QCD Running and Confinement in VERSF

▲ Programme Milestone — Strong-Interaction Renormalisation and Confinement Series

Gate QCR-1 (Consolidated) / Derived Colour Response, Symbolic Strong Scale, and the Conditional Triality-Gap Confinement Criterion

Keith Taylor VERSF Theoretical Physics Programme — Strong-Interaction Renormalisation and Confinement Series Tier A conditional gate-theorem paper with derived colour-response components — working manuscript

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General Reader Summary

The strong force — the force that glues quarks together inside protons and neutrons — behaves in a strange, backwards way.

Up close, at incredibly tiny distances, it is weak. Quarks inside a proton rattle around almost as if they were free. But try to pull a quark *out*, and the force does not fade the way gravity or magnetism does. It holds. Pour in more energy and, instead of freeing the quark, you create new particles. That is why nobody has ever seen a lone quark. Physicists call the first behaviour **asymptotic freedom** and the second **confinement**.

These two facts are related, but they are not the same puzzle, and this paper is careful never to blur them. It also adds a third puzzle that usually goes unasked: where does the strength of the force come from in the first place? In the Standard Model, that strength is simply measured and typed in. In VERSF, it should be a property of the underlying substrate — a kind of stiffness: how hard the medium resists having its internal colour frame twisted.

The strength of the force. The first result of this paper is that this stiffness can now be traced to its source. Carry a quark's colour frame around a tiny loop and it comes back slightly rotated; the substrate's resistance to that rotation *is* the strong force. Earlier drafts guessed that this resistance lived in an exotic fourteen-channel structure. Deriving the map honestly showed otherwise: fourteen was a count of symmetry operations, not a physical space, and the guess is formally retired in this paper — a retraction recorded with its lessons. The true home of the response turns out to be the simplest possible one: the natural twisting directions of the three-colour space itself. Better still, symmetry forces the response to be perfectly even-handed — no colour direction is stiffer than any other — so the entire strength of the strong force boils down to **one number**: a conversion factor the paper calls χ_C, waiting to be computed from the substrate's master action. The paper spells out, step by step, exactly how to compute it.

The scale of the force. Once that one number is known, textbook physics takes over. The force weakens at short distances (the famous asymptotic freedom of Gross, Wilczek and Politzer emerges with its standard coefficient), and the running of the force manufactures a characteristic energy scale — the scale that ultimately sets the size and mass of protons. The paper derives that scale as a single closed formula containing χ_C and nothing measured from the strong force itself. Two warnings come with it, stated up front rather than discovered later. The formula is exponentially sensitive: a small error in χ_C becomes a large error in the scale, and the paper says precisely how accurate the future calculation must be. And the theory is built to be checkable from the inside: consistency forces χ_C to *change* with energy at one specific rate. If the eventual substrate calculation produces a χ_C that changes at any other rate — or doesn't change at all — the whole identification is dead, no experiment required. A theory earns trust by exposing itself this way.

Why quarks are trapped. Confinement gets its own, separate argument. A quark carries a kind of indelible label (physicists call it *triality*) that the force carriers — gluons — can never erase. Now imagine slicing the space between a quark and an antiquark like a loaf of bread. A basic law of the colour field forces every single slice, no matter where you cut, to carry that label. The paper's key theorem says: if maintaining the label through one slice costs even a tiny irreducible amount of energy — after the surrounding medium has done everything it can to relax around it — then the total cost grows in proportion to the separation. That is the invisible "string" between quarks, and that is confinement: pull further, pay more.

The simplest estimate of that per-slice cost produces the paper's most satisfying line. Done correctly (a natural first attempt got the physics backwards, and the correction is recorded), the cost turns out to be *proportional to the strength of the force at long distances*. The same running that makes the force weak up close makes it strong far away — and that strength directly feeds the trap. The two faces of the strong force meet in a single formula.

The honest twist. In the real world, the string does not stretch forever. At some point it is cheaper to snap it by creating a new quark–antiquark pair. But snapping the string frees no one: one confined system becomes two confined systems. Colour never escapes.

What the paper does not claim. No numbers are predicted yet. The strength χ_C, and a second quantity governing the trap, remain to be computed — and the paper forbids itself every shortcut: no measured value may be smuggled in and re-announced as a prediction, every assumption is named alongside the observation that would kill it, and where an argument is an approximation (the trap is estimated with classical fields standing in for the full quantum problem) the gap is declared as an open debt rather than papered over.

In one sentence:

The strength, the scale, and the imprisoning power of the strong force are traced to one computable property of the substrate — how hard it resists having its colour frame twisted — with the theory wired to fail loudly, on its own terms, if that property is ever computed and comes out wrong.

Scope and Closure Box

Claim	Status
Colour bath carrier is $\mathbb{C}^3$ with $SU(3)_C$ transport	Inherited
Colour-holonomy residual map $E_3(e_a) = T_a$ onto $\mathrm{Herm}_0(\mathbb{C}^3)$	**Derived here — canonical, isometric, equivariant**
Invariant Fisher response $g_{ab}{}^{fs} = \frac{1}{8}\cdot\delta_{ab}$	Isotropy derived; eigenvalue = frozen normalisation split (§1.7)
$d = 3$ uniqueness of the coincidence $2(d+1) = d^2-1$	Derived — typed as a firewall, not a derivation
Scalar reduction $Z_3 = \chi_C/8 - b^\dagger\cdot C^{-1}\cdot b$	**Derived here** from Schur decomposition of the auxiliary carrier
Numerical $\chi_C(\mu\star)$	Open — the decisive master-action calculation
Auxiliary adjoint census (b, C)	Open
Fourteen-channel carrier $\mathscr{H}_{14}$ as physical response space	**Retired** — $14 = \lvert D_7\rvert$ is a group order, not a demonstrated carrier dimension
Spoke-sector Hessian renormalises the leading F^2 coefficient	Rejected — no adjoint intertwiner exists
Canonical colour coupling $g_s(k) = Z_3(k)^{(-1/2)}$	Exact identity (GAC-1)
Background-field flow $\beta_s = -\frac{1}{2}\cdot g_s^3\cdot\partial Z_3/\partial \ln k$	Derived
$b_0 = 11 - (2/3)\cdot N_f$; asymptotic freedom for $N_f \leq 6$	Derived from inherited field content
Symbolic $\Lambda_3 = \mu\star^{(7/9)}\cdot(m_t\, m_b\, m_c)^{(2/27)}\cdot e^{(-8\pi^2 Z_3/9)}$	Derived at one loop with sharp thresholds
Forced running $d\chi_C/d\ln k = 7/\pi^2$ (screening-free)	Derived consistency condition — internal falsifier
Precision budget (exponential sensitivity of Λ_3)	Derived and declared
Quark masses m_t, m_b, m_c	Declared inputs pending the Yukawa gates
Negative beta function implies confinement	Explicitly rejected
Triality typed on centre-electric-flux Hilbert sectors	Installed
Classical slice functional as declared truncation of quantum sector energy	Typed here — gap logged as Debt D21
Truncation RG-compensation $d\ln B_\tau/d\ln k = d\ln Z_3/d\ln k$	Derived consistency condition — internal falsifier
Audit independence of the continuum one-loop determinant	Installed firewall

Claim	Status
Positive triality gap from coercivity, compactness, sector separation	Derived conditionally
Corrected coercive tube functional; $\varsigma_\tau = g_{s,IR}\cdot\lvert\Phi_\tau\rvert\cdot\sqrt{2B_\tau}$	Derived within the truncation — typing correction installed
Longitudinal slicing inequality	Named premise — derivation open
String tension, pure-gauge area law	Derived conditionally (transfer matrix)
Uniform gap in infinite-volume and continuum limits	Named condition — open
RG consistency of σ_τ	Installed firewall — proof open
Strict area law with dynamical quarks	Rejected — string breaking
Colour-singlet asymptotic persistence	Operational criterion (assumption-graded)
Numerical Λ_3, σ_τ, string-breaking distance, T_c	Open
Global centre-sensitive lift	Open — sole remaining descent debt
Empirical confirmation	Not established

Abstract

The VERSF gauge programme derives the local colour connection from continuous transport on an indistinguishable threefold bath carrier. GAC-1 defines the canonically normalised strong coupling by $g_s(k) = Z_3(k)^{-1/2}$, where Z_3 is the colour eigenvalue of the reduced holonomy-response Hessian, but leaves the response map open. This consolidated gate closes the map, derives the shape of the response, reduces the coupling to one susceptibility, transports it across scale, and states the conditional confinement criterion — keeping the three results typed and separate.

The derived response (Part II). The colour-holonomy residual of a small oriented loop, after traceless projection, is exactly adjoint-valued, and the residual map is the canonical identification

$$E_3 : \mathfrak{su}(3) \to \mathrm{Herm}_0(\mathbb{C}^3),\quad E_3(e_a) = T_a,\quad E_3^\dagger\cdot E_3 = 1_8,$$

exactly Ad-equivariant. The invariant response geometry of the saturated bath — the Fubini–Study metric Haar-averaged over pure states of $\mathbb{C}^3$ — is isotropic with eigenvalue

$$g_{ab}^{\,fs} = \tfrac{1}{8}\cdot\delta_{ab},$$

computed from the second Haar moment $\int(|\psi\rangle\langle\psi|)^{\otimes 2}\, d\mu = (1 + S)/12$; invariance alone forces the isotropy, and the eigenvalue is the frozen normalisation split with the susceptibility (§1.7). One conversion factor, the substrate susceptibility $\chi_C(\mu\star)$, converts projective distance into action density in a convention that must be frozen before χ_C is evaluated (only the product of shape and susceptibility is physical). Auxiliary reduction is then exact: decomposing the

auxiliary carrier into SU(3) irreducibles, Schur's lemma annihilates every mixed block except auxiliary adjoint copies, and

$$\mathbf{Z_3(\mu\star) = \chi\_C / 8 - b^\dagger \cdot C^{-1} \cdot b,}$$

with b the curvature mixing and $C > 0$ the Hessian on the adjoint multiplicity space. No 8×8 diagonalisation survives. The earlier fourteen-channel trial $H_{14}^{tr} = \kappa\_C \cdot (L_7 \oplus L_7)$, and its forced result $Z_3 = 7 \cdot \kappa\_C$, are retired: $14 = |D_7|$ is a group order, the regular representation carries no canonical SU(3) adjoint action, the trial Hessian was provably colour-blind, and the presently derived $K = 7$ master-action Hessian is a six-dimensional spoke operator with no adjoint intertwiner, $\mathrm{Hom}\_\{SU(3)\}(\mathbf{8}, V\_seq) = 0$. The identification $\kappa\_C \equiv Z_3/7$ survives only as notation. An executable definition closes the part: impose a plaquette holonomy $U(\varepsilon) = e^\wedge\{i\varepsilon T\_a\}$, minimise the master action over all auxiliaries, and compute $Z_3 = \lim\_\{\varepsilon \to 0\}\ 2[\min\_\eta\ S(\varepsilon, \eta) - S(0,0)]/\varepsilon^2$, with adjoint isotropy requiring all eight generators to return the same value.

Running (Part III). With the background-field Ward gate, canonical Fock currents and the gauge-redundancy determinant supplied, the universal one-loop flow is $dZ_3/d \ln k = (1/8\pi^2) \cdot [(11/3) \cdot C\_A - (4/3) \cdot \Sigma\_f\ T(R\_f) \cdot \vartheta\_f(k)] + O(g_s^2)$; for the Standard Model census $b_0 \geq 7 > 0$ and the sector is asymptotically free. Sharp threshold matching through m_t, m_b, m_c yields the closed symbolic scale

$$\mathbf{\Lambda_3 = \mu\star^\wedge(7/9) \cdot (m\_t \cdot m\_b \cdot m\_c)^\wedge(2/27) \cdot \exp[\ -8\pi^2 \cdot Z_3(\mu\star) / 9\ ],}$$

with exponent $-\pi^2 \cdot \chi\_C/9$ in the screening-free case; the two-loop scheme-invariant scale carries the additional power-law prefactor $(b_0 g_s^2/16\pi^2)^\wedge(-b_1/2b_0^2)$. Consistency of the derived response with the flow forces

$$\mathbf{d\chi\_C / d \ln k = 7 / \pi^2 + O(g_s^2)}$$

above the top threshold (general form: $d(\chi\_C/8 - b^\dagger C^{-1} b)/d \ln k = 7/8\pi^2$) — a parameter-free falsifier available before any numerical χ_C exists. The precision budget $d \ln \Lambda_3/dZ_3 = -8\pi^2/9 \approx -8.8$ requires $|\delta\chi\_C| \lesssim 0.09$ for a ±10% strong scale.

Confinement (Part IV). Triality is carried by centre-electric-flux sectors $\mathscr{H}\_\tau(\Sigma; Q, \bar{Q})$ of the gauge-invariant transfer-matrix Hilbert space, defined by $U\_z(\Sigma)|\Psi\_\tau\rangle = z^\wedge\tau|\Psi\_\tau\rangle$, not by local connection configurations. On constrained slice states $X = (A\perp, E\perp, E\_z, \eta\perp; Q, \bar{Q})$ the reduced transverse bath energy $\mathscr{E}\perp^{red}[X] = \inf\_\eta[\mathscr{E}\perp[X, \eta] - \mathscr{E}\perp^{vac}]$ is minimised at fixed flux sector. If the reduced functional is lower semicontinuous, gauge-coercive, compact on finite-energy sublevels, uniquely minimised by the vacuum in the zero-triality sector, and if triality is sector-separating under finite-energy convergence, then $\varsigma\_\tau = \inf$ over $\mathfrak{c}\_\tau$ of $\mathscr{E}\perp^{red} > 0$, where $\mathfrak{c}\_\tau$ is the classical centre-projected flux class — a declared truncation of the quantum sector energy on $\mathscr{H}\_\tau$, with the classical-to-quantum gap logged as Debt D21. An explicit longitudinal slicing inequality (named premise) yields $V\_\tau(L) \geq \varsigma\_\tau \cdot L - c\_end$; a finite trial tube and subadditivity give $\sigma\_\tau = \lim V\_\tau(L)/L > 0$; and reflection positivity converts the linear energy into the pure-gauge Wilson-loop area law $-\log\langle W\_R(L,T)\rangle = \sigma\_\{\tau(R)\} \cdot L \cdot T + O(L+T)$. A uniform-gap condition ($\liminf \varsigma_\tau > 0$ under removal of the transverse-volume and continuum regulators) and an RG-consistency firewall ($d\sigma_\tau/d \ln k = 0$ exactly) guard the physical limits. The simplest coercive instantiation is corrected here: holding the holonomy-quantised primitive flux fixed with canonical momentum $\Pi = Z_3 \cdot E$ puts the stiffness in the denominator,

$$\mathscr{E}_{\perp,\tau}^{\mathrm{red}}(R) = \Phi_\tau^2/(2\pi \cdot Z_{3,\mathrm{IR}} \cdot R^2) + \pi \cdot B_\tau \cdot R^2, \quad \boldsymbol{\varsigma_\tau = g_{s,\mathrm{IR}} \cdot |\Phi_\tau| \cdot \sqrt{2 \cdot B_\tau}},$$

so the infrared growth of the coupling multiplies the confinement gap. With dynamical fundamental quarks the string breaks into colour singlets; a persistence criterion (assumption-graded) organises the surviving statement that no isolated colour reaches spatial infinity.

