes; the Indistinguishability Lemma (equal invariant tuples entail genuine indistinguishability); the Class Multiplicity Proposition (the species ontology presupposes non-degenerate classes, with singletons admissible); the grounding pair's conditionality at the identity programme. Consumed throughout — the Lemma is the engine of the capacity ceiling (§6).

The transport mechanism and the division of labor [Inherited — *The Bath Criterion*]: $U(k)$ per class of size k , conditional on $W0/W1$, $Q1$, $U1$, $S2$ at their statuses; the census named as slot $H1$; *geometry proposes, transport disposes*. This paper is the proposal; it adds nothing to the mechanism and borrows nothing from it except the demand for a k .

The new imports, named once and carried throughout: X3 and X2 (channel indistinguishability — §4), **G-A** (geometric anchoring — §6), **XC** (inventory exhaustiveness — §7), and the occupancy slot **O1** (§8). Each is a slot, none a silent crossing. The paper's epistemic shape is thereby fixed in advance: one conditional theorem (capacity), one factorization reading (conditional, with its own slot), one open identification (occupancy), one fenced entanglement (chirality), one prediction (the ceiling).

3. The Census Problem, Stated Exactly — Capacity and Occupancy

The Bath Criterion's mechanism outputs $U(k)$ for whatever class size k exists, so the arc requires a census. But "which k exist?" conflates two questions of different epistemic kinds, and the paper's first act is to separate them.

Capacity: which class sizes can the architecture house? A class of k mutually indistinguishable sectors requires the architecture to support k -fold structure whose components are genuine copies — and the geometry either contains such structure or does not. Capacity is a question about HG's inventory, decidable by reading the geometry papers against named criteria, and it is where this paper's theorem lives.

Occupancy: which capacities does nature use, and which physical multiplets sit in which geometric channels? Whether the colour triplet is the axis class is a question about how Persistent Fold Defects actually anchor to the architecture — a construction fact at the PFD and

species-inventory papers, not a geometry fact, and no amount of counting decides it. Occupancy is held [Open] at O1 (§8).

The split matters because its halves fail differently. A capacity error is a counting error — a missed structure in the inventory, an unjustified indistinguishability — and strikes the theorem. An occupancy error relocates a multiplet — the colour triplet anchored elsewhere than the axes — and strikes only an identification, with the theorem intact. The predecessors' discipline (the refinement clauses, the graceful degradations) is built from exactly this kind of separation, and the census inherits it whole.

4. The Geometric Candidates and the Indistinguishability Imports

The axis triple. The hexagonal boundary carries six directions — six rays. The rays do not form a six-member class, and the disposal is inherited rather than invented: opposite rays are identified under the reversal structure [Inherited — the reversibility of transport, which pairs each direction with its inverse], so the boundary's directional inventory at the grain of admissible distinction is **three axes**. The candidate $k = 3$ channel structure is the axis triple.

The orientation pair. The hexagon's vertices partition into two interleaved triangles — the two triangular orientations of the closure structure [HG] — supplying a two-fold organization at the coarsest level. The candidate $k = 2$ channel structure is the orientation pair.

The two layers, and the reversal trap. The two candidates do not live on the same six-set, and the paper must say so before a referee draws the hexagon and finds the pair collapsing. (i) *The trap*: work naively on the six rays alone and implement reversal as the half-turn $r_i \mapsto r_{i+3}$. The half-turn that identifies opposite rays into the axis triple simultaneously *exchanges* the interleaved triples — $r_1 \in \{r_1, r_3, r_5\}$ maps to $r_4 \in \{r_2, r_4, r_6\}$ — so on a single-layer reading the quotient that births the triplet identifies the two orientations into one channel. Identification is fatal where indistinguishability is load-bearing: an identified pair is not two indistinguishable channels but one channel, and the doublet would die at the stroke that creates the triplet. (ii) *The resolution*: the two structures live on different geometric layers. The axis triple is a quotient of the *transport-direction* layer — rays identified with their inverses, a transport identification — while the orientation pair lives on the *vertex/sector* layer, the triangular closure sectors of HG, which the reversal of transport directions does not quotient. A direction and its inverse are one axis; the two triangular sectors are not a sector and its inverse, and no inherited identification merges them. (iii) *The slot, sharpened*: whether the source geometry's orientation sector indeed survives the reversal quotient — whether the layers separate in the construction as they must for the pair to exist — is added explicitly to the X2 and factorization slots, since it is precisely the kind of structural fact only the source can confirm. [The trap: Proven on the single-layer reading — which is why that reading must be rejected; the resolution: the source's structure as inherited from HG, with the layer separation now a named condition at the slot.]

The bridge, which must be paid for. A geometric k -structure is not a class capacity until its components are *mutually indistinguishable* — a class, per the inherited machinery, is a family of

genuine copies, and three distinguishable directions are three singletons, not a triple. The bridge is therefore an indistinguishability claim about the geometry itself, and the paper names it as the import it is, once per candidate:

X3 (Axis Indistinguishability) — *the three axes are mutually indistinguishable absent realized configuration differences: no invariant of the architecture singles out an axis, and axis-labels are unwitnessed facts.* This is an isotropy claim — the geometry is even-handed among its own directions — and it is exactly the kind of claim the source papers can verify or refute: a preferred axis anywhere in the construction (in the closure rules, the commitment geometry, the emergence of the continuum) breaks X3 and reduces the triple to singletons or a smaller family. [*cite: hexagonal closure papers — the symmetry of the construction among boundary axes*]. *Fallback if broken:* the axis class shrinks to the surviving indistinguishable subset, and the capacity list revises accordingly — a census change, not an architecture change.]

