The Closure-Interface Potential Necessity and Higgs-Radial Theorem in VERSF

▲ Programme Milestone — Higgs-Radial and Mass-Generation Series Gate HR-1 / Closure-Interface Potential Necessity, Radial-Mode Necessity, and Scalar-Sector Non-Insertion Closure Potential-necessity and radial-admissibility gate — not the numerical Higgs-potential gate

Deriving the Higgs-radial degree from closure-interface stiffness rather than importing a scalar field by convention

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General Reader Summary

The Standard Model contains the Higgs field. In ordinary language, the Higgs field gives particles the structure through which mass can appear. From the point of view of a first-principles programme, this creates a serious question:

Did we derive the Higgs field, or did we simply put it in because the Standard Model needs it?

This paper addresses that question inside VERSF.

VERSF treats physical reality as arising through ordered commitment, closure, admissibility, and frame-invariant readout. Earlier papers establish that physical objects are not placed into the theory as labels. They must be earned by typed structure.

This paper applies that discipline to the Higgs-radial mode.

The central idea is simple:

If a physical world has a stable boundary between committed physical support and uncommitted exterior possibility, then that boundary must have stiffness.

A boundary with stiffness has a displacement mode. If the boundary is pushed away from its equilibrium position, there is a cost. That cost is measured by a potential. The simplest physical displacement of the boundary is not angular, directional, or gauge-labelled. It is radial: the boundary breathes inward or outward.

That radial breathing mode is the Higgs-radial mode.

In plain language:

The Higgs-radial field is the stiffness of the world's closure boundary.

The paper is careful about one further step that is easy to overlook. A displacement coordinate is not yet a field. To become a physical field, the radial displacement must be allowed to vary from region to region and must carry positive kinetic support. HR-1 names this step explicitly — the field-promotion premise — so that the promotion from geometric coordinate to propagating degree of freedom is audited rather than smuggled.

This paper does not claim to compute the numerical Higgs mass. It does not derive the full fermion hierarchy. It does not derive CKM or PMNS mixing. It does not claim that the whole electroweak sector is completed here.

It proves something prior:

A Higgs-like radial scalar is not optional once VERSF requires a stable closure interface with finite return stiffness and locally varying displacement.

The paper firewalls six common errors.

It blocks: "The Higgs field was inserted by hand."

It blocks: "The scalar potential was simply borrowed from the Standard Model."

It blocks: "Angular or gauge directions are extra physical Higgs scalars."

It blocks: "The radial mode is a free fitting parameter."

It blocks: "A displacement coordinate was silently promoted to a quantum field."

It blocks: "Mass generation can be claimed without identifying the closure-stiffness bridge."

The central theorem is:

Whenever VERSF contains a stable, regular, codimension-one closure interface separating committed physical support from exterior non-committed support, and whenever that interface must preserve positivity, frame invariance, finite stiffness, and return-to-closure stability, the interface necessarily admits a scalar normal displacement mode; and when that displacement is promoted to a locally varying carrier with positive kinetic normalisation, it is the unique minimal Higgs-radial degree of freedom.

In one sentence:

The Closure-Interface Potential Necessity and Higgs-Radial Theorem proves that the Higgs-radial mode is the frame-invariant return-stiffness mode of the VERSF closure interface, not an imported scalar field or hidden Standard Model assumption.

Contents

Scope Box — What HR-1 Closes and What It Does Not

Claim	Status
A physical sector requires a closure interface	Closed
The interface separates committed support from exterior non-committed support	Closed
Stable closure requires a return potential	Closed
The potential must be built from frame/gauge-invariant interface data	Closed
Tangential/interface relabelling modes are not independent scalar mass support	Closed
A normal/radial displacement mode is forced at a regular interface	Closed
The radial mode is the unique minimal scalar stiffness mode (codimension one)	Closed
The displacement-to-field promotion step is named and audited	Closed
The possibility-space metric premise is named and audited	Closed
The Higgs-radial degree is admissible as closure-interface stiffness	Closed
The scalar is not inserted by hand	Closed
A bare Standard Model Higgs potential may not be imported without bridge	Closed
Radial stiffness must have a positive Hessian at equilibrium	Closed
The Higgs-radial carrier must preserve physical-sector positivity	Closed
A bridge map from radial stiffness to mass-generation channels is required	Closed
Gauge/angular components require a separate quotient/eating ledger	Closed
Derivation of the microscopic potential shape of V_CI	Not closed
Derivation of the kinetic normalisation Z_H from substrate primitives	Not closed
Exact numerical Higgs mass	Not closed
Exact electroweak vev	Not closed
Exact quartic coupling	Not closed
Full fermion Yukawa hierarchy	Not closed
Full CKM/PMNS prediction	Not closed
Full electroweak precision fit	Not closed
Proof that the observed 125 GeV scalar is numerically derived	Not closed
Empirical confirmation	Not closed

HR-1 is a **Higgs-radial admissibility and necessity theorem**. It closes the structural question: why a scalar radial stiffness mode is required, and under exactly which named conditions. It does not yet close the numerical Higgs sector.

HR-1 Versus the Full Scalar-Sector Gate

HR-1 should be read as the first scalar-sector gate, not as the completed numerical Higgs-sector derivation.

It closes the prior structural question:

Why is there a Higgs-radial degree of freedom at all, rather than no scalar sector or an inserted scalar field?

It does not close the later quantitative questions:

What is the exact closure-interface potential? What fixes the electroweak scale? What fixes the Higgs mass and self-coupling?

Those questions require later gates. HR-1 establishes the admissible carrier: the invariant normal stiffness mode of a stable closure interface. A later closure-potential gate must derive the microscopic form of the potential, its equilibrium radius, its kinetic normalisation, and its numerical Hessian.

Thus HR-1 must not be cited downstream as a derivation of the electroweak vev, the observed 125 GeV scalar, or the quartic coupling. It may be cited only as the theorem that prevents the Higgs-radial mode from being inserted by hand.

The scalar-sector chain is:

closure interface → radial normal coordinate → field-promoted stiffness mode → derived potential shape → equilibrium radius → Higgs mass/self-coupling → electroweak and Yukawa mass channels .

HR-1 closes the first two steps unconditionally and the third conditionally, under the named field-promotion premise (P15). The remaining steps are explicitly owed (§10′).

Abstract

The Standard Model uses a scalar Higgs field whose radial excitation survives as the physical Higgs boson after gauge/angular directions are removed from the scalar count. A first-principles

VERSF derivation cannot import this field, its potential, its symmetry-breaking form, or its surviving radial degree as assumptions. They must arise from typed VERSF structure.

This paper proves that a Higgs-radial degree of freedom is forced by the existence of a stable closure interface, under named regularity, minimality, and field-promotion premises. In VERSF, physical commitment requires a boundary between committed admissible support and exterior non-committed support. Such a boundary cannot be a mere label: it must possess a closure-defect functional, a stable equilibrium condition, finite return stiffness, and a physical-sector positivity constraint. The invariant normal displacement of this interface supplies a scalar radial coordinate. Under the field-promotion premise, that coordinate becomes a locally varying carrier with positive kinetic normalisation. Tangential deformations correspond to frame, gauge, or interface-coordinate motion and cannot be counted as independent scalar radial support.

The paper defines the closure interface Σ as the equilibrium-level support of a closure-defect functional Δ_C. A perturbation of the interface decomposes canonically into normal and tangential parts,

$$\delta q = h\, n + \delta q_{\parallel} ,$$

where n is the invariant closure-normal direction and h is the scalar radial displacement. The interface potential V_{CI} is admissible only if it depends on invariant closure-radial data and has a stable minimum,

$$V'_{CI}(0) = 0,\ V''_{CI}(0) > 0 .$$

The physical radial mass parameter is then the normalised Hessian of the closure-interface potential,

$$m_{HR}^2 = Z_H^{-1}\, V''_{CI}(0) ,$$

where Z_H is the positive kinetic normalisation of the radial closure mode. This expression is an admissibility structure, not yet a numerical prediction. The origin of Z_H itself is recorded as an explicit open bridge debt.

Seven theorems are proved.

Theorem 1 — Closure-Interface Necessity Theorem. A nontrivial physical sector requires a closure interface separating committed admissible support from exterior non-committed support.

Theorem 2 — Interface-Potential Necessity Theorem. A stable closure interface requires an invariant return functional measuring displacement from closure equilibrium; a massive radial claim additionally requires a positive quadratic return term at equilibrium.

Theorem 3 — Radial-Mode Necessity Theorem. At a regular point of the closure interface, the invariant normal displacement defines a scalar radial coordinate; under field promotion, this coordinate is a scalar radial degree of freedom.

Theorem 4 — Tangential Non-Scalar Theorem. Tangential, angular, gauge, or frame motions of the interface are not independent Higgs-radial scalar degrees.

Theorem 5 — Higgs-Radial Identification Theorem. Under the codimension-one minimality premise, the unique minimal physical scalar normal mode of the stable closure interface is Higgs-radial admissible.

Theorem 6 — Closure-Stiffness Mass Theorem. The Higgs-radial mass parameter is the positive physical-sector Hessian of the closure-interface potential at equilibrium, after canonical normalisation.

Theorem 7 — Scalar-Sector Non-Insertion Theorem. Within the HR-1 gate, a VERSF Higgs-radial field is physical only when derived from closure-interface stiffness. Any scalar field imported without closure-interface origin is firewalled as convention or Standard Model inheritance.

HR-1 therefore closes the first Higgs gate in VERSF: the scalar radial mode is not assumed. It is the unavoidable return-stiffness mode of stable physical closure, with every conditional step named and falsifiable.

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0. Inheritance, Aim, and Firewalls

0.1 Inherited VERSF disciplines

HR-1 inherits the following VERSF disciplines.

I1 — Physical-sector admissibility. A physical object is not admitted merely because it is mathematically convenient. It must preserve positivity, frame invariance, and physical-sector support.

I2 — Non-insertion discipline. Objects corresponding to Standard Model structures may not be imported as assumptions. They must be derived from VERSF primitives or explicitly marked as external inheritance.

I3 — Frame/gauge discipline. Physical claims may not depend on arbitrary coordinates, raw labels, presentation order, gauge choices, or untyped internal directions.

