NSTC-1 — The Same-Branch Neutral-Sector Return and Exact Current-Corpus Physical-Certificate Boundary

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Designation: NSTC-1 · Complete Neutral-Sector Return Attempt / Exact Current-Corpus Physical-Certificate Boundary

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General reader summary

The neutral sector is the first place in the VERSF Standard Model programme where every major layer must arrive at once and agree. To claim a physical neutrino result, one needs the mixed active–singlet coupling, the direct right-handed Majorana block, any irreducible left-handed Majorana block, the charged-lepton frame, the absolute physical scale, the branch orientation, the heavy thresholds, and a joint covariance — all descending from one admitted history action at one regulator branch. Diagonalising a matrix is the easy part. Proving that the matrix was not chosen is the whole problem.

The recent papers remove several standing excuses for not attempting the calculation. K7-CBR-1 replaces the one-shot absorbing neutral instrument with a record-preserving recurrent process and lifts the asymptotic neutral Fisher rank from zero to 21, with the K = 7 product process at rank 867; a rank-96 pullback is therefore no longer impossible on capacity grounds. HCMR-MR1 executes the strongest available 102-dimensional parent at four regulators and demonstrates genuine second-order convergence of the reduced response, the scalar ratio and the charged tensors. ASAC-1 returns a conditional geometric length and an exact traversal-clock law. D36-R1 repairs the clock reproducibility conflict.

What the source stack does **not** contain is any numerical array from which the neutral mass operator could be built. There is no direct Yν, no direct MR, no ML return or ML-vanishing theorem, no non-trivial neutral readout selector, no oriented branch, no independent same-action descent pass, and no completion residue Zcomp. Those are not precision gaps. They are missing information.

This paper is therefore written in two layers, and only one of them is populated.

The **theorem layer** is complete and does not depend on any unsupplied number. It proves the exact route from a same-action neutral block to the Takagi spectrum, the effective charged-current matrix, the active nonunitarity defect, the heavy thresholds and the covariance; it fixes exactly which quantities survive rephasing and degeneracy and are therefore promotable; and it proves, with an exact parameter count, why the present corpus cannot be promoted to a physical certificate no matter how much numerical precision is added.

The **execution layer** is specified but empty. Every array, residual, branch comparison, regulator test and machine-readable field required for the numerical certificate is fixed in advance, so that the calculation can be populated later without moving a single admission criterion after the result is seen.

Two results deserve emphasis for the general reader, because they explain why the discipline is not merely cautious. First, an exact counting theorem: the neutral block carries thirty independent physical real parameters, and the current corpus supplies none of them numerically — so the set of neutral spectra compatible with everything presently established is a thirty-dimensional continuum, not a narrow band. Second, a forgery theorem: for *any* target neutrino spectrum and *any* target mixing matrix, there exists a six-real-parameter family of neutral couplings reproducing it exactly. Agreement with observation therefore carries no evidence about provenance whatsoever. The non-insertion firewall is not a stylistic preference; it is the only thing separating a derivation from a fit.

The strongest acceptable outcomes of a populated run are deliberately plural: PASS, FAIL, EXACT CONJUGATE DEGENERACY or UNRESOLVED. A non-viable hierarchy, excessive nonunitarity, an unstable threshold structure or a failed same-action descent would each be a decisive scientific result. The paper is built to terminate in a verdict, not to protect a spectrum.

Abstract

We formulate the complete same-branch neutral-sector return required by the VERSF Standard Model closure programme. At regulator levels n = 3, 4, 5, 6 and on every surviving HCMR branch k, the target is a direct same-action return of the complex neutral Yukawa matrix $Y\nu$, the Dirac block MD, the right-handed Majorana block MR, any irreducible left-handed Majorana block ML, the full complex symmetric neutral operator $M\nu$, its Takagi spectrum, the effective charged-current matrix, the active nonunitarity defect, the heavy thresholds and the joint covariance. The construction must consume one HCMR parent, one set of chiral embeddings, one scalar response, one charged-lepton frame, one branch and one scale convention. No neutrino observable may select an input.

The uploaded source stack materially narrows the task without populating it. K7-CBR-1 constructs a recurrent record-preserving neutral instrument, restores non-zero asymptotic neutral information and removes the previous rank-capacity obstruction. HCMR-MR1 executes a positive four-regulator 102-dimensional candidate with stable charged tensors, but its Yukawa projector is full-support and hence non-selective, its independent CY3′ test fails at order one, its surviving non-zero branch is the exact conjugate class {1, 6}, and Zcomp is absent. ASAC-1 returns a conditional Route-M length and an exact K = 7 traversal-clock law while proving that the numerical action unit and Λ★ do not follow from the current invariants. D36-R1 resolves the

ln 2 versus 0.185 reproducibility discrepancy but leaves the physical clock convention unselected.

We prove seven source-independent results:

1. a same-branch assembly theorem for the complete neutral operator;
2. a machine-checkable Takagi certificate theorem, with the exact residual gauge group and hence the exact list of promotable quantities;
3. an exact nonunitarity identity, in closed non-perturbative form, together with the conjugation-convention trap that makes the naive perturbative identification wrong at leading order;
4. a corrected conjugate-branch theorem, with the explicit Takagi map on the conjugate branch;
5. an absolute-scale non-identifiability theorem;
6. an exact modulus-count theorem — the neutral block carries 30 physical real parameters in general and 18 under an exact ML = 0 theorem; and
7. an inverse-construction forgery theorem showing that any target light spectrum and mixing matrix is exactly reproducible by a six-real-parameter family of same-shaped inputs.

Combining these with the declared source boundary yields an exact current-corpus physical-admission no-go: without direct Yν, MR and ML returns, a unique oriented branch, a non-trivial readout or equivalent differentiated map, an independent same-action descent pass, and Zcomp, a 30-dimensional family of inequivalent neutral spectra remains compatible with the present architecture, and no downstream precision can reduce it. NSTC-1 is therefore graded **COMPLETE EXECUTION CONTRACT / PHYSICAL NEUTRAL CERTIFICATE REFUSED**. The numerical certificate becomes fully determined, in one frozen algorithm, once the named arrays and covariance objects are supplied.

Principal claim

NSTC-1 principal claim. Given a frozen four-regulator same-action source stack that directly returns Yν, MR, ML, the charged-lepton left frame UeL and the completion scale on one admitted branch, the full neutral mass operator possesses a unique machine-checkable Takagi certificate up to the exact residual gauge group $\bigoplus_i O(k_i, \mathbb{R}) \oplus U(k_0)$ and field rephasings. The effective charged-current matrix, the active nonunitarity defect, the threshold operators and the covariance then follow with no readback and no free choice. The current uploaded corpus supplies none of those load-bearing numerical inputs, so it supports the theorem layer and the execution package but not the numerical physical certificate — and, by the modulus-count and forgery theorems, correctly so.

Verdict at a glance

Audit item	Verdict	Meaning
Recurrent neutral history capacity	**PASS / IMPORTED**	K7-CBR-1 restores non-zero long-history neutral information and clears the rank-capacity obstruction.
Four-regulator HCMR parent	**PASS AS CANDIDATE**	Levels 3–6 positive and second-order convergent; branch and provenance conditional.
Same-branch charged-lepton frame	**AVAILABLE AS CANDIDATE**	Required for the relative mixing frame; not physically admitted.
Direct Yν return	**NOT SUPPLIED**	No machine-readable same-action neutral mixed block exists in the corpus.
Direct MR and irreducible ML	**NOT SUPPLIED**	Polar support and recurrent capacity do not determine the Majorana blocks.
Full Takagi spectrum	**EXECUTION READY**	Deterministic once the complete complex symmetric block is supplied.
Mixing, nonunitarity, thresholds	**EXECUTION READY**	Follow from the same-branch Takagi return and UeL; not inferable from support maps.
Absolute neutral mass scale	**OPEN BY EXACT NO-GO**	A★, Zcomp and Λ★ are not identified by the current invariant data.
Oriented physical branch	**OPEN / EXACT PAIR**	The unrestricted candidate returns $k = 0$; the exact non-zero conjugate class is $\{1, 6\}$.
Independent same-action descent	**FAIL IN CURRENT CANDIDATE**	Independent CY3′ residues remain ≈ 0.37–0.49 despite internal functorial agreement.
Physical neutral certificate	**REFUSED FROM CURRENT CORPUS**	Sufficient for an execution contract and a non-closure theorem; not for a mass certificate.

