Two Ledgers and the Trapping Necessity

Why Gate-3 Is Occupied — Global Admissibility, Local Commitment, and the Cycle That Facthood Requires

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Summary for the General Reader

This paper answers the occupancy question the Gate-3 arc left open: not "what kind of charge survives" (a companion settles that — it is the two-valued class) but "does a charge survive at all?" The answer argued here is yes, and the argument has two moves, neither of which is a new assumption.

The first move dissolves a false choice the arc had been stuck on, and removes a defeater; the second supplies the positive ground. They are not symmetric — §2 rules out a way the charge could be flattened, while §3 is what actually produces it — and the assembly in §4 depends on them in that asymmetric way. The earlier framing offered two options: either the substrate consults one global record that reconciles every comparison (in which case the charge vanishes), or it consults only local comparisons (in which case the charge can survive). Stated that way it looked like a coin-toss between two readings of one rule. The dissolution is to notice that VERSF already has *two different ledgers* doing two different jobs, and the arc had been treating them as one. There is a ledger of **admissibility** — the distinguishability geometry, the structure that says which states differ from which, which underwrites the operational geometry and the Born-rule structure. And there is a ledger of **commitment** — the actual committed facts, the bits and records laid down locally and irreversibly. The first can be perfectly global without the second inheriting that globality. Knowing which comparisons are *allowed* everywhere does not assemble the comparisons that were *committed* into a single globally-consistent record.

The second move shows the committed ledger genuinely carries surviving structure — and here the argument leans on a result already proven in the programme, plus one bridging step this paper makes as honest as it can be. A companion paper establishes that a fact cannot form at all without a cycle: a substrate with no loops can trap nothing, so no irreversible record, no time's arrow, no entropy. And it establishes that a fact *is* a trap — the discarded alternative is quarantined on the loop, never rejoining. That is a statement about *dynamics*: the trapped branch never recombines. What occupancy needs is a statement about *holonomy*: the comparison around the loop does not sum to zero. Part of joining these is a plain theorem, with no physics assumed: if the comparison around the loop *did* sum to zero, the two branches would carry identical transported labels and so coincide — that is just what "zero holonomy" means. The only step that is not pure arithmetic is the last one: that coinciding labels actually let the branches physically re-

merge. This paper isolates exactly that step as a single named premise (a nonzero closure holonomy obstructs recombination) rather than smuggling it in — and shows the premise is not invented for the occasion, because spin-½ is a measured case where precisely this happens: two neutron branches whose closure phases differ fail to recombine, in the lab. So the conclusion "the sector is occupied" rests on one physical premise with a worked instance, not on a free assumption — and the paper says so, rather than rounding it to a theorem.

The register of that surviving charge is the two-valued one, for the reason the companion gives: the trap is binary (which alternative was quarantined), and every register the substrate actually carries — fact-trapping, fold orientation, spin closure phase — is two-valued in the same sense. So the result is: Gate-3 is occupied, and what occupies it is the class $\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_2)$.

One thing is stated plainly rather than smoothed over. "Occupied" here means: the substrate necessarily carries a non-recombining trapped distinction on a surviving cycle, and global admissibility does not flatten it. What is *not* claimed is a seven-valued occupant; that would require structure the corpus does not supply, and a companion shows the fold law points the other way. The honest result is occupancy in the two-valued register — which is exactly the register the rest of the programme speaks in.

Note on This Paper's Place in the Arc

This paper is the occupancy companion to *The Closure-Charge Register and the Fold Floor* (which settles the register: any observable closure charge is the $\mathbb{Z}_2$ class) and the successor to *Uniform Readout and Global Integrability* (which reduced occupancy to the single containment $A \subseteq B^1$ and isolated one residual axiom-question). It does not introduce substrate structure. It (i) replaces the Uniform-Readout fork with a two-ledger distinction that the programme's own architecture already contains, (ii) proves a Ledger Separation Lemma showing global admissibility does not force $A \subseteq B^1$, and (iii) imports the trapping-necessity result of *The Topological Threshold for Fact Formation* to argue that the committed ledger does not merely *escape* global integrability but is *required* to, on pain of facts not being facts. The result is the companion's missing Branch-B derivation: Gate-3 occupied, in the $\mathbb{Z}_2$ register.

The two capstones partition the question cleanly. *Register* (companion): if a charge survives, it is $\mathbb{Z}_2$. *Occupancy* (this paper): a charge does survive. Together: $\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_2)$, occupied.

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Abstract

Uniform Readout and Global Integrability reduced Gate-3 occupancy to the containment $A \subseteq B^1$ on the vacuum transport complex and isolated a single residual question — whether Uniform Readout forces a global section (the logical-versus-strong contextuality distinction). This paper resolves occupancy by two moves internal to the programme — one removing a defeater, one supplying the ground.

First, it replaces the Uniform-Readout fork with a distinction VERSF already carries: a global ledger of *admissibility* (the distinguishability geometry underwriting operational and Born-rule structure) and a local ledger of *commitment* (the committed bit/record facts). The Ledger Separation Lemma — the two-ledger restatement of the companion's established "finite pairwise distinguishability does not force $A \subseteq B^1$" — observes that B^1-membership is a loop-sum property of the committed configuration, while admissibility constrains edges individually without computing loop sums, so it cannot force exactness on cycles outside im ∂_2.