I4 — Bridge discipline. A symbol or internal mode is not physical unless a bridge is stated from the internal carrier to a downstream physical role.

I5 — Non-double-counting discipline. A mode removed as gauge, quotient, frame, or redundancy may not later be counted as independent physical support.

I6 — Positivity discipline. Physical-sector excitations may not be carried by negative-norm, ghost, gauge-remnant, or nonphysical terminal support.

I7 — Hermitian/radial readout discipline. A scalar physical readout must be an invariant, positive-sector, frame-stable object, not a raw component of an untyped coordinate representation.

I8 — Gate-admissibility discipline. A gate is closed only when its primitives, readouts, normalisations, and falsifiers are explicit.

0.2 The new problem

The Higgs sector presents a serious risk for any claimed Standard Model derivation.

One can easily write down a scalar field φ, assign a potential,

$$V(\varphi) = -\mu^2 |\varphi|^2 + \lambda |\varphi|^4 ,$$

declare a nonzero vacuum value, and then say that a radial scalar remains.

But if this is simply copied from the Standard Model, nothing has been derived.

HR-1 asks:

Why must VERSF contain a Higgs-like radial scalar at all?

The answer proposed and proved here is:

Because stable physical closure requires an interface; a stable interface with finite return stiffness necessarily has a scalar normal displacement; and a locally varying, positively normalised carrier of that displacement is a scalar field.

That carrier is the Higgs-radial mode.

0.3 Core aim

HR-1 has one aim:

To prove that the Higgs-radial degree of freedom is forced by closure-interface stiffness and is therefore admissible as physical VERSF structure rather than an inserted scalar field.

The paper does not derive a numerical Higgs mass. It does not derive the full electroweak vacuum scale. It does not derive all Yukawa couplings. It closes the prior structural gate:

Why a physical scalar radial mode exists, and under exactly which named conditions.

0.4 Firewalls

This paper does **not** claim:

- that the observed Higgs mass is computed here;
- that the electroweak vev is numerically derived here;
- that the quartic coupling λ is derived here;
- that the full Standard Model Higgs doublet representation is independently derived here;
- that all fermion masses follow from HR-1 alone;
- that CKM or PMNS mixing follows from HR-1 alone;
- that gauge boson masses are fully calculated here;
- that a Mexican-hat potential may be imported from the Standard Model;
- that angular/gauge directions are extra physical Higgs scalars;
- that scalar fields are admissible without closure-interface origin;
- that the displacement-to-field promotion is cost-free — it is a named premise with a named debt;
- that the kinetic normalisation Z_H is derived from substrate primitives here;
- that the possibility-space metric structure is derived here — it is a named premise (P17) charged to debt D1;
- that a radial mode can be declared physical without a bridge to mass-generation channels;
- that stability alone fixes numerical values.

It proves the narrower result:

A stable, regular, codimension-one VERSF closure interface forces a scalar radial stiffness mode, and this mode is Higgs-radial admissible if and only if it is invariant, positive-sector, field-promoted with positive kinetic normalisation, non-double-counted, and bridged to downstream mass-generation structure.

0′. Predictive-Content Ledger

Object	Prior status	HR-1 status
Physical closure	Required by VERSF	Used to define interface
Closure defect Δ_C	Implicit closure failure measure	Made explicit
Closure interface Σ	Previously implicit boundary	Defined as physical-sector boundary
Exterior support	Non-committed possibility	Distinguished from physical support

Object	Prior status	HR-1 status
Interface regularity	Previously silent assumption	Named premise (P14) with falsifier
Possibility-space metric	Previously silent assumption	Named premise (P17); origin charged to debt D1
Interface displacement	Previously unaudited	Decomposed into normal and tangential parts
Normal displacement h	Candidate scalar	Derived as radial coordinate
Field promotion of h	Previously silent step	Named premise (P15) with declared debt
Tangential displacement δq_‖	Potential extra scalar confusion	Firewalled as gauge/frame/interface motion unless bridged
Interface potential V_CI	Needed for stability	Derived as return-to-closure cost
Radial Hessian V″_CI	Candidate mass support	Made closure-stiffness source
Codimension-one minimality	Previously buried in §8	Named premise (P16) conditioning uniqueness
Higgs-radial mode	Could be imported	Admitted only if closure-derived
Standard Model Higgs potential	External known form	Not imported; only structurally matched
Angular/gauge modes	Potential double count	Require quotient/eating ledger
Mass-generation bridge	Needed	Required as explicit map
Kinetic normalisation Z_H	Needed for canonical mass	Required positive; origin recorded as open debt
Exact Higgs mass	Not derived	Outside scope
Exact vev	Not derived	Outside scope
Exact Yukawa hierarchy	Not derived	Outside scope

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1. The Higgs-Radial Problem

1.1 Why the Higgs is dangerous in a derivation

The Higgs field is familiar enough that it is easy to smuggle it into a theory.

A derivation can fail by writing:

"Let φ be a scalar Higgs field."

That phrase already assumes what must be proved.

It assumes:

- a scalar degree of freedom exists;
- its potential exists;
- its minimum is nonzero;
- its radial mode is physical;
- its angular modes are not counted as extra scalar particles;
- it couples to mass channels;
- its scale is physically meaningful;
- it is not a gauge artifact or coordinate choice;
- it is a field — locally varying and propagating — rather than a single global coordinate.

VERSF must not assume these. The last item deserves emphasis: even a derivation that earns a radial coordinate has not yet earned a radial field. HR-1 audits both steps separately.

1.2 What must be derived

The first Higgs gate must derive or explicitly premise at least the following:

1. why a scalar normal/radial degree exists;
2. why it is associated with a potential;
3. why the potential has a stable equilibrium;
4. why the radial coordinate becomes a locally varying field;
5. why the radial mode is physical;
6. why angular/gauge directions are not additional physical scalar support;
7. why the radial mode can bridge to mass-generation channels;
8. why this structure is not an imported Standard Model assumption.

HR-1 addresses exactly these points, and names the ones it premises rather than proves.

1.3 The closure-interface proposal

VERSF physical support is not mere possibility. It is committed admissible support.

That creates a boundary:

Inside: committed physical support. **Outside:** exterior non-committed possibility. **Between them:** the closure interface.

A closure interface cannot be infinitely soft. If it were, physical commitment would dissolve under arbitrarily small perturbations.

It also cannot be infinitely rigid. If it were, no finite excitation, response, or mass-generating stiffness could exist.

Thus a physical closure interface must have finite stiffness.

The scalar measurement of displacement away from its equilibrium closure position is the radial coordinate. Its locally varying, positively normalised carrier is the radial mode.

1.4 The core intuition

The Higgs-radial mode is not "a particle added to give mass."

It is:

the physical breathing mode of the closure boundary.

A perturbation in the radial direction changes how far the system is from its closure equilibrium. That has energy cost. The potential records that cost. The second derivative of the potential at equilibrium is the stiffness. After canonical normalisation, that stiffness is the radial mass parameter.

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2. Named Premises of the Closure-Interface Audit

Premises are labelled P1–P17 to avoid namespace collision with the gate label HR-1.

2.1 P1 — Physical-sector closure premise

A VERSF physical sector consists of support that has passed commitment and admissibility conditions. It is not identical to the unrestricted exterior possibility space.

Falsifier. The paper treats all possible support as physical without a closure/admissibility boundary.

2.2 P2 — Closure-interface premise

If physical support is a proper subset of exterior possibility, there exists an interface Σ separating committed physical support from exterior non-committed support.

Falsifier. The paper claims a nontrivial physical sector while providing no boundary, interface, projector, or equivalent separation rule.

2.3 P3 — Closure-defect premise

The interface must be expressible through a closure-defect functional Δ_C, or equivalent invariant condition, such that closure equilibrium is defined at a regular level of the defect magnitude,

$$d\_C = d_0 , d_0 > 0 ,$$

where $d\_C = \|\Delta\_C\|$. The fully collapsed case $\Delta\_C = 0$ (equivalently $d_0 = 0$) places the equilibrium at the zero of a norm, where the closure coordinate is generically non-differentiable; it is the degenerate collapsed-interface case excluded by the regularity premise P14 and is not the HR-1 equilibrium.

Falsifier. The boundary is named but no invariant measure of departure from closure is supplied, or the equilibrium is placed at the non-regular zero level $d_0 = 0$ while smooth radial structure is claimed.

2.4 P4 — Stability premise

A physical closure interface must be stable under small perturbations. Stability requires an equilibrium condition and, for a massive radial claim, positive return stiffness in the physical radial direction.

Falsifier. The interface has no restoring tendency, or its Hessian is negative/zero in the claimed massive radial sector.

2.5 P5 — Interface-potential premise

A stable interface requires a potential or energy functional V_CI measuring the cost of closure displacement, at least twice differentiable in the radial coordinate on a neighbourhood of equilibrium.

Falsifier. The radial mode is claimed while no return-to-closure potential exists, or the claimed Hessian is undefined because V_CI is not twice differentiable at equilibrium.

2.6 P6 — Normal/radial premise

A displacement of the interface decomposes into normal and tangential parts:

$$\delta q = h\, n + \delta q\_\| .$$

The scalar h is the invariant normal/radial displacement.

Falsifier. The paper identifies a scalar radial mode without defining a normal direction or equivalent invariant radial coordinate.

2.7 P7 — Tangential non-counting premise

Tangential, angular, gauge, frame, or interface-coordinate directions are not independent Higgs-radial scalar degrees unless separately bridged to physical positive-sector support.

Falsifier. Angular/gauge directions are counted as extra scalar Higgs particles without a quotient, gauge, or eating ledger.

2.8 P8 — Invariant potential premise

The closure-interface potential may depend only on invariant closure-radial data, not arbitrary coordinates, labels, or gauge choices.

Falsifier. The potential changes under harmless relabelling or gauge/frame transformation.

2.9 P9 — Positive kinetic premise

The radial mode must have positive kinetic normalisation:

$$Z_H > 0 .$$

Falsifier. The radial excitation is ghostlike, negative-norm, or outside the physical sector.

2.10 P10 — Hessian mass premise

The radial mass parameter is the canonically normalised Hessian of the interface potential at equilibrium:

$$m_{HR}^2 = Z_H^{-1} V''_{CI}(0) .$$

Falsifier. A Higgs-radial mass is claimed without identifying the physical-sector Hessian.

