GQSC-1R — The Faithful-Quotient Retained Global Measure and Strong-Phase Confinement Theorem in VERSF

Designation: GQSC-1R — Strong Finite-Retained Global Theorem / Exact Physical-Confinement Non-Identifiability Result

Source status: This paper consumes the exact FGQT-1 faithful quotient and topological-charge lattice, the frozen D36 discrete-Fold kernel and stationary law, and the four-regulator HCMR-MR1 retained operators and charged tensors. Every new numerical statement is rebuilt from the accompanying package. No measured strong phase, string tension, hadron mass, lattice potential or experimental branch orientation is used.

General reader summary

Earlier papers in this series did two things. They checked that the candidate Standard Model behaves well numerically at short range, and they pinned down the exact global shape of its gauge symmetry — which twisted configurations of the colour, weak and hypercharge fields can exist, and how they are correlated with one another. This paper asks the next question: can the theory's own microscopic engine be turned into a genuine quantum theory of those fields — and if so, what does it say about two famous features of the strong force, the mysteriously absent "strong CP" effect and the confinement of quarks?

The microscopic engine is a small seven-state machine: a hub connected to a ring of six states, around which probability flows with a built-in bias — the arrow of history. A machine with an arrow of time cannot be used directly as quantum machinery, which needs a perfectly time-symmetric core. The surprise is that for this particular machine the repair is unique. Its fading part and its circulating part turn out not to interfere with each other at all, so there is exactly one natural time-symmetric core — no arbitrary choice anywhere — and its energy levels come out as exact formulas rather than numerical accidents. One of those formulas explains a coincidence that had been sitting unnoticed in the tables: the third energy level is exactly twice the first, at every strength of the bias.

Feeding that core into the global structure gives the first physics payoff. The Standard Model contains a dial, the strong CP phase, which experiment finds to be zero to extraordinary precision and which conventional theory leaves unexplained. Here the dial is removed. In any completion of this framework built from ordinary positive probabilities, the phase is forced to be exactly zero — not tuned small, but zero because there is no dial to turn. If nature were found to have a nonzero value, that would not be a different setting of this theory; it would be evidence against this way of completing it.

The same analysis proves two limits with equal force. First, nothing built from ordinary positive probabilities can tell the theory apart from its mirror image; that choice must come from a new kind of ingredient. Second, and more strikingly, the present body of theory provably cannot decide whether quarks are confined, or how strongly. Every possible strength of confinement — including none at all — is equally compatible with everything derived so far. This is not a calculation still waiting to be done; it is a theorem that a new physical ingredient is required.

To see exactly what is missing, the paper then runs the confinement machinery with the missing number simply granted. Everything works: the energy between two static quarks grows in exact proportion to their separation, the signature of a flux tube. So the machinery is sound, and only one number — the cost of a wall in the colour field — lacks a derivation. A proposed shortcut that tried to obtain this cost from entropy is examined and fails three separate mathematical checks. A bookkeeping repair also falls out along the way: two of the supposedly unknown quantities in the quark-pair-creation problem always shift together, so they are really one unknown rather than two, and the paper now tracks the single honest combination.

The most intriguing development is that the missing ingredient may have been inside the theory all along. If winding around the six-state ring is identified with colour charge — an identification containing no adjustable constants — then colour-charged disturbances are automatically penalised: they cannot pass through the hub, and they pay the hub's coupling cost at every step. That one mechanism yields a definite, positive confinement strength, about 0.22 in the machine's natural units, with nothing dialled in — and, by taking a logarithm, the wall operator itself, now derived at candidate grade. A second thread of evidence points the same way: a phase that an earlier calculation derived and then discarded — because closed loops cannot see it — splits arithmetically into exactly two parts, one matching this colour label and one matching the already-derived orientation label. The theory may have written down both missing quantities once before, in a single object it threw away.

None of this is accepted yet. The identification of ring-winding with colour is the one unproven step, and the theory contains a rival reading of the very same structure — as a label of particle generations rather than of colour. So a decisive test has been written down in advance, with its pass-or-fail rule sealed so it cannot be bent after the fact: examine the microscopic transport along the ring and ask whether it acts on colour or shuffles generations. A first machine audit has since rerun the mechanism on the archived data and confirmed every number exactly — while also showing that the packages currently available do not contain the piece of machinery that could answer the colour question either way, so the decisive input still has to be derived and the verdict remains open. One thing the audit did settle is that the next calculation cannot be wasted: deriving that missing piece from the theory's fundamental equations must return one of two answers — a genuine colour action, which feeds the sealed test, or a proven absence of one, which retires the candidates outright — so either result decides something. A negative verdict retires both candidates. A positive verdict advances them — but even then, two further named steps remain before anything is called physical: converting a cost-per-tick into a cost-per-distance, and deriving the coupling that picks between the theory and its mirror image. The retained model is proved to contain no such coupling — its value there is exactly zero — which is precisely why it must be genuinely new.

The bottom line is deliberately two-sided. What is settled: a unique, mathematically healthy quantum core exists; the strong CP phase is exactly zero within it; and the confinement machinery works. What is proven impossible with the current ingredients: deciding confinement, its strength, or the mirror choice. And what is now on the table: two specific, falsifiable candidates for the missing ingredients, one decisive pre-registered test, and a short, explicitly ordered list of what would remain open even if that test passes.

RETAINED POSITIVE TRANSFER AND STRONG-PHASE ZERO: PASS. PHYSICAL CONFINEMENT AND ORIENTED-BRANCH ADMISSION: REFUSED BY EXACT NON-IDENTIFIABILITY. The frozen D36 one-Fold kernel is proved not to be a positive Euclidean transfer matrix. It is, however, normal with respect to its stationary inner product, so its two polar squares coincide and define one canonical positive reversible transfer with a finite spectral gap in exact closed form. Combining this transfer with the FGQT-1 full-support positive sector measure gives a finite retained reflection-positive global measure. The HCMR quark determinants are exactly positive in the frozen construction, and the complete faithful-lattice topological character is forced to be trivial on every ordinary positive full-support pushforward; hence the retained physical strong phase is exactly zero. The same positive construction cannot select the exact conjugate HCMR branch pair. Moreover, the exhaustive family of positive faithful-centre transfers realizes every triality gap — of either sign — without changing the imported local data. The physical confinement or string-breaking result is therefore not merely uncalculated: it is not identifiable from the present corpus. A merged centre-sensitive static-source execution closes the reference member of the exhaustive family — positive transfer, exact area law $W_{N,M} = e^{-NM}$ — while refusing physical admission because its coefficient is inverse-matched; it rejects the proposed entropic shortcut on three independent mathematical grounds; and it registers two quarantined candidates for the missing wall and orientation blocks, whose shared colour-typing prerequisite is gated by the sealed RWCT-1 verdict — with the wall candidate further gated by a derived temporal-to-longitudinal bridge, and physical branch selection further gated by a derived charged coupling λ_κ. Three sharpenings are merged: the finite reduced wall generator is derived at candidate grade, $\mathcal{K}_{\mathrm{wall,Fold}} = \Delta(P_1 + P_2)$; the static-light pair $(M_{Q\bar{q}}, \mu_{\mathrm{end}})$ is retyped to the single renormalisation-invariant threshold $E_{\mathrm{br}} = 2M_{Q\bar{q}} - \mu_{\mathrm{end}}$; and the retained model's orientation coupling is proved exactly zero, $\lambda_\kappa^{\mathrm{retained}} = 0$. A stage-1 machine audit verifies both candidate mechanisms exactly on the archived frozen arrays, grades the typing question NOT TESTED, and certifies the shipped packages insufficient as the typing input — the required package must carry an edge-resolved colour edge action derived from the master action, either outcome of which is decision-grade.

Audit item	Verdict	Meaning
FGQT-1 faithful quotient and bundle lattice	**EXACT PASS**	The group is $[SU(3) \times SU(2) \times U(1)_q]/\mathbb{Z}_6$, with the complete spin-T^4 charge and period lattices.
Raw D36 Fold kernel as Euclidean transfer	**EXACT FAIL**	It is non-reversible and has negative and complex eigenvalues.
π-normality of the raw kernel	**EXACT PASS** (new)	P commutes with its stationary adjoint; $\mathbb{Z}_6$ covariance plus two-state reversibility of the hub sector prove it, residual 3.3×10^{-16}.

Audit item	Verdict	Meaning
Stationary-adjoint polar transfer	**EXACT PASS**	$T\_OS = P\sharp P = PP\sharp$ is positive, self-adjoint in $L^2(\pi)$, Markov and contractive; left and right polar squares coincide by normality.
T_OS spectrum in closed form	**EXACT PASS** (new)	spec $T\_OS = \{\lvert\lambda(P)\rvert^2\}$; spectral gap $3/D(\alpha)$ exactly; energy doubling $E_3 = 2E_1$ exactly.
Finite retained reflection positivity	**PASS**	The symmetric kernel $\pi\_x(T\_OS)\_\{xy\}$ and the positive faithful-sector weights define a reflection-positive cylinder measure; complete conditional-expectation proof supplied.
Full continuum chiral gauge measure	**NOT CLAIMED**	The paper does not replace the missing chiral Dirac-family and bordism calculation with the retained matrix model.
HCMR quark determinant phase	**EXACT ZERO IN THE FROZEN CONSTRUCTION**	$\det Y\_u > 0$ and $\det Y\_d > 0$ at every regulator by the traceless anti-Hermitian orientation theorem.
Faithful-lattice topological character	**TRIVIAL ON EVERY ORDINARY POSITIVE FULL-SUPPORT COMPLETION**	Sectorwise positivity on the three FGQT generators forces the total character to unity.
Physical strong phase	**ZERO-OR-OBSTRUCTION THEOREM; ZERO IN THE RETAINED COMPLETION**	An ordinary positive global completion has $\bar\theta = 0$; a nonzero $\bar\theta$ would signal failure of the positive-pushforward description.
$k = +1$ versus $k = -1$ branch	**EXACT POSITIVE-MEASURE DEGENERACY**	The branch operators are related by an explicit orthogonal complex-conjugation map at all four regulators; the total orientation winding of the history sector is also exactly trivial, $\det U = 1$.
Triality gap	**EXACTLY NON-IDENTIFIED, EXHAUSTIVELY** (strengthened)	The positive centre-transfer family with arbitrary gap Δ is proved exhaustive within the centre algebra, and even the sign of Δ is unconstrained.
String breaking	**EXACTLY NON-IDENTIFIED**	Positive two-channel transfer families have arbitrary tension, threshold and mixing.
NQTC MC4 reference centre transfer (merged execution)	**PASS at reference grade**	Positive finite transfer; $W\_\{N,M\} = e\^\{-NM\}$; $\sigma_0 = \Lambda_3^2 = 0.0510413\ \mathrm{GeV}^2$. Coefficient inverse-matched, not derived.
Sensitivity-table pipeline typing	**CORRECTED**	The $\Lambda_3(Z_3)$ formula with the frozen FRGM thresholds returns 0.216517 GeV at $Z_3 = 1$, not 0.225923 GeV; the table is reprinted with reproducible FRGM inputs, and publication

Audit item	Verdict	Meaning
		of the NQTC-internal tuple is an open debt (D-GQ13).
Entropic wall-coefficient route	**FAIL**	Wrong Cauchy–Schwarz area scaling; $1/\delta$ dilation escape; mixture inequality reversed by convexity.
Screened-channel reduction	**PASS as algebra**	D equal channels reduce to one avoided-crossing block plus $D - 1$ spectators; threshold and mixing unreturned.
Candidate reduced wall generator	**DERIVED at candidate grade**	$\mathcal{K}$_wall,Fold = $\Delta(P_1 + P_2)$ from $-\ln$ T_C2; physical admission still gated by typing and bridge.
Static-light threshold typing	**RETYPED**	(M_{$Q\bar{q}$}, μ_end) → invariant E_br = 2M_{$Q\bar{q}$} − μ_end; μ_end = 0 admissible; value open.
Mixing coupling g_0	**FORMULA AND SIGN CLOSED**	$\lvert g_0\rvert = \Delta E\_min/(2\sqrt{D})$; sign unphysical; magnitude open.
Retained-model orientation coupling	**EXACT ZERO**	λ_κ^retained = 0 by exact branch conjugacy; physical λ_κ open.
Registered candidates C1/C2	**REGISTERED, quarantined**	CRT branch-to-orientation map; winding-lift wall block with derived $\Delta = \ln(D/(D-1))$. RWCT-1 gates the shared typing prerequisite; C2 further requires the D-GQ10 bridge; branch selection further requires λ_κ.
Stage-1 candidate audit	**MECHANISMS MACHINE-VERIFIED / TYPING NOT TESTED**	C2 mechanism and C1 arithmetic reproduce exactly on the archived frozen arrays; typing inputs absent; the shipped packages are insufficient as the RWCT input; a derived identity action would instead be REFUTED-grade (F-GQ24).
Overall GQSC-1R grade	**STRONG RETAINED THEOREM / PHYSICAL CONFINEMENT CERTIFICATE REFUSED**	The measure/phase question is sharply advanced; the current corpus provably does not determine confinement or branch orientation.

Contents

Abstract

We perform a finite-regulator global-measure and strong-sector return using the exact FGQT-1 faithful quotient and topological-charge lattice, the frozen D36 one-bit history kernel, and the four-regulator HCMR-MR1 retained operators. The raw D36 Fold kernel P is row stochastic and stationary but non-reversible. Its spectrum contains one negative symmetric-mode eigenvalue, one negative rim eigenvalue and two complex-conjugate pairs; it cannot be a positive self-adjoint Euclidean transfer matrix.

The kernel is, however, exactly normal with respect to the stationary inner product: with $D_\pi = \mathrm{diag}(\pi)$ and stationary adjoint $P^\sharp = D_\pi^{-1} P^T D_\pi$,

$$P^\sharp P = P P^\sharp,$$

proved structurally from $\mathbb{Z}_6$ rim covariance and two-state reversibility of the hub-symmetric sector, and confirmed numerically at residual 3.3×10^{-16}. The canonical polar transfer

$$T_{OS} = P^\sharp P$$

is therefore unambiguous (left and right polar squares coincide), Markov, self-adjoint in $L^2(\pi)$, positive and contractive, with weighted self-adjoint residual 2.25×10^{-17}. Normality gives its spectrum exactly as the squared eigenvalue moduli of P:

$$\mathrm{spec}\, T_{OS} = (1, 0.3990823414, 0.3990823414, 0.1592667152, 0.1589603612, 0.1589603612, 0.0033003581),$$

with dimensionless transfer energies $E_j = -\ln \lambda_j(T_{OS}) = -2 \ln|\lambda_j(P)|$:

$$(0, 0.9185875140, 0.9185875140, 1.8371750280, 1.8391004087, 1.8391004087, 5.7137242975).$$

Three of these numbers are closed-form and drive-independent identities: the spectral gap is exactly $3/D(\alpha)$ with $D(\alpha) = 2 + 2\cosh\alpha$; the third energy is exactly twice the first, $E_3 = 2E_1$, because $|\lambda_1|^2 = -\lambda_3 = (2\cosh\alpha - 1)/D$ identically; and the splitting between the third and fourth levels is $(3 - 2\cosh\alpha)/D^2$, which changes sign at $\alpha\bigstar = \operatorname{arccosh}(3/2) = 0.9624236501$ — the frozen one-bit drive $\alpha = 0.9590011097$ sits 0.36 % below the crossing.

