The Charged-Lepton Absolute Scale Theorem in VERSF

▲ Programme Milestone — Flavour-Magnitude and Mass-Hierarchy Series Gate CLAS-1 / Dimensionful-Origin No-Go, Primitive-Scale Normal Form, Canonical Chiral Lift, Generalised Singular-Mass Spectrum, Scale-Locking Identity, Explicit Localisation–Closure Magnitude Operator, Colourless Projection Scale, Planck-Basis Anchor Equivalence, and Numerical Non-Insertion Audit

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General Reader Summary

The electron, the muon and the tau are three copies of the same kind of particle. They behave identically in every interaction — except that the muon weighs about 207 times as much as the electron, and the tau about 17 times as much as the muon. Nobody knows why. The Standard Model simply writes these three masses in by hand, as measured numbers with no explanation.

This paper is about two questions that sound like one question but are not.

The first is the question of *pattern*: why 207, why 17? The second is the question of *absolute size*: why is the electron's mass 0.511 MeV rather than twice that, or a millionth of it? To see why these are different questions, imagine knowing three people's heights only as ratios — one is twice as tall as another, four times the third. You still have no idea whether they are dolls or giants. Ratios, however precise, can never tell you an actual size.

The first half of the paper proves this rigorously. No amount of dimensionless structure — counting arguments, symmetry patterns, hierarchies — can ever produce a mass in physical units. Multiply all three masses by the same number and every ratio, every pattern, every ordering survives untouched. So something carrying actual units must enter from somewhere, and the paper proves that exactly one such anchoring quantity, honestly derived, is also enough to pin all three masses at once. This sounds obvious, but proving it formally turns it into a trap-detector: any theory claiming to get absolute masses from pure structure has smuggled a number in somewhere, and the theorem tells you where to look.

The second half builds VERSF's candidate answer, from three pieces.

The ladder. Each generation sits one "refinement step" deeper in the underlying structure than the last, and each step multiplies the mass by the same factor — about 207, a number derived from geometry (written e^(16/3)), not fitted to anything. Prediction: the muon should be about 207 times the electron. Measurement: 206.8. That is a match to about a sixth of one percent, with no adjustable knob. In fact, run backwards, the measured ratio pins the geometric exponent to three hundredths of a percent of its derived value.

The tau's extra toll. The tau's step gets divided by 12. That number comes from counting: the structure that maintains a particle's identity has twelve working channels, and at the tau's depth the machinery saturates, leaving only one channel's worth of effectiveness — one twelfth. Prediction: tau over muon should be 207/12, about 17.3. Measurement: 16.8.

The overall size. The electron's absolute mass comes out as a simple combination of two famous quantities — the fine-structure constant squared, times the electroweak energy scale, divided by 8π. That gives 0.522 MeV against the measured 0.511 MeV.

Putting it together, all three masses land high by 2 to 5 percent: electron +2.1%, muon +2.3%, tau +5.0%. And because the construction is a ladder times one toll times one overall size, those errors split cleanly into exactly three independent discrepancies — a 2.1% error in the overall size, a 0.17% error in the ladder step, and a 2.6% error in the tau's toll — each pointing at a specific unfinished piece of the theory.

The single cleanest test needs no units and no constants at all. Because the ladder steps are equal and the only asymmetry is the factor 12, the construction predicts a pure relationship between the three masses: **muon^2 = 12 × electron × tau**. Nature's answer: 12.29. That one line is the sharpest way the whole idea could die — if the refined theory can't close that last 2.5%, there is no dial anywhere to rescue it.

The paper is deliberately blunt about what has and has not been achieved. No measured mass and no fitted coefficient goes into the formula — with one honestly flagged exception: the "exactly one channel survives" count behind the factor 12 is currently the value the tau's mass *requires*, not yet a value computed independently, and the paper forbids itself from calling the tau relation a prediction until that count is derived on its own. The two famous quantities in the overall size (the fine-structure constant, the electroweak scale) are imported measured values for now, not yet VERSF outputs. And the comparisons are made before the standard bookkeeping corrections of quantum field theory have been applied, so the 2–5% gaps must not yet be blamed on any particular ingredient — and, above all, must not be papered over with a fudge factor. The paper proves a theorem that makes doing so a formal act of self-falsification.

Finally, the paper anchors everything at the deepest scale physics has: the Planck mass, the natural unit built from gravity and quantum mechanics. The electron is smaller than the Planck mass by a factor of about 10^{23}, and the paper converts the entire "why this size?" question into deriving that single pure number — with strict firewalls against the tempting shortcuts.

So in one sentence: **if a handful of VERSF structures hold, the three lepton masses follow from geometry and counting with no adjustable parameters, landing within a few percent of reality — and this paper is the complete audit of that claim: what is proved, what is a candidate, what would kill it, and exactly which four calculations remain between here and a genuine prediction.**

Scope and Closure Box

Claim	CLAS-1 status
Dimensionless flavour data cannot determine an absolute mass	Closed
A common multiplicative mass freedom exists without a scale anchor	Closed
A dimensionful input or invariant is necessary	Closed
The VERSF primitive-scale normal form is derived	Closed conditionally on the listed primitive set
Left- and right-handed charged-lepton carriers are distinguished	Closed structurally
Positive kinetic metrics admit canonical normalisation	Closed
The physical tree-level masses are singular values	Closed
The corresponding generalised eigenvalue problem is basis invariant	Closed
A positive magnitude operator admits a chiral lift	Closed
Chiral orientation does not alter the mass singular values	Closed
One independent homogeneous scale anchor fixes the common scale	Closed
Absolute masses follow from scale times dimensionless spectral magnitudes	Closed
Explicit magnitude operator	Explicit candidate constructed; derivation open on E1–E5
Explicit scale from the colourless completion-interface projection	Explicit symbolic candidate supplied; derivation open on E6–E8, D12, D14′
Scale-free cross-ratio identity $m_\mu^2 = 12\, m_e m_\tau$	Algebraically proved under E3–E5 and E9; independent predictive status open
Higgs-mediated and direct-bilinear descriptions are equivalent after matching	Closed
The Higgs vacuum value and Yukawa normalisation are separately identifiable from masses alone	Disproved
Closure suppression alone fixes a mass operator	Disproved

Claim	CLAS-1 status
A monotone closure-to-mass map is automatic	Disproved
A derived monotone bridge can transfer maintenance ordering to mass ordering	Closed conditionally
The electron–muon split follows from tau suppression alone	Disproved
Branch labels must be fixed independently	Closed as requirement
Basis covariance of the complete construction	Closed
Stability under small positive perturbations	Closed
A fitted mass used as the scale anchor is a prediction	Disproved
One-point calibration may test remaining ratios	Closed as audit distinction
Running masses equal pole masses without radiative matching	Disproved
Residuals of the explicit audit may be absorbed by fitted coefficients	Disproved
Numerical derivation of v_{cl}, α_{int} and its readout scale	Open
Proof of the two-gate readout law and the $1/(8\pi)$ kernel	Open
Inverse-area law and exponential nesting derived (Sector Rescaling Theorem)	Closed conditionally on scale homogeneity
$\kappa = 8/3$ doubly derived (CP^2 focusing; QN-2 substrate anomaly descent)	Closed conditionally; stiffness identification open
$\dim(\Gamma_{min}) = 14$ theoremed (unique minimal pair (3, 4))	Closed conditionally
Boundary/interior split $14 = 2_\partial + 12_{int}$ and $N_{survive} = 1$	Conjectural — three open steps
Three-generation census doubly derived (binding growth; loop-class exhaustion)	Closed conditionally
Plateau condition on admissible continuous gating laws	Closed as constraint
Count-versus-gate discriminator for $\theta^\star$ installed	Closed as audit
Mechanism attribution (localisation-dominant vs σ-dominant)	Open degeneracy with named discriminators
Participation collapse at saturation (conditional theorem)	Closed conditionally on P_{maint} existence and location

Claim	CLAS-1 status
Step at exact closure forced by charged-lepton data	Disproved — natural but observationally identical to a tuned crossover in-domain
Ladder validity outside the charged-lepton sector	Disproved for a single same-form κ_q (opposite-direction quark failures)
In-domain second observable for the suppression	Absent — owned as predictive cost
Static–temporal duality of the two cost decompositions	Conjectural — structured in Appendix G with dictionary (D33) and gauge-asymmetry test
Planck-basis normal form and three-route cross-anchor test	Closed conditionally; running descent $\Phi_\ell(\mu\star)$ open; 4.18546×10^{-23} is the unmatched on-shell target pending $\mathcal{R}_e$ (D34)
Two candidate tau-suppression routes converge numerically (census 0.0833, variational 0.0846)	Physical identity and independent derivational closure remain open
Uniform κ across generation steps	Open pending generation-independence confirmation
κ-route reconciliation (8/3 vs ln 14 vs ln(16π/3))	Open
Koide invariant $Q_{VERSF} = 0.66929$ computed as scale-free cross-check	Closed as audit
Proof of single tau-step activation of the W7 cell weight	Open
Charged-lepton self-energy and pole-mass matching	Open
Empirical confirmation	Not established

Abstract

A dimensionless flavour hierarchy does not determine an absolute mass scale. This paper establishes the Charged-Lepton Absolute Scale Theorem in VERSF in two stages: an abstract stage that separates the scale-free charged-lepton magnitude structure from the unique dimensionful factor required to convert that structure into physical masses, and an explicit stage that instantiates both objects with auditable VERSF candidate operators whose remaining derivational premises are separately recorded.

Let $\mathcal{H}_L$ and $\mathcal{H}_R$ be the left- and right-handed charged-lepton flavour carriers, equipped with positive kinetic metrics $K_L > 0$ and $K_R > 0$, and let $\mathcal{B}_\ell : \mathcal{H}_R \to \mathcal{H}_L$ be a dimension-one bilinear generated after electroweak symmetry breaking. Canonical normalisation yields $M_\ell = K_L^{-1/2} \mathcal{B}_\ell K_R^{-1/2}$, whose physical tree-level masses are its singular values, equivalently the positive square roots of the generalised eigenvalues of the pencil $(\mathcal{B}_\ell^\dagger K_L^{-1} \mathcal{B}_\ell, K_R)$.

The paper first proves the common-scale no-go theorem: if all inherited VERSF flavour, closure, census and participation data are dimensionless, then $M_\ell \mapsto aM_\ell$ preserves every scale-free prediction, so absolute masses are unidentifiable until a nonzero dimensionful invariant breaks the rescaling symmetry. It then proves that every chiral lift of a positive dimensionless magnitude operator $\mathscr{H}_\ell > 0$ has the polar form $M_\ell = \mathfrak{m}_\ell W_\ell \mathscr{H}_\ell^{1/2}$ with W_ℓ unitary, so that $m_a = \mathfrak{m}_\ell \sqrt{\lambda_a(\mathscr{H}_\ell)}$; that every admissible mass scale built from the primitives $\hbar$, c, ξ, τ_s, Λ has the normal form $\mathfrak{m}_\ell = (\hbar/(c\xi))\cdot F_\ell(c\tau_s/\xi, \Lambda\xi^2, \mathbf{\mathcal{I}})$; and that any independently derived nonzero scalar anchor A_ℓ, homogeneous of degree q in the mass operator, locks the scale uniquely as $\mathfrak{m}_\ell = [A_\ell^{VERSF}/A_\ell(\hat{M}_\ell)]^{1/q}$.

The explicit stage replaces $\mathscr{H}_\ell$ and F_ℓ. The refinement-depth operator $G_\ell = \Pi_\mu + 2\Pi_\tau$ and the Role-4 inverse-area localisation law $m_g \propto e^{2\kappa g}$ with $\kappa = 8/3$ supply the unsuppressed ladder $e^{2\kappa G_\ell}$; the W7 primitive closure-cell weight $\theta\star = 1/12$, activated only on the tau branch through $R_\ell\star = \Pi_e + \Pi_\mu + (1/12)\Pi_\tau$, supplies the tau suppression; and the colourless completion-interface readout with two source gates and the proposed projection kernel $1/(8\pi)$ defines the explicit candidate common scale $m_{(\ell,0)} = \alpha_{int}^2 v_{cl}/(8\pi)$. The resulting operator

$$M_\ell^{VERSF} = (\alpha_{int}^2 v_{cl}/(8\pi))\cdot[J_e + e^{16/3} J_\mu + (e^{32/3}/12) J_\tau]$$

has singular masses $m_e = \alpha_{int}^2 v_{cl}/(8\pi)$, $m_\mu = m_e e^{16/3}$, $m_\tau = m_e e^{32/3}/12$ and the scale-free identity $m_\mu^2 = 12 m_e m_\tau$, free of continuously adjustable parameters, conditional on the discrete equal-step and one-surviving-channel premises (E3–E5, E9). When G is admitted to the primitive set, the same scale takes the equivalent Planck-basis form $\mathfrak{m}_\ell = m_P \Phi_\ell$ with $\Phi_\ell = (\ell_P/\xi)F_\ell = (\alpha_{int}^2/(8\pi))(v_{cl}/m_P)$; this converts the absolute-scale problem into the independent derivation of one dimensionless Planck-to-lepton descent invariant and supplies a three-route cross-anchor consistency test. A first numerical audit places all three masses 2–5% high with an exactly decomposable residual structure, and the paper proves that absorbing these residuals with fitted coefficients falsifies the construction's non-insertion status.

The paper also establishes basis covariance, branch-label independence requirements, perturbative stability with explicit spectral-gap margins, a closure-to-mass monotonicity criterion, a Higgs–Yukawa product-degeneracy theorem, a strict running-to-pole firewall, and numerical non-insertion and inverse-calibration theorems.

CLAS-1 therefore closes the structural passage

dimensionless VERSF magnitude operator + independently derived mass anchor → unique canonical charged-lepton mass spectrum,

and supplies an explicit candidate for every object in that passage, while withholding all numerical pole-mass and empirical claims until the completion scale, interface coupling, projection kernel, localisation exponent, closure-cell activation and radiative matching are independently derived.

Contents

Part I — Scale-Free Obstruction and Canonical Structure

1. Aim, Inheritance and Novelty

1.1 Inherited structural results

The charged-lepton programme has already separated several structural layers:

- the existence and counting of charged-lepton branches;
- the distinction between census and dynamical participation;
- the construction of positive maintenance allocations;
- the definition of the closure-deficit operator;
- the localisation of unresolved closure under a finite-capacity variational principle;
- the exact conversion between closure anisotropy and tau-sector suppression;
- the Role-4 localisation law and the inverse-area mass-localisation law;
- the W7 primitive closure-cell weight;
- the electroweak completion-interface (Higgs-radial) structure.

These results concern the structure and distribution of maintained support and the interface through which mass terms become gauge-admissible. Individually, none of them is a physical charged-lepton mass operator. In particular, the VCS suppression identity

$$1/3 - f_\tau = (2\delta_\tau - \delta_0)/(3(3 - \delta_\tau - \delta_0))$$

contains no unit of mass.

Consolidated inheritance statement. With the upstream corpus now mapped, the gate's inheritance can be stated in one sentence: CLAS-1 inherits a fixed three-branch chiral charged-lepton carrier from SC-1 and SC-2; singular-spectrum and frame-invariance discipline from SF-1; a conditionally derived canonical localisation stiffness $\kappa = 8/3$ from QN-1 and QN-2; a conditionally supported localisation ladder and tau-compression mechanism from the Role-4 and saturated-participation papers; and a structurally necessary Higgs-radial carrier from HR-1. These inherited results substantially support the explicit dimensionless magnitude operator of Part II. They do not derive the numerical completion scale v_{cl}, the interface coupling α_{int} at a specified readout scale, the $1/(8\pi)$ projection kernel, the exact one-twelfth surviving-channel count, or the running-to-pole bridge — which remain the load-bearing open debts of the absolute-scale claim.

1.2 The unresolved absolute-scale question

A complete charged-lepton derivation must answer two logically independent questions:

1. What fixes the dimensionless spectral pattern?
2. What fixes the common dimensionful scale?

Writing

$$m_a = \mathfrak{m}_\ell \, \eta_a$$

makes the distinction explicit. The dimensionless numbers η_a determine ratios. The scalar $\mathfrak{m}_\ell$ determines the common scale.

1.3 Central aim

To prove the necessary and sufficient structural conditions under which a dimensionless VERSF charged-lepton magnitude operator becomes a unique, canonically normalised and basis-invariant absolute mass spectrum — and to instantiate every object required by those conditions with an explicit VERSF operator, so that the gate contains no unspecified function and no unspecified positive operator.

1.4 Core novelty

CLAS-1 proves:

1. a common-rescaling no-go theorem for all dimensionless flavour constructions;
2. a primitive-scale normal form for any mass built from the available VERSF dimensionful primitives;
3. a canonical-normalisation theorem for nontrivial flavour kinetic metrics;
4. a basis-invariant generalised singular-value characterisation of physical masses;
5. a minimal chiral-lift theorem separating magnitude from orientation;
6. a homogeneous scale-anchor theorem and an exact scale-locking identity;
7. an absolute charged-lepton mass theorem;
8. a Higgs–Yukawa product-degeneracy theorem;
9. an explicit magnitude operator $\mathscr{H}_\ell = \Pi_e + e^{32/3}\Pi_\mu + (e^{64/3}/144)\Pi_\tau$ built from the refinement ladder and the W7 tau activation;
10. an explicit scale $m_{(\ell,0)} = \alpha_{int}^2 v_{cl}/(8\pi)$ built from the colourless completion-interface projection, eliminating the free function F_ℓ;
11. a scale-free, κ-independent cross-ratio identity $m_\mu^2 = 12\, m_e m_\tau$ and parameter-isolation audits for κ and $\theta\star$;
12. a residual-non-absorption theorem forbidding fitted repair of the numerical audit;
13. a conditional closure-to-mass monotonicity theorem;
14. perturbative stability bounds with explicit spectral-gap margins;
15. inverse-calibration and numerical non-insertion theorems;
16. a strict firewall between a running mass operator and measured pole masses.

1.4′ Provenance note — narrowing the novelty claims

Several items above restate and extend discipline already established upstream rather than introducing it. SF-1 already establishes that physical fermion magnitudes are extracted from Hermitian lifts and singular values (not raw entries or ordinary eigenvalues), and already supplies polar decomposition, basis covariance, positive kinetic-metric discipline and the separation of magnitude from frame orientation. Items 3–5 of the list — canonical normalisation, the singular-value characterisation, and the minimal chiral lift — together with the basis-covariance theorem are therefore **inherited and extended**, with CLAS-1's contribution being their assembly into the absolute-scale argument, not their introduction. The genuinely new Part I content of this gate is narrower and should be claimed as such: the common-scale no-go theorem, the primitive-scale normal form, the homogeneous scale-anchor and scale-locking theorems, and their application to an absolute charged-lepton scale. The Part II instantiation, the scale-free identity apparatus of §18, and the audit architecture are new to this gate.

1.5 Standard mathematics and VERSF-specific content

The following mathematics is standard: finite-dimensional positive operators; polar decomposition; singular values; generalised eigenvalue problems; positive kinetic forms; dimensional analysis; homogeneous functionals; spectral perturbation theory.

The VERSF-specific content is the identification of the charged-lepton magnitude operator and its dimensionful anchor with independently derived closure, localisation, spectral and substrate structures: the refinement-depth ladder, the Role-4 localisation exponent, the W7 closure-cell weight, the VCS tau-localised maintenance optimum, and the colourless completion-interface projection.

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2. Typed Chiral Carrier and Electroweak Setting

2.1 Left- and right-handed carriers

Let $\mathscr{H}_L$ denote the three-dimensional flavour carrier of the charged components of the left-handed electroweak lepton doublets, and let $\mathscr{H}_R$ denote the three-dimensional flavour carrier of the right-handed charged-lepton singlets.

The two spaces have equal flavour rank but are not the same gauge representation.

2.2 Flavour branch projectors

Let

$\Pi_a^L : \mathscr{H}_L \to \mathscr{H}_L$, $\Pi_a^R : \mathscr{H}_R \to \mathscr{H}_R$, $a = e, \mu, \tau$,

be rank-one physical branch projectors satisfying

$\Sigma_a \Pi_a^L = I_L$, $\Sigma_a \Pi_a^R = I_R$.

These projectors must be inherited or independently derived. They may not be assigned retrospectively by sorting the observed masses. Where the chiral label is immaterial the common symbol Π_a is used.

Inheritance from SC-1/SC-2. The carrier and branch structure now have named upstream sources. SC-1 supplies exactly three complete generation packages with the charged-lepton chiral structure (L_L, e_R) and the Higgs-mediated closure route — direct support for the rank-three left and right carriers of §2.1. SC-2 fixes the unique framework **addresses** of the electron, muon and tau slots and separates left/right chirality, weak basis and mass basis — direct support for P7's requirement that branch labels precede the masses.

The occupancy-to-projector bridge (open). One limit must be kept visible: SC-2 fixes occupancy *addresses*, not the spectral projectors of a derived mass operator. CLAS-1 must still prove that its Π_e, Π_μ, Π_τ — the spectral projectors of $\mathscr{H}_\ell$ — coincide with the independently occupied SC-2 branches, rather than labelling three eigenvalues after diagonalisation. This identification is the refined content of debt D2; until it is closed, P7 is structurally supported but not discharged.

2.3 Pre-breaking gauge structure

Before electroweak symmetry breaking, a charged-lepton mass term of the form $\bar{L}E_R$ is not gauge invariant. A Higgs-type field or an equivalent VERSF closure-interface object with the required gauge transformation is necessary. The pre-breaking interaction has the schematic form

$$-\bar{L}\, Y_\ell\, \Phi\, E_R + \text{h.c.},$$

where $Y_\ell : \mathscr{H}_R \to \mathscr{H}_L$ is dimensionless.

2.4 Post-breaking bilinear

If the physical radial mode acquires a vacuum value $v^\star$, the post-breaking bilinear is

$$\mathscr{B}_\ell = (v^\star/\sqrt{2})\, Y_\ell.$$

The operator $\mathscr{B}_\ell$ has mass dimension one and maps $\mathscr{H}_R \to \mathscr{H}_L$. CLAS-1 allows either a direct derivation of $\mathscr{B}_\ell$, or a separate derivation of $v^\star$ and Y_ℓ. Only the product is required for the mass spectrum. In the explicit construction of Part II the interface scale is denoted v_{cl} and plays the role of $v^\star$.

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3. Kinetic Metrics and Canonical Normalisation

3.1 General quadratic charged-lepton action

At a specified matching scale $\mu\star$, the quadratic charged-lepton Lagrangian is

$$\mathcal{L}_\ell^{(2)} = i \bar{L} K_L \not\partial L + i \bar{E}_R K_R \not\partial E_R - \bar{L} \mathcal{B}_\ell E_R - \bar{E}_R \mathcal{B}_\ell^\dagger L,$$

with covariant derivatives understood, where

$$K_L > 0, K_R > 0$$

are Hermitian positive-definite kinetic metrics.

