The Chiral Matter Representation Theorem in VERSF

▲ Programme Milestone — Standard Model Matter-Representation Series

Deriving the Lepton–Quark, Weak-Doublet, Colour-Support, and Charge-Assignment Skeleton from the Spinorial Fock Sector

Keith Taylor *VERSF Theoretical Physics Programme — Standard Model Matter-Representation Series Programme-marker paper*

Successor to *The Fermionic Fock-Space Reconstruction in VERSF*.

Headline result — the representation skeleton, not the mass spectrum.

The predecessor paper reconstructs the generic fermionic Fock-space layer from spinorial commitment loops. The present paper takes the next Standard-Model question: not what fermions weigh, but what representation slots exist.

Given the spinorial Fock sector, persistent winding charge, weak-commitment chirality, and a threefold confined support module, the admissible matter sector decomposes into a one-generation Standard-Model-like chiral skeleton

$$L_L = (\nu_L, e_L), \quad e_R, \quad Q_L = (u_L, d_L), \quad u_R, \quad d_R,$$

together with opposite-winding conjugate sectors. The charge and hypercharge ledger is the anomaly-safe pattern

$$L_L : (1, 2, -\tfrac{1}{2}), \;\; e_R : (1, 1, -1), \quad Q_L : (3, 2, \tfrac{1}{6}), \;\; u_R : (3, 1, \tfrac{2}{3}), \;\; d_R : (3, 1, -\tfrac{1}{3}).$$

Strengthened claim. Given the lepton charge normalization, the weak-doublet/singlet representation content, and perturbative anomaly freedom, the quark hypercharges are the unique completion (over $\mathbb{C}$), which the positive discriminant guarantees is real:

$$Y_Q = \tfrac{1}{6}, \;\; Y_u = \tfrac{2}{3}, \;\; Y_d = -\tfrac{1}{3}.$$

The quark charge ledger is therefore rigid *within the representation/anomaly audit*: once lepton normalization and anomaly freedom are imposed, the quark hypercharges are forced, not fitted — not chosen by mass, mixing, or phenomenological fitting. Deriving anomaly inadmissibility itself from substrate cohomology is the next debt, not a result claimed here.

The claim is deliberately narrow. This paper does **not** derive fermion masses, Yukawa couplings, CKM angles, PMNS angles, generation hierarchies, or the Higgs-sector mass map. It derives the representation skeleton on which those already-separate mass and mixing modules act.

General Reader Summary

The predecessor paper reaches an important point: it shows how the programme gets from tiny spin-carrying loops in the substrate to the mathematical machinery physicists use for fermions — the particles that make up matter. That is a major step, but it yields only a single, generic fermion field. The Standard Model needs more: electrons, neutrinos, quarks, antiparticles, weak doublets, colour-supported quark states, and precise charge assignments.

This paper asks the next question.

It does **not** ask why the electron has its mass, why the strange quark outweighs the down quark, or why quarks mix by the angles they do. Those belong to the mass and mixing programme developed elsewhere. This paper is about something prior: before asking how heavy each particle is, one must ask what kinds of particle slot exist at all.

The proposal is that the generic spinorial fermion sector branches because different loop states carry different kinds of unresolved commitment. Some remain weakly paired, giving the two-member structures seen in weak doublets. Some are fully committed, giving weak singlets. Some carry charge as a single isolated support, giving lepton-like sectors. Others carry charge only through a threefold internal support structure, giving quark-like sectors with fractional charge and colour-like multiplicity. Opposite winding supplies the conjugate sector, the natural home for antiparticles.

The result is not yet the full Standard Model. It does not prove full SU(3) Yang–Mills colour, the Higgs mechanism, exact charge conjugation, three generations, or the fermion mass spectrum. But it does something structural: it turns the single generic fermion field into a matter table resembling one generation of the Standard Model.

One point sharpens the earlier draft. The quark charges ($+\frac{2}{3}$ and $-\frac{1}{3}$) can look like a lucky choice among several third-integer options. They are not a choice. Once the lepton charges are fixed and the ledger is required to be free of quantum anomalies — internal inconsistencies that would break the theory — the quark charges are the only solution the equations allow. The lepton sector and the quark sector lock together.

The milestone is the move from

spinorial loops → fermion algebra

to

fermion algebra → chiral matter representation skeleton.

In plain terms: VERSF no longer merely has *fermions*. It has a candidate route to why the fermion sector branches into lepton-like, quark-like, weak-doublet, weak-singlet, colour-supported, and conjugate matter roles, with a charge ledger that is forced rather than fitted.

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Abstract

The preceding VERSF paper reconstructs the fermionic Fock-space layer from spinorial commitment loops: inherited Finkelstein–Rubinstein antisymmetry confines the spinorial sector to the exterior algebra over a positive one-particle Hilbert space; creation is wedge insertion; annihilation is its Hilbert adjoint; and the full canonical anticommutation relations follow, conditional on one-particle positivity, antisymmetric-sector exactness, Fock density, and adjoint-removal admissibility.

That result supplies a generic fermionic field sector. It does not supply Standard Model matter content. The present paper addresses the next representation-level problem: how the generic spinorial Fock sector decomposes into chiral matter branches corresponding to leptons, quarks, weak doublets, weak singlets, colour-supported modes, and conjugate sectors.

The construction is firewalled from the mass programme. No fermion mass, Yukawa coupling, CKM element, PMNS element, Higgs ratio, or generation hierarchy enters as input or is claimed as output. The target is the representation skeleton prior to mass assignment.

Let the one-particle spinorial source-carrier space carry four additional structures: a persistent winding/charge grading; a weak-commitment projector P_W selecting unresolved two-branch modes; a chirality projector P_L inherited from the spinorial Dirac-admissibility layer; and a confined threefold support module C_3 for fractional source support. Under the chiral-locking condition $P_W = P_L$ on the weak-active low-energy sector, weak-active modes form left-chiral doublets while fully committed right-chiral modes remain weak singlets. Under the colour-support condition, single-support modes form lepton-like sectors, while modes requiring threefold confined support form quark-like sectors.