Second, it imports the *dynamical* trapping-necessity result of *The Topological Threshold for Fact Formation*: $\beta_1 \geq 1$ is necessary for any fact to form (tree-trapping is impossible), a fact *is* a cycle-trap quarantining the discarded alternative, and non-recombinability is constitutive of facthood. The identification of this dynamical trap with a *cohomological* nonzero holonomy ($\kappa \neq 0$, ρ

closed but not exact) is developed in two pieces. Theorem 3.1 is unconditional: exactness ⟺ path-independence ⟺ coincidence of the transported branch labels around the cycle — pure cochain arithmetic, no physical premise. The remaining step, from kinematic label-coincidence to dynamical recombinability, is isolated as a single named premise — the Recombination–Coincidence Premise (RCP): a nonzero closure holonomy obstructs branch recombination. Corollary 3.2 composes the two into the Trap–Holonomy Correspondence (dynamical trap ⟺ $\Sigma_\gamma \rho \neq 0$ ⟺ closed-non-exact). RCP is the entire residue of the former "closure-holonomy hypothesis," now shrunk to one sufficiency clause and realized — in a sister register, stronger than motivation but short of measuring κ — by spin-½, where a $\mathbb{Z}_2$ closure holonomy is measured (neutron interferometry) to obstruct branch recombination.

Assembling, under RCP: facts exist; facts require trapped cycles, forced onto non-bounding cycles by closedness (a bounding cycle has zero holonomy, hence coincident labels, hence no trap); the trap's non-recombination is nonzero holonomy (Cor. 3.2); and the separation lemma shows global admissibility does not flatten it. Therefore the committed ledger realizes a closed-non-exact offset on a surviving cycle, and the sector is occupied — conditional on the existence of facts and on RCP (a premise with a worked empirical instance, not a bare posit).

The register of the occupant is $\mathbb{Z}_2$: the trap is binary (T2), storing one Landauer bit, and that realized bit *forces* the trap cycle to be $\mathbb{Z}_2$-visible — hence free, not order-7 torsion (an order-7 class is $\mathbb{Z}_2$-invisible, $\mathbb{Z}_7 \otimes \mathbb{Z}_2 = 0$, so would carry no bit). This establishes $\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_2)$ occupied, in agreement with the register capstone. It reselects the *occupant's* coefficient, not the transport connection's: the prior arc's $\mathbb{Z}_7$ connection and its flat classes in $H^1(\Gamma\_vac; \mathbb{Z}_7)$ are not retracted, but the realized charge a fact lays down is binary. Since $\mathrm{Hom}(\mathbb{Z}_7, \mathbb{Z}_2) = 0$, this $\mathbb{Z}_2$ trap charge cannot be the reduction of a $\mathbb{Z}_7$ holonomy — it is a distinct homological object on the same complex, and the casualty is specifically *Uniform Readout* §7's order-7 torsion *occupant*, not the connection.

The one open seam is symmetric with the register capstone's: a seven-valued occupant would require a $\mathbb{Z}_7$ connection increment on the fold interface, which the corpus does not supply and which the Fold Interface Law (giving a $\mathbb{Z}_2$-oriented inner-product matching) argues against. Occupancy is established in the two-valued register, conditional on its stated premises; the seven-valued reading remains a posit against the grain of the substrate.

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1. The Two Ledgers

The occupancy arc reached, in the companion, a fork phrased as two readings of Uniform Readout: either readout consults a single global record (UR-frame: every committed offset descends from a global vertex labelling, $A \subseteq B^1$, $\kappa = 0$), or it consults only local comparisons (UR-comp: offsets live in C^1 with no global section forced, $\kappa \neq 0$ possible). The fork was real but, as posed, it treated a single object — "the readout" — as the thing whose quantifier had to be fixed.

The dissolution is to observe that the programme already distinguishes two ledgers, and the fork conflated them.

The admissibility ledger D. The distinguishability structure: which states are distinguishable from which, the operational geometry, the Hilbert geometry, the Born-rule structure, the global consistency of *allowed* comparisons. This ledger has every appearance of being global — the geometry papers read as discovering a single coherent admissibility structure on the substrate. Call this ledger global; nothing here disputes it.

The commitment ledger R. The committed facts: the bits, the records, the irreversible local events. A fact is local (it isolates a region). A fold is local. A record is local. Nothing in the primitive generative chain lays down a *global* assignment; the chain commits local comparisons between neighbours. This ledger is, by the ontology of commitment, a ledger of edge data — comparisons $\rho : E \to \mathbb{Z}_7$ — not of a global vertex function $a : V \to \mathbb{Z}_7$.

The two ledgers do different jobs. D says *which comparisons are admissible*. R records *which comparisons were committed*. The companion's UR-frame reading is the assertion that these coincide — that the committed record inherits the global structure of the admissibility geometry, so that every committed ρ descends from a global section. The two-ledger view exposes that as a substantive claim about *inheritance*, not a reading of a single rule:

The occupancy question is no longer "is there a global ledger?" (the admissibility ledger may well be global) but "**does the committed bit-structure inherit the global ledger automatically?**" — and that is a much less obvious thing.

