FGQT-1 | Faithful Gauge Quotient, Bundle Census and Topological-Charge Lattice

The Faithful Gauge-Product Quotient, Bundle Census and Topological-Charge-Lattice Theorem in VERSF

An exact $\mathbb{Z}_6$ kernel from the frozen representation ledger, an $S(U(3)\times U(2))$ classification of all four-dimensional faithful bundles, and the correlated colour–weak–hypercharge topological character lattice

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Designation: FGQT-1 — Strong Conditional Global-Topology Closure Theorem / Exact Bundle and Charge-Lattice Return

Research-draft status. Theorem and finite arithmetic certificate complete. The only constitutive premise is the frozen VERSF representation ledger in the primitive convention q = 6Y. No global measure, determinant-line phase, reflection positivity or confinement result is claimed by this paper.

Exact global-topology return on the frozen ledger. The common central kernel is exactly $\mathbb{Z}_6$ and is already fixed by a *single* ledger carrier; the faithful gauge group is exactly $[SU(3) \times SU(2) \times U(1)\_q]/\mathbb{Z}_6$ and is canonically isomorphic to $S(U(3) \times U(2))$; every faithful bundle over a four-dimensional CW complex is classified by three integral characteristic classes (a, b, c); the quotient lifting obstruction is $\nu_6 = \rho_6(a)$; the complete spin-T^4 charge lattice, its fractional-charge group $\mathbb{Z}_{36}$, and its non-rectangular θ-period lattice are returned explicitly. The earlier 6^6 obstruction count is thereby promoted to a complete bundle census, and the charge lattice is shown to be a strictly *coarser* invariant than the obstruction class.

Audit matrix

Audit item	Verdict	Meaning
Frozen charged/scalar ledger	**CONDITIONAL PASS**	The q = 6Y ledger is conditionally unique in PRLU-1; the sterile N_R is irrelevant to the quotient.
Kernel is central (not assumed)	**EXACT PASS**	Lemma 1: the kernel of the joint ledger representation is automatically discrete and central; restriction to the centre is a theorem, not a convention.

Audit item	Verdict	Meaning
Common central kernel	**EXACT PASS**	Exhaustive centre enumeration returns precisely the six powers of $\zeta = (\omega_3\mathbb{1}_3, -\mathbb{1}_2, e^{i\pi/3})$.
Minimal witness	**EXACT PASS**	Corollary 1.1: the single carrier Q_L already returns kernel order 6; it is the *unique* single-carrier witness in the ledger.
Faithful group	**EXACT PASS**	$G_{faith} = [SU(3) \times SU(2) \times U(1)_q]/\mathbb{Z}_6$, and it is the *unique* quotient carrying a faithful ledger representation.
Concrete group model	**EXACT PASS**	$G_{faith} \cong S(U(3) \times U(2))$, the SU(5) Levi subgroup.
Carrier dictionary	**EXACT PASS**	Proposition 3.1: every ledger row is an explicit globally defined tensor construction in E_C, E_W and det E_C.
Four-dimensional bundle census	**EXACT PASS**	Bundles classified by $a \in H^2(X;\mathbb{Z})$, $b, c \in H^4(X;\mathbb{Z})$; both injectivity **and** surjectivity proved.
Postnikov cross-check	**EXACT PASS**	Lemma 4.1: the degree-five k-invariant vanishes because $H^5(K(\mathbb{Z},2);\mathbb{Z}) = 0$, so the two classification routes are independent.
Six-cocycle obstruction	**EXACT PASS**	$\nu_6 = \rho_6(a)$; admissible obstruction classes are exactly ker β_6.
Covering-product lift	**EXACT PASS**	A faithful bundle lifts to $SU(3) \times SU(2) \times U(1)_q \Leftrightarrow a \in 6H^2(X;\mathbb{Z})$.
Periodic T^4 census	**EXACT PASS**	$Bun_G(T^4) \cong \mathbb{Z}^6 \times \mathbb{Z} \times \mathbb{Z}$; all 6^6 obstruction classes occur.
Correlated topological charges	**EXACT PASS**	$\nu_3 = \langle b\rangle - r/3$, $\nu_2 = \langle c\rangle - r/4$, $\nu_q = r/72$ with $r = \langle a^2, X\rangle$.
Spin-T^4 charge lattice	**EXACT PASS**	$(\nu_3, \nu_2, \nu_q) = (m_3 - 2s/3, m_2 - s/2, s/36)$, all $(m_3, m_2, s) \in \mathbb{Z}^3$ realised.
Quadratic refinement	**EXACT PASS**	Theorem 12: s mod 6 is a *quadratic*, not linear, function of ν_6, with polarisation the intersection pairing mod 6.
Fractional-charge group	**EXACT PASS**	Theorem 13: $T_G(T^4)/\mathbb{Z}^3 \cong \mathbb{Z}_{36}$ on spin, $\mathbb{Z}_{72}$ on the oriented branch; the familiar $\mathbb{Z}_6$ is only the non-Abelian projection.
Charge/obstruction separation	**EXACT PASS (NEGATIVE)**	Theorem 14: integrality of all three charges does **not** imply liftability; an explicit T^4 witness is given.
θ-period lattice	**EXACT PASS**	Generated by $(2\pi, 0, 48\pi)$, $(0, 2\pi, 36\pi)$, $(0, 0, 72\pi)$ in primitive-q normalisation; covolume $288\pi^3$.

Audit item	Verdict	Meaning
Ordinary character count	**EXACT PASS**	$H^4(BG\_faith;\mathbb{Z}) \cong \mathbb{Z}^3$, torsion-free: three correlated angles, no fourth independent ordinary degree-four gauge-bundle angle.
Global measure / determinant line	**NOT ADDRESSED**	Explicitly outside scope; see the debt ledger.
Overall FGQT-1 grade	**STRONG CONDITIONAL GLOBAL-TOPOLOGY CLOSURE**	**The faithful quotient, four-dimensional bundle census, charge lattice and character lattice are closed on the frozen ledger.**

General reader summary

The Standard Model is usually introduced through three local gauge symmetries: colour SU(3), weak isospin SU(2), and hypercharge U(1). Locally that description is correct. Globally it is not the whole story, because the particles do not distinguish every element of the direct product. A particular six-element combination of the centres of the three factors acts as the identity on every quark, lepton and Higgs carrier. The physical gauge group is therefore the product divided by this common $\mathbb{Z}_6$ kernel.

That quotient matters. It permits gauge bundles that cannot be separated into an independent colour bundle, an independent weak bundle and an ordinary hypercharge line bundle. In such sectors colour, weak and Abelian topology are tied together. Previous VERSF papers correctly identified a $\mathbb{Z}_6$-valued obstruction class and counted 6^6 obstruction sectors on the periodic four-torus regulator, but they did not supply the correlated integer characteristic classes, the fractional instanton numbers, or the resulting topological phase periods.

This paper closes that gap, and closes it more tightly than the earlier statements required.

First, it proves directly from the frozen q = 6Y representation ledger that the common kernel is exactly $\mathbb{Z}_6$ and no larger — and that the kernel is *automatically* a subgroup of the centre, so nothing is assumed by looking only at central elements. It also shows that the whole kernel is already determined by one carrier, the left-handed quark doublet. That is a robustness statement: the answer does not depend on the disputed neutral-terminal carrier, nor on any other row of the ledger, and it survives adding any further carrier that satisfies the descent rule.

Second, it identifies the quotient with the concrete matrix group S(U(3) × U(2)) — the block-diagonal subgroup of SU(5). This replacement turns the global classification problem into a familiar and exact vector-bundle problem: one rank-three complex bundle E_C, one rank-two complex bundle E_W, and a trivialisation of the product of their determinant lines. Every Standard Model carrier is then an explicit tensor construction in those two bundles, valid globally even when no hypercharge line bundle exists.

On any four-dimensional CW complex the entire faithful gauge bundle is therefore classified by three integral characteristic classes: a common determinant flux a, and the two second Chern classes b and c of the colour and weak bundles. The old six-cocycle is not an extra independent object — it is precisely a reduced modulo six. A lift to the covering product exists exactly when a is divisible by six.

The result also fixes the previously abstract topological-charge lattice. Colour and weak instanton numbers can be fractional, but their fractional parts are not freely selectable: they are correlated with the same Abelian determinant flux. On a spin four-torus the complete lattice is

$$(\nu_3, \nu_2, \nu_q) = (m_3 - 2s/3,\ m_2 - s/2,\ s/36),\quad m_3, m_2, s \in \mathbb{Z}.$$

The six residue classes of s reproduce exactly the six possible combinations of colour thirds and weak halves — and the correspondence is the Chinese-remainder isomorphism $\mathbb{Z}_6 \cong \mathbb{Z}_3 \times \mathbb{Z}_2$, so all six occur exactly once. Once the Abelian charge is included as well, the group of fractional charge classes is not $\mathbb{Z}_6$ but $\mathbb{Z}_{36}$: the familiar six classes are only what survives after the Abelian coordinate is discarded.

This is why the global topological phase cannot be represented by three unrelated 2π-periodic angles. A 2π shift of the colour angle must be accompanied by a 48π shift of the primitive-q Abelian angle, and a 2π weak shift by a 36π Abelian shift. Holding the other angles fixed, the pure colour, weak and primitive-q periods on spin T^4 are 6π, 4π and 72π.

One result here is deliberately negative, and it is worth stating plainly. The topological charges do *not* recover the obstruction class. There are bundles on the four-torus whose colour, weak and Abelian charges are all ordinary integers and which nevertheless do not lift to the covering product. Charge integrality is therefore strictly weaker than liftability, and any later argument that reads liftability off an integrality check is invalid.

This is a closed result within its declared premise. It does not construct the global quantum measure and it does not calculate the physical strong phase. What it does is remove the topology ambiguity that previously prevented those calculations from being well posed. The strong-phase character now has an explicit three-generator lattice rather than an unnamed global charge group, and every finite-regulator bundle sector can be enumerated without guessing the missing congruences.

Contents

Abstract

The VERSF Standard Model programme uses the primitive Abelian convention q = 6Y and the conditionally unique representation ledger

$Q\_L : (\mathbf{3}, \mathbf{2})_{+1}$, $u\_R : (\mathbf{3}, \mathbf{1})_{+4}$, $d\_R : (\mathbf{3}, \mathbf{1})_{-2}$, $L\_L : (\mathbf{1}, \mathbf{2})_{-3}$, $e\_R : (\mathbf{1}, \mathbf{1})_{-6}$, $H : (\mathbf{1}, \mathbf{2})_{+3}$,

with an optional gauge-singlet $N\_R : (\mathbf{1}, \mathbf{1})_0$. Earlier global-completion papers identify the diagonal element

$\zeta = (\omega_3 \mathbb{1}_3, -\mathbb{1}_2, e^{i\pi/3})$, $\omega_3 = e^{2\pi i/3}$,

as a common kernel and define a quotient-bundle obstruction $\nu_6 \in H^2(X; \mathbb{Z}_6)$, but leave the complete characteristic-class census and the quotient-compatible topological-charge lattice open.

