The Closure-Operator Yukawa Construction Theorem in VERSF

▲ Programme Milestone — Fermion-Magnitude and Yukawa-Completion Series Gate CMSY-1 / Substrate-Hessian Inheritance, Scalar-Readout Closure Operator, Canonical Chiral-Carrier Embeddings, Radial-Derivative Yukawa Definition, Spectral-Isotropy Theorem, Leading Closure-Functional Uniqueness, Response-Form Insufficiency and Embedding Gauge, Declared-Order Effective Channel Carrier, Closure-Projector Texture-Zero Taxonomy, Rank and Hierarchy-Transfer Theorems, Generalised-CP Phase Audit, Post-Breaking Four-Sector Assembly, Relative-Frame Mixing, and Numerical Non-Insertion Audit

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General Reader Summary

Every particle of matter has a mass, and the pattern of those masses is strange. The top quark weighs roughly as much as an entire gold atom; the electron is about a third of a million times lighter; the three "generations" of matter repeat the same charges at wildly different weights; and quarks blend into one another in a small, oddly skewed pattern when they decay. In the Standard Model, all of this is controlled by a set of numbers called Yukawa couplings — and the theory explains none of them. They are measured and typed in by hand. This is one of the deepest unsolved puzzles in particle physics.

This paper proposes where those numbers come from, using nothing VERSF does not already possess.

The picture in plain terms. Think of the deep closure structure of VERSF — the layer beneath space, time and fields — as a stiff elastic medium. Push on it and it pushes back, and like any elastic body it is stiffer in some directions than in others. Mathematically, that whole pattern of resistance is captured by a single object, the closure Hessian: a table recording how strongly the medium resists every possible small deformation. VERSF already requires this object to exist. Nothing new is invented here.

The claim of this paper is that the fermion masses are readings of that stiffness table.

The chain has three links. First, only some of the medium's stiffness directions are of the right kind to connect a left-handed particle to a right-handed one — and that connection is what "having a mass" means for a fermion. So the table is projected down to just that mass-readout channel. Second, each kind of particle — up quark, down quark, electron, neutrino — plugs into the medium through its own socket, a connection fixed by earlier papers in the programme, not chosen by hand. Third, the Yukawa numbers for each particle are then simply the stiffness of the medium as felt through that particle's socket:

mass pattern = (particle's socket) × (stiffness of the medium) × (particle's socket).

That is the entire construction. Once the closure Hessian, the scalar-readout projector, the carrier embeddings and the completion–Hessian matching are independently calculated and frozen, no phenomenological dial remains to be turned.

What the paper actually proves. A proposal like this is only worth something if it cannot be quietly tuned to fit the known answer. The paper identifies every place such tuning could hide and closes each one with a theorem:

No adjustable strengths. In looser constructions of this kind, each contribution carries its own free weight, and the observed masses can simply be hidden inside the weights. Here a theorem forces the weights to be the stiffness values themselves — the eigenvalues of the closure Hessian. No free weight survives.

No hidden shape for the response. One might worry the medium could respond in some complicated, adjustable way rather than plainly. A theorem shows that if the response respects every symmetry of the stiffness table, it must be a plain function of that table; and at the leading level of approximation the function must be the simplest one possible — the identity. The medium answers with its stiffness, nothing more.

No cheating through the sockets. Another theorem makes an uncomfortable fact explicit: if the sockets were allowed to be arbitrary, then any mass pattern whatsoever could be dressed up in this language, and the construction would say nothing at all. The sockets must therefore arrive fixed and finished from the earlier occupancy programme, and the paper builds a fence — with named failure conditions — around every route by which freedom could creep back into them.

Where the mass hierarchy comes from. If the medium's stiffness values are themselves spread out — one large, one smaller, one smaller still — a theorem guarantees the particle masses inherit that spread: the second mass cannot exceed the second stiffness value, the third cannot exceed the third, and the heavy ones are protected from below. The famous fermion hierarchy would then be a portrait of the closure spectrum. Whether the spectrum really is spread out that way is a question about the medium itself, and the paper says plainly that this remains to be calculated.