This reframing is the contribution of §1. It converts the fork from a choice between two quantifiers on one axiom into a question about whether one ledger (global admissibility) forces a property on another (committed integrability). The next two sections answer it: §2 shows admissibility does *not force* inheritance; §3 shows facthood *requires its failure*.

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2. The Ledger Separation Lemma

The first half of the answer is that the global admissibility ledger does not, by itself, force the committed record into B^1.

Lemma 2.1 (Ledger Separation) (structural restatement). Global consistency of the admissibility ledger D does not entail $A \subseteq B^1$.

This is not a fresh independent theorem; it is the two-ledger restatement of the companion's established result that finite pairwise distinguishability does not force $A \subseteq B^1$ (*Contextual Distinguishability and Coboundary Confinement*). Its contribution is the framing — locating the unforced inheritance in the structural mismatch between an edge-level ledger and a loop-level property — not new mathematical force.

Proof. $B^1 = \text{im } d^0$ is the space of exact offsets, $\rho = d^0 a$ for some $a : V \to \mathbb{Z}_7$. Membership of a committed configuration ρ in B^1 is equivalent to the vanishing of every loop sum: $\Sigma\_\gamma\ \rho \equiv 0$ for every cycle γ (since the exact offsets are exactly those whose holonomy vanishes on all cycles). This is a *global*, loop-level property of ρ.

The admissibility ledger D constrains comparisons *edge by edge*: it determines, for each oriented adjacency, which offset values are allowed across that interface. It contains no operation that forms or constrains loop sums — telescoping is a property of $d^0 a$ (a coboundary), and the face/closure conditions (BCB) impose vanishing only on *bounding* loops (the boundaries of 2-cells), giving flatness $A \subseteq Z^1$, not exactness. Nothing in D computes $\Sigma\_\gamma\ \rho$ on a cycle outside im ∂_2.

Therefore D cannot force $\Sigma\_\gamma\ \rho \equiv 0$ on a surviving (non-bounding) cycle. Exactness on such cycles — $A \subseteq B^1$ — does not follow from D being globally consistent. It would require a *separate* principle that acts at the loop level (the UR-frame reading: $\rho = d^0 a$ imposed for every committed ρ); D does not supply it.

What the lemma establishes, and what it does not. This is a *separation*, not a realization. It proves: global admissibility does not *force* integrability — the inheritance is unforced from the geometry side. It does **not** prove: admissibility *permits* a frustrated cycle (that some admissible committed ρ has nonzero loop sum). The distinction is exactly the one the arc has been disciplined about. A proof of the stronger statement by exhibiting a frustrated triangle and noting it has no global section is circular: it assumes the frustrated configuration is admissible rather than deriving it. Lemma 2.1 avoids that circle by arguing from the *structure* of the two ledgers — admissibility constrains edges, integrability is a loop property, D computes no loop sums — and so it earns only the separation, honestly. The realization is supplied not by D but by the trapping necessity of §3.

The consequence for the fork. With Lemma 2.1, the UR-frame reading is no longer a neutral coin-flip alternative to UR-comp. UR-frame requires a global-section-forcing principle over and above global admissibility, and the companion's scoping found no such principle among the enumerated axioms (with conservation and closure-consistency proven *not* to supply it). The two-ledger view explains *why* none was found: the only ledger that is global is the admissibility ledger, and admissibility is structurally the wrong kind of object to force a loop-level identity. The burden sits squarely on anyone asserting $A \subseteq B^1$ to name the loop-level principle that does the forcing.

Facts are edge data, not vertex data. There is a further, ontological reason UR-frame looks like imported structure rather than substrate physics. A global ledger is a vertex function $a : V \to G$ — a value assigned to each site. A committed fact is a comparison $\rho : E \to G$ — a value on an *interface* between sites. The generative chain of the programme commits facts on interfaces: a fold is an interface structure, a commitment is an irreversible event *across* an adjacency, a record registers a *relation* between committed sites. Nothing in the primitive chain assigns a freestanding value to an isolated vertex; there is no generative act whose output is a global site-labelling. The natural home of committed data is therefore C^1 (edge cochains), and a global section $a \in C^0$ is a derived, non-primitive object — exactly the kind of structure UR-frame would need to impose from outside. This does not *prove* $A \not\subset B^1$ (an edge cochain can still happen to be exact, $\rho = d^0 a$, even if a is not itself committed). But it shifts the prior: the committed ledger lives in C^1 by construction, and the claim that every committed C^1 configuration descends from a global C^0 labelling is the claim that the substrate's edge-level commitments always conspire to be a coboundary — a strong, unforced coincidence, not a structural default. The default, from the ontology of commitment, is that facts are edge data that need not globalize.

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3. The Trapping Necessity

The separation lemma clears the global-geometry obstruction; it does not yet realize a surviving charge. Realization comes from a result already proven in the programme, in *The Topological Threshold for Fact Formation*, which supplies exactly the loop-level content the admissibility ledger lacks — and supplies it as a *necessity*, not an option.

A note on coefficients before we begin. The commitment ledger of §1 was written $\mathbb{Z}_7$-valued, following the prior arc's framing coefficient. But the trapped distinction a fact lays down is binary (T2 below: *which* alternative was quarantined), so the offset that carries the realized charge is $\mathbb{Z}_2$-valued. We therefore work in $\mathbb{Z}_2$ from here, and return in §5 to why this is the full register of the occupant and how it stands to the $\mathbb{Z}_7$ transport connection of the prior arc — which is not retracted.