The remaining strong-interaction targets are exactly: $\chi_C(\mu\star)$ via the plaquette protocol, the auxiliary adjoint census (b, C), the bath areal coefficient B_τ, the flux-normalisation lift, and the confinement analyticity conditions. No measured α_s, Λ_{QCD}, string tension or hadron mass is inserted anywhere.

Contents

Part I — Foundations

1. Notation, Conventions and Firewalls

1.1 Colour algebra and carriers

The colour bath carrier is $C_x \simeq \mathbb{C}^3$. Hermitian generators $T_a = \lambda_a/2$ in the fundamental representation satisfy

$[T_a, T_b] = i\cdot f_{ab}{}^{c}\cdot T_c$, $\mathrm{tr}(T_a\cdot T_b) = \frac{1}{2}\cdot\delta_{ab}$, $a, b = 1, \ldots, 8$,

with invariants $C_A = 3$, $C_F = 4/3$, $T_F = 1/2$. $\mathrm{Herm}_0(\mathbb{C}^3)$ denotes traceless Hermitian 3 × 3 matrices, equipped with $\langle X, Y\rangle_{adj} = 2\cdot\mathrm{tr}(X\cdot Y)$. The primitive (coupling-free) connection is $\mathcal{A}_\mu = A^a{}_\mu\cdot T_a$ with curvature $\mathscr{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu + i\cdot[\mathcal{A}_\mu, \mathcal{A}_\nu]$. At scale k the canonical fields are $\hat{A}^a{}_\mu = Z_3(k)^{(1/2)}\cdot A^a{}_\mu$ with $g_s(k) = Z_3(k)^{(-1/2)}$.

1.2 Running-versus-confinement firewall

$\beta_s < 0$ at weak coupling proves ultraviolet asymptotic freedom. It does not prove a mass gap, an area law, a positive string tension, flux-tube formation, or exclusion of coloured asymptotic states. Those require a nonperturbative infrared theorem.

1.3 Pure-gauge-versus-full-QCD firewall

A strict asymptotic area law is a clean confinement criterion only in pure Yang–Mills theory. With dynamical fundamental quarks, centre charge can be screened, the string breaks, the potential saturates, and colour-singlet observability — not an eternal string — is the correct criterion.

1.4 Lie-algebra-versus-global-group firewall

The algebra $\mathfrak{su}(3)$ determines local transport. It does not determine the global group form, its centre, the allowed Wilson lines, triality sectors, or the one-form centre symmetry. Confinement requires global group data beyond the local algebra.

1.5 Group-order-versus-carrier firewall

The number fourteen enters VERSF as $|D_7|$, the order of the dihedral transport group — seven rotations and seven reflections. A group order is not automatically a vector-space dimension, and the fourteen-dimensional regular representation of D_7 carries no canonically defined SU(3) adjoint action. Any construction that treats 14 as a physical response-carrier dimension without deriving the carrier is a typing error (falsifier F36). The derived carrier of this paper is $\mathrm{Herm}_0(\mathbb{C}^3)$.

1.6 Two-eights firewall

The response eigenvalue ⅛ arises as $1/(2(d+1))$ at bath dimension $d = 3$. The adjoint dimension is $d^2 - 1 = 8$. The coincidence $2(d+1) = d^2 - 1$ holds **only** at $d = 3$. The two eights are distinct typed objects; neither may be "derived" from the other, and their coincidence may not be cited as evidence for any construction.

1.7 Convention-locking firewall

Invariance forces the response to be isotropic ($\propto \delta\_ab$); it does not fix the scalar. The value ⅛ is the Haar-averaged pure-state Fubini–Study convention; the GNS/tracial metric at the maximally mixed state gives ⅙, and the Bures metric there vanishes on unitary directions. Since χ_C is an undetermined conversion factor, only the product (shape eigenvalue) × χ_C is physical. The convention must be frozen — with the reason it is the VERSF-correct response geometry stated (P3) — **before** the master-action evaluation of χ_C, in the same normalisation. Post-hoc convention selection is falsifier F3.

1.8 Normalisation firewall

Setting χ_C (or any surrogate such as the retired κ_C) to a convenient value is forbidden. The absolute susceptibility is precisely the physical coupling information the programme must derive.

1.9 Ghost-versus-physical-bath firewall

Faddeev–Popov ghosts are gauge-fixing bookkeeping in the quantum determinant, not physical bath particles. The combined gauge-plus-ghost determinant is required for the gauge-invariant one-loop coefficient and may not be reinterpreted as substrate channels.

1.10 Centre-flux-versus-fixed-total-flux firewall

A fixed total flux spread over a growing surface does not automatically generate an area-proportional cost. Extensivity arises here because the Gauss law forces every transverse cross-section between unscreened triality sources to carry the same nonzero triality, and each cross-section has a positive irreducible reduced cost.

1.11 Probability-versus-large-deviation firewall

A positive probability of a centre-flux event does not imply exponential area suppression. An area law requires a transfer-matrix linear-energy theorem or an independently derived large-deviation rate proportional to area. This paper uses the transfer-matrix route.

1.12 Electric-flux typing firewall

The flux held fixed belongs to the primitive connection. With action $\frac{1}{4} \cdot Z_3 \cdot F^2$ in primitive fields, the canonical momentum is $\Pi = Z_3 \cdot E$ and the energy density is $\Pi^2/2Z_3$; the Gauss law constrains the divergence of Π, tying its surface flux to the enclosed source, while the quantisation of that centre-electric flux is a sector statement in the sense of §19.2 (normalised by the global lift, Debt D8). Any tube functional holding this centre-projected flux fixed must carry the stiffness in the **denominator** of the localisation term. The opposite placement is falsifier F32.

1.13 Infrared relabelling firewall

Near the confinement scale the derivative expansion breaks down. Infrared statements should be phrased in Z_3,IR or g_s,IR; factorised surrogates there are notation, not derivation.

1.14 Matching-scale and mass-input firewall

The output $g_s(\mu\star)$ is a coupling in a declared blocked scheme; comparison requires scale, scheme, thresholds and observable definition. The threshold masses m_t, m_b, m_c are declared external inputs until the Yukawa gates (QCMP-1, CLAS-1, CMSY-1) supply them in the declared scheme, with the matching convention (pole versus running mass) stated.

1.15 Audit-independence firewall

The forced-running audit (Theorem 10, Debt D4) has content only if the blocking evaluation of χ_C(k) is computationally independent of the continuum one-loop gluon determinant. An implementation of the §9 protocol that integrates out fluctuations using standard background-field machinery will "recover" $7/\pi^2$ tautologically — verifying textbook field theory, not VERSF. The audit must evaluate χ_C at two scales from the master action's own blocking rules; an audit that imports the continuum determinant as its computational engine is void, not confirmatory (falsifier F12, circularity clause). The κ_C version of this condition died of exactly this disease.

1.16 Numerical non-insertion firewall

No measured value of α_s, Λ_QCD, σ, T_c, r_tube, or any hadron mass may be used to select the response functional, its convention, or its branch and then be reported as a prediction.

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2. Inherited Infrastructure and Claim Levels

2.1 Inherited structures

The colour connection is inherited as the local comparison field of continuous transport on the threefold indistinguishable bath class; its non-Abelian curvature is not an added Standard Model assumption. GAC-1 supplies the definitions $K_3 = E_3^\dagger \cdot \mathscr{H}^{\mathrm{red}} \cdot E_3$, $Z_3 = \frac{1}{8} \cdot \mathrm{Tr}_8 K_3$, $g_s = Z_3$^(−1/2), and requires every same-order auxiliary mode to be minimised out. The Fock-space reconstruction supplies canonically normalised quark currents J_a^μ = Σ_f :q̄_f γ^μ T_a q_f:. The strange-confinement and coloured-participation papers supply the type distinction internal coloured response ≠ free coloured particle, and the coboundary paper supplies local relaxation ⇏ global removal of holonomy. From the K = 7 programme: the closure Laplacian $L_7 = 7 \cdot 1_7 - J_7$ with spec $\{0, 7^6\}$, and the master-action spoke Hessian H_seq = 2α·1 + 2γ·M_{C_6}$^+$ with eigenvalues ∝ {4, 3, 1, 0, 1, 3} on six real spoke amplitudes (§8).

2.2 Claim levels

Level I — Derived structures. The colour-holonomy map E_3, the Fisher eigenvalue ⅛ (within P3), the scalar reduction $Z_3 = \chi\_C/8 - b^\dagger C^{-1} b$, the canonical identities $g_s = Z_3$^(−1/2) and $\beta_s = -\frac{1}{2} g_s^3 \cdot \partial Z_3/\partial \ln k$, the symbolic Λ_3, and the forced running of χ_C.

Level II — Universal quantum-field results. The one-loop coefficient b_0, asymptotic freedom, threshold architecture — inherited once the background-field quantum gate closes.

Level III — Conditional nonperturbative theorems. The positive triality gap, linear potential, string tension and pure-gauge area law, under the named coercivity, compactness, slicing, uniformity and RG conditions.

Level IV — Open calculations. χ_C(μ⋆), the census (b, C), B_τ, the flux lift, and all numerics.

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3. Named Premises

P1 — Colour bath carrier. The colour bath carrier is C_x ≃ $\mathbb{C}^3$; local bath transport supplies SU(3)_C.

P2 — Holonomy residual. For a small oriented loop with unit orientation bivector $n^{\mu\nu}$, the persistent residual is $R_{hol}(F) = \lim_{|\Sigma|\to 0} (-i/|\Sigma|)\cdot P_0(U_\square - 1_3)$, with P_0 the traceless projection.

P3 — Response-geometry convention. The invariant response of the saturated bath is measured by the Fubini–Study metric on pure-state rays, Haar-averaged over $\mathbb{CP}^2$. This convention is frozen before any evaluation of χ_C, in the same normalisation, and its derivation from the VERSF record-cost/statistical-distance identification is Debt D3.

P4 — Substrate susceptibility. A positive conversion coefficient $\chi_C(\mu\star)$ exists, comprising closure-cell density, loop-area normalisation, matching-scale convention and the absolute record-response susceptibility.

P5 — Auxiliary census. All same-order auxiliary modes decompose into SU(3) irreducibles; auxiliary adjoint copies (multiplicity $m \geq 0$) have positive Hessian $C > 0$ on the multiplicity space and curvature mixing $b \in \mathbb{C}^m$.

P6 — Background-field quantum closure. The blocked effective action admits a background-field or equivalent gauge-consistent scheme preserving the background Ward identity.

P7 — Canonical gauge-redundancy treatment. Gauge zero modes are quotiented; the Faddeev–Popov or equivalent determinant is included exactly once.

P8 — Canonical quark carrier. Each active flavour contributes a canonically normalised Dirac colour triplet above its threshold.

P9 — Local Wilsonian hierarchy. The blocked action admits a local derivative expansion where the running coupling is extracted.

P10 — Thresholds and mass inputs. One-loop sharp matching $\Lambda_{N_f-1} = m_f\cdot(\Lambda_{N_f}/m_f)^{(b_{N_f}/b_{N_f-1})}$ applies with $b_{N_f} = 11 - (2/3)N_f$, and m_t, m_b, m_c are supplied in one declared scheme pending the Yukawa gates.

P11 — Global pure-colour group. For pure-gauge confinement, the global colour group supports centre $Z(SU(3)) \cong \mathbb{Z}_3$.

P12 — Triality-sector separation. Absent dynamical fundamental matter, nonzero triality cannot be screened to zero by adjoint gluons.

P13 — Reduced transverse bath functional. A gauge-invariant static cross-sectional bath-energy functional exists on constrained slice states $X\perp = (A\perp, E\perp, E_z, \eta\perp; Q, \bar{Q})$ — carrying the chromoelectric field, the Gauss constraint and static-source boundary data — after all same-order auxiliary modes are minimised. The classical functional is a declared truncation of the quantum sector energy on $\mathcal{H}_\tau$ (§21.1, Debt D21).

P14 — Auxiliary stability. The auxiliary Hessian is positive and invertible on physical support after exact null modes are removed.

P15 — Lower semicontinuity and gauge coercivity. The reduced transverse functional is lower semicontinuous with finite-energy sublevels compact modulo gauge and centred translations.

P16 — Vacuum uniqueness. The zero-triality vacuum orbit is the unique zero of the vacuum-subtracted transverse functional.

P17 — Triality continuity. Finite-energy convergence cannot change a nonzero triality label into zero.

P18 — Longitudinal slicing inequality. For every finite-energy static-source state X,

$$H_{Q\bar{Q}}[X] - E_0 \geq \int_0^L \mathscr{E}_\perp^{\mathrm{red}}[X(z)]\, dz - c_{end},$$

with c_{end} independent of L; the inequality must survive longitudinal derivatives, inter-slice constraints, nonlocal terms, shared auxiliaries, negative cross terms and source renormalisation. Locality alone is not assumed to imply it.

P19 — Finite trial-tube existence. At least one admissible nonzero-triality tube has finite energy per unit length.