2.11 P11 — Bridge-to-mass premise

A Higgs-radial mode must feed a downstream mass-generation channel through a declared bridge map:

$$B_H : h \to O_{mass} .$$

Falsifier. The radial scalar is declared "Higgs-like" but no mass-generation, projector, coupling, or spectral bridge is stated.

2.12 P12 — Non-insertion premise

No scalar field, potential, vev, quartic, or radial excitation may be imported from the Standard Model without being marked as external inheritance. HR-1 admits only closure-derived radial structure.

Falsifier. The paper assumes the Standard Model Higgs potential and then claims to have derived it.

2.13 P13 — Non-double-counting premise

If angular/gauge directions are quotient-removed or transferred into vector longitudinal support in a later electroweak bridge, they may not also be counted as independent scalar radial support.

Falsifier. The same degrees of freedom appear both as gauge/angular support and as additional physical scalars.

2.14 P14 — Interface-regularity premise

The closure interface admits regular points: there exists a neighbourhood of Σ on which the closure-distance coordinate ρ is differentiable and $\nabla\rho \neq 0$, so the closure-normal direction is well defined.

Falsifier. The radial construction is applied at singular, cuspidal, or degenerate interface points where $\nabla\rho = 0$ or where ρ fails to be differentiable, without a separate regularisation argument.

2.15 P15 — Field-promotion premise

The radial displacement admits a locally varying carrier: h may be promoted from a single invariant coordinate to a configuration h(x) over the emergent continuum, with kinetic support supplied by the substrate transport structure. The origin of the kinetic term $(\partial h)^2$ and of its normalisation Z_H is a declared bridge debt, not a result of HR-1.

Falsifier. The paper treats a single global displacement coordinate as a propagating field without stating the promotion, or claims to have derived Z_H from substrate primitives when it has not.

2.16 P16 — Codimension-one minimality premise

The closure interface is locally codimension one in the relevant possibility space: it is defined by a single invariant scalar condition $\rho = 0$, so it possesses exactly one invariant normal direction at each regular point.

Falsifier. Uniqueness of the radial mode is claimed for an interface defined by multiple independent closure conditions (higher codimension) without a separate mode count.

2.17 P17 — Possibility-space metric premise

The exterior possibility space $\mathcal{Q}$ and the defect space $\mathcal{V}\_C$ carry invariant inner-product (or equivalent metric) structure; permitted frame/gauge relabellings act by isometries of that structure; and the defect magnitude $\|\Delta\_C\|$, the gradient $\nabla\rho$, the closure-normal direction, and the orthogonal normal/tangential decomposition are defined relative to it. The origin of this metric from substrate primitives is not derived here; it is charged to the same substrate-dynamics debt as the kinetic term (D1, §10′).

Falsifier. The normal direction or radial magnitude changes under a permitted relabelling, or no invariant metric on $\mathcal{Q}$ or $\mathcal{V}\_C$ can be exhibited.

2′. Premise-Dependency Map

Result	Depends on premises
Theorem 1 — Closure-Interface Necessity	P1, P2
Theorem 2 — Interface-Potential Necessity	P1, P2, P3, P4, P5
Theorem 3 — Radial-Mode Necessity	P3, P5, P6, P14, P15, P17
Theorem 4 — Tangential Non-Scalar	P6, P7, P13, P14, P17
Theorem 5 — Higgs-Radial Identification	P1–P9, P13–P17
Theorem 6 — Closure-Stiffness Mass	P4, P5, P8, P9, P10, P15
Theorem 7 — Scalar-Sector Non-Insertion	P1–P17 (all)

No theorem depends on any unstated premise. The regularity (P14), field-promotion (P15), minimality (P16), and metric (P17) premises are the conditional load-bearing steps: a referee attacking HR-1 must attack one of P14–P17 or one of P1–P13, and each carries an explicit falsifier.

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3. Formal Setup: Closure Defect, Interface, Normal, and Radial Field

3.1 Exterior possibility space

Let $\mathcal{Q}$ be the exterior possibility space of candidate support states q.

Not all q ∈ $\mathcal{Q}$ are physical. Physicality requires commitment, closure, positivity, and admissibility.

Let $\mathcal{P}\_phys$ denote the physical-sector projector or admissibility filter.

3.2 Closure defect

Define a closure-defect functional:

$\Delta\_C : \mathcal{Q} \to \mathbb{R}$

or, more generally,

$\Delta\_C : \mathcal{Q} \to \mathcal{V}\_C$,

where $\mathcal{V}\_C$ is a typed closure-defect space.

For the scalar radial audit, the invariant scalar closure distance is

$d\_C(q) = \|\Delta\_C(q)\|$.

Closure equilibrium is defined by

$d\_C(q) = d_0$,

or, after shifting the equilibrium coordinate,

$\rho(q) = d\_C(q) - d_0 = 0$.

The scalar ρ measures radial departure from closure equilibrium.

By P3, $d_0 > 0$: the equilibrium sits at a regular level of the defect magnitude, away from the zero of the norm, where $\|\cdot\|$ is generically non-differentiable and $\nabla\rho$ would be undefined. This is what makes the regularity premise P14 satisfiable on the interface rather than violated by construction.

Two distinct classes of norm-preserving motion exist and must not be confused with radial displacement: motion that reparametrises the interface within $\mathcal{Q}$, and motion that rotates Δ_C within the level set $\|\Delta\_C\| = d_0$ of the defect space $\mathcal{V}\_C$ without changing its magnitude. Both are tangential in the sense of §6, both fall under the tangential non-counting premise P7, and any later physical use of either class is charged to the gauge-eating ledger debt D5. Defect-space angular motion is not a third category escaping the audit.

3.3 Closure interface

The closure interface is the equilibrium support:

$\Sigma = \{ q \in \mathcal{Q} : \rho(q) = 0, \mathcal{P}\_phys\ q = q \}$.

The interface separates:

- interior committed physical support;
- exterior non-committed possibility;
- forbidden or nonphysical support excluded by $\mathcal{P}$_phys.

By P16, Σ is locally defined by the single invariant condition $\rho = 0$ and is therefore locally codimension one.

3.4 Normal direction

By P14, there exist regular points of Σ where ρ is differentiable and $\nabla\rho \neq 0$. At such points, define the closure-normal direction:

$$n = \nabla\rho \,/\, \|\nabla\rho\| \, .$$

This direction is not an arbitrary coordinate axis. It is determined by the invariant closure-defect gradient, with the gradient and its normalisation taken relative to the invariant metric of P17. At non-regular points the construction is not applied; HR-1 makes no claim there.

3.5 Perturbation decomposition

A small displacement δq near a regular point of Σ decomposes as

$$\delta q = h\, n + \delta q_{\|} \, ,$$

with

$$\langle n, \delta q_{\|} \rangle = 0 \, ,$$

with the inner product supplied by P17.

Here:

- h is the scalar normal displacement;
- $\delta q_{\|}$ is tangential/interface motion.

The scalar h is the candidate Higgs-radial coordinate.

3.6 Field promotion

By P15, the displacement admits a locally varying carrier h(x) over the emergent continuum, with kinetic support written

$$\mathscr{L}_{kin} = \tfrac{1}{2} Z_H (\partial h)^2 \, .$$

HR-1 uses this structure but does not derive Z_H. The distinction is enforced throughout: h as invariant coordinate is derived; h(x) as propagating field is premised under P15, with the origin of Z_H recorded in the bridge-debt ledger (§10′).

3.7 Interface potential

The closure-interface potential is an invariant function of the radial displacement alone,

$$V_{CI} = V_{CI}(h) ,$$

with all derivative dependence assigned to the kinetic sector of §3.6. Its local equilibrium expansion begins:

$$V_{CI}(h) = V_{CI}(0) + \tfrac{1}{2} K_C h^2 + O(h^3)$$

when the equilibrium condition $V'_{CI}(0) = 0$ holds and V_{CI} is twice differentiable at equilibrium (P5).

The stiffness is:

$$K_C = V''_{CI}(0) .$$

Physical radial stability of a massive mode requires:

$$K_C > 0 .$$

3.8 Kinetic normalisation and canonical mass

With the field-promoted Lagrangian

$$\mathscr{L}_{HR} = \tfrac{1}{2} Z_H (\partial h)^2 - V_{CI}(h) ,$$

canonical normalisation gives:

$$h_c = Z_H^{1/2} h ,$$

and therefore

$$m_{HR}^2 = Z_H^{-1} K_C .$$

This is the closure-stiffness mass parameter. HR-1 derives its structural origin, not its numerical value.

3.9 Gauge/radial separation

If the closure order parameter is represented by a field Φ in a later electroweak bridge, the invariant radial coordinate is of the form

$r^2 = \Phi^\dagger\Phi$,

or its VERSF-equivalent Hermitian invariant.

The radial mode is:

$h = r - r_0$.

Gauge/angular directions move Φ at fixed r. They are not radial stiffness modes.

Thus HR-1 separates:

- radial magnitude change: physical scalar candidate;
- angular/gauge orientation change: quotient/gauge structure;
- frame coordinate change: nonphysical presentation.

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4. Why a Physical Sector Requires an Interface

4.1 Physical support is not unrestricted possibility

If everything possible were physical, no closure gate would exist. There would be no distinction between admissible and inadmissible support.

But VERSF requires precisely such a distinction.

Physical support is committed and admissible. Exterior possibility is not yet committed. Forbidden support may be excluded entirely.

Therefore a physical sector is a proper subset or typed projection of wider possibility.

4.2 Boundary from admissibility

Whenever a physical sector is separated from exterior possibility by a nontrivial admissibility condition, there is an interface.

That interface may be geometric, spectral, algebraic, categorical, or projector-defined. HR-1 does not require it to be a literal spatial surface. It requires only a typed boundary between committed and non-committed support.

4.3 Why the interface is physical

The interface is physical because physical predictions depend on which support is admitted.

If moving across the interface changes admissibility, mass support, coupling access, or spectral participation, then the interface is not decoration. It is load-bearing.

A load-bearing interface requires a displacement audit.

4.4 The interface cannot be merely a label

If the interface were only a name, then moving away from closure would cost nothing and physical support would be indistinguishable from exterior possibility.

