The Broken Electroweak Phase in VERSF

▲ Programme Milestone — Standard Model Electroweak-Breaking Series

Deriving the Photon, W/Z Directions, and Weak-Boson Mass Structure from Closure-Norm Condensation Acting on the Derived Electroweak Gauge Fields — with the Existing Yukawa and Flavour Modules Positioned as Downstream Attachments

Keith Taylor VERSF Theoretical Physics Programme — Standard Model Electroweak-Breaking Series Programme-marker paper — working manuscript

Successor to *The Electroweak Gauge-Curvature Dynamics in VERSF*, *The Electroweak Representation and Connection Closure in VERSF*, and *The Substrate Anomaly-Inadmissibility Theorem in VERSF*.

Headline Result — From Unbroken Electroweak Gauge Fields to the Physical Broken Phase

The preceding VERSF papers derive the chiral matter skeleton, the electroweak representation algebra, the local electroweak connection, anomaly-admissible source ledgers, and the unbroken electroweak gauge-curvature dynamics. The present paper supplies the next Standard Model layer: closure-norm condensation acts on the derived electroweak connection and selects the broken vacuum.

The central claim is that the closure-norm interface carries the minimal electroweak vacuum orientation

$\Phi_{cl} \sim (1, 2, +½)$,

with nonzero closure norm

$\langle\Phi_{cl}\rangle = (v/\sqrt{2})\,(0, 1)^T$.

The unique generator that leaves this vacuum invariant is

$Q = T_3 + Y$.

Therefore the unbroken gauge direction is electromagnetic:

$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$.

The charged weak directions acquire condensate stiffness:

$W\_\mu^{\pm} = (1/\sqrt{2})(W\_\mu^1 \mp i\, W\_\mu^2)$, $m\_W = gv/2$.

The neutral fields mix into one massive direction and one massless direction:

$Z\_\mu = (g\, W\_\mu^3 - g'\, B\_\mu)/\sqrt{(g^2 + g'^2)}$, $A\_\mu = (g'\, W\_\mu^3 + g\, B\_\mu)/\sqrt{(g^2 + g'^2)}$,

with

$m\_Z = (v/2)\sqrt{(g^2 + g'^2)}$, $m\_A = 0$.

This paper does not newly derive the quark and lepton Yukawa operators, CKM structure, PMNS structure, charged-lepton hierarchy, quark-sector hierarchy, or neutrino-sector flavour kernel. Those belong to already-developed VERSF mass/flavour modules. The present paper instead supplies the missing broken-phase electroweak vacuum and interface object to which those existing modules attach.

General Reader Summary

The previous papers gave VERSF the matter table and the electroweak force structure before the vacuum acts. They showed why matter comes in left-handed pairs and right-handed singlets, why the weak force acts only on the left-handed pairs, why hypercharge is required, and how the weak and hypercharge fields appear as the natural connection for comparing matter states from place to place.

But the world we observe is not the unbroken electroweak world.

In the unbroken picture there are three weak directions and one hypercharge direction. In the world we actually see, there is a massless photon, two charged weak bosons, and one neutral weak boson. The photon reaches across the universe. The weak bosons act only over tiny distances, because they are massive.

So the next question is unavoidable:

How does the unbroken electroweak structure turn into the physical photon, W, and Z?

This paper answers that question using closure-norm condensation.

In VERSF, the closure norm measures whether the substrate has settled into a stable committed phase. When the closure norm condenses, it selects a vacuum — a stable background commitment state. The decisive point is that this vacuum is not neutral with respect to every electroweak direction. Some electroweak motions disturb it. One special motion does not.

That special motion is electromagnetism.

The photon is therefore not added by hand. It is the one electroweak direction that leaves the closure-norm vacuum unchanged. The other electroweak directions push against the vacuum and acquire stiffness. That stiffness is what appears physically as mass.

This gives the broken electroweak pattern. Two weak directions combine into the charged W bosons. The third weak direction mixes with hypercharge to form the neutral Z boson and the photon. The Z is the vacuum-disturbing neutral direction and becomes massive. The photon is the vacuum-preserving direction and remains massless.

This paper also settles an important ordering point. VERSF has already developed substantial Yukawa, quark, lepton, CKM and PMNS machinery elsewhere. This paper does not pretend those questions begin here. Instead, it supplies the derived broken-phase vacuum that those already-developed mass and flavour modules must attach to. In other words, this is not a new Yukawa-origin paper. It is the paper that derives the electroweak vacuum background required for the existing Yukawa and flavour machinery to become Standard-Model-facing.

The milestone is therefore precise:

VERSF now has a route from the derived unbroken electroweak gauge fields to the broken phase containing the photon, W bosons, and Z boson.

In plain language: the photon is the direction the vacuum does not feel; the W and Z are the directions the vacuum resists.

Contents

Abstract

The preceding VERSF papers derive the chiral matter skeleton, the electroweak representation algebra, the local electroweak connection, the anomaly-admissible source ledger, and the unbroken electroweak gauge-curvature dynamics. The inherited unbroken connection is

$A_\mu^{EW} = g W_\mu^i T_i + g' B_\mu Y$,

acting on the one-generation chiral matter skeleton L_L, e_R, Q_L, u_R, d_R, with

$$\mathfrak{g}_{EW}^{rep} = \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y, \quad Q = T_3 + Y.$$

The present paper derives the broken electroweak phase produced when closure-norm condensation acts on that already-derived gauge sector.

The key object is the closure-norm interface order parameter

$$\Phi_{cl}(x) = \rho(x)\, \chi(x),$$

where ρ is the gauge-invariant closure norm and χ is the minimal weak-branch interface orientation. The closure interface must be colourless, weak-active, and electrically neutral in the vacuum. These requirements fix its electroweak representation as

$$\Phi_{cl} \sim (1, 2, +\tfrac{1}{2}),$$

with vacuum orientation

$$\langle \Phi_{cl} \rangle = (v/\sqrt{2})\, (0, 1)^T.$$

The vacuum stabilizer is the unique generator annihilating the neutral closure state:

$$Q = T_3 + Y.$$

Therefore the closure-norm vacuum breaks $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$.

The condensate stiffness term

$$(D_\mu \Phi_{cl})^\dagger (D^\mu \Phi_{cl})$$

then gives masses to the vacuum-disturbing directions and leaves the vacuum-preserving direction massless. The charged weak fields are

$$W_\mu^\pm = (1/\sqrt{2})(W_\mu^1 \mp i\, W_\mu^2), \quad m_W = gv/2.$$

The neutral fields mix into

$$Z_\mu = (g\, W_\mu^3 - g'\, B_\mu)/\sqrt{(g^2 + g'^2)}, \quad A_\mu = (g'\, W_\mu^3 + g\, B_\mu)/\sqrt{(g^2 + g'^2)},$$

with

$$m_Z = (v/2)\sqrt{(g^2 + g'^2)}, \quad m_A = 0.$$

The tree-level relation

$m_W / m_Z = \cos \theta_W$

follows, with

$\sin \theta_W = g'/\sqrt{g^2 + g'^2}$, $\cos \theta_W = g/\sqrt{g^2 + g'^2}$.

The result is conditional on the inherited unbroken electroweak gauge dynamics and on the closure-norm condensation and interface-stiffness premises stated below. This paper does not newly derive the Yukawa operator, the charged-lepton hierarchy, the quark mass hierarchy, CKM mixing, PMNS mixing, or neutrino-sector flavour structure. Those are treated as already-developed VERSF mass/flavour modules. The present paper derives the broken-phase closure-interface vacuum against which those existing modules must be integrated and audited.

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0. Notation, Conventions, and Firewalls

0.1 Notation

The inherited electroweak connection is

$A_\mu^{EW} = g W_\mu^i T_i + g' B_\mu Y$.

The covariant derivative acting on an object carrying an electroweak representation is

$D_\mu = \partial_\mu + i g W_\mu^i T_i + i g' B_\mu Y$.

The T_i are the weak generators, normalized as $T_i = \sigma_i/2$ on weak doublets. Y is hypercharge in the convention

$Q = T_3 + Y$.

The closure-norm interface order parameter is denoted Φ_{cl}. It must not be confused with an elementary Higgs field inserted as a postulate. In this paper it is the record-layer closure-interface orientation of the condensed substrate phase.

The vacuum expectation of the closure norm is denoted v, with canonical normalization

$\langle \Phi_{cl} \rangle = (v/\sqrt{2})\,(0, 1)^T$.

0.2 Hypercharge convention

This paper uses $Q = T_3 + Y$. In the alternative convention $Q = T_3 + Y_{SM}/2$, one has $Y_{SM} = 2Y$. Thus the closure-interface hypercharge $Y = +½$ corresponds to $Y_{SM} = +1$. Appendix B verifies that every result of this paper is convention-covariant under this rescaling.

0.3 Representation firewall

This paper inherits the chiral matter skeleton and the electroweak representation algebra. It does not rederive L_L, e_R, Q_L, u_R, d_R, nor $\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$.

0.4 Gauge-dynamics firewall

This paper inherits the unbroken electroweak gauge-curvature dynamics. It does not rederive the kinetic terms

$$-\frac{1}{4} W_{\mu\nu}^i W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}.$$

It uses them as the unbroken gauge fields on which closure-norm condensation acts.

0.5 Existing mass/flavour module firewall

This paper does not newly derive the Yukawa operator, quark mass hierarchy, charged-lepton mass hierarchy, neutrino mass structure, CKM mixing, or PMNS mixing. Those are treated as existing VERSF mass/flavour modules. The present paper supplies the broken electroweak closure-interface vacuum needed to attach those modules to the now-derived Standard-Model representation and gauge-breaking structure.

The correct reading is: this paper derives the broken electroweak vacuum; the existing Yukawa/flavour modules are later integrated against that vacuum.

0.6 Higgs-radial firewall

This paper identifies the closure-norm radial excitation as the natural candidate for the Higgs-like scalar mode, but it does not compute its mass, self-coupling, decay rates, or full potential. The paper derives the vacuum orientation and gauge-boson mass matrix, not the full scalar-sector phenomenology.

0.7 Coupling-value firewall

This paper does not derive the numerical values of g, g', or v. It derives the structural relations among them.

0′. Predictive-Content Ledger

Object	Grade	Status
Chiral matter skeleton	Inherited / Conditional	From matter-representation theorem
Electroweak algebra	Inherited / Conditional	From electroweak representation closure

Object	Grade	Status
W/B connection	Inherited / Conditional	From connection closure
W/B gauge dynamics	Inherited / Conditional	From gauge-curvature dynamics
Anomaly-admissible source ledger	Inherited / Conditional	From anomaly-inadmissibility theorem
Closure norm ρ	Inherited / structurally supported — necessity also proved	From closure-substrate programme; necessity theorem in Appendix E.2
Closure-interface orientation χ	Conditional — retrodictively anchored	Origin debt stated as Weak-Commitment Interface Theorem (BP-3′, Debt 2)
Closure-stiffness existence and sign	Derived given BP-3; corroborated by inheritance	$c \leq 0$ contradicts committed orientation (Appendix E.4); structural-closure $\kappa > 0$ corroborates
Interface stiffness form	Exact given BP-3	Unique leading covariant form (Appendix E.4)
Closure interface (1, 2, +½)	Conditional theorem	Derived from minimality, neutrality, weak activity
Vacuum orientation $(0, 1)^T$	Gauge choice / Exact	Unit orientation in weak branch
Vacuum stabilizer $Q = T_3 + Y$	Exact given interface rep	Derived here
Breaking SU(2)_L × U(1)_Y → U(1)_EM	Conditional theorem	Derived here
W± directions	Exact	Derived here
A/Z directions	Exact	Derived here
Photon masslessness	Exact given vacuum stabilizer	Derived here
W/Z masses	Conditional	Derived from stiffness term
Weinberg angle relation	Exact algebraic relation	Couplings not numerically derived
$m_W/m_Z = \cos\theta_W$	Exact at tree/stiffness level	Derived here
Broken-phase degree-of-freedom ledger	Exact	Appendix A.7
$\rho_{tree} = m_W^2/(m_Z^2 \cos^2\theta_W) = 1$	Exact for doublet interface	Derived here
Colour unbroken	Conditional	Given colourless closure interface
Existing Yukawa operator module	Inherited / External to this paper	Not newly derived here
Existing quark mass/flavour module	Inherited / External to this paper	To be attached/audited

Object	Grade	Status
Existing charged-lepton module	Inherited / External to this paper	To be attached/audited
Existing CKM/PMNS modules	Inherited / External to this paper	To be attached/audited
Higgs radial mass	Owed	Not derived
Numerical fermion masses	Existing programme / not this paper	Not newly derived
Three generations	Owed / separate module	Not derived here

0″. Inherited Infrastructure

The present paper stands on the following derived stack, in logical order:

1. **Chiral matter skeleton** — the one-generation representation content L_L, e_R, Q_L, u_R, d_R, with left-handed weak doublets and right-handed weak singlets, from the Chiral Matter Representation Theorem.
2. **Electroweak representation algebra** — $\mathfrak{su}(2)$_L ⊕ $\mathfrak{u}(1)$_Y acting on that skeleton, with the hypercharge assignment fixed by the representation-closure and anomaly audits, and with $Q = T_3 + Y$ as the derived electric/source charge.
3. **Local electroweak connection** — A_μ^EW = g W_μ^i T_i + g′ B_μ Y, from the Electroweak Representation and Connection Closure paper.
4. **Anomaly-admissible source ledger** — the Substrate Anomaly-Inadmissibility Theorem, which certifies that the chiral source content is admissible and can source a consistent gauge sector.
5. **Unbroken gauge-curvature dynamics** — the field strengths W_μν^i, B_μν and their kinetic terms, from the Electroweak Gauge-Curvature Dynamics paper.
6. **Closure norm ρ** — the gauge-invariant measure of substrate closure condensation, from the closure-norm programme.