Where zeros and complex phases come from. An entry of a mass matrix is exactly zero when no stiffness direction connects the two sockets involved — and the paper carefully separates genuine, physical zeros from mere bookkeeping zeros. Likewise, the stiffness values are plain positive numbers, so any complex phase — the seed of the matter–antimatter asymmetry — must enter through a short, closed list of doors, and a theorem shows that changing bookkeeping conventions can neither create nor destroy genuine CP violation.

What the paper does not claim. No number is computed here. The stiffness table, the mass-readout channel and the particle sockets must still be calculated from VERSF's deeper dynamics, and each of those calculations is recorded as an explicit, named debt. What the paper establishes is the exact, tamper-proof form the answer must take — so that when those three calculations are done, the fermion masses either come out right or the framework is falsified. There is no third option, and that is the point.

In one sentence:

The masses and mixings of the fundamental fermions are proposed to be readings of one inherited object — the stiffness of the closure structure, felt through each particle's independently fixed connection to it — with theorems closing every channel through which the observed answer could be smuggled in, and an explicit ledger of the calculations — and the single completion–Hessian matching derivation — that remain.

Scope and Closure Box

Claim	CMSY-1 status
Yukawa matrices may be inserted as arbitrary fundamental data	Rejected as a VERSF derivation
Unspecified mode coefficients γ_a^(f) are an acceptable resting point	Rejected — eliminated by the construction
A new independent Yukawa mediator is required at declared order	Rejected conditionally under CY1, CY3′ and CU — the microscopic completion–Hessian matching remains open (D21)
𝒞_Y = P_Y Ĥ_cl P_Y is Hermitian and positive	Closed
Commutation [𝒢, 𝒞_Y] = 0 suffices for functional-calculus form	Disproved — fails inside degenerate eigenspaces
The resolved response may carry a sector index at leading closure-side order	Excluded under premise FR — sector data enter only through the embeddings; microscopic closure-side factorisation remains to be verified (D8)
Commutant invariance forces 𝒢 = F(𝒞_Y)	Closed — Spectral-Isotropy Theorem
The leading closure functional is F_Yuk(x) = x	Closed under FR, SI, CY1–CY5, CY2A and CY3′/CU
Y_f^c = ℰ_fL† 𝒞_Y ℰ_fR at leading linear-sector order	Closed conditionally
Mode coefficients equal closure eigenvalues, γ = λ	Closed

Claim	CMSY-1 status
$F(\lambda)$ values are reconstructible or observable from Y_f^c alone	Disproved — projected channels may vanish or become dependent
The factorised form $Y = \mathscr{E}_L^\dagger \mathcal{G} \mathscr{E}_R$ constrains Y by itself	Disproved — Response-Form Insufficiency Theorem
Individual embedding entries are physical	Disproved — embedding gauge under the commutant of $\mathcal{C}_Y$
A function of the one-mode generator captures composite channels	Disproved — composites live on $\mathscr{H}_{eff}^{(N)}$
The composite extension is typed on a declared-order effective carrier	Closed as typing; its derivation open
Degenerate closure levels require arbitrary basis choices	Disproved — spectral projectors suffice
Texture zeros follow from projector-level pairing conditions on the canonical operator	Closed
Coordinate zeros are automatically physical zeros	Disproved — zero taxonomy required
The preservation condition $[P, K] = 0$ is necessary as well as sufficient	Disproved — sufficient only
$\sigma_{k+1}(Y_f^c) \le \|\mathscr{E}_{fL}\|_2 \|\mathscr{E}_{fR}\|_2 \lambda_{k+1}$, contraction case $\sigma_{k+1} \le \lambda_{k+1}$	Closed — as hierarchy transfer
The closure spectrum is dynamically hierarchical	Open — the transfer theorem does not decide it
A large closure eigenvalue guarantees a fermion mass	Disproved — surviving projections required
Kinetic metrics inject physical CP violation	Disproved — Canonical CP-Equivalence Theorem
Kinetic metrics belong in the joint generalised-CP audit	Closed as requirement
Four-edge plaquettes generate all rephasing invariants	Disproved — general even-cycle invariants required
Quark CP violation is a relative two-sector property	Closed under stated three-generation premises
The pre-breaking Yukawa interaction is a carrier direct sum	Disproved — direct sum holds post-breaking
$\mathbf{M}_{Dirac} = M_u \oplus M_d \oplus M_e \oplus M_\nu$ post-breaking	Closed
A common closure spectrum forces universal sector matrices	Disproved — embeddings differ