The symmetric nonnegative kernel D_π T_OS defines a reversible two-Fold cylinder measure and satisfies the finite-dimensional reflection-positivity criterion by a complete conditional-expectation argument. Tensoring it with a positive full-support measure on the FGQT-1 spin-T^4 charge lattice produces a normalisable finite retained global measure.

The HCMR charged candidate has the exact factorisation

$$Y\_u = \exp(-\Omega/2)D\_u,\quad Y\_d = \exp(+\Omega/2)D\_d,$$

with Ω anti-Hermitian and traceless and D_u, D_d positive. Hence det Y_u and det Y_d are positive exactly and arg det(Y_uY_d) = 0. On the full FGQT lattice, any sectorwise positive full-support pushforward forces the complete topological character to be trivial. Therefore the combined physical strong phase is $\bar{\theta} = 0$ modulo the FGQT period lattice in every ordinary positive completion of the frozen candidate. Equivalently, a nonzero $\bar{\theta}$ is an obstruction to the ordinary positive-pushforward description.

At all four regulators, the k = 6 retained parent and reduced response are exactly related to the k = 1 objects by a fixed orthogonal complex-conjugation operator C: $H^{(6)} = C^T H^{(1)} C$ and $G\_red^{(6)} = C\_red^T\, G\_red^{(1)}\, C\_red$, with zero floating residual. Every positive spectral measure constructed from the retained quadratic operators is therefore branch degenerate. The history sector reproduces the same verdict independently: its orientation unitary U satisfies det U = 1 exactly, with eigenphases cancelling in conjugate pairs, so no scalar spectral functional of the history transfer carries orientation either.

Finally, let P_0, P_1, P_2 be faithful $\mathbb{Z}_3$ centre projectors. Every positive, conjugation-even transfer in the centre algebra has the form

$$T(\Delta) = P_0 + e^{-\Delta}(P_1 + P_2),\quad \Delta \in \mathbb{R},$$

up to overall normalisation — the family is exhaustive, not merely exhibited — and every member is a positive reflection-compatible centre transfer with exact triality gap Δ. Tensoring $T(\Delta)$ with the imported retained transfer preserves all zero-triality local data. Since every real Δ is admissible, including $\Delta < 0$, the current corpus does not identify the physical triality gap or even its sign. A parallel two-channel transfer family proves that string tension, static-light threshold and mixing are likewise non-identified. Final verdict: **retained positive-transfer and strong-phase-zero pass; physical confinement/string-breaking and oriented-branch certificates refused by exact non-identifiability.**

A merged static-source execution closes the reference member of the exhaustive family: on the NQTC branch, $\sigma_0 = \Lambda_3^2 = 0.05104128357$ GeV^2, $T\_cell = \mathrm{diag}(1, e^{-1}, e^{-1})$ from $a\_t\, a\_z\, \sigma_0 = 1$

exact, and $W_{N,M} = e^{-NM}$ — at reference grade only, the coefficient being inverse-matched. The proposed entropic origin of the coefficient is rejected on three independent grounds, the D-channel screened basis reduces exactly to one avoided-crossing block plus $D - 1$ spectators, and the sensitivity table is corrected to separate the two Λ_3 pipelines (0.225923 vs 0.216517 GeV, 4.34 %). Two quarantined candidates are registered for the missing objects: the winding-lift wall block with derived gap $\Delta = \ln(D/(D-1)) = 0.223525982190$ by exact hub avoidance, and the CRT branch-to-orientation map $(k_\Omega = \pm 1) \mapsto (\mathbb{Z}_3 = \pm 1, \mathbb{Z}_7 = r)$. The sealed RWCT-1 calculation decides their shared colour-typing prerequisite: refutation there vacates both, while admission of C2 further requires the temporal-to-longitudinal bridge and physical branch selection further requires a derived charged coupling λ_κ. Neither candidate populates any boundary here.

Three further returns are merged. The finite reduced wall generator is derived at candidate grade: $\mathcal{K}_{\text{wall,Fold}}^{\text{red,cand}} = -\ln T_{C2} = \Delta(P_1 + P_2)$, equal to $\Delta \mathbb{1}_2$ on the charged subspace, with per-cell energy $\Delta\Lambda_3 = 0.0504997$ GeV and tension $\sigma_{\text{cand}} = \Delta/(a_t a_z) \to \Delta\Lambda_3^2 = 0.0114090530$ GeV^2 under the inserted isotropic bridge. The static-light pair $(M_{Q\bar{q}}, \mu_{\text{end}})$ is retyped to the single renormalisation-invariant threshold $E_{br} = 2M_{Q\bar{q}} - \mu_{\text{end}}$ — a common static-source self-energy shifts both channels equally, so $\mu_{\text{end}} = 0$ is an admissible convention and the extraction target is the self-energy-cancelling correlator ratio. The mixing coupling's sign is proved unphysical and its extraction formula closed, $|g_0| = \Delta E_{\min}/(2\sqrt{D})$; and the retained construction's orientation coupling is exactly zero, $\lambda_\kappa^{\text{retained}} = 0$, so a physical selector must be a genuinely new charged orientation-odd matrix element.

A stage-1 machine audit against the archived frozen arrays reproduces both candidate mechanisms exactly — the C2 radius, gap, hub-avoidance and normality-failure numbers, and the C1 bijection — upgrading them to machine-verified grade; grades the typing question NOT TESTED, the shipped instrument's trivial field action being a null by construction; and certifies the shipped packages insufficient as the RWCT-1 input, leaving the derived-identity branch live and narrowing the typed-carrier debt to the master-action edge resolution. All quarantines stand.

Principal theorem

Theorem A (GQSC-1R retained global completion and physical non-identifiability). Fix the FGQT-1 quotient and spin-T^4 charge lattice, the D36 frozen discrete-Fold kernel and stationary law, and the HCMR-MR1 four-regulator retained candidate. Then:

1. the raw D36 Fold operator is not a positive Euclidean transfer matrix;
2. the raw D36 Fold operator is normal with respect to its stationary inner product, so its left and right polar squares coincide;
3. $T_{OS} = P^\sharp P$ is the canonical positive polar transfer, with a unique stationary vacuum, spectrum exactly $\{|\lambda(P)|^2\}$, spectral gap exactly $3/D(\alpha)$, and the certified numerical spectrum above;
4. its cylinder measure is reflection positive at finite retained regulator;
5. a full-support positive measure exists on every finite truncation of the FGQT charge lattice and has a convergent infinite-lattice limit for every positive-definite quadratic sector energy;

6. the complete topological character of any sectorwise positive full-support completion is trivial;
7. the frozen HCMR quark determinant phase is exactly zero, hence $\bar{\theta} = 0$ in the positive completion;
8. the $k = +1$ and $k = -1$ retained branches are exactly orthogonally conjugate and cannot be selected by any real positive spectral measure built from the frozen quadratic stack; independently, the history orientation unitary has $\det U = 1$ with conjugate-pair eigenphases, so scalar spectral functionals of the history sector are orientation-blind;
9. the triality gap and the string-breaking spectrum are not identifiable, because the exhaustive family of positive faithful-centre transfers reproduces all imported local data while returning inequivalent strong-sector spectra of every gap value and sign;
10. the NQTC reference member of that family is explicitly executed — positive transfer, exact linear potential, $W_{N,M} = e^{-NM}$ — at reference grade, the proposed entropic origin of its coefficient is rejected on three independent mathematical grounds, and the equal-coupling screened basis reduces exactly to one avoided-crossing block plus spectators;
11. two quarantined candidates for the missing wall and orientation objects are registered — the winding-lift block with derived closed-form gap, and the CRT branch-to-orientation map — with their shared colour-typing prerequisite decided exclusively by the sealed RWCT-1 rule, and the residual admission conditions — the temporal-to-longitudinal bridge for the wall block, a derived charged coupling for branch selection — named as debts.

Items 1–10 are closed mathematical or machine-certified results at their declared grades; item 11 is a registration under sealed admission rules, not a result. They do not promote the retained transfer to the full continuum chiral gauge measure, whose Dirac-family and bordism audit remain separate physical objects.

Part I — Source boundary, premises, notation and firewalls

1. The exact question

The earlier GQSC-1 draft stated the global admission contract but did not execute it. FGQT-1 subsequently closed the faithful quotient, complete four-dimensional bundle census and topological-charge lattice on the frozen representation ledger. The question is now narrower and executable:

- Can the actual frozen history kernel supply a positive transfer matrix?
- Does a positive full-support measure on the complete faithful lattice select a topological phase?
- What is the exact quark determinant contribution on the same frozen HCMR candidate?
- Can the resulting positive global completion select the conjugate HCMR branch?
- Does the same input determine a triality gap or string-breaking spectrum?

2. Named premises

The claims of this paper are conditional on the following named premises. Each is mapped to a falsifier in §39.

- **P-GQ1 (frozen topology ledger).** The FGQT-1 faithful quotient, spin-T^4 charge lattice and θ-period lattice are consumed exactly as frozen; no representation content is re-derived here.
- **P-GQ2 (frozen kernel and clock convention).** The D36 one-Fold kernel, its drive $\alpha = 0.959001109719975$ fixed by the one-bit condition, its stationary law and the D36-R1 clock-convention interpretation are consumed as frozen. The two-Fold composite P♯P is declared the dimensionless Euclidean unit.
- **P-GQ3 (frozen HCMR arrays).** The four-regulator H_full, G_red, charged matrices and branch classes are consumed as frozen arrays; the physical-HFB refusal recorded in HCMR-MR1 is retained, not overturned.
- **P-GQ4 (ordinary positive full-support pushforward).** The load-bearing premise of the strong-phase theorem: the physical global object is an ordinary positive measure whose pushforward assigns a real nonnegative weight to every supported topological sector, with support containing a generating set of the FGQT lattice.
- **P-GQ5 (invariant typing).** The strong phase is typed, as in SCPθ-1 and SGMC-1R, as the invariant holonomy of the combined topological character and quark determinant line — not as three independent 2π-periodic factor angles.
- **P-GQ6 (non-insertion).** No measured value of $\bar{\theta}$, no neutron electric dipole bound, no lattice string tension, no hadron mass, no static potential and no observed CP orientation enters any selection.
- **P-GQ7 (benchmark quarantine).** The topological stiffness matrix K used in the finite-sum demonstration is arbitrary and explicitly non-physical; the character theorem is independent of it, and no benchmark number may populate a physical boundary.
- **P-GQ8 (reference typing).** Every NQTC MC4 quantity in Part IX is consumed at its source grade: the centre coefficient is inserted, B_τ^implied is an inverse-matched diagnostic, and no Part IX output may be quoted as a physical derivation.
- **P-GQ9 (pipeline separation).** The NQTC repair value $\Lambda_3 = 0.225923$ GeV and the FRGM cascade value $\Lambda_3 = 0.216517$ GeV are distinct pipeline outputs agreeing to 4.34 %; no formula, table or certificate may present one as the output of the other's inputs.
- **P-GQ10 (candidate quarantine).** The candidates of Part X are typed CANDIDATE, populate no boundary, and are admitted or refuted exclusively by the sealed RWCT-1 decision rule, which may not be altered after typed arrays are examined.
- **P-GQ11 (audit-package provenance).** The stage-1 verification package is consumed as an independent rerun of the archived frozen arrays under its own manifest. Its certificate values are admitted only where this paper's own rebuild reproduces them; discrepancies would fall to the package, not the paper.

3. Imported objects

Source	Object consumed	Grade retained
FGQT-1	Exact $\mathbb{Z}_6$ quotient, S(U(3) × U(2)) bundle census, spin-T^4 charge lattice and θ-period lattice	Strong conditional topology closure on the frozen ledger
D36-R1	Frozen row-normalised one-Fold kernel, stationary law and clock-convention interpretation	Reproducible conditional clock branch
HCMR-MR1	Four-regulator H_full, G_red, charged matrices and exact branch class	Complete candidate execution; physical HFB refused
SCPθ-1	Strong phase as the combined topological and quark determinant character	Exact invariant architecture
SGMC-1R	Positive-pushforward no-go and determinant-line typing	Conditional global measure theorem
QGC-1	Local anomaly cancellation and even-doublet Witten audit	Local exact; faithful bordism audit separate
NQTC-1 / QCR-1	Triality projectors, conditional gap theorem and string-breaking bounds	Structural/conditional strong-sector input

4. Three distinct measures

The word measure has been used for three different objects and must be typed.

1. **Oriented history law:** the irreversible D36 Markov kernel P. It is positive entrywise but not time-reversal symmetric.
2. **Euclidean absolute transfer:** the positive self-adjoint operator T_OS derived from P. It forgets the orientation carried by the polar factor.
3. **Full chiral gauge measure:** a measure over faithful gauge bundles and fields, including a globally consistent determinant-line section. This paper does not pretend that the finite retained matrix model is object 3.

The new result closes object 2 and derives exact consequences for any object 3 that is an ordinary positive full-support pushforward.

5. Non-insertion boundary

The calculation uses no measured value of $\bar{\theta}$, no neutron electric dipole bound, no lattice string tension, no hadron mass, no static potential and no observed CP orientation. The benchmark topological stiffness matrix used in the numerical finite-sum demonstration is arbitrary and explicitly non-physical; the character theorem is independent of it.

Part II — Faithful quotient and full topological lattice

6. Imported FGQT-1 result

In primitive $q = 6Y$ normalisation, the common central kernel is generated by

$\zeta = (\omega_3 \mathbb{1}_3, -\mathbb{1}_2, e^{i\pi/3})$,

and the faithful gauge group is

$G_{faith} = [SU(3)_C \times SU(2)_L \times U(1)_q]/\mathbb{Z}_6 \cong S(U(3) \times U(2))$.

On a four-dimensional CW complex, a faithful bundle is classified by

$a = c_1(E_W) = -c_1(E_C) \in H^2(X; \mathbb{Z})$, $b = c_2(E_C) \in H^4(X; \mathbb{Z})$, $c = c_2(E_W) \in H^4(X; \mathbb{Z})$.

The lifting obstruction is $\nu_6 = a \bmod 6$. On spin T^4, the complete charge lattice is

$T_G = \{ (\nu_3, \nu_2, \nu_q) = (m_3 - 2s/3, m_2 - s/2, s/36) : m_3, m_2, s \in \mathbb{Z} \}$.

The period lattice of the three topological angles is generated by

$(2\pi, 0, 48\pi), (0, 2\pi, 36\pi), (0, 0, 72\pi)$.

The important point is not merely the fractional colour and weak charges. It is that the topological phase is one character of this correlated lattice. Three independently chosen 2π-periodic factor angles do not describe the complete quotient-bundle sum.

7. Positive full-support sector measure

Let $q(m_3, m_2, s)$ be the charge vector above, and let K be any real positive-definite 3×3 matrix. Define

$W_K(m_3, m_2, s) = \exp[-q^T K q]$.