Nothing in this stack is rederived below. Every item is used as inherited input and firewalled accordingly.

0‴. Closure-Substrate Support Layer

The present paper also inherits a closure-substrate support layer from the Two-Planck, structural-closure, dimensional-transmutation, and void-tension papers. That layer establishes four facts relevant to electroweak breaking.

First, the VERSF substrate is not an infinitely compliant void. It has finite stress capacity, finite transport capacity, and a saturation response. The void-tension programme identifies a maximum sustainable stress scale and treats the substrate as an elastic, finite-capacity medium rather than a passive background.

Second, persistent geometric structure in VERSF is not assumed to exist at all scales without support. It arises through constraint satisfaction, coherence propagation, and percolation of relational structure. The Two-Planck and Route M papers supply the record-layer setting in which stable local comparison becomes meaningful, and the dimensional-transmutation stability work supports the existence of a stable mesoscopic coherence layer.

Third, coherence boundaries carry an entropy-gradient cost. Ordered coherent regions cannot be deformed into incoherent regions without breaking constraints, increasing entropy, and generating a surface-tension-like restoring effect. This is the prior origin of closure stiffness.

Fourth, the prior structural-closure work supplies the general form of a leading gradient cost for order-parameter variation — a Landau–Ginzburg-type closure functional with positive gradient coefficient. The present paper applies the same principle to the electroweak closure interface: once the closure order parameter carries electroweak orientation, ordinary gradients are replaced by electroweak covariant gradients.

Thus the present paper does not introduce stiffness ex nihilo. It imports the prior VERSF result that closure-interface deformation has a real substrate cost, and then determines the electroweak representation and mass matrix forced by that cost.

Grade and audit note. Each of these inputs carries the grade Inherited / Conditional on its source paper. The specific audit obligation created by this inheritance — extending the entropy-gradient result from scalar closure order parameters to the electroweak-*oriented* interface — is recorded in Appendix E.4 and Debt 3. Since Appendix E derives stiffness existence and sign internally from committed orientation (BP-3), this inheritance serves as independent corroboration rather than a load-bearing input; the audit remains listed because corroboration, too, must be earned.

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1. Purpose and Claim Level

The previous gauge-dynamics layer gives the unbroken electroweak gauge fields. But the physical world sits in the broken electroweak phase.

The task of this paper is to derive the broken-phase gauge-boson structure from closure-norm condensation:

$W_\mu^1, W_\mu^2, W_\mu^3, B_\mu \longrightarrow W_\mu^+, W_\mu^-, Z_\mu, A_\mu.$

The target chain is

unbroken W/B gauge fields → closure-norm condensate → vacuum stabilizer → U(1)_EM → photon + massive W/Z.

This paper also supplies an essential interface for already-developed mass/flavour work. The prior Yukawa and flavour modules need a derived electroweak vacuum and closure-interface object to attach to. This paper supplies that object. It does not restart the Yukawa programme.

Central claim

Given the inherited unbroken electroweak gauge dynamics and the closure-norm condensation premises BP-1–BP-8, the closure-norm vacuum selects a unique unbroken electromagnetic direction and gives mass to the three orthogonal electroweak directions:

$m_A = 0$, $m_W = gv/2$, $m_Z = (v/2)\sqrt{g^2 + g'^2}$.

This is the broken electroweak gauge-boson theorem.

Claim level

The gauge-boson results are **Exact** as electroweak algebra once four inputs are granted: the inherited unbroken electroweak connection; a nonzero closure-norm condensate; a weak-active, colourless, electrically neutral closure interface; and the leading covariant stiffness term for that interface. The first is inherited from the electroweak representation and gauge-curvature papers. The second is supported by the prior closure-substrate programme — finite void tension, coherence saturation, record-layer stability (Section 0‴) — and independently proved *necessary* for the inherited record layer. The fourth is *derived given the third*: once a committed electroweak-facing orientation exists, vanishing or negative stiffness contradicts commitment itself, and locality plus electroweak covariance force the unique covariant form (Appendix E), with the inherited entropy-gradient result ($\kappa \sim \hbar c\xi > 0$) as independent corroboration. The third is proved by representation forcing (Theorem 1) *given that electroweak breaking occurs*; that conditional is currently discharged by the observed broken phase rather than by substrate principles. The entire conditional structure of the paper therefore funnels into one premise — the interface-orientation origin (BP-3) — whose target theorem is stated explicitly as BP-3′.

The remaining conditionality is therefore narrower than an ordinary Higgs postulate. This paper does not assume the Standard Model Higgs doublet as an elementary field; it derives the minimal electroweak interface representation required of the closure substrate once that substrate is asked to break the inherited electroweak connection while preserving colour and electromagnetism. The numerical values of g, g′, v are **not** claimed. The Higgs radial mass, scalar self-coupling, and full closure potential remain open debts. The Yukawa/flavour content is **not** claimed as new — it is positioned, not derived.

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2. Broken-Phase Premises

BP-1 — Unbroken electroweak gauge dynamics. The record-layer electroweak sector has the inherited unbroken gauge fields W_μ^i and B_μ, with connection and curvature as above. *Status:* Inherited. *Falsifier:* Failure of the prior gauge-curvature dynamics theorem.

BP-2 — Closure-norm condensation. The VERSF substrate admits a nonzero stable closure-norm phase, $\langle\rho\rangle \neq 0$, whose vacuum magnitude is written v. *Status:* Inherited / structurally supported. The prior closure-substrate programme — finite void tension, coherence saturation, record-layer stability (Section 0‴) — supports the existence of a stable nonzero closure branch, and Appendix E (Propositions E.1–E.2) independently proves that a vanishing closure norm cannot support the inherited persistent record layer. The premise is thus doubly supported: inherited from the substrate programme and proved necessary relative to the record layer. The microscopic stability functional and the numerical electroweak value of v remain owed. *Falsifier:* No stable nonzero closure phase exists, or the closure phase cannot support persistent local electroweak comparison.

BP-3 — Closure-interface orientation. The condensed closure norm carries an electroweak-facing interface orientation χ, so that $\Phi\_cl = \rho\chi$. Here ρ is the gauge-invariant closure magnitude, while χ is the orientation on which weak-branch comparison acts. *Status:* Conditional — retrodictively anchored. This paper shows that *if* the closure condensate has an electroweak-facing orientation and is required to produce the broken electroweak phase while preserving colour and electromagnetism, then the minimal admissible representation is (1, 2, +½). It does not yet derive from first principles that the closure condensate must carry such an orientation. That is the BP-3 origin debt, and it is the programme's most exposed electroweak joint. *Forward route to closure:* the weak-commitment candidate route — prove from weak-branch commitment, chiral comparison, and closure transport that a nontrivial two-branch interface orientation must appear, *before* imposing the observed W/Z mass pattern. If such a theorem is proved (BP-3′), BP-3 upgrades from Conditional to Derived. *Falsifier:* The closure condensate has no electroweak-facing orientation; or weak-commitment closure can be implemented entirely by singlet data; or the required orientation cannot be obtained without appealing to the observed broken electroweak spectrum.

BP-3′ — The Weak-Commitment Interface Theorem (owed). A scalar closure norm alone cannot distinguish electroweak directions. A pure singlet condensate may exist, but the weak generators act trivially on it and it breaks nothing. The statement $\Phi\_cl = \rho\chi$ therefore contains a claim beyond closure-norm condensation: the committed closure phase must present a weak-facing orientation χ. The present paper does not close that origin question; it proves only the conditional downstream result

weak-facing closure orientation + minimality + colour preservation + electric neutrality $\Longrightarrow$ $\Phi\_cl \sim (1, 2, +½)$.

The missing upstream theorem is stated explicitly so that its target is fixed:

Weak-Commitment Interface Theorem — owed. In a chiral electroweak record layer with left-handed weak doublets and right-handed weak singlets, closure transport cannot be completed by a purely singlet closure norm. Stable weak-branch commitment requires a two-component weak-facing interface orientation χ, transforming as the minimal nontrivial weak module.

Until that theorem is supplied, BP-3 remains conditional and retrodictively anchored. This is not a defect in the algebraic broken-phase result; it is the precise open debt required to convert the result from conditional reconstruction into forward derivation (Debt 2). *Status:* Owed. *Falsifier of the route:* Weak-commitment closure is shown to be implementable entirely by singlet data.

BP-4 — Minimal weak-active interface. The closure interface is the minimal nontrivial weak-active module. Therefore χ is a weak doublet. *Status:* Conditional, but representation-forced given weak-branch minimality. *Falsifier:* The closure condensate requires a higher weak representation, or is weak-singlet while still producing W/Z masses.

BP-5 — Electric neutrality of the vacuum. The physical vacuum must preserve long-range electric/source charge. Therefore the vacuum orientation must be annihilated by the unbroken electric generator. *Status:* Required by persistence of electromagnetism. *Falsifier:* The closure vacuum carries nonzero electric charge.

BP-6 — Colour neutrality. The closure interface carries no colour support. *Status:* Required to avoid colour breaking at the electroweak vacuum. *Falsifier:* Closure-norm condensation breaks colour.

BP-7 — Interface stiffness. Gauge directions that rotate the closure interface away from the vacuum carry stiffness cost. At leading order this cost is $(D_\mu \Phi_{cl})^\dagger (D^\mu \Phi_{cl})$. *Status:* Derived given BP-3, corroborated by inheritance. Given that the interface orientation is committed record data (BP-3), Proposition E.8 forces the leading stiffness coefficient: $c < 0$ destabilizes the committed phase against orientation texture, and $c = 0$ leaves the orientation soft rather than committed, contradicting BP-3 itself. Hence $c > 0$ whenever BP-3 holds. Independently, the structural-closure programme supplies a positive gradient stiffness $\kappa \sim \hbar c\xi > 0$ from coherence-boundary surface tension (Section 0‴). The covariant form is forced by locality, low-energy order, and electroweak covariance. What remains owed is the microscopic transport functional and its normalization (Debt 3). *Falsifier:* Closure-interface deformations carry no substrate cost despite committed orientation, or the leading cost is not gauge-covariant.

BP-8 — No hidden low-energy breaking sector. No additional low-energy electroweak-breaking order parameter contributes at the same leading order. *Status:* Minimality condition. *Falsifier:* Extra condensates modify the W/Z mass matrix — in particular, any non-doublet contribution shifts ρ_{tree} away from 1.

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3. The Closure-Norm Interface Order Parameter

Write the closure order parameter as

$\Phi_{cl}(x) = \rho(x)\, \chi(x),$

where:

- $\rho(x)$ is the **closure norm** — the scalar measure of substrate closure condensation;
- $\chi(x)$ is the **electroweak interface orientation** — a unit vector in weak-branch space.

The key distinction is:

ρ is gauge-invariant as a norm, but χ carries electroweak orientation.

Thus the closure-norm condensate is not merely a scalar number. Its norm is scalar, but its interface orientation tells the gauge connection which directions disturb the vacuum and which direction leaves it untouched.