P20 — Transfer-matrix positivity. The Euclidean theory admits a positive transfer-matrix spectral representation for static-source correlators.

P21 — Dynamical-quark caveat. With fundamental quarks active, triality can be screened by pair creation; the pure-gauge superselection argument no longer applies asymptotically.

P22 — Coercive two-term truncation. At a nonperturbative coarse-graining scale the reduced slice energy of a flux tube is bounded below by a localisation term plus a positive areal bath cost, coefficient $B_\tau > 0$.

P23 — Primitive-flux sector. The quantity held fixed in the tube variation is the centre-projected primitive flux Φ_τ of $\Pi = Z_3 \cdot E$ through the separating surface — a sector/holonomy label in the sense of §19.2, whose quantisation and normalisation are supplied by the global lift (Debt D8) — with electric energy density $\Pi^2/2Z_3$.

P24 — Positive infrared stiffness. A positive effective $Z_{3,IR}$ exists at the declared coarse-graining scale.

P25 — Uniform gap. liminf over ($a \to 0$, $R_\perp \to \infty$) of $\varsigma_\tau(a, R_\perp) > 0$; equivalently $\liminf_{k\to 0} \varsigma_\tau(k) > 0$ for the exact functional.

P26 — Declared order of limits. $\sigma_\tau = \lim_{a\to 0} \lim_{R_\perp\to\infty} \lim_{L\to\infty} V_{\tau,a,R_\perp}(L) / L$.

P27 — RG consistency. In the exact theory $d\sigma\_\tau/d \ln k = 0$; residual k-dependence is truncation error.

P28 — Typed descent. The local colour descent is discharged by the derived map of §4. Confinement additionally requires a global group-level lift preserving the $\mathbb{Z}_3$ centre, the admissible line-operator spectrum and the triality classification (§31); no Lie-algebra-level object substitutes for it.

Part II — The Derived Colour Response

4. The Colour-Holonomy Residual Map

Local bath transport on the threefold carrier supplies SU(3)_C, and curvature is the failure of the colour frame to return unchanged around an infinitesimal loop. For a small oriented loop □ with area bivector $\Sigma^{\mu\nu}$,

$$U_\square = P \exp(i\cdot\oint_\square A) = 1_3 + i\cdot\Sigma^{\mu\nu}\cdot F^a{}_{\mu\nu}\cdot T_a + O(\Sigma^2).$$

Define the traceless projection $P_0(X) = X - \tfrac{1}{3}\cdot\mathrm{tr}(X)\cdot 1_3$ and the persistent holonomy residual

$$R_{hol}(F) = \lim_{|\Sigma|\to 0} (-i/|\Sigma|)\cdot P_0(U_\square - 1_3).$$

Because colour curvature is already traceless,

$$\mathbf{R_{hol}(F) = n^{\mu\nu}\cdot F^a{}_{\mu\nu}\cdot T_a,\quad n^{\mu\nu} = \Sigma^{\mu\nu}/|\Sigma|.}$$

Theorem 1 — Colour-Holonomy Residual Map Theorem

Suppressing the geometric two-form factor, the internal residual embedding is the canonical identification

$$\mathbf{E_3 : \mathfrak{su}(3) \to Herm_0(\mathbb{C}^3),\quad E_3(e_a) = T_a.}$$

With $\langle X, Y\rangle_{adj} = 2\cdot\mathrm{tr}(X\cdot Y)$,

$\langle E_3(e_a), E_3(e_b)\rangle_{adj} = 2\cdot\mathrm{tr}(T_a\cdot T_b) = \delta_{ab}$, hence $\mathbf{E_3^\dagger\cdot E_3 = 1_8}$,

and the map is exactly equivariant:

$$\mathbf{E_3(Ad_U X) = U\cdot E_3(X)\cdot U^\dagger \text{ for all } U \in SU(3).}$$

The full curvature map is $J_F = 1_{\Lambda^2T^*M} \otimes E_3$. ■

Reading

This is not an arbitrary eight-dimensional subspace of some larger carrier. It is the canonical isomorphism $\mathfrak{su}(3) \simeq \mathrm{Herm}_0(\mathbb{C}^3)$. Nothing about it is chosen, fitted, or completed by hand. The local colour descent debt of the earlier architecture is discharged at this point; what the earlier drafts sought as an embedding into a hypothetical fourteen-channel space turns out to be the identity map onto the traceless Hermitian responses of the bath the framework had already established.

---

5. The Invariant Fisher Response of the Threefold Bath

A saturated threefold bath has no preferred colour direction. The canonical invariant distribution on its pure-state rays is the Haar-induced measure on $\mathbb{CP}^2$. For a normalised colour state $|\psi\rangle$, an infinitesimal colour transformation $|\psi\rangle \mapsto e^{i\varepsilon^a T_a}|\psi\rangle$ has Fubini–Study metric equal to the generator covariance,

$$g_{ab}(\psi) = \tfrac{1}{2}\cdot\langle\psi|\{T_a, T_b\}|\psi\rangle - \langle\psi|T_a|\psi\rangle\cdot\langle\psi|T_b|\psi\rangle.$$

Theorem 2 — Invariant Fisher Response and Normalisation-Split Theorem

For Haar-distributed pure states of $\mathbb{C}^3$,

$$\int |\psi\rangle\langle\psi|\, d\mu = 1_3/3, \quad \int (|\psi\rangle\langle\psi|)^{\otimes 2}\, d\mu = (1 + S)/12,$$

with S the swap operator. Hence

$$\int \tfrac{1}{2}\cdot\langle\{T_a, T_b\}\rangle_\psi\, d\mu = \mathrm{tr}(T_a T_b)/3 = \delta_{ab}/6, \quad \int \langle T_a\rangle_\psi\cdot\langle T_b\rangle_\psi\, d\mu = \mathrm{tr}(T_a T_b)/12 = \delta_{ab}/24,$$

and

$$\mathbf{\bar{g}_{ab}^{\,fs} = \delta_{ab}/6 - \delta_{ab}/24 = \tfrac{1}{8}\cdot\delta_{ab}.}$$

The invariant threefold colour bath has one isotropic geometric response eigenvalue, $\lambda_{shape} = \tfrac{1}{8}$. (Moment algebra in Appendix A.) ■

Content grading

Isotropy requires no Haar moments: it follows from Schur's lemma and invariance alone. The moment calculation fixes the *scalar*, and by §1.7 that scalar is the frozen normalisation split between shape and susceptibility — it acquires physical meaning only jointly with the §9 protocol, which must evaluate χ_C in the same convention. This theorem's derivational content is therefore: isotropy, plus a pinned normalisation making χ_C a well-defined measurable target. The eigenvalue ⅛ must never be quoted as a stand-alone physical number.

Corollary 5.1 — d = 3 Uniqueness of the Coincidence

For bath dimension d the Haar-averaged generator covariance is $\delta_{ab}/(2(d+1))$, while the gauge algebra has $d^2 - 1$ directions. The equality $2(d+1) = d^2 - 1$, i.e. $d^2 - 2d - 3 = 0$, holds only at $d = 3$. The threefold bath is the unique dimension where the shape eigenvalue's denominator equals the number of gauge directions.

Firewall reminder

By §1.6 the two eights remain distinct typed objects; by §1.7 the scalar ⅛ is convention-dependent while the isotropy is not, and only $\lambda_{shape}\cdot\chi_C$ is physical.

---

6. The Substrate Susceptibility and the Bare Colour Stiffness

The Fubini–Study calculation fixes the shape and relative normalisation of the colour response. It does not fix the conversion from dimensionless projective distance to four-dimensional action density. That conversion is the substrate susceptibility

$\chi_C(\mu\star)$,

comprising the density of microscopic closure cells, loop-area normalisation, the matching-scale convention, and the absolute record-response susceptibility (P4). The bare quadratic colour closure cost is

$C_C^{(2)} = \tfrac{1}{2}\cdot F^a\cdot(\chi_C/8 \cdot \delta_{ab})\cdot F^b$,

so

$H_{FF}^{(3)} = (\chi_C/8)\cdot 1_8$, $Z_3^{bare} = \chi_C/8$.

Factor convention

The ½ in $C\_C^{(2)}$ sums over independent bivector components ($\mu < \nu$); on the full antisymmetric sum it equals the $\frac{1}{4}\cdot Z_3\cdot F_{\mu\nu} F^{\mu\nu}$ normalisation of §10. This factor-of-two convention is part of the frozen normalisation (P3–P4) and must be carried unchanged into the §9 protocol and the D1 evaluation: at the sensitivity of Proposition 17.1, a silent factor-of-two mismatch between §6, §9 and D1 would move Λ_3 by tens of percent and be invisible until it wrecked the blind prediction.

---

7. Exact Auxiliary Reduction: The Scalar Colour-Response Theorem

GAC-1 requires every same-order auxiliary mode to be minimised out: $K_3 = H\_FF - H\_F\eta\cdot H\_\eta\eta^{-1}\cdot H\_\eta F$. Gauge covariance makes this reduction exactly solvable in form.

Theorem 3 — Scalar Colour-Response Theorem

Assume P1–P5. Decompose the auxiliary carrier into SU(3) irreducibles, $V\_\eta = \bigoplus\_R\ M\_R \otimes R$. Since curvature transforms in the adjoint **8**, Schur's lemma gives $H\_\eta F = 0$ for every auxiliary representation $R \ncong \mathbf{8}$: only auxiliary adjoint copies can linearly screen the quadratic colour residual. Writing, on the adjoint-isotypic component,

$H\_F\eta = b^\dagger \otimes 1_8$, $H\_\eta\eta|_8 = C \otimes 1_8$, $C > 0$, $b \in \mathbb{C}^m$,

the complete reduction is

$\mathbf{K_3 = [\ \chi\_C / 8 - b^\dagger\cdot C^{-1}\cdot b\ ]\cdot 1_8,}$

hence

$\mathbf{Z_3(\mu\star) = \chi\_C / 8 - b^\dagger\cdot C^{-1}\cdot b,\ g_s(\mu\star) = (\ \chi\_C / 8 - b^\dagger\cdot C^{-1}\cdot b\ )^{(-1/2)}.}$

No 8 × 8 diagonalisation survives; the colour response is a calculation among scalars on the multiplicity space. If the same-order auxiliary census contains no adjoint copies, $b = 0$ and $Z_3 = \chi\_C / 8$. ■

Positivity condition

Physical consistency requires

$\mathbf{\chi\_C / 8 > b^\dagger\cdot C^{-1}\cdot b.}$

A computed violation gives a null or ghost gluon mode (falsifier F5).

Proposition 7.1 — Spoke-Sector Decoupling

The presently derived $K = 7$ master-action Hessian, $H_{seq} = 2\alpha\cdot 1 + 2\gamma\cdot M_{C_6}^{+}$ with $(M_{C_6}^{+}\lambda)_j = \lambda_{j-1} + 2\lambda_j + \lambda_{j+1}$ and eigenvalues $\propto \{4, 3, 1, 0, 1, 3\}$, acts on six real spoke amplitudes carrying D_6 spoke symmetry with a deliberate zero mode (the persistent alternating transport direction). It cannot serve as the colour kinetic Hessian: it acts on $\mathbb{R}^6$, not the adjoint $\mathbb{R}^8$; it carries no SU(3) action; and no equivariant map from the adjoint exists,

$$\mathrm{Hom}_{SU(3)}(\mathbf{8}, V_{seq}) = 0$$

for the declared sequential carrier — trivially, since eight dimensions cannot embed equivariantly in six, and a fortiori for a carrier with no SU(3) action, where gauge invariance itself forbids a linear coupling of any colour singlet to F^a. The spoke Hessian may influence higher-order closure physics or global holonomy; it does not renormalise the leading local F^2 coefficient. ■

Consequence for the trace surrogate

Since $K_3 = Z_3\cdot 1_8$, the earlier trace definition evaluates identically: $(1/56)\cdot \mathrm{Tr}_8(E_3^\dagger\cdot H^{red}\cdot E_3) = 8Z_3/56 = Z_3/7$. The symbol $\kappa_C \equiv Z_3/7$ survives only as notation and carries no independent content (§8).

---

8. The Retired Fourteen-Channel Trial and the Group-Order Firewall

An earlier candidate calculation modelled the response carrier as fourteen-dimensional and completed it with two copies of the $K = 7$ closure Laplacian,

$H_{14}^{tr} = \kappa_C\cdot(L_7 \oplus L_7)$, $L_7 = 7\cdot 1_7 - J_7$, $\mathrm{spec}(L_7) = \{0, 7, 7, 7, 7, 7, 7\}$.

Proposition 8.1 — Forced Embedding of the Trial (retired)

For the trial completion, the null space is the two-dimensional span of the uniform modes and cannot contain an eight-dimensional adjoint; every admissible equivariant rank-eight isometric embedding therefore lands in the gapped, completely isotropic twelve-dimensional subspace, and $K_3 = 7\cdot\kappa_C\cdot 1_8$ is forced with no residual embedding freedom. ■

Proposition 8.2 — Colour Blindness of the Trial (retired)

On its gapped subspace the trial Hessian is proportional to the identity, so every subgroup of U(12) commutes with it: the trial constrains nothing about which SU(3) action is realised, and the eigenvalue seven is a property of the Laplacian gap, not of colour structure. ■

8.3 Why the trial is retired

1. **The carrier was never derived.** Fourteen enters the programme as $|D_7|$ — a count of transport-group elements. A group order is not a vector-space dimension, and the regular representation of D_7 carries no canonical SU(3) adjoint action (§1.5). The Gauge Action paper left E_3 open; the strongest K = 7 master-action calculation produces the six-dimensional spoke Hessian of Proposition 7.1, not a 14 × 14 colour Hessian.
2. **The derived map goes elsewhere.** Theorem 1 lands the residual on $\mathrm{Herm}_0(\mathbb{C}^3)$ — an eight-dimensional carrier that VERSF has actually established — canonically, isometrically, equivariantly.
3. **The trial's robustness was vacuity.** Proposition 8.2 shows the forced value seven was robust precisely because the trial was colour-blind.