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- Appendix D — Source ledger and imported grades
- Appendix E — Open-debt ledger and mandatory ledger wording

Part 0 — Premises, falsifiers and vocabulary

0.1 Named premises

Premise	Statement
P-N1	The retained neutral basis is $\Psi L = (\nu L, NR^c)$ with three active and three singlet endpoints, so $M\nu$ is complex symmetric of order 6.
P-N2	$Y\nu$, MR and ML are direct derivatives of one frozen reduced action at one regulator level n and one branch k; no block is reconstructed from a desired spectrum.
P-N3	MD is obtained from $Y\nu$ by one scalar-response convention $MD = c\_H \cdot v(n,k) \cdot Y\nu$, with c_H frozen before execution and identical at every (n, k).
P-N4	The charged-lepton left frame UeL originates from the same action, regulator and branch as the neutral blocks.
P-N5	Branch labels $k \in \mathbb{Z}_7$ are those of the HCMR parent; the unrestricted candidate returns k = 0 and the surviving non-zero minimum is the exact conjugate class {1, 6}.
P-N6	Regulator levels n = 3, 4, 5, 6 are nested and share one algorithm, one tolerance ledger and one hash manifest.
P-N7	The Takagi factorisation is computed by a degeneracy-aware singular-value routine with explicit phase repair, never by a Hermitian eigenvalue routine.
P-N8	Absolute masses require the admitted triple (A★, Zcomp, Λ★); until then the primary returns are dimensionless.
P-N9	No neutrino observable enters upstream of the sealed certificate, directly or as a tie-breaker.
P-N10	Discrete branch ambiguity is carried as a discrete object and is never folded into a continuous covariance.

0.2 Falsifiers

Falsifier	Fires when
F-N1	Any neutral block is supplied as a matrix rather than as a differentiated same-action return with primitive variables named.
F-N2	ML is set to zero by convention rather than by a regulator-stable exact symmetry, support or null-ideal theorem.
F-N3	A support projector, polar factor or rank statement is cited as a selector.
F-N4	$PY = \mathbb{1}$ is cited as evidence that a distinguished neutral subspace exists.

Falsifier	Fires when
F-N5	A Hermitian eigen-decomposition is substituted for the Takagi factorisation, or $U\nu$ is exported without the reconstruction residual.
F-N6	A Takagi column phase, rather than a rephasing- and subspace-invariant quantity, is promoted to a physical result.
F-N7	The nonunitarity defect is computed with the un-conjugated mixing convention $\Theta = MD\ MR^{-1}$ (see Theorem 3 and §19.2).
F-N8	The conjugate-branch map is applied as $U\nu(6) = C\ U\nu(1)^*$ rather than the corrected form of Theorem 4.
F-N9	A light/heavy 3 + 3 partition is imposed rather than returned, or states are relabelled to recover an expected census.
F-N10	A Richardson extrapolation is reported without a demonstrated asymptotic regime, or with the erroneous denominator $2p - 1$ in place of $2^p - 1$.
F-N11	An exact branch difference is absorbed into a continuous error bar, or a CP phase is averaged across the conjugate pair.
F-N12	Absolute masses in eV or GeV appear outside a quarantined diagnostic table while $\Lambda\bigstar$ is open.
F-N13	The Schur reduction uses MR^{-1} when MR is singular or ill-conditioned, without a declared rank-revealing treatment.
F-N14	The source manifest hashes a superseded revision; integrity is not currency.

0.3 Vocabulary of grades

DERIVED — returned by one differentiated same-action route and independently reproduced. **CANDIDATE** — internally consistent and convergent, but not independently descended. **IMPORTED** — accepted at the grade declared by its source paper, without strengthening. **CONDITIONAL** — correct given a stated unproved premise, and labelled with that premise. **DIAGNOSTIC ONLY** — computed for orientation, quarantined from every ledger. **NOT TESTED** — the reported quantity is a theorem of the construction, not evidence. **NON-IDENTIFIABLE** — proved to be unfixed by the current axiom set. **REFUSED** — a certificate that could have been issued is withheld because a load-bearing gate is open.

Part I — Task contract, source boundary and non-insertion firewall

1. The Task-5 contract

The programme's fifth critical-path task is not to produce a neutrino fit. It is to return the complete neutral operator from the same branch and action already used for the charged, gauge and scalar sectors, and then to propagate that immutable object through diagonalisation, charged-current readout, threshold transport and covariance **before** any neutrino datum is consulted. The target package is:

- a direct complex 3×3 $Y\nu$ with provenance and regulator certificate;
- the corresponding MD under one frozen scalar/Yukawa normalisation;
- a direct complex symmetric 3×3 MR, not reconstructed from a desired seesaw spectrum;
- a direct complex symmetric ML, or an exact theorem forcing ML = 0;
- the complete 6×6 Takagi spectrum and the light/heavy census;
- the same-branch charged-current matrix, mixing observables and CP invariants;
- the active nonunitarity defect and the active–heavy mixing;
- heavy thresholds, exact Schur reductions and transport operators;
- a joint covariance covering regulator, branch, scale, scheme and numerical conditioning.

2. What the current source stack supplies

Source	Imported result	Exact limitation retained in NSTC-1
July 2026 Master Ledger	Defines Task 5 and the strict same-branch HFB/RPC admission logic.	The neutral physical mass block, mixing package, scale and provenance were open.
D36-R1	Repairs the ln 2 versus 0.185 reproducibility conflict; supplies a revision and tolerance protocol.	The primitive substrate has not selected the physical clock convention or the attempt rate.
K7-CBR-1	Constructs a recurrent record-preserving neutral instrument; neutral rank 21, K = 7 product rank 867.	Attaching map, reset map, finite-mean physical hazards, the full Jacobian and the cross-block Hessian remain open.
ASAC-1	Conditional ξ = 84.071785948 µm and the exact Route-M law $\Delta t_0 = (8\pi/21)\, \xi/c$.	The numerical action unit, the fold-to-traversal map, the physical TPB clock and $\Lambda\bigstar$ remain open.
HCMR-MR1	The strongest 102-dimensional candidate at four regulators; positivity and second-order convergence pass.	$P_Y = \mathbb{1}$; independent CY3′ fails; k = 0 is global and {1, 6} is the exact non-zero pair; Zcomp absent.

The ASAC-1 law evaluates, at the conditional length, to

$$\ell\_M = (8\pi/21)\ \xi = 1.1967972013675 \times 84.071785948\ \mu m = 100.616878137\ \mu m,$$
$$\Delta t_0 = \ell\_M / c = 3.35621779173 \times 10^{-13}\ s.$$

Both numbers are **CONDITIONAL** on the Route-M premise and carry no action-unit content: they fix a length and a traversal time, not $\Lambda\bigstar$, and therefore cannot be promoted into a mass scale (§28).

3. Claim-grade firewall

No silent promotion. A recurrent instrument is not a mass matrix. Rank capacity is not a differentiated embedding. A support projector is not a selector. Regulator convergence is not primitive provenance. A dimensionless scalar ratio is not an absolute completion scale. A Takagi decomposition is not a physical certificate unless the operator it factors is source-complete.

Each clause of the firewall corresponds to a proved proposition in Part VI, so the firewall is a set of theorems and not a set of cautions.

4. Non-insertion boundary

The following quantities are forbidden upstream of the sealed numerical certificate: observed neutrino mass splittings; the absolute neutrino mass scale; mixing angles or phases; cosmological mass bounds; beta-decay limits; neutrinoless-double-beta constraints; nonunitarity bounds; and sterile-threshold exclusions. They may be consulted only after the branch, matrices, scale, thresholds and covariance are frozen and hashed.

The rule applies directly and indirectly. A branch may not be chosen because its mixing matrix looks closer to observation; a sign convention may not be fixed by the observed CP orientation; a singular-value ordering may not be relabelled to improve agreement; and a tolerance may not be widened until a residual passes. Exact degeneracies are declared as degeneracies.

Appendix B proves that this discipline is load-bearing rather than decorative: the inverse construction reproduces *any* target exactly, so post-hoc agreement contains zero information about provenance.

5. What "same branch" means after HCMR-MR1

HCMR-MR1 does not supply one oriented physical branch. The unrestricted Gaussian candidate selects $k = 0$. If an upstream primitive-Fact condition excludes the identity sector, the surviving minimum is the exact conjugate class $\{k = 1, k = 6\}$. NSTC-1 therefore executes both non-zero branches unless an orientation-odd action term is derived before the neutral calculation, consistent with the corpus finding that sign is carried only by winding orientation under an irreversible commitment current.

The two branches may prove physically equivalent by Theorem 4, but exact equality of free energies and parent spectra is not sufficient. The neutral operators *and* the charged frame must satisfy the explicit conjugation map, with the *same* C. If they do not, the branch class contains physically distinct predictions and HFB uniqueness remains failed.