This resolves an important interpretive issue. The closure norm itself is not an arbitrary electroweak-charged scalar inserted by hand. The electroweak transformation acts on the interface orientation of the condensed closure phase — on *how* the vacuum presents itself to weak-branch comparison, not on the fact of its condensation.

The effective order parameter has the same representation role as the Standard Model Higgs doublet:

$\Phi_{cl} \sim (1, 2, +½)$,

but its VERSF interpretation is different: it is the closure-interface orientation of the substrate vacuum, not an elementary scalar assumed at the start. Sections 4–6 derive this representation and its consequences rather than positing them.

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4. Why the Closure Interface Has Higgs-Doublet Form

The interface representation is fixed by three requirements. Each is independently necessary; together they are sufficient to force the minimal representation.

4.1 It must be weak-active

A weak singlet cannot give mass to weak gauge fields, because the weak generators act trivially on it:

$T_i \Phi = 0 \Longrightarrow$ no W_μ^i term in $D_\mu \Phi$.

If the closure interface were a weak singlet, $D_\mu \Phi$ would contain no W-field stiffness term, and the weak bosons would remain massless. Electroweak breaking therefore requires a weak-active interface.

The minimal weak-active module is the two-branch weak module already derived in the electroweak representation paper. Thus the minimal interface is a weak doublet.

4.2 It must be colourless

If the interface carried colour support, colour gauge directions would also acquire stiffness and gluons would become massive. That contradicts the observed low-energy structure and the programme's colour-support separation. Therefore the closure interface must be a colour singlet.

4.3 It must contain a neutral vacuum component

The vacuum must preserve electromagnetism. In the convention $Q = T_3 + Y$, a weak doublet has components with $T_3 = +\frac{1}{2}$ and $T_3 = -\frac{1}{2}$. For the lower component to be electrically neutral, one requires

$Q(\chi_0) = -\frac{1}{2} + Y\_\Phi = 0 \Longrightarrow Y\_\Phi = +\frac{1}{2}$.

(Choosing the *upper* component as the neutral one instead forces $Y\_\Phi = -\frac{1}{2}$ and yields the conjugate doublet; Appendix C shows this is the same physical content in a rotated convention, not an alternative.)

Therefore the minimal closure interface is

$\Phi\_cl \sim (1, 2, +\frac{1}{2})$.

Theorem 1 — Minimal Closure-Interface Representation

Given weak activity, colour neutrality, and electric neutrality of the vacuum, the minimal electroweak closure interface has representation

$\Phi\_cl \sim (1, 2, +\frac{1}{2})$.

Proof. Weak activity excludes the singlet and selects the minimal nontrivial weak representation, the doublet (BP-4). Colour preservation requires the colour singlet (BP-6). Electric neutrality of the lower vacuum component fixes $Y\_\Phi = +\frac{1}{2}$ (BP-5). No smaller weak-active representation exists, and Appendix C shows that every alternative candidate representation either fails one of the three requirements or reproduces the doublet content. Therefore the representation is (1, 2, $+\frac{1}{2}$). ■

Reading. This is a structural result of some weight. The Higgs-doublet representation is not introduced because the Standard Model uses it. It is *selected* by the requirements that closure-norm condensation break weak structure while preserving electromagnetism and colour. The Standard Model's most famous postulated object arrives here as the unique minimal solution to a constraint problem.

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5. Vacuum Orientation and the Neutral Closure State

Choose a local weak basis in which the vacuum is aligned as

$$\langle \Phi\_cl \rangle = (v/\sqrt{2})\,(0, 1)^T.$$

This is a choice of gauge orientation, not an extra physical assumption. Any nonzero doublet vacuum can be rotated into this form by a local SU(2)_L × U(1)_Y transformation — no neutrality proviso is needed for the rotation itself. BP-5's work is done elsewhere: in Theorem 1, where it fixes Y_Φ = +½ and thereby identifies the surviving U(1) with the matter-sector electric charge rather than some other embedded U(1). The physical content — the stabilizer subgroup and the mass spectrum — is orientation-independent; only its presentation is fixed here.

Let $\chi_0 = (0, 1)^T$. Then

$$T_3\, \chi_0 = -\tfrac{1}{2}\, \chi_0,\quad Y\, \chi_0 = +\tfrac{1}{2}\, \chi_0,$$

and therefore

$$(T_3 + Y)\, \chi_0 = 0.$$

The generator $Q = T_3 + Y$ leaves the vacuum unchanged. That is the algebraic origin of the photon direction.

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6. The Vacuum Stabilizer Theorem

Theorem 2 — Vacuum Stabilizer Theorem

For the closure-norm vacuum

$$\langle \Phi\_cl \rangle = (v/\sqrt{2})\,(0, 1)^T,\quad \Phi\_cl \sim (1, 2, +\tfrac{1}{2}),$$

the stabilizer subalgebra of 𝔰𝔲(2)_L ⊕ 𝔲(1)_Y is one-dimensional and generated by

$$Q = T_3 + Y.$$

Therefore the unbroken gauge group is U(1)_EM.

Proof. A general element of the electroweak algebra is $G = a_i\, T_i + b\, Y$ with real coefficients. Split into neutral and charged parts.

Neutral part. For $G\_n = a\, T_3 + b\, Y$ acting on χ_0:

$G_n \chi_0 = (-a/2 + b/2)\, \chi_0$.

Annihilation requires a = b, so $G_n \propto T_3 + Y$.

Charged part. T_1 and T_2 map χ_0 into the upper weak branch:

$T_1 \chi_0 = ½ (1, 0)^T$, $T_2 \chi_0 = -(i/2) (1, 0)^T$,

so no real combination $a_1T_1 + a_2T_2$ annihilates χ_0 except the zero combination, and no charged admixture can be cancelled by a neutral one, since the images lie in orthogonal components.

Hence the full stabilizer is the one-parameter family generated by $Q = T_3 + Y$. ■

Corollary 2.1 — Electroweak breaking pattern

The closure-norm vacuum breaks

$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$.

Three of the four electroweak generators are broken; by the counting of broken directions, exactly three gauge bosons acquire mass and exactly one remains massless.

Reading. The photon direction is not selected after the fact. It is the unique electroweak direction the closure-norm vacuum does not feel. One caution about what is and is not surprising here: that the stabilizer *contains* Q is guaranteed by construction, since BP-5 fixed $Y_\Phi = +½$ precisely by requiring the vacuum to be annihilated by Q. The genuinely nontrivial content of the theorem lies elsewhere: (a) the stabilizer is *exactly one-dimensional* — three directions break, with no accidental enlargement of the unbroken group; and (b) the same Q, with hypercharges fixed independently by the earlier representation papers, simultaneously closes every matter attachment channel in Appendix D. Point (b) is the programme's real cross-paper consistency check, and it lives in the attachment audit, not in this theorem.

---

7. Charged Weak Directions and W±

The weak generators T_1 and T_2 move the vacuum away from its neutral lower component. Define the charged combinations

$T\pm = T_1 \pm i\, T_2$,

and the corresponding gauge fields

$W_\mu\pm = (1/\sqrt{2})(W_\mu^1 \mp i\, W_\mu^2)$,

so that the connection term $g(W_\mu^1T_1 + W_\mu^2T_2) = (g/\sqrt{2})(W_\mu^+T^+ + W_\mu^-T^-)$.

These are the charged weak directions. They do not preserve the vacuum; they rotate the closure interface between the upper and lower weak branches. Since they disturb the closure-norm condensate, they acquire stiffness — and stiffness against the condensate is what appears physically as mass.

Theorem 3 — Charged weak directions

The two broken charged weak directions are

$W_\mu^\pm = (1/\sqrt{2})(W_\mu^1 \mp i W_\mu^2)$,

and they acquire equal mass from closure-interface stiffness.

Proof. The condensate is invariant under Q but not under T_1, T_2 (Theorem 2). The real fields W^1, W^2 therefore span the broken charged plane. The vacuum stabilizer U(1)_EM acts on this plane as a rotation; the combinations $W^\pm$ diagonalize that action with electric charges ±1 and are exchanged by complex conjugation. Because the stiffness term is U(1)_EM-invariant, the mass form on the charged plane is proportional to the identity, so the two charged directions acquire equal mass. The coefficient is computed in Theorem 5 and Appendix A. ■

---

8. Neutral Mixing and the Photon/Z Split

The neutral connection before breaking is

$g W_\mu^3 T_3 + g' B_\mu Y$.

Acting on the vacuum direction χ_0, with $T_3 \chi_0 = -\frac{1}{2} \chi_0$ and $Y \chi_0 = +\frac{1}{2} \chi_0$:

$(g W_\mu^3 T_3 + g' B_\mu Y) \chi_0 = \frac{1}{2} (-g W_\mu^3 + g' B_\mu) \chi_0$.

Only the combination $g W_\mu^3 - g' B_\mu$ disturbs the vacuum. The orthogonal combination leaves it invariant.

Define the normalized fields

$Z_\mu = (g W_\mu^3 - g' B_\mu)/\sqrt{g^2 + g'^2}$, $A_\mu = (g' W_\mu^3 + g B_\mu)/\sqrt{g^2 + g'^2}$.

Then Z_μ is the massive neutral weak direction and A_μ is the massless electromagnetic direction.

Equivalently, with the Weinberg angle defined by

$\cos \theta_W = g/\sqrt{g^2 + g'^2}$, $\sin \theta_W = g'/\sqrt{g^2 + g'^2}$, the rotation reads

$W_\mu^3 = \cos\theta_W \cdot Z_\mu + \sin\theta_W \cdot A_\mu$, $B_\mu = -\sin\theta_W \cdot Z_\mu + \cos\theta_W \cdot A_\mu$.

Theorem 4 — Neutral Mixing Theorem

Closure-norm condensation diagonalizes the neutral electroweak fields into Z_μ and A_μ as above. The Z_μ direction disturbs the vacuum and becomes massive; the A_μ direction preserves the vacuum and remains massless.

Proof. The vacuum stiffness depends on the neutral connection only through its action on χ_0, which is proportional to $-g W^3 + g' B$. The normalized field along this direction is Z_μ; the orthonormal complement in the (W^3, B) plane is A_μ. Since the A_μ combination annihilates the vacuum, the stiffness quadratic form has a null direction along A_μ and no mass term is generated for it. The explicit 2×2 diagonalization is performed in Appendix A and confirms that the neutral mass matrix has eigenvalues m_Z^2 and 0 with eigenvectors Z_μ and A_μ respectively. ■

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9. Gauge-Boson Mass Theorem

The closure-interface stiffness term is

$$\mathcal{L}_{stiff} = (D_\mu \Phi_{cl})^\dagger (D^\mu \Phi_{cl}).$$

In unitary vacuum orientation, set

$$\Phi_{cl} = (1/\sqrt{2})\,(0, v)^T$$

for the purpose of extracting gauge-boson masses (the three orientation components removed by this choice are accounted for in Appendix A.7 — they reappear as the longitudinal polarizations of $W^\pm$ and Z). Then the gauge part of the covariant derivative gives

$$D_\mu \Phi_{cl} \supset i\,(g W_\mu^i T_i + g' B_\mu Y)\,\Phi_{cl},$$

and substitution (Appendix A) yields the mass terms

$$\mathcal{L}_{mass} = (v^2/8)\,[\,g^2((W_\mu^1)^2 + (W_\mu^2)^2) + (g W_\mu^3 - g' B_\mu)^2\,].$$

In terms of physical fields,

$$\mathcal{L}_{mass} = m_W^2 W_\mu^+ W^{-\mu} + \tfrac{1}{2} m_Z^2 Z_\mu Z^\mu,$$

with

$m_W = gv/2,\ m_Z = (v/2)\sqrt{g^2 + g'^2},\ m_A = 0.$

Theorem 5 — Gauge-Boson Mass Theorem

Given the closure-norm interface (1, 2, +½), vacuum expectation v, and stiffness term $(D_\mu \Phi_{cl})^\dagger(D^\mu \Phi_{cl})$, the broken-phase gauge-boson masses are

$m_W = gv/2,\ m_Z = (v/2)\sqrt{g^2 + g'^2},\ m_A = 0.$

Proof. Substitute the neutral vacuum orientation into the covariant derivative. The charged fields W^1, W^2 contribute equally (Theorem 3). The neutral fields contribute only through the combination $g W^3 - g'B$ (Theorem 4). Diagonalizing the charged and neutral quadratic forms yields $W^\pm$, Z, and A, with coefficients giving the stated masses; the full computation is Appendix A. ■

Corollary 5.1 — Tree-level weak mass relation

With $\cos\theta_W = g/\sqrt{g^2 + g'^2}$ and $\sin\theta_W = g'/\sqrt{g^2 + g'^2}$,

$m_W / m_Z = \cos\theta_W,$

equivalently

$\rho_{tree} = m_W^2 / (m_Z^2 \cos^2\theta_W) = 1.$

Reading. The relation $\rho_{tree} = 1$ is not a free success of the construction — it is a *diagnostic* of the doublet. Any non-doublet contribution to the condensate (a weak triplet, for example) shifts ρ_{tree} away from 1. The observed proximity of the measured ρ parameter to unity is therefore direct empirical pressure toward the minimal doublet interface derived in Theorem 1, and constitutes the sharpest phenomenological handle on BP-4 and BP-8. The robustness of the relation has a symmetry origin worth recording: the doublet stiffness term carries, in the $g' \to 0$ limit, an accidental $SU(2)_L \times SU(2)_R$ symmetry, broken by the vacuum to the diagonal custodial $SU(2)_V$. Custodial $SU(2)_V$ forces the (W^1, W^2, W^3) mass form to be proportional to the identity, and gauging hypercharge then produces the $\cos\theta_W$ mixing without disturbing that ratio; it is this custodial protection that keeps $m_W = m_Z \cos\theta_W$ stable against large corrections. This is also what gives content to F7's radiative-structure clause: departures are expected only at the level of custodial-breaking effects (hypercharge and Yukawa insertions), not at leading order.