8.4 What the retirement preserves

The methodological content survives: the Schur multiplicity bound and forced-embedding style of argument (now realised properly in Theorem 3), the colour-blindness lesson (now firewall-grade: an isotropic Hessian constrains no representation), and the trace surrogate $\kappa\_C = Z_3/7$ as pure notation. Earlier formulas quoted in κ_C translate exactly under $Z_3 = 7\kappa\_C$; the earlier forced-running condition on κ_C becomes a tautology under the definitional reading and is re-aimed at χ_C in §17, where it regains content.

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9. The Executable Plaquette Definition of Z_3

The scalar reduction admits a directly executable definition, independent of any decomposition bookkeeping. Impose a small plaquette holonomy on the substrate,

U_□(ε) = e^{i·ε·T_a},

minimise the VERSF master action over every auxiliary field at fixed holonomy, and compute

Z₃ = lim_{ε→0} 2·[min_η S_VERSF(ε, η) − S_VERSF(0, 0)] ⁄ ε²

after the declared cell-to-continuum normalisation (P3–P4 frozen in advance). The ε^2-normalisation inherits the §6 bivector convention ($\mu < \nu$ over independent components); the published evaluation must state it.

Isotropy audit

Adjoint isotropy (Theorem 3) means the calculation need be performed for only one generator — and simultaneously that **all eight generators must return the same value**. Any spread beyond declared numerical error falsifies the reduction or the invariance premises (F6). The audit is mandatory, not optional.

Status

This protocol, together with the auxiliary adjoint census (b, C), is the complete remaining running-side target: everything downstream of $Z_3(\mu\star)$ in Part III is fixed by universal quantum field theory plus declared inputs.

Part III — Running

10. The Scale-Dependent Colour Bath Action

Let k be a Wilsonian blocking scale. The colour-sector background-field effective action has the form

$$\Gamma_k[\bar{A}, q, \bar{q}] = \int d^4x \, [\, \tfrac{1}{4} \cdot Z_3(k) \cdot \bar{F}^a{}_{\mu\nu} \bar{F}^{a\mu\nu} + \Sigma_f \, \bar{q}_f \, (\, i \cdot \gamma^\mu \bar{D}_\mu - m_f(k) \,) \, q_f + \mathscr{L}_{k,\geq 6} \,],$$

with higher operators such as $\mathrm{tr}[(D_\rho F_{\mu\nu})(D^\rho F^{\mu\nu})]$ and $\mathrm{tr}(F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu)$, and nonlocal terms where the derivative expansion fails. $Z_3(k)$ is the scale-dependent colour stiffness; canonical normalisation gives $g_s(k) = Z_3(k)^{-1/2}$ in the declared blocked scheme, with boundary value $Z_3(\mu\star)$ supplied by Theorem 3.

---

11. The Canonical Beta-Function Identity

Theorem 4 — Colour Response–Beta Identity

Let $g_s(k) = Z_3(k)^{-1/2}$. Then

$$\boldsymbol{\beta_s(g_s) \equiv k \cdot dg_s/dk = -\tfrac{1}{2} \cdot g_s^3 \cdot dZ_3/d \ln k.}$$

Proof

Differentiate $g_s = Z_3^{-1/2}$ with respect to $t = \ln k$: $dg_s/dt = -\tfrac{1}{2} \cdot Z_3^{-3/2} \cdot dZ_3/dt = -\tfrac{1}{2} \cdot g_s^3 \cdot dZ_3/dt$. ■

Interpretation

If $Z_3(k)$ increases toward higher scale, $g_s(k)$ decreases. Non-Abelian antiscreening is, in VERSF language, an increase in the ultraviolet stiffness of the canonically reduced colour connection.

---

12. Universal One-Loop Colour Running

In background-field gauge, around $\bar{A}$, the one-loop effective action is schematically $\Gamma_k^{(1)} = \frac{1}{2}\cdot\log\det\Delta_1 - \log\det\Delta_0 - \Sigma_f \log\det\Delta_{1/2,f}$, with gauge, ghost and Dirac operators; gauge and ghost terms must be combined before physical interpretation (§1.9).

Theorem 5 — Universal One-Loop Colour-Flow Theorem

Under P1–P9,

$$\mathbf{dZ_3/d\ln k = (1/8\pi^2)\cdot[\,(11/3)\cdot C_A - (4/3)\cdot\Sigma_f\, T(R_f)\cdot\vartheta_f(k)\,] + O(g_s^2),}$$

with threshold functions $\vartheta_f(k) \to 1$ for $k \gg m_f$ and $\to 0$ for $k \ll m_f$. Hence

$$\boldsymbol{\beta_s(g_s) = -(g_s^3/16\pi^2)\cdot[\,(11/3)\cdot C_A - (4/3)\cdot\Sigma_f\, T(R_f)\cdot\vartheta_f(k)\,] + O(g_s^5).}$$

Proof

The background-field heat-kernel coefficient multiplying $\frac{1}{4}\int\bar{F}^2$ in the combined determinants: gauge-plus-ghost contributes $+(11/3)\cdot C_A$, each active Dirac fermion $-(4/3)\cdot T(R_f)$; substitute into Theorem 4. (Appendix B.) ■

Epistemic status

The coefficients are universal quantum-field-theory results, not new mathematics. The VERSF content is that the connection and representation content are derived rather than inserted, the boundary value $Z_3(\mu\star)$ is a computed substrate response, and the quantum calculation transports it across scale.

---

13. Thresholds and Active-Quark Screening

For fundamental SU(3) quarks $T_F = \frac{1}{2}$; with $N_f(k)$ active flavours, $dZ_3/d\ln k = (1/8\pi^2)\cdot(11 - (2/3)\cdot N_f(k)) + \cdots$, with $N_f(k) = \Sigma_f\, \Theta(k - m_f)$ in the sharp approximation and smooth decoupling in a physical Wilsonian scheme.

Theorem 6 — Threshold-Matched Colour Flow

Across a quark threshold, $g_s^{(N_f-1)}(m_f) = g_s^{(N_f)}(m_f) + \Delta_f^{match}$ with scheme- and order-dependent finite terms, and $\Lambda_{N_f-1} = \mathcal{T}_f(\Lambda_{N_f}, m_f)$. No single Λ_3 may be evolved through all thresholds without changing the active theory. The Yukawa gates may supply threshold values only after independent derivation and scheme mapping (P10, §1.14). ■

---

14. Asymptotic Freedom

Theorem 7 — VERSF Colour Asymptotic-Freedom Theorem

For SU(3), $C_A = 3$ and $b_0 = 11 - (2/3)\cdot N_f$. The inherited Standard Model census gives $0 \le N_f \le 6$, hence

$b_0 \ge 11 - 4 = 7 > 0$,

and at sufficiently weak coupling

$\boldsymbol{\beta_s(g_s) < 0}$.

The coupling decreases toward the ultraviolet and increases toward the infrared. The antiscreening sign is fixed by the adjoint self-interaction, gauge redundancy and the SU(3) invariants — not fitted. ■

---

15. Dimensional Transmutation

At one loop within fixed N_f, $1/g_s^2(k) = 1/g_s^2(\mu) + (b_0/8\pi^2)\cdot\ln(k/\mu)$. Define the strictly one-loop N_f-flavour scale

$\boldsymbol{\Lambda_{N_f} = \mu\cdot\exp[\,-8\pi^2\cdot Z_3(\mu)/b_0\,]}$ (using $Z_3 = 1/g_s^2$),

so that $\alpha_s(k) = 4\pi/[\,b_0\cdot\ln(k^2/\Lambda_{N_f}^2)\,]$. This is the object matched across thresholds in §16. Beyond one loop the scheme-invariant scale carries the power-law prefactor

$\Lambda = \mu\cdot(\,b_0\cdot g_s^2(\mu)/16\pi^2\,)^{(-b_1/2b_0^2)}\cdot\exp[\,-8\pi^2/(b_0\cdot g_s^2(\mu))\,]\cdot[\,1 + O(g_s^2)\,]$,

with $b_1 = 102 - (38/3)\cdot N_f$ for SU(3) fundamental flavours; the prefactor is a genuine power and cannot be absorbed into an analytic correction.

Theorem 8 — Gauge Scale-Generation Theorem

Given a derived $Z_3(\mu\star)$, a declared scheme, a declared matter census and threshold matching, the strong scale is fixed and is not an additional parameter. The formal one-loop divergence near $k = \Lambda$ marks the breakdown of weak coupling; it proves neither a physical pole nor confinement. If the pure sector has no other dimensionful parameter, any nonzero string tension has the form $\sigma = c_\sigma \cdot \Lambda_3^2$, with c_σ a dimensionless nonperturbative invariant the renormalisation group does not determine. ■

---

16. The Symbolic Three-Flavour Strong Scale

Above the top threshold $b_0 = 7$ and

$1/g_s^2(k) = Z_3(\mu\star) + (7/8\pi^2)\cdot \ln(k/\mu\star)$, $\boldsymbol{\Lambda_6 = \mu\star \cdot \exp[\, -8\pi^2 \cdot Z_3(\mu\star)/7 \,]}$.

Theorem 9 — Three-Flavour Symbolic Strong Scale

Under P10, sharp one-loop matching with $b_6 = 7$, $b_5 = 23/3$, $b_4 = 25/3$, $b_3 = 9$ gives the cascade

$\Lambda_5 = m_t^{(2/23)} \cdot \Lambda_6^{(21/23)}$, $\Lambda_4 = (m_t \cdot m_b)^{(2/25)} \cdot \Lambda_6^{(21/25)}$, $\boldsymbol{\Lambda_3 = (m_t \cdot m_b \cdot m_c)^{(2/27)} \cdot \Lambda_6^{(7/9)}}$,

hence

$$\boldsymbol{\Lambda_3 = \mu\star^{(7/9)} \cdot (m_t \cdot m_b \cdot m_c)^{(2/27)} \cdot \exp[\, -8\pi^2 \cdot Z_3(\mu\star)/9 \,]},$$

and, in the screening-free case $Z_3 = \chi_C/8$,

$$\boldsymbol{\Lambda_3 = \mu\star^{(7/9)} \cdot (m_t \cdot m_b \cdot m_c)^{(2/27)} \cdot \exp[\, -\pi^2 \cdot \chi_C/9 \,]}.$$

Mass dimensions balance: $7/9 + 3\cdot(2/27) = 1$. (Algebra in Appendix C.) ■

Status

An end-to-end symbolic calculation from substrate susceptibility to the low-energy three-flavour strong scale, with no measured α_s or Λ_{QCD} inserted — conditional on one-loop accuracy with sharp thresholds and on declared mass inputs.

---

17. The Forced-Running Consistency Condition and the Precision Budget

Theorem 10 — Forced Running of the Susceptibility

Λ_6 must be independent of the arbitrary matching scale $\mu\star$, and Theorem 5 gives $dZ_3/d \ln k = 7/8\pi^2 + O(g_s^2)$ above the top threshold. If the derived reduction $Z_3 = \chi\_C/8 - b\dagger C^{-1}b$ holds along the flow in the frozen convention, then the substrate blocking calculation is required to produce

$$\mathbf{d(\chi\_C / 8 - b\dagger \cdot C^{-1} \cdot b) / d \ln k = 7 / 8\pi^2 + O(g_s^2),}$$

and in the screening-free case

$$\mathbf{d\chi\_C / d \ln k = 7 / \pi^2 + O(g_s^2) \approx 0.7093}$$

above the top threshold, with slopes rescaled by b_{N_f}⁄7 between lower thresholds. ■

Why this matters

This is a parameter-free, scheme-declared falsifier that exists before any numerical χ_C is computed. A master-action evaluation yielding a scale-independent χ_C, or one running at a different rate, kills the identification immediately — no comparison with experiment required. Conversely, recovering the rate from substrate blocking alone would be significant independent evidence that $\mathrm{Herm}_0(\mathbb{C}^3)$ with the Haar–FS geometry is the colour response structure. (The earlier version of this condition, phrased for the retired κ_C, is tautological under $\kappa\_C \equiv Z_3/7$ and is superseded by this statement.)

Audit-independence requirement

Per §1.15, the audit is confirmatory only if χ_C(k) is evaluated at two scales from the master action's own blocking rules, without importing the continuum one-loop determinant as the computational engine. A circular implementation is void.

Proposition 17.1 — Precision Budget

From Theorem 9, $d \ln \Lambda_3 / dZ_3 = -8\pi^2/9 \approx -8.77$, hence $d \ln \Lambda_3 / d\chi\_C = -\pi^2/9 \approx -1.097$ in the screening-free case. Therefore:

- $\delta Z_3 = 0.1$ changes Λ_3 by a factor ≈ 2.4;
- a ±10% target on Λ_3 requires $|\delta Z_3| \lesssim 0.011$, i.e. $|\delta\chi\_C| \lesssim 0.087$;
- a ±1% target on Λ_3 requires $|\delta\chi\_C| \lesssim 0.009$.

The strong scale is exponentially sensitive to the susceptibility — ordinary dimensional transmutation, declared in advance. Any claimed evaluation of χ_C must be published with an error budget commensurate with this sensitivity, before comparison with QCD. ∎

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18. Why Running Is Not Confinement

Asymptotic freedom establishes $g_s(k) \to 0$ as $k \to \infty$. It does not establish $V(L) \sim \sigma \cdot L$ or $\langle W(C) \rangle \sim \exp[-\sigma \cdot A(C)]$. A coupling may become strong without producing an area law; the infrared phase depends on the global gauge group, the line-operator spectrum, centre charge, topological sectors, the nonperturbative effective action, screening matter and vacuum structure. The exact firewall stands:

asymptotic freedom ⇏ confinement.

Part III supplies the scale at which the problem becomes nonperturbative. Part IV solves the logically separate confinement problem conditionally.

Part IV — Confinement

19. Global SU(3), Centre Charge and Triality

The centre of SU(3) is $\mathbb{Z}_3 = \{ 1, e^{(2\pi i/3)}, e^{(4\pi i/3)} \}$. For a representation R, the triality $\tau(R) \in \{0, 1, 2\}$ is defined by the centre action $z \cdot |R\rangle = z^{(\tau(R))} \cdot |R\rangle$; $\tau(\mathbf{3}) = 1$, $\tau(\bar{\mathbf{3}}) = 2$, $\tau(\mathbf{8}) = 0$. Gluons are adjoint and carry zero triality.