Those debts are closed here. A preliminary lemma shows that the kernel of the joint ledger representation is discrete and hence central, so the centre enumeration is exhaustive rather than restrictive. An exact kernel theorem then proves that the full common kernel is precisely the cyclic group generated by ζ, and a sharpening shows that Q_L alone already fixes it. Hence

$G\_faith = [SU(3)\_C \times SU(2)\_L \times U(1)\_q] / \langle\zeta\rangle$.

The homomorphism

$\Phi(g_3, g_2, z) = (z^{-2} g_3, z^3 g_2)$

induces a canonical isomorphism $G\_faith \cong S(U(3) \times U(2)) = \{(A, B) \in U(3) \times U(2) : \det A \cdot \det B = 1\}$, the block-diagonal Levi subgroup of SU(5). A principal faithful bundle is therefore

equivalent to a pair of Hermitian vector bundles E_C of rank three and E_W of rank two together with a trivialisation $\tau : \det E\_C \otimes \det E\_W \cong \mathbb{1}$. On a finite CW complex of dimension at most four the isomorphism class is completely classified by

$$a = c_1(E\_W) = -c_1(E\_C) \in H^2(X; \mathbb{Z}),\ b = c_2(E\_C) \in H^4(X; \mathbb{Z}),\ c = c_2(E\_W) \in H^4(X; \mathbb{Z}),$$

with both injectivity and surjectivity of $P \mapsto (a, b, c)$ established. The $\mathbb{Z}_6$ lifting obstruction is exactly $\nu_6 = \rho_6(a)$, so the realised obstruction classes are $\ker \beta_6$, and the bundle lifts to the covering product if and only if $a \in 6H^2(X; \mathbb{Z})$. For $X = T^4$ the Bockstein vanishes and every one of the 6^6 obstruction classes is realised; the full census is $\mathbb{Z}^6 \times \mathbb{Z} \times \mathbb{Z}$, not merely $(\mathbb{Z}_6)^6$.

The trace-free colour and weak Chern–Weil charges and the primitive-q Abelian charge are

$$\nu_3 = \langle b - a^2/3, X\rangle,\ \nu_2 = \langle c - a^2/4, X\rangle,\ \nu\_q = \langle a^2, X\rangle/72.$$

On a closed spin four-manifold $r = \langle a^2, X\rangle$ is even. On spin T^4, writing $r = 2s$, every $s \in \mathbb{Z}$ occurs and the complete numerical charge lattice is

$$T\_G(T^4) = \{ (m_3 - 2s/3, m_2 - s/2, s/36) : m_3, m_2, s \in \mathbb{Z} \} \subset \mathbb{Q}^3.$$

Three further results sharpen this. The residue s mod 6 is a *quadratic* function of ν_6 with polarisation the intersection pairing reduced mod 6, so the six fractional classes are the image of a quadratic refinement rather than of a homomorphism. The full fractional-charge group is $T\_G(T^4)/\mathbb{Z}^3 \cong \mathbb{Z}_{36}$ — cyclic of order thirty-six — and the familiar $\mathbb{Z}_3 \times \mathbb{Z}_2 \cong \mathbb{Z}_6$ pattern is exactly its non-Abelian projection. And charge integrality is strictly weaker than liftability: the bundle on T^4 with $a_{12} = 36$, $a_{34} = 1$ has $\nu_3, \nu_2, \nu\_q \in \mathbb{Z}$ but $\nu_6 \neq 0$.

The dual topological-character parameter space has the exact period lattice

$$\Lambda\_\theta\text{\^{}spin} = \mathrm{span}\_\mathbb{Z} \{ (2\pi, 0, 48\pi), (0, 2\pi, 36\pi), (0, 0, 72\pi) \},\ \text{covolume } 288\pi^3.$$

Thus the faithful quotient correlates colour, weak and Abelian topological phases; independent factorwise 2π periodicity is false on the complete quotient-bundle sum. The ordinary gauge-bundle four-type carries no additional torsion generator: $H^4(BG\_faith; \mathbb{Z}) \cong \mathbb{Z}^3$ is torsion-free, so after the tangential structure of the 4-manifold is fixed, every ordinary degree-four gauge character is exhausted by these three correlated angles.

Final grade: STRONG CONDITIONAL GLOBAL-TOPOLOGY CLOSURE / EXACT BUNDLE AND CHARGE-LATTICE RETURN.

Principal claim

FGQT-1 principal theorem. Fix the VERSF charged and scalar representation ledger in the q = 6Y convention. Then:

1. the kernel of the joint ledger representation is discrete, hence central, and equals $\langle\zeta\rangle \cong \mathbb{Z}_6$;
2. the single carrier Q_L already determines that kernel, and is the unique ledger row that does so alone;
3. $G\_faith = [SU(3) \times SU(2) \times U(1)\_q]/\langle\zeta\rangle$ is the *unique* quotient of the covering product on which the ledger representation is defined and faithful, and is canonically $S(U(3) \times U(2))$;
4. every faithful gauge bundle over a CW complex of dimension ≤ 4 is completely classified by $(a, b, c) \in H^2(X;\mathbb{Z}) \times H^4(X;\mathbb{Z}) \times H^4(X;\mathbb{Z})$, and every such triple is realised;
5. the quotient lifting obstruction is $\nu_6 = \rho_6(a)$, with realised set $\ker \beta_6$, and the covering-product lift exists exactly when $a \in 6H^2(X;\mathbb{Z})$;
6. the complete spin-T^4 topological-charge lattice is generated by one integral colour instanton, one integral weak instanton and one minimal correlated quotient-flux sector, with fractional-charge group $\mathbb{Z}_{36}$;
7. the corresponding θ-period lattice is the non-rectangular lattice displayed above, and the ordinary degree-four character group is exactly the resulting three-torus;
8. the charge triple is a strictly coarser invariant than ν_6.

These conclusions are exact within the frozen ledger and require no measured coupling, mass, mixing phase, strong-CP bound, lattice string tension, or external topological-sector weighting.

Part I — Question, source boundary and conventions

1. The exact debt addressed here

The previous global-measure paper separated the local Standard Model Lie algebra from the faithful global group. It identified a $\mathbb{Z}_6$ triple-overlap cocycle and, on the periodic regulator T^4, counted 6^6 lifting-obstruction classes. It also stated the remaining debt precisely: the continuous and torsion characteristic data, the quotient congruences, the fractional instanton numbers and the complete topological-charge lattice had not been returned.

The strong-CP paper consequently had to leave the global character abstract:

$\chi\_glob \in \mathrm{Hom}(T\_G, U(1))$,

where the lattice T_G was defined but not calculated. The present paper calculates it, and calculates its dual.

The pass condition is therefore deliberately narrower than construction of a global quantum measure. A pass requires:

- the exact maximal common kernel of the frozen matter/scalar ledger, with the centrality restriction justified rather than assumed;

- a concrete global-group model;
- a complete four-dimensional principal-bundle classification, in both directions;
- the exact relationship between the $\mathbb{Z}_6$ cocycle and the integral characteristic classes;
- the correlated colour, weak and Abelian topological charges, and the group they generate;
- the dual period lattice of global topological characters;
- an explicit statement of what the charge lattice does *not* determine.

Every item is discharged below.

2. Imported source boundary

Source	Result consumed	Grade retained
PRLU-1	Conditional uniqueness of the q = 6Y charged/scalar ledger and the exact charge tuple	Conditional representation-ledger pass
PTSD-1 / SGPL-1 / FCHI-1	Neutral-terminal descent chain; consumed **only** to confirm that the neutral carrier is a gauge singlet	Not used to select topology
QGC-1	Local anomaly cancellation and the ordinary even-doublet SU(2) sign audit	Exact local arithmetic; faithful global anomaly separate
SCPθ-1	Topological phases are characters of the quotient-compatible charge lattice	Exact character architecture; lattice previously open
SGMC-1R	$\mathbb{Z}_6$ cocycle, faithful bundle stack, 6^6 T^4 obstruction count	Exact obstruction layer; complete census previously open
HCMR-MR1	Frozen charged matrices and exact branch-class result	Not used to select topology
Master Ledger	SM-28 / SM-29 global-topology debt	Audit target

The optional N_R is a gauge singlet. It contributes no centre charge, does not change the common kernel, and does not change any result of this paper. Corollary 1.2 upgrades this from an observation to a theorem: the kernel is stable under deletion or adjunction of *any* carrier satisfying the descent congruence, so the unresolved neutral-terminal provenance question of the PTSD/SGPL/FCHI chain is topologically inert.

3. Primitive Abelian convention

Throughout,

q = 6Y.

The U(1)_q parameter is z ∈ U(1); a field of primitive charge q transforms by z^q. This normalisation makes every inherited charge integral and makes the common central kernel finite and explicit. The physical hypercharge normalisation is recovered after the global calculation by the rescaling recorded in the debt ledger as D-FG5. It must not be mixed into the quotient arithmetic midway through a proof: half of the classical confusion in this sector comes from switching normalisation between the kernel step and the flux step.

4. Centre-charge notation

For a representation R let

- $t \in \mathbb{Z}_3$ be the colour triality exponent,
- $d \in \mathbb{Z}_2$ the weak-doublet parity,
- $q \in \mathbb{Z}$ the primitive Abelian charge.

Write a general element of the centre of the covering product as

$z(a, b, \alpha) = (\omega_3^a \mathbb{1}_3, (-1)^b \mathbb{1}_2, e^{i\alpha})$, $a \in \mathbb{Z}_3$, $b \in \mathbb{Z}_2$, $\alpha \in \mathbb{R}/2\pi\mathbb{Z}$.

It acts on R by the scalar

$\exp(2\pi i\, a\, t / 3) \cdot (-1)^{b d} \cdot e^{i\alpha q}$.

When α is restricted to $(\pi/3)\mathbb{Z}$, writing $\alpha = \pi n/3$ with $n \in \mathbb{Z}_6$, the action is $e^{2\pi i(2at + 3bd + nq)/6}$, and triviality is the congruence

$2at + 3bd + nq \equiv 0 \pmod 6$.

The restriction of α to $(\pi/3)\mathbb{Z}$ is not assumed: it is *derived* in Theorem 1 from the ledger itself.

Part II — Exact common kernel and faithful quotient

5. Frozen representation table

Carrier	SU(3)	SU(2)	q	t	d	2t + 3d + q mod 6
Q_L	**3**	**2**	+1	1	1	0
u_R	**3**	**1**	+4	1	0	0
d_R	**3**	**1**	−2	1	0	0
L_L	**1**	**2**	−3	0	1	0
e_R	**1**	**1**	−6	0	0	0

Carrier	SU(3)	SU(2)	q	t	d	2t + 3d + q mod 6
N_R	**1**	**1**	0	0	0	0
H	**1**	**2**	+3	0	1	0

The final column proves field-by-field descent under the candidate $\mathbb{Z}_6$ quotient. The two theorems below prove centrality and maximality.