Because $K > 0$, the infinite sum over $\mathbb{Z}^3$ is convergent and every lattice sector has strictly positive weight.

Theorem 1 (full-lattice positive measure). For every $K > 0$, W_K defines a normalisable, orientation-even, full-support probability measure on the FGQT charge lattice. No topological sector is removed by a liftability assumption.

Proof. The map $(m_3, m_2, s) \mapsto q$ is an injective affine image of $\mathbb{Z}^3$, so $q^T K q \geq \varepsilon \|(m_3, m_2, s)\|^2 - C$ for some $\varepsilon > 0$, $C \geq 0$, and the sum is dominated by a convergent three-dimensional Gaussian sum. Positivity of every weight and the orientation symmetry $W(q) = W(-q)$ are immediate. ■

Its value is conceptual: the complete faithful lattice admits ordinary positive measures without appending a phase character.

Numerical certificate (finite truncation). The benchmark uses $|m_3|, |m_2|, |s| \le 3$ and $K = \mathrm{diag}(1, 3/2, 36)$, producing 343 strictly positive sectors, truncated partition sum 16.0921147349 and exact orientation-symmetry residual 0.

Remark 1.1 (truncation typing — correction). The number 16.0921147349 is a certificate of the finite 343-sector computation, not an approximation to the infinite-lattice normaliser: at cutoff $|m_3|, |m_2|, |s| \le 9$ the same weight sums to 26.6018182265, so the cutoff-3 benchmark is far from converged. The convergence claim of Theorem 1 is unconditional (Gaussian domination); the specific truncated value must never be quoted as the limit. This typing prevents a future misuse of the benchmark number.

Part III — Why the raw D36 kernel is not a transfer matrix

8. Frozen D36 kernel

The frozen kernel has one hub and six cyclic rim states. Its hub row is uniform. A rim row contains a hub move, a self move, a forward move and a backward move. Writing

$$D(\alpha) = 2 + 2 \cosh \alpha,$$

those nonzero rim probabilities are

$$p_{hub} = p_{self} = 1/D,\quad p_{forward} = e^{\alpha}/D,\quad p_{backward} = e^{-\alpha}/D,$$

with $\alpha = 0.959001109719975$.

The kernel is row stochastic. Its stationary law is, in exact closed form,

$$\pi_{hub} = 7/(6D + 7) = 0.1894237309,\quad \pi_{rim} = D/(6D + 7) = 0.1350960448,$$

each rim state carrying the same weight. Its maximum stationary edge current is, again in closed form,

$$J_{max} = \pi_{rim} (e^{\alpha} - e^{-\alpha})/D = \pi_{rim} \tanh(\alpha/2) = 0.0602316978.$$

Thus it is a genuine non-equilibrium kernel. Both closed forms are new to this revision and were verified against the frozen arrays to machine precision.

9. Exact spectral obstruction

For a nonconstant rim Fourier mode $m = 1, \ldots, 5$ with $\theta_m = 2\pi m/6$, the eigenvalue is

$$\lambda_m = [1 + 2 \cosh \alpha \cos \theta_m + 2i \sinh \alpha \sin \theta_m]/D(\alpha).$$

The hub/rim-symmetric two-dimensional sector has eigenvalues

$$1, \mu = 1/7 - 1/D = (D - 7)/(7D).$$

At the frozen α, the full spectrum is

$-0.05744874335, 1, 0.5 \pm 0.38611182497\, i, -0.39908234138, -0.09938822759 \pm 0.38611182497\, i.$

Two exact features are worth displaying. First, $\mathrm{Re}\, \lambda_1 = (1 + 2 \cosh \alpha \cos(\pi/3))/D = (1 + \cosh \alpha)/(2 + 2 \cosh \alpha) = 1/2$ identically — the $m = 1, 5$ pair sits on the vertical line Re = 1/2 for every drive α. Second, $\mathrm{Im}\, \lambda_1 = \mathrm{Im}\, \lambda_2 = \sqrt{3} \sinh \alpha/D$ identically, because $\sin(\pi/3) = \sin(2\pi/3)$ — the two conjugate pairs share their imaginary part at every α, which is why 0.38611182497 appears four times in the list.

Theorem 2 (raw-clock transfer no-go). For every nonzero drive α, the D36 row-normalised Fold kernel is not a positive self-adjoint Euclidean transfer matrix. At the frozen one-bit root it is non-reversible, has two non-real conjugate pairs and has two negative real eigenvalues.

This is not a defect in the D36 calculation. It is the mathematical signature of an oriented dissipative history law. The error would be to identify that law directly with reflection-positive Euclidean evolution.

Part IV — Normality, the canonical positive polar transfer and reflection positivity

10. Stationary adjoint

Let $D_\pi = \mathrm{diag}(\pi)$, and equip $\mathbb{C}^7$ with

$$\langle f, g\rangle_\pi = f^* D_\pi g.$$

The stationary adjoint is

$$P^\sharp = D_\pi^{-1} P^T D_\pi.$$

It is itself a Markov kernel: positivity is obvious, row sums are one because π is stationary, and it represents the time-reversed transition law.

11. Normality of the frozen kernel

The following result is new to this revision. It upgrades the polar construction from a convention to a theorem.

Theorem 3 (π-normality of the D36 kernel). For every drive α, the D36 Fold kernel commutes with its stationary adjoint:

$P^\sharp P = PP^\sharp$.

Equivalently, P is a normal operator on $L^2(\pi)$.

Proof. Decompose $L^2(\pi) = \mathcal{S} \oplus \mathcal{F}$, where $\mathcal{S}$ is the two-dimensional hub-symmetric sector (functions constant on the rim) and $\mathcal{F} = \bigoplus_{m=1}^{5} \mathbb{C}v_m$ is spanned by the rim Fourier modes v_m (hub component 0, rim components $e^{i\theta_m j}$). Because π is uniform on the rim, the v_m are mutually orthogonal in $L^2(\pi)$ and orthogonal to $\mathcal{S}$.

(i) Each v_m is an eigenvector of P with eigenvalue λ_m: the hub row is uniform, so $(Pv_m)_{hub}$ is proportional to the rim average of v_m, which vanishes for $m \neq 0$; and on the rim P acts as a circulant plus a rank-one hub coupling that annihilates v_m. Since the v_m span mutually orthogonal one-dimensional invariant subspaces, P restricted to $\mathcal{F}$ is diagonal in a π-orthogonal basis, hence normal on $\mathcal{F}$, and $P^\sharp$ acts on v_m by $\bar{\lambda}_m$.

(ii) $\mathcal{S}$ is invariant under P (constancy on the rim is preserved by the $\mathbb{Z}_6$-covariant rim rows and the uniform hub row), and P restricted to $\mathcal{S}$ is the two-state effective chain

$M = [[1/7, 6/7], [1/D, 1 - 1/D]]$

with induced stationary weights $(\pi_{hub}, 6\pi_{rim})$. Every two-state Markov chain satisfies detailed balance with respect to its stationary law — explicitly, $\pi_{hub} \cdot (6/7) = 6\pi_{rim} \cdot (1/D)$ reduces to $\pi_{hub} = 7\pi_{rim}/D$, which is the closed form of §8. Hence P is self-adjoint on $\mathcal{S}$.

A direct sum of a self-adjoint block and scalar blocks on orthogonal one-dimensional subspaces is normal. ■

Numerical certificate. $\|PP^\sharp - P^\sharp P\|_{max} = 3.3 \times 10^{-16}$ on the frozen arrays.

Remark 3.1 (what normality buys). Normality is exactly the statement that the orientation and the magnitude of the history law commute. Three consequences follow immediately and are exploited below: the left and right polar squares coincide, so "canonical" needs no convention; the positive transfer spectrum is exactly the set of squared eigenvalue moduli of P; and the orientation is carried by a genuine unitary commuting with the transfer, whose spectral phases can be listed in closed form.

Remark 3.2 (structural origin). The proof consumed only two structural facts — $\mathbb{Z}_6$ rim covariance and the two-state form of the hub-symmetric sector — neither of which depends on the drive value. Normality is therefore a property of the D36 architecture, not of the one-bit calibration. A future kernel that breaks rim covariance (for example, a defect-decorated rim) would generically destroy it; falsifier F-GQ10 monitors this.

12. Positive transfer

Define

$T\_OS = P♯P.$

Theorem 4 (canonical positive polar transfer). T_OS is Markov, self-adjoint in $L^2(\pi)$, positive and contractive, and by Theorem 3 equals the left polar square PP♯. Moreover,

$\langle f, T\_OS\, f\rangle\_\pi = \|Pf\|\_\pi^2 \geq 0,\ 0 \leq T\_OS \leq \mathbb{1},\ T\_OS\ 1 = 1.$

Proof. Self-adjointness follows from (P♯P)♯ = P♯P. Positivity is the displayed norm identity. Jensen's inequality gives $|Pf|^2 \leq P|f|^2$ pointwise; stationarity of π then gives $\|Pf\|\_\pi \leq \|f\|\_\pi$, proving contractivity. The constant vector is fixed because P and P♯ are Markov. Equality of the two polar squares is Theorem 3. ■

On the support of T_OS,

$P = U\ T\_OS^\wedge\{1/2\},$

where U is the polar orientation factor. Because T_OS is strictly positive at the frozen α (minimum eigenvalue $\mu^2 > 0$), U is a genuine π-unitary on all of $\mathbb{C}^7$, and by normality it commutes with T_OS. T_OS contains the time-even magnitude; U contains the orientation that generates the irreversible current.

13. Exact spectrum and closed-form identities

Theorem 5 (spectral transfer identity). spec T_OS = { $|\lambda|^2 : \lambda \in$ spec P }, with matching eigenspaces. In particular the transfer energies are

$E\_j = -\ln \lambda\_j(T\_OS) = -2 \ln|\lambda\_j(P)|,$

exactly twice the modulus decay rates of the oriented kernel.

Proof. Immediate from normality: on each spectral subspace of P with eigenvalue λ, P♯ acts by $\bar{\lambda}$, so T_OS acts by $|\lambda|^2$. ■

Proposition 5.1 (exact identities, all α). With c = cosh α and D = 2 + 2c:

1. **Energy doubling.** $|\lambda_{1,5}|^2 = -\lambda_3 = (2c - 1)/D$ identically. Hence the T_OS eigenvalue of the $m = 1, 5$ pair equals $|\lambda_3|$, and $E_3 = 2E_1$ exactly. The tabulated values 1.8371750280 = 2 × 0.9185875140 realise this identity; it is not a numerical coincidence.
2. **Closed-form spectral gap.** gap(T_OS) = $1 - (2c - 1)/D = 3/D(\alpha)$. At the frozen α: $3/D$ = 0.6009176586.
3. **Level-3/4 splitting and crossing.** $|\lambda_{2,4}|^2 = (4c^2 - 2c - 2)/D^2$, so $\lambda_3^2 - |\lambda_{2,4}|^2 = (3 - 2c)/D^2$. The splitting is positive iff $c < 3/2$, i.e. $\alpha < \alpha\bigstar = \operatorname{arccosh}(3/2) = 0.9624236501$, and vanishes — a triple degeneracy of levels 3, 4, 5 — exactly at $\alpha\bigstar$. The frozen one-bit drive $\alpha = 0.9590011097$ sits 0.36 % below the crossing; the observed splitting 3.06354×10^{-4} is the closed form evaluated there.
4. **Extremal mode.** The minimum T_OS eigenvalue is $\mu^2 = ((D - 7)/(7D))^2$ = 0.0033003581, and $\mu < 0$ for all $\alpha < \operatorname{arccosh}(5/2) = 1.5668$, so at the frozen drive the slowest-decaying symmetric excitation carries orientation phase π.
5. **Determinant.** $\det P = \mu\, \lambda_3\, |\lambda_1|^2\, |\lambda_2|^2 = 0.0014544353$ in closed form, matching the frozen array determinant to machine precision.

Proof. All five are elementary algebra on the eigenvalue formulas of §9; each was additionally verified numerically on the frozen arrays at residual $\leq 4 \times 10^{-16}$. For (1): $|\lambda_1|^2 = [(1 + c)^2 + 3(c^2 - 1)]/D^2 = (4c^2 + 2c - 2)/D^2 = 2(2c - 1)(c + 1)/D^2 = (2c - 1)/D$ since $D = 2(c + 1)$. ■

Remark 5.2 (cross-corpus consistency). The modulus gap of the oriented kernel, $1 - |\lambda_1|$ = 0.3682703574, is the value recorded independently in the ISG-1 execution (0.368270). The relation $E_1 = -2 \ln(1 - 0.368270) = 0.9185875140$ ties the ISG-1 modulus gap and the present transfer energy into a single exact identity, as Theorem 5 requires.

14. Orientation unitary and its exact winding

Theorem 6 (orientation spectrum and trivial total winding). The polar factor $U = P\, T_{OS}^{-1/2}$ is π-unitary, commutes with T_OS, and has spectrum

$$\{ 1, e^{\pm i\varphi_1}, e^{\pm i\varphi_2}, -1, -1 \},$$

with $\varphi_1 = \arctan(2\sqrt{3} \sinh \alpha/D) = 0.6575731940$ and $\varphi_2 = \pi - \arctan(2\sqrt{3} \sinh \alpha/(2c - 2))$ = 1.8227348715 at the frozen α. Consequently

$$\det U = 1 \text{ exactly:}$$

the conjugate-pair phases cancel and the two -1 modes square to $+1$. The total orientation winding of the history sector vanishes; orientation exists only pairwise, mode by mode.

Proof. Unitarity and commutation follow from normality and strict positivity of T_OS. On each eigenmode, U acts by $\lambda/|\lambda|$: phase $e^{\pm i\varphi}$ on the conjugate pairs, -1 on the two negative real modes ($\mu < 0$ and $\lambda_3 < 0$ at the frozen α by Proposition 5.1(4) and $c > 1/2$), $+1$ on the vacuum. The determinant is the product of these phases. Numerical certificate: unitarity residual 1.1×10^{-14}, commutator residual 5.8×10^{-16}, $\det U = 1 + 7 \times 10^{-15}$. ■

Remark 6.1 (orientation-blindness of scalar functionals). Theorem 6 is the history-sector twin of the branch-degeneracy theorem of Part VII: any real functional of U built from its characteristic polynomial coefficients with real weights — traces of powers, determinants, spectral moments — is invariant under $U \mapsto U\sharp$ because the spectrum is closed under conjugation. Orientation is present ($\varphi_1, \varphi_2 \neq 0, \pi$) but no scalar spectral invariant of the history transfer sees it. This is the same structural verdict that the assembly-layer modulus-tie theorem reached for the record fibre: sign lives only in winding orientation, and only an orientation-odd coupling can read it.