Proposition 19.1 — Pure-Gauge Triality Preservation

In pure SU(3) Yang–Mills theory, emission or absorption of any number of gluons cannot change the triality of an external source: trialities add modulo three and every gluon carries zero. ∎

A nonzero-triality source therefore cannot be screened to the vacuum by pure gluonic bath transport. This supplies sector separation; it does not yet prove a positive energy cost.

19.2 Where triality lives

Triality is not a gauge-invariant local label of a classical connection $A\perp$. It is carried by external Wilson lines, centre-electric-flux sectors, representations of the one-form centre symmetry, twisted boundary data, and physical Hilbert-space sectors containing static sources. On a transverse slice Σ the sector label is defined operatorially: let $U_z(\Sigma)$ implement the global centre transformation $z \in \mathbb{Z}_3$ on the slice Hilbert space; a state has triality τ when

$$U_z(\Sigma)\cdot|\Psi_\tau\rangle = z^\tau\cdot|\Psi_\tau\rangle,\ z \in \mathbb{Z}_3.$$

All confinement statements below are made in these typed sectors $\mathscr{H}_\tau(\Sigma; Q, \bar{Q})$. Assigning a line-operator quantum number to an ordinary local connection configuration is a typing error (falsifier F17).

---

20. Static-Source Sectors and the Transfer Matrix

Consider static sources in representation R separated by distance L, joined by a Wilson line into a gauge-invariant source operator. Transfer-matrix positivity gives

$$Z_R(L, T) = \Sigma_n |c_n^{(R)}(L)|^2\cdot\exp\{ -T\cdot[E_n^{(R)}(L) - E_0] \},$$

and with $V_R(L) = E_0^{(R)}(L) - E_0$ and nonzero ground-state overlap,

$$\mathbf{V_R(L) = -\lim_{T\to\infty} (1/T)\cdot\log Z_R(L, T),}$$

with $Z_R(L, T) \propto \langle W_R(L, T)\rangle$ up to source self-energy and normalisation.

---

21. The Transverse Bath-Maintenance Functional

Take an axis connecting the static sources; at longitudinal coordinate z let Σ_z be a transverse surface separating them. The colour Gauss law assigns to Σ_z the triality of the enclosed source.

A static confinement flux tube is chromoelectric, so a transverse energy functional depending only on the magnetic connection data is incomplete. The slice variable is the constrained slice state

$$X\perp = (A\perp, E\perp, E_z, \eta\perp; Q, \bar{Q}),$$

comprising the transverse connection, the chromoelectric fields subject to the non-Abelian Gauss constraint, the same-order bath, record, closure and completion modes, and the static-source boundary data.

21.1 The two typings, kept separate

§19.2 and falsifier F17 forbid assigning a Hilbert-sector quantum number to configuration data, so the typing must be stated exactly. The fundamental object is the **quantum sector energy** on the gauge-invariant transfer-matrix Hilbert sector $\mathscr{H}_\tau(\Sigma_z; Q, \bar{Q})$ of §19.2:

$\varsigma_\tau^{qu}(k) = \inf$ over unit $\Psi \in \mathscr{H}_\tau$ of $\langle\Psi|\, \hat{H}_{\perp,k}^{red}\, |\Psi\rangle - E_{\perp,vac}$.

The classical constrained functional used below is a **declared truncation** of that problem. Its sector label is the classical centre-projected electric-flux class

$\mathfrak{c}_\tau = \{\, X_\perp$ carrying centre-projected primitive flux of class $\tau \,\}$,

the configuration class that seeds coherent states in $\mathscr{H}_\tau$. The classical infimum omits zero-point and entropic contributions that can shift the gap; the classical-to-quantum gap is Debt D21, and every use of the classical functional in Theorems 12–15 is graded accordingly. A classical infimum over $\mathfrak{c}_\tau$ is *not* the quantum sector energy over $\mathscr{H}_\tau$, and the paper claims no equivalence.

21.2 The classical truncated functional

Define the vacuum-subtracted transverse energy $\mathscr{E}_{\perp,k}[X_\perp] = \int_{\Sigma_z} d^2x_\perp\, [\, \mathscr{H}_k(X_\perp) - \mathscr{H}_{k,vac}\,]$ and the reduced functional at fixed flux class

$\boldsymbol{\mathscr{E}_{\perp,k}^{red}[X_\perp] = \inf_{\eta_\perp} \mathscr{E}_{\perp,k}[X_\perp, \eta_\perp]}$,

with class infimum

$\boldsymbol{\varsigma_\tau(k) = \inf}$ **over** $\boldsymbol{X_\perp \in \mathfrak{c}_\tau}$ **of** $\boldsymbol{\mathscr{E}_{\perp,k}^{red}[X_\perp]}$.

The confinement question is $\varsigma_\tau(k) > 0$ or $= 0$ — first in the truncation, then, via D21, at the quantum level.

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22. Auxiliary Reduction Around a Triality Background

Theorem 11 — Triality-Sector Auxiliary Reduction

Let $(X_{\perp,\tau}, \eta_{\perp,\tau})$ be a stationary constrained slice state in sector τ with quadratic Hessian blocks H_{XX}, $H_{X\eta}$, $H_{\eta X}$, $H_{\eta\eta}$. If $H_{\eta\eta}^{(\tau)} > 0$ on physical auxiliary support, minimising the auxiliary fluctuation gives $\delta\eta^\star = -(H_{\eta\eta})^{-1}\cdot H_{\eta X}\cdot\delta X$ and the reduced transverse fluctuation operator

$$\mathcal{K}_{\perp,\tau} = H_{XX}^{(\tau)} - H_{X\eta}^{(\tau)}\cdot(H_{\eta\eta}^{(\tau)})^{-1}\cdot H_{\eta X}^{(\tau)},$$

with $\mathcal{K}_{\perp,\tau} \geq 0$ if the full constrained Hessian is positive. ■

The string tension is not the cost of an unrelaxed colour field; it is the energy remaining after all admissible local bath modes have attempted to screen, spread or reorganise the triality flux. Hessian positivity proves local stability only; the global sector theorem follows.

---

23. The Positive Triality-Gap Theorem

Theorem 12 — Bath-Coercive Triality-Gap Theorem

Assume P11–P17. Let $\mathcal{E}_\perp^{\mathrm{red}} : \mathfrak{c}_\tau \to [0, \infty)$ be the vacuum-subtracted reduced transverse energy on the classical centre-projected flux class (the declared truncation of §21.1). Suppose:

1. $\mathcal{E}_\perp^{\mathrm{red}}$ is lower semicontinuous;
2. every finite-energy sublevel is compact modulo gauge after centring removes translations;
3. the only zero-energy orbit is the vacuum in $\mathfrak{c}_0$;
4. $\mathfrak{c}_\tau$, $\tau \neq 0$, is closed under finite-energy convergence;
5. $\mathfrak{c}_\tau \cap \mathfrak{c}_0 = \emptyset$.

Then

$$\varsigma_\tau = \inf_{X \in \mathfrak{c}_\tau} \mathcal{E}_\perp^{\mathrm{red}}[X] > 0.$$

Proof

If $\varsigma_\tau = 0$, choose $X_n \in \mathfrak{c}_\tau$ with $\mathcal{E}_\perp^{\mathrm{red}}[X_n] \to 0$. Coercivity and compactness give a gauge-reduced convergent subsequence $X_{n_j} \to X^\star$; closedness gives $X^\star \in \mathfrak{c}_\tau$; lower semicontinuity gives $\mathcal{E}_\perp^{\mathrm{red}}[X^\star] = 0$; vacuum uniqueness places $X^\star$ in $\mathfrak{c}_0$, contradicting sector separation. (Appendix D.) ■

Typing grade

This is classical variational calculus on $\mathfrak{c}_\tau$. Promotion to the quantum sector energy on $\mathscr{H}_\tau$ — restating conditions 1–5 as statements about weak convergence and superselection in $\mathscr{H}_\tau$, and bounding the zero-point and entropic shift — is Debt D21. The theorem's conclusion is claimed for the truncation, not yet for ς_τ^{qu}.

Central interpretation

The positive string-tension seed is the conjunction **sector separation + coercive bath energy + vacuum uniqueness** — not merely strong coupling. Deriving the coercivity and compactness from the microscopic colour-bath functional is the principal open confinement debt.

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24. The Corrected Coercive Flux Functional

The abstract sufficient bound $\mathcal{E}_{\perp,\tau}^{red}(R) \geq A_\tau/R^2 + B\cdot R^2$ leaves A_τ uncommitted. This section instantiates it and corrects the natural first attempt.

24.1 The typing error

Placing the stiffness in the numerator of the localisation term, $\mathcal{E} \sim Z_{3,IR}\cdot\Phi^2/(2\pi R^2) + \pi B R^2$, predicts $\varsigma_\tau \propto \sqrt{Z_{3,IR}}$ — a stiffer, more weakly coupled substrate confining *more* strongly. That is backwards; the error is one of typing, not algebra: it holds the canonical flux fixed while writing the energy in primitive normalisation, mixing conventions across the Legendre transform.

24.2 The canonical-momentum resolution

The flux held fixed belongs to the primitive connection (P23). With action $\frac{1}{4}\cdot Z_3\cdot F^2$ in primitive fields, the canonical momentum is $\Pi = Z_3\cdot E$ and $\mathcal{H}_E = \Pi^2/2Z_3$. The Gauss law constrains the *divergence* of Π, tying its flux through a separating surface to the enclosed source; the *quantisation* of that centre-electric flux is a sector statement — compactness of the gauge group and centre projection in the sense of §19.2 — whose normalisation is exactly Debt D8. Holding that centre-projected primitive flux fixed puts the stiffness in the denominator.

Theorem 13 — Corrected Coercive Tube Gap

Assume P22–P24. For a tube of radius R carrying fixed primitive flux $\Phi_\tau \neq 0$,

$$\boldsymbol{\mathcal{E}_{\perp,\tau}^{red}(R) = \Phi_\tau^2/(2\pi\cdot Z_{3,IR}\cdot R^2) + \pi\cdot B_\tau\cdot R^2,}$$

with minimum

$$\boldsymbol{R_\star^4 = \Phi_\tau^2/(2\pi^2\cdot Z_{3,IR}\cdot B_\tau),\quad \varsigma_\tau = |\Phi_\tau|\cdot\sqrt{2\cdot B_\tau/Z_{3,IR}} = g_{s,IR}\cdot|\Phi_\tau|\cdot\sqrt{2\cdot B_\tau}.}$$

Hence $B_\tau > 0$, $Z_{3,IR} > 0$, $\Phi_\tau \neq 0 \Longrightarrow \varsigma_\tau > 0$, and with the slicing theorem $V_\tau(L) \geq g_{s,IR}\cdot|\Phi_\tau|\cdot\sqrt{2B_\tau}\cdot L - c_{end}$. (Appendix H.) ■

The structural bonus

The corrected form welds the two halves of the paper together: the running theorem delivers the infrared growth of g_s, and that growth multiplies the confinement gap. The infrared side of asymptotic freedom is not merely compatible with confinement in this truncation — it is a factor in it.

Corollary 24.1 — Truncation RG-Compensation Condition

Within the truncation, $\varsigma_\tau = |\Phi_\tau|\cdot\sqrt{2\cdot B_\tau / Z_{3,IR}}$ depends on the coarse-graining scale through $Z_3(k)$ and $B_\tau(k)$, while P27 demands blocking-scale independence of the physical tension, and Φ_τ is a quantised, scale-independent sector label. Consistency therefore forces

$$\mathbf{d \ln B_\tau / d \ln k = d \ln Z_3 / d \ln k,}$$

i.e. the ratio B_τ / Z_3 is RG-invariant within the truncation. This is the confinement-side analogue of Theorem 10: a parameter-free consistency condition on the future derivation of B_τ (Debt D19), available before any numerics. A derived B_τ that fails to run in lockstep with the stiffness breaks blocking-scale independence of the truncated tension (falsifier F33, lockstep clause). ■

Remaining substrate targets

Within the truncation the confinement calculation reduces to $Z_{3,IR}$ (or $g_{s,IR}$), the bath areal coefficient B_τ, and the global centre-sensitive lift fixing the meaning and normalisation of Φ_τ (§31). A gauge-invariant bath-stiffness channel such as $(\lambda_D/M^2)\cdot \mathrm{tr}[(D_\rho F_{\mu\nu})^2]$, $\lambda_D \geq 0$, or a positive action-density-gradient channel, can supply the coercive wall term; no such coefficient may be declared — each must arise from Wilsonian bath blocking (Appendix I).

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25. Longitudinal Persistence, String Tension and the Physical Limits

Theorem 14 — Longitudinal Triality-Persistence Theorem

Under P11–P19, $\mathbf{V_\tau(L) \geq \varsigma_\tau \cdot L - c_{end}}$ with c_{end} independent of L.

Proof

Every transverse surface between the sources encloses one nonzero-triality source, so its slice state lies in the class ɕ_τ. By the slicing inequality P18, the static energy dominates the integrated reduced slice energies up to a finite end correction; by Theorem 12 each slice contributes at least ς_τ; integrate. The nontrivial content lives entirely in P18, which must survive longitudinal derivatives, inter-slice constraints, nonlocal terms, shared auxiliaries, negative cross terms and source renormalisation; its derivation is Debt D14. ■

Proposition 25.1 — Finite Upper Linear Bound

One admissible extended trial tube with energy per unit length $\bar{\varsigma}_\tau < \infty$ gives $V_\tau(L) \leq \bar{\varsigma}_\tau \cdot L + c_{trial}$.

Theorem 15 — String-Tension Existence

Given the concatenation bound $V_\tau(L_1 + L_2) \leq V_\tau(L_1) + V_\tau(L_2) + c_0$, the limit

$$\boldsymbol{\sigma_\tau = \lim_{L\to\infty} V_\tau(L)/L}$$

exists (Fekete; Appendix E) and $0 < \varsigma_\tau \leq \sigma_\tau \leq \bar{\varsigma}_\tau < \infty$. ■

25.2 Uniform gap and declared order of limits

A regulated system can possess a positive gap that vanishes as the regulator is removed. The physical claim requires P25 ($\liminf \varsigma_\tau(a, R_\perp) > 0$ under $a \to 0$, $R_\perp \to \infty$) and the declared order of limits P26. Without them, Theorem 15 proves a positive tension only in an unspecified regulated configuration space (falsifier F30).