The minimal charge ledger then follows from three requirements: integer winding for isolated lepton-like support; third-unit effective charge for threefold confined support; and the weak relation $Q = T_3 + Y$. This fixes the lepton sector as $(0, -1)$ with $Y = -\frac{1}{2}$. Perturbative anomaly freedom, applied to the derived representation content, then determines the quark hypercharges **uniquely** as $(Y_Q, Y_u, Y_d) = (\frac{1}{6}, \frac{2}{3}, -\frac{1}{3})$; the quark charges $(\frac{2}{3}, -\frac{1}{3})$ follow from $Q = T_3 + Y$. The one-generation chiral skeleton is

$$L_L : (1, 2, -\tfrac{1}{2}),\quad e_R : (1, 1, -1),\quad Q_L : (3, 2, \tfrac{1}{6}),\quad u_R : (3, 1, \tfrac{2}{3}),\quad d_R : (3, 1, -\tfrac{1}{3}),$$

with opposite-winding conjugates. The ledger passes the standard one-generation anomaly audits — $SU(3)^3$, $SU(3)^2U(1)$, $SU(2)^2U(1)$, gravitational-$U(1)$, $U(1)^3$, and the even-doublet Witten $SU(2)$ condition — and yields integer-charge three-channel composites ($uud \to +1$, $udd \to 0$).

The result is conditional. The paper does not derive full non-abelian gauge dynamics, continuous $SU(3)$ colour from the C_3 support module, exact charge conjugation/CPT, three generations, or mass terms. It derives the first chiral matter-representation skeleton, given the named VERSF representation premises. The remaining debts are cleanly exposed: prove chiral locking, lift the threefold support module to continuous colour, derive anomaly freedom as substrate-

inadmissibility rather than imposing it, establish exact conjugation, and then attach the already-separate mass and mixing modules.

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0. Notation, Conventions, and Firewall

Notation

$\mathscr{H}_1$ denotes the one-particle spinorial source-carrier Hilbert space inherited from the fermionic Fock-space reconstruction.

$\mathscr{F}$_A($\mathscr{H}_1$) denotes the antisymmetric Fock space.

P_L and P_R denote spinorial chirality projectors, with P_L + P_R = 1.

P_W denotes the weak-commitment projector selecting modes that retain unresolved weak two-branch access.

C_3 denotes the threefold confined support module. It is not identified here with full continuous SU(3) gauge theory. It is the discrete/support-level precursor whose continuous internal-rotation lift remains owed.

W denotes the persistent winding/charge grading.

Q denotes electric/source charge. T_3 denotes the third weak-isospin generator on a weak doublet. Y denotes hypercharge, with the convention

$$Q = T_3 + Y.$$

Representation labels are written in the usual shorthand (SU(3)_C, SU(2)_W, Y), but the SU(3)_C entry is graded as a representation skeleton, not yet as a full Yang–Mills derivation. Anti-triplet is written $\bar{3}$.

Mass firewall

This paper does not compute, explain, fit, or revise fermion masses. It uses no observed masses. It derives no Yukawa couplings. It does not alter the CKM or PMNS programme. It does not explain generation mass hierarchy.

The purpose is prior in representation order — *what matter representation slots exist?* — not *what are their masses?* The mass/Yukawa/mixing sector is downstream; this paper derives the stage on which those operators later act.

Claim firewall

No measured Standard Model mass, mixing angle, decay rate, or scattering amplitude enters the construction. The representation ledger is constrained only by inherited spinorial Fock structure, persistent winding charge, weak-commitment chirality, threefold confined support, charge normalization, and anomaly consistency. The paper distinguishes throughout between exact algebra, conditional representation reconstruction, and owed dynamical derivation.

0′. Predictive-Content Ledger

Object	Grade	Status
Spinorial commitment loops	Inherited	From spinorial source-carrier papers
Fermionic Fock space $\mathscr{F}$_A($\mathscr{H}_1$)	Inherited / Conditional	From Fock-space reconstruction
Full CAR	Inherited / Conditional	Given positivity, antisymmetry, adjoint-removal
Persistent winding charge	Inherited	From record-current / topological-charge programme
Opposite-winding conjugacy	Candidate	Conjugate-sector interpretation
Chirality projectors P_L, P_R	Inherited	From spinorial / Dirac-admissibility layer
Weak-commitment projector P_W	Inherited / Conditional	From weak-commitment programme
Chiral locking P_W = P_L	Owed	Used here as representation premise (MR-5)
Weak doublets	Conditional	Derived given chiral locking
Weak singlets	Conditional	Derived as fully committed / chiral-complement sectors
C_3 threefold support module	Inherited / Conditional	From quark/confinement support programme
Lepton-like single-support sectors	Conditional	Derived as non-C_3 isolated supports
Quark-like three-support sectors	Conditional	Derived given confined C_3 support
Fractional third-charge support	Conditional	Derived from C_3 charge distribution
Confinement → integer-charge composites	Conditional	Derived from C_3-neutrality of record-stable states
$Q = T_3 + Y$	Audit condition	Adopted electroweak decomposition; substrate derivation owed
Lepton hypercharge sector	Conditional	Follows from single-support integer minimality

Object	Grade	Status
Quark-hypercharge rigidity (⅙, ⅔, −⅓)	Conditional / Exact within ledger	Unique anomaly-free completion given lepton normalization
One-generation hypercharge table	Conditional / Exact within ledger	Follows from the above
Anomaly cancellation	Exact (audit)	Passed for the representation skeleton
Continuous SU(3) Yang–Mills	Owed	Not derived
Full electroweak gauge dynamics	Owed	Not derived
Anomaly freedom as substrate-inadmissibility	Owed	Imposed here as consistency filter
Charge conjugation / CPT	Owed	Not derived
Right-handed neutrino ν_R	Candidate	Not required by minimal charged skeleton
Three generations	Owed	Not derived here
Fermion masses	Firewalled	Not addressed
Yukawa couplings	Firewalled	Not addressed
CKM / PMNS	Firewalled	Not addressed

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1. Purpose and Claim Level

The Fock-space reconstruction gives VERSF a generic fermionic source-carrier field. That is necessary but insufficient. A theory with a single undifferentiated fermion field is not a Standard Model matter theory.

The Standard Model matter sector carries a specific representation structure: lepton doublets; charged-lepton singlets; quark doublets; up-type and down-type quark singlets; colour triplicity; fractional quark charges; conjugate sectors; and anomaly-safe charge assignments. The purpose of this paper is to derive the first representation-level skeleton of that structure from VERSF commitments, without entering the mass sector.

The target chain is

spinorial Fock sector → winding charge → weak chirality → single vs threefold support → lepton/quark split → one-generation chiral matter ledger.