(T1) A fact requires a cycle. The Topological Threshold paper proves that no bit can be irreversibly fixed on a substrate with first Betti number $\beta_1 = 0$. On a tree, any separation of a committed alternative from its discarded counterpart requires global surgery — there is no local rerouting that traps one branch while the other continues (the tree-impossibility theorem). A cycle provides the necessary slack: one alternative is rerouted into the loop while the other proceeds. Hence $\beta_1 \geq 1$ is *necessary* for any fact, and from this the paper derives Landauer's $k_B \ln 2$ and the arrow of time. Cycles are not one branch of a topological option (the companion's "T1 branch"); they are *required* for facthood to exist at all.

(T2) A fact is a cycle-trap. The discarded alternative does not vanish — it is quarantined, circulating on the loop, inaccessible to recombination (Topological Threshold, §5.2). Forming a fact *is* trapping the discarded alternative in a cycle. There is therefore no separate "is the cycle realized?" question to settle: the isolating fact realizes its cycle by definition. This closes the realization gap that the separation lemma left open — the gap is closed not by admissibility *permitting* a frustrated cycle, but by facthood *being* a trapped cycle.

(T3) Non-recombination is constitutive of facthood (dynamical, imported). A fact is a non-recombinable separation: the two alternatives never rejoin under forward iteration (Topological Threshold, §5.2). Non-recombination is not a property a fact might happen to have; it is what makes a committed distinction a fact rather than a reversible tick. This is the *dynamical* statement, and it is what the Topological Threshold paper proves.

What that paper proves is dynamical (recombination of trapped branches); what occupancy needs is cohomological (nonzero loop sum, ρ not exact). These are not the same statement, and the bridge between them is the load-bearing step of this paper. It must be proved as far as it can be and its irreducible residue stated openly, not smuggled through a word that does double duty. This section does that: one direction is a theorem with no physical premise; the other rests on a single, named, corpus-realized premise, and nothing more.

Theorem 3.1 (Exactness ⇔ branch-label coincidence) (proven, no physical premise). Let ρ be a closed $\mathbb{Z}_2$ offset on Γ_vac. Then ρ is exact ($\rho = d^0 a$, $\rho \in B^1$) if and only if transport governed by ρ is path-independent — equivalently, the committed and discarded branches carry equal transported labels around every cycle γ (their closure labels coincide after traversal).

Proof. (⟹) Exactness means $\rho_{uv} = a(v) - a(u)$ for a vertex function $a : V \to \mathbb{Z}_2$. The transported label accumulated along any path from u to w is the telescoping sum Σ ρ = a(w) − a(u), which depends only on the endpoints, not the path — this is the definition of path-independence. Around a closed cycle γ (u = w) the accumulated label is a(u) − a(u) = 0: Σ_γ ρ = 0. Two branches that diverge and rejoin at the cycle's far vertex therefore accumulate the same transported label (both equal a(far) − a(start)); carried around the full loop, the discarded branch's label returns to coincide with the committed branch's. (⟸) Conversely, suppose every loop sum vanishes, Σ_γ ρ = 0 for all cycles γ. Fix a spanning tree T of Γ_vac and a root r; define a(v) as the accumulated label along the unique tree-path r → v. For any edge ⟨u,v⟩, tree or not, the loop formed by the tree-paths r → u, the edge, and v → r is a cycle, on which the vanishing loop sum forces $\rho_{uv} = a(v) - a(u)$; hence $\rho = d^0 a$ is exact. (Equivalently, over the field $\mathbb{Z}_2$ the universal-coefficient theorem gives $H^1(\Gamma\_vac; \mathbb{Z}_2) \cong \mathrm{Hom}(H_1(\Gamma\_vac; \mathbb{Z}_2), \mathbb{Z}_2)$, so [ρ] = 0 iff ρ pairs to zero with every cycle class.) This is pure cochain arithmetic; no claim about dynamics is used.

The biconditional is unconditional. In particular its contrapositive: **if the two branches do not coincide in transported label around γ, then ρ is not exact, Σ_γ ρ ≠ 0** — nonzero holonomy. So *label non-coincidence* ⇔ *non-exactness* is a theorem, full stop. The hypothesis-laden step is not here; it is the move from this *kinematic* non-coincidence to the *dynamical* non-recombination that facthood (T3) asserts.

The irreducible premise, isolated and named. The Topological Threshold paper defines a trap *dynamically*: the discarded branch never rejoins the committed one under forward evolution. Theorem 3.1 speaks *kinematically*: transported closure labels coincide or do not. To identify them — to conclude that label-coincidence around γ is what permits the branches to physically re-merge — requires one substrate claim that no purely formal argument supplies:

Recombination–Coincidence Premise (RCP). Two branches can dynamically recombine across γ only if their transported closure labels coincide around γ; equivalently, a nonzero closure holonomy Σ_γ ρ ≠ 0 is an obstruction to recombination, and (in the $\mathbb{Z}_2$ setting) coincidence is sufficient for the closure structure to present no such obstruction.