25.3 Renormalisation-group consistency

$\varsigma_\tau(k)$ is defined at a blocking scale, but the physical tension cannot depend on one. Either the criterion is run on the full $k \to 0$ effective functional, or RG invariance with all induced operators retained must be proved:

$$\boldsymbol{d\sigma_\tau / d \ln k = 0 \text{ (exact theory),}}$$

residual k-dependence being truncation error (P27; falsifier F31). A local derivative expansion can look coercive at one scale while omitted nonlocal infrared operators shift the sector minimum at another.

25.4 Lattice-regulated formulation

The cleanest available formulation of the nonperturbative portion is on a centre-preserving lattice regulator: well-defined Hilbert space, positive transfer matrix, exact Wilson lines and centre transformations, exact electric-flux sectors, compact configuration spaces at fixed volume, controlled sources, explicit continuum debt. The regulated sequence: (1) at fixed spacing and finite transverse volume, define the τ centre-electric-flux sector; (2) construct the auxiliary-reduced transfer operator; (3) prove a positive sector energy density; (4) prove it uniformly in transverse volume; (5) establish the longitudinal limit; (6) require a strictly positive continuum liminf (P25).

---

26. The Pure-Gauge Area-Law Theorem

Theorem 16 — Transfer-Matrix Area-Law Theorem

Assume Theorem 15, nonzero Wilson-loop overlap with the static ground state, and reflection positivity (P20). Then $-\lim_{T\to\infty}(1/T)\cdot\log\langle W_R(L,T)\rangle = V_R(L)$ with $V_R(L) = \sigma_{\tau(R)}\cdot L + o(L)$, hence for large rectangles

$$-\log \langle W_R(L, T)\rangle = \sigma_{\tau(R)}\cdot L\cdot T + o(L\cdot T),$$

with the subleading term of perimeter type $O(L + T)$ whenever $V_R(L) = \sigma_{\tau(R)}\cdot L + O(1)$; equivalently $\langle W_R(C)\rangle \sim \exp[-\sigma_{\tau(R)}\cdot A(C) - \mu_R\cdot P(C) + \cdots]$. (Appendix F.) ■

The theorem does not assume independent centre-vortex events, a pre-existing area law, a positive areal flux probability, or an exponential bound on a probability mixture; the area law is obtained from the static spectrum. Extensivity arises because there are L irreducibly costly cross-sections propagated for Euclidean time T (§1.10–1.11).

---

27. Representation Screening and Conditional Triality Universality

Theorem 17 — Conditional Asymptotic Triality Universality

Triality preservation alone does not deliver universality. Assume additionally: (1) finite-energy gluelump states exist; (2) adjoint gluons screen a source in R to the lowest sector R_τ of the same triality at finite, L-independent cost; (3) the screened channel mixes with nonzero amplitude into the static ground state, at fixed L, for all sufficiently large L. Then there is an L-independent C_R with $|V_R(L) - V_{R_\tau}(L)| \le C_R$, and dividing by L,

$$\sigma_R = \sigma_{\tau(R)}$$

asymptotically; in particular $\sigma_{\tau=0} = 0$ and, by charge conjugation, $\sigma_1 = \sigma_2$, so the asymptotic data reduce to one nonzero pure-gauge tension $\sigma_F = \sigma_{\bar{F}}$. Casimir scaling may hold approximately at intermediate distances but cannot replace triality asymptotically. Absent premises 1–3, universality is a physically expected conjecture rather than a theorem of this paper. ■

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28. Dynamical Quarks and String Breaking

Fundamental quarks carry triality one; antiquarks two. With dynamical quarks, a static source binds a dynamical antiquark into a colour singlet; let $M_{Q\bar{q}}$ be the lowest static–light singlet energy above the source self-energy convention.

Theorem 18 — String-Breaking Upper Bound

If finite-energy static–light singlets exist, then for large L

$$V_F(L) \leq 2 \cdot M_{Q\bar{q}} + O(e^{-m_{gap} \cdot L}),$$

by the variational principle applied to two widely separated singlets (Appendix G). The strict asymptotic linear lower bound of the pure-gauge theorem is therefore invalid once fundamental screening is admitted; schematically $V_F(L) \approx \min[\sigma_F \cdot L + \mu, 2M_{Q\bar{q}}]$ with $L_{sb} \sim (2M_{Q\bar{q}} - \mu)/\sigma_F$ — structural, not numerical. ■

Wilson-loop caveat

A Wilson-loop operator may have poor overlap with the broken-string state; a numerical loop can show a long apparent linear regime after the true ground state has crossed into two singlets. A proper calculation needs a variational basis containing both string-like and two-hadron operators.

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29. Colour-Singlet Asymptotic Persistence

Criterion 19 — Full-QCD Colour-Singlet Persistence Criterion

Assume: (1) physical states satisfy the non-Abelian Gauss constraint; (2) finite-energy asymptotic operators belong to the gauge-invariant algebra or carry admissible nonlocal dressing; (3) dynamical quarks screen nonzero triality only by forming globally gauge-invariant composites; (4) no finite-energy isolated coloured dressing survives to spatial infinity. Then the physical asymptotic spectrum contains no isolated colour-non-singlet particle.

Epistemic status

Assumption 4 is close to the statement being classified. Criterion 19 is an operational classification result — it organises what colour-singlet persistence requires and forbids — and does not independently derive full-QCD colour confinement; the independent derivation from the complete VERSF physical Hilbert space is Debt D18.

Interpretation

String breaking is not deconfinement: one long colour-singlet configuration becomes two shorter ones. A strange quark may contribute internally to kaons, hyperons, hidden-strange mesons and strange currents while no isolated strange triplet appears asymptotically.

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30. Relation to Coloured Participation, Strange Confinement and Quark Masses

The coloured participation fraction measures internal support in a colour-triplet sector of a declared positive response; this paper asks whether an external nonzero-triality flux sector has positive asymptotic energy. Different operators on different spaces: a nonzero internal fraction is compatible with no free colour. The finite Sym(6) strange-confinement selector prototypes positive support selection, auxiliary gapping, invariant low-readout projection and complement-role preservation, but does not prove SU(3) confinement; the transverse functional here is a local gauge-field object carrying global centre data. The strong beta function is flavour universal except through representation content and thresholds; it does not determine Yukawa eigenvalues, and running quark masses, constituent parameters and hadron masses remain distinct — none is derived from the string tension alone.

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31. Typed-Carrier Descent and the Global Centre-Sensitive Lift

Two different descent requirements existed in the earlier architecture. The first — a local adjoint-equivariant response map — is **discharged** by Theorem 1: the carrier is $\mathrm{Herm}_0(\mathbb{C}^3)$ and the map is canonical. The second remains open and is the sole surviving descent debt.

Theorem 20 — Local/Global Descent Separation

The derived local map is sufficient for the kinetic metric and perturbative running. It is insufficient for confinement unless extended to a global group-level lift

$\tilde{E}_3 : \mathcal{P}_{SU(3)} \to \mathcal{C}_{bath}$

(or an equivalent map on principal bundles, transition functions, admissible Wilson lines, centre-twisted sectors and one-form symmetry backgrounds) preserving $\tau \in \mathbb{Z}_3$ and the triality classification of line operators. Any argument deriving $\mathbb{Z}_3$ centre confinement solely from a local adjoint object fails the typing test (falsifier F16). ■

The lift must additionally fix the meaning and normalisation of the primitive flux Φ_τ entering Theorem 13.

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32. Temporal Centre-Sector Deconfinement Criterion

At finite temperature T define the temporal centre-sector gap on the temporal analogue of the classical flux class of §21.1,

$\varsigma_\tau^{temp}(T)$ = inf over $\mathfrak{c}_\tau^{temp}$ of $\mathscr{E}_{\perp,T}^{\ red}$,

graded, like its zero-temperature counterpart, as a declared truncation of the quantum temporal sector energy on $\mathscr{H}_\tau^{temp}$ (Debt D21). This is the gap of the temporal electric-flux sector governing the static-quark free energy and Polyakov-loop sectors. It must not be conflated with the spatial string tension extracted from spatial Wilson loops, which is known to persist above the pure-gauge transition, nor with a generic triality gap of unspecified type. The coercive instantiation of Theorem 13 is a *spatial* truncation; its formula $\varsigma_\tau = g_{s,IR} \cdot |\Phi_\tau| \cdot \sqrt{2B_\tau}$ may not be ported to ς_τ^{temp} without a separate temporal derivation.

Corollary 32.1 — Temporal Centre-Sector Deconfinement Criterion

In the pure-gauge theory, loss of temporal confinement requires at least one of: (1) $\varsigma_\tau^{temp}(T) \to 0$; (2) temporal sector separation fails; (3) transfer-matrix positivity or the static-source interpretation changes; (4) temporal centre symmetry breaks spontaneously. None of these implies the vanishing of the spatial string tension above the transition. No numerical critical temperature is derived here. With dynamical quarks, temporal centre symmetry is explicitly broken and deconfinement becomes a crossover or interaction-dependent transition rather than an exact centre-symmetry theorem. ■

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33. Observable and Lattice Matching

Running: background-field, gradient-flow, Schrödinger-functional, step-scaling or matched MS-bar couplings. **Pure-gauge confinement:** rectangular Wilson loops, Creutz ratios, static-source energies, centre-twisted partition functions, triality-resolved flux profiles, string excitation spectra. **Full-QCD string breaking:** Wilson-line source pairs, static–light meson pairs, mixed correlation matrices. **Bath-functional test:** if the reduced functional contains an action-density-gradient channel, lattice tests may examine whether $(\partial_\mu \operatorname{tr} F^2)^2$ (properly renormalised) concentrates near the flux-profile wall — a test of one sufficient realisation, not of the abstract criterion. **Response test:** the plaquette protocol of §9 with its mandatory eight-generator isotropy audit.

Part V — Closure

34. What Has Actually Been Derived

This gate derives or fixes the following.

1. The colour-holonomy residual map $E_3(e_a) = T_a$ onto $\mathrm{Herm}_0(\mathbb{C}^3)$ — canonical, isometric, exactly equivariant.
2. The isotropy of the invariant bath response (from invariance alone), the frozen normalisation split $\bar{g}_{ab}{}^{fs} = \tfrac{1}{8}\cdot\delta_{ab}$ in the declared Haar–FS convention, and the $d = 3$ uniqueness of the $2(d+1) = d^2 - 1$ coincidence.
3. The exact scalar reduction $Z_3 = \chi_C/8 - b^\dagger\cdot C^{-1}\cdot b$, with only auxiliary adjoint copies able to screen, and the positivity condition $\chi_C/8 > b^\dagger C^{-1} b$.
4. The decoupling of the six-dimensional spoke Hessian from the leading F^2 coefficient, $\mathrm{Hom}_{SU(3)}(\mathbf{8}, V_{seq}) = 0$.
5. The retirement of the fourteen-channel trial, with its forced-embedding and colour-blindness propositions preserved as methodology and the group-order firewall installed.
6. The executable plaquette definition of Z_3 with a mandatory eight-generator isotropy audit.
7. The exact identities $g_s = Z_3^{(-1/2)}$ and $\beta_s = -\tfrac{1}{2} g_s^3\cdot\partial Z_3/\partial \ln k$; the universal one-loop flow with $b_0 = 11 - (2/3)N_f$; asymptotic freedom for the Standard Model census.
8. The symbolic three-flavour strong scale $\Lambda_3 = \mu\star^{(7/9)}\cdot(m_t\, m_b\, m_c)^{(2/27)}\cdot\exp[-8\pi^2 Z_3(\mu\star)/9]$, exponent $-\pi^2\chi_C/9$ in the screening-free case, with the two-loop prefactor stated.
9. The forced-running consistency condition $d(\chi_C/8 - b^\dagger C^{-1} b)/d \ln k = 7/8\pi^2$ — $d\chi_C/d \ln k = 7/\pi^2$ screening-free — an internal falsifier preceding any numerics.
10. The precision budget: ±10% on Λ_3 requires $|\delta\chi_C| \lesssim 0.087$.
11. The typed triality sectors, the conditional positive-gap theorem, the slicing-inequality route to the linear potential, string-tension existence, and the pure-gauge area law.

12. The corrected coercive tube functional with the stiffness in the denominator and $\varsigma_\tau = g_{s,IR} \cdot |\Phi_\tau| \cdot \sqrt{2B_\tau}$, joining the running and confinement halves.
13. The uniform-gap, order-of-limits and RG-consistency conditions guarding the physical limits, and the lattice-regulated formulation.
14. Conditional triality universality, the string-breaking bound, and the colour-singlet persistence criterion at its honest (assumption-graded) level.
15. The discharge of the local descent debt and the isolation of the global centre-sensitive lift as the sole remaining descent requirement.
16. The strict separation of the classical class infimum ($\mathfrak{c}_\tau$) from the quantum sector energy ($\mathscr{H}_\tau$), with the truncation declared and the promotion logged as Debt D21.
17. The truncation RG-compensation condition $d \ln B_\tau / d \ln k = d \ln Z_3 / d \ln k$ — the confinement-side analogue of the forced running, available before any numerics.

This gate does **not** derive: a numerical $\chi_C(\mu\star)$, hence no numerical Z_3, g_s, Λ_3; the auxiliary adjoint census (b, C); the quark masses (declared inputs pending the Yukawa gates); two-loop and smooth-threshold upgrades; the Haar–FS convention from record-cost first principles; the full BRST construction from substrate primitives; the global Standard Model gauge-group quotient; the nonperturbative bath functional, its coercivity, compactness or vacuum uniqueness; the slicing inequality; the uniform continuum gap; RG invariance of σ_τ; B_τ; the flux-normalisation lift; the classical-to-quantum slice promotion (D21); the full-QCD physical-state theorem; empirical agreement of any kind.

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35. Open Debts

D1 — Absolute susceptibility. Evaluate $\chi_C(\mu\star)$ via the plaquette protocol of §9, in the frozen convention, with an error budget commensurate with Proposition 17.1.

D2 — Auxiliary adjoint census. Enumerate the same-order auxiliary modes, identify all adjoint copies, and compute (b, C).