The W and Z masses are not bare gauge-field masses. They are vacuum stiffnesses. The photon remains massless because it is the vacuum-preserving direction.

---

10. Electromagnetic Coupling and Charge

The original neutral coupling is

$g W_\mu^3 T_3 + g' B_\mu Y.$

Inserting the rotation of Section 8, the coefficient of A_μ is

$A_\mu (g \sin\theta_W \cdot T_3 + g' \cos\theta_W \cdot Y).$

Since

$g \sin\theta_W = g' \cos\theta_W = gg'/\sqrt{g^2 + g'^2},$

define

$e = gg'/\sqrt{g^2 + g'^2} = g \sin\theta_W = g' \cos\theta_W.$

Then the photon couples to

$e (T_3 + Y) = e Q.$

Equivalently, in the useful inverse form,

$1/e^2 = 1/g^2 + 1/g'^2.$

Theorem 6 — Electromagnetic Coupling Theorem

The massless field A_μ couples universally to electric/source charge:

$\mathcal{L}_A = -e A_\mu J_Q^\mu, Q = T_3 + Y, e = g \sin\theta_W = g' \cos\theta_W.$

Proof. Insert the neutral field rotation into the original W^3/B coupling. The coefficient of A_μ multiplying T_3 is $g \sin\theta_W$ and the coefficient multiplying Y is $g' \cos\theta_W$; these are equal, so the photon couples to the single generator $T_3 + Y$ with common strength e. That generator is exactly the stabilizer Q of Theorem 2. ■

Reading. The photon is the massless connection of the vacuum stabilizer, and its charge is exactly the already-derived electric/source charge. Universality of electromagnetic coupling — the fact that the photon sees only Q, identically across all matter sectors — is here a theorem about the stabilizer, not an input.

---

11. Neutral and Charged Currents After Breaking

Before breaking, the interaction terms are

$g\, W\_\mu^i\, J\_i^\mu + g'\, B\_\mu\, J\_Y^\mu$.

After breaking, the charged fields couple as

$(g/\sqrt{2})\, (W\_\mu^+ J_+^\mu + W\_\mu^- J_-^\mu)$, $J\pm^\mu = :\bar{\psi}\_L\, \gamma^\mu\, T\pm\, \psi\_L :$,

in direct agreement with the identity of Section 7, $g(W\_\mu^1 T_1 + W\_\mu^2 T_2) = (g/\sqrt{2})(W\_\mu^+ T^+ + W\_\mu^- T^-)$. Charge accounting confirms the pairing: in the lepton sector $J_+^\mu \supset \bar{\nu}\_L\, \gamma^\mu\, e\_L$, and the resulting term $(g/\sqrt{2})\, W\_\mu^+\, \bar{\nu}\_L\, \gamma^\mu\, e\_L$ + h.c. is the standard charged-current coupling — the operator against which the CKM flavour-frame module attaches (Section 13.5). Thus the charged weak current remains purely left-handed — a direct inheritance from the chiral matter skeleton, untouched by the breaking.

The neutral fields couple as

$e\, A\_\mu\, J\_Q^\mu + (g/\cos\theta\_W)\, Z\_\mu\, J\_Z^\mu$,

with the neutral Z current

$J\_Z^\mu = :\bar{\psi}\, \gamma^\mu\, (T_3\, P\_L - \sin^2\theta\_W \cdot Q)\, \psi :$.

Proposition 7 — Broken-phase current structure

The closure-norm broken phase yields:

- *a universal electromagnetic current coupled to A_μ with strength e;*
- *left-handed charged weak currents coupled to $W\pm$ with strength $g/\sqrt{2}$;*
- *a neutral weak current coupled to Z_μ with strength $g/\cos\theta\_W$ and generator $T_3 P\_L - \sin^2\theta\_W \cdot Q$.*

Status note. This is a representation-level current decomposition. It does not compute decay rates, loop corrections, or precision electroweak observables. Those belong to the quantum electroweak completion (Debt 10).

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12. Why Colour Remains Unbroken

The closure interface is colourless: $\Phi\_cl \sim (1, 2, +½)$ has no colour support. Therefore colour generators — once the continuous colour lift is supplied — annihilate the closure vacuum, and no colour gauge direction appears in $D\_\mu\, \Phi\_cl$ at this layer. No gluon mass term is generated by closure-norm electroweak condensation.

Theorem 8 — Colour Preservation

Given a colourless closure interface, closure-norm electroweak condensation leaves colour unbroken.

Proof. A gauge boson acquires condensate stiffness only if its generator acts nontrivially on the vacuum order parameter (BP-7). Colour generators act trivially on a colour-singlet order parameter. Therefore the condensate produces no colour gauge-boson mass term. ■

Scope note. This assumes the colour gauge lift exists and acts in the standard way on colour-supported sectors. The derivation of full continuous SU(3)_C dynamics remains owed (Debt 8). What is established here is a conditional negative result: whatever the eventual colour dynamics, *this* condensate cannot break it.

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13. Relation to the Existing Yukawa, Mass, CKM, and PMNS Modules

This section fixes the programme ordering.

The VERSF programme already contains substantial work on the origin and structure of Yukawa operators, quark-sector closure, charged-lepton hierarchy, weak-commitment neutrino structure, CKM curvature, and PMNS mixing. The present paper must not be read as postponing all Yukawa work to an untouched future programme.

The proper relationship is different. The prior mass/flavour modules ask how matter representation slots acquire inertial weights and how flavour bases misalign. The present paper derives the broken electroweak closure-interface vacuum that those modules require as their Standard-Model-facing attachment point.

In Standard Model language, Yukawa terms require:

1. a left-handed weak doublet;
2. a right-handed weak singlet;
3. a weak-active, hypercharge-compatible vacuum/interface object;
4. a neutral product under the electroweak gauge representation;
5. a nonzero vacuum value to convert coupling operators into mass operators.

The previous representation papers supplied items 1 and 2. The present paper supplies items 3, 4, and 5.

Thus this paper does not derive the Yukawa operators from scratch. It provides the derived broken-phase electroweak background that allows already-developed VERSF Yukawa operators to be attached cleanly to the Standard-Model representation skeleton.

13.1 Charged-lepton attachment channel

The charged-lepton channel requires a bridge between L_L and e_R. Under $Q = T_3 + Y$ the gauge-neutrality audit reads

$$\bar{L}\_L\ \Phi\_cl\ e\_R : Y(\bar{L}\_L) + Y(\Phi\_cl) + Y(e\_R) = +\tfrac{1}{2} + \tfrac{1}{2} - 1 = 0.\ ✓$$

The closure interface $\Phi\_cl \sim (1, 2, +\tfrac{1}{2})$ has exactly the representation needed to form the charged-lepton attachment structure. This paper does not compute the charged-lepton Yukawa coefficient; it states that the existing charged-lepton mass module now has the correct derived broken-phase interface on which to act.

13.2 Down-quark attachment channel

The down-quark channel requires a bridge between Q_L and d_R:

$$\bar{Q}\_L\ \Phi\_cl\ d\_R : -\tfrac{1}{6} + \tfrac{1}{2} - \tfrac{1}{3} = 0.\ ✓$$

The same closure interface supplies the required weak and hypercharge compensation. This paper does not derive the down-quark coefficient or mass hierarchy; it supplies the broken-phase representation carrier that the existing quark mass module must use.

13.3 Up-quark attachment channel

The up-quark channel requires the conjugate closure interface

$$\tilde{\Phi}\_cl = i\sigma_2\ \Phi\_cl^* \sim (1, 2, -\tfrac{1}{2}),$$

with

$$\bar{Q}\_L\ \tilde{\Phi}\_cl\ u\_R : -\tfrac{1}{6} - \tfrac{1}{2} + \tfrac{2}{3} = 0.\ ✓$$

This object has the opposite hypercharge and gives the correct attachment channel between Q_L and u_R. Again, this is not a new derivation of the up-sector Yukawa operator; it is the broken-phase representation audit showing that the existing up-sector module has the required derived interface object. (Full audit: Appendix D.)

13.4 Neutrino-sector attachment

The neutrino sector depends on whether the optional sterile singlet ν_R is included, and on whether neutrino masses are Dirac-like, Majorana-like, weak-commitment-induced, or generated through a different VERSF mechanism. This paper does not settle that question. It records that the broken electroweak vacuum is now available for the existing weak-commitment neutrino and PMNS modules to be tested against, and that the Dirac channel $\bar{L}\_L\ \tilde{\Phi}\_cl\ \nu\_R$ is gauge-neutral if and only if ν_R is a full electroweak singlet.

13.5 CKM and PMNS

CKM and PMNS belong to basis misalignment, not to gauge-boson symmetry breaking itself. This paper does not compute CKM or PMNS. But it supplies the broken electroweak background in which the already-developed flavour-frame and mixing modules must be read — in particular, the charged-current coupling of Section 11 is the operator against which flavour-basis misalignment becomes physically visible.

The point is not: *Yukawa begins here.* The point is: *Yukawa and flavour now have a derived broken-phase electroweak background.*

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14. What This Paper Adds to the Prior Yukawa Programme

The prior Yukawa/flavour work can now be upgraded in four ways.

14.1 The Higgs-like interface is no longer assumed

Earlier mass modules may refer to a Higgs, closure-norm mode, or completion-channel interface. This paper derives the electroweak representation of that interface: (1, 2, +½). The mass modules no longer need to borrow the Standard Model Higgs representation as a background assumption. They inherit it from closure-norm electroweak breaking.

14.2 The vacuum direction is fixed

The neutral vacuum direction is $(0, 1)^T$. Any mass module attaching to the closure interface must respect the same vacuum orientation. This fixes which component participates in charged-lepton and down-sector attachment, and which conjugate interface participates in up-sector attachment.

14.3 The photon and charge operator are derived before mass attachment

Because the vacuum stabilizer is $Q = T_3 + Y$, mass attachment must preserve electric charge. This gives a clean audit rule:

any Yukawa or completion-channel operator that changes electric/source charge is inadmissible in the broken electroweak phase.

The already-developed VERSF Yukawa modules can therefore be checked against a *derived* charge-preserving vacuum, rather than an assumed one.

14.4 The updated programme role

This paper is therefore not a Yukawa-origin paper. It is a broken-phase interface theorem. It gives the existing Yukawa and flavour programme its derived electroweak vacuum, its Higgs-like representation object, its conjugate interface, and its charge-preserving attachment rule.

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15. Relation to Anomaly-Inadmissibility

The closure interface is bosonic. It does not alter the chiral anomaly ledger. The anomaly-inadmissibility theorem remains a condition on the chiral source sectors L_L, e_R, Q_L, u_R, d_R. The closure condensate acts only after the anomaly-admissible electroweak ledger exists.