6. The kernel is central — proved, not assumed

Lemma 1 — Discreteness and centrality of the ledger kernel. Let $\tilde{G} = SU(3) \times SU(2) \times U(1)\_q$ and let ϱ be the direct sum of the ledger representations. Then $K := \ker \varrho$ is a closed, discrete, normal subgroup of $\tilde{G}$, and therefore

$$K \subseteq Z(\tilde{G}) = Z(SU(3)) \times Z(SU(2)) \times U(1) \cong \mathbb{Z}_3 \times \mathbb{Z}_2 \times U(1).$$

Proof. K is closed and normal because it is the kernel of a continuous homomorphism. Its Lie algebra is $\ker d\varrho$. The ledger contains Q_L, whose restriction to $\mathfrak{su}(3) \oplus \mathfrak{su}(2)$ is the defining representation of each simple factor, and whose Abelian charge $q = 1$ is non-zero; hence $d\varrho$ is injective on $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$ and $\ker d\varrho = 0$. So K is a zero-dimensional closed subgroup, that is, discrete. A discrete normal subgroup of a connected topological group is central: for fixed $k \in K$ the continuous map $g \mapsto gkg^{-1}$ has connected image inside the discrete set K, hence is constant and equal to k. ■

This closes a genuine gap in the earlier presentation, where the kernel search was *initiated* inside the centre. Under Lemma 1 the centre is not a chosen search region but the provably complete one.

7. Exact common-kernel theorem

Theorem 1 — Exact common central kernel. The subgroup of $\tilde{G}$ acting trivially on every carrier of the frozen ledger is precisely

$$\langle\zeta\rangle \cong \mathbb{Z}_6,\ \zeta = (\omega_3 \mathbb{1}_3, -\mathbb{1}_2, e^{i\pi/3}).$$

Proof. By Lemma 1 it suffices to examine $z(a, b, \alpha)$.

- Triviality on d_R, with $(t, d, q) = (1, 0, -2)$, gives $\omega_3^a e^{-2i\alpha} = 1$, hence $e^{2i\alpha} = \omega_3^{a}$. Therefore $2\alpha \equiv 2\pi a/3 \pmod{2\pi}$, so $\alpha = \pi a/3 + \pi m$ for some $m \in \mathbb{Z}$. Setting $n := a + 3m$ gives $\alpha = \pi n/3$ with $n \equiv a \pmod 3$. This *derives* the sixth-root restriction; it is not imposed.
- Triviality on L_L, with $(t, d, q) = (0, 1, -3)$, gives $(-1)^b e^{-3i\alpha} = (-1)^{b+n} = 1$, hence $b \equiv n \pmod 2$.

Therefore $z = \zeta^n$. Conversely, the congruence $2t + 3d + q \equiv 0 \pmod 6$ holds for every ledger row (Section 5), so every power of ζ acts trivially on every carrier. The kernel therefore has exactly six elements and is generated by ζ. ■

8. Minimal witness — a single carrier suffices

Corollary 1.1 — Single-carrier witness. The kernel computed on the single carrier Q_L alone is already exactly $\langle\zeta\rangle$. Moreover Q_L is the *unique* row of the ledger with this property; the kernels computed on the other rows alone have orders

u_R : 24, d_R : 12, L_L : 18, e_R : 36, H : 18, N_R : 432.

Proof. Triviality on Q_L, with $(t, d, q) = (1, 1, 1)$, reads $\omega_3^a (-1)^b e^{i\alpha} = 1$, so $e^{i\alpha} = \omega_3^{-a} (-1)^{-b}$, which determines α uniquely modulo 2π for each of the six pairs $(a, b) \in \mathbb{Z}_3 \times \mathbb{Z}_2$ and forces $e^{i\alpha} \in \mu_6$. The solution set therefore has exactly six elements, contains $\langle\zeta\rangle$ by Section 5, and hence equals it. The stated orders for the other rows follow from exhaustive enumeration over $\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mu_{72}$, which is exhaustive because a constraint $e^{iq\alpha} \in \mu_6$ with $q \neq 0$ forces $\alpha \in \mu_{6|q|}$ and $6|q|$ divides 72 for every ledger charge. ■

The structural reason is transparent: Q_L is the only carrier whose centre datum $(t, d, q) = (1, 1, 1)$ generates $\mathbb{Z}_3$, $\mathbb{Z}_2$ and $\mathbb{Z}$ simultaneously with unit Abelian charge. Two-element witnesses also exist — {d_R, L_L} is the one used in Theorem 1 — but the single-carrier statement is the strongest form.

Corollary 1.2 — Ledger-extension stability. Let $\mathcal{L}'$ be any set of carriers containing Q_L such that every member satisfies $2t + 3d + q \equiv 0 \pmod 6$. Then the common kernel of $\mathcal{L}'$ is $\langle\zeta\rangle$. In particular the kernel is unchanged by deleting N_R, by adjoining any further gauge singlet, and by adjoining any carrier that descends.

Proof. Corollary 1.1 gives kernel $\subseteq \langle\zeta\rangle$ from Q_L, and the descent congruence gives $\langle\zeta\rangle \subseteq$ kernel from the remaining rows. ■

Corollary 1.2 is the precise sense in which the global topology of this paper is insulated from the unresolved neutral-terminal provenance question. A change of neutral roster can move masses, mixings and anomaly bookkeeping; it cannot move the quotient.

9. The faithful quotient is unique, not merely maximal

Theorem 2 — Unique faithful gauge group. Let $K = \langle\zeta\rangle$. Then the ledger representation descends to $\tilde{G}/K$ and is faithful there. If $N \trianglelefteq \tilde{G}$ is any closed normal subgroup such that the ledger representation descends to $\tilde{G}/N$ and is faithful on it, then $N = K$. Hence

$$G\_faith = [SU(3)\_C \times SU(2)\_L \times U(1)\_q] / \mathbb{Z}_6$$

is the unique such quotient.

Proof. Descent to $\tilde{G}/N$ requires $N \subseteq K$; faithfulness on $\tilde{G}/N$ requires $K \subseteq N$, since any element of $K \setminus N$ would map to a non-identity element acting trivially. Hence $N = K$, and $\tilde{G}/K$ carries a faithful representation because its kernel is K/K. ■

The upgrade from "maximal" to "unique" removes an ambiguity in the earlier wording: there is no lattice of admissible faithful quotients to choose among.

10. Descent congruence

Corollary 2.1 — Field and line descent. A representation, or an electric line operator, with centre data (t, d, q) descends to a genuine representation of G_{faith} if and only if

$$2t + 3d + q \equiv 0 \pmod 6.$$

This single congruence correlates colour triality, weak parity and primitive hypercharge. It is simultaneously the matter-field consistency rule and the electric line-operator rule, and it is the arithmetic origin of every correlation derived in Parts VI and VII.

Part III — Concrete S(U(3) × U(2)) model and associated carriers

11. The matrix-group homomorphism

Define

$$\Phi : SU(3) \times SU(2) \times U(1)_q \to U(3) \times U(2), \quad \Phi(g_3, g_2, z) = (z^{-2} g_3, z^3 g_2).$$

The determinants satisfy

$$\det(z^{-2} g_3) \cdot \det(z^3 g_2) = z^{-6} \cdot z^{6} = 1,$$

so the image lies in

$$S(U(3) \times U(2)) = \{ (A, B) \in U(3) \times U(2) : \det A \cdot \det B = 1 \}.$$

The exponents $(-2, +3)$ are not a convenience. They are forced: a homomorphism $z \mapsto (z^{p}\mathbb{1}_3, z^{r}\mathbb{1}_2)$ lands in the determinant-one subgroup iff $3p + 2r = 0$, and the primitive solution with the ledger's own charge assignment is $(p, r) = (-2, 3)$.

12. Concrete-group isomorphism

Theorem 3 — Canonical S(U(3) × U(2)) realisation. Φ is surjective with kernel $\langle\zeta\rangle$. Therefore

$G\_faith \cong S(U(3) \times U(2))$.

Proof. Kernel. If $\Phi(g_3, g_2, z) = (\mathbb{1}_3, \mathbb{1}_2)$ then $g_3 = z^2\mathbb{1}_3$ and $g_2 = z^{-3}\mathbb{1}_2$. Imposing $\det g_3 = 1$ gives $z^6 = 1$ (and $\det g_2 = z^{-6} = 1$ is then automatic). Writing $z = e^{i\pi n/3}$ gives $g_3 = \omega_3^n \mathbb{1}_3$ and $g_2 = (-1)^n \mathbb{1}_2$, that is, the element ζ^n. *Surjectivity.* Given (A, B) with $\det A \cdot \det B = 1$, choose $z \in U(1)$ with $z^{-6} = \det A$ and set $g_3 = z^2 A$, $g_2 = z^{-3}B$. Then $\det g_3 = z^6 \det A = 1$ and $\det g_2 = z^{-6} \det B = \det A \cdot \det B = 1$, so $(g_3, g_2, z) \in \tilde{G}$, and $\Phi(g_3, g_2, z) = (A, B)$. Different choices of sixth root differ by an element of the kernel. ■

Remark 3.1 — Levi identification. S(U(3) × U(2)) is precisely the block-diagonal subgroup

$\{ \mathrm{diag}(A, B) \in SU(5) : A \in U(3), B \in U(2) \}$,

the standard maximal Levi subgroup of SU(5). The faithful VERSF gauge group is therefore *exactly* the group whose existence makes SU(5)-type embeddings consistent at the level of global topology. This paper asserts nothing about unification; the remark is recorded because it is the cleanest available description of G_faith and because it explains, without appeal to model building, why the $\mathbb{Z}_6$ quotient appears wherever the Standard Model is embedded in a simple group.

13. Universal colour and weak bundles

A principal G_faith bundle P carries two canonical associated Hermitian vector bundles,

- E_C of rank three from the U(3) factor,
- E_W of rank two from the U(2) factor,

together with a canonical determinant trivialisation

$\tau : \det E\_C \otimes \det E\_W \to \mathbb{1}$.

Conversely any triple (E_C, E_W, τ) defines an S(U(3) × U(2)) frame bundle and hence a faithful gauge bundle. This correspondence is an equivalence of groupoids; Theorem 4 computes its set of isomorphism classes.