3.2 Why kinetic normalisation matters

A raw matrix entry of $\mathcal{B}_\ell$ is not a physical mass if the kinetic terms are not canonical. Rescaling the fields changes the matrix representation of the bilinear. Only the canonically normalised singular spectrum is invariant.

3.3 Premise P1 — Positive kinetic forms

The effective charged-lepton quadratic action must possess positive-definite kinetic metrics.

Falsifier: a zero or negative kinetic eigenvalue, an uncontrolled higher-derivative ghost, or a nonlocal term for which the finite-dimensional reduction is invalid.

3.4 Premise P2 — Common matching scale

The operators K_L, K_R and $\mathcal{B}_\ell$ must be evaluated at the same renormalisation or matching scale.

Falsifier: combining a high-scale Yukawa operator with low-scale kinetic or mass data without renormalisation-group transport.

3.5 Theorem 1 — Canonical-Normalisation Theorem

Because K_L and K_R are positive definite, they possess unique positive square roots. Define the canonical fields

$$L_c = K_L^{1/2} L, \quad E_c = K_R^{1/2} E_R.$$

In canonical fields, the quadratic Lagrangian becomes

$$\mathcal{L}_\ell^{(2)} = i \bar{L}_c \not\partial L_c + i \bar{E}_c \not\partial E_c - \bar{L}_c M_\ell E_c - \bar{E}_c M_\ell^\dagger L_c,$$

where

$$\mathbf{M_\ell = K_L^{-1/2}\, \mathscr{B}_\ell\, K_R^{-1/2}.}$$

Proof. Substitute the inverse field transformations $L = K_L^{-1/2} L_c$ and $E_R = K_R^{-1/2} E_c$ into the kinetic and bilinear terms. Positivity and Hermiticity of the kinetic metrics make their square roots self-adjoint, producing unit kinetic coefficients and the stated transformed bilinear. ■

3.6 Canonical-normalisation firewall

The physical mass matrix is M_ℓ, not $\mathscr{B}_\ell$, unless $K_L = I_L$ and $K_R = I_R$. A nonunitary flavour transformation may alter the entries and eigenvalues of the raw bilinear; canonical normalisation removes this ambiguity.

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4. Physical Masses as Generalised Singular Values

4.1 Singular-value spectrum

The physical tree-level charged-lepton masses are the nonnegative singular values of M_ℓ:

$$m_a^2 \in \sigma(M_\ell^\dagger M_\ell).$$

4.2 Theorem 2 — Generalised Mass-Eigenvalue Theorem

The squared physical masses solve

$$\mathbf{\mathscr{B}_\ell^\dagger K_L^{-1} \mathscr{B}_\ell v_a = m_a^2 K_R v_a,}$$

equivalently

$$\mathbf{\mathscr{B}_\ell K_R^{-1} \mathscr{B}_\ell^\dagger u_a = m_a^2 K_L u_a.}$$

Proof. Since $M_\ell^\dagger M_\ell = K_R^{-1/2} \mathscr{B}_\ell^\dagger K_L^{-1} \mathscr{B}_\ell K_R^{-1/2}$, let w_a be an eigenvector of $M_\ell^\dagger M_\ell$ with eigenvalue m_a^2, and put $v_a = K_R^{-1/2} w_a$. Multiplication by $K_R^{1/2}$ gives the first generalised eigenvalue equation. The second follows analogously from $M_\ell M_\ell^\dagger$. ■

4.3 Why ordinary eigenvalues are insufficient

A general complex Dirac mass matrix need not be Hermitian. Its ordinary eigenvalues are not the invariant physical masses. The singular values are nonnegative and invariant under independent unitary transformations of the left- and right-handed flavour frames.

4.4 Mass positivity

All three charged leptons are massive at the gate level only if rank $M_\ell = 3$, equivalently $\det(M_\ell^\dagger M_\ell) > 0$.

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5. The Common-Scale No-Go Theorem

5.1 Scale-free charged-lepton data

Let $\mathcal{Z}_\ell$ denote the complete set of inherited dimensionless charged-lepton data, including any of

Π_a, R_C, Δ_C, $D_M/\mathrm{Tr}\, D_M$, χ, Q_τ, f_τ, dimensionless spectral ratios.

Assume every element of $\mathcal{Z}_\ell$ is dimensionless.

5.2 Theorem 3 — Common-Scale Orbit

Let $M_\ell \neq 0$ be any candidate mass operator compatible with $\mathcal{Z}_\ell$. For every $a > 0$ define $M_\ell^{(a)} = a\, M_\ell$. Then

$m_i^{(a)}/m_j^{(a)} = m_i/m_j$,

the singular vectors are unchanged, every branch projector derived from the nondegenerate spectral eigenspaces is unchanged, and every degree-zero spectral functional is unchanged.

Proof. The singular values of aM_ℓ are am_i, so ratios are invariant. Moreover $(M_\ell^{(a)})^\dagger M_\ell^{(a)} = a^2 M_\ell^\dagger M_\ell$, so the eigenvectors and spectral projectors are unchanged. Every homogeneous functional of degree zero is invariant. ■

5.3 Theorem 4 — Dimensionless-Origin No-Go

No collection of purely dimensionless charged-lepton inputs can determine an absolute nonzero mass scale.

Proof. Suppose the dimensionless inputs uniquely determined M_ℓ. By Theorem 3, every aM_ℓ has identical dimensionless data. For $a \neq 1$, this contradicts uniqueness. Therefore a dimensionful datum or invariant is necessary. ■

5.4 Corollaries

Corollary 4.1 — Closure suppression is not an absolute mass. The closure-deficit operator Δ_C, its trace, sector fractions, spectral threshold and normalised suppression identity cannot by themselves determine a physical mass.

Corollary 4.2 — Ratios do not fix scale. Even exact derivations of m_μ/m_e and m_τ/m_μ leave the transformation $(m_e, m_\mu, m_\tau) \mapsto a(m_e, m_\mu, m_\tau)$ undetermined.

5.5 Interpretation

The missing absolute scale is not a technical inconvenience. It is a continuous symmetry of every scale-free construction. A valid absolute-scale theorem must break that symmetry using an independently derived dimensionful quantity.

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6. The Primitive-Scale Normal-Form Theorem

6.1 Available VERSF dimensionful primitives

Assume the charged-lepton gate has access to the dimensionful constants

$\hbar$, c, ξ, τ_s, Λ,

where ξ has dimensions of length, τ_s has dimensions of time, and Λ has dimensions of inverse length squared. Let $\mathcal{J}_1, \ldots, \mathcal{J}_N$ denote additional dimensionless VERSF invariants.

6.2 Dimensionless primitive combinations

Define

$\rho_s = c\tau_s/\xi$, $\lambda_\Lambda = \Lambda\xi^2$.

Both are dimensionless. A convenient primitive mass unit is

$m_\xi = \hbar/(c\xi)$.

An equivalent time-derived mass is $m_{(\tau_s)} = \hbar/(c^2\tau_s) = m_\xi/\rho_s$, so m_ξ and $m_{(\tau_s)}$ are not independent once ρ_s is included.

6.3 Theorem 5 — Primitive-Scale Normal Form

Let $\mathfrak{m}$ be any scalar mass constructed from $\hbar$, c, ξ, τ_s, Λ, $\mathcal{I}_1, \ldots, \mathcal{I}_N$ through a dimensionally homogeneous relation. Then there exists a dimensionless function F such that

$$\mathfrak{m} = (\hbar/(c\xi)) \cdot F(c\tau_s/\xi, \Lambda\xi^2, \mathcal{I}_1, \ldots, \mathcal{I}_N).$$

Proof. Divide the candidate mass by $m_\xi = \hbar/(c\xi)$. The quotient is dimensionless. Any remaining dependence on τ_s and Λ must occur through dimensionless combinations. A complete independent choice is $c\tau_s/\xi$ and $\Lambda\xi^2$, together with the already dimensionless invariants. The full exponent bookkeeping is given in Appendix B. ■

6.4 Consequence

The absolute charged-lepton scale cannot be generated by a free dimensionless number written beside m_ξ. The dimensionless amplification F_ℓ must be independently derived from VERSF structure. Part II supplies exactly such a derivation candidate: the colourless completion-interface projection fixes

$$F_\ell = (\alpha_{int}^2/(8\pi)) \cdot (v_{cl}\, c\, \xi/\hbar),$$

so that F_ℓ is no longer a freely selectable function.

6.5 Extended primitive set

If an additional independent dimensionful primitive X enters — in Part II the interface scale v_{cl} plays this role — Theorem 5 extends by including the corresponding dimensionless ratio formed from X, $\hbar$, c and ξ. The theorem must not be applied after silently omitting a dimensionful input used elsewhere in the derivation.

6.6 Premise P3 — Primitive completeness

The declared primitive set must include every dimensionful quantity used in constructing the charged-lepton bilinear.

Falsifier: an unlisted mass, energy, length, stiffness, vacuum value or cutoff appears in an intermediate step. In particular, using v_{cl} in Part II while omitting it from the declared primitive set falsifies P3.

6.7 Planck-Basis Normal Form and Anchor Equivalence

If Newton's constant G — and hence the conventional Planck units defined from G — is admitted to the declared primitive set, define the unreduced Planck quantities ($m_P^{(N)}$ and the

VERSF certification scale $m_P^{(cert)}$ remain separate objects until D34 is closed; see the Planck-Identification Firewall below)

$\ell_P = \sqrt{\hbar G/c^3}$, $t_P = \sqrt{\hbar G/c^5}$, $m_P = \sqrt{\hbar c/G}$.

These satisfy the exact bridge identities

$m_P \ell_P = \hbar/c$, $m_P = \hbar/(c\ell_P) = \hbar/(c^2 t_P)$,

and therefore the closure mass unit obeys, exactly,

$$\mathbf{m_\xi = \hbar/(c\xi) = m_P \cdot (\ell_P/\xi).}$$

The ξ-based normalisation of Theorem 5 is thus already expressible as the Planck mass followed by a dimensionless Planck-to-closure descent.

Theorem 5A — Planck-Basis Normal Form

Let $\mathfrak{m}$ be any scalar mass constructed through a dimensionally homogeneous relation from $\hbar$, c, G, ξ, τ_s, Λ and dimensionless invariants $\mathcal{J}_1,\ldots,\mathcal{J}_N$. Then there exists a dimensionless function Φ such that

$$\mathbf{\mathfrak{m} = m_P \cdot \Phi(\xi/\ell_P, \tau_s/t_P, \Lambda\ell_P^2, \mathcal{J}_1,\ldots,\mathcal{J}_N),}$$

and equivalently a dimensionless function F such that

$$\mathfrak{m} = (\hbar/(c\xi)) \cdot F(c\tau_s/\xi, \Lambda\xi^2, \ell_P/\xi, \mathcal{J}_1,\ldots,\mathcal{J}_N),$$

the two forms related exactly by

$$\mathbf{F = (\xi/\ell_P)\, \Phi.}$$

Proof. Divide $\mathfrak{m}$ by m_P. The quotient is dimensionless, and every remaining dimensionful dependence may be expressed through the independent ratios ξ/ℓ_P, τ_s/t_P and $\Lambda\ell_P^2$, together with the declared dimensionless invariants. The ξ-basis form follows from $m_\xi = m_P(\ell_P/\xi)$. The Planck and closure normal forms are two bases for one dimensionally homogeneous construction. ■

Corollary 5A.1 — Planck-Anchored Charged-Lepton Scale

The charged-lepton common scale may be written

$$\mathbf{\mathfrak{m}_\ell = m_P\, \Phi_\ell, \quad \Phi_\ell = (\ell_P/\xi)\, F_\ell > 0,}$$

so that

$m_a = m_P \Phi_\ell \sqrt{\lambda_a}(\mathscr{H}_\ell)$.

The introduction of m_P does not by itself determine the charged-lepton scale: it transfers the entire unresolved freedom into the dimensionless **Planck-to-lepton descent invariant** Φ_ℓ. Selecting Φ_ℓ from the observed electron mass is inverse calibration (Theorem 21) and is not an absolute-scale prediction (falsifier F66). What the basis change supplies is a sharp **unmatched on-shell audit target**: the observed descent is

$m_e^{pole}/m_P = 4.18546 \times 10^{-23}$.

The running descent $\Phi_\ell(\mu\star)$ may be compared with this number only after the radiative bridge $\mathcal{R}_e$ is derived (see the Planck-to-Pole Matching Firewall below); CLAS-1's scale problem is the derivation of $\Phi_\ell(\mu\star)$ and of that bridge.

Planck-to-Pole Matching Firewall

The Planck-normalised descent carries the same matching-scale discipline as the mass operator itself. Define the running descent invariant

$\Phi_\ell(\mu\star) = \mathfrak{m}_\ell(\mu\star)/m_P$.

The on-shell electron ratio

$\Phi_e^{pole} = m_e^{pole}/m_P = 4.18546 \times 10^{-23}$

is a **distinct observable target**, related to the running invariant only through the radiative bridge

$m_e^{pole} = \mathcal{R}_e(\mu\star, \text{scheme}, \text{couplings}) \cdot m_P \Phi_\ell(\mu\star)$.

Until $\mathcal{R}_e$ is derived, the difference between the explicit interface descent and Φ_e^{pole} is an **unmatched benchmark residual**, not a direct error in the Planck-to-lepton descent; 4.18546×10^{-23} is the unmatched on-shell audit target, not the value $\Phi_\ell(\mu\star)$ must equal before running and self-energy corrections (falsifier F71). D34 carries this matching requirement explicitly.

Planck-Identification Firewall

Two potentially distinct objects must not be silently identified: the conventional Planck mass $m_P^{(N)} = \sqrt{\hbar c/G_N}$ built from the measured low-energy Newton constant, and a VERSF metric-certification scale $m_P^{(cert)}$. This paper uses their identification as an **audit convention**; it has not been proved within VERSF, and D34 includes deriving the matching between the two, up to any renormalisation or order-one conversion (falsifier F70). Otherwise VERSF could derive a Planck-like certification floor without having derived its equality to the experimentally inferred m_P.

Corollary 5A.2 — Planck–Closure Descent and the Independence Firewall

If an independent VERSF theorem establishes $\ell_P/\xi = \mathcal{D}_{(P\to\xi)}$, then $m_\ell = m_P\, \mathcal{D}_{(P\to\xi)}\, F_\ell$. For the Route-M candidate

$$\ell_P/\xi = (1/2)e^{(-\Lambda_M)}, \text{ hence } m_\xi = (m_P/2)e^{(-\Lambda_M)},$$

this is an elegant descent — but because Route M obtains ℓ_P *from* ξ, the Planck and closure routes are then a **change of basis and internal consistency identity, not two independent scale determinations** (falsifier F67). A genuinely independent Planck anchor requires ℓ_P, G or m_P fixed without using ξ, v_{cl}, α_{int}, or any charged-lepton mass. The Planck-certification result supplies strong structural support for Planck scaling but obtains it only up to order-one factors and does not prove nature saturates the bound; an order-one-uncertain bound cannot normalise a lepton mass at the two-percent level (falsifier F68). The Fold–Planck–Closure distinction is inherited explicitly: **ℓ_P is the metric-certification floor and ξ the coherence floor** — different thresholds, never mere notational variants of one another.

Corollary 5A.3 — Three-Route Cross-Anchor Identity

The base scale admits three candidate representations,

$$\mathbf{m_{(\ell,0)} = m_P\, \Phi_\ell^{(P)} = (\hbar/(c\xi))\, F_\ell^{(\xi)} = \alpha_{int}^2\, v_{cl}/(8\pi),}$$

reading respectively as: Planck route (metric-certification scale × dimensionless descent); closure route (coherence-scale unit × dimensionless amplification); interface route (Higgs-radial scale × coupling and projection factors). If — and only if — the three are *independently* derived, their equality is a nontrivial cross-anchor test:

$$\mathbf{\Phi_\ell^{(P)} = (\ell_P/\xi)\, F_\ell^{(\xi)} = (\alpha_{int}^2/(8\pi))\, (v_{cl}/m_P).}$$

Failure of independently derived routes to satisfy this identity falsifies their claimed common-scale equivalence. Routes rendered dependent by shared inputs (Corollary 5A.2) participate as consistency identities only.

Planck-Convention Firewall

Throughout CLAS-1, m_P denotes the **unreduced** Planck mass $\sqrt{\hbar c/G}$. The reduced Planck mass $\bar{m}_P = m_P/\sqrt{8\pi}$ is a distinct convention; it may not be substituted while retaining unchanged factors of 8π in the completion-interface kernel, and any convention switch must be propagated through the complete scale formula (falsifier F69). The coincidence of the kernel's 8π with the reduced-mass 8π is, until derived otherwise, exactly that — a coincidence to be firewalled, not exploited.

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7. Positive Magnitude Operators and Minimal Chiral Lifts

7.1 Canonical magnitude operator

Define the dimensionless positive charged-lepton magnitude operator $\mathscr{H}_\ell > 0$ on the canonical right-handed flavour carrier, with spectral decomposition

$$\mathscr{H}_\ell = \Sigma_{(a=e,\mu,\tau)} \eta_a^2 \Pi_a^R, \eta_a > 0.$$

7.2 Separation of magnitude and chiral orientation

A positive operator on one chiral carrier determines mass magnitudes but does not by itself determine how the right-handed carrier is oriented relative to the left-handed carrier. Let $W_\ell : \mathscr{H}_R^c \to \mathscr{H}_L^c$ be unitary.

7.3 Theorem 6 — Minimal Chiral-Lift Theorem

Every canonical mass operator satisfying

$$M_\ell^\dagger M_\ell = \mathfrak{m}_\ell^2 \mathscr{H}_\ell$$

has the form

$$\mathbf{M_\ell = \mathfrak{m}_\ell W_\ell \mathscr{H}_\ell^{(1/2)}}$$

for some unitary W_ℓ. Conversely, every operator of this form satisfies the magnitude equation.

Proof. Apply the polar decomposition $M_\ell = W_\ell (M_\ell^\dagger M_\ell)^{(1/2)}$. Because $\mathscr{H}_\ell > 0$, the mass operator has full rank and the polar partial isometry is unitary. Substitution gives the result. The converse follows by direct multiplication. ■

7.4 Corollary — Orientation Independence of Masses

The physical masses are

$$\mathbf{m_a = \mathfrak{m}_\ell \eta_a},$$

independent of W_ℓ.

7.5 Meaning of W_ℓ

The unitary W_ℓ may matter for flavour-frame alignment, lepton mixing after comparison with the neutrino sector, phases, and the embedding of branch projectors across the two chiral spaces. It does not change the charged-lepton singular values.

7.6 Premise P4 — Independent magnitude operator

The operator $\mathscr{H}_\ell$ must be derived without using the observed charged-lepton masses or Yukawa couplings.

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8. The Scale Anchor and Scale-Locking Theorem

8.1 Dimensionless normalised mass operator

Define $\hat{M}_\ell = W_\ell \mathscr{H}_\ell^{(1/2)}$, so that $M_\ell = \mathfrak{m}_\ell \hat{M}_\ell$.

8.2 Homogeneous scalar functionals

Let $A[M]$ be a nonzero scalar functional satisfying

$A[aM] = a^q A[M]$, $a > 0$,

for some $q > 0$. Examples:

$A_2[M] = \mathrm{Tr}(M^\dagger M)$, $q = 2$; $A_{\det}[M] = |\det M|$, $q = 3$; $A_\infty[M] = \|M\|_\infty$, $q = 1$.

8.3 Theorem 7 — Scale-Locking Identity

Suppose VERSF independently derives a dimensionful scalar $A_\ell^{\mathrm{VERSF}} > 0$ and proves $A[M_\ell] = A_\ell^{\mathrm{VERSF}}$. Then the common charged-lepton scale is uniquely

$$\mathfrak{m}_\ell = [A_\ell^{\mathrm{VERSF}} / A[\hat{M}_\ell]]^{(1/q)}.$$

Proof. Homogeneity gives $A[M_\ell] = A[\mathfrak{m}_\ell \hat{M}_\ell] = \mathfrak{m}_\ell^q A[\hat{M}_\ell]$. Since both quantities are positive and $q > 0$, solve uniquely for $\mathfrak{m}_\ell$. ■

8.4 Standard anchors

Trace anchor. If VERSF derives $S_{2,\ell}^{\mathrm{VERSF}} = \mathrm{Tr}(M_\ell^\dagger M_\ell)$, then $\mathfrak{m}_\ell = [S_{2,\ell}^{\mathrm{VERSF}} / \mathrm{Tr}\, \mathscr{H}_\ell]^{(1/2)}$.

Determinant anchor. If VERSF derives $P_\ell^{VERSF} = |\det M_\ell|$, then $\mathfrak{m}_\ell = [P_\ell^{VERSF} / \sqrt{\det \mathcal{H}_\ell}]^{1/3}$.

Curvature anchor. If a derived charged-lepton closure potential has canonically normalised quadratic curvature $\mathcal{C}_\ell = \mathfrak{m}_\ell^2 \mathcal{H}_\ell$ of mass dimension two, any nonzero eigenvalue, trace or determinant of $\mathcal{C}_\ell$ locks the scale.

Cross-anchor consistency. Two independent anchors must agree:

$$[S_{2,\ell}^{VERSF} / \mathrm{Tr}\, \mathcal{H}_\ell]^{1/2} = [P_\ell^{VERSF} / \sqrt{\det \mathcal{H}_\ell}]^{1/3}.$$

Failure of this equality falsifies the claimed common-scale factorisation.

8.5 Necessity and sufficiency

Theorem 4 proves that some dimensionful anchor is necessary. Theorem 7 proves that one nonzero independently derived homogeneous anchor is sufficient.

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9. The Charged-Lepton Absolute Scale Theorem (Abstract Form)

9.1 Premises

P1. Positive kinetic metrics K_L, K_R are derived at a common scale. **P2.** The physical left- and right-handed flavour carriers and branch structure are defined independently. **P3.** The dimensionful primitive set is complete. **P4.** A positive dimensionless magnitude operator $\mathcal{H}_\ell > 0$ is independently derived. **P5.** A chiral lift exists with the correct electroweak transformation properties. **P6.** A nonzero scale anchor or an equivalent primitive amplification function is independently derived. **P7.** Physical branch projectors are independently fixed. **P8.** No charged-lepton mass or Yukawa coupling is used to select the scale, spectrum, branch labels or functional form. **P9.** A valid renormalisation and on-shell matching bridge must be supplied before any matched empirical prediction, precision comparison or confirmation claim. (An explicitly designated *unmatched benchmark*, such as Section 19, is admissible without P9; its interpretation, not its computation, is what P9 gates.)

9.2 Theorem 8 — Charged-Lepton Absolute Scale Theorem

Under P1–P8, the canonically normalised charged-lepton mass operator has the form

$$\mathbf{M_\ell = (\hbar/(c\xi))\, F_\ell(c\tau_s/\xi, \Lambda\xi^2, \mathcal{I})\, W_\ell\, \mathcal{H}_\ell^{1/2},}$$

its physical tree-level masses are

$$\mathbf{m_a = (\hbar/(c\xi))\, F_\ell(c\tau_s/\xi,\ \Lambda\xi^2,\ \mathcal{I})\ \sqrt{\lambda_a(\mathcal{H}_\ell)},\ a = e, \mu, \tau,}$$

and the mass ratios are

$$\mathbf{m_a/m_b = \sqrt{\lambda_a(\mathcal{H}_\ell)/\lambda_b(\mathcal{H}_\ell)}.}$$

The common absolute scale is unique once F_ℓ, or any equivalent scale anchor, is independently fixed.