X2 (Orientation Indistinguishability) — *the two orientations are mutually indistinguishable absent realized configuration differences: no invariant of the architecture singles out a triangle, and orientation-labels are unwitnessed.* The same isotropy form — with one flag that the paper raises here, at the import that creates it, and develops in §10: the two orientations are exchanged by a parity-like reflection, and the physical force whose doublet structure this channel would house is the one force that violates parity. X2 therefore sits in direct contact with the chirality question the corpus has fenced since *The Price of Copies*, and the contact is either the census's deepest vulnerability or its most valuable byproduct. [*cite: hexagonal closure papers — the symmetry of the construction between orientations*]. *Fallback if broken:* the orientation class fails, the $k = 2$ capacity is unhoused, and the weak doublet must anchor elsewhere or the framework owes the doublet an origin it has not supplied — stated plainly in §10.]

The imports' form deserves one remark: X2 and X3 are not new *kinds* of premise. They are the geometric instances of exactly what S3's Indistinguishability Lemma runs on — equal invariant structure entailing genuine indistinguishability — applied to the architecture's own channels rather than to the defects that occupy them. The census's bridge is the arc's oldest bridge, crossed one level down.

5. The Factorization — Six Equals Three Times Two

Here is the paper's centerpiece, and it is the reason the two candidates of §4 should not be read as two lucky finds in one geometry.

The observation. The axis triple and the orientation pair are not independent structures that happen to coexist. They are the two factors of *one* structure: the six-fold closure boundary itself — read at the honest grain §4 fixed, on its two layers. The 3 lives on the transport layer: six rays identified in opposite pairs by reversal. The 2 lives on the sector layer: the two interleaved triangular orientations, untouched by the transport quotient. The factorization

$6 = 3 \times 2$ — three axes \times two orientations

is then not two readings of one six-set but one decomposition relating the layers, with the group theory below as its backbone. One caution the backbone owes §10 in advance: in the abstract hexagon the orientation exchange is implemented by *both* element types — the half-turn (a rotation, the C_2 factor below) and the edge-type reflections both swap the triangles, while the vertex-type reflections preserve them — so nothing in the bare drawing fixes whether the physical exchange is rotation-borne or parity-borne; that question belongs to the source and is taken up, as a condition, in §10. (The full symmetry of the hexagon is the dihedral group D_6 , with C_6 its rotation subgroup; the rotations suffice for the factorization, and the reflections are exactly where §10's question lives.)

The backbone. The decomposition is not an accident of drawing but a fact of group theory: the hexagon's rotation group is the cyclic group C_6 , and

$$C_6 \cong C_3 \times C_2,$$

uniquely, by the coprimality of 3 and 2 [Imported-External — the standard factorization of cyclic groups of coprime composite order]. The C_3 factor is the rotation among the axes (equivalently, within each triangle); the C_2 factor is the half-turn exchanging the orientations. The hexagonal architecture does not merely *contain* a 3-structure and a 2-structure: it **is** their product, and the product structure is the only one available — there is no reading of six as anything other than three times two among nontrivial coprime factors.

Factorization Uniqueness Proposition. *The six-fold closure boundary possesses exactly two nontrivial indistinguishable decompositions: the axis decomposition (three reversal-paired directions) and the orientation decomposition (two interleaved triangles). Every other partition fails indistinguishability.* The argument is orbit counting against the bridge X2/X3 instantiate: a family of genuine copies is a family on which the architecture's symmetries act transitively — an unwitnessed labeling *is* a symmetry orbit, since a family not related by any symmetry of the construction carries the orbit structure itself as a distinguishing invariant. One assumption inside that identification is made explicit rather than left riding: it takes the orbit structure to be a *realized invariant of the architecture* in the Lemma's sense — the construction's symmetry relations count among the facts that witness or fail to witness a labeling — and that assumption is added to the X2/X3 slots as part of what the source reading confirms. Granted it, the hexagonal symmetry's orbits on the boundary, at the admissible grain (rays identified in opposite pairs by reversal), come in sizes one, two, and three only: the axis triple (one C_3 orbit), the orientation pair (one C_2 orbit), and fixed singletons. A 4 + 2 partition, a 5 + 1 partition, a 3 + 1 + 1 + 1 partition — each requires a family of four or five boundary elements, and no subgroup of the hexagonal symmetry has an orbit of size four or five on this structure: such a family necessarily contains members no symmetry relates, hence members distinguished by invariant structure, hence non-members. The conclusion at full strength: **3 and 2 are not merely factors of six — they are the only factors compatible with indistinguishable-family structure.** Arithmetic permits many partitions; indistinguishability permits exactly the two the Standard Model uses. [Proven, conditional on HG and the orbit reading of indistinguishability — the reading X2/X3 instantiate, with the orbit-as-invariant assumption now named at their slots.]

Why orbit structure is physically relevant. The orbit criterion is not an arbitrary mathematical preference, and the referee question it invites — why should symmetry orbits matter *physically*? — has a programme-level answer. In this framework, a physical copy-class is defined by the absence of a witnessed internal distinction. If two candidate members differ only by a transformation the architecture itself treats as unwitnessed, their difference is gauge-like or label-like rather than physical — exactly the unwitnessed-label structure classes are built from. If no architecture symmetry relates them, the failure of relation is itself a witnessed invariant: the members do not occupy the same structural role, and the architecture says so in its own vocabulary. Symmetry orbits therefore supply the exact bridge the Indistinguishability Lemma needs — they are not convenient mathematical groupings but the architecture's own test for whether a label is physical or redundant. The orbit-as-invariant assumption named above is then the modest residue: that the construction's symmetry relations count among its realized invariants, which is what the X2/X3 source readings confirm. [The bridge's physical content: stated at the Lemma's grain; the residue: slotted.]