Claim	CMSY-1 status
Y_ν exists without an independently admitted N_R carrier	Disproved
Majorana blocks are part of Y_ν^c, or leave the Dirac massless count final	Disproved on both counts
"Full" means leading linear-sector single-insertion renormalisable order	Closed as scope convention
$c_1 = 1$ follows from CY1–CY3 alone	Disproved — requires the CY3′ matching identity
Canonical units render $\hat{H}_{cl}$ dimensionless (CU, QN-series inheritance)	Closed conditionally; otherwise $\mathcal{N}_Y$ is debt D19
Numerical calculation of H_{cl}, P_Y, $\mathcal{E}_{fL/R}$	Open — load-bearing
Higher-order coefficients c_n and the composite generator $\mathcal{A}_{eff}^{(N)}$	Open
Right-handed neutral carrier multiplicity	Open unless separately inherited
Absolute fermion masses	Open pending H_{cl}, P_Y, $\mathcal{E}_{fL/R}$, v_{cl}, running
Empirical confirmation	Not established

Abstract

The Standard Model Yukawa matrices encode the complete observed pattern of charged-fermion masses, quark mixing, charged-lepton magnitudes and any Dirac contribution to neutrino mass, yet their entries are conventionally treated as unexplained dimensionless inputs. A VERSF derivation must construct them from objects the framework already possesses, and must close every channel through which the answer could be smuggled back in: unspecified coefficients, an assumed functional form, free embedding matrices, silently omitted composite channels, or phase bookkeeping that mistakes basis artefacts for physics.

This gate establishes the Closure-Operator Yukawa Construction Theorem with those channels closed by named theorems.

Let G_{sub} be the substrate free-energy Hessian at a stable admissible configuration (C0), $H_{cl} = P_{cl} G_{sub} P_{cl}$ its closure restriction, $\hat{H}_{cl} = K_{cl}^{-1/2} H_{cl} K_{cl}^{-1/2} \geq 0$ the canonical closure Hessian, and P_Y the independently derived scalar mass-readout projector. The scalar-readout closure operator is

$$\mathcal{C}_Y = P_Y \hat{H}_{cl} P_Y = \Sigma_\alpha \lambda_\alpha \Pi_\alpha, \quad \lambda_\alpha \geq 0.$$

Let $\mathscr{E}_{fL} = K_{cl}^{1/2}\, \iota_{fL}\, K_{fL}^{-1/2}$ and $\mathscr{E}_{fR} = K_{cl}^{1/2}\, \iota_{fR}\, K_{fR}^{-1/2}$ be the canonical chiral-carrier embeddings, mapping fermion carriers into the closure space — derived partial isometries or contractions inherited from the occupancy map, never free matrices.