Part II — Same-action return of Yν, MD, MR and ML

6. Neutral basis and block typing

In the left-handed basis $\Psi L = (\nu L, NR^c)$ the complete mass operator is complex symmetric,

$$M\nu(n,k) = \begin{bmatrix} ML(n,k) & MD(n,k) \\ MD^T(n,k) & MR(n,k) \end{bmatrix}, \qquad M\nu^T = M\nu.$$

Each block carries a distinct source type, and substituting one type for another is the principal failure mode.

Block	Required type	Permitted source	Forbidden substitute
Yν	general complex 3×3	mixed left-active / right-singlet derivative of the frozen common action	support map, fitted texture, charged-sector analogy
MD	general complex 3×3	the frozen scalar/Yukawa conversion applied to Yν	directly fitted Dirac masses
MR	complex symmetric 3×3	direct singlet-completion Hessian or equivalent same-action second derivative	inverse-seesaw reconstruction from desired light masses
ML	complex symmetric 3×3	direct active Majorana return, or an exact symmetry/null theorem	setting ML = 0 by convention

Theorem 1 (same-branch neutral assembly). Fix a regulator n and a branch k. Let Yν(n,k), MR(n,k) and ML(n,k) be direct outputs of one frozen reduced action; let v(n,k) and the Dirac normalisation convention be fixed by the same convention ledger; and let MD(n,k) be the resulting Dirac block. Then the complete neutral operator in the basis ΨL is the complex symmetric matrix displayed above, and no further physical input is required to define the Takagi problem.

Proof. The block form is fixed by the declared active and singlet endpoints and by the Majorana character of the retained fields. Direct same-action provenance removes the continuous freedom to re-choose blocks after the spectrum has been inspected; that freedom is quantified exactly in Theorem 6. ■

7. Direct Yν return and the PY obstruction

HCMR-MR1 reports a full-rank charged support projector with $PY = \mathbb{1}_9$ to floating-point precision. That certifies that the candidate charged tensor has support everywhere in the declared readout space. It does not identify a distinguished neutral mixed derivative or a physical Yukawa subspace, and by Proposition 27.1 it cannot.

NSTC-1 accepts either of two source-clean routes:

1. a non-trivial, gapped and regulator-stable neutral readout projector PY,ν is derived before any spectrum is inspected; or

2. the mixed active–singlet bilinear is obtained directly by differentiating the same marked master action, making an additional selector unnecessary.

A supplied 3×3 matrix, a polar support unitary or a reverse-engineered factorisation is insufficient (F-N1, F-N3). The result must travel with the primitive variables differentiated, the exact projection and dual-basis convention, all cross-blocks, and the regulator manifest:

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Yν(n,k) ≔ EνL(n,k)† · [ direct mixed neutral derivative of the HCMR action ]
· ENR(n,k).
```

8. Dirac mass conversion

Once Yν is returned, MD uses the same scalar response and normalisation convention as the charged sectors:

```
MD(n,k) = c_H · v(n,k) · Yν(n,k),   c_H frozen before execution.
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Under the usual frozen convention c_H = 1/√2. If the VERSF convention ledger fixes a different coefficient, that coefficient is part of the source hash and is applied identically at every regulator and branch.

Until Λ★ and the action normalisation are admitted, the primary output is the dimensionless ratio MD/Λ★, or its native action-normalised equivalent. Printing a value in eV or GeV under an adopted identity completion is permitted only in a quarantined diagnostic table (F-N12).

9. Direct MR return

K7-CBR-1 uses the polar decomposition of supplied neutral matrices to construct recurrent route support. The positive strain and unitary support maps are useful diagnostics, but they do not establish that the primitive history action returns the same MR. NSTC-1 requires the direct complex symmetric singlet block before any Takagi reduction.

The certificate reports symmetry residual, rank, singular values, condition number, regulator drift, branch relation and the exact source derivative. If MR is singular or ill-conditioned, the threshold and Schur treatments must use a declared rank-revealing reduction; substituting an inverse silently fires F-N13.

10. Irreducible ML and the zero theorem

The physically strongest result is not automatically ML = 0. The active–active Majorana block must be calculated. If it vanishes, the zero must follow from an exact symmetry, support or null-ideal theorem that is stable at all four regulators. If a non-zero irreducible block survives, it remains in the full Takagi matrix and in every threshold reduction.

Zero is a result, not a default. Numerically small entries may be truncated only under a predeclared tolerance justified by regulator scaling and covariance. A block set to zero because the intended phenomenology is type-I violates the non-insertion rule and fires F-N2.

The distinction is not cosmetic: by Theorem 6, an exact ML = 0 theorem removes twelve real components and reduces the physical parameter count from 30 to 18. It is the single largest reduction available anywhere in the neutral sector, which is precisely why it may not be assumed.

11. Same-action provenance test

The current independent CY3′ test fails between the completion-derivative and closure-Hessian routes. The neutral paper must repeat the genuinely independent descent test **for every block**, not merely show that two algebraic lifts of one supplied parent agree. The pass condition is one common differentiated action returning the mixed and Majorana blocks within the declared retained order and covariance.

Provenance test	Pass condition	Current source status
Internal functorial lift	Two implementations of the same parent agree.	**PASS in HCMR-MR1; implementation fidelity only.**
Independent mixed-vs-Hessian descent	Separately typed derivatives agree at the declared order.	**FAIL; residues ≈ 0.37–0.49.**
Cross-block completion	Every allowed cross-block is calculated or proved zero.	**OPEN.**
Neutral readout	Non-trivial selector or direct mixed derivative fixed upstream.	**OPEN; current PY is the identity.**

The first row is explicitly labelled implementation fidelity, and by the corpus falsifier on factorised corroboration it may not be cited as independent evidence.

Part III — Full Takagi spectrum and light/heavy census

12. Pre-Takagi validation

Before diagonalisation, the calculation certifies the matrix it is about to factor:

- complex symmetry: $\|M\nu - M\nu^{T}\| / \|M\nu\|$ below the declared tolerance;
- finite entries with declared units or dimensionless normalisation;
- block reconstruction from the exported Yν, MD, MR and ML;

- source-hash agreement, revision currency, and branch/regulator labels;
- rank-revealing singular values and conditioning;
- no Hermitian eigenvalue routine anywhere in the path (F-N5).

13. Robust Takagi algorithm

The reference execution uses a singular-value-based Takagi routine with explicit phase repair and degeneracy-aware subspace alignment, verified by reconstructing $M\nu$ from the exported $U\nu$ and Σ rather than by re-importing the construction function.

1. Symmetrise only at the declared floating tolerance: $M\_sym = (M\nu + M\nu^T)/2$, and report the correction norm.
2. Compute a high-accuracy singular-value decomposition and identify degenerate or near-degenerate clusters by a predeclared relative gap.
3. Within each cluster, solve the symmetric phase/alignment problem so that $U\nu^T M\nu U\nu$ is real, diagonal and non-negative.
4. Sort states by a frozen rule independent of observation: primary key mass, secondary key exact-degeneracy invariant, source-space overlap only within a declared degenerate class.
5. Export $U\nu$, Σ, cluster labels and all residuals; reconstruct $M\nu$ independently as $M\nu = U\nu^* \Sigma U\nu^\dagger$.

Theorem 2 (Takagi certificate and residual gauge). For every finite complex symmetric $M\nu$ there exist a unitary $U\nu$ and a non-negative diagonal $\Sigma = \mathrm{diag}(m_1, \ldots, m_6)$ with

$$U\nu^T M\nu U\nu = \Sigma .$$

The factorisation is unique exactly up to the residual gauge group

$$G\_res = \oplus_i O(k_i, \mathbb{R}) \oplus U(k_0) ,$$

acting on the right of $U\nu$, where the sum runs over the distinct non-zero Takagi values with multiplicities k_i and $k_0 = \dim\ker M\nu$. In particular a **simple non-zero** Takagi value fixes its column up to a sign ±1 and not up to an arbitrary phase.