Central claim

Given the spinorial Fock-sector reconstruction and the representation premises MR-1–MR-7 below, the VERSF matter sector decomposes into the chiral one-generation Standard-Model representation skeleton

$$L\_L : (1, 2, -\tfrac{1}{2}),\ \ e\_R : (1, 1, -1),\quad Q\_L : (3, 2, \tfrac{1}{6}),\ \ u\_R : (3, 1, \tfrac{2}{3}),\ \ d\_R : (3, 1, -\tfrac{1}{3}),$$

together with opposite-winding conjugate sectors, and the quark hypercharges are the unique anomaly-free completion of the lepton normalization. This is a representation theorem, not a mass theorem.

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2. Representation Premises

Seven named premises carry the construction.

MR-1 — Spinorial Fock inheritance

The matter sector inherits the spinorial fermionic Fock space $\mathscr{F}\_A(\mathscr{H}_1)$ with CAR creation and annihilation operators from the preceding reconstruction. **Status:** Inherited, conditional on the previous paper's FF premises. **Falsifier:** failure of positive Fock/CAR reconstruction.

MR-2 — Persistent winding charge

The one-particle space decomposes by persistent winding/source charge,

$$\mathscr{H}_1 = \bigoplus\_w \mathscr{H}_{1,w},$$

and the winding label contributes a conserved source charge under the record-current coupling. **Status:** Inherited from the topological-charge / record-current programme. **Falsifier:** winding fails to define a conserved source grading.

MR-3 — Opposite-winding conjugacy

For every charged winding sector w, there exists an opposite-winding sector −w with opposite source coupling. **Status:** Candidate conjugate-sector condition. **Falsifier:** opposite winding fails to reverse source charge.

MR-4 — Weak-commitment projector

There exists a weak-commitment projector P_W selecting states that retain unresolved two-branch access under the weak projection. **Status:** Inherited from the weak-commitment programme. **Falsifier:** weak projection does not define a stable two-branch support.

MR-5 — Chiral locking

On the low-energy weak-active spinorial source-carrier sector, P_W = P_L. Weak unresolved commitment acts on the left-chiral spinorial sector, while right-chiral modes are weak-committed singlets. **Status:** Owed; used here as a representation premise. **Falsifier:** weak activity does not correlate with chirality.

MR-6 — Threefold confined support

There exists a confined threefold support module C_3 for quark-like source-carriers. Modes carrying this support are not isolated as single record-stable particles; physical record-stable composites must be neutral under the internal support accounting. **Status:** Inherited / Conditional from the quark-confinement support programme. **Falsifier:** no threefold support module exists, or it fails to confine.

MR-7 — Minimal charge normalization

The isolated lepton-like support carries integer source charge in units of e; the threefold confined support permits effective third-unit charge distribution. With $Q = T_3 + Y$, the minimal nontrivial charge normalization fixes the lepton sector; anomaly freedom then completes the quark sector. **Status:** Conditional / Audit. **Falsifier:** charge normalization cannot be obtained from winding/support structure without importing measured charge assignments.

2′. Why This Is Not a Mass Paper

The mass firewall of §0 is a claim-level constraint; the following is a reading-level clarification, since the paper uses familiar particle names. The labels u, d, e, ν denote representation roles — positions in the chiral matter skeleton — not mass eigenstates. No claim is made here about electron, neutrino, up-quark, or down-quark masses, and no familiar name should be read as importing its measured mass, lifetime, or mixing. A "u_R" slot is precisely the right-chiral, C_3-supported, weak-singlet, charge-⅔ representation; whether and how it acquires inertial weight is the concern of the downstream mass/Yukawa module (§19), not of this paper.

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3. The Representation Problem

The Fock-space construction supplies operators $a^\dagger(f)$, $a(f)$, $\psi(x)$, but a generic fermionic field still lacks the internal labels that distinguish matter types. The Standard Model matter problem separates into two layers.

Layer A — Representation skeleton. Which fermion branches exist? Which are weak doublets, which weak singlets, which colour-supported? Which carry integer charge, which fractional? Which conjugate sectors exist? Do the charge assignments pass anomaly consistency? This is the concern of the present paper.

Layer B — Mass and mixing. What are the Yukawa operators? Why these masses, these CKM and PMNS structures, these hierarchy ratios, these three generations? This paper does not address Layer B.

The representation skeleton must be available before the mass operator can act. In Standard Model terms, one first needs the slots Q_L, u_R, d_R, L_L, e_R before one asks how Yukawa couplings connect them.

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4. One-Particle Matter-Space Decomposition

Begin with the inherited one-particle spinorial source-carrier space $\mathscr{H}_1$. The representation claim is that $\mathscr{H}_1$ is not irreducible as a matter space: it carries independent admissible gradings, recorded schematically as

$$\mathscr{H}_1 \cong \mathscr{H}\_\text{spin} \otimes \mathscr{H}\_\text{wind} \otimes \mathscr{H}\_\text{weak} \otimes \mathscr{H}\_\text{support}.$$

The tensor notation is schematic. It asserts only that an admissible source-carrier state carries four independent quantum-number labels — spin, winding, weak-commitment, and support — not that $\mathscr{H}_1$ possesses a primitive microscopic tensor factorization into these factors. The physical content is the independence of the four gradings, not a claimed Hilbert-space product structure.

The relevant decompositions are

$$\mathscr{H}\_\text{wind} = \bigoplus_w \mathscr{H}\_w, \quad \mathscr{H}\_weak = \mathscr{H}(\text{W-doublet}) \oplus \mathscr{H}\_(\text{W-singlet}), \quad \mathscr{H}\_\text{support} = \mathscr{H}\_\text{single} \oplus \mathscr{H}\_(C_3).$$

The first split yields charge and conjugacy; the second yields weak doublets and singlets; the third yields lepton-like and quark-like sectors. The Standard Model representation question thereby becomes a VERSF support question: which spinorial source-carriers are single-support, weak-active, weak-inactive, or C_3-supported?

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5. Winding Charge and Conjugate Sectors

Lemma 1 — Winding defines source charge

Given MR-2, persistent winding w defines a conserved source grading; the record-current coupling reads this grading as electric/source charge. A one-particle state $f\_w \in \mathscr{H}_{1,w}$ contributes to the current with charge proportional to w,

$$J^\mu\_w \sim q(w)\, \rho\_w\, u^\mu,$$

with the proportionality fixed by the primitive charge unit.