15. Numerical certificate

The weighted self-adjoint residual is 2.25×10^{-17}, the ordinary symmetric-similarity residual is 8.78×10^{-17}, the normality residual is 3.3×10^{-16}, and the minimum matrix entry of T_OS is 0.0286151. The eigenvalues and energies $E_j = -\ln \lambda_j$ are:

mode	λ_j	E_j	closed form
0	1.0000000000	0	1
1	0.3990823414	0.9185875140	$(2c - 1)/D$
2	0.3990823414	0.9185875140	$(2c - 1)/D$
3	0.1592667152	1.8371750280 = $2E_1$	$((2c - 1)/D)^2$
4	0.1589603612	1.8391004087	$(4c^2 - 2c - 2)/D^2$
5	0.1589603612	1.8391004087	$(4c^2 - 2c - 2)/D^2$
6	0.0033003581	5.7137242975	$((D - 7)/(7D))^2$

16. Reflection-positive cylinder measure

Define the two-time kernel

$K(x, y) = \pi_x (T_{OS})_{xy}$.

Weighted self-adjointness gives $K(x, y) = K(y, x)$, and every entry is nonnegative.

Theorem 7 (finite retained reflection positivity). The stationary T_OS chain is reflection positive on the complete finite positive-time cylinder algebra: for every observable F of the positive-time chain,

$\langle \Theta F, F\rangle = \Sigma_z\, \pi_z\, |\mathbb{E}[F \mid X_0 = z]|^2 \geq 0$,

where Θ is time reflection through the plane at time zero. Its null space is exactly the set of observables with vanishing conditional expectation at the reflection plane.

Proof. Condition on the state z at the reflection plane. By the Markov property, the expectation of $\Theta\bar{F} \cdot F$ factorises through z into the product of the negative-time and positive-time conditional amplitudes. Because T_OS is reversible with respect to π, the reflected negative-time conditional amplitude is the complex conjugate of the positive-time one, $\mathbb{E}[\Theta\bar{F} \mid z] = \mathbb{E}[F \mid z]^{-}$. Summing over z with weights π_z gives the displayed sum of squares. The null-space characterisation is immediate. Completion of the quotient by zero-norm observables gives the finite OS Hilbert space. ■

This is a complete finite-dimensional theorem on the full cylinder algebra, not a sampled Gram test. The earlier draft's proof sketch omitted the left-hand side of the key display; the complete argument is now in the text and reproduced in Appendix B.

Part V — Finite retained global measure

17. Product construction

At regulator level n and branch k, let $G\_red^{(n'k)}$ be the positive 87-dimensional retained HCMR response. For any $\tau > 0$,

$$T\_quad^{(n'k)}(\tau) = \exp[-\tau\, G\_red^{(n'k)}]$$

is a positive self-adjoint contraction. It can be used as the one-particle transfer of the finite quadratic retained sector, or second-quantised on the appropriate finite carrier.

Let T_top(K) be the diagonal transfer on the FGQT charge lattice with eigenvalue $\exp[-q^T K q]$, and let T_OS be the positive history transfer. The finite-cutoff retained transfer is

$$T\_ret^{(n'k)} = T\_OS \otimes T\_quad^{(n'k)} \otimes T\_top(K).$$

Theorem 8 (finite retained global transfer). For every regulator, branch, $\tau > 0$ and $K > 0$, T_ret is positive, self-adjoint and contractive. Its product cylinder measure is reflection positive. The infinite charge-lattice limit exists in trace norm after normalisation of the topological factor.

Proof. Each factor is positive, self-adjoint and contractive (Theorems 4 and 1; positivity of G_red is a frozen HCMR certificate), and these properties pass to tensor products. Reflection positivity of a product of reversible factors follows by applying Theorem 7 factorwise: the conditional-expectation identity respects tensor structure. Trace-norm convergence of the topological factor is Gaussian domination as in Theorem 1. ■

The theorem closes a concrete retained global measure. It does not claim that G_red is already the full continuum Hamiltonian, nor that the full chiral determinant line over every faithful bundle has been constructed.

18. What the construction teaches

The measure separates three structures that previous notation sometimes compressed:

- irreversible history orientation: U — now a genuine unitary with closed-form phases and exactly trivial total winding;
- positive Euclidean history magnitude: T_{OS} — now with closed-form gap $3/D$;
- retained field response: $\exp(-\tau G_{red})$.

A physical continuum derivation must explain how these objects descend from one master action. But the finite construction already proves that irreversibility and reflection positivity are not mutually exclusive: they live in commuting factors of one polar decomposition, and normality is the precise statement that they commute.

Part VI — Exact strong-phase result

19. Exact quark determinant orientation

The HCMR charged construction has

$$Y_u = U_u D_u, \quad Y_d = U_d D_d,$$

where

$$U_u = \exp(-\Omega/2), \quad U_d = \exp(+\Omega/2), \quad \Omega^\dagger = -\Omega, \quad \mathrm{tr}\,\Omega = 0,$$

and D_u, D_d are positive diagonal magnitude matrices.

Theorem 9 (exact positive quark determinants). At every HCMR regulator,

$$\det Y_u = \det D_u > 0, \quad \det Y_d = \det D_d > 0,$$

and therefore $\arg \det(Y_u Y_d) = 0$ exactly.

Proof. $\det \exp(A) = \exp(\mathrm{tr}\,A)$. Since $\mathrm{tr}\,\Omega = 0$, $\det U_u = \det U_d = 1$. The diagonal magnitudes are strictly positive. ■

The level-6 values are

$$\det Y_u = 2.5992738787273088 \times 10^{-8}, \quad \det Y_d = 7.921449758008737 \times 10^{-11}, \quad \det(Y_u Y_d) = 2.059001743764287 \times 10^{-18}.$$

The imaginary residuals are below 1.4×10^{-29} and 5.1×10^{-32} respectively. These are floating reconstructions of an exact algebraic statement.

20. Full-lattice character selection

A topological phase is a character

$\chi : T\_G \to U(1)$.

Suppose an ordinary physical pushforward assigns a real nonnegative weight $W(q)$ to every supported sector and that the support contains a generating set of the FGQT lattice. Appending χ changes the weight to $\chi(q)W(q)$.

Theorem 10 (full faithful-lattice trivial-character theorem). If $\chi(q)W(q)$ is real and nonnegative on a generating positive-support set, then χ is the trivial character on all of T_G.

Proof. $W(g) > 0$ on each generator g, so $\chi(g) \in U(1) \cap [0, \infty) = \{+1\}$. Multiplicativity fixes $\chi = 1$ on the integer span of the generators, which is the complete lattice. ■

Remark 10.1 (economy of hypotheses). Full support is more than the proof needs: positivity on any single generating set suffices. Full support is retained as the premise (P-GQ4) because it is what the physical pushforward delivers, but the theorem's strength should be quoted at the generating-set level — a completion supported on a finite-index sublattice containing a generating triple is equally constrained.

This theorem uses the actual non-rectangular period lattice. It does not assume three independent 2π periods.

21. Strong-phase synthesis

The physical strong phase is the invariant holonomy of the combined topological and anomalous quark determinant line. On the frozen HCMR candidate, the quark determinant factor is positive. On every ordinary positive full-support completion, Theorem 10 makes the topological character trivial.

Theorem 11 (strong-phase zero-or-obstruction theorem). For the frozen HCMR charged construction and the FGQT faithful lattice:

- if the full global quantum object is an ordinary positive full-support pushforward with a global real section, then $\bar{\theta} = 0$ modulo the FGQT period lattice;
- if $\bar{\theta}$ is nonzero, the ordinary positive-pushforward description is globally obstructed or an additional non-positive line phase has entered.

The finite retained measure constructed in Part V lies in the first branch, so its strong phase is exactly zero.

This is stronger than reporting a small numerical phase. It closes the positive-completion answer and identifies precisely what a nonzero answer would mean.

Part VII — Exact conjugate-branch equivalence

22. Orthogonal conjugation operator

For a complex matrix A, define its realification

$$R(A) = [[\mathrm{Re}\, A, -\mathrm{Im}\, A], [\mathrm{Im}\, A, \mathrm{Re}\, A]].$$

Let $J_m = \mathrm{diag}(\mathbb{1}_m, -\mathbb{1}_m)$. Then

$$R(\bar{A}) = J_m^T R(A) J_m.$$

The HCMR $k = 1$ and $k = 6$ completion and neutral blocks are complex conjugates. Define the 102-dimensional orthogonal operator

$$C = \mathbb{1}_{12} \oplus \mathbb{1}_6 \oplus J_9 \oplus \mathbb{1}_{36} \oplus (J_6 \oplus J_6) \oplus \mathbb{1}_6,$$

block diagonal in the ordered carrier.

Theorem 12 (exact full-parent branch conjugacy). At every regulator $n = 3, 4, 5, 6$,

$$H_{full}^{(n'6)} = C^T H_{full}^{(n'1)} C$$

exactly. After removing the same nine auxiliary directions, the induced 87-dimensional C_{red} satisfies

$$G_{red}^{(n'6)} = C_{red}^T G_{red}^{(n'1)} C_{red}$$

exactly. The calculation package returns zero residual in every matrix entry.

23. Positive-measure branch no-go

Theorem 13 (positive spectral branch-selection no-go). Any branch weight depending only on real spectral functions of the frozen HCMR quadratic operators — determinants, traces, positive Gaussian integrals, heat kernels or transfer spectra — is identical for $k = 1$ and $k = 6$.

Proof. Orthogonal conjugation preserves every spectral function and its trace. ■

The strong-phase result does not lift the tie: its character is trivial and its quark determinant is positive on both branches. The history sector independently corroborates the verdict: by Theorem 6, the orientation unitary of the retained history transfer has conjugation-closed spectrum and $\det U = 1$, so scalar spectral functionals of the history factor are equally orientation-blind. Thus the positive retained global measure selects a conjugate equivalence class, not an orientation.

An oriented branch requires new structure that is not a real spectral function of the present quadratic stack: an orientation-odd real term, an irreversible current coupled asymmetrically to the physical field sector, or a proven physical equivalence theorem.

Part VIII — Triality-gap and string-breaking non-identifiability

24. Faithful centre sectors

Let P_0, P_1, P_2 be the projectors onto the three $\mathbb{Z}_3$ centre characters in the correctly dressed faithful line-operator sector. The centre operator is

$$U_\omega = P_0 + \omega P_1 + \omega^2 P_2, \quad \omega = e^{2\pi i/3}.$$

The quotient fixes the allowed dressing and line lattice. It does not fix the transfer eigenvalue in each sector.

25. Positive gap-deformation family and its exhaustiveness

For every $\Delta \in \mathbb{R}$ define

$$T_{trial}(\Delta) = P_0 + e^{-\Delta}(P_1 + P_2).$$

This transfer is positive, self-adjoint, contractive after normalisation, and commutes with U_ω. Its Hamiltonian is

$$H_{trial}(\Delta) = \Delta(P_1 + P_2),$$

and its exact nonzero-triality gap is Δ.

Proposition 14.1 (exhaustiveness — new). Within the centre algebra generated by U_ω, the set of positive operators covariant under charge conjugation (the swap $P_1 \leftrightarrow P_2$ induced by complex conjugation of the centre character) is exactly

$$\{ a P_0 + b (P_1 + P_2) : a, b \geq 0 \}.$$

After normalising $a = 1$, this is precisely the family $T_{trial}(\Delta)$ with $\Delta = -\ln b \in (-\infty, \infty]$, the boundary $\Delta = \infty$ ($b = 0$) being excluded by full support. The family of Theorem 14 is therefore exhaustive, not merely exhibited, and the admissible gap set is the entire real line: the corpus fails to determine even the sign of Δ.

Proof. A general element of the centre algebra is a $P_0 + b_1 P_1 + b_2 P_2$ with the P_r the spectral projectors of U_ω (Appendix E). Positivity forces a, b_1, $b_2 \geq 0$; conjugation covariance forces $b_1 = b_2$. ■

Theorem 14 (triality-gap non-identifiability). Tensoring T_trial(Δ) with the retained transfer of Part V gives, for every $\Delta \in \mathbb{R}$, a positive reflection-compatible faithful-centre transfer that reproduces all imported zero-triality local data. $\Delta = 0$ is gapless, $\Delta > 0$ is gapped, $\Delta < 0$ is centre-inverted. Therefore the FGQT topology, D36 history transfer and HCMR local quadratic response do not identify the physical triality gap — and by Proposition 14.1 the ambiguity is exhaustive rather than exemplary.

The inserted centre-penalty construction in the earlier NQTC paper corresponds to choosing one positive Δ or σ_0. Its subsequent projector inequality is exact, but the present corpus does not derive the coefficient from the physical colour-projected master action. The family above turns that source debt into a formal non-identifiability theorem, and the exhaustiveness result closes the loophole that some unexamined positive centre transfer might be forced.

26. String-breaking family

A minimal transfer-compatible string-breaking Hamiltonian in the flux-tube/two-singlet basis is

$$H\_sb(R) = [[\sigma R + \mu_0, g], [g, 2M]].$$

Its eigenvalues are

$$E\_\pm = (\sigma R + \mu_0 + 2M)/2 \pm \sqrt{[((\sigma R + \mu_0 - 2M)/2)^2 + g^2]}.$$

For $g > 0$ it has an avoided crossing; for $g = 0$ it has an exact crossing. The unmixed crossing is at

$$R_0 = (2M - \mu_0)/\sigma,$$

and the minimum avoided-crossing gap is $2|g|$.

Theorem 15 (string-breaking non-identifiability). For every $\sigma > 0$, $M > 0$ and real g for which the lowest energy is bounded below, exp[−H_sb(R)] is a positive transfer. The imported VERSF source stack does not determine σ, M or g. Hence it does not determine the string-breaking distance, mixing gap or asymptotic screened energy.

27. Consequence for confinement

A physical confinement certificate requires more than selecting $\Delta > 0$ in an admissible family. It requires a substrate-derived centre-sensitive transfer or static-source correlator, regulator and volume convergence, and a proof that the asymptotic physical spectrum contains no isolated coloured state.

The current corpus supplies none of the coefficients that distinguish the positive transfer families — and Proposition 14.1 shows there is nothing else in the centre algebra to supply them. The correct verdict is therefore not "calculation pending." It is:

Theorem 16 (current-corpus confinement no-go). No theorem using only the FGQT quotient/lattice, the frozen D36 kernel, the HCMR retained quadratic operators and sectorwise positivity can determine whether the physical theory has a positive triality gap, a gapless centre sector, a centre-inverted sector, or a particular string-breaking spectrum. Additional centre-sensitive dynamical data are mathematically necessary.

Part IX — Centre-sensitive static-source execution

28. Scope and imported grades

This Part merges the centre-sensitive static-source execution into the present paper. It executes the reference member of the exhaustive positive centre-transfer family of Proposition 14.1 as far as the sources allow, audits the proposed entropic source of the missing coefficient, and reduces the string-breaking algebra. Premises P-GQ8–P-GQ10 govern this Part and Part X.

Additional imports beyond §3: NQTC-1 (Λ_3, Φ_1, centre projectors, reference σ_0, B_τ formula — reference/conditional grades as labelled by the source); QCR-1 (static-source transfer definition, triality sectors, wall-Hessian target, slicing and string-breaking structure); FRGM-1C (frozen thresholds $(m_t, m_b, m_c) = (171.783689943, 4.213223810, 1.228341125)$ GeV); the strange-confinement support operator (twenty oriented support atoms — candidate operator result); the contextual cohomology criterion (exact criterion; occupancy and coupling open); and the entropic-confinement proposal (audited rather than inherited). No measured α_s, Λ_{QCD}, string tension, flux-tube radius, hadron mass, static-light mass or branch orientation sets any coefficient.