The consequence, at its honest grain. If X3 and X2 both hold — if the architecture is even-handed along both grains of its own factorization — then the two nontrivial class capacities of §7 are the two factors of the closure boundary, and the Standard Model silhouette assembled downstream, $SU(3) \times SU(2)$ with the abelian circle alongside, is **the factorization of hexagonal closure**, read through the Bath Criterion's mechanism: one shape, two grains of indistinguishability, two forces. Stated once at the width it deserves: **the Standard Model numbers 3 and 2 are not explained independently — they are the two factors of a single geometric decomposition.** The claim is stated at exactly the conditionality it has — it consumes X3, X2, the anchoring and exhaustiveness of §§6–7, the occupancy identifications of §8, and the entire predecessor chain at W0 — and it carries its own slot, since the reading of the orientation pair and axis triple as the architecture's operative factorization must be confirmed against the source geometry rather than against the drawing. [⟨cite: hexagonal closure papers — the boundary's product structure⟩. The factorization as geometry: Imported-External plus HG; the factorization as the *origin of the silhouette*: Conditional on the full chain, and the sentence the paper exists to make precise.]

The reframing this performs should be recorded, because it changes what kind of question the multiplicities are. Before: *why 3 and why 2?* — two separate numerological accidents. After: *why 6?* — one question about one shape, whose answer the corpus's hexagonal papers already claim to own, with 3 and 2 arriving together as its unique coprime factorization. Two mysteries become one, and the one is already addressed.

Where the reading is strongest, said plainly. The factorization reading is strongest when the two nontrivial capacities occupy *different* factors of the decomposition — the triplet on the axes, the doublet on the orientations, each factor with its tenant. The amended theorem (§7) permits the alternative: a doublet housed by two axes, the orientation factor idle. If both doublets and triplets ultimately occupy the axis family, the census survives intact — the capacity list is unchanged and the predictions stand — but the explanatory power of the factorization is reduced: "one shape, two grains, two forces" weakens to "one shape, one grain doing double duty," and the centerpiece demotes from the silhouette's origin to its container. The paper says this plainly rather than letting the reader discover it; which configuration nature chose is exactly O1's

question (§8), and G-S, if the source supplies saturation, restores the strong reading as geometry. [The dependence: stated; the verdict: occupancy's.]

6. Geometric Anchoring and the Capacity Ceiling

The exclusion of larger classes requires a premise the count cannot supply by itself, and the paper names it rather than assuming it in motion:

G-A (Geometric Anchoring) — *members of one closure class are realized through distinct indistinguishable channels of the Fold architecture: each member anchors to its own channel, no two members share one, and the class's existence as a class consumes the channels' mutual indistinguishability (X2/X3). Class size is therefore bounded by the count of mutually indistinguishable channels.* The premise's physical content is that closure sectors are not free-floating labels but geometrically housed configurations — which is the Persistent-Fold-Defect ontology's own picture, a defect being a pattern *somewhere* in the architecture — and its slot is the PFD papers' account of how defects anchor. [cite: PFD papers — defect realization and its geometric support]. *Fallback if broken:* if class members need no distinct channels — if multiplicity can float free of geometric housing — the capacity ceiling lapses entirely and the census reverts to [Open] with no geometric bound; the paper's theorem is exactly as strong as G-A.]

The Distinguishability Ceiling (general principle). Before the geometric instance, the principle at corpus width, because it is not a local counting trick: *a class may expand only while additional members can be realized without introducing new invariants. The moment a proposed member requires a new channel, a new label, or a new geometric distinction — any realized invariant the existing members lack — it ceases to be a member of the class it was proposed for. Indistinguishable multiplicity is bounded by indistinguishable structure.* This is the Indistinguishability Lemma's contrapositive, stated as a growth bound rather than a membership test, and it is exportable: wherever the corpus counts copies — sectors, defects, channels, generations — the available indistinguishable structure caps the count, by this principle and no further argument. [Proven at the Lemma's grain, with the grounding pair's conditionality at source.]

The Capacity Ceiling (geometric instance). *Under G-A and the inherited class machinery: no class can exceed the size of its housing channel family, and any proposed additional member is self-excluding.* The argument is the general principle run through G-A. Let a class of k members occupy a family of k mutually indistinguishable channels, and propose a $(k+1)$ -th member. By G-A it requires its own channel, distinct from the k occupied ones; by XC's inventory (§7) no further channel of the *same indistinguishable family* exists; so the new member's channel is distinguishable from the others — it differs in some invariant of the architecture. But then the new member differs from the original k in realized invariant structure, and by the Indistinguishability Lemma it is *not their copy*: it has relocated itself to another class in the act of finding housing. The exact joint where the Lemma turns deserves one explicit sentence: members anchored to different channels of one *indistinguishable* family differ only in realized configuration — which-channel is an unwitnessed fact, so their invariant tuples remain equal and membership survives — whereas anchoring to a *distinguishable* channel imports that channel's

witnessing invariant into the member's own tuple, and equal tuples fail. The ceiling is not a wall that excludes a possible member; it is the observation that "a fourth identical copy" is a contradiction in an architecture with three identical channels — the candidate's own housing requirement destroys its membership. [Proven, conditional on G-A, X2/X3, XC, and the grounding pair at source.]

7. The Capacity Census Theorem

One further import completes the premise set, and it is the one a hostile referee would otherwise supply as an objection:

XC (Inventory Exhaustiveness) — *the mutually indistinguishable channel families of the hexagonal closure architecture are exactly two: the axis triple and the orientation pair.* The six rays are disposed of by reversal (§4 — they reduce to the axes); the vertices are disposed of by the same factorization (they are the orientation pair's triangles, not an independent six-family); and the carrier's internal complex dimensionality [Inherited — the Fold papers] is a property of the amplitude space, not a channel family of the closure boundary. What XC asserts beyond these disposals is the *negative*: no further indistinguishable multiplicity hides in the architecture's full inventory — faces, cells, composite structures, whatever the source construction contains. The negative is exactly the kind of claim only a source reading can fix. [⟨cite: hexagonal closure papers — the full structural inventory, read for indistinguishable families⟩. *Fallback if broken*: an additional family of size k' adds k' to the capacity list and a candidate force factor downstream — the census expands rather than collapses, and the discovery would be a prediction, not an embarrassment.]