Proof sketch. Persistent winding is topological and conserved under admissible refinement. The record-current construction couples only to persistent, conserved current components; winding sectors therefore cannot be freely mixed by source-admissible evolution and supply charge superselection sectors. ■

Lemma 2 — Opposite winding gives conjugate source charge

Given MR-3, the sector $\mathscr{H}_{1,-w}$ carries opposite source coupling, $q(-w) = -q(w)$. This is the representation-level conjugate sector.

Status note. This is a source-coupling conjugacy, not yet a full derivation of antiparticles, CPT, or charge conjugation. A charge-conjugation operator C intertwining the spinorial, winding, and local-field structures remains owed.

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6. Weak Commitment and Chirality

The Standard Model weak interaction is chiral: left-handed fermions sit in weak doublets, right-handed charged fermions are weak singlets. VERSF must account for this at the representation level.

The spinorial source-carrier sector already supplies chirality projectors P_L, P_R, while the weak-commitment programme supplies P_W. The essential representation premise is chiral locking, $P_W = P_L$, on the weak-active low-energy source-carrier sector.

Proposition 1 — Weak doublets arise from unresolved left commitment

Given MR-4 and MR-5, weak-active source-carrier states form left-chiral two-branch modules; right-chiral fully committed states are weak singlets:

$$\mathscr{H}_{(\text{W-active})} = P_L\, \mathscr{H}_1,\quad \mathscr{H}_{(\text{W-singlet})} = P_R\, \mathscr{H}_1.$$

Proof. By MR-4, weak-active states are precisely those retaining unresolved two-branch access under P_W. By MR-5, $P_W = P_L$, so weak-active states are left-chiral. The orthogonal chiral complement $P_R\, \mathscr{H}_1$ retains no weak two-branch access and appears as a weak singlet. ■

The statement is labelled a Proposition rather than a Theorem because it is a direct restatement of MR-4 and MR-5; its entire load-bearing content lives in the chiral-locking premise MR-5 (Debt 1). It derives the pattern *left mode → weak doublet*, *right mode → weak singlet*, but not weak-boson dynamics, weak coupling strength, or electroweak symmetry breaking.

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7. Single Support and Threefold Support

The next split is lepton versus quark. Leptons appear as isolated colourless particles; quarks do not — they carry colour and appear only inside colour-neutral physical states. VERSF represents the distinction through support structure.

Single-support source-carrier. A persistent spinorial loop mode whose source charge is carried by one isolated record-stable support channel, with no internal C_3 multiplicity. These are lepton-like.

Threefold-support source-carrier. A persistent spinorial loop mode whose source charge is distributed across a confined C_3 support module; its individual support channels are not independently record-stable as isolated asymptotic matter states. These are quark-like.

Theorem 2 — Lepton/quark support split

Given MR-6, the matter sector decomposes as $\mathscr{H}_1 = \mathscr{H}_\ell \oplus \mathscr{H}_q$, with $\mathscr{H}_\ell = \mathscr{H}_{single}$ carrying lepton-like single-support modes and $\mathscr{H}_q = \mathscr{H}_{(C_3)}$ carrying quark-like threefold confined modes.

Proof. The support module is an admissibility label on persistent source-carrier states. A state carries source support either through one isolated channel or through a confined threefold module. These sectors cannot be identified: the first permits isolated record-stable source-carriers, the second does not. The support decomposition therefore induces a matter-sector decomposition into lepton-like and quark-like branches. ■

Status note on colour. This paper does not prove full SU(3) Yang–Mills colour. It proves, conditionally, the representation need for a threefold confined internal support index. The continuous internal-rotation lift $C_3 \to SU(3)_C$ remains owed. The label 3 in the tables below means *threefold colour-support skeleton*, not a derivation of the gluon field or colour gauge dynamics.

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8. Fractional Charge and Confinement

The lepton-like sector carries charge on a single support channel, so its minimal nonzero charge unit is integral: $Q_\ell \in \mathbb{Z}e$. The quark-like sector carries source support through a confined threefold module, so its effective support fraction is a third-unit: $Q_q \in (\frac{1}{3})\mathbb{Z}\, e$. A quark is not literally one third of an electron; rather the admissible source-current unit is distributed over a threefold confined structure, so the effective charge read by the electromagnetic/source current appears in thirds.

Lemma 3 — C_3 support permits third-unit charge; confinement restores integer composites

Given MR-6 and MR-7:

1. an isolated single-support state carries source current through one record-stable support, so its primitive charge unit is indivisible and integral;
2. a C_3-supported state carries source current through three internally related channels, so the smallest support-resolved contribution is one third of the combined unit;
3. because individual C_3 channels are confined, they do not appear as isolated single-support particles; a record-stable physical state must be C_3-neutral, i.e. assembled from three channels (or channel/anti-channel pairs);
4. therefore every isolated record-stable state carries **integer** total charge, while the underlying support fractionation supplies the third-integer quark ledger.

Proof sketch. Points 1–2 follow from the single vs threefold channel count and the indivisibility of the primitive support unit. Point 3 is the confinement condition of MR-6: a lone e/3 support is not record-stable. Point 4 is the arithmetic consequence — a C_3-neutral three-channel composite sums three third-integer supports to an integer. ■

This is the representation-level origin of the lepton/quark charge difference, and it yields a charge-quantization statement: no isolated record-stable state carries fractional charge. The consistency of the derived quark charges with this requirement is checked in §15.7 (uud → +1, udd → 0).

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9. The Weak Charge Relation

For a weak doublet set $T_3 = +\frac{1}{2}$ on the upper branch and $T_3 = -\frac{1}{2}$ on the lower branch. Electric/source charge relates to weak isospin and hypercharge by

$$Q = T_3 + Y.$$

This paper treats the relation as the representation-level electroweak charge decomposition; a substrate derivation from the full gauge-commitment algebra remains owed.

Given a doublet with branch charges Q_upper, Q_lower,

$$Q_{upper} = +\tfrac{1}{2} + Y, \quad Q_{lower} = -\tfrac{1}{2} + Y,$$

so the hypercharge is the branch average,

$$Y = (Q_{upper} + Q_{lower}) / 2,$$

and the branches always differ by one unit,

$$Q_{upper} - Q_{lower} = 1.$$

This unit separation is a weak-representation charge splitting, not a mass splitting.