29. Exact reference centre transfer

NQTC-1's reference branch fixes $\ell_C = \Lambda_3^{-1}$ and $\sigma_0 = \ell_C^{-2} = \Lambda_3^2$. The numerical returns, all re-verified independently:

Quantity	Return
Λ_3 (reference branch)	0.2259231807 GeV
ℓ_C	4.426283292 GeV^{-1}
σ_0	0.05104128357 GeV^2
$\lVert\Phi_1\rVert_{adj} = 4\pi/\sqrt{3}$	7.255197457
$B_\tau^{implied} = 3\sigma_0^2/(32\pi^2)$	$2.474655255 \times 10^{-5}$ GeV^4

Quantity	Return
$R\bigstar = (4/\Lambda_3)\sqrt{(\pi/3)}$	$18.11813564\ \mathrm{GeV}^{-1} = 3.575197$ fm
$\mathcal{E}(R\bigstar)$	σ_0 to machine precision (grid-confirmed minimum of the tube functional)

The tube functional $\mathcal{E}(R) = \|\Phi_1\|^2\_{adj}/(2\pi Z_3 R^2) + \pi B\_\tau$^implied R^2 has minimum σ_0 at $R\bigstar$, and the identity $g\_s \|\Phi_1\|\_{adj} \sqrt{(2B\_\tau\text{\^{}implied})} = \sigma_0$ is reproduced to machine precision — a consistency audit of the inverse matching, not an independent prediction.

On the MC4 isotropic primitive-cell realisation $a\_t = a\_z = \Lambda_3^{-1}$, so $a\_t\ a\_z\ \sigma_0 = 1$ exactly, and on the local $\mathbb{Z}_3$ centre basis with $P\_\{\neq 0\} = \mathrm{diag}(0, 1, 1)$,

$T\_cell = \exp[-a\_t\ a\_z\ \sigma_0\ P\_\{\neq 0\}] = \mathrm{diag}(1, e^{-1}, e^{-1})$.

The nonzero-triality cost per longitudinal cell is $E\_cell = \sigma_0 \ell\_C = \Lambda_3$, so N cells give $V\_N = N\Lambda_3$ and an N × M static Wilson rectangle in the reference ground channel obeys

$W\_\{N,M\}$^ref = e^$\{-NM\}$.

N	L_N (fm)	V_N (GeV)	One-time-cell factor
1	0.8734	0.2259	0.367879
2	1.7469	0.4518	0.135335
4	3.4938	0.9037	0.018316
8	6.9876	1.8074	0.000335

Theorem 17 (reference centre-transfer execution). Under the NQTC MC4 centre-projector reference completion and the isotropic primitive-cell realisation, the finite centre transfer is positive, self-adjoint and contractive, and the static ground channel obeys $W\_\{N,M\}$ = e^$\{-NM\}$, with physical-unit slope $\sigma_0 = 0.0510413\ \mathrm{GeV}^2$.

The theorem is exact inside the assembled reference model. It is not the strict VERSF confinement theorem, and by Theorem 16 it could not be: it executes one member of the exhaustive family whose coefficient the corpus provably does not determine. The execution demonstrates the mechanics; it cannot, by construction, demonstrate the provenance. Together the two results form a pincer: everything is ready except the coefficient.

30. Sensitivity to the uncomputed colour response, with pipeline typing

The NQTC conditional one-copy model reduces the colour boundary to $Z_3(\gamma\_\eta) = 6\gamma\_\eta/(1 + \gamma\_\eta)$, and the threshold formula is

$\Lambda_3(Z_3) = \mu\bigstar$^$\{7/9\}$ (m_t m_b m_c)^$\{2/27\}$ $\exp(-8\pi^2 Z_3/9)$.

Correction (pipeline typing). Evaluated with the frozen FRGM-1C thresholds and $\mu\bigstar = 5.80$ TeV, this formula returns $\Lambda_3(1) = 0.216516653$ GeV — exactly the FRGM cascade value, not the 0.225923 GeV of the reference branch. The two values differ by the corpus-flagged 4.34 % and belong to different pipelines with no shared intermediate (premise P-GQ9). The sensitivity table is therefore printed with the reproducible input tuple stated in §28 — the frozen FRGM-1C thresholds and $\mu\bigstar = 5.80$ TeV — so every row below recomputes from data contained in this paper. The reference-branch value 225.923 MeV is not the output of this formula with any input set printed here; it enters the paper only as the independent NQTC pipeline output consumed in §29, and publication of the NQTC-internal threshold tuple that generates it is recorded as debt D-GQ13. The sensitivity conclusion is unaffected — the existence of two admissible pipelines moving the central value by 4.34 % is itself a small instance of the sensitivity being demonstrated.

γ_η	Z_3	Λ_3 (FRGM inputs, reproducible)
0.10	0.5455	11.677 GeV
0.20	1.0000	216.517 MeV
0.50	2.0000	0.033530 MeV
1.00	3.0000	5.1925×10^{-6} MeV
5.00	5.0000	1.2453×10^{-13} MeV

Near $\gamma\_\eta = 1/5$, keeping Λ_3 within ten per cent requires $|\delta\gamma\_\eta| \lesssim 2.7 \times 10^{-3}$, about 1.4 % relative precision: the local intertwiner calculation decides the scale exponentially.

31. The entropic route fails to supply the coefficient

Three independent failures close the proposed entropic derivation of B_τ.

Fixed flux has no area cost. Dividing a surface of area $A = N\_p\ell^2$ into N_p patches carrying flux Φ/N_p each, Cauchy–Schwarz gives $\int\_p|F|^2 \geq (\Phi/N\_p)^2/\ell^2$ per patch, summing to $\Phi^2/(N\_p\ell^2) = \Phi^2/A$ — not the claimed $\Phi^2 A/\ell^2$, which exceeds the correct bound by N_p^2 (a factor 4096 at $N\_p = 64$). A fixed total flux spread over a larger surface does not produce area extensivity; every longitudinal slice must carry a separate irreducible cost.

The gradient wall has a dilation escape. For $E\_grad[\varphi] = (c_6/\Lambda\bigstar^2)\int dx\, (\partial_x\varphi)^2/(\varphi + \varepsilon)$ and a fixed-shape wall $\varphi\_\delta(x) = \varphi\_vac + \Delta\varphi\, h(x/\delta)$, the cost scales as δ^{-1}, so $\inf\_\delta\, E\_grad = 0$ (numerical profile audit: log slope -1.0000000000, constant to 6×10^{-17}). A finite wall coefficient requires a second same-action term whose cost grows with width — $E(\delta) = C/\delta + D\delta$ has minimum at $\delta\bigstar = \sqrt{(C/D)}$ — and the sources do not return the required D.

The mixture inequality is reversed. The route uses $(1 - p) + pe^{-x} \leq e^{-px}$; convexity of the exponential gives the opposite, $e^{-px} \leq (1 - p) + pe^{-x}$ (at $p = ½$, $x = 1$: $0.68394 > 0.60653$), and the mixture tends to $1 - p$, not zero, as $x \to \infty$.

Theorem 18 (entropic coefficient-route failure). The fixed-flux patch argument, the gradient-only wall functional and the probability-mixture inequality do not derive a positive physical B_τ or an area law. The result does not exclude every possible entropy-based confinement mechanism; it rejects this derivation as written.

32. Screened-channel reduction and the unreturned breaking distance

A minimal static Hamiltonian with D equivalent screened channels,

$$H_D(L) = [[\sigma L + \mu_{end}, g_0 \mathbb{1}_D^T], [g_0 \mathbb{1}_D, 2M\, \mathbb{1}_{D\text{-block}}]],$$

couples the flux tube only to the symmetric direction $s_D = D^{-1/2} \mathbb{1}_D$ with effective coupling $g_{eff} = \sqrt{D}\, g_0$; the remaining $D - 1$ combinations are exact threshold spectators, and the spectrum is that of the 2×2 block $[[\sigma L + \mu_{end}, \sqrt{D}\, g_0], [\sqrt{D}\, g_0, 2M]]$ plus $D - 1$ copies of 2M, with avoided-crossing gap $2\sqrt{D}|g_0|$.

Theorem 19 (screened-basis reduction). An equal-coupling D-channel static-light basis reduces exactly to one flux/symmetric avoided-crossing block and $D - 1$ threshold spectators. The supplied sources reduce the algebra but do not determine the physical threshold or mixing.

The strange-confinement operator's twenty oriented support atoms give the diagnostic case $D = 20$; direct diagonalisation of the 21×21 Hamiltonian agrees with the reduction to 8.0×10^{-15} (independently reproduced at 4.0×10^{-15}). $D = 20$ is not a physical static-light census.

Two structural returns close parts of this box.

Proposition 32.1 (renormalisation-invariant threshold). Both channels of H_D contain the same static source pair, so a static-source self-energy renormalisation shifts both diagonal entries equally: $\mu_{end} \to \mu_{end} + 2\delta m_Q$ and $2M_{Q\bar{q}} \to 2M_{Q\bar{q}} + 2\delta m_Q$. Neither $M_{Q\bar{q}}$ nor μ_{end} is separately scheme-independent; the invariant is

$$E_{br} = 2M_{Q\bar{q}} - \mu_{end},$$

and the convention $\mu_{end} = 0$ (whence $M_{Q\bar{q}}^{ren} = E_{br}/2$) loses no physics. The unmixed candidate distance is $L_{sb}^{(0)} = E_{br}/\sigma$. The pair $(M_{Q\bar{q}}, \mu_{end})$ is therefore retyped throughout this paper to the single physical unknown E_{br}.

Proposition 32.2 (mixing-coupling extraction and sign closure). At the unmixed crossing $\sigma L + \mu_{end} = 2M_{Q\bar{q}}$, the minimum mixed-level separation is $\Delta E_{min} = 2\sqrt{D}|g_0|$, so

$$|g_0| = \Delta E_{min}/(2\sqrt{D}) \quad (D = 20: |g_0| = \Delta E_{min}/8.94427191).$$

The sign of g_0 is unphysical — rephasing the symmetric screened state by −1 sends $g_0 \to -g_0$ — so the sign ambiguity and the extraction formula are closed; only the magnitude remains open pending the off-diagonal correlator or the avoided-crossing spectrum.

The quantity to be calculated is therefore preferably the self-energy-cancelling ratio

$$E_{br} = -\lim_{t\to\infty} (1/t) \ln [C_{Q\bar{q},Q\bar{q}}(t) / C_{tube,tube}(L = 0, t)],$$

with the gauge-invariant static-light operator and normalisation declared. No existing calculation supplies this correlator in a declared static-source renormalisation convention. As a deliberately non-admitted illustration only, $M_{Q\bar{q}} \to (20/9)m_\pi \simeq 0.3102$ GeV with $\mu_{end} = 0$ and the reference σ_0 gives $L_{sb} \simeq 2.40$ fm — not a VERSF prediction: a constituent quark scale is not a static-light singlet energy, and binding and source self-energy are omitted (the omission of the common self-energy is exactly what Proposition 32.1 shows the invariant target avoids).

Part X — Registered candidates for the missing strong-sector objects

33. Orientation boundary and Candidate C1

Tensoring either HCMR branch with the positive reference centre transfer preserves the exact conjugacy of Theorem 12, so the reference confinement calculation is branch-blind — the constructive twin of Theorems 13 and 6. The contextual criterion supplies the exact survival condition for a class $\kappa = [\rho] \in H^1(\Gamma_{vac}; \mathbb{Z}_7)$: it survives iff a closed admissible offset has nonzero pairing with a cycle left open by completion. This is the right type of object to carry orientation, because it is not a real spectral invariant of the HCMR quadratic form. A minimal orientation-odd term $H_{odd} = \lambda_\kappa \kappa$ would split the branches by $2\lambda_\kappa$ with one-step transfer ratio $e^{-2a_t\lambda_\kappa}$ — if the map $k \mapsto \kappa$ and the coupling λ_κ were derived, which they are not.

Theorem 20 (orientation-coupling boundary). The contextual class can lift the HCMR conjugate degeneracy only through an independently derived orientation-odd coupling. Without that coupling, the exact positive-transfer branch equivalence remains.

Proposition 33.1 (retained-model orientation coupling vanishes exactly). Normalise $\kappa|k = \pm1\rangle = \pm|k = \pm1\rangle$, so that $H_{odd} = \lambda_\kappa\kappa$ produces $E_+ - E_- = 2\lambda_\kappa$ and $\lambda_\kappa = (E_+ - E_-)/2$. The retained construction has exact branch conjugacy at every regulator, hence zero branch splitting in every real spectral functional, and therefore

$$\lambda_\kappa^{retained} = 0 \text{ exactly.}$$

This is an exact statement about the present retained positive construction, not a prediction that the physical theory has $\lambda_\kappa = 0$: it certifies, from a third independent direction (after Theorems 13 and 6), that a nonzero selector must be a genuinely new charged, orientation-odd matrix element absent from the frozen quadratic stack. The charged-sector normality residual of

Proposition 34.2 is dimensionless evidence that orientation becomes visible to charged probes; it cannot be converted into λ_κ, which is an energy splitting.

Candidate C1 (CRT branch-to-orientation map — registered, quarantined under P-GQ10). The superseded terminal edge phase $e^{2\pi ik/21}$ lives in $\mathbb{Z}_{21} \cong \mathbb{Z}_3 \times \mathbb{Z}_7$. Fix the terminal-coordinate CRT identification

$k \equiv 7a + 3b \pmod{21}$, $a \in \mathbb{Z}_3$, $b \in \mathbb{Z}_7$,

a bijection whose coordinates are, since $7 \equiv 1 \pmod 3$ and $3^{-1} \equiv 5 \pmod 7$,

$a = k \bmod 3$, $b = 3^{-1}k \bmod 7 = 5k \bmod 7$,

with reconstruction rule $k = 7a + 3b \pmod{21}$. The $\mathbb{Z}_3$ coordinate is therefore canonical, $a = k\_\Omega \bmod 3$; the $\mathbb{Z}_7$ coordinate is NOT the canonical residue $k \bmod 7$ but the transformed terminal coordinate $b = 3^{-1}k\_\Omega \bmod 7$ — which is exactly the derived terminal representative r satisfying $3r \equiv k\_\Omega$ (and hence $6r \equiv k\_vac$) mod 7. Under this identification: $k\_\Omega = +1 \mapsto (a, b) = (+1, 5)$ and $k\_\Omega = -1 \mapsto (-1, 2)$, each verified by the reconstruction rule. Over a full six-step rim cycle the $\mathbb{Z}_3$ part cancels ($6 \equiv 0 \bmod 3$) while the $\mathbb{Z}_7$ part returns $6b \equiv 6 \times 5 \equiv 2 = k\_vac \pmod 7$ — which is why the closed-loop Wilson audit that superseded the edge phase retained only its $\mathbb{Z}_7$ content. C1 therefore proposes: (i) the branch-to-cohomology map $k\_\Omega = \pm 1 \mapsto$ per-step $\mathbb{Z}_3$ charge ± 1, and (ii) the identification of the contextual $\mathbb{Z}_7$ class and the centre $\mathbb{Z}_3$ cocycle as the two CRT factors of one edge datum. If admitted, the branch-sign degeneracy and centre conjugation are one degeneracy, and any coupling that selects the branch simultaneously orients the centre. C1's typing prerequisite stands or falls with the RWCT-1 verdict on the driven object; a passing verdict validates the map at the typing level but does not by itself select a branch, which additionally requires the charged coupling λ_κ (Theorem 20, D-GQ3). The dihedral symmetry of the undriven primitive transfer (automorphism order 12, conjugating drive sign and twist sign together, against order 6 for the driven kernel) forces a null there by symmetry, so an undriven null is not evidence.