Proof. Existence is the Autonne–Takagi factorisation. For uniqueness, let $U\nu$ and $U\nu Q$ both be Takagi with the same Σ. Then $Q^T \Sigma Q = \Sigma$, which forces Q to be block diagonal with respect to the multiplicity decomposition. On a block of a non-zero value m, the condition reads $m Q_i^T Q_i = m \mathbb{1}$, so $Q_i^T Q_i = \mathbb{1}$; combined with unitarity $Q_i^\dagger Q_i = \mathbb{1}$ this gives $\bar{Q}_i = Q_i$, i.e. $Q_i \in O(k_i, \mathbb{R})$. On the kernel block the condition is vacuous, leaving $U(k_0)$. For $k_i = 1$ this is $O(1, \mathbb{R}) = \{\pm1\}$. ■

Corollary 2.1 (promotable quantities). A quantity may be promoted from the certificate only if it is invariant under G_res together with the physical rephasing group $U(1)^3$ acting on the active rows. Column phases, individual entries of $U\nu$ and any function of a within-cluster basis choice are **not** promotable (F-N6). Takagi values, cluster labels, the moduli $|N\_\alpha i|$ outside degenerate clusters, ΔNU, and rephasing invariants of the form $\mathrm{Im}[N\_\alpha i\, N\_\beta j\, N\_\alpha j^*\, N\_\beta i^*]$ are promotable.

The residual-gauge statement is not pedantry. Because a simple Takagi value fixes its column only up to a real sign, an implementation that "repairs phases" by an arbitrary continuous convention will produce sign flips that migrate between regulator levels and manifest as spurious CP-odd differences. The certificate therefore reports the sign convention as a hashed field.

Takagi certificate field	Required condition
Symmetry residual	below declared tolerance, stable or decreasing under refinement
Unitarity residual	$\|U\nu^\dagger U\nu - \mathbb{1}\|$ within tolerance
Diagonal residual	off-diagonal norm of $U\nu^T M\nu\, U\nu$ within tolerance
Reconstruction residual	$\|M\nu - U\nu^* \Sigma\, U\nu^\dagger\| / \|M\nu\|$ within tolerance
Non-negativity	all $m_i \geq 0$ after phase repair
Degeneracy ledger	clusters, gaps and the allowed G_res rotations recorded explicitly
Sign/phase convention	hashed, identical at every (n, k)
Regulator tracking	subspaces matched by invariant overlaps, never by column phases

14. Light/heavy census without insertion

The calculation may not assume that exactly three states are light. The full six-state spectrum is returned first. A light/heavy partition is then defined by an upstream scale-separation rule or a clear spectral gap, with the rule frozen before any comparison to observed scales (F-N9).

The certificate distinguishes: exact zero modes; parametrically light states; intermediate states; heavy states. If no stable 3 + 3 partition exists across regulators and branches, Task 5 fails rather than relabelling states to recover the expected census.

15. Regulator convergence of spectra and subspaces

Mass convergence alone is insufficient. Near-degenerate Takagi vectors can rotate strongly while singular values remain stable, so NSTC-1 tracks both. For each state or cluster, report successive differences at $n = 3\to4$, $4\to5$ and $5\to6$, the inferred order and a Richardson estimate **only** where the asymptotic regime is demonstrated:

$$p \approx \log_2(\delta_{4\to5} / \delta_{5\to6}) \ , \qquad X^\infty \approx X_6 + (X_6 - X_5)/(2^p - 1) \ .$$

The denominator is $2^p - 1$, not $2p - 1$; using the linear form silently biases every extrapolation and fires F-N10.

Subspace convergence is certified quantitatively, not by inspection. Let $\mathcal{S}_n$ be the invariant subspace of a cluster at level n, let $E = M\nu(n+1) - M\nu(n)$, and let γ be the absolute separation between the cluster's Takagi values and the rest of the spectrum. Then the Wedin sinΘ bound gives

$$\|\sin \Theta(\mathcal{S}_n,\ \mathcal{S}_{n+1})\| \leq \|E\| / \gamma \ ,$$

so a cluster with $\gamma \lesssim \|E\|$ carries **no** convergent subspace information and its internal structure is reported as NOT TESTED rather than as a converged result. This is the quantitative form of the requirement that near-degenerate structure not be over-read.

Branch differences and regulator errors are separate objects. An exact $k = 1$ versus $k = 6$ difference may never be folded into a continuous numerical error bar (F-N11).

16. Same-branch and cross-branch Takagi tests

Test	Question answered	Promotion rule
Within-branch regulator test	Does one source branch approach a stable Takagi limit?	Required for any candidate spectrum.
Cross-branch spectrum test	Do $k = 1$ and $k = 6$ have identical Takagi values?	Necessary but not sufficient for physical equivalence.
Conjugation-map test	Are the full matrices related by the corrected map of Theorem 4?	If yes, the branch class may be physically equivalent up to CP orientation.
Charged-frame compatibility	Does the *same* C relate UeL(1) and UeL(6)?	Required before comparing mixing outputs.
CP-invariant test	Do CP-even invariants agree and CP-odd invariants reverse?	Confirms the physical meaning of the conjugate class.

Part IV — Same-branch mixing, active nonunitarity and CP structure

17. Charged-current extraction

Let $A = [\mathbb{1}_3\ 0]$ select the active rows of the six-dimensional Takagi matrix and let UL denote the light-state columns selected by the source-clean census. The active light block is $N_0 = A\ UL$. With the charged-lepton left frame UeL from the same regulator and branch,

```
N(n,k) = UeL(n,k)† · A · UL(n,k) .
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NSTC-1 calls N the effective charged-current matrix. It is **not** silently projected back to a unitary matrix. A unitary factor from its polar decomposition may be reported as a diagnostic; the physical charged-current block remains N.

18. Mixing observables and phase convention

The full complex matrix is published before any reduction to angles and phases. Row and column rephasings are fixed by a deterministic convention acting identically at every regulator and branch. In exact or near degeneracies, only G_res- and rephasing-invariant quantities are promoted (Corollary 2.1). The returns are:

- the full complex N and its modulus;
- row and column unitarity defects;
- the three standard angles, when the adopted parameterisation is non-singular;
- the Dirac CP invariant and phase;
- Majorana-sensitive rephasing invariants and phases where identifiable;
- the branch relation of every CP-even and CP-odd invariant;
- regulator drift and covariance.

19. Active nonunitarity

19.1 Exact identities

Theorem 3 (active nonunitarity). Partition the active rows of the unitary Takagi matrix into the light columns N and the heavy complement S. Then

$$N N^\dagger + S S^\dagger = \mathbb{1}_3 , \qquad \Delta NU \coloneqq \mathbb{1}_3 - N N^\dagger = S S^\dagger \geq 0 .$$

Moreover, writing the exact block-diagonalising parametrisation with mixing operator Θ (the solution of the associated algebraic Riccati equation), the defect has the closed non-perturbative form

$$\Delta NU = \Theta \Theta^\dagger (\mathbb{1}_3 + \Theta \Theta^\dagger)^{-1} ,$$

so $\mathrm{spec}(\Delta NU) \subset [0, 1)$, $\mathrm{Tr}\,\Delta NU = 3 - \|N\|_F^2$, and $\Delta NU = 0$ if and only if $\Theta = 0$, i.e. exact decoupling.

Proof. The first identity is orthonormality of the first three rows of a unitary matrix. For the second, the exact block-diagonalising unitary may be written with upper-left block $(\mathbb{1} + \Theta\Theta^\dagger)^{(-1/2)}$, whence $N = (\mathbb{1} + \Theta\Theta^\dagger)^{(-1/2)} U_0$ for a unitary U_0 and $N N^\dagger = (\mathbb{1} + X)^{-1}$ with $X = \Theta\Theta^\dagger \geq 0$. Then $\Delta NU = \mathbb{1} - (\mathbb{1} + X)^{-1} = X(\mathbb{1} + X)^{-1}$, whose eigenvalues are $x/(1 + x) \in [0, 1)$ for $x \geq 0$. ■

19.2 The conjugation-convention trap

The perturbative identification of Θ is where implementations fail. In the complex-symmetric (Takagi) convention adopted here, the leading off-diagonal cancellation condition for

$$M\nu = \begin{bmatrix} 0 & MD \\ MD^T & MR \end{bmatrix}$$

reads $MD + \Theta^* MR = 0$, so that

$$\Theta = -(MD\, MR^{-1})^* , \qquad \Delta NU = (MD\, MR^{-1} (MD\, MR^{-1})^\dagger)^* + O(\|\Theta\|^4) .$$

The complex conjugate is not optional. Using the un-conjugated form $\Theta\Theta^\dagger$ with $\Theta = M_D M_R^{-1}$ produces an error in ΔNU that is **O(1) in relative norm and does not vanish in the decoupling limit** — direct evaluation on random complex symmetric MR gives relative discrepancies of order 0.57 that are stable as the seesaw parameter is driven to zero, whereas the conjugated form converges with relative error scaling as $\|\Theta\|^2$. Because the wrong convention converges to a wrong answer smoothly, it cannot be detected by regulator stability, only by the identity ΔNU = S S† of Theorem 3. This is why the residual of ΔNU − S S† is a mandatory certificate field and why F-N7 exists.

19.3 Required returns

Nonunitarity output	Required return
Matrix	all complex entries of ΔNU, and optionally η = ΔNU/2, never mixed
Positivity	minimum eigenvalue and the negative-eigenvalue tolerance
Norms	spectral, Frobenius and maximum-entry
Heavy-mixing check	residual of ΔNU − S S† (primary convention test)
Closed-form check	residual of $\Delta NU - \Theta\Theta^\dagger(\mathbb{1}_3 + \Theta\Theta^\dagger)^{-1}$ with Θ from the exact unitary
Branch relation	equality or conjugation map across k = 1, 6
Covariance	joint covariance with masses and mixing quantities

20. CP orientation and the branch pair

Theorem 4 (conditional conjugate-branch equivalence). Suppose the two surviving branches satisfy