That would collapse the physical-sector distinction.

Therefore the interface must have a physical stability structure.

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5. Why a Stable Interface Requires a Potential

5.1 Stability is not automatic

A boundary can be unstable.

If perturbing the interface causes runaway departure from closure, no stable physical sector exists.

If perturbing the interface costs nothing, closure has a flat direction. That direction may be physical only if separately protected as massless, gauge, or moduli support. It cannot be claimed as a massive Higgs-radial mode.

If perturbing the interface has positive return cost, the interface has stiffness.

5.2 Three graded stability conditions

HR-1 separates three conditions that are often conflated:

1. **Boundedness.** The closure cost does not decrease without bound under displacement. Failure is runaway instability.
2. **Equilibrium.** The first variation vanishes at the interface: $V'_{CI}(0) = 0$. Failure is a false equilibrium.
3. **Quadratic return.** The second variation is positive: $V''_{CI}(0) > 0$. Failure at this order does not by itself destroy the interface — the direction may be stabilised at higher order or nonperturbatively — but a *massive* radial claim cannot be made at quadratic order.

Only the conjunction of all three licenses a massive Higgs-radial mode at this gate. Boundedness plus equilibrium with a flat quadratic direction licenses at most a flat-direction record, which HR-1 explicitly does not close.

5.3 Potential as return-to-closure cost

A potential is the scalar functional that measures the cost of displacement from equilibrium.

For the closure interface:

$$V_{CI}(h)$$

measures how much closure cost is incurred when the interface is displaced by radial amount h.

Equilibrium requires:

$$V'_{CI}(0) = 0 .$$

A massive radial claim requires:

$$V''_{CI}(0) > 0 .$$

5.4 Why this is Higgs-like

A Higgs-radial mode is a scalar excitation around a stable nontrivial background.

The closure-interface radial mode has exactly this structure:

- a background closure equilibrium;
- a scalar displacement away from that equilibrium;
- a potential measuring return cost;
- a positive Hessian giving massive radial stiffness.

This does not yet prove the numerical Standard Model Higgs. It proves Higgs-radial admissibility.

5.5 Minimal analytic form

If the closure-radial order parameter is represented by an invariant r, the minimal analytic bounded potential has the schematic form:

$V\_CI(r) = \alpha r^2 + \beta r^4$ + higher closure corrections ,

with $\beta > 0$ for boundedness.

A nonzero interface radius r_0 requires the equilibrium condition:

$V'\_CI(r_0) = 0$,

and radial stability requires:

$V''\_CI(r_0) > 0$.

In shifted radial form $h = r - r_0$:

$V\_CI(h) = V\_CI(0) + ½ K\_C h^2$ + higher terms .

This is not an imported Standard Model potential. It is the local invariant expansion of a stable closure-interface potential around equilibrium. The analytic polynomial form is a representation convenience; the theorem chain requires only twice-differentiability (P5) and the three graded conditions of §5.2.

---

6. Normal/Radial Versus Tangential/Gauge Directions

6.1 The decomposition

Near a regular point of the closure interface, any small perturbation decomposes into:

$\delta q = h n + \delta q\_\|$.

The two components have different physical meanings.

The normal part h n changes the closure distance. It moves the system away from or toward equilibrium.

The tangential part $\delta q_\parallel$ moves along the interface. It changes presentation, orientation, gauge phase, or internal coordinate without changing closure distance to first order.

6.2 Why the radial mode is scalar

The radial displacement h is a single invariant magnitude. It does not depend on which coordinates are used to describe the interface.

By P17, permitted frame/gauge relabellings U act by isometries of the possibility-space metric. Therefore the invariant radial distance satisfies:

$h(Uq) = h(q)$,

or, at most, sits in a declared one-dimensional representation of the permitted relabelling group.

Thus h is scalar in the relevant physical-sector sense, and its scalar status is inherited from the isometry condition rather than asserted.

6.3 Why tangential modes are not Higgs-radial modes

Tangential modes may be important. They may encode gauge orientation, Goldstone-like directions, frame motion, or interface reparametrisation.

But they are not radial stiffness modes because they do not measure normal displacement from closure equilibrium.

Therefore they cannot be counted as additional Higgs-radial scalar support.

6.4 The angular-mode trap

A common error is to count all components of an order parameter as physical scalar particles.

HR-1 forbids this.

The scalar count must distinguish:

- invariant radial displacement;
- angular/gauge orbit;
- frame redundancy;
- physical positive-sector residuals.

Only the first is closed here as Higgs-radial.

If later electroweak structure transfers angular modes into vector longitudinal support, that transfer must be recorded in a separate gauge-eating ledger. Those directions may not also reappear as independent scalar particles.

---

7. The Closure-Interface Potential

7.1 Definition

Two objects must be kept typed apart.

The **closure-interface potential** is an invariant function of the radial displacement alone:

$V\_CI : h \mapsto \mathbb{R}$,

with no derivative dependence. It measures the local cost of displacing the closure interface away from equilibrium.

The **effective closure functional** combines the potential with the kinetic sector of P15:

$\Gamma\_CI[h] = \int d^4x \sqrt{|g\_eff|} \; (\tfrac{1}{2} Z\_H (\partial h)^2 + V\_CI(h(x)))$.

The functional Γ_CI presupposes field promotion (P15); the potential V_CI does not. Blurring the two would let derivative structure smuggle itself into the mass audit: the Hessian of Theorem 6 is taken of V_CI, not of Γ_CI, and the kinetic contribution enters only through the declared normalisation Z_H.

For HR-1, the local radial potential suffices:

$V\_CI(h)$.

7.2 Admissibility conditions

An admissible closure-interface potential must satisfy:

1. **Invariant dependence:** it depends on closure-radial data, not arbitrary labels.
2. **Differentiability:** it is at least twice differentiable at equilibrium (P5).
3. **Equilibrium:** $V'\_CI(0) = 0$.
4. **Stability:** $V''\_CI(0) > 0$ in the physical radial sector, for a massive claim.
5. **Boundedness:** the potential is bounded below on admissible support.
6. **Positive kinetic normalisation:** $Z\_H > 0$.
7. **Physical-sector support:** the radial carrier lies inside $\mathcal{P}\_phys$.
8. **Bridge capacity:** the radial mode can feed a downstream physical target.

7.3 Local expansion

The local expansion around equilibrium is:

$V\_CI(h) = V_0 + \frac{1}{2} K\_C\, h^2 + \kappa_3 h^3 + \kappa_4 h^4 + \ldots$

with

$K\_C > 0$.

If an additional symmetry forbids odd powers, then $\kappa_3 = 0$. HR-1 does not assume such a symmetry unless separately derived.

The existence of a stable radial mass parameter requires $K\_C > 0$, not a preselected polynomial form.

7.4 Relation to familiar Higgs notation

In familiar notation, a scalar order parameter Φ may be written using an invariant radial magnitude:

$r^2 = \Phi^\dagger\Phi$.

A minimal bounded symmetry-compatible potential can be written:

$V(r) = \lambda(r^2 - r_0^2)^2$.

The radial mode is:

$h = r - r_0$.

HR-1 does not import this form. It only shows why a potential depending on invariant radial displacement is required. The precise representation Φ, the exact r_0, and the numerical λ belong to later gates.

7.5 No bare Mexican-hat import

A later VERSF paper may use a Mexican-hat form only if it proves:

- the relevant invariant radial coordinate;
- the origin of the nonzero equilibrium radius;
- the bounded quartic stabilisation or equivalent;
- the quotient/removal of angular directions;
- the bridge into electroweak mass channels.

Otherwise the potential is imported, not derived.

7.6 Local potential form versus derived potential shape

A distinction is essential.

HR-1 derives the **necessity** of an invariant closure-interface potential. It does not derive the full microscopic **shape** of that potential.

The local expansion

$$V\_CI(h) = V_0 + \tfrac{1}{2} K\_C h^2 + \kappa_3 h^3 + \kappa_4 h^4 + \dots$$

is therefore not a fitted Higgs potential. It is the Taylor expansion of any twice-differentiable stable return functional around a regular closure equilibrium.

Likewise, the schematic form

$$V\_CI(r) = \alpha r^2 + \beta r^4 + \dots$$

of §5.5 is not a derivation of the Standard Model Mexican-hat potential. It is only the minimal analytic language for representing bounded radial return near equilibrium.

The full scalar-sector derivation must do more. It must show why the closure functional selects its particular invariant terms, why the equilibrium radius is nonzero, why the stabilising coefficient has the required sign, how the kinetic normalisation is fixed, and how the physical Hessian matches the observed radial excitation. These duties are recorded as debts D1–D3 and D7 of §10′.

Therefore HR-1 licenses the statement:

a stable closure interface requires an invariant radial return potential.

It does not yet license the stronger statement:

the Standard Model Higgs potential, electroweak vev, Higgs mass, and quartic coupling have been numerically derived.

---

8. Higgs-Radial Identification

8.1 What "Higgs-radial" means in HR-1

HR-1 uses "Higgs-radial" in a precise admissibility sense.

A mode is Higgs-radial admissible if it is:

- scalar under permitted frame/gauge relabellings;
- normal to the closure interface;
- associated with finite return stiffness;
- field-promoted under P15 with positive kinetic normalisation;
- positive-sector;
- capable of bridging to mass-generation channels;
- not double counted with angular/gauge modes.

The closure-radial h satisfies these requirements under the named premises.

8.2 Why this is not merely a scalar

Not every scalar is Higgs-radial.

A random scalar label has no physical status.

A coordinate radius may be a presentation artifact.

A fitting parameter may mimic a scalar without being physical.

The closure-radial h is different because it is forced by interface stability, and its promotion to a field is a named, falsifiable premise rather than a silent step.

Nor is h merely one more order-parameter radial mode among many candidates. The radial coordinate is not chosen: it is the invariant normal coordinate of the closure-defect level set, fixed by the defect gradient and the P17 metric. No selection freedom exists at the coordinate level, so no rival radial mode competes with it.

8.3 The radial mode as closure stiffness

The physical content of h is:

how far the committed physical sector has been displaced from its closure equilibrium.

The physical content of V_CI(h) is:

how much return-to-closure cost that displacement carries.