34. Candidate C2 — the winding-lift wall block

Registered, doubly quarantined under P-GQ10; populates no boundary.

Lifting the frozen D36 one-Fold kernel over its own rim winding reduced mod 3 — the cocycle forward +1, backward −1, hub and self neutral, with no adjustable coefficients — yields a centre-graded transfer. The following statements are exact mathematics of the lift, verified on the frozen kernel; only their physical reading is conditional.

Proposition 34.1 (closed-form centre gap by hub avoidance). The lifted spectrum decomposes exactly into charge sectors. Both nonzero triality sectors acquire the same strictly positive Perron gap

$\Delta = \ln(D(\alpha)/(D(\alpha) - 1)) = -\ln(1 - p\_hub) = 0.223525982190$

per Fold at the frozen one-bit drive, with $p_{hub} = 1/D$ the rim-to-hub coupling probability. The twisted Perron eigenvector is a pure rim Fourier mode with identically zero hub amplitude: a centre-charged excitation cannot enter the centre-neutral hub and pays exactly the hub-escape probability per step. Conjugate twists tie in modulus exactly; sectorwise reflection positivity holds; and $\Delta(\alpha)$ decreases monotonically from $\ln(4/3)$ at zero drive to zero at infinite drive.

Remark 34.1a (charge-blindness of the full lift). The identical closed-form gap is verified to hold for every nontrivial sector of the full $\mathbb{Z}_6$ winding lift, not only the two triality sectors: at this order the gap is charge-blind (in particular the $\mathbb{Z}_3$ and $\mathbb{Z}_2$ centre gaps coincide). This is recorded as a fingerprint for the centre-versus-generation discrimination of the typing test, since a genuinely colour-sensitive substrate block would generically split the two.

Proposition 34.2 (charged-sector normality failure). The π-normality of Theorem 3 fails in every twisted sector (residual 0.15468094255 at the frozen drive; quoted at reduced precision in earlier revisions): the convention-free coincidence of the polar squares is a neutral-sector property, and the orientation of the history law becomes operationally visible exactly and only to centre-charged probes. This sharpens debt D-GQ3: the orientation-odd coupling it demands must be charged.

Proposition 34.3 (candidate reduced wall generator). Write the centre-projected lift transfer as $T_{C2} = P_0 + e^{-\Delta}(P_1 + P_2)$. Its logarithm gives the finite reduced generator per Fold,

$$\mathcal{K}_{wall,Fold}^{red,cand} = -\ln T_{C2} = \Delta(P_1 + P_2) = \mathrm{diag}(0,\ 0.223525982190,\ 0.223525982190)$$

in the triality basis $(0, +1, -1)$; restricted to the charged subspace, $\mathcal{K} = \Delta \mathbb{1}_2 = 0.223525982190\ \mathbb{1}_2$. This is an exact result for the lifted finite kernel — the operator shape of the wall block is now derived at candidate grade — not yet the physically admitted master-action wall Hessian. For a Euclidean time step a_t the per-cell wall Hamiltonian is $H_{wall,cell} = (\Delta/a_t)(P_1 + P_2)$ and the corresponding tension is $\sigma_{cand} = \Delta/(a_t\, a_z)$.

What admission would deliver. Under the currently inserted isotropic identification $a_t = a_z = \Lambda_3^{-1}$ of §29, Proposition 34.3 evaluates to: per-cell twisted transfer factor $e^{-\Delta} = 0.799694$ replacing the reference e^{-1}; energy per primitive longitudinal cell $E_{cell} = \Delta\Lambda_3 = 0.0504997$ GeV replacing Λ_3; and

$$\sigma_{cand} = \Delta\Lambda_3^2 = 0.0114090530\ \mathrm{GeV}^2,\quad \sqrt{\sigma_{cand}} = 106.813\ \mathrm{MeV},$$

against the reference $\sqrt{\sigma_0} = 225.923$ MeV. These numbers display the arithmetic consequence of admission only; they are diagnostic and non-promotable at the present grade. The honest statement of standing: the centre-projected finite reduced wall generator is derived at candidate grade; the physical same-action $\mathcal{K}_{wall}^{red}$ is not yet admitted because colour typing (RWCT-1) and the temporal-to-longitudinal bridge (D-GQ10) remain open.

What blocks admission — two named conditions. (1) *Typing:* the identification of rim winding with colour-centre charge must be derived, not assumed. The corpus carries a rival reading of the same grading — the driven kernel is strongly lumpable at every α under exactly the antipodal mod-3 quotient, which the primitive-ledger analysis reads as the generation quotient — so the question is decided only by the substrate typing of the forward rim edge: does its transport act on colour indices, or permute generation blocks? The decision rule is sealed in the companion protocol RWCT-1 (outcomes COLOUR-DERIVED / GENERATION / MIXED / NULL-BY-SYMMETRY / REFUTED) and requires the typed 102-carrier instrument and the E_C typing of the charged block (debt D-GQ11). (2) *Bridge:* Δ is a temporal (per-Fold) gap of the history sector, while the wall block requires a per-longitudinal-cell cost; the isotropic identification a_t = a_z converting one into the other is at present an insertion (debt D-GQ10).

35. Consistency with Theorem 16

The registrations do not contradict the confinement no-go; they instantiate its escape clause. Theorem 16 forbids determination of the gap from the current corpus's local, topological and positivity data, and names "additional centre-sensitive dynamical data" as mathematically necessary. Candidates C1 and C2 propose precisely such data — a typing of the rim winding and an orientation assignment — whose admissibility is a new derivation, not a re-reading of existing data. RWCT-1 decides the shared colour-typing prerequisite for both candidates, not their full admission. A COLOUR-DERIVED verdict advances both: it promotes Δ to conditionally derived — conditional further on the instrument provenance and the bridge debt D-GQ10 — and validates C1's map at the typing level, while physical branch selection remains conditional on the derived charged coupling λ_κ. A GENERATION or REFUTED verdict demotes Δ to a numerical curiosity, vacates both registrations, and returns D-GQ1 to fully open status. Either outcome is progress: the refutation side of the ambiguity Theorem 16 proved exhaustive is reduced to one sealed calculation, and the admission side to a short, explicitly ordered chain of named debts.

The revised position of the boxed objects after Parts IX–X:

Quantity	What can now be returned
$\mathcal{K}$_wall^red (finite reduced block)	Candidate generator derived: Δ($P_1 + P_2$) (Proposition 34.3)
Physical same-action $\mathcal{K}$_wall^red	Still conditional on RWCT-1 and the D-GQ10 bridge
M_{Qq̄}	Not numerically derivable; not separately scheme-independent
μ_end	Not independent; μ_end = 0 admissible by convention (Proposition 32.1)
E_br = 2M_{Qq̄} − μ_end	Correct invariant target; value open
g_0	Sign unphysical and extraction formula closed (Proposition 32.2); magnitude open
λ_κ in the retained model	Exactly zero (Proposition 33.1)
Physical λ_κ	Open; requires a new charged orientation-odd matrix element

The strongest genuine advances of this Part are therefore the derived candidate generator $\mathcal{K}$_wall,Fold^red,cand = 0.223525982190 ($P_1 + P_2$) and the retyping (M_{Qq̄}, μ_end) → E_br:

the finite operator shape is closed and one falsely independent parameter is removed, but the physical static-light threshold and mixing dynamics are still not produced.

36. Stage-1 machine audit of the registered candidates

A stage-1 verification package was executed against the archived frozen arrays — the K7 kernel and stationary law, SHA-matched to the manifest and bit-identical to the canonical D36 rebuild (maximum deviation 1.1×10^{-16}) — and its certificate reproduces under independent rerun with zero differences. Findings, at their sealed grades:

Mechanisms machine-verified. Candidate C2's mechanism reproduces exactly on the archived kernel: charged-sector Perron radius $(D - 1)/D = 0.799694113793042$ with $D = 4.992364522761903$ read from the matrix itself, gap $\ln(D/(D - 1)) = 0.223525982190$ in both charged sectors (errors $\leq 7.5 \times 10^{-16}$), hub amplitudes $\leq 1.1 \times 10^{-16}$, conjugate-sector residual 3.2×10^{-16}, drive-sweep maximum gap error 1.6×10^{-15}, positive polar transfers (minimum twisted-sector transfer eigenvalue 0.0861), and charged normality residual 0.15468094255, consistent with the rounded value 0.155 recorded in Proposition 34.2; the full-precision value is archived in the stage-1 certificate. Candidate C1's terminal-coordinate map $\Phi(k) = (k \bmod 3, 3^{-1}k \bmod 7)$ is machine-verified as a bijective homomorphism on all twenty-one elements with inverse $k = 7a + 3b \pmod{21}$, returning $(+1, 5)$ and $(-1, 2)$; the six-step cycle classes are $(0, 6b \bmod 7) = (0, 2)$ for the + branch and $(0, 5)$ for the − branch, i.e. $6 \times 5 \equiv 2$ and $6 \times 2 \equiv 5 \pmod 7$, conjugate as required by branch conjugation. An independent census of all fifteen perfect matchings of the rim confirms exactly one strongly lumpable pairing, the antipodal one. Both mechanisms are thereby upgraded from session-verified to machine-verified on the archived frozen arrays.

Typing verdict: NOT TESTED. The inputs the sealed rule requires — the typed carrier with move assignment, the E_C typing of the charged block, the edge-resolved transfer — were absent from the materialised packages, so no RWCT-1 outcome is issued. The package's provisional GENERATION label is retyped here on two grounds of protocol discipline. First, its principal finding — that the shipped full-carrier instrument extends the history instrument to the retained field carrier by the identity — is a null by construction: an instrument built as history $\otimes$ $\mathbb{1}$_field cannot exhibit a colour action whether or not one exists in the master action, and the instrument's own premise records it as supplied rather than forced. Such a null is the evidential twin of the protocol's NULL-BY-SYMMETRY outcome and carries no weight against, or for, the candidates. Second, the documentary generation typing of the antipodal layers was already recorded at registration (§34) as the rival reading; it is the motivation for the test, not a new discriminating measurement. The documentary lean toward the generation reading is recorded; the candidates neither advance nor retreat, and every quarantine stands unchanged.

Proposition 36.1 (current-package instrument insufficiency). In the materialised stage-1 packages, the full-carrier instrument acts on the retained field carrier as the identity on colour indices, and no edge-resolved, colour-nontrivial transport operator assigned to the forward rim move is present in any package. Consequently the sealed RWCT-1 computation cannot be executed on these packages: Steps 2–4 have no nontrivial colour action to evaluate, and any GENERATION or REFUTED grade issued from them would be a null-by-construction promotion (F-GQ23). The stage-1 packages are therefore insufficient as the RWCT-1 typed input, and the typing verdict remains NOT TESTED.

Scope note. This is an insufficiency statement about the materialised packages, not a disqualification of the full-carrier instrument as such. Whether the master action forces a colour-nontrivial edge action on the retained carrier — or genuinely forces the identity, which would itself be a decisive REFUTED-grade datum if derived rather than supplied — is exactly the content of debt D-GQ11. The audit closes the shipped-package route; it does not adjudicate the underlying physical question.

Proposition 36.1 is the audit's genuine addition: it closes the shipped-package route to the typing verdict and narrows D-GQ11 to the master-action edge resolution, with both outcomes of that resolution — nontrivial action or derived identity — now typed as decision-grade.

Part XI — Certificate, premises, falsifiers, debts and programme impact

37. Numerical certificate

Quantity	Return
D36 α	0.959001109719975
Stationary law (closed form)	$\pi_{hub} = 7/(6D + 7) = 0.1894237309$, $\pi_{rim} = D/(6D + 7) = 0.1350960448$
Raw-kernel maximum stationary current (closed form)	$\pi_{rim} \tanh(\alpha/2) = 0.0602316978168342$
Raw-kernel non-real eigenvalues	four
Raw-kernel negative real eigenvalues	two
Normality residual $\|PP^\sharp - P^\sharp P\|_{max}$	3.3×10^{-16}
T_OS weighted self-adjoint residual	2.25×10^{-17}
T_OS spectral gap (closed form)	$3/D(\alpha) = 0.600917658621$
T_OS minimum eigenvalue (closed form)	$((D - 7)/(7D))^2 = 0.00330035811247$
T_OS first transfer energy	0.918587514012

Quantity	Return
Energy-doubling identity	$E_3 = 2E_1$ exact, from $\|\lambda_1\|^2 = -\lambda_3 = (2\cosh\alpha - 1)/D$
Level-3/4 splitting (closed form)	$(3 - 2\cosh\alpha)/D^2 = 3.06354 \times 10^{-4}$; crossing at $\alpha\bigstar = \operatorname{arccosh}(3/2) = 0.9624236501$
Orientation unitary	det U = 1 exactly; phases 0, ±0.6575731940, ±1.8227348715, π, π
det P (closed form)	0.00145443527
Level-6 det Y_u	$2.59927387873 \times 10^{-8}$
Level-6 det Y_d	$7.92144975801 \times 10^{-11}$
Level-6 det(Y_u Y_d)	$2.05900174376 \times 10^{-18}$
Maximum k = 1/k = 6 conjugacy residual	0
Positive benchmark lattice sectors	343
Benchmark truncated partition sum	16.0921147349 (cutoff 3; cutoff 9 gives 26.6018182265 — not converged, certificate only)
Benchmark orientation-symmetry residual	0
Strong phase in retained positive completion	0
Physical triality gap	not identified; exhaustive T(Δ) family, $\Delta \in \mathbb{R}$
Physical string-breaking spectrum	not identified; arbitrary (σ, M, g) family
Reference execution (merged)	$\sigma_0 = 0.0510412835712$ GeV², B_τ^implied $= 2.47465525471 \times 10^{-5}$ GeV⁴, R★ = 3.57519699623 fm, $\mathcal{E}(R\bigstar) = \sigma_0$ grid-confirmed
Pipeline typing	$\Lambda_3(Z_3 = 1)$ with FRGM thresholds = 0.216516653 GeV vs reference 0.225923180686 GeV (4.34 %, distinct pipelines)
Entropic audit	patch bound Φ^2/A; dilation slope −1.0000000000; $\frac{1}{2} + \frac{1}{2}e^{-1} = 0.68394 > e^{-1/2} = 0.60653$
Screened reduction residual	7.99×10^{-15} (package); 4.0×10^{-15} (independent rebuild)
Candidate C2 gap (quarantined)	$\Delta = \ln(D/(D-1)) = 0.223525982190$ per Fold; $\sigma = \Delta\Lambda_3^2 = 0.011409$ GeV² if admitted
Candidate reduced wall generator (quarantined)	$\mathcal{K}$_wall,Fold^red,cand $= \Delta(P_1 + P_2)$, Δ = 0.223525982190; charged restriction $\Delta\mathbb{1}_2$; $E\_cell = \Delta\Lambda_3 = 0.0504997$ GeV and $\sigma\_cand = \Delta\Lambda_3^2 = 0.0114090530$ GeV² under the inserted isotropic bridge
Invariant threshold retyping	$(M_{Q\bar{q}}, \mu\_end) \to E\_br = 2M_{Q\bar{q}} - \mu\_end$; μ_end = 0 admissible; value open
g_0 extraction	$\|g_0\| = \Delta E\_min/(2\sqrt{D})$; D = 20: ΔE_min/8.94427191; sign unphysical
λ_κ in the retained model	0 exactly (branch conjugacy); physical λ_κ open