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10. Leptonic Representation Ledger

The single-support lepton-like sector carries no C_3 colour-support index. Its weak doublet consists of two integer-charge branches differing by one unit, i.e. a pair $(n, n-1)$ with $n \in \mathbb{Z}$. Select the pair minimizing the largest charge magnitude, $\max(|n|, |n-1|)$. This is minimized at max = 1 by exactly two pairs, $(0, -1)$ at $n = 0$ and $(1, 0)$ at $n = 1$; every other pair has max ≥ 2. The two minimizers are related by the charge-orientation convention (Residual Freedom 1 of §12), which the lepton convention $Q(e) = -1$ fixes to the pair containing the neutral upper branch:

$$Q(\nu_L) = 0,\ \ Q(e_L) = -1,$$

so

$$Y(L_L) = (0 + (-1)) / 2 = -\tfrac{1}{2},$$

giving

$$L_L = (\nu_L, e_L) : (1, 2, -\tfrac{1}{2}).$$

The corresponding right-chiral charged lepton is a weak singlet with $T_3 = 0$, so

$$Y(e_R) = Q(e_R) = -1,\ \ e_R : (1, 1, -1).$$

The lepton sector is therefore fixed by single-support integer minimality alone. This is the normalization input to the quark-sector rigidity theorem of §12.

Neutral right mode. A right-chiral neutral singlet would have $Q = 0$, $T_3 = 0$, $Y = 0$, giving a possible sterile sector $\nu_R : (1, 1, 0)$. It is not required by the minimal charged skeleton and is not forced here; its presence is a separate question tied to neutrino mass and the weak-neutral branch, firewalled from the present result. (It contributes zero to every anomaly sum, so its optional inclusion does not affect §12–§15.)

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11. Quark Representation Ledger — Constructive Route

The quark-like sector carries the threefold support index, so its effective charges come in thirds, and its weak doublet branches differ by one unit. The minimal third-integer pair with unit separation and a positive upper branch is

$Q(u\_L) = +\frac{2}{3}$, $Q(d\_L) = -\frac{1}{3}$, (difference $+\frac{2}{3} - (-\frac{1}{3}) = 1$),

with common hypercharge

$Y(Q\_L) = (\frac{2}{3} + (-\frac{1}{3})) / 2 = \frac{1}{6}$,

so

$Q\_L = (u\_L, d\_L) : (3, 2, \frac{1}{6})$.

The right-chiral quark branches are weak singlets ($T_3 = 0$), so hypercharge equals source charge:

$u\_R : (3, 1, \frac{2}{3})$, $d\_R : (3, 1, -\frac{1}{3})$.

This is the constructive route: minimality proposes a candidate. It leaves an obvious objection — why $(\frac{2}{3}, -\frac{1}{3})$ rather than another third-integer doublet such as $(\frac{1}{3}, -\frac{2}{3})$ or $(-\frac{1}{3}, -\frac{4}{3})$? Section 12 removes the objection: the candidate is not merely minimal but is the unique anomaly-free completion of the lepton normalization.

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12. Rigidity of the Quark Ledger

The constructive route selects the quark charges by minimality. The stronger statement is that, once the representation content and the lepton normalization are fixed, perturbative anomaly freedom **determines** the quark hypercharges uniquely, with no appeal to minimality.

Theorem 2′ — Quark-hypercharge rigidity

Inputs. (i) Representation content, as derived in §6–§7: one colour-triplet weak doublet Q_L, two colour-triplet weak singlets u_R, d_R, one lepton doublet L_L, one charged-lepton singlet e_R, with hypercharges free except as fixed below. (i′) Minimality / no-exotics condition. The theorem is restricted to exactly this representation content, with no additional anomaly-carrying chiral matter fields admitted at this representation layer. If extra chiral exotics were allowed, anomaly freedom could be restored by other charge assignments and uniqueness would not hold; the claim is therefore uniqueness *given the derived one-generation skeleton and no chiral exotics*. A sterile $\nu\_R : (1, 1, 0)$ is permitted because it contributes zero to every anomaly sum. (ii) Lepton normalization, from §10: $Y(L\_L) = -\frac{1}{2}$ and $Y(e\_R) = -1$. (iii) Perturbative anomaly freedom for $SU(2)^2U(1)$, $SU(3)^2U(1)$, gravitational-U(1), and $U(1)^3$ (imposed as a consistency filter; substrate origin owed, Debt 3).

Conclusion. The quark hypercharges are uniquely

$Y(Q\_L) = \frac{1}{6}$, $\{Y(u\_R), Y(d\_R)\} = \{\frac{2}{3}, -\frac{1}{3}\}$.

Proof. Work with left-handed Weyl content, right-handed fields represented by conjugates: $u_R \to u_L^c$ ($Y = -Y_u$), $d_R \to d_L^c$ ($Y = -Y_d$), $e_R \to e_L^c$ ($Y = -Y_e$). Let Y_Q, Y_u, Y_d be the unknown quark hypercharges; $Y_L = -\frac{1}{2}$, $Y_e = -1$ are fixed.

Step 1 (linear, $SU(2)^2U(1)$). Only doublets contribute, weighted by colour multiplicity: $3 \cdot Y_Q + 1 \cdot Y_L = 0$, so $Y_Q = -Y_L/3 = \frac{1}{6}$.

Step 2 (linear, $SU(3)^2U(1)$). Only coloured fields contribute: $2 \cdot Y_Q - Y_u - Y_d = 0$, so $Y_u + Y_d = 2Y_Q = \frac{1}{3}$.

Step 3 (cubic, $U(1)^3$). Summing Y^3 over all left-handed Weyl degrees,

$$6Y_Q^3 - 3Y_u^3 - 3Y_d^3 + 2Y_L^3 - Y_e^3 = 0.$$

With $Y_Q = \frac{1}{6}$, $Y_L = -\frac{1}{2}$, $Y_e = -1$ this gives $3(Y_u^3 + Y_d^3) = 7/9$, i.e. $Y_u^3 + Y_d^3 = 7/27$.