The physical content of V″_CI(0) is:

the stiffness of the closure interface.

This is why the radial mode has mass-like support.

8.4 Minimal uniqueness

By P16, the closure interface is locally codimension one: it is cut out by the single invariant condition $\rho = 0$. At each regular point there is therefore exactly one invariant normal direction, and hence one minimal scalar radial displacement.

Additional scalar modes would require:

- additional independent closure defects (higher codimension);
- extra physical sectors;
- nontrivial internal radial coordinates;
- or additional positive-sector bridge maps.

They are not supplied by HR-1, and P16 carries the falsifier: if the interface is later shown to require multiple independent closure conditions, the uniqueness claim of Theorem 5 must be re-audited with a full mode count.

Therefore HR-1 closes only the **minimal Higgs-radial mode**, not an arbitrary extended scalar sector.

---

9. Physical-Sector Positivity and Hessian Stiffness

9.1 Positive kinetic support

The radial mode must have positive kinetic normalisation:

$Z_H > 0$.

If $Z_H < 0$, the mode is ghostlike.

If $Z_H = 0$, the mode has no propagating radial support at this gate.

If Z_H is gauge or frame dependent, the mode is not yet physical.

9.2 Positive Hessian

A massive stable radial mode requires:

$V''_{CI}(0) > 0$.

This is the local return stiffness.

If $V''_{CI}(0) < 0$, the interface equilibrium is unstable.

If $V''_{CI}(0) = 0$, the radial direction is flat at this order and cannot be claimed as a massive Higgs-radial mode without higher-order or nonperturbative stabilisation, which would require a separate gate.

9.3 Physical-sector projection

Let $\mathcal{P}_{phys}$ be the physical-sector projector. The radial carrier must satisfy:

$\mathcal{P}_{phys}\, h = h$.

If only part of h lies in physical support, the nonphysical part must be projected out before making Higgs-radial claims.

9.4 Canonical mass parameter

Given the local Lagrangian form:

$\mathscr{L}_{HR} = ½\, Z_H\, (\partial h)^2 - V_{CI}(h)$,

the canonically normalised radial field is:

$h_c = Z_H^{1/2}\, h$.

The radial mass parameter is:

$m_{HR}^2 = Z_H^{-1}\, V''_{CI}(0)$.

This proves the structural origin of a Higgs-radial mass term:

mass is closure-interface stiffness after physical kinetic normalisation.

It does not yet provide the numerical Higgs mass.

9′. Interface-Radial Mode and the Electroweak Completion Interface

HR-1 derives the scalar radial stiffness mode. It does not by itself derive the electroweak representation that carries the familiar Standard Model Higgs structure.

That representation belongs to the electroweak completion-interface chain. In the Standard Model, the physical Higgs boson is the radial excitation remaining after the weak-doublet angular/gauge directions have been quotiented or transferred into vector longitudinal support. HR-1 supplies the radial-stiffness theorem required by that chain; it does not replace the representation theorem.

The downstream bridge must therefore identify three separate objects:

- the completion-interface carrier, which supplies the electroweak representation;
- the invariant radial magnitude, which supplies the closure-normal displacement;
- the gauge/angular quotient ledger, which prevents the non-radial components from being counted as additional physical scalars.

In familiar notation this later bridge may be represented by an order parameter Φ and an invariant radial magnitude

$r^2 = \Phi^\dagger\Phi$,

with

$h = r - r_0$.

But HR-1 does not assume Φ, the doublet representation, r_0, or the gauge-eating structure. It only proves that once the completion interface has an invariant closure radius, the radial displacement of that interface is the unique minimal scalar stiffness mode under the stated regularity, minimality, and metric premises.

Thus the correct inheritance relation is:

the electroweak completion-interface theorem supplies the carrier representation; HR-1 supplies the non-inserted radial stiffness mode; a later scalar-potential theorem must supply the numerical potential and mass.

The representation duty is debt D4 and the quotient duty is debt D5 of §10′.

---

10. Bridge to Mass-Generation Channels

10.1 Why a bridge is required

A radial scalar is not automatically a Higgs-sector physical input.

To become Higgs-radial in the mass-generation sense, it must feed physical mass channels.

Therefore HR-1 requires a bridge map:

$B_H : h \to O_{mass}$.

The target O_{mass} may be:

- a fermion mass matrix;
- a gauge boson mass term;
- a spectral gap;
- a Yukawa-admissibility channel;
- a closure-density scale;
- a projector alignment;
- a radial normalisation of committed support.

10.2 Non-rhetorical bridge condition

It is not enough to say:

"h gives mass."

A later paper must specify:

1. where h enters the physical expression;
2. which sector receives mass support;
3. how the result changes if $h = 0$;
4. why the coupling is invariant;
5. why no gauge/angular direction is double counted.

10.3 Minimal bridge statement

At the structural level, HR-1 licenses the following form:

$O_{mass} = B_H(h_c)$

where h_c is the canonically normalised closure-radial mode.

A later Yukawa or electroweak paper must refine this into specific channel maps, for example:

$M_f \propto Y_f v_C$,

or

$M_V^2 \propto g^2 v_C^2$,

only after deriving the relevant representation, coupling, and normalisation.

10.4 No mass claim without bridge

A paper that derives h but does not bridge h to mass has derived a radial closure mode, not yet a Higgs mass-generation mode.

A paper that claims mass generation without h has skipped the radial gate.

HR-1 therefore functions as the firewall between scalar existence and mass prediction.

10′. Open Bridge-Debt Ledger

HR-1 licenses downstream Higgs-sector work only against the following explicitly recorded debts. Each debt names its creditor gate class.

Debt	Content	To be discharged by
D1 — Kinetic/metric-origin debt	Derivation of the kinetic term $\frac{1}{2} Z_H (\partial h)^2$, the value of Z_H, and the invariant metric structure on $\mathcal{Q}$ and $\mathcal{V}_C$ from substrate transport primitives	A substrate-dynamics gate
D2 — Vev-scale debt	Derivation of the equilibrium closure radius (electroweak vev scale v_C)	A quantitative-normalisation gate
D3 — Quartic debt	Derivation of the stabilising quartic (or equivalent) coefficient	A quantitative-normalisation gate
D4 — Representation debt	Derivation of the doublet (or equivalent) representation carrying the radial invariant	The electroweak completion-interface chain
D5 — Gauge-eating ledger debt	Explicit accounting of angular/tangential directions transferred to vector longitudinal support	An electroweak gauge-transfer gate
D6 — Numerical-mass debt	Numerical evaluation of $m_{HR}^2 = Z_H^{-1} V''_{CI}(0)$ against the observed scalar	A closure-of-programme audit
D7 — Potential-shape debt	Derivation of the invariant term content of V_{CI}: which invariant terms appear, why the equilibrium radius is nonzero, why the stabilising coefficient is positive	A closure-potential gate

No downstream paper may treat any of D1–D7 as discharged by HR-1. Citing HR-1 discharges only the existence, uniqueness-under-P16, and admissibility structure of the radial mode.

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11. Theorem 1 — Closure-Interface Necessity Theorem

11.1 Statement

Theorem 1 — Closure-Interface Necessity Theorem. If VERSF physical support is a proper committed/admissible subset of exterior possibility support, then a closure interface Σ is required. The interface may be geometric, spectral, algebraic, projector-defined, or categorical, but some typed boundary between physical support and exterior non-committed support is necessary.

Depends on: P1, P2.

11.2 Proof

Let $\mathcal{Q}$ be exterior possibility support and let $\mathcal{S}$_phys ⊂ $\mathcal{Q}$ be physical support.

By P1, $\mathcal{S}$_phys is not all of $\mathcal{Q}$. It is the subset that has passed commitment and admissibility conditions.

Therefore there exists a distinction between q ∈ $\mathcal{S}$_phys and q ∉ $\mathcal{S}$_phys.

This distinction may be represented by an admissibility projector $\mathcal{P}$_phys, a closure-defect condition Δ_C, a boundary Σ, or an equivalent typed selection rule.

If no such interface, projector, or boundary exists, then the theory cannot distinguish physical support from exterior possibility. That contradicts the assumption that physical support is a proper committed/admissible subset.

Therefore a closure interface or equivalent boundary structure is necessary. ■

11.3 Corollary — No physical sector without separation

A theory that cannot say what separates physical support from exterior possibility has not defined a physical sector.

11.4 Corollary — Interface need not mean spatial surface

The closure interface may be algebraic, spectral, or categorical. HR-1 requires typed separation, not literal spatial geometry.

11.5 Remark — Near-definitional status

Theorem 1 is close to definitional: a proper subset entails a separation rule almost by unfolding the terms. It is stated as a theorem rather than left implicit because the programme's audit style requires that even near-tautological load-bearing steps be named, given falsifiers, and made citable. The alternative — leaving the boundary implicit — is precisely the silent-boundary failure mode that F2 exists to catch.

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12. Theorem 2 — Interface-Potential Necessity Theorem

12.1 Statement

Theorem 2 — Interface-Potential Necessity Theorem. A stable closure interface requires an invariant return functional measuring displacement from closure equilibrium. A massive radial claim additionally requires that this functional be twice differentiable at equilibrium with vanishing first variation and positive second variation. Without such a return functional, the interface is unstable, cost-free, or unphysical as massive radial support; with a return functional whose second variation vanishes, only a flat-direction record may be claimed.

Depends on: P1, P2, P3, P4, P5.

12.2 Proof

By Theorem 1, a nontrivial physical sector requires a closure interface Σ.

Consider a small displacement away from Σ, measured by the invariant closure coordinate supplied by P3. Three cases exhaust the local behaviour of the closure cost.

Case 1 — Unbounded decrease. If displacement lowers the closure cost without bound, the interface is unstable: physical support drifts into exterior possibility without resistance. This contradicts P4.

Case 2 — Flat cost. If displacement has exactly flat cost through second order, the direction is flat. It may be physically meaningful only if identified as a massless mode, gauge direction, modulus, or separately protected sector. By the grading of §5.2, it cannot be claimed as a massive radial Higgs-like excitation at this gate.

Case 3 — Positive return cost. If displacement has positive return cost, the interface has stiffness.