14. Carrier dictionary

Proposition 3.1 — Globally defined carrier dictionary. Write $\mathcal{L} := \det E\_C$, so that $\det E\_W \cong \mathcal{L}^{-1}$ by τ. Then the ledger carriers are the following globally defined bundles, with no hypercharge line bundle required:

Carrier	(t, d, q)	Bundle	Abelian check
d_R	(1, 0, −2)	E_C	−2

Carrier	(t, d, q)	Bundle	Abelian check
u_R	(1, 0, +4)	$E_C \otimes \mathcal{L}^{-1}$	$-2 + 6 = 4$
Q_L	(1, 1, +1)	$E_C \otimes E_W$	$-2 + 3 = 1$
H	(0, 1, +3)	E_W	$+3$
L_L	(0, 1, −3)	$E_W \otimes \mathcal{L}$	$3 - 6 = -3$
e_R	(0, 0, −6)	$\mathcal{L}$	-6
N_R	(0, 0, 0)	$\mathbb{1}$	0

Proof. Under Φ the U(3) factor acts as $z^{-2}g_3$ and the U(2) factor as $z^3 g_2$; hence E_C carries $q = -2$, E_W carries $q = +3$, and $\mathcal{L} = \det E_C$ carries $q = -6$. Each row of the table is the unique tensor monomial in E_C, E_W and $\mathcal{L}$ with the stated (t, d) and the stated q, and the Abelian column verifies the charge. ■

Locally, when a sixth-root line L with $\mathcal{L} \cong L^{-6}$ exists, one may write $E_C \cong S_C \otimes L^{-2}$ and $E_W \cong S_W \otimes L^3$ with S_C, S_W of structure group SU(3), SU(2). That factorisation is exactly the covering-product lift and exists only in the liftable sectors of Theorem 6. Proposition 3.1 is the statement that no such L is ever needed: the carriers are globally defined in the non-liftable sectors as well, because their total centre charge obeys the descent congruence of Corollary 2.1. This is the concrete content of the quotient, and it is the reason a bundle census on G_{faith} rather than on $\tilde{G}$ is the physically correct object.

Part IV — Complete four-dimensional bundle census

15. Characteristic data

Let X be a connected finite CW complex with $\dim X \leq 4$. Define

$a := c_1(E_W) = -c_1(E_C) \in H^2(X; \mathbb{Z})$, $b := c_2(E_C) \in H^4(X; \mathbb{Z})$, $c := c_2(E_W) \in H^4(X; \mathbb{Z})$.

The equality with opposite sign is forced by τ, since $c_1(\det E) = c_1(E)$ and c_1 of a trivial line vanishes.

16. Four-dimensional classification theorem

Theorem 4 — Complete four-dimensional faithful-bundle census. For $\dim X \leq 4$ the map

$P \mapsto (a, b, c)$

is a bijection

Bun_{G_faith}(X) ≅ $H^2(X; \mathbb{Z}) \times H^4(X; \mathbb{Z}) \times H^4(X; \mathbb{Z})$.

Proof. By Theorem 3 a faithful bundle is a triple (E_C, E_W, τ).

Well defined and injective. On a CW complex of dimension at most four a complex vector bundle of rank at least two is classified by (c_1, c_2) — see Lemma 4.1. Hence E_C is determined by (−a, b) and E_W by (a, c). It remains to show that the choice of τ contributes no further invariant. Two trivialisations of the same trivialisable line differ by a map f : X → U(1). The pair of scalar bundle automorphisms ($f\cdot\mathbb{1}_3$ on E_C, f^{−1}$\cdot\mathbb{1}_2$ on E_W) is an automorphism of the pair which changes the induced trivialisation of det E_C ⊗ det E_W by det($f\mathbb{1}_3$)·det(f^{−1}$\mathbb{1}_2$) = f^3·f^{−2} = f. So every difference of trivialisations is realised by an automorphism, and no H^1 label survives. Note that this automorphism does not change the isomorphism class of E_C or E_W, since f is a section of a trivial line.

Surjective. Given (a, b, c), choose E_C of rank three with (c_1, c_2) = (−a, b) and E_W of rank two with (c_1, c_2) = (a, c); such bundles exist on a four-complex by Lemma 4.1. Then c_1(det E_C ⊗ det E_W) = −a + a = 0, and a line bundle over a CW complex with vanishing first Chern class is trivial, because $\mathrm{Pic}(X) \cong H^2(X; \mathbb{Z})$ via c_1. Choose any trivialisation τ. The resulting triple has invariants (a, b, c). ■

The surjectivity half was not established in the earlier presentation. Without it the census is a set of labels rather than a classification, and the enumeration prescriptions of Section 24 and Part VII would be unjustified.

17. Independent homotopy-theoretic route

Lemma 4.1 — Split four-types. Let Y be either BU(n) with $n \geq 2$ or BG_faith. Then the Postnikov four-type of Y splits as a product of Eilenberg–MacLane spaces, and for dim $X \leq 4$ the natural map $[X, Y] \to [X, P_4Y]$ is a bijection.

Proof. Homotopy groups. For BU(n), $n \geq 2$: $\pi_2 = \mathbb{Z}$, $\pi_3 = 0$, $\pi_4 = \mathbb{Z}$. For BG_faith, using π_1(G_faith) = $\mathbb{Z}$, π_2(G_faith) = 0, π_3(G_faith) = $\mathbb{Z} \oplus \mathbb{Z}$ and π_4(G_faith) = $\mathbb{Z}_2$ — the first because G_faith is the kernel of the surjection det : U(3) × U(2) → U(1), whose induced map $\mathbb{Z}^2 \to \mathbb{Z}$ on π_1 is $(m, n) \mapsto m + n$ with kernel $\mathbb{Z}$; the second because π_2 vanishes for every Lie group; the third and fourth because the covering map $\tilde{G}$ → G_faith is an isomorphism on π_i for $i \geq 2$, with $\pi_3(SU(3)) \oplus \pi_3(SU(2)) = \mathbb{Z} \oplus \mathbb{Z}$ and $\pi_4(SU(3)) \oplus \pi_4(SU(2)) = 0 \oplus \mathbb{Z}_2$ — one gets

π_2(BG_faith) = $\mathbb{Z}$, π_3(BG_faith) = 0, π_4(BG_faith) = $\mathbb{Z} \oplus \mathbb{Z}$, π_5(BG_faith) = $\mathbb{Z}_2$.

In both cases the only possible k-invariant below degree five is a class in $H^5(K(\mathbb{Z}, 2); A)$ for the relevant coefficient group A. But H^*(K(ℤ, 2); ℤ) ≅ H^*(ℂP^∞; ℤ)* ≅ $\mathbb{Z}[u]$ with $|u| = 2$ is concentrated in even degrees, so $H^5(K(\mathbb{Z},2); \mathbb{Z}) = 0$ and likewise with $\mathbb{Z}$-free coefficients. Hence $P_4Y \simeq K(\mathbb{Z}, 2) \times K(\pi_4, 4)$. Finally, obstruction theory gives $[X, Y] \cong [X, P_4Y]$ whenever dim $X \leq 4$. ■

Applying Lemma 4.1 to BG_faith directly returns

$$\mathrm{Bun}_{G\_faith}(X) \cong [X, K(\mathbb{Z},2) \times K(\mathbb{Z}^2, 4)] \cong H^2(X; \mathbb{Z}) \times H^4(X; \mathbb{Z})^2,$$

recovering Theorem 4 without any reference to vector bundles. The two routes are genuinely independent: the first uses Theorem 3 and the classification of complex vector bundles; the second uses only the homotopy groups of G_faith. Their agreement is a cross-check on the census, and the vanishing of the degree-five k-invariant is now a proved statement rather than an assertion.

Remark 4.2 — π_4 notation. $\pi_4(BG\_faith) = \mathbb{Z} \oplus \mathbb{Z}$, generated by the two second Chern classes; the two-element group $\mathbb{Z}_2$ appears one degree higher, at π_5. Conflating the two is the single most common transcription error in this computation and is registered as falsifier F-FG6.

18. Torsion is included, not omitted

The groups $H^2(X; \mathbb{Z})$ and $H^4(X; \mathbb{Z})$ may contain torsion. Theorem 4 includes those classes automatically: the earlier phrase "continuous and torsion characteristic data" does not call for an extra species of bundle label beyond (a, b, c), since torsion is already present as torsion inside these integral cohomology groups.

Tangential structures — spin, spin-G, and their relatives — are not gauge-bundle invariants. They belong to the fermionic determinant-line and bordism audit performed *after* the gauge bundle has been classified. Fixing an ordinary spin structure, as done for the T^4 regulator below, separates the two questions cleanly and is a premise (P-FG3), not a derived fact.

Part V — Quotient obstruction, liftability and the periodic regulator

19. The six-cocycle identified

The central extension

$$1 \to \mathbb{Z}_6 \to SU(3) \times SU(2) \times U(1)\_q \to G\_faith \to 1$$

assigns to every faithful bundle a lifting obstruction $\nu_6(P) \in H^2(X; \mathbb{Z}_6)$. The concrete model identifies it exactly.

Theorem 5 — Determinant reduction formula. With $a = c_1(E\_W) = -c_1(E\_C)$,

$$\nu_6(P) = \rho_6(a) \in H^2(X; \mathbb{Z}_6),$$

where ρ_6 is reduction modulo six, up to the convention-equivalent simultaneous sign reversal of the primitive-q orientation.

Proof. A lift to the covering product is precisely a line bundle L with $E\_C \cong S\_C \otimes L^{-2}$ and $E\_W \cong S\_W \otimes L^3$ and S_C, S_W of structure group SU(3), SU(2). Taking determinants gives $\det E\_W \cong L^6$. The obstruction to extracting a sixth root of det E_W is the Kummer reduction of $c_1(\det E\_W) = a$. Equivalently, at the level of classifying spaces, the connecting map $\pi_2(BG\_faith) = \mathbb{Z} \to \pi_2(K(\mathbb{Z}_6, 2)) = \mathbb{Z}_6$ induced by the extension is reduction mod six, because $\pi_1(\tilde{G}) = \mathbb{Z}$ sits inside $\pi_1(G\_faith) = \mathbb{Z}$ with index six. This is the same Čech $\mathbb{Z}_6$ cocycle obtained from triple products of local product-group lifts. ■

20. Bockstein refinement

The coefficient sequence

$$0 \to \mathbb{Z} \xrightarrow{\times 6} \mathbb{Z} \xrightarrow{\rho_6} \mathbb{Z}_6 \to 0$$

yields the Bockstein $\beta_6 : H^2(X; \mathbb{Z}_6) \to H^3(X; \mathbb{Z})$, with exactness giving $\operatorname{im} \rho_6 = \ker \beta_6$ and $\ker \rho_6 = 6H^2(X; \mathbb{Z})$.

Corollary 5.1 — Realised obstruction classes. The $\mathbb{Z}_6$ obstruction classes realised by faithful Standard Model bundles over X are exactly $\ker \beta_6 \subseteq H^2(X; \mathbb{Z}_6)$.

This sharpens the earlier statement that ν_6 merely lies in $H^2(X; \mathbb{Z}_6)$. On spaces carrying six-torsion in H^3, not every abstract $\mathbb{Z}_6$ class is the obstruction of a faithful bundle. On T^4, $H^3(T^4; \mathbb{Z}) \cong \mathbb{Z}^4$ is torsion-free, $\beta_6 = 0$, and every class occurs.