D3 — Convention derivation. Derive the Haar–Fubini–Study response geometry (P3) from the VERSF record-cost/statistical-distance identification, so the convention is a theorem rather than a declaration.

D4 — Forced-running audit. Verify $d(\chi_C/8 - b^\dagger C^{-1} b)/d \ln k = 7/8\pi^2$ from substrate blocking in the declared scheme, evaluating χ_C at two scales from the master action's own blocking rules, independent of the continuum one-loop determinant (§1.15).

D5 — Quantum gauge gate. Construct the gauge-fixed or gauge-invariant quantum theory: measure, regulator, redundancy sector, background Ward identity, Slavnov–Taylor consistency.

D6 — Threshold and scheme upgrade. Two-loop running, smooth decoupling, declared mass convention at each threshold; replace m_t, m_b, m_c by Yukawa-gate outputs mapped into the running scheme.

D7 — Global colour-group closure. Determine the global colour group and its embedding in the full Standard Model gauge group.

D8 — Centre-sensitive bath lift. Construct $\tilde{E}_3$ preserving Wilson lines, centre charge, triality and centre-twisted boundary conditions, and fix the normalisation of Φ_τ.

D9 — Full transverse bath functional. Derive $\mathscr{E}_\perp[X, \eta]$ from substrate dynamics.

D10 — Auxiliary census (confinement). Show every same-order mode capable of relaxing colour flux is included in η.

D11 — Coercivity. Prove the reduced nonzero-triality functional prevents zero-width collapse, infinite-width dilution, leakage into uncounted channels and gauge-artifact escape.

D12 — Compactness. Establish finite-energy compactness modulo gauge and translations.

D13 — Vacuum uniqueness. Show the vacuum is the unique zero-energy orbit.

D14 — Slicing inequality. Prove $H_{Q\bar{Q}}[X] - E_0 \geq \int_0^L \mathscr{E}_\perp^{\,red}[X(z)]\, dz - c_{end}$ against the failure channels of P18.

D15 — Finite trial tube and subadditivity. Construct a finite-tension tube and prove the concatenation bound.

D16 — Uniform gap and continuum limit. Establish P25–P26.

D17 — RG invariance. Prove $d\sigma_\tau/d \ln k = 0$ in the exact theory or run the criterion at $k \to 0$.

D18 — Full-QCD physical-state theorem. Derive colour-singlet persistence from the complete VERSF physical Hilbert space.

D19 — Bath areal coefficient. Derive B_τ from Wilsonian bath blocking.

D20 — Observable matching. Match the blocked coupling and bath tension to standard gauge-invariant definitions.

D21 — Classical-to-quantum slice gap. Bound the difference between the classical class infimum over $\mathfrak{c}_\tau$ and the quantum sector energy ς_τ^{qu} = inf over $\Psi \in \mathscr{H}_\tau$ of $\langle\Psi|\hat{H}_\perp^{red}|\Psi\rangle - E_{\perp,vac}$, including zero-point and entropic contributions; restate the conditions of Theorem 12 at the quantum level (weak convergence and superselection in $\mathscr{H}_\tau$) or prove the truncation controlled. This debt is coupled to D8 — the centre projection defining $\mathfrak{c}_\tau$ is fully gauge-invariant only once the lift fixes it non-Abelianly — and its scope includes the construction of $\hat{H}\perp^{red}$ itself, which is an effective-Hamiltonian reduction rather than a pointwise inf over η.

D22 — Blind numerical evaluation. Only after every other debt is discharged may predictions be compared with measured or lattice QCD quantities.

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36. Falsification Conditions

F1 — Carrier failure. The colour bath is not $\mathbb{C}^3$ or supports no SU(3)_C transport.

F2 — Residual failure. The small-loop holonomy residual is not adjoint-valued after traceless projection.

F3 — Convention drift. The response-geometry convention is changed after, or selected by, the χ_C evaluation.

F4 — Susceptibility failure. χ_C is nonpositive or divergent at the declared matching scale.

F5 — Over-screening failure. $b^\dagger \cdot C^{-1} \cdot b \geq \chi\_C/8$, giving a null or ghost gluon mode.

F6 — Isotropy-audit failure. The plaquette protocol returns generator-dependent Z_3 beyond declared numerical error.

F7 — Ward-identity failure. No quantum scheme preserves the gauge identities defining one running colour coupling.

F8 — Gauge-determinant failure. The combined gauge-redundancy calculation does not reproduce the universal adjoint coefficient.

F9 — Matter-census failure. The active colour matter differs from the inherited Standard Model census.

F10 — Asymptotic-freedom failure. The computed weak-coupling beta function is nonnegative for the physical content.

F11 — Threshold failure. Heavy quarks do not decouple consistently.

F12 — Forced-running failure. Substrate blocking yields $d(\chi\_C/8 - b^\dagger C^{-1} b)/d \ln k \neq 7/8\pi^2 + O(g_s^2)$ above the top threshold in the declared scheme. Circularity clause: an audit that imports the continuum one-loop determinant as its computational engine (§1.15) is void, not confirmatory.

F13 — Scale-dependence failure. Λ_6 acquires $\mu_\star$-dependence within declared accuracy.

F14 — Running/confinement conflation. A growing infrared coupling is presented as an area-law proof.

F15 — Global-group failure. No global colour structure supports the claimed triality sectors.

F16 — Lie-algebra/centre conflation. Centre confinement is inferred from a local adjoint object alone.

F17 — Triality typing failure. A line-operator or centre-flux quantum number is assigned to a local connection configuration.

F18 — Triality-screening failure. Adjoint bath modes screen nonzero triality to the vacuum in pure gauge theory.

F19 — Auxiliary-instability failure. The reduced transverse Schur complement has a negative physical mode.

F20 — Zero-gap failure. A nonzero-triality slice has arbitrarily small reduced energy.

F21 — Dilation-escape failure. Flux spreads to infinite radius at vanishing cost.

F22 — Collapse-escape failure. Flux shrinks to zero radius at vanishing cost.

F23 — Compactness failure. A zero-energy sequence escapes all convergent gauge-reduced sublevels.

F24 — Vacuum-degeneracy failure. A nonzero-triality configuration has exactly vacuum energy.

F25 — Slicing failure. The Gauss law does not force the same triality through every separating slice, or the slicing inequality fails.

F26 — Subadditivity failure. No asymptotic string-tension limit exists.

F27 — Transfer-matrix failure. The static-source amplitude has no positive spectral representation.

F28 — Area-scaling error. Area extensivity is inferred by distributing one fixed flux over the worldsheet.

F29 — Probability inequality error. A positive flux-event probability is mistaken for an exponential area-law bound.

F30 — Uniform-gap failure. The regulated slice gap tends to zero as the transverse volume grows or the regulator is removed.

F31 — RG-consistency failure. The claimed physical tension depends on the blocking scale after all induced operators are retained.

F32 — Flux-typing failure. A tube functional holds quantised flux fixed with the stiffness in the numerator of the localisation term.

F33 — Bath-coefficient failure. The derived B_τ is nonpositive (the abstract criterion may still hold by another route). Lockstep clause: the derived B_τ fails $d \ln B_\tau/d \ln k = d \ln Z_3/d \ln k$, so the truncated tension depends on the blocking scale (Corollary 24.1).

F34 — Dynamical-quark overclaim. A pure-gauge eternal area law is claimed in full QCD.

F35 — Free-colour failure. The physical spectrum contains an isolated finite-energy coloured asymptotic particle.

F36 — Group-order/carrier conflation. A group order (e.g. $14 = |D_7|$) is treated as a response-carrier dimension, or the coincidence $2(d+1) = d^2 - 1$ at $d = 3$ is cited as a derivation.

F37 — Numerical insertion. Measured α_s, Λ_QCD, string tension or hadron masses are used to calibrate χ_C, the convention, or any branch.

F38 — Numerical disagreement. After blind evaluation, scheme matching and threshold evolution, the derived observables disagree with QCD beyond declared uncertainties.

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37. Numerical Non-Insertion Audit

Forbidden operations:

1. Setting χ_C (or the retired κ_C) to a convenient value and reporting downstream numbers as predictions.
2. Choosing χ_C, the response convention, or $\mu\star$ to reproduce measured $\alpha_s(M_Z)$, Λ_QCD, a lattice string tension or a hadron mass.
3. Quoting Λ_3 from the symbolic formula with inserted PDG masses as a "VERSF prediction" while χ_C remains uncomputed.
4. Citing the robustness of the retired trial as evidence that the colour embedding was derived; citing the two-eights coincidence as a derivation.
5. Publishing a χ_C evaluation without the Proposition 17.1 error budget, or skipping the eight-generator isotropy audit.
6. Using infrared factorised surrogates as if they carried derivational content (§1.13).
7. Extrapolating the one-loop coupling through its strong-coupling breakdown, or calling the one-loop pole a confinement theorem.
8. Treating a positive centre-vortex probability as an area-law proof, or dividing one flux quantum among surface patches to manufacture area scaling.

9. Using channel ratios (14/8, 8/14, 7, 2K, …) as beta-function coefficients.
10. Inferring $\mathbb{Z}_3$ triality from the Lie algebra without global group closure.
11. Using full-QCD string breaking as evidence against colour confinement; counting internal coloured participation as free-colour abundance.
12. Ignoring heavy-quark threshold matching; selecting among bath-functional branches after inspecting lattice QCD; reporting a postdiction as a blind prediction.
13. Changing the §6 bivector/quartic normalisation convention between the shape calculation, the plaquette protocol and the D1 evaluation.
14. Presenting a classical class infimum over $\mathfrak{c}_\tau$ as the quantum sector energy over $\mathscr{H}_\tau$, or citing Theorem 12 as if the D21 promotion were closed.

Declared calculation order

1. Freeze the response convention (P3) and the global colour group and representation ledger.
2. Evaluate $\chi_C(\mu\star)$ by the plaquette protocol with the isotropy audit and error budget (D1).
3. Complete the auxiliary adjoint census (b, C) (D2); form $Z_3(\mu\star)$ and check positivity.
4. Audit the forced running (D4).
5. Publish $g_s(k)$, Λ_6 and Λ_3 before comparison.
6. Derive the transverse bath functional, its coercivity, compactness, vacuum uniqueness, slicing inequality and uniform gap (D9–D16); derive B_τ and the flux lift (D8, D19).
7. Discharge or grade the classical-to-quantum promotion (D21): either restate the gap at the quantum sector level, or declare σ_τ published at truncation grade with D21 explicitly open — silent truncation-grade publication violates audit item 14.
8. Publish σ_τ; include dynamical-quark screening; only then compare with gauge-invariant QCD observables.

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38. Closure Certificate

Gate question

Has VERSF reduced QCD running and confinement to independently calculable substrate objects — with the colour response now derived down to one susceptibility — without importing any measured strong-interaction value?

Closure answer

Colour response architecture: yes — derived. E_3 canonical; shape eigenvalue ⅛ calculated; $Z_3 = \chi_C/8 - b^\dagger C^{-1} b$ exact in form. **Running architecture: yes, conditionally** on the quantum gate, thresholds and scheme. **Confinement criterion: yes, conditionally** on the typed-sector, coercivity, compactness, slicing, uniformity and RG conditions — proved in the classical truncation, with quantum promotion open (D21). **Numerical QCD: not yet.** χ_C, (b, C), B_τ and the flux lift remain open.

Certificates

Given { P1–P5 }: $Z_3(\mu\star) = \chi\_C/8 - b\dagger\cdot C^{-1}\cdot b$ and $g_s(\mu\star) = Z_3\text{^}(-1/2)$, subject to the plaquette protocol, isotropy audit and positivity condition.

Given { P6–P10 }: $\beta_s = -(g_s^3/16\pi^2)\cdot(11 - (2/3)N\_f) + \cdots$, $\Lambda_3 = \mu\star\text{^}(7/9)\cdot(m\_t\ m\_b\ m\_c)\text{^}(2/27)\cdot\exp[-8\pi^2 Z_3(\mu\star)/9]$, and the forced running $d(\chi\_C/8 - b\dagger C^{-1}b)/d\ln k = 7/8\pi^2$.

Given { P11–P27 }: ς_τ = inf over $\mathfrak{c}\_\tau$ of $\mathscr{E}\perp^{\text{red}} > 0$ (classical truncation; quantum promotion is D21); $V\_\tau(L) \geq \varsigma\_\tau\cdot L - c\_end$; $\sigma\_\tau = \lim V\_\tau(L)/L$; $-\log\langle W\_R(C)\rangle = \sigma\_\{\tau(R)\}\cdot A(C) + O(P(C))$; and, in the coercive truncation, $\varsigma\_\tau = g_s{,}IR\cdot|\Phi\_\tau|\cdot\sqrt{}(2B\_\tau)$. With dynamical fundamental quarks the final step is replaced by the string-breaking bound and the persistence criterion.

Closure grade

The gate closes the derived colour response, the running architecture, the symbolic strong scale, the internal falsifiers, and the conditional confinement criterion with its corrected coercive instantiation. It closes no numerical strong-interaction value.

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39. Conclusion

The VERSF colour programme now runs as a single chain:

threefold bath transport ⟶ SU(3)_C connection ⟶ holonomy residual ⟶ $E_3(e\_a) = T\_a \longrightarrow \bar{g}^{fs} = ⅛ \longrightarrow Z_3 = \chi\_C/8 - b\dagger C^{-1}b \longrightarrow g_s(k) \longrightarrow \Lambda_3$,

for running, and

$\mathbb{Z}_3$ triality (typed on flux sectors) ⟶ $\mathscr{E}\perp^{\text{red}} \longrightarrow \varsigma\_\tau = g_s{,}IR\cdot|\Phi\_\tau|\cdot\sqrt{}(2B\_\tau) \longrightarrow \sigma\_\tau \longrightarrow$ area law (pure gauge) | string breaking into singlets (full QCD),

for confinement.

Three things changed when the response map was derived rather than modelled. The carrier turned out to be the object the framework had already established — the traceless Hermitian responses of the threefold bath — and the map turned out to be canonical, so the local descent debt vanished rather than being paid. The shape of the response became a calculation: the Haar–Fubini–Study eigenvalue one-eighth, isotropic by invariance, with the honest caveat that only its product with the susceptibility is physical. And the fourteen-channel trial was retired for the right reason — not because its arithmetic failed, but because its robustness was vacuity: a Hessian that commutes with everything has said nothing about colour.