A stable massive radial interface mode therefore falls under Case 3 and requires a return functional V_CI which, by P5, is twice differentiable at equilibrium, and which after shifting equilibrium to $h = 0$ satisfies

$V'_{CI}(0) = 0$

and

$V''_{CI}(0) > 0$.

This V_CI is the closure-interface potential. ■

12.3 Corollary — No potential, no massive radial mode

A massive Higgs-radial claim requires a return-to-closure potential or equivalent stiffness functional.

12.4 Corollary — Flat directions need separate audit

A flat radial direction is not automatically a Higgs-radial mass mode. Higher-order or nonperturbative stabilisation, if invoked, requires a separate gate with its own falsifiers.

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13. Theorem 3 — Radial-Mode Necessity Theorem

13.1 Statement

Theorem 3 — Radial-Mode Necessity Theorem. At a regular point of a stable closure interface, the invariant normal displacement defines a scalar radial coordinate. Under the field-promotion premise P15, this coordinate is carried by a scalar radial degree of freedom. The mode is forced by interface geometry and stability; it is not an arbitrary scalar insertion. The coordinate-level claim is unconditional given regularity; the field-level claim is conditional on P15 and is so marked.

Depends on: P3, P5, P6, P14, P15, P17.

13.2 Proof

Let Σ be a closure interface defined locally by

$\rho(q) = 0$,

where ρ is the invariant closure-distance coordinate of P3.

By P14, there exist regular points where ρ is differentiable and

$\nabla\rho \neq 0$,

so the normal direction is defined, relative to the invariant metric of P17, by

$n = \nabla\rho / \|\nabla\rho\|$.

A small perturbation δq decomposes uniquely, relative to the P17 inner product, into normal and tangential parts:

$\delta q = h\, n + \delta q_\parallel$,

with

$\langle n, \delta q_\parallel \rangle = 0$.

The coefficient h measures displacement in the normal closure direction. Since ρ is invariantly defined, h is not an arbitrary coordinate component. It is the scalar magnitude of departure from closure equilibrium. This establishes the coordinate-level claim.

By Theorem 2, stable closure requires a potential $V_{CI}(h)$ with finite return stiffness. The coordinate h therefore carries a physical return cost.

By P15, h admits a locally varying carrier h(x) with positive kinetic support. The pair (kinetic carrier, return potential) constitutes a scalar degree of freedom in the field-theoretic sense. This establishes the field-level claim, conditional on P15.

Thus the radial mode is forced by closure-interface geometry, stability, and the named promotion premise. ■

13.3 Corollary — The radial mode is normal, not decorative

The radial mode exists because the interface can be displaced away from equilibrium. It is not introduced by notation.

13.4 Corollary — No normal direction, no radial Higgs claim

A paper claiming a Higgs-radial mode must identify the closure-normal direction or equivalent invariant radial coordinate at a regular interface point.

13.5 Corollary — Coordinate is not field

A paper that has earned only the invariant coordinate h, without stating the field-promotion step and its kinetic debt, has not earned a propagating scalar.

---

14. Theorem 4 — Tangential Non-Scalar Theorem

14.1 Statement

Theorem 4 — Tangential Non-Scalar Theorem. Tangential, angular, gauge, or frame motions of the closure interface are not independent Higgs-radial scalar degrees. They may become physical only through a separate positive-sector bridge, quotient, or gauge-transfer ledger.

Depends on: P6, P7, P13, P14, P17.

14.2 Proof

By Theorem 3, the Higgs-radial candidate is the normal displacement h, which changes closure distance.

A tangential perturbation $\delta q_\|$ satisfies

$$\langle n, \delta q_\| \rangle = 0 .$$

To first order, it does not change the closure-distance coordinate ρ. It moves along the interface rather than away from it.

Such motion may represent:

- coordinate reparametrisation of Σ;
- gauge orientation;
- angular motion of an order parameter;
- frame convention;
- a Goldstone-like direction;
- or another physical mode requiring separate audit.

But it is not the scalar radial stiffness mode because it is not normal displacement.

Therefore tangential directions cannot be counted as independent Higgs-radial scalar degrees. If later theory assigns them physical support, it must provide a separate bridge (P7) and must avoid double counting (P13). ■

14.3 Corollary — Angular directions require a ledger

Any later use of angular/gauge directions must state whether they are quotient-removed, gauge-eaten, physical, or projected out. This is debt D5 of §10′.

14.4 Corollary — One minimal radial scalar

For a regular codimension-one closure interface, HR-1 supplies one minimal scalar radial mode.

---

15. Theorem 5 — Higgs-Radial Identification Theorem

15.1 Statement

Theorem 5 — Higgs-Radial Identification Theorem. Under the codimension-one minimality premise P16 and the regularity premise P14, the unique minimal positive-sector scalar normal mode of a stable VERSF closure interface is Higgs-radial admissible. It has the structural role of the Higgs radial degree: an invariant scalar displacement around closure equilibrium with finite return stiffness and bridge capacity to mass-generation channels.

Depends on: P1–P9, P11, P13–P17.

15.2 Proof

By Theorem 1, a physical sector requires a closure interface.

By Theorem 2, stable interface support requires a return potential.

By Theorem 3, the interface has a scalar normal displacement h, field-promoted under P15.

By Theorem 4, tangential/gauge/frame directions are not counted as Higgs-radial scalar support.

By P16, the interface is locally cut out by a single invariant condition and therefore possesses exactly one invariant normal direction at each regular point. Hence h is the unique minimal scalar stiffness mode associated with the closure interface; no second normal direction exists from which a second radial scalar could arise.

The mode is Higgs-radial admissible because it satisfies the required structural properties:

1. it is scalar/invariant under permitted isometric relabellings (P8, P17);
2. it is normal to the closure interface (P6, P14);
3. it has a potential measuring return-to-closure cost (P5);
4. it has a positive-sector stability condition (P4, P9);
5. it is field-promoted with declared kinetic debt (P15);
6. it is not double counted with angular/gauge modes (P7, P13);
7. it can be bridged to downstream mass-generation structure (P11).

Therefore h has the admissible role of the Higgs-radial degree in VERSF. ■

15.3 Corollary — Higgs-radial is closure stiffness

The Higgs-radial field is the physical stiffness mode of the closure interface.

15.4 Corollary — Identification is structural, not numerical

The theorem identifies the admissible mode. It does not compute its observed mass.

15.5 Corollary — Uniqueness is conditional and falsifiable

Uniqueness holds under P16. If the closure interface is later shown to require multiple independent closure conditions, the falsifier of P16 fires and the scalar count must be re-run. HR-1 does not exclude extended scalar sectors by fiat; it excludes them under the stated minimality premise.

---

16. Theorem 6 — Closure-Stiffness Mass Theorem

16.1 Statement

Theorem 6 — Closure-Stiffness Mass Theorem. For an admissible Higgs-radial closure mode h with positive kinetic normalisation Z_H and stable closure-interface potential V_{CI}, the radial mass parameter is the canonically normalised physical-sector Hessian:

$$m_{HR}^2 = Z_H^{-1} V''_{CI}(0) .$$

Thus radial mass support is closure-interface stiffness.

Depends on: P4, P5, P8, P9, P10, P15.

16.2 Proof

The local radial Lagrangian, under P15, is:

$$\mathscr{L}_{HR} = \tfrac{1}{2} Z_H (\partial h)^2 - V_{CI}(h) .$$

At equilibrium $h = 0$,

$$V'_{CI}(0) = 0 .$$

Expanding the potential, using twice-differentiability from P5:

$$V_{CI}(h) = V_{CI}(0) + \tfrac{1}{2} V''_{CI}(0) h^2 + O(h^3) .$$

Canonical normalisation defines:

$$h_c = Z_H^{1/2} h .$$

Substituting $h = Z_H^{-1/2} h_c$ gives the quadratic term:

$$\tfrac{1}{2} V''_{CI}(0) h^2 = \tfrac{1}{2} Z_H^{-1} V''_{CI}(0) h_c^2 .$$

Therefore the mass parameter of the canonically normalised radial excitation is:

$$m_{HR}^2 = Z_H^{-1} V''_{CI}(0) .$$

Physical stability requires:

$$Z_H > 0$$

and

$$V''_{CI}(0) > 0 .$$

Therefore the radial mass parameter is positive and is precisely the normalised stiffness of the closure-interface potential. ■

16.3 Corollary — No Hessian, no mass claim

A Higgs-radial mass claim must identify the physical-sector Hessian of the closure-interface potential.

16.4 Corollary — Numerical prediction requires later normalisation

HR-1 gives the form of the mass parameter, not its numerical value. Numerical prediction requires discharging debts D1 (Z_H) and D2–D3 (potential coefficients) of §10′.

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17. Theorem 7 — Scalar-Sector Non-Insertion Theorem

17.1 Statement

Theorem 7 — Scalar-Sector Non-Insertion Theorem. Within the HR-1 closure-interface gate, a VERSF Higgs-radial field is physically admissible if and only if it is derived from stable closure-interface stiffness, is positive-sector, is invariant under permitted relabellings, is field-promoted under a declared kinetic premise, is not double counted with angular/gauge modes, and is bridged to downstream mass-generation channels. The biconditional is relative to the named premises P1–P17; it is not an unconditional claim about all possible scalar sectors. Any scalar field imported without this structure is firewalled as convention, bookkeeping, or external Standard Model inheritance.

Depends on: P1–P17.

17.2 Proof

Necessity. Suppose a Higgs-radial field is physically admissible in VERSF.

Theorem 1 requires a closure interface.

Theorem 2 requires a return potential with the graded stability conditions.

Theorem 3 requires a scalar normal/radial displacement, field-promoted under P15.

Theorem 4 excludes tangential/gauge directions from independent Higgs-radial scalar counting.

Theorem 5 identifies the normal mode as the unique Higgs-radial admissible candidate under P16.

Theorem 6 identifies the mass parameter as closure-interface Hessian stiffness.

Therefore any physical Higgs-radial field must pass through the chain:

closure interface → invariant normal displacement → return potential → positive Hessian → field promotion → non-double-counted radial scalar → mass-generation bridge .

If a scalar field is introduced without this chain, its physical status is not derived. It may be a useful borrowed Standard Model object, but it is not a VERSF-derived Higgs-radial mode.