Proof. The primitive-scale normal form gives $\mathfrak{m}_\ell = (\hbar/(c\xi))F_\ell$. The minimal chiral-lift theorem gives $M_\ell = \mathfrak{m}_\ell\, W_\ell\, \mathcal{H}_\ell^{(1/2)}$. The singular values of a unitary times a positive operator are the eigenvalues of the positive operator, so $m_a = \mathfrak{m}_\ell \sqrt{\lambda_a(\mathcal{H}_\ell)}$. Uniqueness follows from the independent fixation of the dimensionless magnitude operator and scale anchor. ■

9.3 Outcome taxonomy

1. **Ratio closure:** $\mathcal{H}_\ell$ is derived but F_ℓ is not — exact ratios, free scale.
2. **One-anchor absolute closure:** one independent VERSF dimensionful invariant fixes the common scale.
3. **Full physical closure:** the running mass operator is matched through electroweak symmetry breaking and radiative corrections to the physical pole masses.

Both components — magnitude and scale — are required for outcome 2; outcome 3 additionally requires P9.

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10. Higgs-Mediated Matching and Product Degeneracy

10.1 Higgs-mediated form

Let $\mathcal{B}_\ell = (v\star/\sqrt{2})Y_\ell$. After canonical normalisation,

$$M_\ell = (v\star/\sqrt{2})\ K_L^{(-1/2)}\ Y_\ell\ K_R^{(-1/2)}.$$

10.2 Yukawa magnitude factorisation

Write the canonical Yukawa operator as

$Y_\ell^c = K_L^{(-1/2)} Y_\ell K_R^{(-1/2)} = y_\star W_\ell \mathscr{H}_\ell^{(1/2)}$.

Then $M_\ell = (v_\star y_\star/\sqrt{2}) W_\ell \mathscr{H}_\ell^{(1/2)}$, so $\mathfrak{m}_\ell = v_\star y_\star/\sqrt{2}$.

10.3 Theorem 9 — Higgs–Yukawa Product Degeneracy

The charged-lepton mass spectrum alone determines only the product $v_\star y_\star$. For every $a > 0$, the substitution $v_\star \mapsto a v_\star$, $y_\star \mapsto y_\star/a$ leaves M_ℓ and every charged-lepton mass invariant.

Proof. Substitute the rescaled quantities into $v_\star y_\star/\sqrt{2}$. The product is unchanged. ■

10.4 Consequence

A charged-lepton mass theorem does not, by itself, derive the Higgs radial scale and the universal Yukawa normalisation separately. A separate gauge-boson, scalar-potential or closure-interface theorem must fix $v_\star$. In Part II this is precisely the status of v_{cl}: it is imported at audit time and remains an open bridge debt (D12).

HR-1 provenance. HR-1 establishes that a Higgs-radial mode is structurally *required* — the normal stiffness mode of a stable closure interface — and correctly separates the derivational steps (scalar coordinate, propagating field, kinetic normalisation, vacuum scale). It thereby supports the existence and admissibility of the v_{cl}-type structure this gate consumes. HR-1 explicitly leaves open the numerical vacuum scale, the microscopic potential, the kinetic normalisation, the Higgs mass and quartic, and the bridge to fermion masses. D12 is therefore **structurally supported, numerically open** — and HR-1 must not be cited as supplying the number 246.21965 GeV (doing so is an instance of F46).

10.5 Direct-bilinear option

If VERSF derives the dimension-one bilinear $\mathscr{B}_\ell$ directly, the charged-lepton masses can be fixed without first decomposing it into $v_\star$ and Y_ℓ. However, interpreting the same result as a Standard Model Yukawa derivation still requires a valid Higgs-mediated factorisation.

Part II — Explicit VERSF Instantiation

Part I establishes what any absolute-scale derivation must contain: a positive dimensionless magnitude operator, a chiral lift, and one independent dimensionful anchor. Part II supplies an explicit VERSF candidate for each object. Every explicit input is named, its inherited source is identified, and its proof-gated status is recorded in the premise map (Section 26) and the bridge-debt ledger (Appendix F).

11. Refinement-Depth Operator and the Localisation Ladder

11.1 Refinement-depth operator

Let Π_e, Π_μ, Π_τ be the independently defined charged-lepton branch projectors, satisfying

$\Pi_a \Pi_b = \delta_{ab} \Pi_a$, $\Pi_e + \Pi_\mu + \Pi_\tau = I_\ell$.

Define the charged-lepton refinement-depth operator

$$\mathbf{G_\ell = 0 \cdot \Pi_e + 1 \cdot \Pi_\mu + 2 \cdot \Pi_\tau.}$$

The three eigenvalues 0, 1, 2 are the three admissible charged-lepton refinement depths. Their equality of step — one unit from electron to muon and one unit from muon to tau — is a structural inheritance from the generation census, not a fitted choice.

Termination of the depth spectrum. The absence of a fourth depth is dynamically backed by the Role-4 binding analysis: the sector binding threshold grows super-linearly in g, while the well capacity $C_g = C_0(1 + \eta_A g)$ grows linearly, so $C_g/C_{crit}(g) \to 0$ and a finite cutoff exists for every admissible parameterisation (analytic); the sign result — if the g = 2 sector binds then g = 3 does not — holds for all $\eta_A \geq 0$ because $\sup(1+3\eta_A)/(1+2\eta_A) = 3/2 < C_{crit}(3)/C_{crit}(2) = 1.830$; and the structural capacity bound $C_0 \leq 14\kappa'$ from $\dim(\Gamma_{min}) = 14$ places C_3 below $C_{crit}(3) = 34.435$ throughout the admitted region. The rank-3 carrier of Section 2 is therefore not merely inherited but conditionally derived; the residual conditionality lies in the capacity-bound chain (debt D26).

A **second, independent census derivation** now exists. The persistent-distinguishability enumeration derives the charged-lepton n-band {1, 2, 3} directly: minimal persistence forces $n \geq 1$ (an open structure dissolves under refinement); loop count is the only independent admissibility-topological label within the sector, so integer minimality forces unit steps; and primitive-function exhaustion closes the band at n = 3 — a fourth loop class is necessarily a sub-leading correction, a quotient-equivalent structure, or a sector change (quark/neutrino), not a fourth charged-lepton generation. The binding-threshold route and the loop-class-exhaustion route share no step, so the three-generation census is now doubly derived, mirroring the two-route convergence of the tau suppression. The exhaustion route also carries the census's direct experimental falsifier: discovery of a genuine fourth charged lepton.

11.2 Premise E1 — Role-4 localisation law

The inherited Role-4 localisation law is

$L_g = L_0 e^{(-\kappa g)}$,

and the inverse-area mass-localisation law gives

$m_g \propto L_g^{-2} = e^{2\kappa g}$.

Derivation source. Both components of E1 are now theorems within the Role-4 variational programme rather than ansätze. The exponential form is a theorem under the scale-homogeneity postulate: because the substrate carries no preferred length, the nesting entropy $F(x) = \alpha \ln(1/x) + \beta x$ depends only on the ratio $x = L_g/L_{g-1}$, whose minimiser $x^\star = \alpha/\beta$ is generation-independent, giving $L_g = L_0 e^{-\kappa g}$ with $\kappa = \ln(\beta/\alpha)$. The −2 exponent is the Sector Rescaling Theorem: under $\tilde{r} = r/L_g$ the sector eigenvalue problem depends only on the dimensionless capacity $C_g = A_g L_g^2$, and the physical mass is exactly $M_g = \tilde{E}_g/L_g^2$. The pure-geometry ladder $m_g \propto e^{2\kappa g}$ then holds exactly when the rescaled energies $\tilde{E}_g$ are equal across branches; the computed light-sector ratio $\tilde{E}_1/\tilde{E}_0 = 0.999$ confirms this to 0.1% for the electron–muon step, and its collapse at the tau step is the mechanism content of E4 below.

Falsifier: a derived localisation law with non-exponential depth dependence, a mass-localisation exponent other than −2, or failure of the scale-homogeneity postulate for the substrate.

11.3 Premise E2 — Localisation exponent

The inherited geometric value of the localisation exponent is

$\kappa = 8/3$.

Derivation source (Route A, adopted). The value descends from CP^2 geometry: the holomorphic sectional curvature fixes $K_{hol} = 4$; the isotropy structure of CP^{n-1} leaves $n-1$ of n directions free, giving the transverse focusing factor $(n-1)/n = 2/3$ at $n = 3$; hence $K_{eff} = K_{hol}\cdot(n-1)/n = 8/3$. The Jacobi equation $J'' + K_{eff} J = 0$ establishes K_{eff} as the unique focusing parameter of the free subspace. The identification $\kappa = K_{eff}$ is the **Sector–Curvature Correspondence**, an explicitly named axiom: the 2D interface analysis fixes the spreading term ($\alpha = 2$) and identifies interface confinement as the mechanism class for the linear looseness term, but the confinement tension $T_{conf} = 2e^{K_{eff}}$ required for $\kappa = K_{eff}$ exactly remains a closure hypothesis (debt D24). No lepton mass enters the chain, so the agreement of $e^{16/3}$ with the observed μ/e ratio is a conditional mass-independent postdiction (stiffness identification pending, D24).

Route A′ — independent substrate derivation of the same value. QN-2 supplies a second, independent conditional derivation of $\kappa^\star = 8/3$ by substrate anomaly descent: a four-obligation substrate commitment descending into three physical support directions, doubled by bidirectional terminality ($4/3 \times 2 = 8/3$). This chain does not pass through CP^2 curvature or the Sector–Curvature Correspondence, so the exponent **value** now carries two-route convergence — the same pattern already established for the census (§11.1) and the tau suppression (§12.5). E2's grade rises accordingly. What Route A′ does *not* close, and must not be over-credited with: the exponential generation law itself, the inverse-area mass relation (both remain E1's burden), the uniformity of κ across steps (E9), and the identification of QN-2's canonical stiffness with the precise exponent appearing in the mass ladder — the **stiffness-identification gap**, now recorded inside D24.

Rival routes and the convention firewall. Two genuinely rival derivations remain: the closure-architecture route gives $\kappa = \ln(N\_loop) = \ln 14 \approx 2.639$, and the interface-Helfrich route gives $\kappa = \ln(16\pi/3) \approx 2.819$. QN-1 supplies the convention firewall for this whole sector: 8/3, 3/8, ln 14 and related quantities are distinct typed objects that may not be silently interchanged (the upstream backing for F54/F60), and QN-1 explicitly keeps the ln 14 ↔ 8/3 relation open rather than resolved. The candidates are audited against the exponent-isolation identity in Section 18. This paper adopts Route A/A′ because 8/3 is the value inherited by the localisation programme and now carries two independent chains — not because it fits best; selecting among routes by fit quality is falsifier F51, and reconciling the routes is debt D24.

Falsifier: an independent derivation of κ yielding a different value, evidence that κ is branch-dependent, or refutation of the Sector–Curvature Correspondence.

11.3′ Premise E9 — Generation-independent exponent

A single κ governs both the e→μ and the μ→τ localisation steps.

Support. The derivation $K\_eff = 4 \times 2/3$ carries no generation index — it is a fixed geometric quantity — so if K_eff is the exponent at every step, uniformity is a consequence of Route A rather than a free assumption. The variational model independently disfavours the alternative: it produces the tau deficit as a collapse of the rescaled energy at fixed κ (capacity supercriticality), whereas a running κ would need an unmotivated generation dependence of the geometry.

Why it is load-bearing. With only two independent ratios, "uniform κ + tau suppression θ⋆" and "running κ (2.666 → 1.411) with no suppression" are observationally degenerate. E9 is what privileges the θ⋆ reading, and Theorem 11 (the cross-ratio identity) — though independent of the *value* of κ — fails under running κ, since it requires equal ladder steps. Confirming that K_eff carries no generation index is debt D28 and the highest-leverage single confirmation for the explicit construction.

Falsifier: a derived generation dependence of the localisation exponent.

11.4 Localisation operator

Because G_ℓ is diagonal in the branch projectors, the charged-lepton localisation operator is

$$\mathcal{L}\_\ell = e^{(2\kappa G\_\ell)},$$

and with κ = 8/3,

$$\boldsymbol{\mathcal{L}\_\ell = \Pi\_e + e^{(16/3)}\, \Pi\_\mu + e^{(32/3)}\, \Pi\_\tau.}$$

This operator supplies the unsuppressed localisation hierarchy: the same exponential step $e^{(16/3)}$ from electron to muon and from muon to tau, before closure saturation is included.

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12. W7 Closure-Cell Activation and the VCS Tau Operator

12.1 Premise E3 — W7 primitive cell weight

The primitive W7 closure-cell suppression weight is

$\theta\star = \lambda\star/\mathrm{Tr}\ \mathcal{L}\_W7 = 2/24 = 1/12$.

Derivation source. The denominator is no longer a bare census entry. The minimal closure algebra of the coupled (CP^2 internal) ⊗ (Role-4 substrate) system has dimension

$\dim(\Gamma\_min) = 2(n\_int + n\_role) = 2(3 + 4) = 14$,

where (n_int, n_role) = (3, 4) is proved to be the unique admissible minimal pair (n_int = 3 from the curvature-coherence bound above and sector-transition completeness below; n_role = 4 from completeness-without-redundancy of the positional, scale, phase and topological constraint channels). The boundary/interior split of the fourteen generators,

$14 = 2\_\partial + 12\_int$,

gives the anchoring channel count N_anchor = 12, and the saturated participation factor is θ⋆ = N_survive/N_anchor = 1/12.

Bookkeeping reconciliation — trace form versus channel form. The premise header states θ⋆ in the inherited W7 trace normalisation, λ⋆/Tr $\mathcal{L}$_W7 = 2/24; the derivation chain states it in the closure-algebra normalisation, N_survive/N_anchor = 1/12. These are two presentations of one weight, and the candidate map between them is a common factor of two: λ⋆ = 2N_survive, Tr $\mathcal{L}$_W7 = 2N_anchor. Whether that factor is the bidirectional-terminality doubling already appearing in Route A′, or an independent convention of the W7 census, is unresolved; it is exactly the "two-factors-of-two obligation" recorded in D31, and until it is derived the two forms are numerically equal but not yet proven co-typed. One typing firewall attaches immediately: the λ⋆ = 2 of the trace form is a doubled single-channel weight, **not** the boundary-generator count 2_∂ of the split 14 = 2_∂ + 12_int; conflating those two 2's is an instance of F54.

Grade correction — what is theoremed and what is conjectured. The chain to 1/12 must not be graded uniformly. dim(Γ_min) = 14 is a theorem, and its *proved* decomposition is 6 internal + 8 substrate (2n_int + 2n_role). The boundary/interior partition 14 = 2_∂ + 12_int is a **different, conjectural** partition of the same fourteen generators, carrying three unproven steps: (i) that the generators split naturally into two boundary and twelve interior; (ii) that boundary generators — enforcing closure orientation and endpoint matching — cannot act as anchoring-effective commitment channels; (iii) that saturation leaves exactly one anchoring-effective interior channel, N_survive = 1. The value of the algebra dimension is therefore derived; the value of the *denominator* rests on steps (i)–(ii) and the *numerator* on step (iii), all three open (debt D25). Describing the split as proved is falsifier F57.

Unification note. The same algebraic object dim(Γ_min) = 14 appears in three roles across the programme: as the rival hierarchy exponent $\kappa = \ln 14$ (Route B of E2), as the capacity bound $C_0 \leq 14\kappa'$ backing the depth-spectrum termination (Section 11.1), and — through its conjectured 2 + 12 split — as the denominator of $\theta\star$. The 12 of the tau suppression and the 14 of Route B are thus not independent numbers but two readings of one algebra; conflating them, however, is a type error (falsifier F54: the 12 counts interior channels, the 14 the full generator set).

Falsifier: a closure-algebra census yielding a different dimension or boundary/interior split, or a demonstration that the weight is not universal across activations.

12.2 Premise E4 — Single tau-step activation

The charged-lepton activation rule assigns no closure-cell suppression to the electron or muon branch and exactly one primitive closure-cell weight to the tau branch.

Onset mechanism. The calibrated Role-4 model supplies a concrete mechanism capable of localising the onset to the tau branch while leaving the muon branch essentially ungated — a mechanism demonstration, not a derivation (the derivation waits on D26). In the model the tau-sector capacity exceeds its critical value, $C_2 = 22 > C\_crit(2) = 18.822$, collapsing the rescaled energy ($\tilde{E}_2/\tilde{E}_1 \approx 0.085$), while the muon sector remains subcritical ($\tilde{E}_1/\tilde{E}_0 = 0.999$, no suppression). The proposed closure reading is consistent with assigning N_survive = 1 at the tau sector, but that kernel dimension remains uncomputed under D25 and D32. The nonlinear SCF computation in the calibrated regime ($\kappa = 2.67$) confirms the pattern is structurally rigid: the entropy term deepens the three bound sectors monotonically while shifting the unbound sector by zero to machine precision — the nonlinearity is a selective stabiliser, not a rescue mechanism.

Calibration caveat. The capacity values C_g themselves are calibrated inputs of the variational model, not yet substrate-derived; the near-threshold status of the tau sector is therefore a consistency demonstration of the mechanism, not a completed derivation of the onset (debt D26). Describing the calibrated capacities as derived is falsifier F52.

Plateau condition on admissible gating laws. Any continuous realisation of the onset — an alignment law p_align(Δ) in the threshold margin $\Delta\_g = C\_g - C\_crit(g)$ — is tightly constrained by the muon's success: a law declining gradually across the whole margin range would gate the muon too and destroy the 0.17% e–μ postdiction. Admissible laws are therefore plateau-plus-cliff: p_align(Δ) ≈ 1 away from the edge, falling sharply only as $\Delta \to 0^+$ (e.g. an essential-singularity profile). With representative margins, no single-parameter monotone law meets the muon tolerance (Θ_1 = 1 to a few parts in 10^3) and the tau target ($\Theta_2 \approx 0.081$) simultaneously; adopting a law that violates the plateau condition is falsifier F58. The cleanest sub-target for D26 is the ordering inequality $dC/dg < dC_{crit}/dg$ over the bound sectors, which settles the tau's near-criticality without requiring the capacities in closed form.

Falsifier: a derived activation count other than (0, 0, 1) across the branches, or activation on a step other than the tau step.

12.3 The VCS tau-localised maintenance operator

Define

$$\mathbf{R_{\ell\star} = \Pi_e + \Pi_\mu + \theta\star\, \Pi_\tau = \Pi_e + \Pi_\mu + (1/12)\, \Pi_\tau.}$$

This is a positive contraction: $0 < R_{\ell\star} \le I_\ell$. Its closure-deficit operator is

$$\mathbf{\Delta_{\ell\star} = I_\ell - R_{\ell\star} = (11/12)\, \Pi_\tau,}$$

so the unresolved closure is entirely tau-localised. This is precisely the operator form required by the partial-tau VCS regime:

$$\Delta_{\ell\star} = \Pi_\tau\, \Delta_{\ell\star}\, \Pi_\tau.$$

For a one-dimensional tau sector, the corresponding maintenance capacity is $C\star = C_0 + d_\tau/12$, where C_0 is the cost of fully maintaining the electron–muon complement and d_τ is the tau maintenance demand; equivalently $C_{full} - C\star = (11/12)d_\tau$.

12.4 Firewall — completion eigenvalue versus participation fraction

The normalised tau-maintenance fraction is

$$f_{\tau\star} = \mathrm{Tr}(\Pi_\tau R_{\ell\star})/\mathrm{Tr}\, R_{\ell\star} = (1/12)/(2 + 1/12) = 1/25.$$

The quantity 1/12 used in the mass operator is the **unnormalised tau completion eigenvalue**, not the normalised participation fraction 1/25. These quantities must not be conflated; conflating them is falsifier F41.

12.5 Premise E5 — Cell weight equals maintenance eigenvalue

The W7 primitive cell weight $\theta\star$ must equal the VCS tau maintenance eigenvalue at the variational optimum.

Two-route convergence evidence. The tau suppression is now reached by two chains sharing no common step:

closure census: $\theta\star = N\_survive/N\_anchor = 1/12 = 0.0833$; variational collapse: $\Theta_2 = \tilde{E}_2/\tilde{E}_1 \approx 0.0846 = 1/11.82$ (from capacity supercriticality); observed: $\Theta_2 = 0.08134 = 1/12.295$.

Baseline note. Two implied suppression values circulate and must be typed. Measured against the *observed* electron–muon step — the cross-ratio reading of §18.1 — the implied value is 1/12.295. Measured against the *ladder* step e^(16/3) at $\kappa = 8/3$ — the like-for-like comparator for the variational ratio $\tilde{E}_2/\tilde{E}_1$, which is defined at fixed κ — it is 16.8177/207.127 = 0.081195 = 1/12.316. The two differ by exactly the +0.17% electron–muon step residual and may not be interchanged inside precision statements.

The two derivations agree to 1.6% and bracket the observed value. The numerical proximity of the two routes is consistent with a shared structural mechanism and motivates the identity conjecture, but does not prove that the channel count and the variational energy ratio represent the same physical object: two routes can agree through shared assumptions, hidden calibration, or coincidence. The natural reading is that the channel count and the energy ratio are two descriptions of one object — "saturation leaves roughly one-twelfth of the mass-generating effectiveness available at the tau" — but that identity is itself a conjecture the agreement supports rather than proves (debt D27, falsifier F55 if asserted as proved).

Anti-double-counting firewall. Because the two readings are one suppression, the mass law carries it **once**: M_ℓ contains the factor $\theta\star\Pi\_\tau$ and nothing else on the tau step. Writing e^(16/3) × (1/12) × ($\tilde{E}_2/\tilde{E}_1$) double-counts and is falsifier F48.

Falsifier: a VCS optimisation yielding a tau eigenvalue different from $\theta\star$, or a demonstration that the two quantities coincide only after using observed mass data.

12.6 Suppression typing — coupling versus anchoring, count versus gate

Two typing questions about $\theta\star$ are open and now named.

Which substrate factor carries it. The coherence–anchoring relation $\tilde{E} \propto p\_eff/K\_c$ decomposes any sector suppression into a coupling part and an anchoring part,

$$\Theta_2 = (p\_eff,2/p\_eff,0) \cdot (K\_c,0/K\_c,2),$$

where $p\_eff = g\_c \cdot p\_align$ is the anchoring-effective coupling (the fraction of phase-aligned states that complete irreversible commitment) and K_c the anchoring depth (successful micro-events required before commitment completes). The mass law of Theorem 10 carries the suppression once — that firewall (F48) is unaffected — but which object the maintenance eigenvalue *reads* is undecided: a gated coupling with stable depth (Route B: $p\_eff,2 \approx 0.081 \cdot p\_eff,0$), a deepened anchoring count with stable coupling (Route A: $K\_c,2 \approx 12 \cdot K\_c,0$), or a mixed split (Route C). The Route B reading is the one investigated, for its direct connection to threshold gating of aligned states; that choice is strategic, not evidential — $K\_c,2 \approx K\_c,0$ is assumed, not shown (debt D29).