```
Mν(n,6) = C Mν(n,1)* Cᵀ
```

for a fixed unitary C, and the charged-lepton frames obey the corresponding relation with the *same* C. Then a Takagi matrix on the conjugate branch is

```
Uν(n,6) = C̄ Uν(n,1)* = ( C Uν(n,1) )* ,
```

with identical Σ. Consequently the Takagi values, the mixing moduli and the nonunitarity eigenvalues agree on the two branches, while every CP-odd rephasing invariant reverses sign. If the relation fails, the exact free-energy degeneracy does not imply physical equivalence and both branches must be separately admitted or separately rejected.

Proof. With $U_6 = \bar{C}\, U_1^*$,

```
U₆ᵀ M₆ U₆ = U₁† C† · C M₁* Cᵀ · C̄ U₁* = U₁† M₁* U₁* = ( U₁ᵀ M₁ U₁ )* = Σ* = Σ ,
```

using $C^\dagger C = \mathbb{1}$ and $C^T \bar{C} = (C^\dagger C)^T = \mathbb{1}$, and Σ real. Unitarity of U_6 is immediate. Complex conjugation preserves singular values; the unitary maps cancel in moduli and spectra; rephasing invariants odd under conjugation reverse. ■

Remark 4.1 (a map that looks right and is not). The form $U\nu(6) = C\, U\nu(1)^*$ — without the conjugation on C — satisfies $U^T M U = \Sigma$ **only** when $C^T C = \mathbb{1}$, i.e. when C is real orthogonal. For a general unitary C it fails at order one; direct evaluation on random 6×6 complex symmetric input gives a residual of order 8 against a machine-precision residual for the corrected form. The two expressions coincide exactly in the real-orthogonal case, which is how the error survives casual checking. F-N8 fires on the un-corrected form.

HCMR-MR1 proves the exact $k \leftrightarrow -k$ degeneracy of the Gaussian branch functional. NSTC-1 tests whether the neutral and charged matrices realise that symmetry as physical CP conjugation. If Theorem 4 passes, masses, mixing moduli and nonunitarity are branch invariant while CP-odd invariants reverse; that closes the branch class as an exact-degenerate physical set **only if** the HFB admission rules accept orientation-conjugate outputs as physically equivalent. Otherwise an upstream orientation-odd current remains mandatory, in agreement with the corpus result that sign is carried only by winding orientation under an irreversible commitment current.

If the conjugation test fails, the exact free-energy tie masks physically distinct neutral sectors, and the programme must return an orientation selector before claiming one mixing matrix or one sign of leptonic CP violation.

Part V — Thresholds, regulator convergence and covariance

21. Exact block reduction and the seesaw firewall

The full Takagi spectrum is primary. When MR is invertible and a controlled separation exists, the static active-sector Schur complement is

$$M_{eff} = ML - MD\, MR^{-1}\, MD^T \, .$$

This is a block-algebra identity, not by itself a complete matching theorem. NSTC-1 compares the exact light Takagi spectrum with the Schur approximation and reports the error under a rigorous bound rather than an impression.

Proposition 21.1 (Schur error control). Let m_i^{exact} be the three smallest Takagi values of $M\nu$ and σ_i the singular values of M_{eff}. Writing ΔM for the difference between the exact reduced operator and M_{eff}, Weyl's inequality for singular values gives

$$| m_i^{exact} - \sigma_i | \le \|\Delta M\|_2 \, ,$$

and in the decoupling regime $\|\Delta M\|_2 / \|M_{eff}\|_2 = O(\|\Theta\|^2)$. Direct evaluation confirms the quadratic law: driving the Dirac scale down by one decade reduces the maximum relative discrepancy by two decades ($\approx 7.8 \times 10^{-4} \to 7.8 \times 10^{-6}$).

If MR is singular or poorly conditioned, a rank-revealing exact reduction is required and the naive inverse is forbidden (F-N13). The Schur complement is a *check* on the Takagi spectrum, never a substitute for it.

22. Heavy thresholds

Each heavy Takagi state defines a candidate decoupling threshold only after the absolute scale is admitted. The threshold ledger records the state mass, branch, regulator, source-space composition, matching order, induced active operator, covariance and round-trip reconstruction residual. Thresholds are ordered by the frozen Takagi census, never by comparison to known phenomenology.

Threshold field	Meaning
m_a	absolute heavy Takagi mass; **conditional** while Λ★ is open
w_a	active/singlet composition and invariant subspace label
K_eff below m_a	returned active operator after exact or controlled reduction
Wilson/matching objects	same-action coefficients with scheme and order
Round-trip residual	difference between full and sequentially reduced observables
Covariance	joint scale, branch, regulator and truncation uncertainty

23. Covariance hierarchy

The covariance is not one undifferentiated error matrix. NSTC-1 separates five layers:

1. numerical conditioning and floating reconstruction;
2. regulator extrapolation and retained-order truncation;
3. continuous source-parameter covariance within one branch;
4. scale and action-normalisation covariance;
5. discrete branch class or exact degeneracy.

Layers 1–4 may enter a joint continuous covariance where their assumptions hold. Layer 5 remains a discrete mixture or equivalence class and may never be represented as a symmetric Gaussian uncertainty around an averaged CP phase (F-N11).

24. Recommended covariance execution

1. Freeze a primitive parameter vector θ and its covariance C_θ from the same action calculation.
2. Propagate θ through Yν, MD, MR, ML and the Takagi map by automatic differentiation or a validated symmetric finite-difference scheme.
3. Use G_res-invariant derivatives for degenerate clusters; never differentiate a column phase or a within-cluster basis choice.

4. Add regulator and truncation envelopes as a separately labelled component unless a probabilistic model is justified.
5. Publish the covariance of masses, mixing moduli, CP invariants, ΔNU and thresholds, together with the full parameter-order ledger.

25. Four-regulator pass condition

Regulator pass. One frozen algorithm must return stable block ranks, a stable light/heavy census, convergent Takagi values, convergent invariant subspaces certified by the Wedin bound of §15, stable mixing moduli and nonunitarity, and a stable branch relation. A convergent but independently inconsistent neutral block remains a failed physical candidate, exactly as HCMR-MR1 showed for the broader parent.

Part VI — Exact current-corpus non-closure theorems

26. Recurrent capacity does not identify the neutral block

K7-CBR-1 proves that the recurrent marked process has enough Fisher-rank capacity to support the non-terminal parent. This removes a necessary-condition obstruction; it does not determine the differentiated embedding J or the complex neutral Hessian.

Proposition 26.1 (capacity ≠ identifiability, with the fibre dimension). The recurrent rank 867 can support a rank-96 pullback, but does not determine $Y\nu$, MR, ML or their phases. The correct one-way inequality is

$$\operatorname{rank}(J^\dagger G J) \le \min(\operatorname{rank} G, \operatorname{rank} J) ,$$

which is necessity only. Moreover the ambiguity is large and countable: if J_1 and J_2 satisfy $J_1^\dagger G J_1 = J_2^\dagger G J_2 = H$ with H of full rank 96 on the pullback space, then $J_2 = Q J_1$ for a G-isometry Q, and the fibre is the homogeneous space

$$U(867) / U(771) , \quad \dim_{\mathbb{R}} = 867^2 - 771^2 = 96 \times 1638 = 157\,248 ,$$

reducing to O(867)/O(771) of real dimension 78 576 in the real symmetric case. Any claim that the rank result alone fixes the neutral spectrum reverses a one-way inequality and discards a fibre of dimension in the tens of thousands.

Proof. The inequality is standard. For the fibre, the isometry group of a non-degenerate form acts transitively on the set of embeddings with a fixed pullback, with stabiliser the isometry group of the orthogonal complement; the dimension follows. ■

27. Full-support PY cannot select a neutral readout

Proposition 27.1 (identity-projector no-selection). If $P_Y = \mathbb{1}$ on the declared readout space, every vector lies in its range. P_Y can certify that no direction is removed, but it cannot select a distinguished neutral mixed subspace, branch or texture, because the identity has no non-trivial spectral gap and no proper invariant range. Any Y_ν extracted by inserting a preferred sub-block after $P_Y = \mathbb{1}$ is a supplied candidate, not a return. ■

28. Absolute-scale non-identifiability

Theorem 5 (absolute-scale orbit). If the source stack returns only a dimensionless neutral operator $\hat{M}$ and leaves the positive completion scale s unidentified, then the family $M_\nu(s) = s\,\hat{M}$, $s > 0$, has identical Takagi vectors, identical effective mixing matrix and identical dimensionless nonunitarity for every s, while every mass and every threshold scales linearly with s. Absolute neutral masses are therefore non-identifiable until the same-action scale and residue are returned.

Proof. $U_\nu^T(s\hat{M})U_\nu = s\Sigma$, so the Takagi subspaces and every dimensionless overlap are unchanged. The statement is independent of branch and regulator. ■

ASAC-1 separates the geometric length ξ from the completion scale $\Lambda\bigstar$ and proves that the numerical action unit is invariant under common positive rescaling; HCMR-MR1 returns only dimensionless scalar response and no Z_{comp}. The scale orbit is therefore live.

Proposition 28.1 (physical-mass refusal). Printing absolute neutrino masses or heavy thresholds from the current source stack would require an adopted identity completion, an imported action unit, or another unproved scale bridge. Such numbers may appear only as quarantined conditional diagnostics and may not enter HFB-1. ■

29. Independent descent obstruction

The independent CY3′ residues reported by HCMR-MR1 remain between approximately 0.37 and 0.49 across regulators. That is a structural discrepancy, not a discretisation tail.

Proposition 29.1 (stable disagreement is a physical blocker). If two independently typed derivatives that are required to represent the same action object converge to *different* non-zero limits, regulator stability certifies the disagreement rather than repairing it. A same-branch neutral spectrum built from only one of the two routes remains a candidate, not an admitted physical return. ■

The neutral paper must either repair the common-action descent or issue a physical-certificate refusal even if the Takagi spectrum is numerically attractive.

30. The exact modulus count

Theorem 6 (neutral modulus count). In the basis of P-N1, with the charged-lepton mass matrix diagonal and positive, the neutral operator $M\nu$ is a complex symmetric 6×6 matrix carrying 2 × 21 = 42 real components. The residual field-redefinition group is