Quantity	Return
Candidate C1 CRT coordinates (quarantined)	terminal-coordinate identification $k \equiv 7a + 3b \pmod{21}$: $(k\_\Omega = +1) \mapsto (a, b) = (+1, 5)$ with $a = k \bmod 3$ canonical, $b = 3^{-1}k \bmod 7$ transformed; checks $3 \cdot 5 \equiv 1$, $6 \cdot 5 \equiv 2 \pmod{7}$
Stage-1 audit: kernel provenance	archived arrays SHA-matched; bit-identical to the canonical rebuild (1.1×10^{-16})
Stage-1 audit: C2 reproduction	gap errors $\leq 7.5 \times 10^{-16}$; hub amplitudes $\leq 1.1 \times 10^{-16}$; conjugacy 3.2×10^{-16}; drive sweep 1.6×10^{-15}; min twisted transfer eigenvalue 0.0861
Stage-1 audit: C1 and census	Φ bijective homomorphism on $\mathbb{Z}_{21}$; unique strongly lumpable matching = antipodal (15 tested); certificate rerun diff none
Stage-1 typing verdict	NOT TESTED — inputs absent; shipped packages insufficient (Proposition 36.1); derived-identity branch live; documentary generation lean recorded

38. Admission ledger

Gate	Result after GQSC-1R
Faithful quotient and charge lattice	**CLOSED on the frozen ledger**
Raw D36 clock as physical transfer	**FAILED**
Normality of the frozen clock	**CLOSED** (new)
Canonical retained positive transfer	**CLOSED, convention-free**
Finite retained reflection positivity	**CLOSED**
Full faithful chiral gauge measure	**NOT PROMOTED from retained model**
Positive-completion strong phase	$\bar{\theta} = 0$
Nonzero strong phase inside ordinary positive pushforward	**NO-GO**
Positive-measure branch orientation	**NO-GO** (two independent routes: branch conjugacy and det U = 1)
Physical triality gap	**NON-IDENTIFIED, exhaustively**
Physical string breaking	**NON-IDENTIFIED**
SM-28 strong phase	**conditional zero-or-obstruction result substantially strengthened**
SM-27 confinement	**physical pass refused by exact non-identifiability**

Gate	Result after GQSC-1R
SM-29 global quantum completion	**finite retained measure pass; full chiral gauge completion not admitted**
NQTC reference centre transfer (merged)	**EXECUTED — reference grade only**
Entropic wall-coefficient route	**REJECTED as written**
Static-light threshold and mixing	**NOT RETURNED**
Candidates C1/C2	**REGISTERED — shared typing prerequisite gated by RWCT-1; C2 further gated by the D-GQ10 bridge; branch selection further gated by λ_κ**
Stage-1 candidate audit	**MECHANISMS MACHINE-VERIFIED; TYPING NOT TESTED; shipped-package route closed, derived-identity branch live**

39. Falsifiers

Falsifier namespace note: the identifiers F-GQ1–F-GQ24 below are local to GQSC-1R and disjoint from RPC-1's F1–F39, SMC-1U's F1–F43, FGQT-1's F-FG1–F-FG14 and the RWCT-1 protocol's F-RW1–F-RW4.

- **F-GQ1 — Kernel mismatch.** The frozen D36 matrix does not equal the archived one-Fold kernel. (Premise P-GQ2.)
- **F-GQ2 — Stationary-adjoint failure.** P♯ is not Markov or T_OS is not positive in $L^2(\pi)$. (P-GQ2.)
- **F-GQ3 — Reflection failure.** The complete cylinder algebra contains a negative OS norm despite the positive transfer construction. (P-GQ2.)
- **F-GQ4 — Determinant-factorisation failure.** The HCMR charged matrices do not have the stated orientation-times-positive-magnitude form. (P-GQ3.)
- **F-GQ5 — Nontrivial positive character.** A multiplicative nontrivial character leaves every positive generator weight real and nonnegative. (P-GQ4.)
- **F-GQ6 — Branch-conjugacy failure.** A frozen array violates $H^{(6)} = C^T H^{(1)} C$ or the reduced analogue. (P-GQ3.)
- **F-GQ7 — Hidden centre coefficient.** The existing source stack contains a uniquely derived centre-sensitive transfer coefficient not consumed by this paper. (Would demote Theorems 14 and 16.)
- **F-GQ8 — Hidden string-breaking data.** The source stack already fixes σ, M and g from one same-action static-source calculation. (Would demote Theorem 15.)
- **F-GQ9 — Experimental insertion.** Any observed strong-phase or confinement datum enters the upstream selection. (P-GQ6.)
- **F-GQ10 — Normality failure (new).** The archived kernel violates $\mathbb{Z}_6$ rim covariance or the two-state form of its hub-symmetric sector, or $\|PP♯ - P♯P\|$ fails to vanish at machine

precision. Firing this falsifier reopens the left/right polar convention and voids Theorems 3, 5, 6 while leaving Theorems 4 and 7 intact for the right polar square.

- **F-GQ11 — Spectral-identity failure (new).** Any of the closed forms of Proposition 5.1 ($E_3 = 2E_1$, gap = 3/D, splitting = (3 − 2 cosh α)/D², minimum = ((D − 7)/(7D))²) disagrees with the frozen arrays beyond machine precision.
- **F-GQ12 — Benchmark promotion (new).** The truncated partition sum 16.0921147349, the benchmark stiffness K, or any T_trial coefficient is quoted as a physical boundary value. (P-GQ7.)
- **F-GQ13 — Reference promotion.** Any Part IX output (σ_0, B_τ^implied, R★, W_{N,M}) is quoted as a physical VERSF derivation. (P-GQ8.)
- **F-GQ14 — Pipeline conflation.** The 225.923 MeV and 216.517 MeV values are presented as outputs of one another's inputs, or any Λ_3 value is quoted as the output of a formula whose printed inputs do not generate it. (P-GQ9.)
- **F-GQ15 — Entropic rehabilitation by citation.** The audited entropic route is cited as supplying B_τ without a repaired derivation addressing all three failures of §31.
- **F-GQ16 — Diagnostic census promotion.** D = 20 is quoted as a physical static-light census, or the L_sb ≃ 2.40 fm illustration is quoted as a prediction.
- **F-GQ17 — Candidate C1 promotion.** Candidate C1 is quoted as a derived branch-to-orientation map, or populates any boundary, before RWCT-1 returns COLOUR-DERIVED and the branch map is explicitly derived from the typed transport. (P-GQ10.)
- **F-GQ18 — Candidate C2 promotion.** Candidate C2, Δ, or $\sigma = \Delta\Lambda_3^2$ is quoted as derived, or populates any boundary, before RWCT-1 returns COLOUR-DERIVED and the temporal-to-longitudinal bridge of D-GQ10 is derived. (P-GQ10.)
- **F-GQ19 — Orientation selection without coupling.** A physical branch selection is asserted before the charged orientation-odd coupling λ_κ is returned from the same action, regardless of the status of C1's map. (P-GQ10; D-GQ3.)
- **F-GQ20 — Rule drift.** The RWCT-1 decision rule is altered after typed arrays are examined, or a NULL-BY-SYMMETRY outcome on the undriven object is cited as evidence against the candidates.
- **F-GQ21 — Hidden coefficient.** The source stack is found to contain a uniquely derived 𝒦_wall, E_br, g_0 or λ_κ not consumed here.
- **F-GQ22 — Manifest currency.** Any package manifest hashes a superseded revision of a frozen input.
- **F-GQ23 — Null-by-construction promotion.** A trivial colour action exhibited by a probe structurally incapable of carrying one — an instrument that is the identity on the field carrier, the undriven kernel, or any equivalent — is cited as a GENERATION or REFUTED verdict under RWCT-1, or as evidence against the candidates. (P-GQ10.)
- **F-GQ24 — Derived-identity suppression.** A colour-identity edge action that is derived from the master action (rather than supplied by an instrument construction) is dismissed as null-by-construction, or the REFUTED grade it would mandate under the sealed rule is withheld. (P-GQ10.)

40. Open debts

- **D-GQ1 — Centre-sensitive transfer block.** The single object Theorem 16 proves necessary: a faithful, centre-sensitive static-source transfer block or correlator matrix derived from the same master action and branch as HCMR. Critical path for SM-27. Candidate C2 (§34) is queued for this slot, and the finite reduced generator shape is now derived at candidate grade (Proposition 34.3); the remaining content of this debt is the physical same-action admission, gated by RWCT-1 and D-GQ10.

- **D-GQ2 — Chiral determinant-line and bordism audit.** The full determinant-line holonomy audit over the FGQT bundle generators, required before the retained measure can be promoted toward object 3 of §4.
- **D-GQ3 — Orientation-odd selector.** The branch tie (Theorems 12, 13) and the trivial total winding (Theorem 6) jointly certify that only an orientation-odd coupling can select k; deriving one from the substrate remains open, in registered alignment with the assembly-layer sign debt. Candidate C1 (§33) supplies the queued branch-to-orientation map, and Proposition 34.2 sharpens the requirement: the coupling must be charged. Proposition 33.1 sharpens it further: $\lambda_\kappa = 0$ exactly in the retained model, so the physical coupling must be a genuinely new charged matrix element, and the charged-sector normality residual is qualitative evidence only, not convertible to an energy splitting. The coupling λ_κ itself remains fully open.
- **D-GQ4 — Substrate sector stiffness.** The physical replacement for the benchmark K: a derived positive quadratic sector energy on the FGQT lattice.
- **D-GQ5 — Second-quantisation carrier.** The declared finite carrier on which T_{quad} is second-quantised, with a positivity audit at the Fock level.
- **D-GQ6 — Regulator and volume limits.** Regulator sequences and infinite-volume control for the retained measure beyond the finite trace-norm statement of Theorem 8.
- **D-GQ7 — Manifest currency.** The reproducibility package must carry a revision-registry check confirming that every hashed input is the current frozen revision of its source, not merely an integral copy.
- **D-GQ8 — Invariant static-light threshold.** The renormalisation-invariant $E_{br} = 2M_{Q\bar{q}} - \mu_{end}$ (Proposition 32.1), preferably via the self-energy-cancelling correlator ratio of §32, with regulator and volume covariance. μ_{end} is removed as an independent debt ($\mu_{end} = 0$ is an admissible convention), and $M_{Q\bar{q}}$ alone is scheme-dependent.
- **D-GQ9 — Mixing magnitude.** $|g_0|$ from off-diagonal flux-tube/static-light correlators or the avoided-crossing spectrum, via the closed extraction formula $|g_0| = \Delta E_{min}/(2\sqrt{D})$ of Proposition 32.2; the sign is unphysical, so only the magnitude remains.
- **D-GQ10 — Temporal-to-longitudinal bridge.** Derivation of the isotropic identification $a_t = a_z$ (or the correct anisotropy) converting a per-Fold gap into a per-cell wall cost; prerequisite for admitting Candidate C2.
- **D-GQ11 — Typed-carrier package assembly.** The RWCT-1 inputs — the typed 102-carrier instrument, the E_C typing of the charged block, the edge-resolved primitive transfer — assembled into one frozen package with a currency-checked manifest. Sharpened by the stage-1 audit (Proposition 36.1): the shipped packages are insufficient as this input; the debt now includes determining whether the master action forces a nontrivial or an identity colour edge action — either outcome is decision-grade.
- **D-GQ12 — Entropic repair.** If the entropic mechanism is to be pursued, a same-action term with cost growing in wall width (the D of $E(\delta) = C/\delta + D\delta$) must be derived; otherwise the route stays closed.

- **D-GQ13 — NQTC input publication.** The NQTC-internal threshold tuple that generates $\Lambda_3 = 0.225923$ GeV through the §30 formula must be published; until then the reference value is consumable only at pipeline grade and is not independently reproducible from this paper.

41. Exact next return

The no-go theorem identifies the smallest new object capable of moving the programme:

1. a faithful, centre-sensitive static-source transfer block or correlator matrix;
2. its derivation from the same master action and branch as HCMR;
3. regulator and volume sequences;
4. the triality-sector eigenvalues or the flux-tube/two-singlet mixing matrix;
5. a full chiral determinant-line holonomy audit over the FGQT bundle generators.

A further regulator refinement of the existing local HCMR matrices cannot determine these quantities, because the entire exhaustive Δ and (σ, M, g) transfer families remain compatible with all such local data.

With Part X in place, the most economical next return is the RWCT-1 typing run on the assembled typed-carrier package (D-GQ11). RWCT-1 decides the shared colour-typing prerequisite for both candidates: a GENERATION or REFUTED verdict vacates both slots definitively, while a COLOUR-DERIVED verdict advances C1 and C2 — with C2 remaining conditional on the temporal-to-longitudinal bridge (D-GQ10), and physical branch selection remaining conditional on a derived charged coupling λ_κ (D-GQ3). Within the boxes, item 4's target is retyped: the physically meaningful pair is $(E\_br, |g_0|)$, the endpoint convention being unphysical (Propositions 32.1–32.2).

42. Terminal verdict ledger

Structural.

- The faithful quotient and complete charge/period lattices are consumed exactly; the retained global measure exists (Theorem 8); the full chiral gauge measure is not claimed.
- Current-corpus confinement no-go (Theorem 16) — the physical pass is refused as non-identifiable, not deferred as pending.

Derived mechanisms.

- π-normality of the frozen kernel (Theorem 3): orientation and magnitude commute; the canonical transfer is convention-free.
- Spectral transfer identity and closed forms (Theorem 5, Proposition 5.1): spec $T\_OS = \{|\lambda(P)|^2\}$; gap = 3/D; $E_3 = 2E_1$; splitting sign change at $\alpha\bigstar = \operatorname{arccosh}(3/2)$, with the frozen one-bit drive 0.36 % below the crossing.
- Orientation unitary with det U = 1 and conjugate-pair phases (Theorem 6).

- Exhaustiveness of the centre family (Proposition 14.1): the admissible triality-gap set is exactly $\mathbb{R}$.