Why XC is not a visual assumption. XC is not the claim that a drawn hexagon visibly contains only a pair and a triple. It is the stronger structural claim that, at the grain relevant to closure-class housing, every candidate multiplicity must arise as an *indistinguishable channel family* of the Fold architecture. Composite arrangements, unions of channels, paths across channels, and higher-order configurations do not create new fundamental class capacities unless their components are mutually interchangeable under the architecture's own invariants. A four-element subset, for example, is not a four-capacity merely because four things can be selected — it is a four-capacity only if the four selected channels form one family of copies, and by the Factorization Uniqueness Proposition no such family exists at the boundary's grain. The source inventory therefore has to be read not as a list of parts but as a list of transitive indistinguishability families, and at that grain the only nontrivial families hexagonal closure supplies are the axis triple and the orientation pair. XC is not "we looked and saw no four"; it is "four does not count unless it forms one indistinguishable orbit, and none does." [The reading discipline: stated; the negative itself: still the slot's to confirm.]

Capacity Census Theorem. *Under HG, X3, X2, G-A, and XC, with the class machinery at its inherited statuses: the admissible closure-class capacities of the unique Fold architecture are exactly*

$$k \in \{1, 2, 3\},$$

with $k = 1$ unconstrained (singleton classes need no indistinguishable housing family and are admissible per the Class Multiplicity Proposition — the ontology's analogue of singlet representations), $k = 3$ housed by the axis triple, $k = 2$ housed by the orientation pair or by any two channels of the axis triple, and every $k \geq 4$ inadmissible by the Capacity Ceiling: a fourth member of any class requires a distinguishing channel and thereby exits the class. [Proven, conditional as stated.]

The $k = 2$ clause's second disjunct is owed to honesty about what the premises do and do not require: nothing in G-A, the ceiling, or the Lemma obliges a class to *saturate* its housing family. Two members anchored to two of the three axes, the third axis vacant, occupy mutually indistinguishable channels with equal invariant tuples — which-axis is unwitnessed, and the vacancy is realized configuration, which the Lemma exempts — so partial-family housing is admissible at every stated premise. A saturation premise (**G-S**: classes fill their housing families), if the source's defect-anchoring account supplies one, would restore the tight pairing of capacities to factors; it is named here as an available import and *not assumed* — absent source support, the amended clause is the theorem. The capacity list itself is unaffected: {1, 2, 3} either way.

Where the weight sits, said plainly. A referee will observe that the Capacity Census Theorem is close to immediate once XC and G-A are granted — bound by channels, list the channels — and the paper agrees rather than resists: the theorem's force lives almost entirely in XC's negative (no further indistinguishable family *anywhere* in the inventory) and G-A's housing requirement, and the census is exactly as strong as those two readings. The paper's genuine labor is elsewhere and is claimed as such: the conversion of an unexamined "the geometry gives 3 and 2" into named, slotted, falsifiable premises; the reversal-trap disposal of §4, without which the doublet candidate dies on the drawing; the Factorization Uniqueness Proposition of §5, which is where the actual mathematics lives; and the ceiling's general form, which the corpus keeps. A theorem that is short once its premises are honest is not a weakness — it is what premise-naming is *for* — and the objection "the theorem is the premises" describes every conditional result in the corpus; the question is whether the premises are named, slotted, and independently readable, and here they are. [The weight distribution: stated; the theorem's standing: unchanged.]

Two consistency checks pass and are recorded. The singleton clause is not an exception smuggled in for the leptons' sake: the Class Multiplicity Proposition of *The Price of Copies* explicitly admits singleton classes while forbidding a singleton-only ontology, and the census delivers exactly that profile — singletons freely, plus the two nontrivial capacities the non-degeneracy claim requires. And the theorem's output is precisely what the Bath Criterion's slot H1 requested: a k -list for the mechanism to consume — $U(2)$ and $U(3)$ available, $U(k \geq 4)$ geometrically unhoused — with the division of labor intact in both directions: nothing here yields a force (transport's job), and nothing in transport yields this list (geometry's).

8. Occupancy — The Open Half

The theorem bounds; it does not identify. Whether the physical multiplets sit in the geometric channels is the occupancy question, and the paper states it as the open node it is:

O1 (Occupancy) [Open] — *the colour triplet is the axis class; the weak doublet is the orientation class*. Stated as candidate identifications, with everything that makes them attractive and nothing that makes them established. Attractive: the sizes match the only nontrivial capacities; the factorization (§5) delivers both from one structure, matching the Standard Model's product silhouette; and the corpus's hexagonal papers already associate three-channel occupancy with the strong sector and orientation structure with the weak — so the identifications are the source's own, awaiting confirmation as Representation-Class anchorings in the S3 sense. Established: nothing — the anchoring of actual PFD species to actual geometric channels is a construction fact at the defect papers, and the census's credit does not transfer to it. [cite: PFD papers and species inventory — which defect classes anchor to which channel families]. The identifications: Conjectural until read; the capacity theorem: independent of them.]

One structural note keeps the half honest: occupancy can fail in two grades, and they cost differently. A *relocation* (the colour triplet anchored to some structure other than the axes — possible only if XC missed a family, so this grade folds into XC's fallback) revises the map but keeps the census. A *vacancy* (a capacity nature declines to use) costs nothing at all — capacity is permission, not obligation, and an unused channel family is the geometry's spare room. The only occupancy outcome that would strain the arc is the Standard Model's multiplets anchoring *nowhere* in the inventory — and that outcome is already excluded conditionally by the conjunction of the predecessors (the multiplets are classes; classes are housed per G-A; the housing list is XC's) — so the occupancy reading is constrained from both sides before it begins.