```
G_red = U(1)³_ν × U(3)_N ,   dim_ℝ G_red = 3 + 9 = 12 ,
```

acting freely on a generic orbit. The physical content of the neutral sector is therefore exactly

```
42 - 12 = 30 real parameters .
```

Under an exact ML = 0 theorem the count reduces to 18 + 12 − 12 = 18, recovering the standard three-generation type-I census of three light masses, three heavy masses, six angles and six phases.

Proof. The component count is immediate. $U(1)^3$ is the residual rephasing of the active doublets compatible with a diagonal positive charged-lepton mass matrix; U(3)_N is a complete field redefinition of the gauge-singlet fields, which carry no other couplings. Generic freeness of the action gives orbit dimension 12. ■

Corollary 6.1 (what an ML theorem is worth). An exact, regulator-stable ML = 0 theorem removes twelve real parameters — forty per cent of the neutral sector's physical content. No other single result available in the neutral sector removes as much. This is exactly why ML = 0 may not be adopted as a convention (F-N2).

31. Current-corpus neutral non-closure theorem

Theorem 7 (NSTC-1 current-corpus non-closure). Under the uploaded source boundary the physical neutral sector is not identifiable, and the failure is exactly quantified. Five independent freedoms remain:

1. the unsupplied complex neutral blocks — 30 real physical parameters by Theorem 6, of which the corpus fixes none numerically;
2. the identity readout, which by Proposition 27.1 selects nothing;
3. the exact orientation pair {1, 6}, a discrete $\mathbb{Z}_2$ ambiguity not resolved by any free-energy comparison;
4. the failed independent action descent, at residue ≈ 0.37–0.49 (Proposition 29.1);
5. the positive absolute-scale orbit of Theorem 5, a one-parameter subgroup of the 30.

Consequently the set of neutral spectra compatible with everything presently established contains a 30-dimensional real manifold, doubled by a discrete conjugate pair. No downstream Takagi, mixing or threshold calculation can remove information absent upstream. A physical certificate requires new same-action returns, not additional precision on the existing candidate.

Proof. Choose any source-compatible dimensionless $Y\nu$ candidate and vary a symmetric MR by $MR(t) = MR + t\mathbb{1}$. For an open set of t the carrier dimensions, the recurrent rank capacity and the already reported charged sector are unchanged, while the Takagi masses and thresholds vary analytically and non-trivially. Theorem 6 gives the dimension of the full family; Theorem 5 supplies an independent continuous orbit inside it; Proposition 26.1 shows that the reported rank data constrain a fibre of dimension 157 248 rather than a point. ■

Corollary 7.1 (new information, not computation). Every remaining freedom in Theorem 7 is an absent object, not an unfinished calculation. D-N1…D-N4 are therefore new-information requirements in the sense already established for the assembly-layer blocker, and no increase in regulator level, precision or run length can close them.

Part VII — Numerical certificate, pass conditions and revised programme ledger

32. Minimum input package

To populate the result layer without revising any admission rule, the calculation package supplies the following machine-readable objects for n = 3, 4, 5, 6 and k = 1, 6, plus k = 0 where the unrestricted branch is retained for diagnosis. Filenames are descriptive and may change if the manifest preserves the typing.

Object	Proposed file	Shape / content
Neutral mixed return	`level_n_branch_k_Ynu.npy`	complex 3×3, direct same-action derivative
Dirac block	`level_n_branch_k_MD.npy`	complex 3×3 with normalisation metadata
Right Majorana block	`level_n_branch_k_MR.npy`	complex symmetric 3×3
Left Majorana block	`level_n_branch_k_ML.npy`	complex symmetric 3×3, or an exact zero certificate
Charged-lepton left frame	`level_n_UeL.npy`	complex unitary 3×3
Neutral embeddings	`level_n_branch_k_EnuL.npy` / `_ENR.npy`	full-carrier isometries
Completion response	`level_n_branch_k_Kcomp_p2.npz`	K_comp(p^2) samples / derivative data for Zcomp
Primitive covariance	`level_n_branch_k_cov_theta.npy`	ordered joint covariance and parameter ledger
Branch conjugation map	`level_n_C_map.npy`	the unitary C of Theorem 4, shared by neutral and charged sectors

Object	Proposed file	Shape / content
Source manifest	`frozen_input_manifest.json`	hashes, revision **currency**, units and conventions

The manifest records revision currency as well as integrity: a hash proves that a file has not changed, not that it is the current file (F-N14).

33. Reference execution order

1. Verify hashes, source **currency**, units, branch labels and regulator nesting.
2. Reconstruct Yν, MD, MR and ML independently from the exported source derivatives where possible.
3. Check symmetry, rank, conditioning and same-action provenance; run the independent CY3′ descent test.
4. Build the full 6×6 complex symmetric Mν on every regulator and branch.
5. Run the degeneracy-aware Takagi factorisation, record G_res per cluster, and reconstruct Mν independently.
6. Establish the source-clean light/heavy census; track invariant subspaces with the Wedin bound and mark any cluster with $\gamma \lesssim \|E\|$ as NOT TESTED.
7. Combine with the same-branch UeL to return N, the mixing observables and ΔNU; verify both the S S† identity and the closed-form identity of Theorem 3.
8. Construct threshold reductions; compare exact and Schur light sectors under Proposition 21.1.
9. Propagate continuous covariance; retain branch ambiguity as a separate discrete object.
10. Seal and hash the certificate **before** any comparison with neutrino data.

34. Numerical certificate template

Certificate item	Level/branch return	Admission status
Yν direct return and provenance	TO BE POPULATED	**OPEN**
MD normalisation and scale typing	TO BE POPULATED	**OPEN**
MR direct symmetric return	TO BE POPULATED	**OPEN**
ML direct return or exact zero theorem	TO BE POPULATED	**OPEN**
Takagi values $m_1 \ldots m_6$	TO BE POPULATED	**OPEN**
Takagi reconstruction residual	TO BE POPULATED	**OPEN**

Certificate item	Level/branch return	Admission status
Residual-gauge / degeneracy ledger	TO BE POPULATED	**OPEN**
Light/heavy census	TO BE POPULATED	**OPEN**
Effective charged-current matrix N	TO BE POPULATED	**OPEN**
CP-even / CP-odd invariants	TO BE POPULATED	**OPEN**
ΔNU, S S† residual and closed-form residual	TO BE POPULATED	**OPEN**
Threshold ledger	TO BE POPULATED	**OPEN**
Joint covariance	TO BE POPULATED	**OPEN**
Branch equivalence or selector	TO BE POPULATED	**OPEN**
Absolute scale, Zcomp and Λ★	TO BE POPULATED	**OPEN**
Independent same-action descent	TO BE POPULATED	**OPEN**
Physical neutral certificate	**NOT ISSUED**	**REFUSED UNTIL ALL LOAD-BEARING GATES PASS**

35. Final pass conditions

1. One frozen common action directly returns Yν, MR and ML, with MD generated under the same scalar/Yukawa convention.
2. Every matrix passes source, symmetry, rank, covariance and independent-descent audits at all four regulators.
3. The complete Takagi spectrum and its invariant subspaces converge, with a stable source-clean light/heavy census and a declared residual-gauge ledger.
4. The charged-lepton and neutral frames come from the same branch, regulator and convention ledger.
5. Mixing observables, CP invariants and ΔNU are returned before comparison and satisfy their exact internal identities, including the S S† and closed-form nonunitarity checks.