This ordering matters. An anomalous source ledger could not define a stable electroweak gauge sector for the condensate to break. Anomaly-admissibility is therefore a *precondition* for the broken phase, not a consequence of it. And because the breaking preserves $Q = T_3 + Y$, the surviving electromagnetic ledger inherits its admissibility from the electroweak one.

The logical order is

chiral matter skeleton → anomaly-admissible ledger → unbroken gauge dynamics → closure-norm breaking.

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16. Relation to Gauge-Curvature Dynamics

The previous gauge-dynamics paper derives the unbroken field strengths and kinetic form. The present paper does not replace that layer; it acts on it.

The unbroken gauge dynamics supply $W_{\mu\nu}^i$, $B_{\mu\nu}$ and the kinetic energy of the gauge fields. The closure-norm condensate supplies the vacuum stiffness terms. Together they produce the broken-phase gauge-boson Lagrangian:

$$\mathcal{L}_{gauge}^{broken} = -\tfrac{1}{2} W_{\mu\nu}^{+} W^{-\mu\nu} - \tfrac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \tfrac{1}{4} A_{\mu\nu} A^{\mu\nu} + m_W^2 W_\mu^{+} W^{-\mu} + \tfrac{1}{2} m_Z^2 Z_\mu Z^\mu + \cdots$$

where the ellipsis includes gauge self-interactions, scalar fluctuations, and gauge-fixing terms not derived here. The photon remains massless.

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17. The Broken Electroweak Phase Theorem

Theorem 9 — Broken Electroweak Phase Theorem

Given:

- *the inherited chiral matter skeleton;*
- *the inherited electroweak representation algebra;*

- *the inherited unbroken W/B gauge dynamics;*
- *anomaly-admissible source ledgers;*
- *closure-norm condensation with an electroweak-facing interface satisfying weak activity, colour neutrality, and vacuum electric neutrality — whence, by Theorem 1, the minimal representation (1, 2, +½);*
- *interface stiffness $(D_\mu \Phi_{cl})^\dagger(D^\mu \Phi_{cl})$;*

the VERSF electroweak sector breaks as

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM},$$

with $Q = T_3 + Y$ as the vacuum stabilizer. The physical gauge directions are

$$W_\mu^\pm = (1/\sqrt{2})(W_\mu^1 \mp i W_\mu^2),\quad Z_\mu = (g W_\mu^3 - g' B_\mu)/\sqrt{g^2 + g'^2},\quad A_\mu = (g' W_\mu^3 + g B_\mu)/\sqrt{g^2 + g'^2},$$

with masses

$$m_W = gv/2,\quad m_Z = (v/2)\sqrt{g^2 + g'^2},\quad m_A = 0,$$

and tree-level relation $m_W/m_Z = \cos\theta_W$. The photon couples universally to electric/source charge Q with

$$e = gg'/\sqrt{g^2 + g'^2}.$$

The existing VERSF Yukawa and flavour modules now have a derived broken-phase closure interface to attach to:

$$\Phi_{cl} \sim (1, 2, +\tfrac{1}{2}),\quad \tilde{\Phi}_{cl} = i\sigma_2 \Phi_{cl}^*.$$

Proof. Theorem 1 fixes $\Phi_{cl} \sim (1, 2, +\tfrac{1}{2})$ from weak activity, colour neutrality, and vacuum electric neutrality. Theorem 2 identifies $Q = T_3 + Y$ as the unique generator annihilating the neutral closure vacuum, so the unbroken group is $U(1)_{EM}$. Theorem 3 gives the charged broken plane and $W^\pm$. Theorem 4 diagonalizes the neutral sector into Z (vacuum-disturbing) and A (vacuum-preserving). Theorem 5 extracts the mass spectrum from the stiffness term, with Corollary 5.1 giving $m_W/m_Z = \cos\theta_W$. Theorem 6 identifies the photon coupling as eQ. Theorem 8 shows colour survives. Finally, the same closure interface and its conjugate supply the representation carriers required by the already-developed Yukawa/flavour modules (Section 13, Appendix D). ■

Status. Exact given BP-1–BP-8 and the inherited electroweak stack. As a VERSF derivation, the conditionality now funnels into a single load-bearing premise. BP-2 is inherited from the finite-capacity substrate programme and independently proved necessary for the record layer. BP-7 is derived given BP-3: a committed orientation with vanishing or negative stiffness is a contradiction, and the covariant form is unique, with the inherited entropy-gradient result as corroboration. What remains is BP-3 — the electroweak interface orientation, retrodictively anchored, with its target theorem stated as BP-3′ — together with the scale v and the bookkeeping items of Debt 3.

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18. What Has Actually Been Derived

This paper derives, conditionally:

- the closure-norm interface representation (1, 2, +½);
- the neutral vacuum orientation;
- the vacuum stabilizer $Q = T_3 + Y$;
- the breaking pattern $SU(2)_L \times U(1)_Y \to U(1)_{EM}$;
- the charged weak fields W±;
- the neutral fields Z and A;
- photon masslessness;
- W and Z mass expressions in terms of g, g′, v;
- the tree-level relation $m_W/m_Z = \cos\theta_W$ and $\rho_{tree} = 1$;
- universal photon coupling to electric/source charge;
- broken-phase charged and neutral current structure;
- colour preservation by a colourless closure interface;
- the broken-phase degree-of-freedom ledger: three interface orientation modes absorbed as longitudinal W±/Z polarizations, one radial scalar surviving (Appendix A.7);
- the necessity of a nonzero closure-norm vacuum for the inherited persistent record layer (Appendix E);
- the existence, positivity, and unique form of the leading interface stiffness, derived given BP-3 and corroborated by the inherited entropy-gradient result (Appendix E);
- the representation objects (Φ_{cl}, $\tilde{\Phi}_{cl}$) required for charged-lepton, down-quark and up-quark Yukawa attachment;
- the audit rule that mass/flavour attachments must preserve the derived electric charge.

This paper does **not** newly derive:

- the Yukawa operator itself;
- the charged-lepton mass hierarchy;
- the quark mass hierarchy;
- CKM mixing;
- PMNS mixing;
- the neutrino mass mechanism;
- the numerical values of g, g′;
- the numerical value of v;
- the Higgs radial mass;
- the scalar self-coupling;
- the full closure-norm potential;
- the microscopic closure-interface transport functional and its normalization (sign and form are fixed; the functional itself is not);

- the substrate origin of the electroweak interface orientation (BP-3 / the Weak-Commitment Interface Theorem, BP-3′);
- three generations;
- full colour dynamics;
- renormalised electroweak precision structure.

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19. Open Debts

Debt 1 — Microscopic closure-norm potential and the scale v. The prior closure-substrate programme supports a stable nonzero closure phase, and Appendix E proves that the inherited persistent record layer cannot be supported on a vanishing closure norm — so nonzero condensation is a necessity, not a free assumption. What remains owed is the microscopic stability functional: why a stable nonzero branch exists dynamically, and why its electroweak value is v.

Debt 2 — The Weak-Commitment Interface Theorem (the BP-3 origin debt). This is the main remaining load-bearing debt of the electroweak chain. The paper requires an electroweak-facing closure orientation χ; a future theorem must derive χ from weak-branch commitment and chiral closure transport, *without* appealing to the observed W/Z mass pattern. The target statement is fixed in BP-3′, with its own falsifier (weak-commitment closure implementable entirely by singlet data). If proved, BP-3 upgrades from Conditional to Derived and the electroweak chain becomes VERSF-native end to end.

Debt 3 — Microscopic closure-interface transport functional. The stiffness question is substantially settled: given BP-3, existence and positivity are forced ($c < 0$ destabilizes the committed phase; $c = 0$ contradicts commitment itself), the covariant form is unique, and the inherited entropy-gradient surface tension ($\kappa \sim \hbar c \xi > 0$) independently corroborates the sign. What remains owed is the full microscopic derivation of the exact electroweak closure-interface transport functional and its normalization, together with the corroborating audit extending the inherited gradient result from scalar closure order parameters to the oriented interface. Neither item is load-bearing for the broken-phase theorem; both are required for a complete substrate account.

Debt 4 — Higgs radial dynamics. The radial closure-norm excitation is the natural Higgs-like scalar candidate, but its mass and potential are not derived here.

Debt 5 — Gauge coupling values. The paper derives structural relations but not the numerical values or running of g, g′.

Debt 6 — Broken-phase integration of existing Yukawa/flavour modules. The Yukawa, quark, lepton, CKM and PMNS modules should now be audited against the derived closure-interface vacuum. This is an integration/audit debt, not an origin-from-scratch debt. The required audit is:

- charged-lepton channel attaches through Φ_{cl};
- down-quark channel attaches through Φ_{cl};
- up-quark channel attaches through $\tilde{\Phi}_{cl}$;
- neutrino channel depends on the sterile/weak-commitment branch selected by the existing neutrino module;
- all attachment operators preserve the derived electromagnetic charge;
- all attachment operators respect the anomaly-admissible matter ledger.

Debt 7 — Neutrino mass sector integration. The optional sterile ν_R, weak-commitment neutrino mechanism, PMNS structure, and possible Majorana/Dirac distinction must be reconciled with the broken-phase closure interface.

Debt 8 — Full colour gauge dynamics. Colour remains unbroken because the closure interface is colourless, but the continuous colour lift and QCD dynamics remain owed.

Debt 9 — Three generations. This paper is one-generation at the electroweak gauge-boson level. It does not derive replication.

Debt 10 — Quantum electroweak completion. Gauge fixing, ghost structure, BRST symmetry, renormalisation, loop corrections, and precision electroweak observables remain outside this paper.

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20. Falsification Conditions

F1 — No closure condensate. If the closure norm does not condense, the broken electroweak phase does not follow.

F2 — Wrong interface representation. If the closure interface is not (1, 2, +½) or an equivalent minimal neutral weak-active representation, the mass and photon results change.

F3 — Charged vacuum. If the closure vacuum is not electrically neutral, electromagnetism is broken and the photon is not recovered.

F4 — Colour-breaking interface. If the closure interface carries colour, colour is broken and the low-energy structure fails.

F5 — No interface stiffness. If gauge directions disturbing the closure vacuum do not acquire stiffness, W/Z masses are not derived.

F6 — Photon mass leakage. If the vacuum-preserving direction nevertheless gains mass, the stabilizer theorem fails.

F7 — W/Z mass-ratio failure. If the minimal doublet closure interface does not produce $m_W/m_Z = \cos\theta_W$ at tree level — equivalently, if $\rho_{tree} \neq 1$ for the pure doublet — the stiffness construction fails. Conversely, a robust experimental departure of the ρ parameter from unity (beyond the established custodial-breaking radiative structure) would falsify the minimal-interface premises BP-4 and BP-8.

F8 — Hidden condensate obstruction. If additional condensates contribute at the same order, the minimal mass matrix is incomplete.

F9 — Anomaly failure. If the inherited source ledger is anomalous, the broken gauge sector lacks an admissible source foundation.

F10 — Yukawa-ordering error. If this paper is read as newly deriving the quark/lepton Yukawa operators or CKM/PMNS structures already developed elsewhere, the programme ordering has been misrepresented.

F11 — Attachment incompatibility. If the existing VERSF Yukawa/flavour modules cannot attach to the derived closure interface without violating electroweak charge, weak representation, or anomaly admissibility, then the mass/flavour integration fails even if the gauge-boson breaking theorem survives.

F12 — Coupling-value overclaim. If the paper is read as computing numerical g, g' or v, the claim has been overstated. It derives relations, not values.

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21. Milestone Statement

The result of this paper is:

derived unbroken electroweak gauge fields + closure-norm condensate + minimal neutral weak-active interface $\Longrightarrow SU(2)_L \times U(1)_Y \to U(1)_{EM}$,

with

$A_\mu = (g' W_\mu^3 + g B_\mu)/\sqrt{g^2 + g'^2}$, $Z_\mu = (g W_\mu^3 - g' B_\mu)/\sqrt{g^2 + g'^2}$, $W_\mu^\pm = (1/\sqrt{2})(W_\mu^1 \mp i W_\mu^2)$,

and mass structure

$m_A = 0$, $m_W = gv/2$, $m_Z = (v/2)\sqrt{g^2 + g'^2}$.

The photon is the vacuum-preserving direction. The W and Z are the vacuum-disturbing directions.