And one item belongs to O1 that a casual reading would assign to the geometry: the *pairing* of capacities to factors. By the theorem's amended clause (§7), $k = 2$ can be housed by the orientation pair or by two axes, so the geometry alone does not force the doublet onto the orientation channel — the factorization reading of §5 is at its most compelling when 2 and 3 are housed by the two factors respectively, and whether nature's doublet actually sits on the orientation layer is occupancy content, decided where O1 is decided (or restored as geometry by G-S, if the source supplies saturation). The centerpiece is stated accordingly: the factorization supplies the channels; O1 assigns the tenants.

The other three — generations, fenced. The Standard Model contains a second famous three: the fermion generations, three copies of the entire multiplet structure, and the census owes a stated position rather than silence. The position is a fence with a named question inside it. The corpus's current standing, fixed in *The Bath Criterion* §9, is that generations are *distinguishable* — they differ in mass — hence distinct classes, cross-class with respect to everything intra-class this paper counts, and therefore outside the census's capacity question, with their transport handled by the flavour machinery at its own slots. But that standing carries a level-dependence the fence must name: masses are breaking-phase facts, and if at the unbroken level the generation labels are unwitnessed — if the three generations are, pre-breaking, genuine copies — then the census's own Distinguishability Ceiling (§6) seizes jurisdiction: a $k = 3$ generation class would require an indistinguishable housing family, the axis triple is the only triple in XC's inventory, and the question of whether G-A permits two tenants on one family at different structural depths (colour and generation both anchored to the axes) is a G-A question the anchoring account has not been asked. The alternatives are exactly two, both open: generations witnessed at every level (the flavour paper's reading, generations outside the census) or generations as an unbroken-level

class whose distinguishing invariants are realized only in the broken phase — in which case generation-distinguishability-breaking is a sibling of X2-B's orientation-breaking, and the census's chirality machinery and the corpus's generation obligation may be one machinery. The fence is named **GEN** [Open — ⟨the question: are generation labels witnessed at the unbroken level, and if not, what houses the generation class?⟩], cross-referenced to the Distinguishability Ceiling as its natural tool, and deliberately not adjudicated here: this paper counts the gauge multiplets; the other three has its own arc. **Why GEN does not refute the census.** The generation triplet is not counted here because the census concerns *intra-class carrier multiplicity*: the number of mutually interchangeable components inside one Representation Class. Generations, by contrast, are copies of whole representation structures across a second level of organization — a colour triplet is three channels inside one class; three generations are three recurrence layers of the class system itself. They may ultimately answer to the same Distinguishability Ceiling, but they do not compete with the present census unless the programme later proves that generation labels are unwitnessed at the *same structural level* as colour labels. Until then, generations are a higher-order recurrence problem, not a counterexample to the closure-class capacity list — and "but there are three generations, so is that colour again?" is answered before it is asked: same number, different storey of the building. [The fence: stated; the level-dependence: the sharp residue; no census claim extends to generations.]

9. The Adjoint Disambiguation and the Prediction

The disambiguation, owed within a paragraph of the theorem. A reader who knows the Standard Model will object on sight: the strong force has *eight* gluons and the weak force *three* bosons — where are 8 and 3 in a census that tops out at 3? The answer is that the census counts **sector classes** — the fundamental multiplets, the carrier on which transport acts — and the gauge bosons are not sectors. They are the transport structure itself: the mechanism's group $U(k)$ (and its $SU(k)$ core) arrives with its generators for free, $\dim SU(k) = k^2 - 1$, so a $k = 3$ class brings $3^2 - 1 = 8$ gluon-like generators and a $k = 2$ class brings $2^2 - 1 = 3$ weak-boson-like generators *as consequences of the census, not entries in it*. The numbers 8 and 3 are derived from 3 and 2; counting them separately would be counting the same structure twice. [Proven — bare group theory above the chain's conditionality.]

The prediction, claimed — as a ladder. The Capacity Census is a standalone falsifiable commitment — the arc's third, beside the charge lattice and the conditional monopole exclusion — and the paper claims it rung by rung, so the exposure is visible at every grain:

- **Prediction A** — no fundamental quartet: no family of four identical fundamental species exists, anywhere, at any energy.
- **Prediction B** — no fundamental quintet.
- **Prediction C** — no fundamental sextet: in particular, no colour-sextet fundamental fermion.
- **Prediction D** — closure of the list: any fundamental multiplet ever observed must be a singleton, a doublet, a triplet, or composite — resolvable into bound structure over the capacity list. There is no fifth option, because the architecture contains no channel family to house one.

A single confirmed discovery violating any rung refutes the census outright, striking XC or G-A at source. **The composite clause, disciplined.** Prediction D's "or composite" is a criterion, not an escape hatch, and the criterion is fixed in advance of any discovery so that no quartet can be retroactively declared composite to save the census: compositeness must be established by its standard independent signatures — substructure form factors, an excitation spectrum, a compositeness scale visible in scattering — and a discovered multiplet of size ≥ 4 that remains point-like at all probed scales, with no excitation spectrum and no substructure signal, *counts as fundamental and kills the census*, full stop [the criterion: Imported-External — the standard experimental tests of compositeness, applied as found]. The clause exists because genuinely composite multiplets (nuclei, hadrons, molecules) are unbounded in size and outside any carrier-level counting; it does not exist to absorb counterexamples. One sharpening is available beyond the census and is marked as what it is: *under O1* — the identification of the weak sector with the orientation channel — additionally no weak multiplet above the doublet exists; this clause is occupancy content, conditional on the identification, and deliberately excluded from the ladder proper, which is census-pure. The commitment's profile is the right kind: it agrees with everything observed (the Standard Model's fundamental multiplets are singlets, doublets, and triplets — exactly the capacity list), it is exposed continuously at every collider energy, and it is a *counting* claim killed by a *counting* observation — the cleanest falsification shape available, now with its fundamentality criterion fixed before it is ever needed. Composite and adjoint structures are outside the claim's scope, per the disambiguation above; the claim governs the fundamental carrier. [The ladder: Conditional on the census's premises, and stated as the census's empirical face; the weak-sector sharpening: additionally conditional on O1.]