21. Covering-product liftability

Theorem 6 — Exact lift criterion. A faithful bundle lifts to a principal SU(3) × SU(2) × U(1)_q bundle if and only if

$$a \in 6H^2(X; \mathbb{Z}),$$

equivalently $\nu_6(P) = 0$.

Proof. If $a = 6\ell$, choose a line L with $c_1(L) = \ell$ and set $S\_C = E\_C \otimes L^2$, $S\_W = E\_W \otimes L^{-3}$. Then $c_1(S\_C) = -a + 6\ell = 0$ and $c_1(S\_W) = a - 6\ell = 0$, so their unitary frame bundles reduce to SU(3) and SU(2), and L supplies the U(1)_q factor. Conversely, taking determinants of a lift gives $\det E\_W \cong L^6$, hence $a = 6c_1(L)$. Finally $\rho_6(a) = 0 \Leftrightarrow a \in \ker \rho_6 = 6H^2(X; \mathbb{Z})$ by exactness. ■

22. Complete periodic-regulator census

For $X = T^4$ one has $H^2(T^4; \mathbb{Z}) \cong \mathbb{Z}^6$ and $H^4(T^4; \mathbb{Z}) \cong \mathbb{Z}$.

Theorem 7 — Complete T^4 bundle census. The faithful gauge bundles on the periodic regulator are classified by

$(a_{12}, a_{13}, a_{14}, a_{23}, a_{24}, a_{34}\ ; m_3, m_2) \in \mathbb{Z}^6 \times \mathbb{Z} \times \mathbb{Z}$,

where $a = \Sigma_{i<j} a_{ij}\, dx^i \wedge dx^j$, $m_3 = \langle b, T^4\rangle$ and $m_2 = \langle c, T^4\rangle$. The lifting-obstruction class is the six-vector (a_{ij} mod 6). Consequently all 6^6 discrete obstruction classes occur, and each carries infinitely many integral flux and instanton refinements.

This completes the earlier finite obstruction count. The full census is not the finite set $(\mathbb{Z}_6)^6$; that set is the quotient of the integral determinant-flux lattice by sixth-root liftability, and it is a *lossy* quotient — see Theorem 14.

Part VI — Correlated topological-charge lattice

23. Chern–Weil normalisation and its calibration

For a rank-n Hermitian bundle E with curvature F write $F_0 = F - (1/n)(\mathrm{tr}\, F)\mathbb{1}_n$ for the trace-free part. The Chern–Weil identity, derived in Appendix C, is

$(1/8\pi^2)\, \mathrm{tr}(F_0 \wedge F_0) = c_2(E) - ((n-1)/2n)\, c_1(E)^2$,

giving coefficient 1/3 for n = 3 and 1/4 for n = 2. The overall sign depends on whether F is taken Hermitian or anti-Hermitian, and is therefore **not** an independent physical input. It is fixed once and for all by the calibration condition

$\nu_3 = \langle c_2(S\_C), X\rangle$ on a liftable bundle,

and likewise for ν_2 with S_W. Premise P-FG4 records this; falsifier F-FG13 fires if any downstream calculation adopts a Chern–Weil sign convention without applying the calibration. The earlier presentation asserted a sign without this step, which is harmless numerically but is a genuine convention debt at the interface with the determinant-line calculation.

24. The three charges

Trace-free colour charge.

$\nu_3 = \langle c_2(E\_C) - (1/3)c_1(E\_C)^2, X\rangle = \langle b - a^2/3, X\rangle$.

Trace-free weak charge.

$\nu_2 = \langle c_2(E\_W) - (1/4)c_1(E\_W)^2, X\rangle = \langle c - a^2/4, X\rangle$.

On a liftable bundle with $a = 6\ell$ these reduce exactly to the integral second Chern numbers of S_C and S_W; the reduction is verified in Appendix C.

Primitive-q Abelian charge. The U(1)_q curvature has rational flux class $y = a/6 \in H^2(X; \mathbb{Q})$, which becomes the integral class $c_1(L)$ precisely when the covering lift exists. Then

$\nu\_q := (1/2)\langle y^2, X\rangle = (1/72)\langle a^2, X\rangle$.

The word *rational* here does not mean arbitrary: both the denominator and the correlation with the non-Abelian fractional parts are fixed by the quotient. Note that on a liftable bundle over a non-spin oriented 4-manifold $\nu\_q = (1/2)\langle c_1(L)^2, X\rangle$ can be half-integral; integrality of ν_q on liftable bundles is a spin statement.

25. General charge-lattice theorem

Set $m_3 = \langle b, X\rangle$, $m_2 = \langle c, X\rangle$, $r = \langle a^2, X\rangle$.

Theorem 8 — Correlated faithful topological-charge lattice. The complete Chern–Weil charge triple of a faithful bundle is

$(\nu_3, \nu_2, \nu\_q) = (m_3 - r/3, m_2 - r/4, r/72)$,

where (a, b, c) ranges over the complete census of Theorem 4. Equivalently,

$\nu_3 + 24\nu\_q \in \mathbb{Z}$, $\nu_2 + 18\nu\_q \in \mathbb{Z}$, $72\nu\_q \in I\_X$,

with $I\_X = \{ \langle a^2, X\rangle : a \in H^2(X; \mathbb{Z}) \}$ the image of the self-intersection form.

Proof. Direct substitution; the three integrality statements return m_3, m_2 and r respectively. ■

The fractional parts of the colour and weak charges are therefore not independent. Both are determined by the same determinant flux, and hence by the same obstruction class.

26. Spin refinement and the T^4 intersection formula

On a closed spin four-manifold the intersection form is even, so $r = 2s$ with $s \in \mathbb{Z}$, and

$(\nu_3, \nu_2, \nu\_q) = (m_3 - 2s/3, m_2 - s/2, s/36)$.

Not every spin manifold realises every integer s, because the intersection form may be constrained. The periodic four-torus does. With the standard orientation,

$s = (1/2)\langle a^2, T^4\rangle = a_{12}a_{34} - a_{13}a_{24} + a_{14}a_{23}$, and every integer s occurs: take $a_{12} = s$, $a_{34} = 1$ and the remaining coefficients zero.

Theorem 9 — Complete spin-T^4 numerical charge lattice.

$T\_G(T^4) = \{ (m_3 - 2s/3, m_2 - s/2, s/36) : m_3, m_2, s \in \mathbb{Z} \} \subset \mathbb{Q}^3$,

a rank-three lattice with $\mathbb{Z}$-basis

$g\_C = (1, 0, 0)$, $g\_W = (0, 1, 0)$, $g\_Q = (-2/3, -1/2, 1/36)$,

and covolume 1/36.

27. Quadratic refinement of the obstruction class

Theorem 12 — The fractional sector is a quadratic refinement of ν_6. On a closed spin four-manifold the assignment

$q_6 : \nu_6 \mapsto s \bmod 6$

is well defined on realised obstruction classes, and is *quadratic*, with polarisation the intersection pairing reduced mod six:

$q_6(\nu_6 + \nu_6') - q_6(\nu_6) - q_6(\nu_6') = \langle a \smile a', X\rangle \bmod 6$.

Proof. Well-definedness: replacing a by a + 6x gives

$s(a + 6x) - s(a) = (1/2)[\langle (a+6x)^2, X\rangle - \langle a^2, X\rangle] = 6\langle a \smile x, X\rangle + 18\langle x^2, X\rangle$,

and every term is divisible by six. Polarisation: $s(a + a') = (1/2)\langle (a+a')^2, X\rangle = s(a) + s(a') + \langle a \smile a', X\rangle$. ■

Two corrections to the earlier presentation are recorded here. First, the divisibility argument does **not** require $\langle x^2, X\rangle$ to be even: $18\langle x^2, X\rangle$ is divisible by six unconditionally. Evenness of the intersection form is used earlier and for a different purpose — to define $s = r/2$ at all. Second, q_6 is not a homomorphism, and it should not be quoted as one; it is the standard quadratic refinement, the half-Pontryagin-square of ν_6, and its non-linearity is precisely what makes Theorem 14 possible.

28. The six fractional classes and the Chinese remainder structure

For s mod 6 the fractional non-Abelian charges are:

s mod 6	ν_3 mod 1	ν_2 mod 1	ν_q representative
0	0	0	0
1	1/3	1/2	1/36
2	2/3	0	1/18
3	0	1/2	1/12
4	1/3	0	1/9
5	2/3	1/2	5/36

Corollary 12.1 — CRT bijection. Since $-2 \equiv 1 \pmod 3$, the fractional part of ν_3 is (s mod 3)/3 exactly, and that of ν_2 is (s mod 2)/2. The map

$$s \bmod 6 \mapsto (\nu_3 \bmod 1, \nu_2 \bmod 1) \in (1/3)\mathbb{Z}_3 \times (1/2)\mathbb{Z}_2$$

is therefore the Chinese-remainder isomorphism $\mathbb{Z}_6 \cong \mathbb{Z}_3 \times \mathbb{Z}_2$. Every combination of colour thirds and weak halves occurs, and each occurs exactly once.

The table is thus not a list of observed cases but the graph of an isomorphism — a completeness statement.

29. The full fractional-charge group is $\mathbb{Z}_{36}$

Theorem 13 — Fractional-charge group. Let $\mathbb{Z}^3 \subset \mathbb{Q}^3$ be the lattice of fully integral charge triples. Then on spin T^4

$$\mathbb{Z}^3 \subset T\_G(T^4) \text{ and } T\_G(T^4)/\mathbb{Z}^3 \cong \mathbb{Z}_{36},$$

generated by the class of g_Q. On the oriented (non-spin) branch of Section 33 the corresponding quotient is $\mathbb{Z}_{72}$.

Proof. A point $(m_3 - 2s/3, m_2 - s/2, s/36)$ is integral iff $s/36 \in \mathbb{Z}$, and $s \in 36\mathbb{Z}$ then makes $2s/3$ and $s/2$ integral automatically; conversely $s = 36k$ with m_3, m_2 free gives every element of $\mathbb{Z}^3$. Hence $\mathbb{Z}^3 \subset T\_G(T^4)$, and $k\cdot g\_Q \in \mathbb{Z}^3$ iff $36 \mid k$, so the quotient is cyclic of order 36. Indices: $\mathrm{covol}(\mathbb{Z}^3)/\mathrm{covol}(T\_G) = 1/(1/36) = 36$. For the oriented branch replace g_Q by $(-1/3, -1/4, 1/72)$, and $k\cdot g\_Q \in \mathbb{Z}^3$ iff $72 \mid k$. ■

Corollary 13.1 — Why $\mathbb{Z}_6$ is not the whole story. The familiar $\mathbb{Z}_6$ pattern is the image of $\mathbb{Z}_{36}$ under the projection that forgets ν_q, that is, $\mathbb{Z}_{36} \twoheadrightarrow \mathbb{Z}_6 \cong \mathbb{Z}_3 \times \mathbb{Z}_2$ of Corollary 12.1. On the oriented branch the corresponding non-Abelian image is $\mathbb{Z}_{12} \cong \mathbb{Z}_3 \times \mathbb{Z}_4$, since ν_2 then depends on r mod 4 rather than on s mod 2. Quoting the fractional-charge group as $\mathbb{Z}_6$ in a calculation that retains ν_q is an error of order six; falsifier F-FG11 fires on it.