RCP is the entire residue of the former "closure-holonomy hypothesis," now reduced to its irreducible core and separated from everything provable. It is a physical statement about VERSF — that the closure connection is what governs branch re-merging — not a definitional identity. It is *necessary*-direction-cheap and *sufficient*-direction-substantive: that holonomy obstructs recombination (necessary: recombination needs coincidence) is close to what "the substrate transports by this connection" means; that coincidence *suffices* for recombinability is the genuine physical content, since equal $\mathbb{Z}_2$ labels are necessary but not obviously sufficient for a dynamical re-merge path to exist.

Corollary 3.2 (Trap–Holonomy Correspondence) (proven, given RCP). For a cycle γ realized by a committed fact, with ρ the closed admissible $\mathbb{Z}_2$ offset:

dynamical trap on γ (non-recombination) ⟺ $\Sigma\_\gamma\ \rho \neq 0$ ⟺ ρ closed but not exact on γ.

Proof. By Theorem 3.1, non-exactness ⟺ label non-coincidence around γ, unconditionally. By RCP, label non-coincidence ⟺ recombination obstructed ⟺ dynamical trap. Composing the two equivalences gives the result. The second equivalence ($\Sigma\_\gamma\ \rho \neq 0$ ⟺ not exact for a closed ρ) is again pure cohomology. RCP enters once, at exactly the kinematic-to-dynamical seam, and nowhere else.

This is the honest structure: everything cohomological is a theorem (3.1); the dynamical bridge is one named premise (RCP); the correspondence (3.2) is their composition. The assumption is exactly one clause — RCP's sufficiency direction — and the upgrade over a bare hypothesis is real: the cohomological content is provable and only the kinematic-to-dynamical sufficiency is posited. It is not elimination, and the paper does not claim elimination.

Spin-½ realizes the RCP mechanism in a sister register. The reason RCP is not ad hoc is that VERSF already contains a worked, experimentally realized case in which a closure holonomy demonstrably *is* a recombination obstruction. Spin-½ (*Half-Integer Spin from Closure*) carries a closure phase $\varphi \in \mathbb{Z}_2$ that flips around a 2π loop. In neutron interferometry the two branches — rotated and unrotated — fail to recombine constructively precisely because their closure phases differ by the nontrivial $\mathbb{Z}_2$ element; the measured interference is the physical signature of non-coincident transported labels obstructing re-merging. This realizes the general form RCP names — a $\mathbb{Z}_2$ closure holonomy obstructing branch recombination — stronger than motivation, short of measuring κ. The carrier space differs: the spin phase lives on $\pi_1(SO(3)) = \mathbb{Z}_2$, not on $H^1(\Gamma\_vac)$, so this is not a measurement of the Gate-3 class κ, nor an instantiation of RCP *on Γ_vac*; identifying the two registers would be a separate claim. What it establishes is that the RCP *schema* — closure-label non-coincidence obstructing recombination — is realized physics in VERSF's most secure application, not a device introduced to close this paper. RCP is therefore the general form of a principle the framework has already paid for empirically in a sister register; it remains, for Gate-3's κ specifically, a premise rather than a measurement — but a premise with a worked instance, not a bare posit.

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4. From Separation to Occupancy

Assembling §§2–3:

1. **(Facts exist.)** The substrate carries committed facts — this is not in dispute; it is the precondition of records, time, and entropy. [given]
2. **(Necessity, T1–T2.)** Each fact requires and realizes a cycle: $\beta_1 \geq 1$ is necessary for facthood, and a fact *is* a trapped cycle. So cycles are realized by the committed ledger — necessarily, not optionally. [proven, Topological Threshold] 2′. **(The trap cycle is non-bounding.)** The trap cycle is not merely a cycle but a *non-bounding* one (outside im ∂_2). This is forced, not assumed: a bounding cycle is the boundary of a 2-cell, and on it the closed offset has forced-zero holonomy (BCB closedness: $\rho(\partial_2\sigma) = 0$). By Theorem 3.1 a cycle with zero holonomy has coincident branch labels, and by Corollary 3.2 (under RCP) carries no trap. So a trap cycle must have nonzero holonomy, which a bounding cycle cannot; the trap is forced onto a non-bounding cycle. (This is the companion's "closedness forces the witness onto an open cycle.") [proven, given closedness and RCP]
3. **(Non-recombination ⟹ nonzero holonomy, Corollary 3.2.)** A committed fact on the trap cycle is dynamically non-recombining (T3); by Theorem 3.1 (unconditional: non-exactness ⟺ label non-coincidence) composed with RCP (label non-coincidence ⟺ recombination obstructed), this is equivalent to nonzero $\mathbb{Z}_2$ closure holonomy, $\Sigma\_\gamma\ \rho \neq 0$, i.e. ρ closed but not exact. The cohomological half is a theorem; RCP enters once, at the kinematic-to-dynamical seam. [proven, conditional on RCP]
4. **(Separation, Lemma 2.1.)** Global admissibility does not force the committed ledger back into B^1: admissibility computes no loop sums and so cannot flatten the nonzero-holonomy cycle. [proven]
5. **(Therefore.)** The committed ledger realizes a closed (flatness, BCB) offset whose loop sum is nonzero on a surviving non-bounding cycle — a closed-non-exact admissible offset. By the companion's survival criterion this represents a nonzero class. **$A \not\subset B^1$; the sector is occupied** — conditional on the existence of facts (step 1) and RCP (steps 2′, 3); the cohomological content (Theorem 3.1) is unconditional. [proven, given 1–4 and RCP]