Sufficiency. Conversely, suppose the chain is supplied, with all premises P1–P17 satisfied and all debts of §10′ recorded. Then each admissibility property excludes a specific insertion route. Closure-interface origin (Theorems 1–3) excludes free scalar insertion: the field's existence is tied to the boundary and cannot be added or removed independently of the physical sector. Invariance under isometric relabellings (P8, P17) excludes coordinate and gauge artifacts: no presentation choice can manufacture the mode. The positive Hessian and positive kinetic conditions (P4, P9) exclude fitting-parameter mimics: a fitted scalar carries no closure-anchored return cost. The tangential firewall and non-double-counting rules (P7, P13) exclude relabelled angular modes posing as new scalars. The named promotion premise (P15) excludes silent coordinate-to-field inflation. The bridge requirement (P11) excludes bookkeeping-only scalars with no downstream physical role. With every insertion route individually blocked, no arbitrary scalar insertion remains: the field is derived as the unique minimal radial stiffness mode of the closure interface, and its physical role is bridged to downstream mass-generation structure.

Therefore the scalar-sector non-insertion gate is closed at the level of Higgs-radial admissibility; the numerical scalar-potential gate remains open. ■

17.3 Corollary — The Higgs-radial field is not optional

If stable closure-interface physics is required, its radial stiffness mode is forced.

17.4 Corollary — The Standard Model form is a later bridge, not a starting assumption

VERSF may recover familiar Higgs-sector forms only after the closure-radial structure has been derived.

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18. Falsification Conditions

#	Failure	Consequence
F1	Physical support is treated as all possibility support	No closure sector
F2	No interface/projector/boundary separates physical from exterior support	No closure interface
F3	No closure-defect functional or equivalent condition is supplied	Undefined interface

#	Failure	Consequence
F4	Interface has no stability condition	No physical closure equilibrium
F5	No potential or return functional exists	No massive radial support
F6	$V'_{CI}(0) \neq 0$ while equilibrium is claimed	False equilibrium
F7	$V''_{CI}(0) < 0$ while stability is claimed	Instability failure
F8	$V''_{CI}(0) = 0$ while massive Higgs-radial support is claimed	Flat-direction failure
F9	No normal/radial direction is defined	No radial mode
F10	Tangential/gauge directions are counted as radial scalars	Scalar-count failure
F11	Angular modes are counted twice	Double-counting failure
F12	Potential depends on arbitrary coordinates or labels	Frame failure
F13	Potential depends on gauge convention	Gauge failure
F14	Radial kinetic normalisation $Z_H < 0$	Ghost failure
F15	$Z_H = 0$ but propagating scalar is claimed	Kinetic failure
F16	Radial carrier lies outside physical sector	Positivity failure
F17	Scalar field is imported without closure-interface origin	Non-insertion failure
F18	Mexican-hat potential is assumed without derivation	SM-import failure
F19	Higgs-radial mode has no bridge to mass-generation channels	Bookkeeping-only failure
F20	Exact Higgs mass is claimed from HR-1 alone	Scope violation
F21	Exact electroweak vev is claimed from HR-1 alone	Scope violation
F22	Full Yukawa hierarchy is claimed from HR-1 alone	Scope violation
F23	Gauge boson mass formulae are claimed without representation/coupling bridge	Bridge failure
F24	Passing HR-1 is treated as empirical confirmation	Truth/admissibility confusion
F25	Additional scalar particles are inferred without extra closure defects	Overcounting failure
F26	Radial construction applied at singular interface points where $\nabla\rho = 0$	Regularity failure (P14)
F27	Global displacement coordinate treated as a field without stating promotion	Field-promotion failure (P15)
F28	Z_H claimed as derived from substrate primitives when debt D1 is undischarged	Kinetic-origin failure
F29	Uniqueness claimed for a higher-codimension interface without a mode count	Minimality failure (P16)
F30	V_{CI} not twice differentiable at equilibrium while a Hessian mass is claimed	Differentiability failure (P5)

#	Failure	Consequence
F31	Normal direction or radial magnitude changes under a permitted relabelling, or no invariant metric on $\mathcal{Q}/\mathcal{V}$_C is exhibited	Metric/isometry failure (P17)
F32	HR-1 is cited downstream as deriving the potential shape rather than its necessity	Potential-shape scope violation

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19. HR-1 Closure Certificate

HR-1 condition	Result
Physical support distinguished from exterior possibility	Closed
Closure interface Σ required	Closed
Closure-defect functional Δ_C required	Closed
Interface regularity premise named with falsifier	Closed
Interface equilibrium condition supplied	Closed
Graded stability conditions separated (bounded/equilibrium/return)	Closed
Interface potential V_CI required	Closed
Invariant radial coordinate supplied	Closed
Normal/tangential decomposition supplied	Closed
Radial scalar h identified	Closed
Field-promotion step named with declared kinetic debt	Closed
Possibility-space metric premise named with falsifier	Closed
Equilibrium placed at regular defect level $d_0 > 0$	Closed
Codimension-one minimality premise named with falsifier	Closed
Tangential/gauge directions firewalled from radial scalar count	Closed
Positive kinetic normalisation required	Closed
Positive Hessian stiffness required	Closed
Canonical radial mass form derived	Closed
Scalar-sector non-insertion rule supplied	Closed
Mass-generation bridge map required	Closed
Bridge-debt ledger recorded (D1–D7)	Closed
Angular/gauge double-counting firewalled	Closed
Potential-necessity versus potential-shape distinction supplied	Closed
Derivation of Z_H from substrate primitives	Outside scope
Microscopic potential shape of V_CI	Outside scope
Exact numerical Higgs mass	Outside scope
Exact electroweak vev	Outside scope
Exact quartic coupling	Outside scope

HR-1 condition	Result
Full fermion hierarchy	Outside scope
Full electroweak fit	Outside scope
Empirical confirmation	Outside scope

HR-1 closure grade: closed as a Higgs-radial admissibility and necessity theorem.

It proves that a Higgs-radial mode is physically admissible in VERSF only when it is the positive-sector radial stiffness mode of a stable, regular, codimension-one closure interface, with the possibility-space metric and the displacement-to-field promotion named and audited, angular/gauge directions excluded from independent scalar counting, and downstream mass-generation bridges explicitly required.

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20. Conclusion

HR-1 closes the first Higgs gate in VERSF.

The paper begins from a simple problem: a first-principles derivation may not simply insert the Higgs field. It must explain why a scalar radial mode exists at all.

The answer is closure-interface stiffness.

A VERSF physical sector is not unrestricted possibility. It is committed, admissible support. Therefore it requires an interface separating physical support from exterior non-committed support.

A stable interface cannot be a mere label. It must have a return-to-closure cost. That cost is encoded in a closure-interface potential:

$V_{CI}(h)$.

The interface displacement decomposes into normal and tangential parts:

$\delta q = h\, n + \delta q_{\parallel}$.

The normal part h changes closure distance. It is scalar, radial, invariant, and physically load-bearing. The tangential part moves along the interface and may represent gauge, angular, frame, or coordinate motion. It is not independent Higgs-radial scalar support.

The radial stiffness is measured by the Hessian:

$V''_{CI}(0)$.

After positive kinetic normalisation, the radial mass parameter is:

$m_{HR}^2 = Z_H^{-1} V''_{CI}(0)$.

This is the structural meaning of the Higgs-radial mode in VERSF:

The Higgs-radial field is the canonically normalised return-stiffness excitation of the closure interface.

The paper therefore blocks the main abuse modes:

- no scalar field inserted by hand;
- no imported Mexican-hat potential;
- no arbitrary radial mode;
- no silent promotion of a displacement coordinate to a field;
- no silent import of possibility-space geometry;
- no gauge/angular double counting;
- no mass-generation claim without a bridge;
- no numerical Higgs claim before discharging the recorded debts.

HR-1 does not compute the observed Higgs mass. It does not complete the electroweak sector. It does not derive the full Yukawa hierarchy.

It proves the necessary prior result:

If VERSF has stable committed physical support, then it has a closure interface; if the interface has finite return stiffness at a regular codimension-one boundary, then it has a scalar radial mode; and that scalar radial mode, once field-promoted under a named premise, is Higgs-radial admissible.

Thus the Higgs-radial degree is not an imported Standard Model object.

It is the stiffness of physical closure itself.

Appendix A — Minimal Algebraic Skeleton

Exterior possibility space:

$\mathcal{Q}$.

Physical-sector projector:

$\mathcal{P}_{phys}$.

Invariant metric structure (P17):

$\langle\cdot,\cdot\rangle$ on $\mathcal{Q}$, $\|\cdot\|$ on $\mathcal{V}$_C ; permitted relabellings act by isometries .

Physical support:

$\mathcal{S}$_phys = { q ∈ $\mathcal{Q}$: $\mathcal{P}$_phys q = q } .

Closure defect:

Δ_C : $\mathcal{Q}$ → $\mathcal{V}$_C .

Invariant closure distance:

d_C(q) = ‖Δ_C(q)‖ .

Shifted radial closure coordinate:

ρ(q) = d_C(q) − d_0 , with d_0 > 0 (P3) .

Closure interface:

Σ = { q ∈ $\mathcal{Q}$: ρ(q) = 0, $\mathcal{P}$_phys q = q } .

Regularity condition (P14):

∇ρ ≠ 0 on a neighbourhood of the regular locus of Σ .

Closure normal:

n = ∇ρ / ‖∇ρ‖ .

Perturbation decomposition:

δq = h n + δq_∥ .

Tangential condition:

⟨n, δq_∥⟩ = 0 .

Closure-interface potential:

V_CI(h) , twice differentiable at h = 0 .

Equilibrium:

$V'_{CI}(0) = 0$.

Radial stiffness:

$K_C = V''_{CI}(0)$.

Stability:

$K_C > 0$.

Field promotion (P15):

$h \mapsto h(x)$, kinetic carrier $\frac{1}{2} Z_H (\partial h)^2$, origin of Z_H = debt D1 .

Field-promoted Lagrangian:

$\mathscr{L}_{HR} = \frac{1}{2} Z_H (\partial h)^2 - V_{CI}(h)$.

Positive kinetic condition:

$Z_H > 0$.

Canonical radial field:

$h_c = Z_H^{1/2} h$.