The paper also supplies the missing broken-phase interface for the already-developed VERSF mass/flavour modules:

$\Phi\_cl \sim (1, 2, +½)$, $\tilde{\Phi}\_cl = i\sigma_2 \Phi\_cl^*$.

That is the intended milestone.

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22. Conclusion

The VERSF programme has already derived the fermionic matter layer, the chiral matter representation skeleton, the electroweak representation algebra, the local W/B connection, anomaly-admissible source ledgers, and unbroken electroweak gauge dynamics.

The present paper moves one structural layer forward.

It shows how closure-norm condensation breaks the unbroken electroweak gauge structure. The closure-norm interface must be weak-active, colourless, and electrically neutral. These requirements fix the minimal interface representation as (1, 2, +½). Its neutral vacuum is preserved by one generator only:

$Q = T_3 + Y$.

That generator is electromagnetism. The photon is the corresponding massless connection direction.

The other electroweak directions disturb the closure-norm vacuum. They acquire stiffness, which appears as mass. The charged broken directions become W±. The neutral broken direction becomes Z. The orthogonal neutral unbroken direction becomes the photon.

The paper therefore derives the broken electroweak gauge-boson architecture — W±, Z, A — with

$m\_W = gv/2$, $m\_Z = (v/2)\sqrt{(g^2 + g'^2)}$, $m\_A = 0$,

and the tree-level relation $m\_W/m\_Z = \cos \theta\_W$, whose experimental proximity to unity in the ρ parameter is the sharpest empirical vote for the minimal doublet interface derived here.

The closure-substrate support layer and Appendix E reduce the paper's three load-bearing premises to a single residue. Nonzero closure-norm condensation is inherited from the finite-capacity substrate programme and independently proved necessary for the record layer. Closure stiffness is no longer even an inherited premise: given a committed electroweak-facing orientation, its existence and positivity are forced — a soft or unstable orientation is not committed — and its covariant form is unique, with the inherited entropy-gradient surface tension as independent corroboration. What genuinely remains assumed is one clause: that the condensed closure phase carries electroweak orientation at all. Its target theorem is stated precisely as the Weak-Commitment Interface Theorem (BP-3′), and the weak-commitment module is the natural place to prove it.

The ordering update made in this paper is important. It does not treat Yukawa, quark, lepton, CKM or PMNS work as absent. Those are already substantial VERSF modules. What this paper does is give them their derived broken-phase electroweak home: the closure-interface doublet, its conjugate, the neutral vacuum, and the charge-preserving attachment rule.

Appendix A — Mass Matrix Calculation

A.1 Setup

In unitary vacuum orientation,

$\Phi_{cl} = (1/\sqrt{2})\,(0, v)^T$,

and the gauge part of the covariant derivative is

$D_\mu \Phi_{cl} = i\,(g\, W_\mu^i\, \sigma_i/2 + g'\, B_\mu\, Y)\, \Phi_{cl}$, $Y = +\frac{1}{2}$ on Φ_{cl}.

A.2 Component evaluation

With σ_i the Pauli matrices,

$(\sigma_1/2)(0, 1)^T = \frac{1}{2}\,(1, 0)^T$, $(\sigma_2/2)(0, 1)^T = -(i/2)\,(1, 0)^T$, $(\sigma_3/2)(0, 1)^T = -\frac{1}{2}\,(0, 1)^T$, $Y\,(0, 1)^T = +\frac{1}{2}\,(0, 1)^T$.

Therefore

$D_\mu \Phi_{cl} = (iv/2\sqrt{2})\,\left(g(W_\mu^1 - i\, W_\mu^2),\, -g\, W_\mu^3 + g'\, B_\mu \right)^T$.

A.3 Stiffness contraction

$(D_\mu \Phi_{cl})^\dagger(D^\mu \Phi_{cl}) = (v^2/8)\,[\, g^2\,((W_\mu^1)^2 + (W_\mu^2)^2) + (g\, W_\mu^3 - g'\, B_\mu)^2\,]$.

A.4 Charged sector

Using $W_\mu^\pm = (1/\sqrt{2})(W_\mu^1 \mp i\, W_\mu^2)$, one has $(W_\mu^1)^2 + (W_\mu^2)^2 = 2\, W_\mu^+ W^{-\mu}$, so the charged term is

$(g^2v^2/4)\, W_\mu^+ W^{-\mu} \Longrightarrow m_W^2 = g^2v^2/4$, $m_W = gv/2$.

A.5 Neutral sector

In the (W^3, B) basis the neutral quadratic form is

$(v^2/8)\,(W_\mu^3, B_\mu)\, M^2\, (W^{3\mu}, B^\mu)^T$, $M^2 = \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}$.

The matrix M^2 has

$\det M^2 = g^2g'^2 - g^2g'^2 = 0$, $\operatorname{tr} M^2 = g^2 + g'^2$,

so its eigenvalues are 0 and $g^2 + g'^2$. The null eigenvector is $(g', g)/\sqrt{g^2 + g'^2}$ — the photon A_μ — and the massive eigenvector is $(g, -g')/\sqrt{g^2 + g'^2}$ — the Z_μ. Restoring the canonical $\frac{1}{2} m_Z^2 Z^2$ normalization gives

$m_Z^2 = (g^2 + g'^2)\, v^2/4$, $m_Z = (v/2)\sqrt{g^2 + g'^2}$, $m_A = 0$.

The vanishing determinant is the algebraic fingerprint of the stabilizer: exactly one exact zero mode, guaranteed by Theorem 2, not tuned.

A.6 Mass ratio

$m_W^2/m_Z^2 = g^2/(g^2 + g'^2) = \cos^2\theta_W \Longrightarrow m_W/m_Z = \cos\theta_W$, $\rho_{tree} = 1$. ■

A.7 Degree-of-freedom audit

The doublet Φ_{cl} carries four real components. In unitary vacuum orientation (Section 9), three of them — the would-be Goldstone orientations along the broken generators T_1, T_2, and the vacuum-disturbing neutral combination — are removed from the scalar spectrum and reappear as the longitudinal polarizations of the massive vectors. The counting closes exactly:

before: 4 massless vectors × 2 states + 4 real scalar components = 12; after: 3 massive vectors × 3 states + 1 massless photon × 2 states + 1 radial scalar = 12.

This ledger is not bookkeeping decoration; it is part of what the paper's thesis physically means. Each massive gauge boson carrying a third polarization state is where the vacuum-disturbing orientation modes of the closure interface *went*: they are not lost in the unitary gauge choice but transferred into the gauge sector as the longitudinal states of $W^\pm$ and Z. "Acquiring stiffness" is, in degree-of-freedom terms, the absorption of an interface orientation mode.

Exactly one scalar mode survives — the radial closure-norm excitation — and it is the object at which the Higgs-radial firewall (Section 0.6) and Debt 4 point. One further honesty note: no potential $V(\Phi_{cl})$ appears in this paper, so at this order the radial direction is flat. Its stabilization at the scale v is precisely the content of Debts 1 and 4, not of the present construction; a reader asking "what fixes the radius?" is asking the right question, and the paper's answer is that it is owed.

Appendix B — Convention Audit

B.1 Hypercharge normalization

This paper uses $Q = T_3 + Y$. The common alternative uses $Q = T_3 + Y\_SM/2$, i.e. $Y\_SM = 2Y$. Under this rescaling:

- the interface hypercharge $Y = +½$ becomes $Y\_SM = +1$;
- the covariant derivative term $g'B\_\mu Y$ becomes $(g'/2)B\_\mu Y\_SM$ with the *same* physical coupling once g' is co-rescaled, or equivalently $g'\_SM\ Y\_SM/2 = g'\ Y$;
- every product $Y\cdot B\_\mu$ appearing in Sections 8–10 and Appendix A is invariant;
- the physical outputs e, θ_W, m_W, m_Z, m_A are convention-independent.

B.2 Sign conventions

The signs in $W\_\mu\pm = (1/\sqrt{2})(W\_\mu^1 \mp iW\_\mu^2)$ fix W^+ as the field that raises electric charge in the current coupling $W\_\mu^+J_+\hat{}\mu$ (Section 11). Flipping the sign convention exchanges $W^+ \leftrightarrow W^-$ and leaves all masses and the charge assignment $|Q(W\pm)| = 1$ unchanged.

B.3 Vacuum component choice

Placing the neutral component in the upper slot instead of the lower forces $Y\_\Phi = -½$ and reproduces the conjugate doublet $\tilde{\Phi}\_cl$. All stabilizer, mass, and coupling results are unchanged (Appendix C.4).

B.4 Metric and normalization

The stiffness term uses canonical kinetic normalization for Φ_cl and the gauge fields, matching the inherited $-¼F^2$ conventions of the gauge-curvature paper. The factor $v/\sqrt{2}$ in the vacuum is fixed by requiring the radial excitation to be canonically normalized; it is a bookkeeping choice that cancels in all mass ratios.

Appendix C — Why Alternative Vacuum Representations Fail

Theorem 1 asserts minimality. This appendix disposes of the alternatives.

C.1 Weak singlet, Y ≠ 0

A weak singlet with hypercharge would break U(1)_Y alone if condensed. Since T_i act trivially, $D\_\mu\Phi$ contains only $g'B\_\mu Y\Phi$: B acquires mass, W^1, W^2, W^3 do not, and — worse — if $Y \neq 0$ the vacuum is not annihilated by $Q = T_3 + Y$, so electromagnetism itself is broken while the weak bosons stay massless. This inverts the observed structure. Excluded by BP-4 and BP-5.

C.2 Weak singlet, $Y = 0$

Fully inert: no gauge boson acquires mass. Not a breaking sector at all. Excluded by BP-4.

C.3 Weak triplet $(1, 3, Y)$

A real triplet with $Y = 0$ or a complex triplet with $Y = 1$ can contain a neutral component and can break $SU(2)_L \times U(1)_Y \to U(1)_{EM}$. But the triplet stiffness produces

$\rho_{tree} \neq 1$

(the real $Y = 0$ triplet gives $m_Z = 0$ with massive W, i.e. $\rho \to \infty$; the complex $Y = 1$ triplet gives $\rho_{tree} = ½$). Both are excluded by the tree-level relation derived in Corollary 5.1 and by the empirical proximity of ρ to unity, and both violate minimality (BP-4) in any case. A triplet may appear only as a subleading admixture, which BP-8 excludes at leading order.

C.4 Conjugate doublet $(1, 2, -½)$

Not an alternative: $\tilde{\Phi} = i\sigma_2\Phi^*$ is the same physical content with the neutral component in the upper slot. Its stabilizer is again $Q = T_3 + Y$ and its mass matrix is identical. It is used constructively as the up-sector attachment carrier.

C.5 Higher representations

Any representation of weak isospin $T > 1$ with a neutral component produces $\rho_{tree} = [T(T+1) - T_3^2]/(2T_3^2)$ evaluated on the neutral slot, which equals 1 only for the doublet values $(T, T_3) = (½, ±½)$ among small representations (the next solution, $T = 3$, $|T_3| = 2$, is far from minimal). Minimality and the ρ-parameter thus jointly select the doublet.

Conclusion. The doublet $(1, 2, +½)$ is not merely the minimal choice; among representations compatible with the observed $\rho_{tree} = 1$ it is essentially the *only* small choice. ■

Appendix D — Broken-Phase Attachment Audit for Existing Yukawa Modules

This appendix records the gauge-representation audit that the existing VERSF mass/flavour modules must pass against the derived closure interface. Hypercharges are in the $Q = T_3 + Y$ convention: $Y(L_L) = -½$, $Y(e_R) = -1$, $Y(Q_L) = +1/6$, $Y(u_R) = +2/3$, $Y(d_R) = -1/3$, $Y(\Phi_{cl}) = +½$, $Y(\tilde{\Phi}_{cl}) = -½$.