The structure of the argument is worth stating plainly because it is the inverse of the move the arc kept refusing. The earlier worry was: "is a frustrated cycle *admissible*?" — and answering it by positing one is circular. The trapping necessity inverts the burden: it is not that admissibility must be shown to *permit* frustration, but that *facthood requires* it. A non-frustrated committed cycle is not a fact (T3); a fact requires a cycle (T1) and is a trap on it (T2). So the frustrated cycle is not a configuration we must argue into admissibility — it is what every committed fact *is*. The separation lemma then guarantees the global geometry does not undo it.

This is the companion's missing Branch-B derivation. Occupancy is not "structurally motivated"; it is established — conditional on the existence of facts, the proven results of the Topological Threshold paper, and RCP (the single physical premise joining the dynamical trap to the cohomological charge, with the cohomological half itself a theorem, 3.1).

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5. The Register of the Occupant

Occupancy establishes that *a* charge survives. Its register is fixed by the same results, and it is $\mathbb{Z}_2$.

The trap of T2 is binary: it stores *which alternative was discarded* — one bit, k_B ln 2, a $\mathbb{Z}_2$ register (Topological Threshold's natural register throughout). The surviving holonomy is the two-valued "this cycle carries a trapped, non-recombined distinction." Three $\mathbb{Z}_2$ registers appear across the substrate — the fact-trapping bit (Topological Threshold), the fold orientation $\sigma, \omega \in \{\pm1\}$ (*Fold Interface Law*), the spin closure phase $\varphi \in \mathbb{Z}_2$ from $\pi_1(SO(3))$ (*Half-Integer Spin*). These are, as established, the *same group* $\mathbb{Z}_2$ in each case; whether they are the *same charge* (a single $\mathbb{Z}_2$ register seen three ways) or three distinct $\mathbb{Z}_2$ registers that coincide in group is not settled here, and the occupancy argument does not need it settled. What it needs is only that the occupant of the Gate-3 trap cycle is $\mathbb{Z}_2$-valued, which T2 supplies directly. The occupant is the class

$\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_2)$, occupied,

in agreement with the register capstone (*The Closure-Charge Register and the Fold Floor*), which reaches the same group from the register side via the Fold Uniqueness and Saturation theorems.

What reselects, and what does not. This paper concludes the *occupant* is $\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_2)$; the prior arc carried the Gate-3 sector at coefficient $\mathbb{Z}_7$, and *Uniform Readout and Global Integrability* §7 located a surviving "sevenness" as an order-7 torsion generator ($7\gamma = \partial_2\sigma$). It is important to be precise about which of these reselects. The $\mathbb{Z}_7$ *transport connection* — the D_7 structure and its flat classes in $H^1(\Gamma\_vac; \mathbb{Z}_7)$ — is *not* retracted; it remains the home of the connection's flat classes, exactly as the prior arc has it. What reselects is the coefficient of the *realized occupant*: the charge a fact actually lays down. That occupant is binary, because the trap is binary (T2), and so it is $\mathbb{Z}_2$-valued and $\mathbb{Z}_2$-detected.

The two are not in competition, and the arithmetic shows why they cannot even be mistaken for one another. Seven is prime, so $\mathrm{Hom}(\mathbb{Z}_7, \mathbb{Z}_2) = 0$: a genuinely $\mathbb{Z}_7$-valued holonomy on the free cycle has *no nonzero $\mathbb{Z}_2$ image*. The $\mathbb{Z}_2$ trap charge therefore cannot be the $\mathbb{Z}_2$-reduction of the $\mathbb{Z}_7$ connection's holonomy — were it such a reduction it would be forced to zero. It is a *distinct homological object on the same complex*: the binary class the committed fact lays down, living alongside (not derived from, not replacing) the connection's $\mathbb{Z}_7$ flat classes. The prior §7 order-7 torsion occupant is the casualty — it was the claim that the *realized* charge is seven-valued, and that is what the trapping necessity overturns by delivering a free cycle carrying a binary charge. The connection's $\mathbb{Z}_7$ classes survive; the seven-valued *occupant* does not.

The mechanism that makes this work is the *direction* of the inference, and getting it right closes a gap. It is tempting to argue "$\beta_1 \geq 1$ delivers a free cycle, and a free generator carries a nonzero $\mathbb{Z}_2$ charge" — but $\beta_1 \geq 1$ only guarantees free generators exist *somewhere*; it does not establish that the trap cycle γ the fact lays down is one of them. Step 2′ proves γ is non-bounding ($[\gamma] \neq 0$ in $H_1(\cdot; \mathbb{Z})$), and non-bounding is compatible with *torsion* — which is exactly what the prior arc's order-7 occupant was ($7\gamma = \partial_2\sigma$: non-bounding, yet seven copies bound). So freeness cannot be asserted; it must be earned, and the earning runs the other way.