Exact zeros.

- arg det(Y_u Y_d) = 0 at every regulator (Theorem 9).
- Trivial topological character on every ordinary positive full-support completion (Theorem 10).
- $\bar{\theta} = 0$ in the retained positive completion; nonzero $\bar{\theta}$ is an obstruction, not a parameter (Theorem 11).
- $\lambda_\kappa = 0$ exactly in the retained positive construction (Proposition 33.1): the frozen stack supports no orientation-odd coupling; a physical selector must be new charged input.

Certified degeneracies.

- k = ±1 branch pair exactly conjugate at all four regulators; positive spectral selection impossible (Theorems 12, 13), independently corroborated by det U = 1.

Corrections and typing.

- The truncated benchmark sum 16.0921147349 is a certificate, not an approximation of the infinite-lattice normaliser (Remark 1.1).
- Theorem 10 needs only a generating positive-support set; full support is convenience, not necessity (Remark 10.1).
- The reflection-positivity display of the earlier draft is completed (Theorem 7).
- The static-light pair (M_{Qq̄}, μ_end) is retyped to the single invariant E_br = 2M_{Qq̄} − μ_end; μ_end is not independently physical, and the g_0 sign ambiguity is closed (Propositions 32.1–32.2).

Executed reference (merged).

- Reference centre transfer executed: positive, exact area law W_{N,M} = e^{−NM}, $\sigma_0 = \Lambda_3^2$ (Theorem 17) — the constructive complement of Theorem 16: mechanics proven, provenance absent.
- Entropic coefficient route rejected on three grounds (Theorem 18); screened-channel algebra reduced exactly (Theorem 19); sensitivity table pipeline-typed (§30).

Registered candidates (no admission).

- C1: CRT branch-to-orientation map, (k_Ω = ±1) ↦ ($\mathbb{Z}_3$ = ±1, $\mathbb{Z}_7$ = r), unifying the contextual class and the centre cocycle as factors of one edge datum (§33).
- C2: winding-lift wall block with derived Δ = ln(D/(D−1)) = 0.2235260 per Fold by exact hub avoidance, with charged-sector normality failure sharpening D-GQ3 (§34); finite reduced generator $\mathcal{K} = \Delta(P_1 + P_2)$ derived at candidate grade (Proposition 34.3); shared typing prerequisite gated by RWCT-1, admission further gated by D-GQ10.

- Stage-1 audit: both mechanisms machine-verified on the archived frozen arrays; typing NOT TESTED (inputs absent); the shipped packages certified insufficient as RWCT input (Proposition 36.1); the derived-identity branch remains live; documentary generation lean recorded; quarantines unchanged.

Blockers.

- D-GQ1 (centre-sensitive dynamical block) — sole critical path for SM-27.
- D-GQ2 (chiral determinant line / bordism) — critical path for SM-29 promotion.
- D-GQ3 (orientation-odd selector) — critical path for branch admission; must be charged (Proposition 34.2).
- D-GQ10 and D-GQ11 (bridge; typed-carrier assembly) — the named prerequisites for deciding the queued candidates.

Final lines.

- RETAINED GLOBAL POSITIVE-TRANSFER AND STRONG-PHASE-ZERO: PASS.
- PHYSICAL CONFINEMENT, STRING-BREAKING AND ORIENTED-BRANCH CERTIFICATES: REFUSED BY EXACT NON-IDENTIFIABILITY — and, by Proposition 14.1, exhaustively so.
- REFERENCE EXECUTION: PASS AT REFERENCE GRADE; ENTROPIC SHORTCUT: REJECTED.
- TWO CANDIDATES REGISTERED — SHARED TYPING PREREQUISITE GATED BY THE SEALED RWCT-1 VERDICT; C2 FURTHER BY THE D-GQ10 BRIDGE; BRANCH SELECTION FURTHER BY λ_κ.

Conclusion

GQSC-1R replaces the earlier architecture-only global paper with an executed finite theorem package. The raw D36 Fold law is not a positive Euclidean transfer matrix, but it is normal with respect to its stationary law, so the positive transfer it induces is canonical without convention: left and right polar squares coincide, the transfer spectrum is exactly the squared modulus spectrum of the oriented kernel, the spectral gap is the closed-form number $3/(2 + 2 \cosh \alpha)$, and the resulting finite retained cylinder measure is reflection positive. This supplies a mathematically clean separation — in fact a commuting separation — between irreversible history orientation and the positive Euclidean magnitude required for Hilbert-space reconstruction.

The faithful quotient and charge lattice then make the strong-phase statement exact. Every ordinary positive full-support global completion has trivial topological character. The frozen HCMR quark determinants are positive by construction, not merely numerically close to real. Consequently $\bar{\theta} = 0$ in the retained positive completion, and any nonzero $\bar{\theta}$ would be an obstruction to the ordinary positive-pushforward description rather than another unconstrained parameter choice.

The same analysis proves two limits. First, the positive construction cannot select the exact $k = \pm 1$ HCMR branch pair: the branch operators are orthogonally conjugate at every regulator, and independently the history orientation unitary has exactly trivial total winding, so no scalar spectral functional anywhere in the retained stack carries orientation. Second, the physical triality gap and string-breaking spectrum are not encoded in the current local or topological data: the positive reflection-compatible transfer families that realize inequivalent strong-sector outcomes are not merely exhibited but exhaustive, so the ambiguity cannot be removed by inspecting further members.

The merged static-source execution completes the picture from the constructive side. The reference centre-projector model performs exactly as the architecture intends — positive transfer, linear potential, exact area law — the moment a coefficient is granted, and Theorem 16 explains why granting it is the entire problem. The entropic shortcut to that coefficient is closed on three independent grounds, and the string-breaking algebra is reduced to a two-state problem awaiting three physical numbers. What is new is that the corpus is no longer empty-handed before the missing objects: a candidate wall block with a derived, sign-definite, closed-form coefficient exists inside the frozen history sector, and a candidate branch-to-orientation map exists inside the frozen holonomy congruences, the two arithmetically linked through the CRT factorisation of a single discarded edge datum. Neither is admitted, both are guarded by named falsifiers against premature promotion, and their shared prerequisite — and with it the survival or demotion of the specific number $\Delta = 0.2235260$ — turns on one pre-registered calculation with a sealed decision rule, the colour typing of the forward rim edge; admission then proceeds, if at all, through the named bridge and coupling debts.

Final scientific verdict. RETAINED GLOBAL POSITIVE-TRANSFER AND STRONG-PHASE-ZERO PASS / REFERENCE CENTRE-TRANSFER EXECUTION PASS AT REFERENCE GRADE / ENTROPIC SHORTCUT REJECTED / FULL PHYSICAL CONFINEMENT, STRING-BREAKING AND ORIENTED-BRANCH CERTIFICATES REFUSED BY EXACT NON-IDENTIFIABILITY — TWO QUARANTINED CANDIDATES REGISTERED, THEIR SHARED TYPING PREREQUISITE GATED BY THE SEALED RWCT-1 VERDICT.

This is not an open-ended conclusion. It proves what the present corpus does determine, what it cannot determine, and the exact new transfer data required to decide the remaining strong-sector gate.

Appendix A — Analytic D36 spectrum

The hub/rim-symmetric block is

$M = [[1/7, 6/7], [1/D, 1 - 1/D]]$,

with eigenvalues 1 and $\mu = 1/7 - 1/D = (D - 7)/(7D)$. The five nonconstant rim Fourier modes give

$\lambda_m = [1 + 2 \cosh \alpha \cos \theta_m + 2i \sinh \alpha \sin \theta_m]/D$, $\theta_m = 2\pi m/6$, $m = 1, \ldots, 5$.

Exact identities used in the text, valid for all α with $c = \cosh \alpha$:

- $\mathrm{Re}\, \lambda_{\{1,5\}} = 1/2$; $\mathrm{Im}\, \lambda_1 = \mathrm{Im}\, \lambda_2 = \sqrt{3} \sinh \alpha/D$;
- $|\lambda_{\{1,5\}}|^2 = 2(2c - 1)(c + 1)/D^2 = (2c - 1)/D = -\lambda_3$;
- $|\lambda_{\{2,4\}}|^2 = (4c^2 - 2c - 2)/D^2$;
- $\lambda_3^2 - |\lambda_{\{2,4\}}|^2 = (3 - 2c)/D^2$, vanishing at $c = 3/2$;
- $\det P = \mu\, \lambda_3\, |\lambda_1|^2\, |\lambda_2|^2$.

The accompanying script matches the analytic and numerical eigenvalue multisets to below 10^{-15}.

Appendix B — Reflection positivity proof

For an observable F of the positive-time chain, condition on the state z at the reflection plane. By the Markov property the two half-lines decouple given z, and because T_{OS} is reversible with respect to π the reflected negative-time factor is the complex conjugate of the positive-time conditional amplitude $A_F(z) = \mathbb{E}[F \mid X_0 = z]$. Therefore

$$\langle \Theta F, F \rangle = \Sigma_z\, \pi_z\, |A_F(z)|^2 \geq 0.$$

This proves positivity on the complete cylinder algebra without sampling, and identifies the null space as $\{F : A_F \equiv 0\}$. Completion of the quotient by zero-norm observables gives the finite OS Hilbert space.

Appendix C — Exact determinant proof

The regulator correction to Ω is a commutator of a real symmetric record matrix with a real diagonal matrix, hence real skew-symmetric. The base Ω is anti-Hermitian and traceless. Their sum is anti-Hermitian and traceless. The determinant result follows from $\det \exp A = \exp \mathrm{tr}\, A$.

Appendix D — Branch conjugation matrix

In the ordered 102-dimensional carrier, C is block diagonal with dimensions

$$12 + 6 + 18 + 36 + 24 + 6.$$

It is identity on history, gauge, charged and record blocks; $\mathrm{diag}(\mathbb{1}_9, -\mathbb{1}_9)$ on the completion realification; and $\mathrm{diag}(J_6, J_6)$ on the two sides of the neutral parent. The reduced C_{red} is obtained by deleting the six history-bath and three gauge-auxiliary coordinates. The certificate returns exact zero residuals.

Appendix E — Triality transfer family

The centre projectors are polynomials in U_ω:

$$P_r = (1/3) \Sigma_{j=0}^{2} \omega^{-rj} U_\omega^{j}.$$

$T_{trial}(\Delta)$ is therefore a positive function of the centre operator. It is compatible with the faithful quotient once the line operator has the correct weak/hypercharge dressing. Proposition 14.1 shows that, within the centre algebra and under conjugation covariance, no other positive form exists: the construction proves non-identifiability of the coefficient exhaustively. It does not claim every member is substrate-derived.

Appendix F — Orientation unitary

The polar factor $U = P\, T_{OS}^{-1/2}$ is π-unitary because $T_{OS} > 0$. In the π-orthogonal eigenbasis of P it acts by $\lambda/|\lambda|$:

- vacuum mode: +1;
- m = 1, 5 pair: $e^{\pm i\varphi_1}$, $\tan \varphi_1 = 2\sqrt{3} \sinh \alpha/D$ ($\mathrm{Re}\, \lambda_1 = 1/2$), $\varphi_1 = 0.6575731940$ at the frozen α;
- m = 2, 4 pair: $e^{\pm i\varphi_2}$, $\varphi_2 = 1.8227348715$ at the frozen α;
- m = 3 mode and symmetric μ-mode: −1 each (both real negative at the frozen drive).

Hence $\det U = e^{i\varphi_1}e^{-i\varphi_1}e^{i\varphi_2}e^{-i\varphi_2}(-1)^2 = 1$ exactly. Certificates: unitarity residual 1.1×10^{-14}, $[U, T_{OS}]$ residual 5.8×10^{-16}.

Appendix G — Reproducibility package

The package contains:

- the frozen FGQT-1 calculation certificate and representation ledger;
- the frozen D36 kernel and stationary law;
- four-regulator HCMR charged matrices and k = 1, 6 parent/reduced operators;
- `scripts/run_gqsc1r.py`, extended with the normality, closed-form and orientation-unitary certificates of this revision;
- an independent verifier;
- machine-readable certificates;
- CSV tables;
- five figures;
- a complete SHA-256 manifest, subject to the D-GQ7 currency check.

Appendix H — Merged static-source execution: reproducibility

The static-source execution ships with `scripts/run_csst1.py`, `scripts/verify_csst1.py`, a frozen-input SHA-256 manifest (subject to the F-GQ22 currency check), the HCMR branch arrays used for the conjugacy recheck, machine-readable certificates and CSV tables. The RWCT-1 protocol and runner skeleton are sealed companions; the typed-carrier inputs of D-GQ11 are not yet part of any package and their assembly is the named prerequisite for deciding Part X. The stage-1 candidate-verification package (archived K7 kernel and stationary law, run and rebuild scripts, CSV tables, JSON certificate, SHA manifest) is consumed in §36; its certificate reproduces under independent rerun with zero differences.

Source ledger

1. Keith Taylor, *The Faithful Gauge-Product Quotient, Bundle Census and Topological-Charge Lattice Theorem in VERSF* (FGQT-1).
2. Keith Taylor, *The D36 Clock-Convention, Reproducibility and Manifest-Currency Theorem in VERSF* (D36-R1).
3. Keith Taylor, *The Four-Regulator Commitment-Maintenance Execution and Physical-Admission No-Go Theorem in VERSF* (HCMR-MR1).
4. Keith Taylor, *The Strong CP and Global Theta-Sector Theorem in VERSF* (SCPθ-1).
5. Keith Taylor, *The Substrate Cohomology, Global Gauge Measure and Positive-Pushforward Strong-Phase No-Go Theorem in VERSF* (SGMC-1R).
6. Keith Taylor, *The Quantum Gauge Completion of the VERSF Standard Model* (QGC-1).
7. Keith Taylor, *The Non-Perturbative QCD Scale and Triality-Gap Completion Theorem in VERSF* (NQTC-1).
8. Keith Taylor, *QCD Running and Confinement in VERSF* (QCR-1).
9. Keith Taylor, *The VERSF History-Measure, Generation-Geometry and Branch-Closure Theorem* (HMGBC-1).
10. Keith Taylor, *The VERSF Master Substrate Action and Constitutive-Descent Theorem* (MASD-1).
11. Keith Taylor, *VERSF Standard Model Derivation Master Ledger*, July 2026.
12. Keith Taylor, *The Fermion RG Matching and Self-Consistent Quark-Threshold Theorem in VERSF* (FRGM-1C).
13. Keith Taylor, *The Strange Confinement Operator in VERSF*; *The Strange Confinement Dynamics Theorem in VERSF*.
14. Keith Taylor, *Contextual Distinguishability and Coboundary Confinement*.
15. Keith Taylor, *Entropic Confinement and Effective Quark Masses in the VERSF Framework* (audited, not inherited).
16. RWCT-1 — *Rim-Winding Colour-Typing Test Protocol* (sealed companion; decision rule frozen before data).

17. *RWCT Stage-1 Candidate Verification Package* — archived-kernel machine audit of Candidates C1 and C2 (SHA-manifested; certificate reproduced under independent rerun).