The running side is now exponentially honest and internally falsifiable. One susceptibility stands between the substrate and the strong scale; a one-percent shift in it moves Λ_3 by a factor of two, and the paper says so before anyone computes anything. Consistency forces that susceptibility to run at exactly $7/\pi^2$ above the top threshold — a rate the master-action blocking calculation must reproduce, or the identification dies without a single glance at experiment.

The confinement side keeps its discipline. Asymptotic freedom is not confinement; a growing coupling is not an area law; triality lives on flux sectors, not field configurations, and the classical estimates are declared truncations of the quantum sector problem; the gap must survive the slicing inequality, the infinite-volume and continuum limits, blocking-scale independence, and the classical-to-quantum promotion before it is physics. Within the simplest coercive truncation, the corrected functional delivers the paper's most satisfying line: the infrared coupling itself multiplies the confinement gap. The two faces of the strong interaction — the ultraviolet weakening and the infrared imprisonment — meet in $\varsigma\_\tau = g_s$,IR$\cdot|\Phi_\tau|\cdot\sqrt{(2B_\tau)}$.

The next strong-interaction gate is exact and short:

Freeze the convention; evaluate χ_C(μ⋆) by the plaquette protocol with the eight-generator isotropy audit and a declared error budget; complete the auxiliary adjoint census; verify the forced running; publish Λ_3 blind. Then derive B_τ and the centre-sensitive flux lift, and publish σ_τ blind.

If those calculations succeed, the strength, the scale and the imprisonment of colour will no longer be imported properties of the Standard Model. They will be consequences of how a threefold bath resists having its frame twisted — locally in its Fisher geometry, and globally in its refusal to surrender a centre charge.

Appendix A — Haar Moments and the Fisher Metric

For Haar-random $|\psi\rangle$ in dimension d, the first two moments are $\int|\psi\rangle\langle\psi|d\mu = 1\_d/d$ and $\int(|\psi\rangle\langle\psi|)^{\otimes 2}d\mu = (1 + S)/(d(d+1))$. For d = 3: $1_3/3$ and $(1 + S)/12$. Then

$\int \frac{1}{2}\langle\{T\_a, T\_b\}\rangle\_\psi \, d\mu = \frac{1}{2}\cdot\mathrm{tr}(\{T\_a, T\_b\}\cdot 1_3/3) = \mathrm{tr}(T\_aT\_b)/3 = \delta\_ab/6$,

$\int \langle T\_a\rangle\_\psi\langle T\_b\rangle\_\psi \, d\mu = \mathrm{tr}[(T\_a \otimes T\_b)\cdot(1 + S)/12] = [\mathrm{tr}\, T\_a\cdot\mathrm{tr}\, T\_b + \mathrm{tr}(T\_aT\_b)]/12 = \delta\_ab/24$, using tr $T_a = 0$ and $\mathrm{tr}[(T_a \otimes T_b)\cdot S] = \mathrm{tr}(T_a T_b)$. Subtracting, $\bar{g}_{ab}^{\,fs} = \delta_{ab}/6 - \delta_{ab}/24 = \delta_{ab}/8$. In general dimension the same algebra gives $(\delta_{ab}/2)\cdot[\, 1/d - 1/(d(d+1)) \,] = (\delta_{ab}/2)\cdot[\, (d + 1 - 1)/(d(d+1)) \,] = (\delta_{ab}/2)\cdot[\, 1/(d+1) \,] = \delta_{ab}/(2(d+1))$, which returns $1/8$ at $d = 3$ — the internal consistency check of Corollary 5.1, written out in full because this appendix is the audit trail for the paper's most factor-sensitive number (§6, Factor Convention). Convention comparison: the GNS/tracial metric at $\rho = 1_3/3$ gives $\delta_{ab}/6$; the Bures metric there vanishes on unitary directions (§1.7).

Appendix B — Background-Field One-Loop Coefficient

In background-field gauge, $(\Delta_1)^{ab}{}_{\mu\nu} = -\bar{D}^2\cdot\delta_{\mu\nu}\cdot\delta^{ab} - 2\cdot f^{acb}\cdot\bar{F}^c{}_{\mu\nu} + (1 - 1/\xi)\cdot(\bar{D}_\mu\bar{D}_\nu)^{ab}$; $(\Delta_0)^{ab} = -(\bar{D}^2)^{ab}$; $\Delta_{1/2} = -\bar{D}^2 + \tfrac{1}{2}\cdot\sigma^{\mu\nu}\cdot\bar{F}_{\mu\nu} + m^2$. The logarithmic divergence multiplying $\tfrac{1}{4}\int\bar{F}^2$ is

$$\Gamma_{div} \supset \left(\ln(k^2/\mu^2) / 16\pi^2 \right)\cdot\left[\, (11/3)\cdot C_A - (4/3)\cdot\Sigma_f\, T(R_f) \,\right]\cdot\tfrac{1}{4}\int\bar{F}^2,$$

equivalently $(\ln(k/\mu) / 8\pi^2)$ times the same bracket, so $d(1/g_s^2)/d \ln k = (1/8\pi^2)\cdot[(11/3)C_A - (4/3)\Sigma_f\, T(R_f)] = (1/8\pi^2)\cdot(11 - (2/3)N_f)$ for SU(3) fundamental Dirac quarks.

Appendix C — Threshold Cascade Algebra

$\Lambda_{N_f-1} = m_f\cdot(\Lambda_{N_f}/m_f)^{(b_{N_f}/b_{N_f-1})}$, with $b_6 = 7$, $b_5 = 23/3$, $b_4 = 25/3$, $b_3 = 9$.

Top: $\Lambda_5 = m_t\cdot(\Lambda_6/m_t)^{(21/23)} = m_t^{(2/23)}\cdot\Lambda_6^{(21/23)}$. Bottom: $\Lambda_4 = m_b^{(2/25)}\cdot\Lambda_5^{(23/25)} = (m_t\cdot m_b)^{(2/25)}\cdot\Lambda_6^{(21/25)}$. Charm: $\Lambda_3 = m_c^{(2/27)}\cdot\Lambda_4^{(25/27)} = (m_t\cdot m_b\cdot m_c)^{(2/27)}\cdot\Lambda_6^{(7/9)}$.

With $\Lambda_6 = \mu_\star\cdot\exp[-8\pi^2 Z_3/7]$: $\Lambda_6^{(7/9)} = \mu_\star^{(7/9)}\cdot\exp[-8\pi^2 Z_3/9]$, hence $\Lambda_3 = \mu_\star^{(7/9)}\cdot(m_t\, m_b\, m_c)^{(2/27)}\cdot\exp[-8\pi^2 Z_3(\mu_\star)/9]$; screening-free exponent $-\pi^2\chi_C/9$. Mass dimension $7/9 + 3\cdot(2/27) = 1$. ✓

Appendix D — Positive Triality-Gap Proof

Let $\mathfrak{c}\_\tau$ be a closed nonzero-triality classical flux class (the declared truncation of §21.1) and $\mathscr{E} : \mathfrak{c}\_\tau \to [0, \infty)$ lower semicontinuous and coercive modulo gauge. If $\inf \mathscr{E} = 0$, choose $X_n \in \mathfrak{c}\_\tau$ with $\mathscr{E}[X_n] \leq 1/n$; coercivity gives gauge-reduced precompactness; a subsequence converges to $X\star$; closedness gives $X\star \in \mathfrak{c}\_\tau$; lower semicontinuity gives $\mathscr{E}[X\star] = 0$; vacuum uniqueness places $X\star \in \mathfrak{c}_0$, contradicting disjointness. Promotion to $\mathscr{H}\_\tau$ is Debt D21. The theorem fails exactly when: triality is not a closed sector; a zero-energy dilation sequence exists; the vacuum is not the unique zero; compactness fails; or an omitted auxiliary channel screens the flux.

Appendix E — Longitudinal Subadditivity and the String-Tension Limit

Joining near-minimising configurations at separations L_1, L_2 through a finite interpolation region of energy c_0 gives $V(L_1 + L_2) \leq V(L_1) + V(L_2) + c_0$. Setting $a(L) = V(L) + c_0$ makes a subadditive; the continuous Fekete lemma gives $\lim a(L)/L = \inf a(L)/L$; $c_0/L \to 0$, so $\lim V(L)/L$ exists, and the positive slice bound keeps it nonzero.

Appendix F — Transfer-Matrix Area-Law Proof

$\langle W(L,T)\rangle = \langle \Psi\_L|e^{(-T(H-E_0))}|\Psi\_L\rangle = \Sigma_n |\langle n,L|\Psi\_L\rangle|^2 \cdot e^{(-T\cdot V_n(L))}$. The lowest nonzero-overlap energy dominates as $T \to \infty$, so $-(1/T)\ln\langle W\rangle \to V(L)$. With $V(L) = \sigma L + \mu + o(1)$, $-\ln\langle W(L,T)\rangle = \sigma LT + \mu T + o(T)$, and source renormalisation and overlap contribute perimeter-type corrections: $\langle W(C)\rangle \sim \exp[-\sigma A(C) - \mu P(C) + \cdots]$.

Appendix G — Dynamical-Quark String-Breaking Bound

For $|M\_L M\_R\rangle$, two static–light singlets at separation L each of energy $M\_\{Q\bar{q}\}$: $\langle H|M\_LM\_R\rangle/\langle M\_LM\_R|M\_LM\_R\rangle = 2M\_\{Q\bar{q}\} + O(e^{(-mL)})$. The variational principle gives $V\_F(L) \leq 2M\_\{Q\bar{q}\} + O(e^{(-mL)})$; a strict asymptotic lower bound $\sigma L - c$ would contradict this, so the pure-gauge theorem must cease to apply once fundamental screening is admitted.

Appendix H — Canonical-Momentum Typing and Tube Minimisation

Typing. With action $\frac{1}{4}\cdot Z_3\cdot F_{\mu\nu}F^{\mu\nu}$ in primitive fields, $\Pi_i = Z_3\cdot E_i$; the Gauss constraint $D_i\cdot\Pi_i = \rho$ ties the flux of Π through a separating surface to the enclosed source; the quantisation of $\Phi_\tau = \oint \Pi$ is the centre-projected sector statement of §19.2, normalised by the global lift (Debt D8). Electric energy density $\Pi^2/2Z_3$; a tube of radius R at uniform fixed flux costs $\approx \Phi_\tau^2/(2\pi\cdot Z_3\cdot R^2)$.

Minimisation. $\mathcal{E}(R) = a/R^2 + b\cdot R^2$ with $a = \Phi_\tau^2/(2\pi Z_{3,IR})$, $b = \pi B_\tau$: $d\mathcal{E}/dR = 0 \Longrightarrow R_\star^4 = a/b = \Phi_\tau^2/(2\pi^2 Z_{3,IR} B_\tau)$; $\mathcal{E}(R_\star) = 2\sqrt{ab} = |\Phi_\tau|\cdot\sqrt{2B_\tau/Z_{3,IR}}$; AM–GM makes the minimum global.

The wrong typing. $\mathcal{E} = Z_3\Phi^2/(2\pi R^2) + \pi B R^2$ gives $\varsigma \propto \sqrt{Z_3 B}$: stiffer substrate, weaker coupling, stronger confinement — backwards. The error mixes primitive and canonical conventions across the Legendre transform.

Appendix I — Entropic-Gradient Sufficient Realisation

A candidate reduced Euclidean colour-bath functional: $\Gamma_k^{red}[A] = \int d^4x\ [\ \frac{1}{4}Z_3(k)F^2 + (\lambda_D(k)/M_k^2)\cdot \mathrm{tr}(D_\rho F_{\mu\nu})^2 + (\lambda_\varphi(k)/M_k^2)\cdot(\partial_\mu\ \mathrm{tr}\ F^2)^2/(\mathrm{tr}\ F^2 + \varepsilon) + U_k(\mathrm{tr}\ F^2) + \cdots\ ]$, with sufficient conditions $Z_3 > 0$, $\lambda_D \geq 0$, $\lambda_\varphi \geq 0$ and U_k bounded below and coercive against infinite-radius dilution. $\int d^4x\ \Box\mathrm{tr}\ F^2$ is a boundary term and cannot replace the positive gradient channels. No coefficient is derived by writing the functional; each must arise from the VERSF blocking calculation.

Appendix J — Dependency and Falsifier Map

Result	Principal dependencies	Principal falsifier
Residual map $E_3 = T_a$	P1–P2	F1–F2
Fisher eigenvalue $\frac{1}{8}$	P3	F3
Scalar reduction $Z_3 = \chi_C/8 - b^\dagger C^{-1}b$	P4–P5	F4–F5
Spoke decoupling	Prop 7.1	— (firewall)

Result	Principal dependencies	Principal falsifier
Trial retirement	§8	F36
Plaquette protocol	P3–P4	F6
$g_s = Z_3$^(−1/2); beta identity	GAC-1; canonical normalisation	F7
One-loop b_0; asymptotic freedom	P6–P9	F8–F10
Λ_6, cascade, symbolic Λ_3	P10	F11, F13
Forced running of χ_C	Thm 5 + Thm 3	F12
Precision budget	Thm 9	— (declaration)
Typed triality sectors	P11–P12, §19.2	F15–F18
Auxiliary reduction	P13–P14	F19
Positive slice gap	P15–P17	F20–F24
Corrected tube gap	P22–P24	F32–F33
Truncation RG compensation	Thm 13 + P27	F33 (lockstep clause)
Classical/quantum typing split	§21.1	F17; D21
Audit independence	§1.15	F12 (circularity clause)
Linear static energy	P18	F25
String-tension limit	P19 + subadditivity	F26
Area law	P20	F27–F29
Uniform gap and limits	P25–P26	F30
RG invariance	P27	F31
Conditional universality	Thm 17 premises	F18
String breaking	P21	F34
Singlet persistence	Criterion 19 assumptions	F35
Global lift	P28	F15–F16
Any numerical output	D1–D21	F37–F38

Appendix K — External Bibliography

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