The functional form is derived, not assumed. Let $\mathcal{G}$ be the resolved closure response operator on the readout channel, prior to any structural assumption — sector-index-free by the **closure-side factorisation premise FR**, which requires the response to be resolved prior to and independently of the fermion compression, with sector data entering only through the embeddings; sector-dependent dressing of the response is typed as a higher-order correction, never absorbed into $\mathcal{G}$. The **Spectral-Isotropy Theorem** proves: if $\mathcal{G}$ is invariant under every unitary preserving $\mathcal{C}_Y$, then $\Pi_\alpha\, \mathcal{G}\, \Pi_\beta = 0$ for $\alpha \neq \beta$ and $\Pi_\alpha\, \mathcal{G}\, \Pi_\alpha = F(\lambda_\alpha)\, \Pi_\alpha$ on every eigenspace — degenerate or not — so $\mathcal{G} = F(\mathcal{C}_Y)$ by unique functional calculus. Mere commutation $[\mathcal{G}, \mathcal{C}_Y] = 0$ is proved insufficient under degeneracy. The **Leading Closure-Functional Uniqueness Theorem** then collapses the function to the identity, $F_{Yuk}(x) = x$: the constant term is removed by the unique-mediator premise, the higher insertions by the single-insertion order with repeated insertions composing by ordinary operator multiplication under the insertion-algebra premise CY2A, and the unit coefficient $c_1 = 1$ is fixed as a **matching identity** — the completion operator is identified with the φ-derivative of the fermion-dressed closure free energy in the same canonical units as $\hat{H}_{cl}$ (premise CY3′), with those units inherited from the quantitative-normalisation programme (premise CU). Dimensional consistency of the central identity is thereby made explicit: absent the CU inheritance, the vertex normalisation $\mathcal{N}_Y$ is a fourth calculable object, ledgered as debt D19, and the matching identity itself is the central physical bridge, ledgered as debt D21. Hence, at leading order on the linear primitive-channel sector,

$$Y_f^c = \mathscr{E}_{fL}^\dagger\, \mathcal{C}_Y\, \mathscr{E}_{fR} = \Sigma_\alpha\, \lambda_\alpha\, \mathscr{E}_{fL}^\dagger\, \Pi_\alpha\, \mathscr{E}_{fR},$$

with mode coefficients $\gamma_\alpha^{(f)} = \lambda_\alpha$, the closure eigenvalues themselves.

The factorisation alone predicts nothing: the **Response-Form Insufficiency Theorem** proves by explicit singular-value construction that every matrix admits a factorisation $\mathscr{E}_L^\dagger\, \mathcal{G}\, \mathscr{E}_R$ with positive $\mathcal{G}$ and contractive maps. Predictive content therefore resides entirely in the independent derivation of $\mathcal{C}_Y$ and the embeddings. An **embedding gauge** accompanies this: for any unitary U commuting with $\mathcal{C}_Y$, the replacement $\mathscr{E}_{fL} \to U\mathscr{E}_{fL}$, $\mathscr{E}_{fR} \to U\mathscr{E}_{fR}$ leaves Y_f^c invariant, so individual embedding components are defined only modulo the commutant unless fixed upstream. Correspondingly, the values $F(\lambda)$ are uniquely specified by the derivation but need not be reconstructible from Y_f^c, since projected channels $\mathscr{E}_{fL}^\dagger\, \Pi_\alpha\, \mathscr{E}_{fR}$ may vanish or become linearly dependent.

Composite channels — mode products, gradients, interface dressings — live on the declared-order effective carrier $\mathscr{H}_{eff}^{(N)} = \mathscr{H}_{cl} \oplus \mathscr{H}_{cl}^{\otimes 2} \oplus \cdots \oplus \mathscr{H}_\nabla \oplus \mathscr{H}_{\Phi\text{-dress}}$ with its own generator $\mathcal{A}_{eff}^{(N)}$; a scalar function of the one-mode operator does not capture them. The leading theorem is accordingly restricted to the linear primitive-channel sector, with the extension $Y_f^c = \mathscr{E}_{fL}^\dagger\, F_f(\mathcal{A}_{eff}^{(N)})\, \mathscr{E}_{fR}$ typed and ledgered.