30. Charge integrality does not imply liftability

Theorem 14 — Strict coarsening. The map from the bundle census to the charge triple does not determine the obstruction class. Explicitly, on T^4 take

$a_{12} = 36$, $a_{34} = 1$, all other $a_{ij} = 0$, $b = c = 0$.

Then $s = 36$ and

$(\nu_3, \nu_2, \nu_q) = (-24, -18, 1) \in \mathbb{Z}^3$,

so all three topological charges are ordinary integers; but $\rho_6(a) \neq 0$ because $a_{34} \equiv 1 \pmod 6$, so by Theorem 6 the bundle does **not** lift to the covering product.

Proof. Direct evaluation using Section 26 and Theorem 6. ■

Corollary 14.1. Liftability is strictly stronger than charge integrality; ν_6 carries information invisible to (ν_3, ν_2, ν_q). Furthermore the weaker condition $s \equiv 0 \pmod 6$ — integrality of the *non-Abelian* charges alone — is realised by $a_{12} = 6$, $a_{34} = 1$, where $\nu_q = 1/6$ is still fractional. Any downstream argument that infers the existence of a covering-product lift, an $SU(3) \times SU(2) \times U(1)$ gauge-field description, or the vanishing of a quotient phase from integrality of instanton numbers is invalid.

This is the paper's principal negative certificate. It is stated positively as a design instruction for the determinant-line and global-measure calculation: iterate over ν_6, never over the charge triple.

Part VII — Topological characters and exact periods

31. Global topological character

For angles $\Theta = (\theta_3, \theta_2, \theta_q)$ define

$$\chi_\Theta(\nu_3, \nu_2, \nu_q) = \exp[\, i(\theta_3\nu_3 + \theta_2\nu_2 + \theta_q \nu_q) \,].$$

On spin T^4, using Theorem 9,

$$\theta_3\nu_3 + \theta_2\nu_2 + \theta_q \nu_q = \theta_3 m_3 + \theta_2 m_2 + s\cdot(-2\theta_3/3 - \theta_2/2 + \theta_q/36),$$

so χ_Θ is determined by its values on the three lattice generators g_C, g_W, g_Q. The previously abstract character group is therefore completely explicit: it is $\mathrm{Hom}(T_G(T^4), U(1)) \cong U(1)^3$.

32. Spin-T^4 period lattice

Theorem 10 — Exact spin-T^4 θ-period lattice. Two angle triples define the same character exactly when their difference lies in

$$\Lambda_\theta^{spin} = \mathrm{span}_\mathbb{Z} \{ (2\pi, 0, 48\pi), (0, 2\pi, 36\pi), (0, 0, 72\pi) \},$$

whose covolume is $288\pi^3$.

Proof. Triviality on g_C and g_W forces $\delta\theta_3 = 2\pi p$ and $\delta\theta_2 = 2\pi q$ with $p, q \in \mathbb{Z}$. Triviality on g_Q then requires

$$-2\delta\theta_3/3 - \delta\theta_2/2 + \delta\theta_q/36 \in 2\pi\mathbb{Z},$$

that is $-4\pi p/3 - \pi q + \delta\theta_q/36 = 2\pi n$, hence

$$\delta\theta_q = 48\pi p + 36\pi q + 72\pi n.$$

These are exactly the stated generators. Covolume: det diag-triangular $= 2\pi \cdot 2\pi \cdot 72\pi = 288\pi^3$, which agrees with $(2\pi)^3$ times the covolume 36 of the dual of $T_G(T^4)$. ■

33. Consequences for individual periods, and the oriented branch

Holding the other two angles fixed, the smallest pure periods on spin T^4 are

$$\theta_3 \sim \theta_3 + 6\pi,\ \theta_2 \sim \theta_2 + 4\pi,\ \theta_q \sim \theta_q + 72\pi.$$

The more fundamental statement is the joint lattice. A compensated 2π colour or weak shift is already an equivalence:

$$(\theta_3, \theta_q) \mapsto (\theta_3 + 2\pi, \theta_q + 48\pi),\ (\theta_2, \theta_q) \mapsto (\theta_2 + 2\pi, \theta_q + 36\pi).$$

The complete quotient theory therefore does not possess three independent factorwise 2π circles.

If odd self-intersections r are admitted — that is, on an oriented 4-manifold that is not spin — the generator becomes $g_Q^{or} = (-1/3, -1/4, 1/72)$ and the same calculation gives

$$\Lambda_\theta^{oriented} = \mathrm{span}_\mathbb{Z} \{ (2\pi, 0, 48\pi), (0, 2\pi, 36\pi), (0, 0, 144\pi) \},$$

with pure periods 6π, 8π and 144π. The pure colour period is unchanged; the pure weak period doubles, and so does the pure Abelian period. This is consistent with Theorem 13: the fractional-charge group grows from $\mathbb{Z}_{36}$ to $\mathbb{Z}_{72}$ and its non-Abelian image from $\mathbb{Z}_6$ to $\mathbb{Z}_{12}$. The principal VERSF regulator branch is spin T^4 (premise P-FG3); the oriented lattice is recorded because it makes the tangential dependence exact rather than tacit, and because the factor of two is precisely the kind of discrepancy that a later determinant-line calculation could otherwise absorb silently.

34. No fourth ordinary gauge-bundle angle

Theorem 11 — Ordinary degree-four character completeness. $H^4(BG\_faith; \mathbb{Z}) \cong \mathbb{Z}^3$ and is torsion-free. Therefore, once the tangential structure of the 4-manifold is fixed, every ordinary multiplicative degree-four gauge-bundle character is exhausted by the three angles above modulo Λ_θ, and there is no fourth independent ordinary gauge-bundle topological angle and no extra discrete character.

Proof. By Lemma 4.1, $BG\_faith \to P_4BG\_faith \simeq K(\mathbb{Z},2) \times K(\mathbb{Z}^2, 4)$ is 5-connected, hence an isomorphism on H^i for $i \leq 4$. Künneth for the product, using $H^*(K(\mathbb{Z},2); \mathbb{Z}) \cong \mathbb{Z}[u]$ with $|u| = 2$ and $H^i(K(\mathbb{Z}^2,4); \mathbb{Z}) = 0$ for $0 < i < 4$ with $H^4 = \mathbb{Z}^2$, gives

$$H^4 \cong H^4(K(\mathbb{Z},2)) \oplus H^4(K(\mathbb{Z}^2,4)) \cong \mathbb{Z}\langle u^2\rangle \oplus \mathbb{Z}^2 \cong \mathbb{Z}^3,$$

with all Tor terms vanishing because the relevant groups are free. The three generators evaluate to $r = \langle a^2, X\rangle$, m_3 and m_2; the change of basis to (ν_3, ν_2, ν_q) is the invertible rational matrix of Theorem 8. ■

This theorem does *not* remove possible η-invariant, spin-bordism or mixed gauge–gravitational phases. Those are determinant-line or tangential characters, not missing principal-bundle labels. Keeping the distinction sharp is what prevents a global anomaly calculation from being confused with the now-complete gauge-bundle census; it is recorded as debt D-FG3.

35. Finite trivial-character test

The positive-pushforward strong-phase theorem can now be evaluated on three explicit generators. A primitive sector character is trivial if and only if

$$\chi(g\_C) = 1,\ \chi(g\_W) = 1,\ \chi(g\_Q) = 1,$$

equivalently

$$e^{i\theta_3} = 1,\ e^{i\theta_2} = 1,\ \exp[\, i(-2\theta_3/3 - \theta_2/2 + \theta\_q/36)\,] = 1.$$

Modulo Λ_θ^{spin} these conditions select the unique trivial character. The abstract topological generator problem of SCPθ-1 is thereby reduced to a three-sector finite test.

Part VIII — Premises, falsifiers, debts and programme consequences

36. What has been closed

On the frozen representation ledger, FGQT-1 closes:

- the centrality of the kernel (proved, not assumed);
- the maximal common central kernel, and its determination by a single carrier;
- the uniqueness of the faithful gauge-product quotient;
- a concrete matrix-group model and its Levi identification;
- the complete carrier dictionary in globally defined bundles;
- the complete principal-bundle census in dimension four, in both directions, by two independent routes;
- the exact relationship between the Čech six-cocycle and the integral determinant class;
- the covering-product lift criterion and the realised-obstruction set ker β_6;
- the complete periodic-regulator bundle census;
- the correlated colour, weak and primitive-q topological charges;
- the spin-T^4 charge lattice, its fractional-charge group $\mathbb{Z}_{36}$, and its CRT structure;
- the quadratic character of the map from obstruction class to fractional sector;
- the strict coarsening of the charge triple relative to ν_6;
- the global θ-period lattice on both the spin and oriented branches;
- the finite generator set for the primitive topological character.

These are outputs, not proposed tasks.

37. What this paper deliberately does not claim

- that the primitive unextended MASD–HMGBC transfer uniquely forces the frozen representation ledger;
- a global chiral determinant-line trivialisation;
- a substrate-derived positive global measure;
- reflection positivity or a transfer matrix;
- a numerical physical strong phase;
- a triality gap, string tension or string-breaking spectrum;
- any statement about grand unification beyond the group-theoretic identification of Remark 3.1.

Their absence does not weaken the bundle theorem. It means the next paper can consume a complete topological input instead of carrying an undefined bundle lattice.

38. Named premises

Premise	Statement
P-FG1	The charged and scalar representation ledger, and the $q = 6Y$ charges, are the frozen VERSF ledger of Section 5.
P-FG2	The global gauge group is required to act faithfully on every frozen carrier.
P-FG3	The regulator geometry is a finite CW complex of dimension at most four; the principal execution branch is spin T^4 with a fixed spin structure.

Premise	Statement
P-FG4	Chern–Weil normalisations, including the overall sign of the trace-free density, are fixed by the calibration $\nu_3 = \langle c_2(S\_C), X \rangle$ on covering-product lifts, and by the analogous condition for ν_2.
P-FG5	No measured strong phase, monopole spectrum or lattice topological weight is used to select a class, a period or a normalisation.
P-FG6	Tangential structure is fixed *before* the gauge-bundle census is quoted; determinant-line and bordism data are not gauge-bundle labels.
P-FG7	The primitive convention q = 6Y is held fixed throughout every step; the restoration to physical hypercharge is applied only after the character lattice is returned.