Whether it is a count or a gate. The quantised and continuous readings of $\theta\star$ make distinguishably different predictions. A pure channel count predicts *exactly* 1/12 up to corrections, independent of the threshold margin Δ_2; a continuous gating law predicts a margin-dependent value near, but generically not at, 1/12. The observed implied value (1/12.295 in the cross-ratio reading; 1/12.316 against the ladder baseline — §12.5) is precisely where they separate: the −2.4% deviation either flags a correction to the counting or favours a continuous value near the integer count. Whether the integer emerges as the $\Delta \to 0$ limit of the gating curve or is an independent quantisation is part of debt D27. The residual audits of Sections 18–19 are therefore not merely error budgets — they are the operating discriminator between the two readings.

12.7 The participation object and the conditional collapse

The participation-gating construction gives the activation premises E3–E5 their proper formal object. Let Γ_anchor be the anchoring-eligible channel space and Γ_maint,g the subspace consumed by closure maintenance at refinement level g. The **channel-participation fraction** — a channel-count object, deliberately distinguished from the normalised maintenance fraction $f_{\tau\star} = 1/25$ of §12.4, whose separation F41 protects — is

$\Pi_g = \dim(\Gamma_{anchor} \ominus \Gamma_{maint,g})/\dim \Gamma_{anchor}$,

and, on the K = 7 register with occupancy ladder $O_0 = 1$, $O_1 = 3$, $O_2 = 7 = K$, the **Sector-Restricted Participation Collapse** holds conditionally: *if* a maintenance projection P_maint is absent below saturation and present at exact closure, then

$\Pi_g = 1$ $(O_g < K)$, $\Pi_2 = \dim \mathrm{Ker}(P_{maint})/\dim \Gamma_{anchor} < 1$ $(O_2 = K)$,

specialising to $\Pi_2 = 1/12$ on the imported dimensions $\dim \Gamma_{anchor} = 12$, $\dim \mathrm{Ker}(P_{maint}) = 1$. Four honesty registers attach:

1. **Data versus conjecture within the hypothesis.** What the muon establishes is only $\Pi_1 \approx 1$ to ~0.1% — the mechanism is negligible at the second occupancy point. The *existence* of P_maint and its *location* at exact closure O = K (rather than somewhere in $O \in (3, 7]$) are conjectural and carry the load. A genuine threshold is the **natural** realisation (a smooth law must be tuned flat through O_1 and then plunge — a step in all but name), but it is not **forced**: with no in-domain occupancy between $O_1 = 3$ and $O_2 = 7$, a true step and a tuned crossover are observationally identical. The step location is therefore not merely unproven but *unprovable within the domain*; it must come from the Role-4 algebra (debt D32).
2. **The numerator is currently imported.** dim Ker(P_maint) = 1 is at present the value consistency with the tau residual requires, not a computed kernel dimension. Until it is derived independently of the tau mass, any use of the 1 as derived is circular (F26/F35 apply); this is the sharpest current statement of D25's numerator link.
3. **The nullity trap — a forbidden shortcut.** The K = 7 architecture carries a nullity-one mode, and the construction needs a kernel of dimension one. Identifying the two is **forbidden**: the architectural nullity is sector-independent, so if it were the surviving

channel, every sector would read $\Pi = 1/12$ — gating the muon and destroying the 0.17% postdiction. The required kernel must be a property distinguishing the saturated sector; any derivation of N_survive = 1 must exclude this identification explicitly (falsifier F62).

4. **Anchoring-space stability.** The denominator carries no sector index — all sector variation is loaded into Γ_maint,g while Γ_anchor is held fixed. This is a named modelling assumption (reasonable, since the register is the same at every g), not a derived fact; if refinement altered N_anchor across generations, Π_2 would shift even with a one-dimensional kernel.

Beneath all of this sits a provenance layer: the K = 7 architecture itself (six-plus-one constraints, fourteen traversals, nullity-one) is inherited **pending source confirmation** within the corpus. The $\theta\star$ denominator is therefore a conjectural split atop a not-yet-pinned architecture (debt D31), one grade below even the corrected E3 register.

12.8 Premise E10 — Domain validity of the localisation law

The ladder of E1–E2 is defined only on sectors where it is independently validated — where it postdicts the observed second-generation ratio without adjustment.

In domain: the charged leptons (muon postdiction to 0.17%).

Out of domain: both quark towers, and decisively so. Applying the same law to quarks fails **two generations before saturation**, at the muon-analogue step, and fails in opposite directions: m_c/m_u ≈ 588 overshoots the predicted 207, while m_s/m_d ≈ 20 undershoots it. Repairing the up tower needs κ_up > 8/3; repairing the down tower needs κ_down < 8/3; no single κ_q within a law of the same exponential form can do both, and the gaps far exceed scheme or running effects. Quark "Θ_2" values computed against the lepton baseline are residuals against an invalid baseline and test nothing.

Three consequences for this gate. First, the domain restriction is **empirical, not stipulated** — drawn where the law's own postdiction succeeds. Second, CLAS-1's explicit construction is charged-lepton-local by data; exporting the ladder, the activation, or $\theta\star$ to quark or neutrino sectors without separately validated localisation constants (κ_up, κ_down, κ_ν) is falsifier F61. Third, the restriction has a real predictive cost that must be owned rather than hidden: within the charged-lepton domain there is no heavier sibling, so the suppression mechanism has **no in-domain second observable** — the numbers 12 and 1 do not recur anywhere independent of the tau residual they reproduce. Out-of-sample corroboration requires either a sector re-admitted under its own validated κ, or the independent kernel computation of D32.

Falsifier: demonstration that the ladder postdicts a second-generation ratio in a sector outside the charged leptons (which would *re-admit* that sector and reactivate the generation-wide test), or conversely any downstream VERSF paper applying the lepton-form law to quarks without sector-specific validation.

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13. The Explicit Magnitude Operator and Its Spectrum

13.1 Construction

Because $R_\ell^\star$, G_ℓ and the branch projectors commute, define the dimensionless charged-lepton mass-magnitude operator

$$\boldsymbol{\mathcal{M}_\ell = R_\ell^\star\, e^{(2\kappa G_\ell)} = \Pi_e + e^{(16/3)}\, \Pi_\mu + (1/12)\, e^{(32/3)}\, \Pi_\tau.}$$

The abstract positive magnitude operator of Part I is then replaced by

$$\boldsymbol{\mathcal{H}_\ell = \mathcal{M}_\ell^\dagger\, \mathcal{M}_\ell = \Pi_e + e^{(32/3)}\, \Pi_\mu + (1/144)\, e^{(64/3)}\, \Pi_\tau.}$$

There is no unspecified spectral function remaining.

13.2 Explicit spectrum

The three dimensionless mass magnitudes are exactly

$$\boldsymbol{\eta_e = 1,\ \eta_\mu = e^{(16/3)},\ \eta_\tau = (1/12)\, e^{(32/3)},}$$

hence

$$\boldsymbol{\eta_e : \eta_\mu : \eta_\tau = 1 : e^{(16/3)} : e^{(32/3)}/12.}$$

Numerically, $\eta_\mu = e^{(16/3)} \approx 207.127$ and $\eta_\tau = e^{(32/3)}/12 \approx 42\,901.6/12 \approx 3575.13$, so the ordering $0 < \eta_e < \eta_\mu < \eta_\tau$ required by Part I holds strictly, and the eigenvalues of $\mathcal{H}_\ell$,

$$\lambda_e = 1,\ \lambda_\mu = e^{(32/3)} \approx 4.290 \times 10^4,\ \lambda_\tau = e^{(64/3)}/144 \approx 1.278 \times 10^7,$$

are widely nondegenerate. The spectral gaps relevant to the stability bounds of Section 23 are

$$\lambda_\mu - \lambda_e \approx 4.290 \times 10^4,\ \lambda_\tau - \lambda_\mu \approx 1.274 \times 10^7.$$

13.3 Positivity and rank

$\mathcal{H}_\ell$ is a positive combination of orthogonal rank-one projectors with strictly positive coefficients; therefore $\mathcal{H}_\ell > 0$ and rank $\mathcal{H}_\ell = 3$, satisfying the full-rank requirement of Section 4.4 and the positivity requirement P4 of Part I structurally. Whether it satisfies P4's independence requirement is exactly the content of premises E1–E5.

14. The Colourless Projection Scale

14.1 Interface inputs

Let v_{cl} be the closure-interface radial vacuum scale and α_{int} the source-readable interface coupling at the declared readout scale.

14.2 Premise E6 — Two-gate readout law

The colourless charged-lepton current readout requires exactly two source-visible current gates, contributing the factor α_{int}^2.

Falsifier: a derived readout law with a gate count other than two, or a coupling power other than α_{int}^2.

14.3 Premise E7 — Factorised projection kernel

The colourless charged-lepton access space is

$\boldsymbol{\mathcal{A}}_\ell = S^1_{phase} \otimes \mathbb{C}^2_{chiral} \otimes \mathbb{C}^2_{weak}$,

with one phase-loop projection, one chiral rank-one selection and one weak-branch rank-one selection. The continuous projection weight is

$\Pi_{cont}^{(\ell)} = (1/2\pi) \cdot (\text{rank } P_{chiral}/2) \cdot (\text{rank } P_{weak}/2) = (1/2\pi)\cdot(1/2)\cdot(1/2) = \mathbf{1/(8\pi)}.$

Falsifier: a derived access-space factorisation with different factors, ranks or normalisations.

What the kernel derivation must actually supply (D21, sharpened). The expression $(1/2\pi)\cdot(1/2)\cdot(1/2)$ does not yet follow merely from naming one phase space and two rank-one projectors. A closing derivation must define: (i) the state or ensemble being projected; (ii) the phase measure; (iii) whether $1/(2\pi)$ is a measure density or an integrated probability; (iv) why the chiral and weak factors enter as statistical fractions rather than amplitudes (which would square them); and (v) why the three operations factorise. Until these are supplied, the Planck-basis reframing of §14.6 makes the kernel more visible, not more derived.

14.4 Premise E8 — Colourless multiplicity

The charged-lepton readout carries no colour multiplicity and no triality multiplicity: $N_\ell = 1$.

Falsifier: any derived multiplicity factor for the colourless readout other than unity.

14.5 The explicit ground scale

Under E6–E8, the charged-lepton ground scale is

$$\mathbf{m_{(\ell,0)}^{VERSF} = N_\ell\, \Pi_{cont}^{(\ell)}\, \alpha_{int}^2\, v_{cl} = \alpha_{int}^2\, v_{cl}/(8\pi).}$$

This replaces the abstract scale function F_ℓ. In the primitive-scale notation of Theorem 5 (with v_{cl} adjoined to the primitive set per Section 6.5),

$(\hbar/(c\xi))\, F_\ell = \alpha_{int}^2\, v_{cl}/(8\pi)$, so $\mathbf{F_\ell = (\alpha_{int}^2/(8\pi)) \cdot (v_{cl}\, c\, \xi/\hbar).}$

F_ℓ is no longer a freely selectable function: it is an explicit dimensionless expression in declared primitives. Its numerical status is exactly the status of v_{cl}, α_{int} and the readout scale — proof-gated, not free.

Complementarity note. The Role-4 variational programme independently proves that its structural results fix ratios only, leaving one genuine dimensionful degree of freedom (the $\varepsilon_{bit}/\varepsilon_{ref}$ conversion, equivalently the absolute electron mass) — exactly the situation Theorem 4 predicts for any dimensionless construction. The colourless kernel above is the CLAS-1 candidate for discharging that single open degree of freedom; conversely, the Role-4 no-go on absolute masses is an independent confirmation of the common-scale orbit theorem. The two programmes meet without overlap: ratios from localisation and closure, scale from the completion interface.

14.6 Planck-basis reading of the explicit scale

In the Planck basis of Theorem 5A, the explicit interface scale is a pure dimensionless descent,

$$\mathbf{\Phi_\ell = m_{(\ell,0)}/m_P = (v_{cl}/m_P) \cdot (\alpha_{int}^2/(8\pi)) = \Psi_H \cdot \Pi_\ell,}$$

factorising the Planck-to-lepton hierarchy into a **Planck-to-electroweak descent** $\Psi_H = v_{cl}/m_P$ and a **colourless interface projection** $\Pi_\ell = \alpha_{int}^2/(8\pi)$. With the imported audit values,

$\Psi_H = 2.01672 \times 10^{-17}$, $\Pi_\ell = 2.11880 \times 10^{-6}$, $\Phi_\ell = 4.27304 \times 10^{-23}$,

against the observed on-shell descent $m_e^{pole}/m_P = 4.18546 \times 10^{-23}$ — the same +2.09% common-scale residual, now expressed as a fully dimensionless Planck-normalised discrepancy. Per the Planck-to-pole firewall of §6.7, this is an *unmatched* on-shell audit target: the tree-level Φ_ℓ and the pole target are related only through the undelivered radiative bridge $\mathcal{R}_e$ (P9). This reframing sharpens the debt structure: the numerical GeV value of v_{cl} is secondary; the real theoretical object HR-1's successor must deliver is the dimensionless hierarchy $\mathbf{\Psi_H = v_{cl}/m_P}$, the interface programme must deliver $\mathbf{\Pi_\ell}$, and CLAS-1 defines and benchmarks the candidate descent $\Phi_\ell = \Psi_H\, \Pi_\ell$ (debt D34).

15. Explicit Chiral Lift, Mass Operator and Yukawa Operator

15.1 Branch partial isometries

Let $J_a : \mathscr{H}_R \to \mathscr{H}_L$, $a = e, \mu, \tau$, be the physical branch partial isometries satisfying

$$J_a^\dagger J_b = \delta_{ab} \Pi_a^R, \quad J_a J_b^\dagger = \delta_{ab} \Pi_a^L.$$

Define the branch-aligned chiral lift

$$J_\ell = J_e + J_\mu + J_\tau.$$

J_ℓ is unitary from $\mathscr{H}_R$ to $\mathscr{H}_L$, so it is an admissible W_ℓ in the sense of Theorem 6.

15.2 The explicit mass operator

The canonically normalised charged-lepton mass operator is

$$\mathbf{M_\ell^{VERSF} = (\alpha_{int}^2 v_{cl}/(8\pi)) \cdot [J_e + e^{(16/3)} J_\mu + (e^{(32/3)}/12) J_\tau] = (\alpha_{int}^2 v_{cl}/(8\pi)) \cdot J_\ell R_{\ell\star} e^{(2\kappa G_\ell)}.}$$

Its positive right-handed magnitude is

$$(M_\ell^{VERSF})^\dagger M_\ell^{VERSF} = (\alpha_{int}^2 v_{cl}/(8\pi))^2 \cdot [\Pi_e^R + e^{(32/3)} \Pi_\mu^R + (1/144) e^{(64/3)} \Pi_\tau^R],$$

which is exactly $\mathfrak{m}_\ell^2 \mathscr{H}_\ell$ with $\mathfrak{m}_\ell = \alpha_{int}^2 v_{cl}/(8\pi)$. This is the complete replacement for the abstract operator equation $M_\ell = \mathfrak{m}_\ell W_\ell \mathscr{H}_\ell^{(1/2)}$.

If the kinetic terms are noncanonical, the pre-canonical bilinear is $\mathscr{B}_\ell^{VERSF} = K_L^{(1/2)} M_\ell^{VERSF} K_R^{(1/2)}$; canonical normalisation then recovers the operator above.

15.3 The explicit Yukawa operator

Using the convention $M_\ell = (v_{cl}/\sqrt{2}) Y_\ell$, the canonically normalised VERSF Yukawa operator is

$$\mathbf{Y_\ell^{VERSF} = (\sqrt{2}\, \alpha_{int}^2/(8\pi)) \cdot [J_e + e^{(16/3)} J_\mu + (e^{(32/3)}/12) J_\tau],}$$

with singular values

$$\mathbf{y_e = \sqrt{2}\, \alpha_{int}^2/(8\pi), \quad y_\mu = y_e\, e^{(16/3)}, \quad y_\tau = y_e\, e^{(32/3)}/12.}$$

No independent charged-lepton Yukawa numbers are inserted. Note that this factorisation identifies $v\star = v_{cl}$ by convention; by Theorem 9 the mass spectrum tests only the product, so this identification is a labelling choice pending the scalar-sector derivation of v_{cl}.

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16. The Explicit Charged-Lepton Absolute Scale Theorem

16.1 Theorem 10 — Explicit Charged-Lepton Absolute Scale

Assume:

1. the physical electron, muon and tau branch projectors are independently derived (P7);
2. the three refinement depths are 0, 1, 2 (census inheritance);
3. the Role-4 localisation law is $L_g = L_0 e^{-8g/3}$, with the same exponent κ governing both refinement steps (E1, E2, E9);
4. physical mass magnitude scales as L_g^{-2} (E1);
5. the W7 primitive closure-cell weight is $2/24 = 1/12$ (E3);
6. the closure-cell activation acts once and only on the tau step (E4);
7. the VCS optimum lies at $R_\ell\star = \Pi_e + \Pi_\mu + (1/12)\Pi_\tau$ (E5);
8. the colourless source-current readout requires two interface gates (E6);
9. the substrate-to-field projection is $\Pi_{cont}^{(\ell)} = 1/(8\pi)$ (E7, E8);
10. the completion-interface scale is v_{cl} (D12);
11. the relevant interface coupling is α_{int} at a declared readout scale (D14');
12. no charged-lepton mass or Yukawa is used to select any of these structures (P8).

Application-status note. The theorem is a valid conditional implication. None of assumptions 3–11 is yet independently discharged — their open debts are tracked in §26.3 and Appendix F. Among them, assumptions 5–7 and 12 carry the sharpest present gap, because the exact one-surviving-channel tau structure remains open under D25 and D32.

Then the positive magnitude operator and the singular-mass spectrum are uniquely fixed, while the full chiral operator is fixed up to the independently admissible branch-aligned unitary equivalence class of J_ℓ:

$$\mathbf{M_\ell^{VERSF} = (\alpha_{int}^2 v_{cl}/(8\pi)) \cdot [J_e + e^{16/3} J_\mu + (e^{32/3}/12) J_\tau],}$$

and its physical tree-level masses are

$$\mathbf{m_e = \alpha_{int}^2 v_{cl}/(8\pi),\ m_\mu = (\alpha_{int}^2 v_{cl}/(8\pi))\, e^{16/3},\ m_\tau = (\alpha_{int}^2 v_{cl}/(8\pi))\, e^{32/3}/12,}$$

with exact internal ratios

$m_\mu/m_e = e^{(16/3)}$, $m_\tau/m_\mu = e^{(16/3)}/12$, $m_\tau/m_e = e^{(32/3)}/12$.

Proof. The inverse-area localisation law and the refinement-depth operator give $e^{(2\kappa G_\ell)} = \Pi_e + e^{(16/3)}\Pi_\mu + e^{(32/3)}\Pi_\tau$. The W7 tau activation gives $R_{\ell\star} = \Pi_e + \Pi_\mu + (1/12)\Pi_\tau$. Because the two operators commute,

$$R_{\ell\star}\, e^{(2\kappa G_\ell)} = \Pi_e + e^{(16/3)}\Pi_\mu + (1/12)e^{(32/3)}\Pi_\tau.$$

The colourless two-gate projection supplies the common scale $\alpha_{int}^2 v_{cl}/(8\pi)$. The branch partial isometries provide the chiral lift; orthogonality of the branch maps makes the singular values equal to the three positive coefficients. The positive magnitude and singular spectrum are unique under the enumerated assumptions; the full chiral operator remains defined up to the admissible branch-aligned unitary equivalence class. ■

16.2 Relation to Theorem 8

Theorem 10 is the abstract Theorem 8 with all four slots filled:

Abstract object (Part I)	Explicit instantiation (Part II)
$\mathscr{H}_\ell$	$\Pi_e + e^{(32/3)}\Pi_\mu + (e^{(64/3)}/144)\Pi_\tau$
W_ℓ	$J_\ell = J_e + J_\mu + J_\tau$
$\mathfrak{m}_\ell$	$\alpha_{int}^2 v_{cl}/(8\pi)$
F_ℓ	$(\alpha_{int}^2/(8\pi))\cdot(v_{cl}\, c\, \xi/\hbar)$

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17. Compliance of the Explicit Construction with the Part I Theorems

The explicit construction is not exempt from the Part I audit apparatus; it must pass it. This section verifies each requirement.

17.1 Positivity and full rank

$\mathscr{H}_\ell > 0$ with strictly positive eigenvalues 1, $e^{(32/3)}$, $e^{(64/3)}/144$ (Section 13.3). Requirement of Sections 4.4 and 7.1: **passed.**

17.2 Nondegeneracy and branch identification

The three eigenvalues are separated by factors of order 10^4 and 10^2, so the spectral projectors of $\mathscr{H}_\ell$ are unique and coincide with Π_e, Π_μ, Π_τ by construction. The branch labels are carried by the refinement-depth assignment (0, 1, 2) and the tau-localised activation — not by mass sorting. Requirement of Section 22: **passed structurally, conditional on P7.**

17.3 Chiral lift admissibility

J_ℓ is unitary and branch-aligned; the singular values of J_ℓ times a positive operator are the eigenvalues of the positive operator (Theorem 6). Status: **an admissible branch-aligned chiral representative has been constructed; the physical derivation of J_ℓ, or of its equivalence class, remains open under D11.**

17.4 Primitive-scale normal form

With v_{cl} adjoined to the primitive set, $F_\ell = (\alpha_{int}^2/(8\pi))(v_{cl}\, c\xi/\hbar)$ is dimensionless and explicit. Status: **the formula is dimensionally consistent with the declared primitive set; primitive completeness itself remains open under D8** — declaring v_{cl} does not prove that no other hidden dimensionful primitive exists.

17.5 Scale-locking consistency

The explicit construction satisfies the scale-locking identity of Theorem 7 with, for example, the trace anchor:

$\mathrm{Tr}[(M_\ell^{VERSF})^\dagger M_\ell^{VERSF}] = \mathfrak{m}_\ell^2\, \mathrm{Tr}\, \mathscr{H}_\ell$, $\mathrm{Tr}\, \mathscr{H}_\ell = 1 + e^{32/3} + e^{64/3}/144 \approx 1.2824 \times 10^7$,

and the determinant anchor:

$|\det M_\ell^{VERSF}| = \mathfrak{m}_\ell^3 \sqrt{\det \mathscr{H}_\ell}$, $\sqrt{\det \mathscr{H}_\ell} = \eta_e\, \eta_\mu\, \eta_\tau = e^{16/3} \cdot e^{32/3}/12 \approx 7.4052 \times 10^5$.