Exact structural theorems follow on the linear sector. **Texture zeros:** $(Y_f^c)_{ij} = 0$ *whenever every projector-level pairing vanishes — with a three-way taxonomy separating coordinate* *zeros, canonical-frame-preserved zeros and physical block zeros between independently fixed invariant subspaces; the preservation condition [P, K] = 0 is sufficient, not necessary.* ***Rank:*** *$\mathrm{rank}(Y_f^c) \leq \min\{\dim \mathcal{S}_f, \mathrm{rank}\, \mathcal{C}_Y, \mathrm{rank}\, \mathcal{E}_{fL}, \mathrm{rank}\, \mathcal{E}_{fR}\}$, counting only closure directions surviving both chiral projections.* ***Hierarchy transfer:*** $\sigma_{k+1}(Y_f^c) \leq \|\mathcal{E}_{fL}\|_2 \|\mathcal{E}_{fR}\|_2 \lambda_{k+1}$, reducing to λ_{k+1} for contractive embeddings, with Weyl protection of the leading magnitudes; whether the closure spectrum is hierarchical remains a separately ledgered dynamical question. **Phases:** every λ_α is real and nonnegative, so complex structure enters only through projectors, embeddings, contractions or transported kinetic normalisation; the **Canonical CP-Equivalence Theorem** proves that canonical normalisation preserves the existence or nonexistence of a generalised CP symmetry — metrics transport phase structure but cannot create a physical CP obstruction — and quark CP violation is the relative invariant $\mathrm{Im\,Tr}([H_{uL}, H_{dL}]^3)$ under named three-generation premises. Rephasing invariants are generated by general even support cycles, of which four-plaquettes are the smallest case.

Assembly holds post-breaking: $\mathbf{M}_{\mathrm{Dirac}} = M_u \oplus M_d \oplus M_e \oplus M_\nu$ with $M_f = (v_{cl}/\sqrt{2})\cdot Y_f^c$, the neutrino block conditional on an independently established N_R, Majorana completion separately typed, and the Dirac-level massless count subject to revision by subsequent Majorana blocks.

CMSY-1 closes the leading operator-level derivation of the Dirac Yukawa matrices as sector-specific compressions of the canonically normalised scalar-readout closure Hessian, with the functional form derived by spectral isotropy and the coefficient, embedding and phase freedoms eliminated or gauged by theorem. It does not yet calculate the numerical matrices: the closure Hessian, the readout projector, the embeddings, the composite generator and the spectrum's dynamical shape remain explicit bridge debts. The Yukawa problem is reduced to three concrete calculations — H_{cl}, P_Y, $\mathcal{E}_{fL/R}$ — under the CU unit inheritance; absent it, the vertex normalisation $\mathcal{N}_Y$ is the fourth.

Notation

The Standard Model fermion sectors are indexed by $f \in \{u, d, e, \nu\}$. Their canonically normalised left- and right-handed carrier spaces are $\mathcal{H}_{fL}^c$, $\mathcal{H}_{fR}^c$; the raw carriers are $\mathcal{H}_{fL}$, $\mathcal{H}_{fR}$ with positive chiral kinetic metrics $K_{fL}, K_{fR} > 0$. For the neutrino sector, $\mathcal{H}_{\nu R} \equiv \mathcal{H}_{NR}$ exists only if a right-handed neutral carrier has been independently admitted.

The substrate free-energy functional is $\boldsymbol{\mathcal{F}}_{sub}$, with admissible closure configuration q_0 and substrate Hessian

$$G_{sub} = \delta^2\boldsymbol{\mathcal{F}}_{sub} / \delta q\, \delta q\, |_{q_0}.$$

P_{cl} projects onto the retained closure carrier $\mathcal{H}_{cl}$; the closure Hessian is $H_{cl} = P_{cl}\, G_{sub}\, P_{cl}$. The closure kinetic metric is $K_{cl} = K_{cl}^\dagger > 0$, and the canonical closure Hessian is

$$\hat{H}_{cl} = K_{cl}^{-1/2}\, H_{cl}\, K_{cl}^{-1/2},$$

with $\hat{H}\_{cl} \geq 0$ at a stable admissible configuration.