The realized premise is T2: the trap stores one genuine bit, k_B ln 2 (Landauer) — a nonzero $\mathbb{Z}_2$ charge, derived rather than assumed. Now the arithmetic bites productively. An order-7 torsion summand of $H_1(\cdot;\mathbb{Z})$ is $\mathbb{Z}_2$-*invisible*: since 2 is invertible mod 7, both $\mathrm{Tor}(\mathbb{Z}_7, \mathbb{Z}_2) = 0$ and $\mathbb{Z}_7 \otimes \mathbb{Z}_2 = 0$, so by the coefficient sequence such a summand contributes nothing to $H_1(\cdot;\mathbb{Z}_2)$; and correspondingly $H^1(\cdot;\mathbb{Z}_2) = \mathrm{Hom}(H_1(\cdot;\mathbb{Z}_2), \mathbb{Z}_2)$ pairs to zero with it. A $\mathbb{Z}_2$-invisible cycle therefore carries *zero* $\mathbb{Z}_2$ holonomy — no bit. Composing: a realized nonzero binary trap (T2) cannot live on a $\mathbb{Z}_2$-invisible cycle; hence the trap cycle is $\mathbb{Z}_2$-visible; hence it is not order-7 torsion; hence it is free. Freeness is a *consequence* of the trap being a genuine bit, not a premise. This is what answers the sharp reader's objection — "what rules out the trap sitting on the prior arc's order-7 torsion cycle, which would be $\mathbb{Z}_2$-invisible and consistent with the empty-side reading?" — directly: a $\mathbb{Z}_2$-invisible trap would carry no bit, contradicting T2 and its Landauer cost. The occupant is therefore the $\mathbb{Z}_2$ charge on a free β_1-generator, and (by $\mathrm{Hom}(\mathbb{Z}_7, \mathbb{Z}_2) = 0$, which here does double duty) a distinct homological object from any order-7 torsion class. This is the same seam §6 marks, made explicit: the *realized* charge is binary on a free cycle, forced there by carrying a bit, and "seven was a count" is the consequence of that, not its justification.

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6. What Is and Is Not Closed

Stated without rounding, in the discipline the arc has held throughout.

Closed (conditional on its premises). Gate-3 is occupied in the $\mathbb{Z}_2$ register, on a free surviving cycle. The argument rests on: the existence of facts; the dynamical trapping-necessity results of the Topological Threshold paper ($\beta_1 \geq 1$ necessary, fact = cycle-trap); Theorem 3.1 (unconditional: exactness $\Leftrightarrow$ branch-label coincidence) composed via RCP into the Trap–Holonomy Correspondence (Corollary 3.2); and the Ledger Separation Lemma (global admissibility does not flatten the committed cycle). The only physical premise beyond the existence of facts is RCP — that a nonzero closure holonomy obstructs recombination — and it is realized in a sister register by spin-½. Under it, the empty-side reading is not merely unforced but incompatible with facthood: a globally-exact committed offset would, by Theorem 3.1, have coincident branch labels everywhere, hence (RCP) every branch recombining, hence no traps, hence no facts.

Not closed — the symmetric seam. A *seven-valued* occupant is not established and is not what this argument delivers. The surviving charge is $\mathbb{Z}_2$ because the trap is binary, and the occupied cycle is free rather than order-7 torsion — forced so by carrying a bit (§5), not assumed. To make the occupant genuinely $\mathbb{Z}_7$ — a seven-valued holonomy on the cycle — would require a $\mathbb{Z}_7$ connection increment τ on the fold interface (the offset would need a transition term beyond the telescoping site-index part; cf. the register capstone §6.5 and the telescoping decomposition $\rho = d^0\chi + \tau$, in which τ is a holonomy representative defined only up to a coboundary, requiring a choice of splitting $Z^1 \to H^1$ — it is not a canonical edge function). The corpus does not supply such a τ: the Fold Interface Law gives a $\mathbb{Z}_2$-oriented inner-product *matching* across the interface, which is the coboundary (reconciling) structure, not a $\mathbb{Z}_7$ twist; and independently of any derivation, "seven" surfaces across the corpus only as a count ($K = 7$ constraints, $N\_loop = 14$ channels, $12 = 8 + 3 + 1$ generators), never as a holonomy value — corroboration, not a second argument of equal standing. So a $\mathbb{Z}_7$ occupant is a posit the substrate does not support, and the fold law argues against it. There is a sharper reason still, from §5: a realized fact carries a bit

(Landauer), which forces its trap cycle to be $\mathbb{Z}_2$-visible, hence free, hence *not* order-7 torsion. A genuinely seven-valued occupant would have to live on an order-7 torsion cycle — which is $\mathbb{Z}_2$-invisible, carrying zero binary charge, i.e. *no bit*. So a $\mathbb{Z}_7$ occupant is not merely unsupported; it is in tension with facthood's own thermodynamic signature: it would require the trap to carry no bit at all, contradicting the very Landauer cost that makes the distinction a fact. This is the same seam the register capstone marks from the other side: it is one seam, seen twice.

The honest joint statement of the two capstones: **occupied (this paper), and $\mathbb{Z}_2$ (register capstone) — $\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_2)$, occupied — with a seven-valued occupant requiring fold-interface structure the corpus neither contains nor, via the Fold Interface Law, points toward.**

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7. The Test

The occupancy argument is operationally checkable at its load-bearing joint — the claim that committed facts are non-recombining traps rather than globally-recombining records.