39. Falsifiers

Namespace note. This paper's falsifiers carry the F-FG prefix and form their own contiguous series. They are disjoint from the RPC-1 series (F1–F39) and the SMC-1U series (F1–F43); map between series only through the assembly-layer cross-reference appendix, never by number.

The paper must be revised or rejected if any of the following occurs.

- **F-FG1** — a frozen ledger row violates $2t + 3d + q \equiv 0 \pmod 6$.
- **F-FG2** — exhaustive centre enumeration returns an element outside $\langle \zeta \rangle$ acting trivially on the whole ledger.
- **F-FG3** — the physical ledger contains a charged carrier not included in the kernel audit.
- **F-FG4** — Φ fails to be surjective, or has kernel larger than $\langle \zeta \rangle$.
- **F-FG5** — a four-dimensional faithful bundle exists with the same (a, b, c) but inequivalent gauge topology not absorbed by a bundle automorphism; or a triple (a, b, c) is exhibited that is not realised.
- **F-FG6** — $\pi_4(BG\_faith)$ is taken to be $\mathbb{Z}_2$ rather than $\mathbb{Z} \oplus \mathbb{Z}$, or the degree-five k-invariant is assumed non-zero without exhibiting it.
- **F-FG7** — the Chern–Weil charges fail to reduce to the integral second Chern numbers of S_C and S_W on a covering lift.
- **F-FG8** — the period generators fail on any bundle in the stated T^4 census.
- **F-FG9** — the single-carrier witness of Corollary 1.1 is cited as a ledger-independence claim rather than as a stability statement under the descent congruence; or the kernel is quoted as depending on the neutral roster.
- **F-FG10** — liftability, or the existence of a covering-product gauge-field description, is inferred from integrality of the topological charges (excluded by Theorem 14).
- **F-FG11** — the fractional-charge group is quoted as $\mathbb{Z}_6$ in a calculation that retains ν_q, or as $\mathbb{Z}_{36}$ on the oriented branch.
- **F-FG12** — a spin-branch lattice, period or fractional group is used on a non-spin oriented geometry, or conversely, without the substitution of Section 33.
- **F-FG13** — a Chern–Weil sign convention is adopted downstream without applying the calibration of P-FG4.
- **F-FG14** — an observed strong-CP value, or an external topological-sector weight, enters upstream of the lattice return.

The supplied arithmetic package passes F-FG1, F-FG2, F-FG7, F-FG8, F-FG9, F-FG10, F-FG11 and F-FG12 on the declared finite tests; Theorems 3–14 and Lemmas 1 and 4.1 settle F-FG4, F-FG5, F-FG6 and F-FG13 analytically.

40. Open-debt ledger

Debt	Statement	Status
D-FG1	Provenance of the frozen representation ledger: does the primitive unextended transfer force it?	OPEN — inherited from PRLU-1 / PTSD-1 / SGPL-1 / FCHI-1; not touched here
D-FG2	Precise relative/boundary form of Theorem 4 when boundary trivialisation data are fixed: which relative groups, and what replaces the automorphism argument	OPEN — stated qualitatively only
D-FG3	Determinant line, η-invariants, spin-bordism and mixed gauge–tangential phases	OPEN — explicitly outside scope, but required before any global phase is quoted
D-FG4	Global positive measure, reflection positivity and a transfer matrix on the census	OPEN — the next paper's target
D-FG5	The exact restoration map from primitive-q normalisation to physical hypercharge at the level of the period lattice	OPEN — the rescaling is elementary but must be executed once, in writing, to prevent silent factor-of-six drift
D-FG6	Selection of the tangential branch: is spin T^4 forced by the substrate, or a regulator choice?	OPEN — currently premise P-FG3
D-FG7	Realisability of every s on spin 4-manifolds other than T^4, and the resulting charge lattice there	OPEN — Section 26 gives T^4 only

41. Revised programme ledger

Programme item	Previous status	FGQT-1 status
Kernel centrality	Tacit	**Proved (Lemma 1)**
Faithful $\mathbb{Z}_6$ quotient	Inherited / conditional	**Exact on frozen ledger**
Kernel robustness	Three-carrier witness	**Single-carrier witness; stable under descent-compatible extension**
Faithful quotient uniqueness	"Maximal"	**Unique**
Carrier dictionary	Sketch	**Complete and globally defined**
Bundle census in dim 4	Labels only	**Bijection, both directions, two independent proofs**
$\mathbb{Z}_6$ lifting obstruction	Defined	**Identified as $\rho_6(a)$**

Programme item	Previous status	FGQT-1 status
Admissible obstruction set	Unspecified subset of $H^2(X; \mathbb{Z}_6)$	**ker β_6**
Periodic T^4 obstruction count	6^6	**Retained and embedded in the full $\mathbb{Z}^6 \times \mathbb{Z} \times \mathbb{Z}$ census**
Correlated c_1/c_2 compatibility	Open D-SG5	**Closed**
Fractional colour/weak charges	Open	**Closed, with CRT completeness**
Fractional-charge group	Implicitly $\mathbb{Z}_6$	**$\mathbb{Z}_{36}$ (spin), $\mathbb{Z}_{72}$ (oriented)**
Obstruction ↔ charge relation	Assumed equivalent	**Strictly coarser (Theorem 14)**
Quotient-compatible lattice T_G	Defined target	**Closed on spin T^4**
θ periodicity	Open	**Closed on both tangential branches, primitive-q normalisation**
Strong-phase generator set	Abstract	**Three explicit generators**
Global measure / transfer matrix	Open	Not addressed here (D-FG4)

Recommended ledger wording:

Faithful quotient and global gauge topology: CLOSED ON THE FROZEN REPRESENTATION LEDGER. The kernel is central by theorem and exactly $\mathbb{Z}_6$, fixed by Q_L alone; G_faith is uniquely S(U(3) × U(2)); four-dimensional bundles are classified by (a, b, c) with the map bijective in both directions; $\nu_6 = \rho_6(a)$ with realised set ker β_6; the periodic spin-T^4 charge lattice, its fractional group $\mathbb{Z}_{36}$ and its θ-period lattice are explicit; and the charge triple is proved strictly coarser than ν_6. Primitive representation-ledger provenance, and the subsequent determinant-line and global-measure execution, remain separate gates.

42. Ordered next calculation

The next global paper no longer needs to guess the bundle stack. It should iterate over

$(a, b, c) \in H^2(X; \mathbb{Z}) \times H^4(X; \mathbb{Z})^2$,

or, on the periodic regulator, over

$(a_{\{ij\}}; m_3, m_2) \in \mathbb{Z}^6 \times \mathbb{Z}^2$, and calculate the determinant-line holonomy and the positive measure on the three topological generator classes g_C, g_W, g_Q. The strong-phase character then has a finite exact admission test.

Two execution instructions follow from the negative results above and should be carried into that calculation verbatim. First, iterate over the obstruction class ν_6 and not over the charge triple, because Theorem 14 shows the latter does not determine the former. Second, fix the tangential branch before quoting any period, because Section 33 shows the weak and Abelian periods double between the two branches while the colour period does not.

Conclusion

The faithful global topology of the frozen VERSF Standard Model ledger is no longer an open placeholder. The kernel of the joint ledger representation is automatically central, is exactly $\mathbb{Z}_6$, and is already determined by the left-handed quark doublet alone. The quotient is the unique faithful one and is canonically S(U(3) × U(2)), the block-diagonal Levi subgroup of SU(5); this concrete model reduces the global classification to an exact vector-bundle problem in four dimensions and gives every Standard Model carrier as a globally defined tensor construction.

Every faithful bundle is classified by one determinant-flux class a and two second Chern classes b and c, and every such triple is realised. The earlier Čech obstruction is exactly a reduced modulo six, and the realised obstruction classes are exactly ker β_6. On the periodic regulator the complete census is $\mathbb{Z}^6 \times \mathbb{Z} \times \mathbb{Z}$; the 6^6 classes previously identified are the discrete residues of the integral determinant-flux lattice.

The topological charges are correspondingly correlated. Colour thirds, weak halves and Abelian flux are three faces of one quotient class, not independently selectable sectors, and the correspondence between the six residues and the six colour–weak combinations is the Chinese-remainder isomorphism. Once the Abelian coordinate is kept, the fractional-charge group is cyclic of order thirty-six. The correlation fixes the complete spin-T^4 charge lattice and the non-rectangular period lattice of global topological phases, so the strong-phase programme now has an explicit generator basis and exact periodicity rather than an unspecified character group.

One result is negative and is the most consequential for what follows: the charge triple is a strictly coarser invariant than the obstruction class. There exist bundles on T^4 with entirely integral colour, weak and Abelian charges that do not lift to the covering product. Any later step that reads liftability, or the availability of an ordinary product-group gauge-field description, off an integrality check is invalid.

Final scientific verdict: STRONG CONDITIONAL GLOBAL-TOPOLOGY CLOSURE / EXACT FAITHFUL BUNDLE AND TOPOLOGICAL-CHARGE-LATTICE RETURN. The result is conditional only on the frozen representation ledger. Within that premise the quotient, the bundle census, the liftability criterion, the fractional instanton structure, the fractional-charge group and the θ-period lattice are complete.

Terminal verdict ledger

Structural results. Kernel centrality (Lemma 1); exact kernel $\langle\zeta\rangle \cong \mathbb{Z}_6$ (Theorem 1); uniqueness of the faithful quotient (Theorem 2); the isomorphism G_faith ≅ S(U(3) × U(2)) (Theorem 3); the four-dimensional census as a bijection in both directions (Theorem 4); split four-types and vanishing degree-five k-invariant (Lemma 4.1); $\nu_6 = \rho_6(a)$ (Theorem 5); the lift criterion (Theorem 6); the T^4 census (Theorem 7); the charge lattice (Theorems 8, 9); the quadratic refinement (Theorem 12); the fractional-charge group (Theorem 13); the period lattice (Theorem 10); degree-four character completeness (Theorem 11).

Derived mechanisms. The descent congruence $2t + 3d + q \equiv 0 \pmod 6$ as the single arithmetic source of every correlation in the paper; the forced exponents (−2, +3) in Φ; the carrier dictionary in E_C, E_W and det E_C; the CRT structure of the six fractional classes.

Negative certificates. Charge integrality does not imply liftability (Theorem 14 and Corollary 14.1); the fractional-charge group is not $\mathbb{Z}_6$ once ν_q is retained (Corollary 13.1); q_6 is quadratic, not a homomorphism (Theorem 12).

Corrections absorbed. Centrality upgraded from assumption to lemma; "maximal" quotient upgraded to "unique"; census surjectivity supplied; k-invariant vanishing proved; $\pi_4(BG)$ corrected to $\mathbb{Z} \oplus \mathbb{Z}$ with $\mathbb{Z}_2$ relocated to π_5; the superfluous evenness appeal in the mod-six divisibility argument removed; the Chern–Weil sign restated as a calibrated convention; the minimal witness sharpened from three carriers to one.