10. The Chirality Entanglement

The flag raised at X2 is developed here, because burying it would be the paper's one unforgivable omission. The two orientations are exchanged by a parity-like reflection of the hexagonal structure: they are mirror twins. X2 asserts their indistinguishability — orientation-labels unwitnessed. And the physical force whose doublet structure the orientation channel would house is the weak force, the one interaction in nature that violates parity maximally: it couples to left-handed particles and not right-handed ones, a fact the corpus has fenced as its largest untouched gap since *The Price of Copies* named it.

The collision admits exactly two readings, and the paper states both without choosing:

Reading one — X2 holds, and its breaking is physics. The architecture is orientation-even-handed at the fundamental level (X2 true; the doublet class exists; the census stands), and the observed chirality of the weak interaction reflects a *breaking* of orientation-indistinguishability at the level of realized configurations — some sector of the world carries a realized invariant that distinguishes the orientations, converting an unwitnessed label into a witnessed one in the broken phase. On this reading the census acquires a research programme rather than a problem: the mechanism of orientation-distinguishability-breaking becomes a candidate origin for electroweak chirality — possibly for electroweak symmetry breaking itself — and is named as the successor obligation **X2-B** [Conjectural — (the obligation: derive or refute a realized orientation-distinguishing invariant and its consequences)].

And on this reading one apparent embarrassment converts into corroboration, and the paper claims the conversion explicitly. A reader will object that the weak doublet's members are manifestly *distinguishable* — the up-type and down-type components differ in charge and in mass — unlike the genuinely identical colour triplet, so the doublet looks like a bad fit for a class of copies from the start. But the objection describes the broken phase, and reading one is precisely the claim that the broken phase is not where classes are counted: at the unbroken level the doublet members differ only by the gauge label itself — the unwitnessed which-channel fact — and their observed distinguishability *is* the realized breaking X2-B is obligated to produce. The asymmetry between the two candidate classes — colour triplet identical in every phase, weak doublet identical only above the breaking — is then not a defect of the census but its fingerprint: the parity-clean family stays unbroken and its members stay copies; the parity-borne family breaks and its members acquire witnessed differences. The census predicts not only where chirality lives but where member-distinguishability lives, and observation agrees on both. [The conversion: stated at reading one's conditionality; it is corroboration of the breaking-as-physics reading, not independent evidence for X2.]

And on this reading the entanglement inverts from vulnerability to anticipation — under one further condition the paper now states rather than assumes, because §5's caution comes due here. In the abstract hexagon the orientation exchange is implemented by both element types: the half-turn (a rotation) and the edge-type reflections both swap the triangles, and the axes, for their part, are permuted by reflections too (a vertex-type reflection fixes one axis and swaps the other two). So "orientations parity-borne, axes rotation-borne" is not a fact of the drawing — it can only be a fact of the *source construction*, holding if the physical operation that exchanges the orientation sectors is genuinely reflection-implemented there (a structure of the closure dynamics, an embedding, a transport orientation that closes the rotation route and leaves the mirror one). That condition is added to the X2 slot. Granted it, set the two channel families beside the two forces they would house:

axes — related by *rotations* — house colour — and the strong force is *parity-clean*; orientations — exchanged by *reflection* — house weak isospin — and the weak force is *parity-violating*.

The census does not merely reproduce the weak multiplicity. It places the weak multiplicity precisely on the only channel family whose defining exchange would be parity-borne. If chirality ultimately emerges from orientation asymmetry, the location of the weak sector is not accidental but anticipated by the census itself: the geometry assigned the mirror-borne family to the mirror-breaking force before anyone asked it to. This is a correspondence, not a proof — two facts aligning is not a derivation, the parity-implementation of the exchange is a named condition at the slot, and the paper marks the whole at exactly that strength — but it is the kind of correspondence a counting argument has no right to produce by luck, and it converts the census's most exposed flank into its most intriguing observation. [The correspondence: stated, Conjectural, conditional on the source's exchange type; its promotion to derivation: X2-B's job, not this paper's.]

Reading two — X2 fails at source. The geometry papers' construction distinguishes the orientations fundamentally (a preferred triangle somewhere in the closure rules). Then the orientation pair is two singletons and the orientation channel is lost. The cost is real but must be

priced at the amended theorem's grain rather than overstated: the $k = 2$ *capacity* survives, because two channels of the axis triple remain an available indistinguishable housing already inside XC's inventory — the weak doublet does not face "anchor elsewhere or owe an origin" but the stranger, live configuration of sharing the axis family with colour, two classes housed at different occupancies of one channel family. What is genuinely lost on reading two is the factorization's elegance — §5's centerpiece reads "one shape, two grains" only while both factors house tenants, and with the orientation factor idle the silhouette's 2 and 3 both draw on the 3-side of the decomposition — together with the §10 correspondence, which dies with its channel. A serious blow to the arc's aesthetic claim and to the chirality programme; not, under the amended clause, a vacancy crisis for the doublet itself.

The honest summary of the entanglement: the doublet candidate sits exactly on top of the Standard Model's deepest unexplained asymmetry, and that is either the census's most dangerous exposure or the corpus's first structural handle on chirality — the X2 source reading decides which, and the paper declines to predict it. [The entanglement: stated; X2-B: Conjectural, named; no marker moves.]