Both anchors return the same $\mathfrak{m}_\ell = \alpha_{int}^2\, v_{cl}/(8\pi)$. This must be graded precisely: because trace and determinant are here computed from the *same already-defined operator*, their agreement is an **internal homogeneous-anchor identity** — algebraically guaranteed — and not an independent anchor test. A genuine cross-anchor test requires VERSF to derive $\mathrm{Tr}(M_\ell^\dagger M_\ell)$ and $|\det M_\ell|$ independently, from different substrate structures, *before* inserting the candidate mass operator; no such second anchor has yet been derived. Status: **internal identity verified; independent second anchor open.**

17.6 Non-insertion

No η_a, no mass, no Yukawa and no ratio appears explicitly among the inputs $\{v_{cl}, \alpha_{int}, 1/(8\pi), \kappa = 8/3, \theta\star = 1/12, \Pi_a, J_a\}$. Requirement P8: **closed for continuous target insertion; open for the discrete tau-selection independence** — the surviving-channel value inside $\theta\star$ is presently target-adjacent (Section 20.0), so P8 is split rather than passed.

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18. Scale-Free Cross-Ratio and Parameter-Isolation Identities

The explicit construction implies identities that are invariant under the common-scale orbit of Theorem 3 and therefore testable without any dimensionful input. These are the sharpest falsifiable consequences of Part II.

18.1 Theorem 11 — Scale-Free Cross-Ratio Identity

Under the assumptions of Theorem 10,

$m_\mu^2/(m_e m_\tau) = 12$, equivalently $m_\mu^2 = 12 m_e m_\tau$,

exactly, independent of α_{int}, v_{cl}, the projection kernel $1/(8\pi)$, **and independent of the localisation exponent κ.**

Proof. $m_\mu/m_e = e^{2\kappa}$ and $m_\tau/m_\mu = e^{2\kappa}\theta\star$. Therefore

$m_\mu^2/(m_e m_\tau) = (m_\mu/m_e)\cdot(m_\mu/m_\tau) = e^{2\kappa}\cdot e^{-2\kappa}\theta\star^{-1} = 1/\theta\star = 12$.

The exponent κ cancels because the refinement ladder has equal steps. ■

The identity is independent of the *value* of κ but not of its *uniformity*: under a running exponent (E9 false) the ladder steps no longer cancel and the identity fails. Theorem 11 is therefore also the sharpest observational consequence of E9.

This identity tests exactly two structural claims and nothing else: the equal-step refinement ladder (depths 0, 1, 2) and the single tau activation of the primitive weight $\theta\star = 1/12$.

18.2 Theorem 12 — Exponent-Isolation Identity

Under the same assumptions,

$\kappa = (1/2) \ln(m_\mu/m_e)$,

independent of $\theta\star$, α_{int}, v_{cl} and the projection kernel.

Proof. Immediate from $m_\mu/m_e = e^{2\kappa}$. ■

18.3 Numerical values of the isolation audits

Using the reference values $m_e = 0.510999$ MeV, $m_\mu = 105.658$ MeV, $m_\tau = 1776.93$ MeV:

Identity	Predicted	Implied by observation	Deviation
$m_\mu^2/(m_e m_\tau) = 1/\theta\star$	12	12.29469	+2.46%
$\kappa = (1/2)\ln(m_\mu/m_e)$	$8/3 \approx 2.66667$	2.66580	−0.033%

The exponent audit is striking: the observed electron–muon ratio implies $\kappa = 2.66580$, within 0.033% of the inherited geometric value 8/3. The cross-ratio audit isolates the tau activation weight: observation implies $\theta\star = 0.0813359$ against the primitive $1/12 = 0.0833333$, a −2.40% deficit. Within the unmatched bare construction, the tau-step discrepancy is algebraically isolated in the suppression factor rather than the ladder; its physical attribution remains open pending radiative matching, since matching corrections can be branch-dependent.

κ-leverage refinement of the tau gap. The tau target carries 4κ of exponential leverage, while κ itself is pinned only to the 0.17% level by the muon postdiction. The $\theta\star$ gap is therefore κ-treatment-dependent: quoted as a clean −2.4% it overstates precision, and its honest range is roughly 2.4–2.9%, with κ-anchoring precision a legitimate candidate absorber of part of the gap alongside the structural candidates (a sub-leading count correction, finite-margin leakage, a residual anchoring-depth share). What κ-precision cannot do is absorb the whole gap: retuning κ toward the tau would pull it off the muon — and the muon postdiction is what validates the localisation law in the first place, so moving κ to the tau dissolves the law's own warrant (this is the structural reason behind falsifier F43). A residual of order a couple of percent survives any honest κ treatment.

Evidential-status caveat. While the numerator of $\theta\star$ remains imported from the tau residual (Section 12.7, D25), the scale-free identities of this section — the cross-ratio and the Koide invariant — function as *consistency identities* on the construction, not as independent corroboration: the 12 and the 1 do not yet recur anywhere independent of the tau datum they reproduce, and the charged-lepton domain contains no second heavy sibling (Section 12.8). They graduate from identities to genuine tests exactly when D25/D32 deliver the kernel dimension independently of the tau mass — at which point $m_\mu^2 = 12\, m_e m_\tau$ becomes a zero-parameter prediction with a live falsifier rather than an internal check.

These implied values are audit diagnostics only. Adjusting κ or $\theta\star$ to the implied values would constitute target insertion (falsifiers F43, F27).

18.4 κ-route audit

Four candidate derivation routes for the localisation exponent produce three numerical values: two independent routes to 8/3, one to ln 14, and one to ln(16π/3). Audited against the exponent-isolation value $\kappa\_implied = 2.66580$:

Route	Derived κ	Deviation from κ_implied
A — CP^2 Jacobi focusing (adopted)	8/3 = 2.66667	+0.033%

Route	Derived κ	Deviation from κ_implied
A′ — QN-2 substrate anomaly descent (independent chain, same value)	8/3 = 2.66667	+0.033%
B — closure algebra, ln dim(Γ_min)	ln 14 = 2.63906	−1.00%
C — interface Helfrich, ln($16\pi/3$)	2.81871	+5.74%

Route A's agreement is two orders of magnitude tighter than Route B's and is a conditional mass-independent postdiction: κ is derived without lepton input, but the identification of the derived stiffness with the mass-ladder exponent remains open under D24. That Routes A and A′ land on 8/3 by chains sharing no step — CP^2 focusing geometry versus substrate anomaly descent — gives the exponent value the same two-route convergence standing as the census and the suppression; under the Appendix G duality both would be static-gauge derivations, leaving Route B as the temporal-gauge candidate. But route selection must remain derivation-driven: adopting A *because* it fits best is falsifier F51, and the A/B reconciliation — or a proof that one supersedes the other — is debt D24. Note that Route B would give the integer prediction (no continuous parameter) $m_\mu/m_e = e^{(2 \ln 14)} = 196$ exactly, against the observed 206.77 (−5.2%); its failure at that level is itself informative for the reconciliation.

18.5 Θ_2 spread audit

The tau suppression admits its own three-way comparison:

Reading of Θ_2	Value	$1/\Theta_2$	τ/μ implied	Deviation
Observed	0.08134	12.295	16.818	—
Closure census (adopted)	0.0833	12.00	17.26	+2.63%
Variational collapse	0.0846	11.82	17.52	+4.19%

The two derivations cluster near, but not on, the observed value — convergence evidence consistent with a structural mechanism (a fit would have returned 12.295 exactly), while remaining insufficient to distinguish mechanism from shared hidden dependence or coincidence. The entire remaining tau tension is the ~2–4% spread inside this bracket; it no longer reopens the order of magnitude.

18.6 Theorem 12′ — Koide Cross-Check Invariant

Define the Koide combination $Q = (m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2$. Q is homogeneous of degree zero, hence invariant under the common-scale orbit of Theorem 3, and is therefore a second scale-free audit target alongside Theorem 11. Under the assumptions of Theorem 10,

$Q_{VERSF} = (1 + e^{16/3} + e^{32/3}/12) / (1 + e^{8/3} + e^{16/3}/\sqrt{12})^2 = 0.66929$, against the observed $Q = 0.66666$ (= 2/3 to one part in 10^5): a +0.39% excess.

Two consequences. First, the construction does **not** reproduce the empirical Koide relation exactly — which converts Koide from a rhetorical comparison into a quantitative cross-check. Second, Q and the cross-ratio of Theorem 11 are **coupled through θ⋆**: any refinement of the tau activation weight that improves τ/μ must simultaneously pull Q toward 2/3. A $\theta^\star$ correction that repairs one invariant while worsening the other falsifies the single-suppression structure (falsifier F56). Together, the cross-ratio and the Koide quantity define a one-dimensional joint consistency curve under variation of $\theta^\star$: a single suppression parameter cannot tune the two observables independently, but must reproduce their correlated pair — the observed (12.29469, 0.66666) either lies on the $\theta^\star$-curve or falsifies the single-suppression structure. One qualification applies before this becomes a physical test: all three masses and the VERSF operator must first be expressed in a common scheme and matched to the same observable convention — bare, running and on-shell quantities are not interchangeable inside a high-precision scale-free comparison.

18.7 Mechanism-attribution degeneracy and its discriminators

A rival decomposition of the same two ratios exists within the programme. The substrate-stiffness reconstruction

$$m_D = D^{\gamma_D} \cdot S_H(D) \cdot S_L(D) \cdot S_P(D) \cdot S_I(D) \cdot v$$

reproduces 1 : 205 : 3450 with localisation **sub-dominant** (7–9% of each log-ratio, through a back-fit length exponent) and the persistent-distinguishability load σ_D **dominant** (~55–66%). The σ values themselves are now derived to band level by the quotient-counting law

$$\sigma(n) = (n - 1) + (\xi + 2\eta)\cdot\mathbb{1}_{(n\geq 2)},\ \xi \in [1, 2],\ \eta \in [1/4, 3/4],$$

whose natural midpoint $\xi + 2\eta = 2.5$ reproduces the working values $\sigma_\mu = 3.5$, $\sigma_\tau = 4.5$ exactly, and whose inter-generational gap $\sigma_\tau - \sigma_\mu = 1$ and saturation shape $\sigma_\tau - \sigma_\mu < \sigma_\mu - \sigma_e$ contain no continuous parameter. With only two observed ratios, the localisation-dominant ladder of this paper and the information-dominant stiffness reading are observationally degenerate — a **three-way** degeneracy once running κ (Section 11.3′) is included.

The degeneracy has discriminators. The ladder makes the continuous-parameter-free identities of §§18.1–18.6 ($m_\mu^2 = 12\, m_e m_\tau$; κ-isolation at 8/3 to 0.033%; Koide coupling), which the stiffness decomposition does not naturally produce. The stiffness counting law makes its own continuous-parameter-free predictions (the forced unit σ-gap; the forced saturation shape; sector-extension consilience if the quark and neutrino n-bands also exhaust at 3), which the ladder produces only through the single tau activation. Adjudicating the attribution — including whether the stiffness σ-sector is a re-description of the closure-maintenance sector in different variables, so that the two pictures are one mechanism twice written — is debt D30; presenting both attributions as jointly confirmed is falsifier F59.

A convention hazard is logged in passing: the stiffness reconstruction employs $\kappa = \sqrt{(8/3)} \approx 1.633$ in a different structural role (a localisation-length rate paired with a back-fit exponent $p \approx 0.233$, not the mass exponent 2κ). The CP^2 object 8/3 must not be silently transported between roles (falsifier F60).

The degeneracy recorded here is not left as a bare contradiction. Section 18.8 points to the structured hypothesis (Appendix G) under which the two attributions are one mechanism in two gauges — with the requirements that would prove it and the outcomes that would kill it.

18.8 The Static–Temporal Cost Duality Conjecture (pointer)

The mechanism-attribution degeneracy above admits a structured resolution hypothesis: that the localisation-dominant and information-dominant decompositions are the **static** and **temporal** gauges of one substrate cost, related through the tick invariant ρ_s and coinciding exactly where a sector's commitment-rate structure is unmodified. Because the conjecture is a programme-level object rather than a charged-lepton scale result, its full statement — the five-entry proof dictionary, the sector gauge-asymmetry prediction, and the firewalls F64–F65 — is placed in **Appendix G**, with debt D33 tracking its discharge. Within the main text it functions only as the recorded organising hypothesis behind D24, D27 and D30.

19. Unmatched Numerical Benchmark and Algebraic Residual Factorisation

Numerical mass audits throughout use natural units $\hbar = c = 1$, so masses are quoted through their energy equivalents (MeV, GeV); explicit factors of $\hbar$ and c are retained in the structural theorems to preserve dimensional typing. This section's algebraic factorisation of the residuals is exact; its *physical* attribution is not yet licensed, because the computation mixes an imported low-energy coupling, an electroweak vacuum value, a tree-level mass operator and on-shell reference masses without a common scheme or matching scale (Section 24). The numbers below are therefore an unmatched benchmark, not a matched comparison.

19.1 Imported readout values

The audit uses the imported first-readout convention

$\alpha_{int}^{-1} = 137.035999084$, $v_{cl} = 246.21965$ GeV.

Data conventions: α^{-1} is the CODATA recommended value of the inverse fine-structure constant; v_{cl} is here **numerically identified with the conventional Fermi-constant electroweak scale**, $v = (\sqrt{2}\, G_F)^{-1/2}$ with G_F from the muon lifetime — an identification that is itself part of debt D12, since VERSF must derive both the scale and the legitimacy of that identification. The on-shell reference masses ($m_e = 0.51099895069$ MeV, $m_\mu = 105.6583755$ MeV, $m_\tau = 1776.93 \pm 0.09$ MeV) are Particle Data Group 2024 values (frozen edition), and $m_P = 1.220890$

$\times 10^{19}$ GeV is the CODATA 2018 unreduced Planck mass (frozen edition). The last quoted digits and their edition conventions are part of the audit record (references 18–19).

Neither α_{int} nor v_{cl} is yet derived within VERSF at this gate; both are declared imports (bridge debts D12, D14'). One provenance caveat: the benchmark is independent of the charged-lepton *masses* being tested, but not of all charged-lepton experimental information, because the imported electroweak scale is inferred from G_F, which is extracted from the muon lifetime. This does not violate the mass non-insertion rule, but it forecloses an overbroad "lepton-data-free" reading of the audit. The audit is therefore a consistency probe, not an empirical closure.

19.2 Projected masses

The colourless kernel gives

$m_e^{proj} = \alpha_{int}^2 v_{cl}/(8\pi) = 0.521691$ MeV,

and the localisation and closure operators then give

$m_\mu^{proj} = 108.056$ MeV, $m_\tau^{proj} = 1865.12$ MeV.

Branch	Explicit VERSF construction	On-shell reference value	Deviation
Electron	0.521691 MeV	0.510999 MeV	+2.09%
Muon	108.056 MeV	105.658 MeV	+2.27%
Tau	1865.12 MeV	1776.93 MeV	+4.96%

19.3 Ratio audit

$e^{(16/3)} = 207.127$ against $m_\mu/m_e = 206.768$: deviation **+0.174%.** $e^{(16/3)}/12 = 17.2606$ against $m_\tau/m_\mu = 16.8177$: deviation **+2.63%.**

19.4 Theorem 13 — Exact Residual Decomposition

Define the multiplicative residuals $r_a = m_a^{proj}/m_a^{ref}$. Then, exactly,

$r_e = r_{scale}$, $r_\mu = r_{scale} \cdot r_{e\mu}$, $r_\tau = r_{scale} \cdot r_{e\mu} \cdot r_{\mu\tau}$,

where $r_{scale} = 1.0209$ is the common-scale residual, $r_{e\mu} = 1.00174$ is the electron–muon step residual, and $r_{\mu\tau} = 1.0263$ is the tau-step residual.

Proof. The construction is a product of one scale and two ladder steps; residuals of a product factorise into residuals of the factors. Numerically $1.0209 \times 1.00174 = 1.0227$ and $1.0227 \times 1.0263 = 1.0496$, reproducing the table. ■

The audit therefore establishes:

1. the localisation operator closely reproduces the electron–muon ratio (+0.17%);
2. the tau residual is *algebraically located* in the tau-step factor of the bare construction (+2.63%), but is not yet physically proven to arise from the closure-cell activation rather than radiative or matching effects;
3. the combined interface-scale product $\alpha_{int}^2 v_{cl}/(8\pi)$ lies within ≈ 2.1% of the on-shell first-generation reference scale; the unmatched benchmark cannot attribute this proximity separately to the coupling power, the projection kernel or the radial scale, nor exclude compensation among them;
4. the residual structure is fully separable: any repair must address the scale channel and the tau channel independently, and the near-exact electron–muon step must not be disturbed.

19.5 Theorem 14 — Residual Non-Absorption

Let the construction of Theorem 10 be augmented by any continuous coefficient $c\star$ (or any continuous deformation of κ, $\theta\star$, or the kernel) whose value is chosen, in whole or in part, to reduce the deviations of Section 19.2–19.3. Then the augmented construction is a fit, not a derivation: it violates P8, triggers falsifiers F9/F27/F31/F42–F44, and forfeits the numerical non-insertion status of Section 20.

Proof. The choice of $c\star$ uses the observed masses as inputs; by the inverse-reconstruction argument of Theorem 21 (Section 25), agreement obtained this way carries no evidential weight, and by definition it inserts a member of the forbidden target set $\mathcal{T}_\ell$ into the input list. ■

The residuals are therefore to be treated as standing falsification pressure on the open debts — above all the readout scale of α_{int}, the derivation of v_{cl}, and the running-to-pole bridge — and not as adjustable slack.

19.6 Comparison-target caveat

The reference values are on-shell reference values; the construction delivers a tree-level operator at an undeclared matching scale. Until the scheme, scale and self-energy bridge are fixed (Section 24), the sign and size of the radiative shift are not available to explain or absorb any part of the residuals, and no claim in either direction is licensed.

19.7 Candidate residual channels (diagnostic only)

The Role-4 variational model identifies a natural candidate for the +0.17% electron–muon step residual: the computed light-sector energy ratio $\tilde{E}_1/\tilde{E}_0 = 0.999$ is not exactly unity, and multiplying the pure-geometry step by it moves the predicted ratio from 207.13 to 206.92, reducing the step residual from +0.174% to roughly +0.07%. Similarly, the variational $\Theta_2 = 0.0846$ versus the closure 1/12 brackets the tau channel. **Neither correction may be inserted numerically at this gate**: the rescaled energies rest on calibrated capacity inputs C_g (Section 12.2 caveat), so importing them would launder a calibration into the mass law (falsifiers F52, F53). They are recorded here as the identified physical channels through which the step residuals are expected to be derived — not fitted — once the capacities are substrate-derived (debt D26).

19.8 Planck-descent audit — the additive cost ledger

Because VERSF expresses hierarchies exponentially, define the Planck-descent cost

$\boldsymbol{\Sigma_\ell = -\ln(m_{(\ell,0)}/m_P)}$.

For the explicit interface construction this is an additive ledger of named costs,

$\Sigma_\ell^{VERSF} = \ln(m_P/v_{cl}) + 2\ln(\alpha_{int}^{-1}) + \ln(8\pi) = 38.4425 + 9.8405 + 3.2242 =$ **51.5071**,

against the observed electron descent

$\Sigma_e^{obs} = -\ln(m_e^{pole}/m_P) =$ **51.5278** (an unmatched on-shell target; §6.7),

so the entire common-scale residual is the additive gap

$\boldsymbol{\Delta\Sigma_{scale} = 0.0207}$ (≡ the +2.09% multiplicative residual).

The ledger reading is: Planck-to-Higgs cost (38.44) + two-gate coupling cost (9.84) + projection cost (3.22), with a 0.021 shortfall to be *derived*. One warning is mandatory: 51.5278 is an audit target, and searching the programme's stock of integers, exponents and closure counts for a combination reproducing it is discrete target insertion (F27, F66) — the ledger exists to be hit by derivations, not mined for coincidences.

20. Non-Insertion Status of the Explicit Inputs

20.0 Numerical and Discrete Non-Insertion Status

No measured charged-lepton mass, Yukawa coupling or continuously fitted coefficient is explicitly inserted into the formula for M_ℓ^{VERSF}. However, full non-insertion has **not** yet been established for the discrete tau-suppression structure: the claimed surviving-channel dimension $\dim \mathrm{Ker}(P_{maint}) = 1$, and therefore the exact factor 1/12, remain to be derived independently of the tau mass (debts D25, D32).

Accordingly:

- the **electron–muon ladder** is presently a genuine mass-independent postdiction, conditional on E1, E2 and E9;

- the **exact tau cross-ratio** $m_\mu^2 = 12\, m_e m_\tau$ is a conditional consistency identity until the one-dimensional saturated kernel is computed without using the charged-lepton spectrum;
- **P8 is therefore split**: closed for continuous target insertion, open for the independence of the discrete tau-selection premises.

Every "parameter-free" claim in this paper is to be read under this status: no continuously adjustable coefficient, conditional on the discrete premises.

20.1 Complete structural input list

The explicit construction contains no unspecified function F_ℓ, no unspecified positive operator $\mathscr{H}_\ell$, and no independently inserted electron, muon or tau Yukawa. Its complete structural input list is

$\{ v_{cl}, \alpha_{int}, 1/(8\pi), \kappa = 8/3, \theta^\star = 1/12, \Pi_e, \Pi_\mu, \Pi_\tau, J_e, J_\mu, J_\tau \}$.

20.2 Conditionality register

A numerical first-principles claim is admissible only if every item is derived without using the charged-lepton masses. The construction is therefore explicit but remains conditional on:

- derivation of v_{cl} from the closure-interface potential;
- derivation of α_{int} and its readout scale;
- proof of the two-gate current-readout law;
- proof of the $1/(8\pi)$ projection kernel;
- proof of $\kappa = 8/3$ and inverse-area mass scaling;
- proof that the W7 1/12 weight activates exactly once and only on the tau step;
- proof that the W7 cell weight equals the VCS tau maintenance eigenvalue;
- running-to-pole matching.

Each item is tracked as a named debt in Appendix F.