Step 4 (solve). With $s = Y_u + Y_d = \frac{1}{3}$ and $p = Y_u Y_d$, the identity $Y_u^3 + Y_d^3 = s^3 - 3ps$ is linear in p (since $s = \frac{1}{3} \neq 0$): $(1/27) - p = 7/27$, so $p = -2/9$. Thus s and p are each fixed by the linear Steps 1–3, and the unordered pair $\{Y_u, Y_d\}$ is the root set of one fixed quadratic,

$$t^2 - \tfrac{1}{3} t - 2/9 = 0.$$

By Vieta this root set is unique **over** $\mathbb{C}$ — no complex branch is discarded; uniqueness is unconditional. That the roots are *real* is a separate, contingent fact: the discriminant is $\frac{1}{9} + 8/9 = 1 > 0$, so a real solution exists at all, giving $t = (\frac{1}{3} \pm 1)/2 = \frac{2}{3}$ or $-\frac{1}{3}$. Hence $\{Y_u, Y_d\} = \{\frac{2}{3}, -\frac{1}{3}\}$. The gravitational-$U(1)$ condition is then satisfied identically (and, per the reduction remark below, can be used in place of Step 2). ■

Residual freedoms and how each is fixed

The solution is unique up to exactly two conventions, both already fixed upstream:

1. **Overall charge orientation.** The sign of the whole ledger is set by the lepton convention $Q(e) = -1$ (§10), which orients the source-charge axis. This is the one-parameter $U(1)$ scale/orientation freedom of the anomaly system, removed by the lepton normalization.
2. **Naming of the two singlet roots.** The quadratic returns the unordered pair $\{\frac{2}{3}, -\frac{1}{3}\}$; the assignment of which root is up-type is fixed by $Q = T_3 + Y$ on the doublet. The upper branch has $T_3 = +\frac{1}{2}$, so $Q_{upper} = +\frac{1}{2} + \frac{1}{6} = \frac{2}{3}$, matching the right partner of hypercharge $\frac{2}{3}$; hence $u_R : (3, 1, \frac{2}{3})$ and $d_R : (3, 1, -\frac{1}{3})$.

With both conventions fixed, the labelling is determined, not chosen.

Reducing the gauge input

Step 2 uses $SU(3)^2U(1)$, which presupposes gauged continuous colour. It is dispensable. Dropping Step 2 and using the gravitational-$U(1)$ condition instead,

$$6Y\_Q - 3(Y\_u + Y\_d) + 2Y\_L - Y\_e = 0 \implies 1 - 3s - 1 + 1 = 0 \implies s = \tfrac{1}{3},$$

recovers the same constraint on s. The system {$SU(2)^2U(1)$, gravitational-U(1), $U(1)^3$} then delivers the identical unique pair, and $SU(3)^2U(1)$ becomes a derived check ($2(\tfrac{1}{6}) - \tfrac{1}{3} = 0$ ✓) rather than an input. Crucially, the gravitational and cubic conditions require only the threefold Weyl multiplicity of the C_3 channels — a state count — not gauged continuous colour. The rigidity result therefore rests on threefold multiplicity + gauged SU(2) + gauged U(1) + a gravitational coupling, and is **independent of the continuous-colour lift (Debt 2)**.

Independent charge quantization

The rigidity route takes inputs (i)–(iii) only; it does not assume the third-unit clause of MR-7. The third-integer charges {⅔, −⅓} emerge from anomaly freedom, integer lepton normalization, and threefold multiplicity alone. That they coincide with the fractional charges obtained independently in the constructive route (§11) and from C_3 fractionation (Lemma 3, §8) is a nontrivial consistency success: the support-fractionation account and the anomaly ledger agree without being wired together.

Reading

The quark charges (⅔, −⅓) are not a fortunate minimal pick. Given the lepton sector and the requirement that the ledger be anomaly-free, they are the only solution the constraints admit. The lepton and quark sectors are rigidly locked: fixing one determines the other. In VERSF language, the matter ledger *closes* only when the single-support and C_3-support branches co-occur with these charges.

Status. Conditional, and doubly so. First, the anomaly conditions of input (iii) are conditions of the gauged continuum theory, so the result presupposes that the representation skeleton lifts to a chiral gauge theory of the assumed type — gauged SU(2)×U(1) with a gravitational coupling, the electroweak gauge dynamics being owed (§17). By the reduction remark, gauged continuous colour (Debt 2) is *not* required. Second, anomaly freedom is imposed as a consistency filter rather than derived; establishing it as a substrate constraint is Debt 3. Within the ledger so conditioned, the completion is Exact.

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13. One-Generation Chiral Matter Table

Sector	VERSF origin	Representation	Charge content	Status
L_L = (ν_L, e_L)	single support + unresolved left commitment	(1, 2, −½)	(0, −1)	Conditional
e_R	single support + right-committed singlet	(1, 1, −1)	(−1)	Conditional

Sector	VERSF origin	Representation	Charge content	Status
Q_L = (u_L, d_L)	C_3 support + unresolved left commitment	(3, 2, ⅙)	(⅔, −⅓)	Conditional / rigid
u_R	C_3 support + right-committed singlet	(3, 1, ⅔)	(⅔)	Conditional / rigid
d_R	C_3 support + right-committed singlet	(3, 1, −⅓)	(−⅓)	Conditional / rigid
conjugate sectors	source-conjugate winding	conjugate reps	opposite charges	Candidate
ν_R	neutral right singlet	(1, 1, 0)	(0)	Optional / not forced

The table is the central output of the paper. It is a representation table — not a mass, generation, or Yukawa table.

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14. Hypercharge as a Consistency Ledger

Once the charge pairings and the lepton normalization are fixed, the hypercharge assignments are not free (§12):

$$Y_L = -\tfrac{1}{2},\ \ Y_{e_R} = -1,\ \ Y_Q = \tfrac{1}{6},\ \ Y_{u_R} = \tfrac{2}{3},\ \ Y_{d_R} = -\tfrac{1}{3}.$$

Thus the one-generation matter skeleton is

$$L_L : (1, 2, -\tfrac{1}{2}),\quad e_R : (1, 1, -1),\quad Q_L : (3, 2, \tfrac{1}{6}),\quad u_R : (3, 1, \tfrac{2}{3}),\quad d_R : (3, 1, -\tfrac{1}{3}),$$

matching the Standard Model chiral matter pattern for one generation, prior to mass assignment.

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15. Anomaly Audit

The audit re-verifies, by direct evaluation, that the rigid ledger of §12–§14 is anomaly-safe. Right-handed fields are represented by their left-handed conjugates:

$$u_R \to u_L^c : (\bar{3}, 1, -\tfrac{2}{3}),\quad d_R \to d_L^c : (\bar{3}, 1, +\tfrac{1}{3}),\quad e_R \to e_L^c : (1, 1, +1).$$

Left-handed content: Q_L (3, 2, ⅙), u_L^c ($\bar{3}$, 1, −⅔), d_L^c ($\bar{3}$, 1, +⅓), L_L (1, 2, −½), e_L^c (1, 1, +1).