11. Position in the Programme

The Multiplicity–Mechanism Principle. The two papers' joint output deserves explicit statement, because separately they understate it. The census supplies multiplicities — triplets and doublets, by counting. The Bath Criterion supplies mechanism — continuous mixing, by conservation. The resulting gauge structures are not independently selected at either stage: *they are the transport realizations of the multiplicities*,

$3 + \text{bath} \Rightarrow U(3) \Rightarrow SU(3)$ beyond the shared phase, $2 + \text{bath} \Rightarrow U(2) \Rightarrow SU(2)$ beyond the shared phase,

with neither paper able to produce a force alone — the census without the bath yields inert side-by-side copies (the predecessor's ledger ceiling), and the bath without the census yields $U(k)$ of no particular k . One derivation, two papers, meeting at the assembly: the numbers from geometry, the dynamics from conservation, the silhouette from their product. [The principle: Proven at the level of the two theorems' statements; the assembly's conditionality: the union of both stacks, at W0.]

The chain, at its true grain. The census and the transport mechanism are inputs at *different nodes*, not links in one chain, and the assembly is drawn as the Bath Criterion's final grain requires — the census feeding the mechanism's k , the mechanism conditional on W0:

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One Fold [Uniqueness, Saturation] – hexagonal closure architecture [HG]
|
|— six rays ÷ reversal ⇒ three axes; two interleaved orientations
|— X3 (axes indistinguishable) + X2 (orientations indistinguishable
– chirality flag, §10)
|   + G-A (anchoring) + XC (exhaustiveness)
|   ⇒ CAPACITY CENSUS:  $k \in \{1, 2, 3\}$  ← this paper

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Against the factorization reading (§5). Show the source geometry's operative structure is not the boundary's 3×2 product — that the axis triple and orientation pair are not the architecture's working decomposition. Effect: the centerpiece demotes from "the silhouette is the hexagon's factorization" to "the architecture contains a 3 and a 2," which is weaker poetry over the same theorem.

Deciders. X3, X2, XC, G-A at the geometry and PFD slots; O1 at the species inventory; X2-B as the named successor at the chirality frontier.

Empirically. One confirmed fundamental multiplet of size ≥ 4 — a quartet, quintet, or sextet of identical fundamental species under any force, at any energy — refutes the census outright (§9); a weak multiplet above the doublet additionally refutes the O1 identification, on its own conditionality. Continued absence is continuous weak corroboration, and the commitment is exposed at every collider, every day. This is the arc's third standalone kill-condition, and the only one that is a pure counting claim.

13. What the Paper Establishes

Established (conditionally, with conditions named):

- The capacity/occupancy split as the census's governing discipline: capacity geometric and bounded here; occupancy constructional and held open at O1 — failing grades separated, costs priced (§3, §8). [The split: definitional; its enforcement: the paper's structure.]
- The geometric candidates with their bridges paid: rays reduced to three axes by inherited reversal; the orientation pair from the interleaved triangles; the two-layer separation named with the reversal trap stated and disposed — the half-turn that creates the axes exchanges the triangles, so the pair survives only because it lives on the sector layer the transport quotient does not touch, a condition added to the X2 and factorization slots; X3 and X2 named as the indistinguishability imports that alone convert geometric multiplicity into class capacity — the arc's oldest bridge, crossed one level down (§4). [Candidates: Inherited at HG; bridges and the layer condition: slotted.]
- The factorization centerpiece: the two nontrivial capacities as the two factors of one structure — $6 = 3 \times 2$, $C_6 \cong C_3 \times C_2$ unique by coprimality [Imported-External] — converting "why 3 and why 2?" into "why 6?", one question the hexagonal papers already address; the silhouette-as-factorization claim stated at full conditionality with its own source slot; and the reading's strength-dependence said plainly — strongest when the two capacities occupy different factors, reduced to container from origin if both occupy the axis family, with the verdict assigned to O1 and the strong reading restorable by G-S (§5). [The geometry: Proven given HG; the origin claim: Conditional on the full chain.]
- The Factorization Uniqueness Proposition: indistinguishable families are symmetry orbits — with the orbit-as-invariant assumption made explicit and added to the X2/X3 slots — the boundary's orbits at admissible grain come only in sizes one, two, and three, and every partition invoking four or five ($4+2$, $5+1$, $3+1+1+1$) contains members no symmetry relates — hence 3 and 2 are not merely factors of six but the *only* factors compatible with indistinguishable-family structure (§5). [Proven, conditional on HG and the orbit reading X2/X3 instantiate.]

- The Distinguishability Ceiling as a general corpus principle: indistinguishable multiplicity is bounded by indistinguishable structure — a class grows only while new members need no new invariants — the Indistinguishability Lemma's contrapositive stated as a growth bound, exportable wherever the corpus counts copies; with the Capacity Ceiling as its geometric instance through G-A, the (k+1)-th member self-excluding (§6). [The principle: Proven at the Lemma's grain; the instance: conditional on G-A, X2/X3, XC.]
- The Capacity Census Theorem, at its amended grain: $k \in \{1, 2, 3\}$ exactly — singletons free (consistent with the Class Multiplicity Proposition, a check that passes), 3 by the axis triple, 2 by the orientation pair *or by any two axes* (partial-family housing admissible at every premise; saturation G-S named as an available import, not assumed), $k \geq 4$ unhoused — with the pairing of capacities to factors thereby assigned to O1 (§7, §8). [Proven, conditional as stated.]
- The Multiplicity–Mechanism Principle: the gauge structures as the transport realizations of the multiplicities — $3 + \text{bath} \Rightarrow \text{SU}(3)$, $2 + \text{bath} \Rightarrow \text{SU}(2)$ — neither paper producing a force alone, one derivation meeting at the assembly (§11). [Proven at the level of the two theorems' statements.]
- The adjoint disambiguation: 8 gluons and 3 weak bosons as $\dim \text{SU}(k) = k^2 - 1$ — consequences of the census, not entries in it; the on-sight objection answered within a paragraph of the theorem (§9). [Proven — group theory.]
- The prediction ladder, claimed: Predictions A–D — no fundamental quartet, quintet, or sextet, and closure of the list (singleton, doublet, triplet, or composite — no fifth option) — the arc's third standalone kill-condition, a counting claim killed by a counting; the composite clause disciplined by an advance criterion [Imported-External — the standard compositeness tests], so that a point-like multiplet of size ≥ 4 with no excitation spectrum counts as fundamental and kills the census, full stop; the weak-sector sharpening (no weak multiplet above the doublet) separately marked as O1-conditional occupancy content, excluded from the census-pure ladder (§9). [Conditional on the census's premises; the empirical face.]
- The weight distribution, said plainly: the theorem is close to immediate once XC and G-A are granted, and the census is exactly as strong as those readings; the paper's labor claimed where it lives — premise-naming, the reversal-trap disposal, the Uniqueness Proposition, the general ceiling (§7). [Stated; the theorem's standing unchanged.]
- The generations fence: GEN named [Open] — the corpus's current standing (generations distinguishable, cross-class, per *The Bath Criterion* §9) holds the other three outside the census, with the level-dependence stated as the sharp residue: if generation labels are unwitnessed at the unbroken level, the Distinguishability Ceiling seizes jurisdiction, the axis triple is the only available triple housing, the multi-tenancy question lands on G-A, and generation-breaking becomes a sibling of X2-B (§8). [The fence: stated; no census claim extends to generations.]
- The distinguishability conversion: the manifest broken-phase differences between weak-doublet members (charge, mass) inverted from objection to fingerprint — at the unbroken level the members differ only by the unwitnessed gauge label, their observed distinguishability being exactly the realized breaking X2-B must produce, so the census predicts where member-distinguishability lives as well as where chirality does (§10). [Stated at reading one's conditionality; corroboration, not independent evidence.]