20.3 Derivation-chain register

Each explicit premise now carries a named derivation chain with a sharply located residual step:

Premise	Derivation chain	Sharply located open step
E1 (localisation law, L^{-2} scaling)	Scale-homogeneity postulate → exponential nesting theorem; Sector Rescaling Theorem → $M_g = \tilde{E}_g/L_g^2$	Scale-homogeneity postulate itself
E2 ($\kappa = 8/3$)	Route A: CP^2, $K_{hol} = 4$ → $K_{eff} = 8/3$; Route A′: QN-2 anomaly descent, 4	Sector–Curvature Correspondence (A); stiffness-identification gap (A′); $T_{conf} = 2e^{K_{eff}}$ (D24)

Premise	Derivation chain	Sharply located open step
	obligations → 3 support directions × 2 terminality = 8/3	
E3 ($\theta\star = 1/12$)	$\dim(\Gamma\_min) = 2(3+4) = 14$ (theorem) → conjectured split $14 = 2\_\partial + 12\_int$ → N_anchor = 12	Split naturalness, boundary exclusion, and N_survive = 1 (D25, three conjectural steps)
E4 (tau-only onset)	Capacity supercriticality $C_2 > C\_crit(2)$; muon subcritical; SCF-confirmed rigidity	Substrate derivation of the capacities C_g, C_crit(g) (D26)
E5 (weight = eigenvalue)	Two-route convergence: census 0.0833 ↔ variational 0.0846 (1.6% agreement)	Identity conjecture: channel count ↔ energy ratio (D27)
E9 (uniform κ)	K_eff carries no generation index; variational mechanism operates at fixed κ	Explicit confirmation in the Role-4 source (D28)
E10 (domain validity)	Empirical fence: muon postdiction validates the charged-lepton sector; opposite-direction quark failures (588 vs 20 against 207) exclude quarks	Sector re-entry via separately validated κ_up, κ_down, κ_ν (out of gate scope)
E6–E8 (kernel and gates)	Completion-interface access-space factorisation	Two-gate law and $1/(8\pi)$ kernel proofs (D21)

The register makes the inheritance-cap discipline auditable: no downstream numerical claim may be graded above the weakest chain it consumes.

Part III — Covariance, Stability, Firewalls and Audit Apparatus

21. Closure-to-Magnitude Bridge Conditions and What VCS-1 Does Not Imply

21.1 Closure data are not automatically mass data

The VCS closure operator $\Delta\_C = I - R\_C$ is dimensionless and positive. It is not automatically the magnitude operator $\mathscr{H}\_\ell$. A bridge map is required:

$\mathscr{H}\_\ell = \mathfrak{G}\_\ell(\Delta\_C, D\_M/\bar{d}, \chi, \Pi\_a, \ldots)$.

Part II supplies exactly such a bridge: $\mathfrak{G}\_\ell$ composes the VCS maintenance optimum $R\_\ell\star$ with the localisation ladder, $\mathscr{H}\_\ell = (R\_\ell\star\, e^{(2\kappa G\_\ell)})^\dagger (R\_\ell\star\, e^{(2\kappa G\_\ell)})$. The admissibility conditions below apply to it as to any candidate.

21.2 Admissibility requirements

A valid bridge $\mathfrak{G}_\ell$ must satisfy:

1. **Dimensionlessness:** $\mathscr{H}_\ell$ is dimensionless.
2. **Positivity:** $\mathscr{H}_\ell > 0$.
3. **Flavour covariance:** $\mathfrak{G}_\ell(UZU^\dagger) = U\mathfrak{G}_\ell(Z)U^\dagger$.
4. **Full rank** if all three charged leptons are to be massive.
5. **Independent construction:** no observed charged-lepton mass may be used to choose the functional form.
6. **Branch compatibility:** the resulting spectral projectors must be consistently matched to the physical branch structure.
7. **Scale separation:** the bridge may determine $\mathscr{H}_\ell$ but must not silently insert the absolute scale.

The Part II bridge satisfies 1–4, 6 and 7 by construction (Section 17); condition 5 is the proof-gated content of E1–E5.

21.3 Theorem 15 — Closure Data Insufficiency

The scalar closure data δ_τ, δ_0, f_τ do not uniquely determine a three-generation positive magnitude operator.

Proof by construction. Fix any admissible closure data. Infinitely many positive diagonal matrices $\mathscr{H}_\ell = \mathrm{diag}(x, y, z)$, $x, y, z > 0$, can be paired with those same scalar data, because the closure identity constrains only maintenance fractions, not the three independent mass magnitudes. ■

This theorem is why the localisation ladder is indispensable in Part II: the closure sector alone contributes only the single tau factor $\theta\star$; the electron–muon split is carried entirely by the refinement-depth ladder.

21.4 Theorem 16 — Monotone Maintenance-to-Magnitude Transfer

Suppose R_C and $\mathscr{H}_\ell$ commute and $\mathscr{H}_\ell = \varphi(R_C)$ for a strictly decreasing positive function φ. Let r_a be the branch eigenvalues of R_C. Then

$$r_a < r_b \Longrightarrow m_a > m_b.$$

Proof. Strict decrease gives $\varphi(r_a) > \varphi(r_b)$; the branch mass magnitudes are proportional to $\sqrt{\varphi(r_a)}$ and $\sqrt{\varphi(r_b)}$. ■

Limitation. The monotonicity direction is not automatic; an increasing bridge would reverse the ordering. The bridge must be dynamically derived. Note that the Part II bridge is **not** a pure function of $R_{\ell\star}$: the electron and muon share the maintenance eigenvalue 1 but differ in magnitude through G_ℓ. This is consistent with Theorem 15 and is the reason no purely maintenance-based monotone bridge could have produced the observed three-way split.

21.5 What VCS-1 supplies and what CLAS-1 adds

VCS-1 supplies: a positive closure-deficit operator; a localisation theorem; a sector-ordering statement; an exact suppression identity. Tau-only closure localisation does not distinguish the electron from the muon, and the value of f_τ does not by itself determine m_τ/m_μ or m_μ/m_e.

CLAS-1 adds: a positive mass-magnitude operator; a chiral lift; a dimensionful anchor; electroweak and renormalisation matching requirements — and, in Part II, explicit candidates for the first three.

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22. Basis Covariance, Uniqueness and Branch Identification

22.1 General flavour transformations

Let $S_L : \mathscr{H}_L \to \mathscr{H}_L$ and $S_R : \mathscr{H}_R \to \mathscr{H}_R$ be invertible flavour-frame transformations. Then

$K_L \mapsto S_L^\dagger K_L S_L$, $K_R \mapsto S_R^\dagger K_R S_R$, $\mathscr{B}_\ell \mapsto S_L^\dagger \mathscr{B}_\ell S_R$.

22.2 Theorem 17 — Basis Covariance

The generalised mass eigenvalues are invariant under simultaneous transformations of K_L, K_R, $\mathscr{B}_\ell$.

Proof. The quadratic action is unchanged under the passive field-coordinate transformation. Canonical mass operators obtained before and after the transformation are related by unitary maps arising from the polar factors of $K^{1/2}S$; singular values are therefore unchanged. Equivalently, the generalised eigenvalue pencil transforms by congruence and has the same roots. ■

22.3 Unitary canonical transformations

Once the kinetic terms are canonical, the remaining passive flavour transformations are unitary: $M_\ell \mapsto U_L^\dagger M_\ell U_R$. The singular values remain invariant. The explicit operator M_ℓ^{VERSF} transforms covariantly, with $J_a \mapsto U_L^\dagger J_a U_R$ and Π_a conjugated in the appropriate carriers.

22.4 Degeneracy caveats

If the magnitude spectrum is nondegenerate — as in Part II, where the eigenvalues are separated by orders of magnitude — the spectral projectors of $\mathscr{H}_\ell$ are unique. If two eigenvalues coincide, the spectral subspace is fixed but individual branch vectors within it are not, and a distinct identification requires an additional commuting observable.

22.5 Branch-label non-insertion

The branch labels must arise from inherited world-to-framework occupancy maps, gauge couplings, weak attachments, refinement-depth assignments or independently derived commuting flavour observables. They may not be assigned using the observed mass ordering. In Part II the labels are carried by the depth assignment (0, 1, 2) and the tau-localised activation; the required independence of that assignment is premise P7 with falsifier F11.

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23. Perturbative Stability

23.1 Perturbed scale and magnitude operator

Let $\tilde{\mathfrak{m}}_\ell = \mathfrak{m}_\ell(1 + \varepsilon_s)$ and $\tilde{\mathscr{H}}_\ell = \mathscr{H}_\ell + E$ with $E = E^\dagger$, and assume $\|E\|_\infty < \lambda_{min}(\mathscr{H}_\ell)$ so positivity is preserved.

23.2 Eigenvalue stability

Weyl's inequality gives $|\tilde{\lambda}_a - \lambda_a| \leq \|E\|_\infty$.

23.3 Theorem 18 — Mass Stability Bound

Assume $1 + \varepsilon_s > 0$ and pair the perturbed and unperturbed eigenvalues in the stated ordering. Then for every branch,

$$|\tilde{m}_a - m_a| \leq \mathfrak{m}_\ell \left[|\varepsilon_s| \sqrt{\tilde{\lambda}_a} + \|E\|_\infty / (\sqrt{\tilde{\lambda}_a} + \sqrt{\lambda_a}) \right].$$

For small perturbations the relative bound is

$$|\tilde{m}_a - m_a| / m_a \lesssim |\varepsilon_s| + \|E\|_\infty / (2\lambda_a).$$

23.4 Hierarchy stability

Three perturbation regimes must be kept distinct, because the binding constraints differ:

1. **Positivity-preserving generic perturbations.** For arbitrary Hermitian E, the norm condition $\|E\|_\infty < \lambda_{min}(\mathscr{H}_\ell) = 1$ (the electron eigenvalue) is a uniform *sufficient* condition for positivity of $\mathscr{H}_\ell + E$ — not a necessary one; positivity may survive larger structured perturbations, but must then be checked from their branch-resolved action. Within the sufficient regime every stability bound of §23.3 applies and all orderings are trivially preserved.
2. **Ordering-preserving perturbations.** The Weyl condition $\lambda_\mu - \lambda_e > 2\|E\|_\infty$ *and* $\lambda_\tau - \lambda_\mu > 2\|E\|_\infty$ would nominally tolerate $\|E\|_\infty$ up to $\approx 2.1 \times 10^4$ and $\approx 6.4 \times 10^6$ respectively — but for *generic* E the positivity constraint of regime 1 binds first, so these large margins are not directly available.
3. **Large structured perturbations.** The spectral gaps are exploitable only under branch-specific bounds. Positivity alone does **not** protect the ordering, and neither does $E \succeq 0$ by itself: a sufficiently large positive perturbation confined to the electron projector ($E = c\Pi_e$, $c > \lambda_\mu - \lambda_e$) preserves positivity while reordering the branches. Structured perturbations therefore require branch-resolved conditions — e.g. $\|\Pi_a E \Pi_a\|$ small on the lower branches — before the gap-scale margins may be invoked.

The correct rigidity statement is therefore: $\mathbf{\|E\|_\infty < (1/2)\cdot\min\{\lambda_\mu - \lambda_e, \lambda_\tau - \lambda_\mu\}}$ is a sufficient norm condition for preservation of both adjacent orderings, and for generic Hermitian perturbations the stronger bound $\|E\|_\infty < 1$ simultaneously guarantees positivity. The hierarchy is guaranteed ordered under the adjacent-gap bounds; positivity alone is insufficient to preserve branch ordering; larger structured perturbations require branch-specific bounds.

23.5 Scale sensitivity

A common fractional error in the scale (for example in $\alpha_{int}^2 v_{cl}$) produces the same common fractional error in every tree-level mass while leaving all mass ratios unchanged. This cleanly distinguishes scale uncertainty from hierarchy uncertainty and is exactly the decomposition observed in the numerical audit (Theorem 13).

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24. Renormalisation-Scale and Pole-Mass Firewall

24.1 Running mass operator

The operator $M_\ell(\mu\star)$ derived at a matching scale is generally a running mass operator; its singular values are running masses in the selected renormalisation scheme. This applies to M_ℓ^{VERSF}, whose matching scale is presently undeclared (debt D14).

24.2 Pole masses

The physical pole masses are determined by the poles of the full renormalised propagator, satisfying schematically

$$\det[\, \not{p} - M_\ell(\mu) - \Sigma_\ell(\not{p}, \mu) \,] = 0,$$

where Σ_ℓ is the renormalised self-energy.

24.3 Theorem 19 — Tree-to-Pole Separation

A derivation of $M_\ell(\mu\star)$ does not by itself constitute a derivation of the measured pole masses unless the renormalisation-group evolution and self-energy matching are also specified.

24.4 Required pole bridge

A complete observable prediction requires: (1) the matching scale $\mu\star$; (2) the renormalisation scheme; (3) gauge and scalar couplings; (4) threshold corrections; (5) running from $\mu\star$ to the relevant low scale; (6) the pole condition.

24.5 Appropriate claims

Before pole matching, the correct claim is: *VERSF constructs a canonically normalised charged-lepton tree-level mass operator whose unmatched benchmark against on-shell reference values shows a decomposable 2–5% residual.* It is not: *VERSF has reproduced the measured pole masses.* Nor may the undeclared radiative shift be invoked qualitatively to excuse the residual (Section 19.6).

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25. Numerical Non-Insertion and Inverse-Calibration Theorems

25.1 Forbidden target set

Define

$\mathcal{T}_\ell = \{ m_e, m_\mu, m_\tau, y_e, y_\mu, y_\tau$, charged-lepton mass ratios $\}$.

A first-principles CLAS-1 derivation requires

$\mathcal{T}_\ell \cap \mathrm{Inputs}(\mathscr{H}_\ell, F_\ell, K_L, K_R, W_\ell, \Pi_a, \kappa, \theta\star, \alpha_{int}, v_{cl}) = \emptyset$,

except for quantities explicitly declared as empirical calibration.

25.2 Theorem 20 — One-Mass Calibration Is Not Absolute Prediction

Suppose $\mathscr{H}_\ell$ is derived but the common scale is set using one observed mass m_b^{obs}: $\mathfrak{m}_\ell = m_b^{obs}/\eta_b$. Then the calibrated branch b is not predicted; the remaining masses $m_a = m_b^{obs} (\eta_a/\eta_b)$ test only the predicted ratios. Such a result is a **one-point calibrated two-ratio prediction**, not a zero-input absolute-scale theorem.

25.3 Theorem 21 — Inverse Scale Reconstruction

Given any target positive mass $m_b\star$ and any nonzero predicted magnitude η_b, the scale $\mathfrak{m}_\ell\star = m_b\star/\eta_b$ reproduces that target exactly. Therefore agreement after choosing $\mathfrak{m}_\ell$ from $m_b\star$ provides no evidence for the scale derivation.

25.4 Multi-parameter fitting and discrete-choice insertion

If free parameters in $\mathscr{H}_\ell$ are adjusted using all three charged-lepton masses, the resulting spectrum is a fit, not a prediction. Selecting an integer, exponent, branch, topological count or closure level because it best reproduces the masses is also insertion unless that choice is independently derived. This applies with full force to $\kappa = 8/3$, $\theta\star = 1/12$, the gate count 2, and the kernel $1/(8\pi)$: their evidential value stands or falls with their independent derivations, not with their numerical success.

25.5 Blind prediction protocol

The strongest empirical protocol is: (1) freeze all VERSF primitives and conventions; (2) derive $\mathscr{H}_\ell$; (3) derive F_ℓ (equivalently v_{cl}, α_{int}, the readout scale and the kernel); (4) declare the matching scale and scheme; (5) publish the three predicted running masses; (6) perform radiative matching; (7) only then compare with observation. The Part II audit deliberately violates step 3 by importing α_{int} and v_{cl}, and says so; that is why it is labelled a consistency probe.

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26. Premise-Dependency Map

26.1 Named premises

Premise	Content
P1	Positive left and right kinetic metrics
P2	Common matching scale and scheme
P3	Complete dimensionful primitive set
P4	Independently derived positive dimensionless magnitude operator
P5	Correct chiral and electroweak lift
P6	Independently derived dimensionful scale anchor or amplification
P7	Independently fixed physical branch projectors
P8	No target-mass insertion
P9	Valid renormalisation and on-shell matching bridge before any matched empirical claim
E1	Role-4 exponential localisation law with inverse-area mass scaling
E2	Localisation exponent $\kappa = 8/3$
E3	W7 primitive closure-cell weight $\theta\star = 2/24 = 1/12$
E4	Single tau-step activation of the cell weight
E5	Cell weight equals the VCS tau maintenance eigenvalue
E6	Two-gate colourless current-readout law (α_int^2 power)
E7	Factorised phase–chiral–weak projection kernel $1/(8\pi)$
E8	Colourless multiplicity $N\_\ell = 1$
E9	Generation-independent localisation exponent (uniform κ)
E10	Domain validity: the ladder applies only where its second-generation postdiction is independently validated (currently charged leptons only)

26.2 Result dependencies

Two dependency types are distinguished: ● marks an *algebraic dependency* (the result is mathematically false or undefined without it); ○ marks a *derivational or predictive-grade dependency* (the result remains algebraically true without it, but its status as an independent prediction requires it). For example, the scale-locking identity holds even for a calibrated anchor — P8 governs only its evidential grade.

Result	P1	P2	P3	P4	P5	P6	P7	P8	P9	E1–E2	E3–E5	E6–E8	E9	G
Common-scale no-go (Thm 3–4)														
Primitive-scale normal form (Thm 5)			●											
Planck-basis normal form (Thm 5A)			●											●
Planck–closure basis equivalence ($F = (\xi/\ell\_P)\Phi$)			●											●
Canonical normalisation (Thm 1)	●	●												

Result	P1	P2	P3	P4	P5	P6	P7	P8	P9	E1–E2	E3–E5	E6–E8	E9	G
Generalised singular spectrum (Thm 2)	●	●			●									
Minimal chiral lift (Thm 6)				●	●									
Scale-locking identity (Thm 7)				●		●		○						
Abstract absolute-scale theorem (Thm 8)	●	●	●	●	●	●	○	○						
Explicit magnitude operator (§13)							●	○		●	●		●	
Explicit scale (§14)			●					○				●		
Explicit absolute-scale theorem (Thm 10)	●	●	●		●	●	●	○		●	●	●	●	
Three-route cross-anchor identity (Cor 5A.3)			●			●						●		●
Cross-ratio identity (Thm 11)							○	○			●		●	
Exponent isolation (Thm 12)							○	○		●			●	
Residual decomposition (Thm 13)										●	●	●	●	
Planck-descent benchmark (§19.8)									○			●		●
Numerical running masses	●	●	●		●	●	●	○		●	●	●	●	
Numerical on-shell masses	●	●	●		●	●	●	○	●	●	●	●	●	
Empirical confirmation	●	●	●		●	●	●	○	●	●	●	●	●	

The G column marks results requiring Newton's constant (equivalently m_P) in the primitive set; they degrade gracefully to the ξ-basis if G is withdrawn. P4 is superseded by the E-columns on explicit rows. Theorem 10's dependence on P6 is discharged, conditionally, by the explicit scale candidate of §14 — that is exactly why its status reads "conditional implication proved" rather than closed.

26.3 Load-bearing open dependencies

The most important unresolved physical inputs are:

1. the derivation of v_cl from the closure-interface potential (discharges P6/D12);
2. the derivation of α_int and its readout scale (D14');
3. the proofs of the two-gate law and the 1/(8π) kernel (E6–E8);
4. the gate-level proofs of κ = 8/3, inverse-area scaling, and the single tau activation (E1–E5);
5. the electroweak matching scale and the pole-mass bridge (P9).

27. Falsification Conditions

#	Failure condition	Consequence
F1	The left- and right-handed carriers are not distinguished	Chiral mass construction invalid
F2	Gauge representations are ignored before electroweak breaking	Non-gauge-invariant mass term
F3	K_L or K_R is not positive	Canonical particle interpretation fails
F4	The raw bilinear is called the physical mass matrix without canonical normalisation	Field-normalisation error
F5	Ordinary eigenvalues of a non-Hermitian mass matrix are called masses	Wrong invariant spectrum
F6	Singular values are not used	Physical mass extraction invalid
F7	Dimensionless closure data are claimed to determine an absolute mass	Violates Theorem 4
F8	A dimensionful constant enters without being listed as a primitive	Hidden scale insertion
F9	F_ℓ (equivalently the kernel, coupling or scale) is selected from observed masses	Numerical insertion
F10	A charged-lepton mass is used to set $\mathfrak{m}_\ell$ and then called predicted	Calibration overclaim
F11	The tau mass is used to choose the tau branch	Retrospective branch labelling
F12	The observed hierarchy is used to choose the bridge monotonicity	Circularity
F13	Δ_C is identified directly with M_ℓ	Operator-type error
F14	A scalar participation fraction is claimed to fix three masses	Underdetermination
F15	Tau suppression is claimed to derive the electron–muon split	Missing flavour information
F16	The Higgs value is inserted while absolute-scale independence is claimed	Hidden empirical input
F17	$v_\star$ and $y_\star$ are claimed separately determined from masses alone	Violates product degeneracy
F18	The chiral lift lacks the correct electroweak transformation	Gauge mismatch
F19	The magnitude operator is not positive	Imaginary or undefined masses
F20	The magnitude operator has a zero eigenvalue while three nonzero masses are claimed	Rank contradiction
F21	Degenerate eigenvalues are assigned distinct branches without another observable	Branch ambiguity

#	Failure condition	Consequence
F22	Operators evaluated at different scales are combined	Renormalisation mismatch
F23	Running masses are compared with on-shell reference masses without either a valid matching bridge or an explicit unmatched-benchmark designation	Observable mismatch
F24	Radiative corrections are omitted from a precision claim	Incomplete prediction
F25	A direct bilinear is called a Yukawa derivation without Higgs factorisation	Interpretation overclaim
F26	A scale anchor is defined using the quantity it is meant to predict	Circular scale locking
F27	A discrete parameter is selected by best fit	Discrete target insertion
F28	The primitive-scale normal form is treated as fixing F_ℓ	Dimensional-analysis overclaim
F29	Approximate numerical agreement is claimed without uncertainty control	Unquantified comparison
F30	Passing CLAS-1 is described as empirical confirmation	Admissibility/truth confusion
F31	A fitted three-mass spectrum is called zero-input	Prediction-grade inflation
F32	The unitary chiral orientation is claimed to alter singular masses	Polar-decomposition error
F33	A nonunitary frame change is treated as a physical mass change	Basis-covariance failure
F34	The full dimensionful primitive set is incomplete	Normal-form theorem misapplied
F35	An inverse reconstruction is presented as a forward derivation	Direction-of-inference error
F36	A scale is quoted without units or matching convention	Type incompleteness
F37	The closure-to-mass bridge is postulated but described as derived	Bridge-debt concealment
F38	The matching scale is absent from a running-mass claim	Running mass undefined
F39	Pole masses are quoted without a self-energy prescription	Experimental claim invalid
F40	A common scale uncertainty is confused with a ratio uncertainty	Error-budget failure
F41	The completion eigenvalue 1/12 is conflated with the participation fraction 1/25	Operator/fraction type error
F42	A continuous coefficient is introduced to absorb the audit residuals	Violates Theorem 14
F43	κ is adjusted toward the observed value 2.66580	Exponent target insertion
F44	The readout scale of α_int is selected to improve the mass fit	Coupling-scale insertion
F45	The 2–5% audit is described as empirical confirmation	Overclaim on a consistency probe

#	Failure condition	Consequence
F46	v_cl = 246.21965 GeV is described as derived while imported	Import mislabelling
F47	Unequal ladder steps are introduced post hoc to repair the tau ratio	Ladder deformation by fit
F48	Closure suppression is counted in both R_ℓ⋆ and the localisation exponent	Double counting
F49	The branch partial isometries J_a are chosen using observed masses or mixing	Lift target insertion
F50	The cross-ratio identity $m_\mu^2 = 12\, m_e m_\tau$ failing beyond derived corrections is ignored	Suppressed falsification
F51	A κ route (8/3, ln 14, ln(16π/3)) is selected among rivals by mass-fit quality	Route-selection insertion
F52	Calibrated variational capacities C_g are described as substrate-derived	Calibration mislabelling
F53	Calibrated-regime energy ratios ($\tilde{E}_1/\tilde{E}_0 = 0.999$, $\tilde{E}_2/\tilde{E}_1 = 0.0846$) are inserted as numerical corrections while their capacity inputs remain calibrated	Laundered calibration
F54	The interior-channel count 12 and the full algebra dimension 14 are conflated	Census type error
F55	The channel-count ↔ energy-ratio identity is asserted as proved	Conjecture-grade inflation
F56	A θ⋆ refinement improves τ/μ while driving the Koide invariant away from 2/3, and is retained	Joint-invariant violation
F57	The boundary/interior split 14 = 2_∂ + 12_int is described as proved rather than conjectural	Grade inflation on the θ⋆ chain
F58	A gating law is adopted whose decline is not confined to the threshold neighbourhood (it gates the muon)	Plateau-condition violation
F59	The localisation-dominant ladder and the σ-dominant stiffness decomposition are presented as jointly confirmed	Mechanism-attribution double claim
F60	The CP^2 object 8/3 is transported between structural roles (mass exponent 2κ vs length rate √(8/3)) without derivation	Convention laundering
F61	The lepton-form ladder, activation, or θ⋆ is exported to quark or neutrino sectors without a separately validated localisation constant	Domain violation (E10)
F62	The surviving-channel dimension is identified with the sector-independent architectural nullity-one mode	Nullity trap — would gate the muon ($\Pi_1 = 1/12$)
F63	The step location at exact closure is described as data-established rather than natural-but-unprovable in-domain	Threshold overclaim
F64	The static–temporal duality is asserted from total-matching of the two decompositions (both were built against the same two ratios)	Duality overclaim — structure-level dictionary required

#	Failure condition	Consequence
F65	The duality dictionary forces stiffness parameters outside their independently derived bands ($\xi + 2\eta \notin [1.5, 3.5]$; incompatible p) and is retained	Dictionary-consistency violation
F66	m_e/m_P is computed from the observed electron mass and then presented as a derived Planck suppression	Inverse calibration in the Planck basis
F67	The Planck and ξ routes are called independent while ℓ_P was obtained from ξ through Route M	Basis change mislabelled as second derivation
F68	A Planck-scale bound known only to order-one factors is used for percent-level lepton normalisation	Precision laundering
F69	Reduced and unreduced Planck masses are interchanged without propagating the 8π convention through the projection kernel	Planck-convention violation
F70	The VERSF certification scale m_P^(cert) and the Newton-constant m_P^(N) are identified without a matching theorem	Planck-identification laundering
F71	The running descent Φ_ℓ(μ⋆) is compared to the pole target 4.18546×10^{-23} as if identical, before the bridge $\mathcal{R}$_e is derived	Matching-scale conflation

The cross-ratio identity deserves emphasis as the construction's cleanest kill switch: it involves no dimensionful input and no localisation exponent. If, after matching, the corrected cross-ratio deviates from observation beyond the derived corrections, the conjunction of E3–E5 and E9 is falsified — there is no continuous parameter available to save it without triggering F42, and the cross-ratio alone does not identify which individual premise failed.