Occupancy test. Inspect the committed-record structure for a fact that isolates a region. Does the discarded alternative remain quarantined — non-recombinable, never rejoining the committed branch under forward iteration (a nonzero trapped distinction on the isolating cycle)? Or does it recombine — admit a forward path returning it to contact, so that the committed distinction could in principle be undone?

The trapping necessity predicts the first: a fact whose branches recombined would be reversible, hence not a fact (T3), and tree-trapping is impossible (T1). The prediction is therefore sharp and falsifiable in the programme's own terms: **no committed fact will be found whose discarded branch recombines** — equivalently, every committed fact sits on a non-bounding cycle with nonzero trapped ($\mathbb{Z}_2$) holonomy. Finding a committed fact whose branches recombine would break the occupancy argument (and, by T3, would not have been a fact). Finding none — the predicted outcome — confirms occupancy.

Spin-½ is the sister-register realization, not a measurement of this class: a committed closure charge measured (neutron interferometry) whose branches fail to recombine around a 2π loop — but living on $\pi_1(SO(3)) = \mathbb{Z}_2$, not on $H^1(\Gamma\_vac)$. It is the existence proof that discrete non-recombining closure charges are physical; it does not directly measure the Gate-3 κ. The test asks whether the generic committed fact exhibits the same kind of non-recombination spin-½ does, and the trapping necessity says it must.

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8. Status and Conclusion

The occupancy arc reached, in *Uniform Readout and Global Integrability*, a single residual axiom-question. This paper resolves it, not by fixing a quantifier on Uniform Readout, but by replacing the fork with a distinction the programme already carries and then importing a result the programme has already proven.

What is established.

- The Uniform-Readout fork dissolves into two ledgers: admissibility (global) and commitment (local). The occupancy question becomes whether the committed ledger inherits global integrability — a question about inheritance, not about one rule's quantifier. [reframing]
- The Ledger Separation Lemma: global admissibility does not entail $A \subseteq B^1$, because B^1-membership is a loop-sum property and admissibility constrains only edges. The inheritance is unforced from the geometry side. [proven]
- The trapping necessity (Topological Threshold): facts require cycles ($\beta_1 \geq 1$ necessary), a fact *is* a cycle-trap, and non-recombination is constitutive of facthood. [proven, imported — dynamical] The cohomological identification has two parts: Theorem 3.1 (exactness $\Leftrightarrow$ branch-label coincidence) is unconditional and proved here; the kinematic-to-dynamical bridge is the single premise RCP (nonzero holonomy obstructs recombination), realized in a sister register by spin-½. Corollary 3.2 composes them. [3.1 proven; 3.2 conditional on RCP]
- Assembling: facts realize non-recombining trapped cycles (necessity), Corollary 3.2 identifies these with nonzero $\mathbb{Z}_2$ holonomy (Theorem 3.1 unconditional, RCP the one premise), and global admissibility does not flatten them (separation); hence $A \not\subset B^1$. **Gate-3 is occupied**, conditional on the existence of facts and RCP. [proven, given those premises]
- The occupant is the $\mathbb{Z}_2$ class, unifying with the fact-trapping, fold-orientation, and spin registers, and agreeing with the register capstone: **$\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_2)$, occupied.** [proven]

What remains open.

- A *seven-valued* occupant. The surviving charge is $\mathbb{Z}_2$ (binary trap). A $\mathbb{Z}_7$ occupant would require a fold-interface connection increment $\tau \in \mathbb{Z}_7$ that the corpus does not supply and that the Fold Interface Law ($\mathbb{Z}_2$-matching) argues against. This is the same seam the register capstone marks; it is not closed here and is, on present evidence, a posit against the grain of the substrate. [open]

The arc set out to decide occupancy. The verdict, carrying its condition: the sector is occupied — given that facts exist and the Recombination–Coincidence Premise (a nonzero closure holonomy obstructs recombination, realized in a sister register by spin-½) — because a fact is then a non-recombining trapped distinction whose nonzero holonomy global admissibility cannot flatten. The cohomological spine of that argument (exactness $\Leftrightarrow$ branch-label coincidence) is an unconditional theorem; RCP enters once, at the single kinematic-to-dynamical seam. The charge that occupies it is the two-valued class on a free surviving cycle — the register the substrate speaks in throughout, and a different homological object from the prior arc's order-7 torsion, which is $\mathbb{Z}_2$-invisible and now relegated. The seven was a count; the surviving holonomy is a bit. Conditional on those stated premises, Gate-3 is occupied, and $\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_2)$ is its occupant.

References (programme)

The load-bearing inputs are established in companion VERSF programme works: the occupancy reduction and the survival criterion (*Uniform Readout and Global Integrability*; *Contextual Distinguishability and Coboundary Confinement*); the register verdict (*The Closure-Charge Register and the Fold Floor*); the trapping-necessity results — $\beta_1 \geq 1$ necessary for facthood, fact-as-cycle-trap, non-recombinability — (*The Topological Threshold for Fact Formation*); the empirical discrete closure charge (*Half-Integer Spin from Closure*); and the fold-interface structure and its $\mathbb{Z}_2$ orientation (*the Fold Interface Law / minimal commitment structure at the void–universe boundary*; *The Fold and the Record*).