Channel	Operator form	SU(3)_C	SU(2)_L	ΣY	Q-preserving	Verdict
Charged lepton	L̄_L Φ_cl e_R	1·1·1 = 1 ✓	2̄ ⊗ 2 ⊃ 1 ✓	+½ + ½ − 1 = 0 ✓	✓	Admissible
Down quark	Q̄_L Φ_cl d_R	3̄ ⊗ 3 ⊃ 1 ✓	2̄ ⊗ 2 ⊃ 1 ✓	−1⁄6 + ½ − 1⁄3 = 0 ✓	✓	Admissible
Up quark	Q̄_L Φ̃_cl u_R	3̄ ⊗ 3 ⊃ 1 ✓	2̄ ⊗ 2 ⊃ 1 ✓	−1⁄6 − ½ + 2⁄3 = 0 ✓	✓	Admissible
Dirac neutrino (optional)	L̄_L Φ̃_cl ν_R	1 ✓	2̄ ⊗ 2 ⊃ 1 ✓	+½ − ½ + 0 = 0 ✓ iff ν_R ~ (1,1,0)	✓	Conditional on ν_R module
Wrong-interface up	Q̄_L Φ_cl u_R	1 ✓	⊃ 1 ✓	−1⁄6 + ½ + 2⁄3 = 1 ✗	✗	Inadmissible
Wrong-interface down	Q̄_L Φ̃_cl d_R	1 ✓	⊃ 1 ✓	−1⁄6 − ½ − 1⁄3 = −1 ✗	✗	Inadmissible

Audit rules for module integration (Debt 6):

1. Every attachment operator must be a full electroweak singlet before condensation, hence Q-preserving after condensation.
2. Charged-lepton and down-quark modules attach through Φ_cl; the up-quark module attaches through Φ̃_cl; no other pairing is admissible.
3. Each admissible channel converts a coupling operator into a mass operator with weight v/√2 upon condensation; the coefficient structure (hierarchies, mixing) belongs entirely to the existing mass/flavour modules.
4. The neutrino row remains open pending the existing neutrino module's branch selection (sterile Dirac, Majorana, weak-commitment, or other mechanism).
5. Any operator failing rows 1–2 is inadmissible in the broken phase regardless of its status in any earlier flavour-frame construction — this is falsifier F11.

The audit shows that the derived closure interface (1, 2, +½) and its conjugate provide exactly — and only — the attachment channels the existing VERSF Yukawa modules require. This table is also the locus of the programme's genuine cross-paper consistency check (cf. the Reading of Theorem 2): the hypercharges entering ΣY were fixed by the earlier representation papers, the vacuum stabilizer was fixed by the breaking analysis, and nothing in either construction guaranteed in advance that the same $Q = T_3 + Y$ would close every channel. ■

Appendix E — Closure-Substrate Support for the Load-Bearing Premises

The Broken Electroweak Phase Theorem rests on three non-inherited inputs:

1. a nonzero closure-norm condensate (BP-2);
2. an electroweak-facing closure-interface orientation with minimal admissible representation (BP-3–BP-6);
3. a leading stiffness cost for gauge directions that disturb that interface (BP-7).

This appendix sharpens the status of each, drawing on the closure-substrate support layer of Section 0‴. The stronger framing established here is:

VERSF already possesses a finite-capacity, stiffness-bearing closure substrate. This paper proves that, when that substrate is coupled to the derived electroweak connection, the minimal admissible broken-phase interface is exactly the Higgs-doublet-form closure interface, and the leading disturbance cost is exactly the covariant stiffness term.

The three premises do **not** end at the same grade, and the honest accounting of the differences is the point of this appendix:

- premise 1 is *inherited* from the finite-capacity substrate programme **and** independently *proved necessary* relative to the inherited record layer;
- premise 2 is *forced given that electroweak breaking occurs* — a retrodictive anchor, with its missing forward theorem stated explicitly as BP-3′, not yet a substrate derivation;
- premise 3 is *derived given premise 2*: a committed electroweak-facing orientation with vanishing or negative stiffness is a contradiction, so existence and sign are forced and the covariant form is unique; the structural-closure entropy-gradient result stands as independent corroboration, with one bookkeeping audit (Debt 3).

The appendix does not compute the numerical value of v, the Higgs radial mass, or the scalar self-coupling. Those remain separate debts.

E.1 Scope and inheritance sources

The proofs below use the inherited stack of Section 0″ together with the closure-substrate support layer of Section 0‴:

- persistent record-layer transport (Two-Planck / Route M setting);
- finite substrate stress and transport capacity with saturation response (void-tension programme);
- entropy-gradient cost at coherence boundaries, with a Landau–Ginzburg-type closure functional (structural-closure programme);
- a stable mesoscopic coherence layer (dimensional-transmutation stability work);
- the derived electroweak representation algebra and local connection;
- anomaly-admissible chiral source sectors and unbroken gauge-curvature dynamics.

The appendix therefore proves its results **relative to that inherited infrastructure**. The precise combined statement is:

finite closure substrate + derived electroweak connection + minimal preservation requirements $\Longrightarrow \Phi_{cl} \sim (1, 2, +\frac{1}{2})$, with leading stiffness $(D_\mu\Phi_{cl})^\dagger(D^\mu\Phi_{cl})$.

The three clauses feeding this implication carry the three distinct grades listed above. They must not be flattened into a single "the premises are proved."

E.2 First load-bearing premise — the closure norm condenses

Prior substrate support. The void-tension and closure-norm papers establish that the VERSF substrate is not infinitely deformable: it has finite stress capacity, finite entropy-transport capacity, and a saturation response. A substrate of this kind supports coherent structure precisely because it resists arbitrary deformation. The existence of a stable nonzero closure branch is therefore not a fresh postulate of this paper; it is the working conclusion of the substrate programme. What the present paper adds is an *independent internal necessity argument*, so that the premise stands even for a reader who brackets the substrate papers:

Proposition E.1 — Persistent record transport requires nonzero closure norm. *A persistent low-energy record layer cannot be supported on a vacuum with identically vanishing closure norm.*

Proof. The closure norm ρ measures the degree to which the substrate has entered a committed, refinement-stable record phase. A phase with $\rho = 0$ is uncommitted: it contains no stable local standard against which relational transport can be persistently compared, and no substrate distinction between vacuum-preserving and vacuum-disturbing deformations. But the inherited electroweak stack already contains persistent local matter representations, a local electroweak connection, and unbroken gauge-curvature dynamics — structures that require a stable record layer on which local comparison is defined. If $\rho = 0$ everywhere, that comparison background does not exist and the inherited stack is unsupported. Therefore $\langle\rho\rangle \neq 0$. ■

Grade note on E.1. The proof is close to definitional: ρ is *defined* in the closure-norm programme as the measure of the committed record phase, so "$\rho = 0 \Longrightarrow$ no persistent records" is nearly analytic. The proposition is therefore exactly as strong as the inherited definition of ρ — no stronger — and its grade is Conditional on that definition doing real dynamical work in the closure-norm programme rather than merely labelling it. The scope note under E.2 applies here with equal force.

Proposition E.2 — The physical vacuum occupies a nonzero closure branch. *Given that persistent record transport exists — an inherited fact, not a new assumption — the physical low-energy vacuum lies on a nonzero closure-norm branch.*

Proof. The low-energy world described by the inherited stack contains persistent matter records, persistent gauge transport, and stable source ledgers: reproducible structures supporting long-lived local comparison, not transient defects in an uncommitted substrate. By Proposition E.1, such structures cannot be supported at $\rho = 0$. Therefore the vacuum on which they are supported occupies a nonzero closure branch. Denote the vacuum value

$$\langle\rho\rangle = v/\sqrt{2},$$

with normalization chosen so that the broken-phase gauge-boson mass formulae take canonical form. ■

Scope note on E.2. This proposition is deliberately conditional. It does *not* derive the dynamical existence or stability of the nonzero branch from a substrate potential — that microphysics is the substrate programme's contribution and Debt 1's residue. The proposition establishes that the world described by the inherited stack must sit on such a branch. BP-2 is thereby doubly supported: inherited from the finite-capacity substrate programme, and proved necessary relative to the record layer.

Corollary E.2.1 — What is and is not established about v. The support above establishes the existence of a nonzero closure-norm vacuum at the level required by this paper. It does not compute the numerical value of v (≈ 246 GeV in Standard Model normalization). The scale-setting problem remains open (Debt 1).

E.3 Second load-bearing premise — the electroweak-facing interface and its representation

Why the electroweak interface is still a conditional bridge. The closure norm ρ is gauge-invariant as a magnitude. If the condensate were only that scalar, the electroweak generators would act trivially on it,

$$T_i\rho = 0,\ Y\rho = 0,$$

so that $D_\mu\rho = \partial_\mu\rho$, and no quadratic mass form for W_μ^i or B_μ could arise. This proves only a *negative* statement: a singlet closure norm cannot do the electroweak-breaking job. It does not by itself prove that the closure condensate must possess an electroweak-facing orientation — to infer χ merely from the fact that the physical world contains massive W± and Z would be retrodictive. The forward VERSF task is stronger: derive χ from weak-commitment closure itself. The candidate route is that a chiral weak-branch record layer cannot complete local comparison using only a scalar closure magnitude; it must also specify a two-branch interface orientation. That target is fixed as the Weak-Commitment Interface Theorem (BP-3′) and recorded as Debt 2; it is not proved in this paper. What the present paper does prove is conditional: *if* such an electroweak-facing interface exists,

$$\Phi_{cl} = \rho\chi,$$

then weak activity, colour preservation, and electromagnetic neutrality force its minimal representation to be (1, 2, +½). This is the exact point at which the present paper extends the prior closure-substrate work: the substrate papers establish finite closure and gradient stiffness; the present paper identifies the electroweak representation the interface must carry, conditionally on its existence.

Representation forcing (compressed — the full result is Theorem 1 with Appendix C).

Proposition E.3 — Weak activity. A weak singlet is annihilated by all weak generators, so the covariant derivative contains no weak stiffness term and the weak bosons remain massless. Since the target broken phase contains massive $W^\pm$ and Z, the interface must transform nontrivially under SU(2)_L; the minimal nontrivial representation is the doublet. ■

Proposition E.4 — Colour neutrality. A colour-supported interface would give gluon directions stiffness from the same mechanism, breaking colour at the electroweak vacuum, contrary to the required low-energy structure. Hence colour singlet. ■

Proposition E.5 — Neutral vacuum component. The photon is the observed long-range massless direction, so the vacuum must be annihilated by $Q = T_3 + Y$. For the lower doublet component: $-½ + Y = 0$, hence $Y = +½$. ■

Theorem E.6 — Minimal closure-interface representation. Given E.3–E.5, the minimal closure interface is $\Phi\_cl \sim (1, 2, +½)$, uniquely at minimal order: no smaller weak-active representation exists, a coloured representation breaks colour, a non-neutral vacuum breaks electromagnetism, and higher representations are non-minimal and generically shift the tree-level mass relation (Appendix C). ■

Status note — retrodictive anchor. The proofs of E.3 and E.5 must be read with care about their epistemic direction. Both reason *from the observed broken phase backwards*: massive W/Z force weak activity; the long-range photon forces vacuum neutrality. This is legitimate anchoring — it converts the interface representation into a forced consequence of the phenomenology plus the inherited algebra — but it is not a substrate derivation. The premise chain runs "breaking occurs, therefore the interface must be (1, 2, +½)," not "the substrate implies an oriented interface, therefore breaking occurs." Coupling to the electroweak connection does not by itself force a nontrivial representation: gauge fields act trivially on singlets, and a purely scalar condensate would simply break nothing. BP-3 accordingly retains the grade *Conditional — retrodictively anchored*, and it is now the single most exposed joint in the electroweak chain. Theorem E.10 below is stated so as not to erase this distinction.

Candidate closure of BP-3 — weak-commitment orientation. *Grade: Candidate; flagged, not used.* In the weak-commitment module (the Weak Commitment Closure Kernel and the Weak-Commitment Leakage Trace), closure events are intrinsically weak-branch commitments: comparison across the two-branch weak module is part of what a commitment is in that construction. If that identification survives audit, then the condensed closure phase carries weak-branch orientation *by construction*, and BP-3 becomes inherited from the weak-commitment module rather than anchored to observed phenomenology. That would close the last substantive gap in the premise stack and make the derivation chain VERSF-native end to end. This appendix flags the route and deliberately does not lean on it; the audit belongs to the weak-commitment integration paper, and the precise target statement is fixed in BP-3′.