Cross-checks. Two independent proofs of the census (vector-bundle and homotopy-theoretic); covolume agreement $288\pi^3 = (2\pi)^3 \cdot 36$ between the period lattice and the dual of the charge lattice; pure-period consistency between the spin and oriented branches and the corresponding fractional groups $\mathbb{Z}_{36}$ and $\mathbb{Z}_{72}$; exhaustive centre enumeration over $\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mu_{72}$ reproducing the analytic kernel.

Blockers. D-FG1 (ledger provenance) ∥ D-FG3 (determinant line and bordism) ∥ D-FG4 (global measure and positivity); with D-FG2, D-FG5, D-FG6 and D-FG7 alongside.

Final lines. The global topology is issued. The global measure is not — and, by the declared scope of this paper, correctly so.

Appendix A — Exact central-kernel arithmetic

Parameterise the centre by $(a, b, n) \in \mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_6$ corresponding to

$(\omega_3{}^a \mathbb{1}_3, (-1)^b \mathbb{1}_2, e^{i\pi n/3})$,

which is legitimate once Theorem 1 has derived $\alpha \in (\pi/3)\mathbb{Z}$ from d_R. The exponent on a slot with centre data (t, d, q), in sixth-root units, is

$2at + 3bd + nq \pmod 6$.

Exhaustive evaluation of all $3 \cdot 2 \cdot 6 = 36$ candidates on all seven ledger rows leaves exactly six:

n	a = n mod 3	b = n mod 2
0	0	0
1	1	1
2	2	0
3	0	1
4	1	0
5	2	1

This is the cyclic graph of ζ.

For Corollary 1.1 the enumeration must be run *without* the sixth-root restriction, since a single carrier need not derive it. Taking $\alpha \in (2\pi/72)\mathbb{Z}$ is exhaustive, because a constraint $e^{iq\alpha} \in \mu_6$ with $q \neq 0$ forces $\alpha \in \mu_{6|q|}$, and $6|q| \in \{6, 12, 18, 24, 36\}$ divides 72 for every ledger charge. Single-row kernel orders are then

$Q_L : 6,\ d_R : 12,\ L_L : 18,\ H : 18,\ u_R : 24,\ e_R : 36,\ N_R : 432$,

confirming that Q_L is the unique single-carrier witness, and that $\{d_R, L_L\}$ is a minimal two-carrier witness.

Appendix B — Vector-bundle classification proof

Let

$S(U(3) \times U(2)) \hookrightarrow U(3) \times U(2) \xrightarrow{\det \otimes \det} U(1)$

be the determinant-product sequence. A principal S-bundle induces U(3)- and U(2)-bundles together with a trivialisation of the product determinant; conversely such a pair with trivialisation reduces the product frame bundle to S.

For rank $n \geq 2$ the Postnikov four-type of $BU(n)$ is $K(\mathbb{Z}, 2) \times K(\mathbb{Z}, 4)$ with coordinates c_1 and c_2; the degree-five k-invariant lies in $H^5(K(\mathbb{Z},2); \mathbb{Z}) = 0$ and hence vanishes. Therefore a rank-two or rank-three complex bundle over a four-complex is classified by (c_1, c_2), and every pair of classes is realised. The determinant relation removes one c_1 and leaves three integral coordinates.

If two determinant trivialisations differ by $f : X \to U(1)$, the scalar automorphisms $f\mathbb{1}_3$ on E_C and $f^{-1}\mathbb{1}_2$ on E_W alter the product determinant by $f^{3-2} = f$. Thus the difference is gauge-equivalent and contributes no H^1 label, while leaving the isomorphism classes of E_C and E_W unchanged because f is a section of a trivial line.

For surjectivity, given (a, b, c) realise E_C with $(c_1, c_2) = (-a, b)$ and E_W with (a, c); then $c_1(\det E\_C \otimes \det E\_W) = 0$, so that line is trivial and admits a trivialisation.

Appendix C — Chern–Weil identities

For a rank-n Hermitian bundle E with curvature F write $F_0 = F - (1/n)(\operatorname{tr} F)\mathbb{1}_n$. With the total Chern form $\det(\mathbb{1} + (i/2\pi)F)$,

$$c_1 = (i/2\pi) \operatorname{tr} F, \quad c_2 = \tfrac{1}{2}[\, c_1^2 + (1/4\pi^2) \operatorname{tr}(F \wedge F) \,],$$

so $\operatorname{tr}(F \wedge F) = 4\pi^2(2c_2 - c_1^2)$. Since

$$\operatorname{tr}(F_0 \wedge F_0) = \operatorname{tr}(F \wedge F) - (1/n)(\operatorname{tr} F)^2 = \operatorname{tr}(F \wedge F) + (4\pi^2/n)\, c_1^2,$$

one obtains

$$(1/8\pi^2) \operatorname{tr}(F_0 \wedge F_0) = c_2(E) - ((n-1)/2n)\, c_1(E)^2.$$

For $n = 3$ the coefficient is 1/3; for $n = 2$ it is 1/4. The overall sign is convention-dependent (Hermitian versus anti-Hermitian curvature) and is fixed by the calibration of premise P-FG4, not chosen independently.

On a covering lift with $a = 6\ell$, write $E\_C \cong S\_C \otimes L^{-2}$ and $E\_W \cong S\_W \otimes L^3$ with $c_1(L) = \ell$. The tensor-product Chern formulas for a rank-n bundle S with $c_1(S) = 0$ twisted by a line M give $c_2(S \otimes M) = c_2(S) + ((n(n-1))/2)\, c_1(M)^2$, hence

$$c_2(E\_C) = c_2(S\_C) + 3\cdot(-2\ell)^2 = c_2(S\_C) + 12\ell^2, \quad c_2(E\_W) = c_2(S\_W) + 1\cdot(3\ell)^2 = c_2(S\_W) + 9\ell^2.$$

Subtracting $a^2/3 = 12\ell^2$ and $a^2/4 = 9\ell^2$ recovers $\nu_3 = \langle c_2(S\_C), X\rangle$ and $\nu_2 = \langle c_2(S\_W), X\rangle$ exactly, which is both the calibration and the verification of falsifier F-FG7.

Appendix D — Period-lattice proof

On spin T^4 let

$Q(m_3, m_2, s) = (m_3 - 2s/3, m_2 - s/2, s/36)$.

A period $\delta\Theta$ must satisfy $\delta\Theta \cdot Q(m_3, m_2, s) \in 2\pi\mathbb{Z}$ for all integers m_3, m_2, s. The m_3 and m_2 directions give $\delta\theta_3 = 2\pi p$ and $\delta\theta_2 = 2\pi q$. The s direction then gives

$$-4\pi p/3 - \pi q + \delta\theta\_q/36 = 2\pi n,$$

hence $\delta\theta\_q = 48\pi p + 36\pi q + 72\pi n$, which is exactly the claimed lattice. Its covolume is $288\pi^3$, equal to $(2\pi)^3$ times the covolume 36 of the dual of $T\_G(T^4)$, confirming that Λ_θ^spin is the full period lattice and not a proper sublattice.

On the oriented branch replace the third generator of the charge lattice by $(-1/3, -1/4, 1/72)$; the same elimination returns $\delta\theta\_q = 48\pi p + 36\pi q + 144\pi n$.

Appendix E — Worked minimal sectors on T^4

Sector	a (nonzero components)	s	ν_6	(ν_3, ν_2, ν_q) with b = c = 0	Lifts?
Trivial	—	0	0	(0, 0, 0)	yes
Minimal quotient flux	$a_{12} = 1, a_{34} = 1$	1	$\neq 0$	$(-2/3, -1/2, 1/36)$	no
Non-Abelian-integral	$a_{12} = 6, a_{34} = 1$	6	$\neq 0$	$(-4, -3, 1/6)$	no
Fully charge-integral	$a_{12} = 36, a_{34} = 1$	36	$\neq 0$	$(-24, -18, 1)$	**no**
Liftable	$a_{12} = 6, a_{34} = 6$	36	0	$(-24, -18, 1)$	yes

The last two rows share an identical charge triple and differ in liftability. That single line is the content of Theorem 14, and it is the reason the enumeration instruction of Section 42 is stated in terms of ν_6.

Appendix F — Machine-readable certificate

The accompanying package contains:

- `run_fgqt1.py` — unrestricted centre enumeration, single-carrier witness scan, charge- and period-lattice checks, and the Theorem 14 witness;
- `results/FGQT1_calculation_certificate.json` — principal pass certificate;
- `tables/frozen_representation_ledger.csv` — field-by-field descent audit;
- `tables/single_carrier_kernel_orders.csv` — kernel order per ledger row under the unrestricted enumeration;
- `tables/fractional_charge_classes_T4.csv` — the six non-Abelian classes and the thirty-six full classes;
- `figures/fractional_charge_classes_T4.png` — charge-class diagram;
- `sha256_manifest.json` — package integrity manifest.

The certificate returns:

- kernel order 6, exact $\mathbb{Z}_6$ kernel pass;
- single-carrier kernel orders (6, 24, 12, 18, 36, 432, 18) for (Q_L, u_R, d_R, L_L, e_R, N_R, H);
- all ledger rows descend;
- liftable charge formulas reduce to integral covering c_2;
- every integer s realised on T^4;
- fractional-charge group order 36 (spin) and 72 (oriented);
- all three period generators pass on the finite integer stress test, with covolume $288\pi^3$;
- the Theorem 14 witness returning integral charges with non-zero ν_6.

Integrity note: hashes certify integrity, not currency. The manifest must be regenerated whenever an upstream ledger paper is revised, and the certificate must record which revision of PRLU-1 supplied P-FG1.

Source ledger

- Keith Taylor, *PRLU-1: Primitive-Transfer Representation-Ledger Uniqueness Attempt*.
- Keith Taylor, *PTSD-1: Primitive Terminal-and-Score Descent Attempt*.
- Keith Taylor, *SGPL-1: Six-Gate Primitive Path-Law Attempt*.
- Keith Taylor, *FCHI-1: Full-Carrier Finite-Regulator Hamiltonian and Primitive Instrument*.
- Keith Taylor, *The Quantum Gauge Completion of the VERSF Standard Model*.
- Keith Taylor, *The Strong CP and Global θ-Sector Theorem in VERSF*.
- Keith Taylor, *The Substrate Cohomology, Global Gauge Measure, and Positive-Pushforward Strong-Phase No-Go Theorem in VERSF*.
- Keith Taylor, *The Non-Perturbative QCD Scale and Triality-Gap Completion Theorem in VERSF*.
- Keith Taylor, *The Four-Regulator Commitment-Maintenance Execution and Physical-Admission No-Go Theorem in VERSF*.
- Keith Taylor, *VERSF Standard Model Derivation Master Ledger, July 2026*.
- FGQT-1 calculation package — exact central-kernel, bundle-census and topological-lattice certificate.

VERSF Theoretical Physics Programme | Exact global-topology return