6. Heavy thresholds and the effective active operator have controlled round-trip and truncation residuals within the Weyl bound.
7. The branch is uniquely selected, or the exact branch class is proved physically output-equivalent under Theorem 4 with a shared C and accepted as such by the HFB rules.
8. Zcomp, Λ★ and the action/clock bridge are admitted before any absolute mass or threshold is promoted.

9. A complete joint covariance and a sealed non-insertion certificate are published with the manifest.

36. Revised Task-5 ledger

Sub-gate	Current grade	Release condition
Recurrent neutral history	**STRUCTURAL PASS**	Finite-mean physical hazard and provenance still required.
Neutral information capacity	**PASS IN PRINCIPLE**	Compute the differentiated embedding J; capacity alone leaves a 157 248-dimensional fibre.
Same-branch Yν	**OPEN**	Direct mixed derivative, or an upstream gapped selector.
Direct MR	**OPEN**	Same-action singlet Majorana block.
Irreducible ML	**OPEN**	Direct return, or a regulator-stable exact zero theorem (worth 12 of 30 parameters).
Full Takagi spectrum	**EXECUTION READY**	Supply the complete symmetric mass block.
Mixing and nonunitarity	**EXECUTION READY**	Supply the Takagi matrix and the same-branch UeL.
Thresholds	**CONTRACT CLOSED / NUMERICAL OPEN**	Scale, matching objects and covariance.
Branch orientation / equivalence	**OPEN**	Selector, or an exact physical conjugacy theorem with shared C.
Absolute physical masses	**OPEN BY EXACT NO-GO**	Return A★, Zcomp and Λ★.
Independent same-action descent	**FAIL**	Repair the common-action descent or refuse the certificate.
Overall Task 5	**EXECUTION CONTRACT CLOSED / PHYSICAL REFUSED**	All load-bearing returns must close on one admitted branch, or on an exact-degenerate set proved output-equivalent.

37. Programme consequence

A populated and passing NSTC-1 certificate would be the first paper in the current sequence capable of moving SM-24 and SM-25 from structural or partial status toward genuine physical closure. It would supply the neutral component of SM-22 and a substantial part of the HFB boundary. It would not by itself close the global quantum measure, the strong phase, confinement, the finite scheme bridge, the poles, or the final RPC certificate.

A failed or non-viable neutral return would be equally decisive. It would show that the strongest recurrent HCMR branch class cannot support the physical neutral sector without new primitive structure, and would prevent further downstream tuning from being misclassified as derivation.

38. Terminal verdict ledger

Structural. The same-branch assembly theorem, the Takagi certificate with its exact residual gauge, the exact nonunitarity identities, the corrected conjugate-branch map, the scale-orbit theorem, the modulus count and the forgery theorem are established and independent of every unsupplied number.

Derived mechanisms. The residual gauge $\bigoplus_i O(k_i,\mathbb{R}) \oplus U(k_0)$ and the promotability rule; the closed form $\Delta NU = \Theta\Theta^\dagger(\mathbb{1}_3 + \Theta\Theta^\dagger)^{-1}$; the Wedin criterion for subspace admissibility; the Weyl bound on the Schur comparison.

Corrections carried as falsifiers. The conjugation convention in Θ (F-N7); the conjugate-branch Takagi map (F-N8); the Richardson denominator $2^p - 1$ (F-N10); the rank inequality direction (Proposition 26.1).

Negative certificates. Identity-projector no-selection; capacity–identifiability separation with a 157 248-dimensional fibre; absolute-scale non-identifiability; stable-disagreement blocking; the 30-parameter non-closure; inverse-construction forgery.

Imported at declared grade. Recurrent neutral capacity and ranks 21 / 867 (K7-CBR-1); four-regulator positivity and second-order convergence (HCMR-MR1); ξ and the Route-M traversal law (ASAC-1, conditional); the clock reproducibility repair (D36-R1).

Blockers. Direct $Y\nu$, MR, ML (D-N1); the neutral readout selector (D-N2); the oriented branch (D-N3); the independent descent repair (D-N4); Zcomp and $\Lambda\bigstar$ (D-N5).

Final lines. The execution contract is closed. The physical neutral certificate is **NOT ISSUED** — and, by Theorem 7, correctly so.

Appendix A — Proof details

A.1 Takagi reconstruction identity

From $U\nu^T M\nu\, U\nu = \Sigma$ and unitarity, multiply on the left by $U\nu^*$ and on the right by $U\nu^\dagger$:

$$U\nu^*\, U\nu^T\, M\nu\, U\nu\, U\nu^\dagger = U\nu^*\, \Sigma\, U\nu^\dagger \quad \Longrightarrow \quad M\nu = U\nu^*\, \Sigma\, U\nu^\dagger \ ,$$

using $U\nu^* \, U\nu^T = (U\nu \, U\nu^\dagger)^* = \mathbb{1}$ and $U\nu \, U\nu^\dagger = \mathbb{1}$. The reconstruction residual therefore tests the full complex matrix, not merely its singular values, and is the only certificate field that cannot be satisfied by a phase-broken $U\nu$.

A.2 Nonunitarity identity

Write the first three rows of $U\nu$ as $[N_0 \; S_0]$, with N_0 the light columns and S_0 the complement. Row unitarity gives $N_0 N_0^\dagger + S_0 S_0^\dagger = \mathbb{1}_3$. Left multiplication by the charged-lepton unitary frame preserves the identity, so

$$\Delta NU = \mathbb{1}_3 - N N^\dagger = S S^\dagger \geq 0 \; .$$

The closed form $\Delta NU = \Theta\Theta^\dagger(\mathbb{1}_3 + \Theta\Theta^\dagger)^{-1}$ follows from the exact block-diagonalising parametrisation whose upper-left block is $(\mathbb{1}_3 + \Theta\Theta^\dagger)^{(-1/2)}$; see Theorem 3.

A.3 Conjugate-branch theorem

Let C be unitary and $M_6 = C \, M_1^* \, C^T$, with $U_1^T \, M_1 \, U_1 = \Sigma$. Set $U_6 = \bar{C} \, U_1^* = (C \, U_1)^*$. Then

$$U_6^T \, M_6 \, U_6 = U_1^\dagger \, C^\dagger \, (C \, M_1^* \, C^T) \, \bar{C} \, U_1^* = U_1^\dagger \, M_1^* \, U_1^* = (U_1^T \, M_1 \, U_1)^* = \Sigma \; ,$$

using $C^\dagger C = \mathbb{1}$ and $C^T \bar{C} = (C^\dagger C)^T = \mathbb{1}$, and Σ real. Hence U_6 is a Takagi matrix of M_6 with the same non-negative Σ.

The alternative expression $C \, U_1^*$ requires $C^T C = \mathbb{1}$ in place of $C^T \bar{C} = \mathbb{1}$, i.e. a real orthogonal C. For a general unitary C it is not a Takagi matrix of M_6 at all, and the failure is of order one rather than of order tolerance. Since the two forms coincide whenever C happens to be real, this is a defect that survives real-valued testing and appears only on the first genuinely complex branch map.

The corresponding charged-frame conjugation maps N_1 to N_6 by unitary rephasings together with complex conjugation. Moduli and CP-even invariants therefore agree; imaginary rephasing invariants reverse sign.

A.4 Scale orbit

For $M(s) = s \, \hat{M}$ with $s > 0$, $U^T \, M(s) \, U = s\Sigma$. Hence U, every dimensionless overlap matrix and ΔNU are unchanged, while masses and thresholds scale linearly. No downstream mixing calculation can determine s.

A.5 Residual gauge of the Takagi factorisation

Let $U\nu^T \, M\nu \, U\nu = \Sigma$ and $(U\nu Q)^T \, M\nu \, (U\nu Q) = \Sigma$ with Q unitary. Then $Q^T \, \Sigma \, Q = \Sigma$. Comparing entries shows Q is block diagonal with respect to the distinct values of Σ. On a block with value

$m > 0$, $Q_i^T Q_i = \mathbb{1}$; with $Q_i^\dagger Q_i = \mathbb{1}$ this gives $\bar{Q}_i = Q_i$ and hence $Q_i \in O(k_i, \mathbb{R})$. On the kernel block the constraint is empty, leaving $U(k_0)$. Therefore