Corollary E.6.1 — The conjugate interface is not an independent alternative. The conjugate object $\tilde{\Phi}\_{cl} = i\sigma_2\Phi\_{cl}^* \sim (1, 2, -\tfrac{1}{2})$ is not a second breaking condensate. The broken vacuum and gauge-boson mass matrix are fixed by Φ_{cl} alone; the conjugate is the representation carrier required for up-sector Yukawa attachment (Section 13.3, Appendix D), not a second breaking direction. ■

E.4 Third load-bearing premise — stiffness: inherited existence, forced form, doubly forced sign

Inherited existence. The structural-closure programme supplies the relevant dynamical principle. Coherence boundaries carry an entropy-gradient cost: deforming coherent relational structure breaks constraints at the boundary, increases accessible microstates, and generates a surface-tension-like restoring response. At the coarse-grained level this stiffness takes the generic order-parameter form

$$C\_{grad} \sim (\partial\_\mu\Psi)^\dagger(\partial^\mu\Psi),$$

or, in the entropy-functional language of the structural-closure paper,

$$S[\Psi] = S_0 - \int d^3x \, [\, V(\Psi) + (\kappa/2)|\nabla\Psi|^2 \,], \ \kappa > 0.$$

The existence of a leading gradient cost for closure-order-parameter deformation is therefore an inherited VERSF result, not an assumption introduced in this paper. What this paper adds is the electroweak specialization: the relevant order parameter here is not the scalar closure norm alone but the oriented interface $\Phi\_{cl} = \rho\chi$, and once χ carries electroweak representation data, ordinary gradients must be replaced by covariant ones:

$$\partial\_\mu\Phi\_{cl} \longrightarrow D\_\mu\Phi\_{cl}.$$

Inheritance-audit note. The inherited gradient result was established for scalar and generic closure order parameters. Its extension to the electroweak-*oriented* interface — the claim that orientation deformation, and not only magnitude deformation, carries the entropy-gradient cost — is a minimal and natural step, but it is a step, and it is recorded as an explicit audit obligation (Debt 3). Because Proposition E.8 below derives stiffness existence and sign internally from committed orientation (BP-3), this inheritance serves as *corroboration* rather than a load-bearing input; the audit is retained because corroboration, too, must be earned.

Proposition E.7 — Closure-interface transport must be covariant. *The local comparison of the closure-interface orientation is governed by the electroweak covariant derivative.*

Proof. The closure interface carries electroweak representation data. Under a local electroweak transformation, $\Phi_{cl}(x)$ rotates in weak-hypercharge space, and the plain derivative $\partial_\mu\Phi_{cl}$ does not transform covariantly under such local rotations. The inherited electroweak connection is $A_\mu^{EW} = gW_\mu^i T_i + g'B_\mu Y$, and the unique first-order local derivative that compares electroweak-oriented objects while preserving local gauge covariance is

$$D_\mu\Phi_{cl} = \partial_\mu\Phi_{cl} + igW_\mu^i T_i\Phi_{cl} + ig'B_\mu Y\Phi_{cl}.$$

Any leading local transport cost for the closure interface must therefore be built from $D_\mu\Phi_{cl}$, not from $\partial_\mu\Phi_{cl}$ alone. ■

Proposition E.8 — Uniqueness, existence, and positivity of the leading stiffness functional. *Given a committed electroweak-facing closure interface (BP-3), the leading local transport cost is*

$$\mathcal{L}_{stiff} = c\,(D_\mu\Phi_{cl})^\dagger(D^\mu\Phi_{cl}),\quad c > 0,$$

with the form unique up to normalization and the coefficient forced: $c \leq 0$ contradicts the committed record phase. Canonical normalization sets $c = 1$.

Proof. *Form.* A leading transport cost must be (i) local, (ii) gauge-invariant, (iii) quadratic at leading order near a stable branch, and (iv) lowest order in derivatives. By Proposition E.7 the derivative must be D_μ. Any term linear in $D_\mu\Phi_{cl}$ is not a scalar cost and provides no quadratic mass form; any higher-derivative term is subleading at low energy; any non-covariant term violates (ii); any quadratic pairing other than the Hermitian contraction fails gauge invariance or reality. The unique candidate at leading order is therefore $c(D_\mu\Phi_{cl})^\dagger(D^\mu\Phi_{cl})$.

Negative coefficient excluded — record persistence. Suppose $c < 0$. Then variation of the interface orientation across the continuum *lowers* the transport cost, and the uniform committed phase is unstable against orientation texture: the substrate prefers orientation gradients rather than suppressing them, arbitrarily fine gradients are energetically favoured, and local weak-facing records do not persist as stable background data. That contradicts the record-layer persistence established in Propositions E.1–E.2.

Vanishing coefficient excluded — commitment itself. Suppose instead $c = 0$. Then orientation gradients cost nothing at leading order: the interface has no restoring response against weak-branch orientation variation, the vacuum orientation is soft rather than committed, and the distinction between vacuum-preserving and vacuum-disturbing electroweak directions is not dynamically supported. A soft orientation is not committed record data — it contradicts BP-3 itself, since commitment in the VERSF sense is refinement-stable resistance to deformation. Therefore, given BP-3, $c > 0$.

Corroboration — inherited surface tension. Independently, the structural-closure programme derives a positive gradient stiffness from coherence-boundary surface tension: broken coherence constraints create an entropy-gradient cost, giving a Landau–Ginzburg-type functional with gradient term $(\kappa/2)|\nabla p|^2$, $\kappa \sim \hbar c\xi > 0$ (Section 0‴). The sign is thus fixed twice over — internally by commitment and persistence, externally by inherited surface tension — and is in no sense an imposed aesthetic condition.

Normalization. The magnitude of c is absorbed into the normalization of Φ_cl and v; canonically, $c = 1$. ■

Scope note on E.8 — the residual. Given BP-3, both the existence and the sign of the leading stiffness are forced; BP-7 is therefore *derived given BP-3*, and the paper's entire conditional structure funnels into the single orientation premise. Two bookkeeping items remain, neither load-bearing: the microscopic transport functional and its normalization, and the corroborating audit extending the inherited entropy-gradient result to the oriented interface (both Debt 3). The case $c = 0$ survives only as falsifier F5 — the scenario in which BP-3's committed orientation somehow exists without the resistance that commitment entails, which would signal an inconsistency in the commitment concept itself rather than a free parameter.

Theorem E.9 — Vacuum-disturbing directions acquire stiffness. *Let the closure vacuum be $\langle\Phi\_cl\rangle = (v/\sqrt{2})(0, 1)^T$ with $\Phi\_cl \sim (1, 2, +\frac{1}{2})$, and let BP-3 hold, so that Proposition E.8 supplies a positive leading transport cost. Then a gauge direction generated by G receives a mass term from closure-interface stiffness if and only if $G\langle\Phi\_cl\rangle \neq 0$, and remains massless if and only if $G\langle\Phi\_cl\rangle = 0$.*

Proof. At the vacuum, radial fluctuations are set aside and $\partial\_\mu\langle\Phi\_cl\rangle = 0$, so

$$D\_\mu\langle\Phi\_cl\rangle = i(gW\_\mu^i T_i + g'B\_\mu Y)\langle\Phi\_cl\rangle,$$

and the stiffness term becomes $(D\_\mu\langle\Phi\_cl\rangle)^\dagger(D^\wedge\mu\langle\Phi\_cl\rangle)$ — a positive semidefinite quadratic form in the gauge fields. A gauge field appears in this form precisely when its generator acts nontrivially on the vacuum. If $G\langle\Phi\_cl\rangle \neq 0$, the contribution is strictly positive (positivity by Proposition E.8) and appears as a gauge-boson mass term. If $G\langle\Phi\_cl\rangle = 0$, the direction lies in the vacuum stabilizer and receives nothing. Gauge-boson mass is exactly closure-interface disturbance. ■

Corollary E.9.1 — The photon is protected by the stabilizer. For the neutral vacuum component, $(T_3 + Y)\chi_0 = 0$ (Theorem 2), so $Q = T_3 + Y$ annihilates the vacuum and the corresponding gauge direction is exactly massless — the photon. The charged generators T_1, T_2 and the orthogonal neutral combination $gW^3 - g'B$ do not annihilate the vacuum; these become $W\pm$ and Z from the same substrate mechanism. Photon masslessness is thus doubly protected: algebraically by the stabilizer (Theorem 2) and dynamically by the null direction of the stiffness form (Appendix A.5). ■

E.5 Consolidated load-bearing premise theorem

Theorem E.10 — Load-Bearing Premise Reduction Theorem. *Relative to the inherited VERSF closure-substrate and electroweak stack, the three load-bearing premises of the Broken Electroweak Phase Theorem reduce to the following graded statements:*

1. *(Inherited from the finite-capacity substrate programme; independently proved necessary for the record layer.)* The physical vacuum lies on a nonzero closure-norm branch (Section 0‴; Propositions E.1–E.2).
2. *(Proved — given that electroweak breaking occurs; substrate derivation open.)* The condensate must carry an electroweak-facing interface orientation, and its minimal admissible representation is $\Phi_{cl} \sim (1, 2, +½)$ (Theorem E.6, restating Theorem 1).
3. *(Derived given BP-3; corroborated by inheritance.)* Given a committed electroweak-facing orientation, the leading disturbance cost exists, is positive, and is uniquely $(D_\mu\Phi_{cl})^\dagger(D^\mu\Phi_{cl})$ (Propositions E.7–E.8) — the electroweak-covariant specialization of the prior closure-gradient stiffness, with the inherited entropy-gradient result (Section 0‴) as independent corroboration.

Proof. Statement 1 is Section 0‴ with Propositions E.1–E.2. Statement 2 is Propositions E.3–E.5 with Theorem E.6. Statement 3 is Propositions E.7–E.8 with Theorem E.9, corroborated by Section 0‴. Substitution of the resulting interface into the inherited electroweak covariant derivative yields the broken-phase structure of the main text: $SU(2)_L \times U(1)_Y \to U(1)_{EM}$, with the photon as the vacuum-preserving direction and $W^\pm$, Z as the vacuum-disturbing directions. ■

Reading — what this theorem does and does not establish. The Broken Electroweak Phase Theorem is not an imported Standard Model Higgs assumption. It is the electroweak specialization of the prior VERSF closure-substrate stiffness mechanism, with the minimal interface representation fixed by Standard-Model-facing preservation requirements. But the reduction is not a completion, and the grades in the statement are load-bearing. The consolidation should be stated exactly: because Statement 3 is derived *given* Statement 2's orientation claim, the paper's entire conditional structure funnels into a single load-bearing item — the *orientation* premise, that the condensed closure phase carries electroweak interface orientation at all (BP-3, currently anchored by the observed broken phase, with its forward target stated as the Weak-Commitment Interface Theorem, BP-3′, Debt 2). The extension audit and the microscopic transport functional (Debt 3) are bookkeeping, not load-bearing. This appendix must not be cited as deriving all three premises from substrate principles: it derives two of them conditionally on the third, and states the third's missing theorem precisely.

E.6 Remaining debt after the strengthening

The strengthening above narrows the open debts but does not eliminate them. In current order of exposure:

1. **The Weak-Commitment Interface Theorem (BP-3′; Debt 2).** The single load-bearing debt of the electroweak chain: derive the electroweak-facing orientation χ from weak-

branch commitment and chiral closure transport, without appealing to the observed W/Z mass pattern. If proved, BP-3 upgrades from Conditional to Derived, BP-7 follows automatically, and the chain becomes VERSF-native end to end. If refuted — weak-commitment closure implementable entirely by singlet data — the broken-phase result survives only as conditional reconstruction.

2. **Microscopic closure-interface transport functional (Debt 3).** Sign and form are settled; the exact substrate transport functional and its normalization are owed, together with the corroborating audit extending the inherited entropy-gradient result to the oriented interface. Bookkeeping, not load-bearing.
3. **Microscopic closure-norm potential and the scale v (Debt 1).** Existence of the nonzero branch is supported and proved necessary; its stability functional and numerical value are not derived.
4. **Higgs radial mass and self-coupling (Debt 4).** Untouched by this appendix.
5. **Loop-level electroweak precision structure (Debt 10).** Gauge fixing, ghosts, renormalisation, precision observables.
6. **Yukawa/flavour integration (Debt 6).** The attachment audit of Appendix D remains to be run against the existing modules.

The key upgrade is claim-status, not numerical completion. The paper no longer merely assumes an electroweak Higgs-like doublet with a kinetic term. Given the inherited finite-capacity closure substrate and the derived electroweak connection, the minimal closure interface capable of breaking $SU(2)_L \times U(1)_Y$ while preserving $U(1)_{EM}$ and colour must have Higgs-doublet form, and — given that a committed orientation exists at all — its leading disturbance cost can only be the positive covariant stiffness term. One theorem now stands between the substrate and the photon, and it is stated. What remains owed is named, graded, and located.