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28. CLAS-1 Closure Certificate

CLAS-1 requirement	Result
Chiral carriers typed	Closed structurally
Gauge-representation distinction installed	Closed
Positive kinetic metrics defined	Closed
Canonical normalisation proved	Closed
Physical singular-mass spectrum proved	Closed
Generalised eigenvalue form proved	Closed
Common-scale orbit proved	Closed
Dimensionless absolute-scale no-go proved	Closed
Primitive mass unit identified	Closed
Primitive-scale normal form proved	Closed

CLAS-1 requirement	Result
Minimal chiral lift proved	Closed
Chiral-orientation independence proved	Closed
Scale-anchor sufficiency proved	Closed
Exact scale-locking identity proved	Closed
Abstract absolute-scale theorem proved	Closed
Explicit magnitude operator	Explicit candidate constructed; derivation open on E1–E5
Explicit scale	Explicit symbolic candidate supplied; derivation open on E6–E8, D12, D14′
Admissible chiral-lift representative constructed	Physical equivalence-class derivation open under D11
Explicit absolute-scale theorem	Conditional implication proved; physical premises not yet discharged
Cross-ratio identity $m_\mu^2 = 12\, m_e m_\tau$	Algebraically proved under E3–E5 and E9; independent predictive status open
Exponent-isolation identity proved	Closed conditionally
Internal homogeneous-anchor identity verified for the explicit operator	Closed as identity; independent second-anchor test open
Three-generation invariant formulae proved	Closed
Higgs-mediated matching formulated	Closed
Higgs–Yukawa product degeneracy proved	Closed
Closure-data insufficiency proved	Closed
Conditional monotone bridge theorem proved	Closed
Electron–muon split firewall installed	Closed
Completion-eigenvalue/participation-fraction firewall installed	Closed
Basis covariance proved	Closed
Degeneracy and branch-identity caveats installed	Closed
Perturbative stability supplied with explicit margins	Closed
Residual decomposition proved	Closed
Residual non-absorption theorem proved	Closed
Running-to-pole firewall installed	Closed
One-mass calibration theorem proved	Closed
Inverse scale reconstruction identified	Closed
Derivation of v_{cl}	Open
Derivation of α_{int} and readout scale	Open
Proof of two-gate law and $1/(8\pi)$ kernel	Open

CLAS-1 requirement	Result
Inverse-area law and exponential nesting derived (Sector Rescaling Theorem, scale homogeneity)	Closed conditionally on the homogeneity postulate
$\kappa = 8/3$ doubly derived (CP^2 focusing; QN-2 anomaly descent)	Closed conditionally; stiffness-identification gap open (D24)
QN-1 convention firewall (8/3 / 3/8 / ln 14 typed as distinct)	Closed as inherited firewall
Chiral carrier inherited from SC-1/SC-2	Closed conditionally
Occupancy-to-projector bridge ($\Pi_a \leftrightarrow$ SC-2 addresses)	Open (D2 refined)
Part I novelty provenance corrected (SF-1 inheritance recorded)	Closed as bookkeeping
Higgs-radial carrier structurally required (HR-1)	Closed structurally; numerical scale open (D12)
Uniform κ across steps (E9)	Open — confirmation of generation-independence (D28)
$\theta\star$ chain: $\dim(\Gamma_{min}) = 14$ theoremed	Closed conditionally
$\theta\star$ chain: $2_\partial + 12_{int}$ split, boundary exclusion, $N_{survive} = 1$	Conjectural — three open steps (D25)
Second independent three-generation census derivation (n-band exhaustion)	Closed conditionally
Plateau condition on gating laws installed	Closed
Count-versus-gate discriminator installed	Closed as audit
Suppression route typing (coupling vs anchoring)	Open (D29)
Mechanism-attribution degeneracy logged with discriminators	Closed as audit; adjudication open (D30)
Participation object Π_g defined; conditional collapse theorem recorded	Closed conditionally
Step location at exact closure	Natural, not forced; unprovable in-domain (D32)
Numerator $\dim \mathrm{Ker}(P_{maint}) = 1$	Imported from the tau — circularity hazard until D32
Nullity-trap exclusion installed	Closed as firewall (F62)
$K = 7$ architecture provenance	Open (D31)
Domain restriction to charged leptons (E10)	Closed as empirical fence
Evidential status of §18 identities	Consistency identities pending D25/D32; tests thereafter
Static–Temporal Cost Duality Conjecture stated with dictionary and falsifiers (Appendix G)	Closed as named conjecture (D33); unproved
Sector gauge-asymmetry prediction (temporal gauge reaches quarks with confinement commitments)	Open — the duality's primary test

CLAS-1 requirement	Result
Planck-basis normal form (Theorem 5A) and anchor equivalence proved	Closed conditionally on admitting G to the primitive set
Three-route cross-anchor identity stated	Closed as test; routes' independence open (D34, F67)
Planck-descent audit ledger ($\Delta\Sigma = 0.0207$) installed	Closed as audit; unmatched on-shell target pending P9
Planck-convention firewall (unreduced vs reduced, 8π propagation)	Closed as firewall (F69)
Independent derivation of the descent Φ_ℓ	Open (D34)
Surviving-channel count $N_{survive} = 1$	Open (D25)
Tau-onset mechanism located (capacity supercriticality)	Closed conditionally; capacities calibrated (D26)
Two-route convergence of Θ_2 (0.0833 vs 0.0846, 1.6%)	Closed as convergence evidence; identity and independent closure open (D27)
Koide cross-check invariant computed ($Q_{VERSF} = 0.66929$)	Closed as audit
κ-route reconciliation (A vs B vs C)	Open (D24)
Matching scale and scheme derived	Open
Radiative pole-mass bridge	Open
Numerical electron mass (unconditional)	Open
Numerical muon mass (unconditional)	Open
Numerical tau mass (unconditional)	Open
Empirical confirmation	Open

Closure grade

CLAS-1 is mathematically closed as a no-go, normal-form, canonical-normalisation, singular-spectrum and scale-locking theorem. It additionally supplies an explicit VERSF candidate mass construction — $M_\ell^{VERSF} = [\alpha_{int}^2 v_{cl}/(8\pi)]\cdot[J_e + e^{16/3} J_\mu + (e^{32/3}/12) J_\tau]$ — and a Planck-basis descent architecture, $\mathfrak{m}_\ell = m_P \Phi_\ell$. The conditional implications of that construction are algebraically closed, but its physical derivation remains open on the exact tau kernel, the colourless projection measure, the interface coupling and radial scale, the matching scale, and the radiative pole bridge. The unmatched benchmark places the electron, muon and tau masses approximately 2.1%, 2.3% and 5.0% above the on-shell reference values, factorising exactly into a +2.09% common-scale residual, a +0.17% electron–muon step residual and a +2.63% tau-step residual; no continuous coefficient may be introduced to absorb these residuals, and no empirical closure may be claimed, until the open items above are derived rather than fitted. The document thereby separates three distinct grades: proved mathematical theorem;

explicit physical candidate; unconditional empirical prediction — of which CLAS-1 delivers the first fully, the second explicitly, and the third not yet.

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29. Conclusion

A charged-lepton hierarchy and a charged-lepton absolute scale are not the same result. The dimensionless flavour structure may determine $m_e : m_\mu : m_\tau$, but every such result is invariant under $M_\ell \mapsto aM_\ell$. Therefore a purely dimensionless census, participation, closure or flavour theorem cannot determine an absolute mass. CLAS-1 proves that the missing ingredient is one independently derived dimensionful anchor — necessary by the common-scale no-go, sufficient by the scale-locking identity.

The physical charged-lepton sector is chiral and may have noncanonical kinetic metrics. The correctly normalised mass operator is $M_\ell = K_L^{(-1/2)} \mathscr{B}_\ell K_R^{(-1/2)}$; its physical tree-level masses are its singular values, equivalently the square roots of the generalised eigenvalues of the pencil $(\mathscr{B}_\ell^\dagger K_L^{(-1)} \mathscr{B}_\ell, K_R)$. Every full-rank chiral lift of a positive dimensionless magnitude operator has the polar form $M_\ell = \mathfrak{m}_\ell W_\ell \mathscr{H}_\ell^{(1/2)}$, the orientation does not alter the magnitudes, and dimensional analysis forces $\mathfrak{m}_\ell = (\hbar/(c\xi)) F_\ell(c\tau_s/\xi, \Lambda\xi^2, \mathfrak{I})$.

The gate then goes further than the abstract theorem. The refinement-depth ladder with $\kappa = 8/3$, the W7 closure-cell weight 1/12 activated once on the tau step through the VCS optimum, and the colourless two-gate completion-interface projection $1/(8\pi)$ together replace the abstract objects exactly:

$\mathscr{H}_\ell = \Pi_e + e^{(32/3)} \Pi_\mu + (e^{(64/3)}/144) \Pi_\tau$, $\mathfrak{m}_\ell = \alpha_{int}^2 v_{cl}/(8\pi)$,

so that

$\boldsymbol{m_e = \alpha_{int}^2 v_{cl}/(8\pi), m_\mu = m_e e^{(16/3)}, m_\tau = m_e e^{(32/3)}/12,}$

with the scale-free identity $m_\mu^2 = 12 m_e m_\tau$ (no continuous parameter; conditional on the discrete premises E3–E5/E9) and the exponent-isolation identity $\kappa = (1/2)\ln(m_\mu/m_e)$. The first audit places the construction 2–5% high with an exactly decomposable residual structure, algebraically places the unmatched tau residual in the tau-step factor of the bare construction — without yet proving that its physical origin is the activation weight — and finds the observed electron–muon ratio within 0.033% of the $\kappa = 8/3$ ladder. These residuals are standing falsification pressure on named open debts; the paper proves that absorbing them with fitted coefficients would forfeit the construction's non-insertion status.

The theorem does not identify the closure deficit with mass. It does not infer a mass ratio from a participation fraction. It does not derive the electron–muon split from tau suppression. It does not use a measured mass to define the scale and then rename the result a prediction. Its genuine programme advance is charged-lepton census → dimensionless flavour magnitude → chiral canonical lift → independent dimensionful anchor → explicit absolute running mass spectrum → auditable residual ledger.

In the Planck basis of §6.7, the entire construction reads as a single descent chain — Planck metric anchor → dimensionless electroweak descent → colourless lepton projection → generation hierarchy — with master formula

$$\mathbf{M_\ell^{VERSF} = m_P\, \Phi_\ell\, [J_e + e^{(16/3)} J_\mu + (e^{(32/3)}/12) J_\tau],\quad \Phi_\ell = (\alpha_{int}^2/(8\pi))(v_{cl}/m_P) = (\ell_P/\xi)\, F_\ell,}$$

so that the absolute-scale problem is now stated as one dimensionless programme: derive the running descent $\Phi_\ell(\mu\star)$, derive the radiative bridge $\mathcal{R}_e$, and only then compare the resulting on-shell descent with 4.18546×10^{-23} (D34), under the independence, identification and convention firewalls of §6.7.

One further object is now on the ledger by name. The gate's deepest open tension — that the programme contains a localisation-dominant and an information-dominant account of the same hierarchy — is recast in Appendix G as the Static–Temporal Cost Duality Conjecture: the two accounts as the static and temporal gauges of one substrate cost, related through the declared tick invariant ρ_s, coinciding exactly where a sector's commitment-rate structure is unmodified. The conjecture carries a five-entry dictionary that would prove it, a sector gauge-asymmetry prediction that would test it (the temporal gauge, extended with confinement commitments, reaching the quark towers the static ladder cannot), and firewalls against claiming it from total-matching alone. If the dictionary is proved, the κ-route bifurcation, the count-versus-gate question and the mechanism-attribution degeneracy resolve as one theorem; if it fails, each stands as separately recorded.

Final Theorems

Structural theorem, conditional on P1–P8. $M_\ell = \mathfrak{m}_\ell W_\ell \mathscr{H}_\ell^{(1/2)}$, with physical masses $m_a = \mathfrak{m}_\ell \sqrt{\lambda_a(\mathscr{H}_\ell)}$. Purely dimensionless data cannot fix $\mathfrak{m}_\ell$; one independently derived homogeneous dimensionful anchor is sufficient, and fixes it uniquely as $\mathfrak{m}_\ell = [A_\ell^{VERSF}/A(\hat{M}_\ell)]^{(1/q)}$.

Explicit conditional corollary (under E1–E10 and the independent branch premises).

$$M_\ell^{VERSF} = (\alpha_{int}^2 v_{cl}/(8\pi)) \cdot [J_e + e^{(16/3)} J_\mu + (e^{(32/3)}/12) J_\tau],$$

with $m_e = \alpha_{int}^2 v_{cl}/(8\pi)$, $m_\mu = m_e e^{(16/3)}$, $m_\tau = m_e e^{(32/3)}/12$, the scale-free identity $m_\mu^2 = 12 m_e m_\tau$ (no continuous parameter; conditional on the discrete equal-step and one-surviving-channel premises), and the split non-insertion status of §20.0: closed for continuous inputs, open for the discrete tau selection.

Planck-basis corollary (with G in the primitive set).

$M_\ell^{VERSF} = m_P \Phi_\ell \cdot [J_e + e^{(16/3)} J_\mu + (e^{(32/3)}/12) J_\tau]$, $\Phi_\ell = (\ell_P/\xi) F_\ell = (\alpha_{int}^2/(8\pi))(v_{cl}/m_P)$,

with the unmatched on-shell audit target $\Phi_e^{pole} = 4.18546 \times 10^{-23}$ reachable only through the radiative bridge $\mathcal{R}_e$, and the three-route cross-anchor identity of Corollary 5A.3 as the equality that independently derived routes must jointly satisfy.

No numerical pole-mass, Yukawa-normalisation or empirical claim is licensed until the completion scale (equivalently $\Psi_H = v_{cl}/m_P$), the interface coupling at its readout scale, the projection kernel, the surviving-channel count, the matching scale and the self-energy bridge are derived without using the charged-lepton masses as inputs.

Appendix A — Minimal Algebraic Skeleton

Chiral carriers: $\mathscr{H}_L$, $\mathscr{H}_R$. Kinetic metrics: $K_L > 0$, $K_R > 0$. Post-breaking bilinear: $\mathscr{B}_\ell : \mathscr{H}_R \to \mathscr{H}_L$. Canonical mass operator: $M_\ell = K_L^{(-1/2)} \mathscr{B}_\ell K_R^{(-1/2)}$. Physical squared masses: $m_a^2 \in \sigma(M_\ell^\dagger M_\ell)$. Generalised eigenvalue form: $\mathscr{B}_\ell^\dagger K_L^{(-1)} \mathscr{B}_\ell v_a = m_a^2 K_R v_a$. Dimensionless magnitude operator: $\mathscr{H}_\ell > 0$. Minimal chiral lift: $M_\ell = \mathfrak{m}_\ell W_\ell \mathscr{H}_\ell^{(1/2)}$. Masses: $m_a = \mathfrak{m}_\ell \sqrt{\lambda_a(\mathscr{H}_\ell)}$. Primitive dimensionless invariants: $\rho_s = c\tau_s/\xi$, $\lambda_\Lambda = \Lambda\xi^2$. Primitive scale normal form: $\mathfrak{m}_\ell = (\hbar/(c\xi)) F_\ell(\rho_s, \lambda_\Lambda, \mathcal{J})$. Homogeneous scale anchor: $A[aM] = a^q A[M]$. Scale locking: $\mathfrak{m}_\ell = [A_\ell^{VERSF}/A[\hat{M}_\ell]]^{(1/q)}$. Higgs-mediated form: $\mathfrak{m}_\ell = v_\star y_\star/\sqrt{2}$; product degeneracy $v_\star \to a v_\star$, $y_\star \to y_\star/a$.

Explicit instantiation: Refinement depth: $G_\ell = \Pi_\mu + 2\Pi_\tau$. Localisation ladder: $e^{(2\kappa G_\ell)} = \Pi_e + e^{(16/3)}\Pi_\mu + e^{(32/3)}\Pi_\tau$, $\kappa = 8/3$. Tau activation: $R_{\ell\star} = \Pi_e + \Pi_\mu + (1/12)\Pi_\tau$; $\Delta_{\ell\star} = (11/12)\Pi_\tau$. Magnitude: $\mathcal{M}_\ell = R_{\ell\star} e^{(2\kappa G_\ell)}$; $\mathscr{H}_\ell = \mathcal{M}_\ell^\dagger \mathcal{M}_\ell = \Pi_e + e^{(32/3)}\Pi_\mu + (e^{(64/3)}/144)\Pi_\tau$. Scale: $\mathfrak{m}_\ell = \alpha_{int}^2 v_{cl}/(8\pi)$; $F_\ell = (\alpha_{int}^2/(8\pi))(v_{cl} c\xi/\hbar)$. Mass operator: $M_\ell^{VERSF} = \mathfrak{m}_\ell[J_e + e^{(16/3)}J_\mu + (e^{(32/3)}/12)J_\tau]$. Masses: $m_e = \mathfrak{m}_\ell$, $m_\mu = \mathfrak{m}_\ell e^{(16/3)}$, $m_\tau = \mathfrak{m}_\ell e^{(32/3)}/12$. Scale-free identities: $m_\mu^2/(m_e m_\tau) = 1/\theta_\star = 12$; $\kappa = (1/2)\ln(m_\mu/m_e)$.

Appendix B — Proof of the Primitive-Scale Normal Form

Let the base dimensions be mass M, length L and time T. The primitives have dimensions

$[\hbar] = M L^2 T^{-1}$, $[c] = L T^{-1}$, $[\xi] = L$, $[\tau_s] = T$, $[\Lambda] = L^{-2}$.

Consider a monomial $X = \hbar^\alpha c^\beta \xi^\gamma \tau_s^\delta \Lambda^\varepsilon$, with dimensions

$M^\alpha L^{(2\alpha+\beta+\gamma-2\varepsilon)} T^{(-\alpha-\beta+\delta)}$.

For X to have dimensions of mass: $\alpha = 1$, $2 + \beta + \gamma - 2\varepsilon = 0$, $-1 - \beta + \delta = 0$. A particular solution is $\hbar/(c\xi)$. The remaining independent dimensionless monomials may be chosen as $c\tau_s/\xi$ and $\Lambda\xi^2$. Therefore every dimensionally homogeneous scalar mass can be written as $(\hbar/(c\xi)) F(c\tau_s/\xi, \Lambda\xi^2)$, with additional dimensionless invariants entering as further arguments of F. If an additional independent dimensionful primitive X' (such as v_{cl}) is adjoined, the corresponding dimensionless ratio (for v_{cl}: $v_{cl} c\xi/\hbar$) enters as a further argument; this is exactly the channel through which the explicit F_ℓ of Section 14.5 arises. ■

Appendix C — Generalised Singular-Value Proofs

C.1 Canonical right-handed magnitude

Define $H_R^c = M_\ell^\dagger M_\ell = K_R^{(-1/2)} \mathscr{B}_\ell^\dagger K_L^{(-1)} \mathscr{B}_\ell K_R^{(-1/2)}$. Because $H_R^c \geq 0$, it has a unique positive square root.

C.2 Generalised eigenvectors

If $H_R^c w_a = m_a^2 w_a$, put $v_a = K_R^{(-1/2)} w_a$. Then $\mathscr{B}_\ell^\dagger K_L^{(-1)} \mathscr{B}_\ell v_a = m_a^2 K_R v_a$, and the vectors may be chosen K_R-orthonormal: $v_a^\dagger K_R v_b = \delta_{ab}$.

C.3 Left-handed equation

Similarly, with $u_a^\dagger K_L u_b = \delta_{ab}$, $\mathscr{B}_\ell K_R^{(-1)} \mathscr{B}_\ell^\dagger u_a = m_a^2 K_L u_a$.