15.1 $SU(3)^3$ (pure colour). The cubic colour anomaly is proportional to the sum of the colour anomaly coefficients A(R) over left-handed Weyl fermions, with $A(\bar{3}) = -A(3)$. The coloured content is 2 triplets (the SU(2) doublet Q_L) and 2 anti-triplets (u_L^c, d_L^c):

$$2\,A(3) + A(\bar{3}) + A(\bar{3}) = 2\,A(3) - 2\,A(3) = 0.$$

The coloured sector is vector-like, so the pure colour anomaly cancels trivially. (There is no $SU(2)^3$ condition: SU(2) admits no independent symmetric cubic invariant, so its cubic anomaly vanishes identically; the relevant SU(2) subtlety is the global Witten condition of 15.6.)

15.2 $SU(3)^2U(1)$. With $T(3) = T(\bar{3}) = ½$ and the doublet supplying two SU(2) components,

$$2(⅙)(½) + (-⅔)(½) + (⅓)(½) = ⅙ - ⅓ + ⅙ = 0.$$

15.3 $SU(2)^2U(1)$. With T(2) = ½ and colour multiplicity 3 on the quark doublet,

$$3(⅙)(½) + (-½)(½) = ¼ - ¼ = 0.$$

15.4 Gravitational-U(1). Summing Y over all left-handed Weyl degrees,

$$6(⅙) + 3(-⅔) + 3(⅓) + 2(-½) + 1(+1) = 1 - 2 + 1 - 1 + 1 = 0.$$

15.5 $U(1)^3$. Summing Y^3,

$$6(⅙)^3 + 3(-⅔)^3 + 3(⅓)^3 + 2(-½)^3 + 1(+1)^3 \quad = 6(1/216) + 3(-8/27) + 3(1/27) + 2(-1/8) + 1 \quad = 1/36 - 8/9 + 1/9 - 1/4 + 1 \quad = (1 - 32 + 4 - 9 + 36)/36 = 0.$$

15.6 Witten SU(2) global. The number of left-handed SU(2) doublets is 3 (coloured Q_L) + 1 (L_L) = 4, which is even; the global anomaly condition is passed.

15.7 Confinement consistency. The rigid quark charges assemble into integer-charge C_3-neutral three-channel composites, consistent with Lemma 3: uud → ⅔ + ⅔ − ⅓ = +1, udd → ⅔ − ⅓ − ⅓ = 0.

Audit conclusion

The representation skeleton is anomaly-safe for one generation, and its charges yield integer-charge composites. The result is not a bare list of charges: the lepton and quark sectors cancel each other's gauge inconsistencies, and — by §12 — this cancellation is what *forces* the quark charges once the lepton sector is set. The matter ledger closes only when the single-support and C_3-support branches are present together.

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16. Chiral Matter Representation Theorem

Theorem 3 — Chiral Matter Representation Theorem

Given MR-1–MR-7, the spinorial Fock-space matter sector decomposes into the minimal one-generation chiral matter representation skeleton

$L_L : (1, 2, -\tfrac{1}{2})$, $e_R : (1, 1, -1)$, $Q_L : (3, 2, \tfrac{1}{6})$, $u_R : (3, 1, \tfrac{2}{3})$, $d_R : (3, 1, -\tfrac{1}{3})$,

with opposite-winding conjugate sectors and the optional, non-forced neutral singlet $\nu_R : (1, 1, 0)$. Given the lepton normalization, the quark hypercharges are the unique anomaly-free completion (Theorem 2′). The ledger passes the local audits $SU(3)^3$, $SU(3)^2U(1)$, $SU(2)^2U(1)$, $U(1)^3$, gravitational-U(1), and the global even-doublet SU(2) audit.

Proof. By MR-1 the sector is fermionic and spinorial; by MR-2 it carries persistent winding charge; by MR-3 opposite-winding sectors carry opposite source coupling and provide conjugate branches. By MR-4–MR-5, weak-active modes coincide with left-chiral unresolved-commitment modes, so left-chiral carriers form weak doublets and right-chiral committed modes form weak singlets (Proposition 1). By MR-6 the support sector splits into single-support (lepton-like) and threefold-support (quark-like) branches (Theorem 2). By MR-7, isolated single-support charge is integral, fixing the lepton doublet at (0, −1) with $Y = -\tfrac{1}{2}$ and the charged singlet at $Y = -1$ (§10). Theorem 2′ then determines the quark hypercharges uniquely as $(\tfrac{1}{6}, \tfrac{2}{3}, -\tfrac{1}{3})$, and $Q = T_3 + Y$ fixes the branch charges $(\tfrac{2}{3}, -\tfrac{1}{3})$. Section 15 verifies all required anomalies and the integer-charge composite condition. ■

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17. What Has Actually Been Derived

Derived, conditionally:

1. the split between lepton-like and quark-like source-carriers;
2. the split between weak doublets and weak singlets;
3. the left-chiral placement of weak doublets;
4. the single-support origin of lepton-like integer charges;
5. the threefold-support origin of quark-like fractional charges, with confinement restoring integer composites;
6. the lepton hypercharge sector from integer minimality;
7. the **uniqueness** of the quark hypercharge ledger under anomaly freedom given the lepton normalization;
8. anomaly consistency of the resulting skeleton;
9. opposite-winding conjugate sectors at the source-charge level.

Not derived:

1. fermion masses; 2. Yukawa couplings; 3. Higgs coupling structure; 4. CKM mixing; 5. PMNS mixing; 6. three generations; 7. full SU(3) gauge dynamics; 8. full electroweak

dynamics; 9. charge conjugation/CPT; 10. anomaly freedom as a substrate-level path-integral theorem.

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18. Relation to the Previous Fock-Space Paper

The predecessor supplies the algebraic fermion layer,

spinorial loops $\to \mathscr{F}_A(\mathscr{H}_1) \to$ CAR $\to \psi(x)$,

answering *why fermionic operators?* The present paper supplies the first representation decomposition,

$\psi(x) \to L_L, e_R, Q_L, u_R, d_R$,

answering *why these matter representation slots?* The two results sit at different layers. The present paper inherits the Fock-space result without reopening it and does not enter the mass programme. The full chain is

persistent loops → spinorial carriers → fermionic Fock space → chiral matter representations → future mass/Yukawa action.