Radial mass parameter:

$m_{HR}^2 = Z_H^{-1} V''_{CI}(0)$.

Mass-generation bridge:

$B_H : h_c \to O_{mass}$.

Higgs-radial admissibility:

$h_c \in \mathcal{P}_{phys}$, invariant, radial, stable, field-promoted, non-double-counted, bridged.

Appendix B — Worked Toy Interface

This appendix gives a minimal example. It is not a full electroweak model. It only illustrates the HR-1 audit, including the regularity and field-promotion checkpoints.

Let the exterior possibility space be two-dimensional:

$q = (x, y)$.

Let the closure interface be the circle:

$x^2 + y^2 = v\_C^2$.

Define the closure-radial coordinate:

$r = \sqrt{x^2 + y^2}$,

and

$h = r - v\_C$.

The closure interface is:

$\Sigma = \{ (x, y) : h = 0 \}$.

Regularity check (P14). $\nabla r = (x/r, y/r)$ is well defined and nonzero everywhere except the origin $r = 0$. All points of Σ (which lies at $r = v\_C > 0$) are regular. The singular point $r = 0$ lies off the interface and the construction is not applied there. This also exemplifies the P3 requirement $d_0 > 0$: the equilibrium level sits away from the non-smooth zero of the radial magnitude.

Minimality check (P16). Σ is cut out by the single condition $h = 0$ in a two-dimensional possibility space. It is codimension one, with exactly one normal direction at each point.

A perturbation decomposes into:

- radial displacement h;
- angular displacement θ.

The potential may be:

$V\_CI(h) = \frac{1}{2} K\_C h^2 + O(h^3)$,

with $K\_C > 0$.

The radial displacement h changes the distance from closure equilibrium. It carries positive return cost.

The angular coordinate θ moves along the interface at fixed r. It does not change closure distance.

Field-promotion check (P15). To treat h as a field, the toy must allow the displacement to vary with position, $h \to h(x)$, and supply a kinetic term $\frac{1}{2} Z_H (\partial h)^2$. In this toy the kinetic term is posited, exactly as P15 requires it to be declared: nothing in the circle geometry alone produces $(\partial h)^2$. This is the point of naming the premise.

Therefore:

- h is the radial stiffness mode;
- θ is not the radial stiffness mode;
- θ may represent gauge/angular support in a later bridge, but it cannot be counted as an additional Higgs-radial scalar.

The canonically normalised radial mass is:

$$m_{HR}^2 = Z_H^{-1} K_C .$$

This toy model captures the HR-1 logic:

radial displacement changes closure; angular motion changes orientation. Only the radial displacement is Higgs-radial support — and only after the promotion step is declared.

Appendix C — Higgs-Radial Audit Table

Object	Meaning	Admissible role	Failure if confused
$\mathcal{Q}$	Exterior possibility space	Candidate support	Everything wrongly physical
$\mathcal{P}_{phys}$	Physical-sector projector	Admissibility filter	No physical boundary
Δ_C	Closure defect	Measures closure failure	Undefined interface
d_C	Closure distance	Invariant radial input	Coordinate artifact
ρ	Shifted closure coordinate	Defines equilibrium	No radial zero
Σ	Closure interface	Physical/exterior boundary	No interface
$\nabla\rho \neq 0$	Regularity condition	Licenses normal direction	Construction at singularity
n	Closure-normal direction	Defines radial displacement	Arbitrary axis
h	Normal/radial displacement	Higgs-radial candidate	Raw scalar insertion
h(x)	Field-promoted carrier	Propagating radial mode	Silent coordinate-to-field jump
$\delta q_\parallel$	Tangential displacement	Gauge/frame/angular support	Scalar overcount
V_{CI}	Interface potential	Return-to-closure cost	Imported potential
V''_{CI}	Hessian stiffness	Radial mass source	No mass derivation

Object	Meaning	Admissible role	Failure if confused
Z_H	Kinetic normalisation	Canonical field scaling	Ghost or nonpropagating mode
h_c	Canonical radial field	Physical radial excitation	Misnormalised scalar
B_H	Mass bridge	Downstream physical role	Bookkeeping only
D1–D7	Bridge debts	Recorded downstream duties	Debts treated as discharged

Appendix D — Referee Objections and Replies

Objection 1 — Is this just renaming the Higgs field?

Reply. No. HR-1 does not begin with a scalar field. It begins with the need for a physical-sector boundary. From that boundary it derives a closure-defect coordinate, a normal direction, a radial displacement, a potential, and a stiffness Hessian. The scalar appears as the normal displacement of the closure interface, not as an assumed field, and the promotion of that displacement to a field is a named, falsifiable premise rather than a silent identification.

Objection 2 — Does HR-1 derive the Standard Model Higgs boson?

Reply. It derives Higgs-radial admissibility, not the full numerical Standard Model Higgs sector. The observed Higgs boson requires later gates deriving representation, vev normalisation, coupling structure, and numerical mass — recorded here as debts D1–D7.

Objection 3 — A displacement coordinate is not a field. Hasn't HR-1 conflated the two?

Reply. This is the strongest available objection, and HR-1 answers it by structure rather than by rhetoric. Theorem 3 separates the two claims explicitly: the coordinate-level result (an invariant radial displacement exists at every regular interface point) is derived unconditionally; the field-level result (a locally varying, propagating carrier) is conditional on the field-promotion premise P15, which carries its own falsifier and whose kinetic origin is logged as debt D1. The conflation the objection fears is exactly what F27 falsifies.

Objection 3′ — Where does the geometry of possibility space come from?

Reply. The gradient, the normal direction, the orthogonal decomposition, and the defect magnitude all presuppose invariant metric structure on $\mathcal{Q}$ and $\mathcal{V}$_C. HR-1 does not derive that structure; it names it as premise P17, requires permitted relabellings to act by isometries, and supplies falsifier F31. The origin of the metric is charged to the same substrate-dynamics debt as the kinetic term (D1): the transport structure that owes the programme ½ Z_H $(\partial h)^2$ owes it the inner product as well, and a single downstream gate should discharge both.

Objection 4 — Why must the interface have a potential?

Reply. Because a stable massive radial mode requires return-to-equilibrium cost. Section 5.2 grades this into boundedness, equilibrium, and quadratic return, and Theorem 2 shows the three cases exhaust the local behaviour. Without a return functional the interface is unstable, flat, or physically unaudited. A massive Higgs-radial claim cannot be made without positive quadratic return.

Objection 5 — Why is the radial mode unique?

Reply. Uniqueness is conditional on the codimension-one minimality premise P16: an interface cut out by a single invariant condition has exactly one normal direction at each regular point. HR-1 states the condition, supplies its falsifier (F29), and Corollary 15.5 records that a higher-codimension interface would force a full mode re-count. The uniqueness claim is therefore falsifiable rather than asserted.

Objection 6 — What about singular interface points?

Reply. The construction is applied only at regular points, where $\nabla\rho \neq 0$ (P14). At cusps, self-intersections, or degenerate loci the normal direction is undefined and HR-1 makes no claim; applying the construction there fires falsifier F26. Whether the physical interface possesses singular loci, and what happens at them, is a legitimate later question outside this gate.

Objection 7 — What about angular or Goldstone-like modes?

Reply. They may be important, but they are not radial stiffness modes. They move along the interface rather than away from it. A later electroweak bridge must state whether they are gauge-removed, eaten, physical, or projected out — debt D5. HR-1 forbids counting them as additional Higgs-radial scalars.

Objection 8 — Is the Mexican-hat potential derived?

Reply. Not in numerical Standard Model form. HR-1 derives the need for an invariant closure-interface potential with stable nonzero equilibrium if a nonzero closure radius is present. A Mexican-hat-like expression may appear as a later minimal analytic representation, but it may not be imported as an assumption; the theorem chain requires only twice-differentiability and the graded stability conditions, not a preselected polynomial.

Objection 9 — Does HR-1 prove mass generation?

Reply. It proves the radial stiffness source needed for mass-generation bridges. It does not by itself derive all mass terms. Later papers must provide explicit bridge maps from the radial mode to fermion, gauge, or spectral mass channels, satisfying the five-point non-rhetorical bridge condition of §10.2.

Objection 10 — Why is the Hessian identified with mass?

Reply. For a canonically normalised scalar excitation around a stable equilibrium, the quadratic coefficient of the potential is the mass parameter. HR-1 expresses that coefficient as the closure-interface Hessian. The physical mass value requires deriving the Hessian and the kinetic normalisation — debts D1–D3.

Objection 11 — Could the radial mode be a gauge artifact?

Reply. Only if the alleged radial coordinate changes under permitted gauge/frame relabellings. HR-1 requires the radial mode to be built from invariant closure-distance data (P8). Gauge/angular directions are explicitly separated and excluded from the radial scalar count.

Objection 12 — Isn't Z_H doing hidden work? The mass formula is empty until Z_H is known.

Reply. The formula $m_{HR}^2 = Z_H^{-1} V''_{CI}(0)$ is an admissibility structure, and the paper says so in the Abstract, in Theorem 6, and in the closure certificate. The origin of Z_H is not hidden: it is debt D1, with its own falsifier (F28) against any downstream paper that pretends the debt is discharged. A structure paper that names its undischarged quantities is doing the opposite of hiding them.

Objection 13 — What is the one-sentence result?

Reply.

The Higgs-radial mode is the positive-sector normal stiffness excitation of the VERSF closure interface; it is forced by stable physical closure at a regular codimension-one boundary, its promotion to a field is a named audited premise, and it cannot be replaced by an imported scalar field, arbitrary potential, or gauge/angular double count.

Appendix E — Final One-Sentence Theorem

Within the HR-1 closure-interface gate, a VERSF Higgs-radial degree of freedom is physically admissible if and only if it is the invariant, positive-sector, canonically normalised normal displacement — taken relative to a declared invariant possibility-space metric — of a stable, regular, codimension-one closure interface, promoted to a locally varying carrier under a declared kinetic premise, with its potential supplied by return-to-closure stiffness, its mass parameter supplied by the physical Hessian of that potential, its angular/gauge directions excluded from independent scalar counting, and its downstream mass role supplied through an explicit bridge map; therefore the Higgs-radial field is not inserted by hand but is forced by the existence of stable physical closure.