C.4 Polar lift

If $M_\ell^\dagger M_\ell = \mathfrak{m}_\ell^2 \mathscr{H}_\ell$, the polar decomposition gives $M_\ell = W_\ell(M_\ell^\dagger M_\ell)^{(1/2)} = \mathfrak{m}_\ell W_\ell \mathscr{H}_\ell^{(1/2)}$.

C.5 Explicit case

For $M_\ell^{VERSF} = \mathfrak{m}_\ell \Sigma_a \eta_a J_a$ with orthogonal partial isometries, $(M_\ell^{VERSF})^\dagger M_\ell^{VERSF} = \mathfrak{m}_\ell^2 \Sigma_a \eta_a^2 \Pi_a^R$, so the singular values are $\mathfrak{m}_\ell \eta_a$ directly, and the polar unitary is J_ℓ.

Appendix D — Three-Generation Invariant Formulae and Explicit Values

Let $\mathscr{H}_\ell$ have eigenvalues η_e^2, η_μ^2, η_τ^2. Then

$\mathrm{Tr}\,\mathscr{H}_\ell = \eta_e^2 + \eta_\mu^2 + \eta_\tau^2$, $\det \mathscr{H}_\ell = \eta_e^2\eta_\mu^2\eta_\tau^2$, $e_2(\mathscr{H}_\ell) = \eta_e^2\eta_\mu^2 + \eta_e^2\eta_\tau^2 + \eta_\mu^2\eta_\tau^2$.

Physical invariants:

$\mathrm{Tr}(M_\ell^\dagger M_\ell) = \mathfrak{m}_\ell^2\, \mathrm{Tr}\,\mathscr{H}_\ell$, $|\det M_\ell| = \mathfrak{m}_\ell^3 \sqrt{\det \mathscr{H}_\ell}$, $e_2(M_\ell^\dagger M_\ell) = \mathfrak{m}_\ell^4\, e_2(\mathscr{H}_\ell)$,

and the squared masses are the roots of

$x^3 - \mathfrak{m}_\ell^2(\mathrm{Tr}\,\mathscr{H}_\ell)x^2 + \mathfrak{m}_\ell^4\, e_2(\mathscr{H}_\ell)x - \mathfrak{m}_\ell^6 \det \mathscr{H}_\ell = 0$.

Explicit values for the Part II operator ($\eta_e = 1$, $\eta_\mu = e^{16/3} \approx 207.127$, $\eta_\tau = e^{32/3}/12 \approx 3575.13$):

$\mathrm{Tr}\,\mathscr{H}_\ell = 1 + e^{32/3} + e^{64/3}/144 \approx 1.2824 \times 10^7$; $\sqrt{\det \mathscr{H}_\ell} = e^{16/3}\cdot(e^{32/3}/12) = e^{48/3}/12 = e^{16}/12 \approx 7.4052 \times 10^5$; $e_2(\mathscr{H}_\ell) \approx 5.4836 \times 10^{11} + 4.2902 \times 10^4 + 1.2782 \times 10^7 \approx 5.4837 \times 10^{11}$.

Cross-anchor consistency (Section 8.4) holds identically for the explicit operator because both anchors are computed from the same $\mathfrak{m}_\ell$ and $\mathscr{H}_\ell$.

Appendix E — Worked Symbolic Models and Residual Arithmetic

These examples are structural illustrations, not numerical VERSF predictions.

E.1 Same hierarchy, different absolute scales

Let $\mathscr{H}_\ell = \mathrm{diag}(\varepsilon^4, \varepsilon^2, 1)$, $0 < \varepsilon < 1$. Then $m_e : m_\mu : m_\tau = \varepsilon^2 : \varepsilon : 1$, and both $M_1 = m_1\, \mathrm{diag}(\varepsilon^2, \varepsilon, 1)$ and $M_2 = m_2\, \mathrm{diag}(\varepsilon^2, \varepsilon, 1)$ have the same ratios for arbitrary $m_1, m_2 > 0$. This illustrates the common-scale no-go.

E.2 Trace scale locking

With the same magnitude operator, if VERSF derives S_2,ℓ^{VERSF}, then $\mathfrak{m}_\ell = \sqrt{(S_2,\ell^{VERSF}/(\varepsilon^8 + \varepsilon^4 + 1))}$ and all three masses are fixed.

E.3 Chiral orientation

Distinct unitaries $W_1 \neq W_2$ give mass operators $\mathfrak{m}_\ell W_1 \mathscr{H}_\ell^{(1/2)}$ and $\mathfrak{m}_\ell W_2 \mathscr{H}_\ell^{(1/2)}$ with identical singular masses but possibly different left–right flavour alignment.

E.4 Monotone closure toy model

With maintenance occupancies $r_e > r_\mu > r_\tau > 0$ and the illustrative bridge $\mathscr{H}_\ell = R^{-2}$, one obtains $m_a \propto r_a^{-1}$: lower maintenance transfers into higher mass. The exponent and functional form are not derived here and cannot be used numerically.

E.5 One-point calibration

If η_e, η_μ, η_τ are derived and the tau mass is used to choose $\mathfrak{m}_\ell = m_\tau^{obs}/\eta_\tau$, then $m_e^{test} = m_\tau^{obs}\, \eta_e/\eta_\tau$ and $m_\mu^{test} = m_\tau^{obs}\, \eta_\mu/\eta_\tau$. Only the two ratios are being tested.

E.6 Residual arithmetic for the Part II audit

Common-scale residual: 0.521691/0.510999 = 1.02092 (+2.09%). Electron–muon step: 207.127/206.768 = 1.00174 (+0.17%). Tau step: 17.2606/16.8177 = 1.02634 (+2.63%). Products: 1.02092 × 1.00174 = 1.02270 (+2.27% muon); 1.02270 × 1.02634 = 1.04963 (+4.96% tau) — reproducing the audit table exactly, as required by Theorem 13. Cross-ratio: observed $m_\mu^2/(m_e\, m_\tau) = 105.6584^2/(0.5109990 \times 1776.93) = 11\,163.69/908.01 = 12.29469$ against the predicted 12 (+2.46%); implied $\theta\star = 1/12.29469 = 0.0813359$ against $1/12 = 0.0833333$ (−2.40%). Exponent isolation: $\kappa_{implied} = (1/2)\ln(206.768) = 2.66580$ against $8/3 = 2.66667$ (−0.033%).

Appendix F — Open Bridge-Debt Ledger

Debt	Required content	Status
D1 — Chiral-carrier debt	Derive the physical left and right charged-lepton carriers	Inherited conditionally from SC-1/SC-2 (three generation packages; chiral structure L_L, e_R)

Debt	Required content	Status
D2 — Branch-projector debt	Derive electron, muon and tau projectors independently	Partially discharged: SC-2 fixes the occupancy addresses; the identification of the spectral projectors Π_a with the occupied branches (occupancy-to-projector bridge, §2.2) remains open
D3 — Kinetic-metric debt	Derive K_L, K_R	Open unless inherited
D4 — Magnitude-operator debt	Derive $\mathscr{H}_\ell$	Explicit candidate supplied by §13; derivational closure open (E1–E5)
D5 — Closure-bridge debt	Derive $\mathfrak{G}_\ell$ from VERSF closure structure	Explicit candidate supplied by §13/§21.1; derivational closure open
D6 — Electron–muon split debt	Derive the first two spectral magnitudes	Explicit candidate supplied by the refinement ladder; closure open (E1–E2, E9)
D7 — Tau-magnitude debt	Derive the third spectral magnitude without using observed tau data	Explicit candidate supplied by the W7 activation; closure open (E3–E5, D25, D32)
D8 — Primitive-completeness debt	Prove no dimensionful primitive has been omitted	Open audit (v_{cl} now declared)
D9 — Amplification debt	Derive F_ℓ	Explicit candidate supplied by §14; derivational closure open (E6–E8, D12, D14'); equivalently the descent Φ_ℓ of §14.6 (D34)
D10 — Scale-anchor debt	Derive a nonzero homogeneous dimensionful invariant	Open alternative route to D9
D11 — Chiral-lift debt	Derive J_ℓ or its physical equivalence class	Open
D12 — Higgs-matching debt	Derive v_{cl} from the closure-interface potential	Open — load-bearing; structurally supported by HR-1 (radial mode required), numerically undelivered
D13 — Yukawa-normalisation debt	Derive $y\star$ if separate Yukawas are claimed	Open (subject to product degeneracy)

Debt	Required content	Status
D14 — Matching-scale debt	Fix μ⋆	Open
D14' — Coupling-readout debt	Derive α_int and its readout scale	Open — load-bearing
D15 — Scheme debt	Specify the renormalisation scheme	Open
D16 — RG debt	Run the mass operator to low scale	Open
D17 — Self-energy debt	Calculate the pole-mass shift	Open
D18 — Uncertainty debt	Quantify primitive and truncation uncertainties	Open
D19 — Blind-comparison debt	Freeze the derivation before comparison	Open procedural requirement
D20 — Empirical debt	Compare the final pole masses with observation	Open
D21 — Kernel debt	Prove the two-gate law and the 1/(8π) projection kernel	Open — load-bearing
D22 — Activation debt	Prove single tau-step activation and θ⋆ = VCS eigenvalue	Open — load-bearing
D23 — Residual debt	Derive (not fit) the +2.09% scale and +2.63% tau-step residuals	Open — standing falsification pressure
D24 — κ-route debt	Reconcile Routes A/A′ (8/3: CP² focusing; QN-2 anomaly descent) with Route B (ln 14, closure) and Route C (ln(16π/3) ≈ 2.819, interface); derive the confinement tension T_conf = 2e^(K_eff); close the stiffness-identification gap (QN-2's canonical stiffness = the ladder exponent); QN-1 convention firewall in force	Open — E2 now doubly supported; facet (i) of the D33 duality (Appendix G): static vs temporal gauge of the exponent, residual 207 vs 196
D25 — θ⋆-chain debt	Derive the boundary/interior split 14 = 2_∂ + 12_int, the non-anchoring status of the boundary generators, and the surviving count N_survive = 1 — three conjectural steps beyond the theoremed dim(Γ_min) = 14	Open — the denominator now carries two open links, the numerator one
D26 — Capacity debt	Derive the sector capacities C_g and thresholds C_crit(g) from substrate dynamics (currently calibrated); cleanest sub-target: the ordering	Open — gates E4's onset location and unlocks the §19.7 residual channels

Debt	Required content	Status
	inequality dC/dg < dC_crit/dg over the bound sectors	
D27 — Identity-conjecture debt	Prove or refute N_survive/N_anchor ↔ $\tilde{E}_2/\tilde{E}_1$ (one suppression, two levels)	Open — conjecture; 1.6% agreement is evidence, not proof; facet (ii) of the D33 duality: static count vs temporal gate
D28 — Uniformity debt	Confirm K_eff carries no generation index (discharges E9; closes the running-κ degeneracy; protects Theorem 11)	Open — highest-leverage single confirmation
D29 — Route debt	Decide which substrate factor the maintenance eigenvalue reads: gated coupling (Route B, p_eff suppressed), anchoring depth (Route A, K_c deepened), or mixed (Route C); Route B currently assumed strategically	Open — types the suppression
D30 — Attribution debt	Adjudicate the mechanism-attribution degeneracy between the localisation-dominant ladder and the σ-dominant stiffness decomposition; determine whether the stiffness σ-sector re-describes the closure-maintenance sector; compute (ξ + 2η) from substrate thermodynamics	Open — programme-level; discriminators listed in §18.7; facet (iii) of the D33 duality: the two gauges of the whole cost
D31 — Architecture-provenance debt	Pin the K = 7 architecture (six-plus-one constraints, fourteen traversals, nullity-one) to a citable closure-geometry result and resolve the two-factors-of-two obligation beneath the 12	Open — the θ⋆ denominator rests on it
D32 — Projection debt	Derive P_maint from the Role-4 closure algebra: its existence, its activation at exact closure O = K rather than within O ∈ (3, 7], and dim Ker(P_maint) = 1, independently of the tau mass and excluding the nullity shortcut (F62); tractability of this computation is itself presently unassessed	Open — the load-bearing link; converts the imported numerator into a computed one and upgrades §18's identities to tests
D33 — Duality-dictionary debt	Prove the static–temporal cost dictionary of Appendix G: (i) exponent map resolving 207 vs 196; (ii) suppression map (discharges D27); (iii) activation map; (iv) reduction of the stiffness back-fits (p, k_D = T(D)) to derived ladder functions; (v) tick factorisation through ρ_s with no hidden primitive; plus the sector gauge-asymmetry test (temporal gauge with confinement commitments reaching the quark towers)	Open — Conjectural; would unify D24/D27/D30 and convert the E10 fence into a prediction; falsified per F64/F65 or by both-gauge quark failure

Debt	Required content	Status
D34 — Planck-descent debt	Independently derive $\Phi_\ell(\mu\star) = \mathfrak{m}_\ell(\mu\star)/m_P$; determine whether it factors through v_{cl}/m_P, ℓ_P/ξ, or a direct certification functional; establish which Planck and closure routes are genuinely independent (Route M renders the ℓ_P/ξ pair dependent); derive the Planck-to-pole bridge $\boldsymbol{\mathcal{R}}_e$ relating $\Phi_\ell(\mu\star)$ to Φ_e^{pole}; derive the matching between the VERSF metric-certification scale $m_P^{(cert)}$ and the conventional $m_P^{(N)}$, including any order-one conversion	Open — the dimensionless restatement of the whole scale problem; unmatched on-shell target $\Phi_e^{pole} = 4.18546 \times 10^{-23}$

Appendix G — The Static–Temporal Cost Duality Conjecture (companion-programme material)

Conjecture (Static–Temporal Cost Duality). The substrate cost of a persistent structure admits two readings of one quantity: a **static gauge** — the committed structure held (inverse localisation area, channel and commitment counts, a configuration property) — and a **temporal gauge** — the running rate the substrate pays per emergent tick to maintain that structure against refinement (commitments per tick, a process property). The two are related through the substrate tick conversion — the static mass unit $\hbar/(c\xi)$ and the temporal mass unit $\hbar/(c^2\tau_s)$ already differ only by the declared primitive invariant $\rho_s = c\tau_s/\xi$ (Section 6.2) — and they coincide exactly on sectors whose commitment-rate structure is unmodified. Since time is emergent from commitment events in VERSF, and physical mass defines a rest-energy or Compton angular frequency $\omega_C = mc^2/\hbar$ — allowing a temporal-rate representation of the same mass scale — the temporal gauge is a natural native reading rather than a mere analogy; the static gauge is its fixed-configuration shadow.

The claim applied to the ledger. Under the conjecture, three of the gate's open tensions are facets of one object:

1. **D24 (κ-routes).** Route A, $\kappa = 8/3$, is the static gauge of the exponent (CP^2 focusing geometry); Route B, $\kappa = \ln 14 = \ln \dim(\Gamma_{min})$, is its temporal gauge (log of the maintenance pathway count). They are not rivals but the same exponent twice written, and their present mismatch is the duality's sharpest quantitative residual: per generation step the static gauge gives $e^{(16/3)} = 207.13$ while the temporal gauge gives $\dim(\Gamma_{min})^2 = 196$ exactly — 5.7% apart, with observation (206.77) siding with the static reading. The dictionary must either derive the correction that carries 196 to 207 (the "inter-sector weight asymmetry" $w \approx 1.055$ already flagged in the closure route is a named candidate) or prove one gauge primary.

2. **D27 (count versus gate).** The quantised 1/12 is the static reading of θ⋆ (a state property, margin-independent); the continuous gating $p_{align}(\Delta)$ is its temporal reading (a rate suppression near threshold). The identity conjecture "channel count ↔ energy ratio" is the duality restricted to the tau term, and the −2.4% implied-θ⋆ deviation is its local residual.
3. **D30 (mechanism attribution).** The localisation-dominant ladder and the σ-dominant stiffness decomposition are the two gauges of the whole charged-lepton cost: mass as inverse coherence area (static) versus mass as informational maintenance rate (temporal). F59's tension dissolves *if and only if* the dictionary below is proved.

Non-triviality firewall. Both decompositions were constructed against the same two observed ratios, so their **totals agree by construction** (ladder $\ln(m_\mu/m_e) = 5.333$ vs stiffness 5.323; ladder $\ln(m_\tau/m_e) = 8.182$ vs stiffness 8.146) and total-matching is evidence of nothing (falsifier F64). The conjecture has content only as a **structure-level dictionary**, each entry derived rather than fitted:

(i) the exponent map: 2κ ↔ per-step maintenance-pathway multiplicity, resolving the 207/196 residual; (ii) the suppression map: the 12 interior channels ↔ the variational energy collapse $\tilde{E}_2/\tilde{E}_1$, i.e. D27 proved rather than conjectured; (iii) the activation map: single tau activation ↔ the forced saturation shape $\sigma_\tau - \sigma_\mu < \sigma_\mu - \sigma_e$; (iv) the back-fit reduction: the stiffness framework's fitted quantities (the length exponent $p \approx 0.233$; the linkage $k_D = T(D)$) become *derived* functions of ladder quantities under the gauge map — the dictionary's internal consistency test; (v) the tick factorisation: every entry of the map factors through the declared conversion ρ_s, with no new dimensionful primitive introduced (P3 discipline).

The discriminating consequence — sector gauge asymmetry. The duality holds where the sector's emergent-time structure is unmodified, and this converts the domain fence of E10 from an embarrassment into a prediction. Colour confinement is an additional *running* maintenance cost with no static-geometric counterpart; the conjecture therefore predicts precisely the observed pattern — the static ladder fails for quarks (E10's opposite-direction failures) — while the **temporal gauge, extended with confinement commitment classes, should still reach the quark towers**, and the admissibility-mode lightness of neutrino commitments should suppress σ_ν in the temporal gauge. *Which gauge survives is sector-diagnostic.* This is the joint prediction neither picture makes alone, and it is the conjecture's primary falsification surface: quark failure in **both** gauges, once confinement commitments are properly included, kills the duality rather than merely restricting it.

Grade and discharge. [Conjectural.] Discharged by proving the dictionary (i)–(v) — debt D33. Falsified by: total-matching offered as evidence (F64); the dictionary forcing stiffness parameters outside their independently derived bands — $\xi + 2\eta$ outside [1.5, 3.5], or a derived p incompatible with the ladder (F65); a tick-conversion factor inconsistent with the declared ρ_s (hidden-primitive violation of P3/F8); or both-gauge failure on the re-admitted quark sector. Until D33 is discharged, the mechanism-attribution degeneracy of §18.7 stands as recorded, and neither gauge may borrow the other's successes (F59 remains in force).

Appendix H — References

Internal VERSF programme papers (author K. Taylor, VERSF Theoretical Physics Programme; versions as supplied to this gate, 2026). The bracketed note records what CLAS-1 inherits from each.

1. SC-1 — "Standard Model Census Series: Generation/Species Census Theorem." [Three complete generation packages; chiral structure (L_L, e_R); Higgs-mediated closure route — §2.]
2. SC-2 — "Standard Model Census Series: World-to-Framework Occupancy Map." [Unique framework addresses of e, μ, τ; separation of chirality, weak and mass bases — §2, P7; the occupancy-to-projector bridge remains open (D2).]
3. SF-1 — "Singular-Value and Frame-Covariance Discipline for Fermion Magnitudes." [Hermitian-lift/singular-value extraction; polar decomposition; basis covariance; kinetic-metric discipline — Theorems 1, 2, 6, 17 restate and extend.]
4. QN-1 — "Quantitative Normalisation Series: κ-Reconciliation and Convention Audit." [Typed distinction of 8/3, 3/8, ln 14; convention firewall behind F54/F60; ln 14 ↔ 8/3 left open — §11.3.]
5. QN-2 — "Quantitative Normalisation Series: Substrate Anomaly-Descent Theorem." [Independent conditional derivation of $\kappa\star = 8/3$ (Route A′); stiffness-identification gap recorded in D24 — §11.3.]
6. HR-1 — "The Closure-Interface Potential Necessity and Higgs-Radial Theorem in VERSF," July 2026. [Structural necessity of the radial mode; numerical vacuum scale explicitly open — §10.4, D12.]
7. VCS-1 — "The Variational–Closure Suppression Identity in VERSF," July 2026. [Closure-deficit operator; tau-localised maintenance optimum $R_\ell\star$; suppression identity — §12.]
8. SMP(τ)-1 — "The Saturated-Maintenance Projection and Tau-Suppression Theorem in VERSF," July 2026. [Saturated-maintenance projection; tau-sector suppression structure — §12.]
9. FH-1 — "The First χ-Increment Magnitude-Bridge and Charm-Hinge Theorem in VERSF," July 2026. [Magnitude-bridge admissibility conditions — §21.]
10. VERSF-TP-I — "Toward a Lepton-Sector Mass Derivation in the Role-4/BCB Framework." [Sector Rescaling Theorem ($M_g = \tilde{E}_g/L_g^2$); exponential nesting under scale homogeneity; calibrated capacity model; $\tilde{E}_1/\tilde{E}_0 = 0.999$, $\tilde{E}_2/\tilde{E}_1 \approx 0.0846$ — §§11–12, E1, E4.]
11. "Structural Completeness in the Role-4 Lepton Sector Model: No-Fourth-Generation, Parameter Dependencies, and Calibrated-Regime Nonlinear Stability." [$\dim(\Gamma_{min}) = 14$ theorem; unique minimal pair (3, 4); capacity bound $C_0 \leq 14\kappa'$; no-fourth-generation results; SCF selective stabilisation — §§11–12.]
12. "A Charged-Lepton Ratio Law from Localization and Saturated Participation: A Convergence of Two Derivation Chains." [Census/variational Θ_2 convergence; Koide computation; CP^2 chain assembly — §§11.3, 12.5, 18.6.]

13. "Threshold Compression in the Charged-Lepton Spectrum: A Muon-Scale Postdiction and the Open Tau Suppression Factor." [Coherence–anchoring decomposition; Routes A/B/C; plateau condition; conjectural status of the $2_\partial + 12_{int}$ split — §§12.2, 12.6, E3 grading.]
14. "First Toy Reconstruction of the Charged-Lepton Hierarchy in VERSF." [Substrate-stiffness rival decomposition; mechanism-attribution degeneracy — §18.7, D30.]
15. "First-Principles Enumeration of Persistent Distinguishability in Charged-Lepton PFDs." [n-band {1, 2, 3} exhaustion; second census derivation; σ-counting law — §§11.1, 18.7.]
16. "Charged-Lepton Participation Gating in Saturated Closure." [Participation object Π_g; conditional collapse; nullity trap; quark domain restriction — §§12.7, 12.8, E10.]
17. Route M / Planck-certification / Fold–Planck–Closure papers. [ℓ_P/ξ descent candidate; certification bound (order-one); metric-certification vs coherence floors — §6.7, D34.]

External references.

18. Particle Data Group, *Review of Particle Physics*, 2024 edition (frozen for this gate): on-shell charged-lepton reference masses $m_e = 0.51099895069$ MeV, $m_\mu = 105.6583755$ MeV, $m_\tau = 1776.93 \pm 0.09$ MeV.
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