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19. Relation to the Mass and Mixing Programme

The mass and mixing papers ask how matter branches acquire different inertial weights and how their flavour bases misalign. Those questions require a representation skeleton first. In Standard Model language, the Yukawa terms

$\bar{L}_L H e_R,\ \ \bar{Q}_L \tilde{H} u_R,\ \ \bar{Q}_L H d_R$

presuppose knowledge of which left and right representations are connected. The present paper supplies exactly those slots; it computes no coefficients, hierarchy, or mixing matrix. The programme ordering is: (1) derive the fermionic field algebra; (2) derive the matter representation skeleton; (3) attach Higgs/Yukawa/mass operators; (4) compute mass and mixing. The present paper is step 2.

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20. Open Debts

Debt 1 — Chiral-locking theorem (now the principal hinge). Derive $P_W = P_L$ on the weak-active sector from weak-commitment dynamics rather than imposing it: *weak unresolved* *commitment acts only on left-chiral spinorial source-carriers.* With §12 settling the charge ledger, the load-bearing premise of the paper has shifted from *why these quark charges?* to this single locking condition (MR-5). If chiral locking fails, the entire Standard-Model chiral matter structure — doublet/singlet placement and everything downstream of it — fails with it. This is the natural subject of the next paper in the series.

Debt 2 — Continuous colour lift. Derive continuous SU(3)_C gauge dynamics from the C_3 confined support module: C_3 confined support → SU(3)_C internal gauge representation.

Debt 3 — Substrate anomaly-inadmissibility theorem. Show that anomalous chiral ledgers are inadmissible in the VERSF substrate before field-theoretic quantisation. This would upgrade anomaly freedom from an imposed consistency filter (Theorem 2′, input iii; §15 audit) to a derived substrate constraint, and would make Theorem 2′ a genuine VERSF derivation of the hypercharge ledger rather than an anomaly-audit completion.

Debt 4 — Exact charge conjugation. Opposite winding is a candidate conjugate sector; construct the full charge-conjugation operator C and establish CPT.

Debt 5 — Three generations. Derive why the representation skeleton repeats threefold.

Debt 6 — Mass attachment. Attach the already-developed mass/Yukawa/mixing programme to the representation slots derived here.

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21. Falsification Conditions

F1 — Fock inheritance failure. If the spinorial Fock-space reconstruction fails, the representation theorem loses its fermionic base. *Retires MR-1.*

F2 — Winding-charge failure. If persistent winding does not define conserved source charge, the charge ledger cannot be substrate-derived. *Retires MR-2.*

F3 — Opposite-winding failure. If opposite winding does not reverse source coupling, the conjugate-sector interpretation fails. *Retires MR-3.*

F4 — Weak-projector failure. If weak commitment does not define a stable two-branch support, weak doublets are not derived. *Retires MR-4.*

F5 — Chiral-locking failure. If weak activity is not locked to left chirality, the chiral structure is not recovered. *Retires MR-5.*

F6 — C_3-support failure. If no threefold confined support module exists, the quark/lepton split is not derived. *Retires MR-6.*

F7 — Fractional-charge failure. If C_3 support does not yield third-unit effective source charges, the quark charge ledger fails. *Retires MR-7.*

F8 — Colour-lift failure. If the threefold support skeleton cannot lift to continuous colour, the result remains a fractional-support model rather than a full QCD representation. *Limits the theorem to a pre-QCD skeleton.*

F9 — Rigidity failure. If a more exact treatment admits a second anomaly-free quark ledger given the lepton normalization, the uniqueness claim of Theorem 2′ fails. *Retires the rigidity result, not the constructive ledger.*

F10 — Anomaly failure. If the derived charge ledger fails anomaly cancellation under a more exact representation treatment, the matter skeleton is inconsistent. *Retires the one-generation representation theorem.*

F11 — Mass contamination. If the representation result requires observed masses or fitted mixing data, the mass firewall fails. *Retires the claim that this paper is representation-prior.*

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22. Milestone Statement

The result of this paper:

spinorial Fock fermion + winding charge + weak chiral commitment + C_3 support $\Longrightarrow$ one-generation chiral Standard-Model-like matter skeleton,

with the precise skeleton

$L\_L : (1, 2, -\tfrac{1}{2}),\ e\_R : (1, 1, -1),\quad Q\_L : (3, 2, \tfrac{1}{6}),\ u\_R : (3, 1, \tfrac{2}{3}),\ d\_R : (3, 1, -\tfrac{1}{3}),$

and with the quark hypercharges forced, not fitted, once the lepton sector is fixed. This is the first VERSF bridge from generic fermionic field algebra to Standard Model matter representation structure.

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23. Conclusion

The Fermionic Fock-Space Reconstruction gives VERSF a generic fermion field sector; it does not explain why the matter sector branches into the Standard Model representation pattern. This paper supplies that next layer.

Under the named premises, the spinorial Fock sector decomposes into lepton-like single-support branches, quark-like threefold-support branches, weak-active left doublets, weak-inactive right singlets, and opposite-winding conjugate sectors. Single-support integer minimality fixes the lepton doublet at (0, −1) with $Y = -\tfrac{1}{2}$ and the charged singlet at $Y = -1$. Perturbative anomaly freedom then determines the quark hypercharges uniquely as ($\tfrac{1}{6}$, $\tfrac{2}{3}$, $-\tfrac{1}{3}$), and $Q = T_3 + Y$ fixes the branch charges ($\tfrac{2}{3}$, $-\tfrac{1}{3}$). The resulting one-generation ledger

L_L, e_R, Q_L, u_R, d_R, hypercharges $-\tfrac{1}{2}$, -1, $\tfrac{1}{6}$, $\tfrac{2}{3}$, $-\tfrac{1}{3}$,

passes every standard anomaly audit and assembles into integer-charge composites.

This is not a numerical mass result. It computes no Yukawas, no CKM, no PMNS, no generation count. It derives the representation skeleton on which those downstream structures act, and it upgrades the quark charge assignment from a minimal choice to a forced consequence. The remaining work is now clear: prove chiral locking, lift C_3 support to continuous colour, derive anomaly freedom from substrate cohomology, construct exact charge conjugation, and connect this representation skeleton to the existing VERSF mass and mixing modules.

The programme has moved forward one structural layer:

fermion algebra → chiral matter representation → future mass and mixing attachment.

That is the intended milestone.