- The chirality entanglement, flagged, developed, and partly inverted: X2's indistinguishability against the weak force's maximal parity violation; both readings stated — breaking-as-physics with X2-B named [Conjectural], or X2's failure with the cost re-priced at the amended grain (the doublet survives via two-axis housing; what dies is the factorization's second tenant and the correspondence) — and the correspondence claimed at its exact strength and full conditionality: in the bare hexagon the orientation exchange is implemented by rotation and reflection alike, so the census places the weak multiplicity on the parity-borne family only if the source's physical exchange is reflection-implemented, a condition added to the X2 slot — anticipated, not accidental, if that condition and X2-B both hold (§10). [The entanglement: stated; the correspondence: Conjectural, conditional on the exchange type; no marker moves.]
- The division fulfilled and the assembly drawn at W0 grain; assignment-neutrality extended (§11). [Proven, given the stated grain.]

Not established (open, out of scope, or pending):

- O1 — the occupancy identifications: colour triplet $\stackrel{?}{\cong}$ axis class, weak doublet $\stackrel{?}{\cong}$ orientation class (§8) [Open at the PFD/species inventory];
- GEN — the generation question: the level at which generation labels are witnessed, and the housing of an unbroken-level generation class if they are not (§8) [Open at its fence];
- the imports at their slots: HG's inventory, X3, X2, G-A, XC — every one a source reading (§§4–7);
- X2-B — the orientation-breaking programme, the chirality frontier's named candidate (§10) [Conjectural];
- hypercharge, symmetry breaking, occupancy patterns within multiplets, mass hierarchy — the arc's interior, fenced exactly as the predecessors fenced it;
- everything inherited: the Bath Criterion's stack at W0, the Price of Copies' machinery, the Fold and hexagonal papers at source.

The honest summary: the paper converts the census from a list of observations into a conditional counting theorem — capacities exactly one, two, and three, because the unique Fold's hexagonal closure is the product of an axis triple and an orientation pair and contains housing for nothing larger — states the factorization $6 = 3 \times 2$ as the origin-shaped reading of the Standard Model's silhouette, claims the no-larger-multiplets prediction as the arc's third kill-condition, and lays the chirality entanglement on the table face-up. The numbers three and two were never two questions; they were one hexagon.

How the theorem should be read. Neither as numerology nor as a completed derivation of the Standard Model — it is a *conditional narrowing result*. Given the Fold's hexagonal closure, given channel indistinguishability, given geometric anchoring, and given inventory exhaustiveness, the admissible fundamental class capacities narrow to singleton, doublet, and triplet; the Standard Model then appears not as an arbitrary selection from all possible $SU(k)$ but as the occupied part of the only nontrivial capacity list the architecture supplies. The claim is strong precisely *because* it is conditional: each condition is named, each has a source slot, and failure of any one condition has a specified cost. A reader who wants to attack the paper has been handed the complete target list; a reader who wants to extend it has been handed the complete

dependency graph. That is the posture the corpus calls honest strength, and the census is offered in it.

14. Conclusion

Every gauge theory begins by choosing numbers, and the Standard Model's choices — three colours, two isospin states — have stood for half a century without a reason. The arc this paper completes proposes that the reason is a shape. The unique Fold closes hexagonally; a hexagon is six directions; and six is, uniquely and without alternative, three times two — three axes when read by reversal, two triangles when read by orientation. If the architecture is even-handed along both grains of its own factorization, then it can house identical copies in exactly two nontrivial ways: by threes and by twos. Nothing chooses the numbers. The shape counts them.

The paper has been careful to claim only what the count supports. The capacities are a theorem, conditional on four named geometric premises with their slots. The identifications — colour to the axes, weak isospin to the orientations — are candidates, open at the defect inventory. The entanglement between the orientation pair's mirror symmetry and the weak force's mirror violation is laid face-up: it is either the census's exposure or the corpus's first structural handle on why exactly one force is chiral, and a single source reading at X2 begins to decide which. And the census pays its way empirically in the cleanest currency available: if the architecture truly contains no family larger than three, then nature contains no fundamental multiplet larger than a triplet — a claim every collider tests continuously and one discovery would kill.

The predecessors showed that copies are expensive and that the bill is paid from a shared bath. This paper counts the copies: one, two, three, and never four. If the count survives its slots, then the most arbitrary-looking numbers in physics were never arbitrary — they were the factors of the first shape the universe knows how to close.