$$G\_res = \bigoplus_i O(k_i, \mathbb{R}) \oplus U(k_0) ,$$

and for a simple non-zero value the freedom is exactly the sign group $\{\pm 1\}$.

A.6 Modulus count

$M\nu$ complex symmetric of order 6 has 21 complex, hence 42 real, components. In the charged-lepton mass basis the surviving field redefinitions are the rephasings $U(1)^3$ of the active doublets and the full U(3) redefinition of the gauge singlets, of total real dimension 12, acting freely on a generic orbit. The physical dimension is $42 - 12 = 30$. Imposing $ML = 0$ removes 12 real components, giving 18 — three light masses, three heavy masses, six angles and six phases.

Appendix B — Inverse-construction forgery and the necessity of the firewall

Theorem B.1 (forgery). Fix any target light spectrum (m_1, m_2, m_3), any target unitary mixing matrix U, and any heavy spectrum (M_1, M_2, M_3) with $M_i > 0$. Then in the type-I case $ML = 0$ the set of neutral Yukawa matrices reproducing that target *exactly* is

$$Y\nu = (1/v)\ \sqrt{M\_R} \cdot R \cdot \sqrt{m\_\nu} \cdot U^\dagger , \quad R \in O(3, \mathbb{C}) ,$$

a family of real dimension $\dim\_\mathbb{R}\, O(3, \mathbb{C}) = 6$. Consequently agreement of a computed neutral sector with observation carries **no** information about provenance: a six-parameter family of same-shaped inputs reproduces any observation to arbitrary precision.

Proof. Direct substitution into the Schur reduction gives $-Y\nu^T M\_R^{-1} Y\nu\, v^2 = U^* \mathrm{diag}(m_i) U^\dagger$, and the complex orthogonality $R^T R = \mathbb{1}$ is exactly the condition for the construction to be consistent. The parameter count is 18 real components in $Y\nu$, less 3 rephasings, less the 9 physical outputs (three light masses and six mixing parameters), leaving 6. ■

Corollary B.2 (the firewall is a theorem). Since the forgery family exists for every target, no post-hoc comparison — however impressive — distinguishes a derivation from a construction. The only discriminator is the provenance of the inputs: whether $Y\nu$, MR and ML were differentiated from one frozen action before any observable was consulted. This places the non-insertion boundary of §4 on the same footing as the other no-go results of Part VI rather than on the footing of methodological preference.

Remark B.3. The forgery family is the neutral-sector instance of the inverse-Yukawa construction already quarantined at the assembly layer. It is recorded here as a theorem rather than as a quarantine so that any future neutral result reporting agreement with data must first state which of its inputs were fixed before the comparison, and by what derivative.

Appendix C — Machine-readable certificate schema

The calculation package emits a JSON certificate containing at least the following fields. Arrays remain separate binary objects referenced by exact hashes.

- `designation`, `source_revision`, `revision_currency`, `regulator`, `branch`, `units`, `conventions_hash`;
- `Ynu_hash`, `MD_hash`, `MR_hash`, `ML_hash`, `UeL_hash`, `C_map_hash`, `covariance_hash`;
- `symmetry_residual`, `Takagi_unitarity_residual`, `Takagi_diagonal_residual`, `reconstruction_residual`;
- `Takagi_masses`, `degeneracy_clusters`, `residual_gauge_per_cluster`, `sign_convention_hash`, `light_indices`, `heavy_indices`;
- `subspace_gaps`, `wedin_bounds`, `subspace_grade` ∈ {CONVERGED, NOT TESTED};
- `mixing_matrix_hash`, `mixing_modulus`, `CP_invariants`, `nonunitarity_hash`, `SSdag_residual`, `closed_form_residual`;
- `thresholds`, `Schur_residuals`, `weyl_bound`, `regulator_order`, `Richardson_estimates`, `Richardson_regime_certified`;
- `branch_conjugation_residuals`, `independent_descent_residuals`, `Zcomp`, `Lambda_star`;
- `noninsertion_seal`, `seal_timestamp_relative_to_data_access`;
- `final_verdict`: PASS / FAIL / EXACT_DEGENERATE / UNRESOLVED.

A certificate lacking `revision_currency`, `residual_gauge_per_cluster`, `SSdag_residual` or `noninsertion_seal` is incomplete by construction and is not eligible for admission.

Appendix D — Source ledger and imported grades

Ref.	Source	Role in NSTC-1	Grade
[1]	VERSF Standard Model Derivation Master Ledger	Baseline gate grades, the Task-5 contract, strict HFB/RPC admission logic.	IMPORTED

Ref.	Source	Role in NSTC-1	Grade
[2]	The D36 Clock-Convention, Reproducibility and Manifest-Currency Theorem (D36-R1)	Reproducibility repair, clock-convention separation, revision protocol.	IMPORTED
[3]	The K = 7 Closure–Born–TPB Renewal Construction (K7-CBR-1)	Recurrent neutral history, 102-dimensional census, rank-capacity result, provenance debt.	IMPORTED
[4]	The Absolute-Scale, Action-Normalisation and Closure-Clock Conditional Theorem (ASAC-1)	Conditional length and clock return; exact action/completion-scale no-go.	CONDITIONAL
[5]	The Four-Regulator Commitment-Maintenance Execution and Physical-Admission No-Go Theorem (HCMR-MR1)	Four-regulator parent, charged frame, branch class, PY identity, independent CY3′ failure, missing Zcomp.	CANDIDATE

Upstream MASD-1, HMGBC-1, FCHI-1, RRHF-1, PRLU-1 and related papers are named by the uploaded sources but were not independently supplied with this drafting request. NSTC-1 treats their imported objects exactly at the grades declared in references [2]–[5] and does not silently strengthen them.

Appendix E — Open-debt ledger and mandatory ledger wording

E.1 Debts

Debt	Statement	Kind
D-N1	Direct same-action returns of Yν, MR and ML at n = 3,4,5,6 and k = 1,6.	New information
D-N2	A non-trivial, gapped, regulator-stable neutral readout selector PY,ν, or a direct mixed derivative making one unnecessary.	New information
D-N3	An orientation-odd action term or equivalent selector resolving the exact {1, 6} pair, or a proof of output equivalence with shared C.	New information
D-N4	Repair of the independent CY3′ descent, currently failing at residue ≈ 0.37–0.49.	Structural
D-N5	Zcomp, Λ★ and the action/clock bridge required for absolute masses.	New information
D-N6	An exact regulator-stable ML theorem, or the direct block; worth 12 of the 30 physical parameters.	Calculation or theorem

Debt	Statement	Kind
D-N7	Finite-mean physical hazards, the attaching and reset maps, the full Jacobian and cross-block Hessian left open by K7-CBR-1.	Calculation
D-N8	The shared conjugation map C for the neutral and charged sectors, as an exported object rather than an assumption.	Calculation
D-N9	A revision-currency check on every hashed input, since integrity is not currency.	Process

D-N1, D-N2, D-N3 and D-N5 are new-information requirements in the sense of Corollary 7.1; they are not closable by additional computation on the present corpus.

E.2 Mandatory ledger wording

Task-5 entry. Physical neutral sector: recurrent terminal history and information capacity are structurally closed, and the complete same-branch Takagi / mixing / nonunitarity / threshold execution contract is fixed, with an exact residual-gauge and promotability rule. The current corpus does not supply direct Yν, MR or ML returns, a non-trivial neutral readout, an oriented branch, an independent same-action descent pass, or Zcomp and Λ★. By the modulus-count theorem the compatible family retains 30 real physical parameters, doubled by an unresolved conjugate pair, and by the forgery theorem no comparison with data can reduce it. No physical neutral mass certificate is issued. **Grade: COMPLETE EXECUTION CONTRACT / PHYSICAL REFUSED.**

End of NSTC-1