P_Y is the independently derived projector onto the scalar closure channel capable of contributing to a left–right fermion completion. It is an **orthogonal canonical projector**,

$$P_Y^2 = P_Y, \quad P_Y^\dagger = P_Y,$$

on the canonical closure carrier: compression by P_Y is orthogonal, never oblique, which is what the positivity proof of Theorem 1 uses. The **scalar-readout closure operator** is

$$\mathcal{C}_Y = P_Y \hat{H}_{cl} P_Y, \quad \mathcal{C}_Y^\dagger = \mathcal{C}_Y \geq 0, \quad \mathcal{C}_Y = \Sigma_\alpha \lambda_\alpha \Pi_\alpha,$$

with distinct eigenvalues $\lambda_\alpha \geq 0$ and orthogonal spectral projectors Π_α.

The **resolved closure response operator** — prior to any functional-form assumption — is

$$\mathcal{G} : \text{im } P_Y \to \text{im } P_Y,$$

extended by zero on the P_Y-complement wherever it appears on $\mathscr{H}_{cl}^c$. It carries no sector index: by the closure-side factorisation premise FR (Section 8.3), the response is resolved on the closure side prior to and independently of the fermion compression, and sector data enter the construction only through the embeddings. The Spectral-Isotropy Theorem (Section 8) reduces $\mathcal{G}$ to a scalar function of $\mathcal{C}_Y$ under a stated invariance premise; the Leading Closure-Functional Uniqueness Theorem then reduces that function to the identity.

The raw world-to-closure embeddings inherited from the occupancy and chiral-carrier maps are $\iota_{fL} : \mathscr{H}_{fL} \to \mathscr{H}_{cl}$ and $\iota_{fR} : \mathscr{H}_{fR} \to \mathscr{H}_{cl}$. The **canonical chiral-carrier embeddings** are

$$\mathscr{E}_{fL} = K_{cl}^{1/2} \iota_{fL} K_{fL}^{-1/2} : \mathscr{H}_{fL}^c \to \mathscr{H}_{cl}^c, \quad \mathscr{E}_{fR} = K_{cl}^{1/2} \iota_{fR} K_{fR}^{-1/2} : \mathscr{H}_{fR}^c \to \mathscr{H}_{cl}^c.$$

Both embeddings map fermion carriers **into** the closure space; the Yukawa operator is the compression $\mathscr{E}_{fL}^\dagger \mathcal{C}_Y \mathscr{E}_{fR} : \mathscr{H}_{fR}^c \to \mathscr{H}_{fL}^c$.

The **declared-order effective channel carrier**, hosting composite channels, is

$$\mathscr{H}_{eff}^{(N)} = \mathscr{H}_{cl} \oplus \mathscr{H}_{cl}^{\otimes 2} \oplus \cdots \oplus \mathscr{H}_{cl}^{\otimes N} \oplus \mathscr{H}_\nabla \oplus \mathscr{H}_{\Phi\text{-dress}},$$

with separately derived effective generator $\mathcal{A}_{eff}^{(N)}$. The leading theorem of this gate operates on the linear primitive-channel sector $\mathscr{H}_{cl}$; the composite extension is typed in Section 15.

The closure-interface radial amplitude is φ, canonically normalised, with vacuum value $\varphi_0 = v_{cl}/\sqrt{2}$. The scalar vacuum direction Φ_0 is unit-normalised; the dimensionful magnitude resides solely in v_{cl}.

The left–right quadratic completion operator in radial background φ is $\mathcal{M}_f(\varphi) : \mathcal{H}_{fR}^c \to \mathcal{H}_{fL}^c$. The canonical Yukawa operator is defined exactly by

$$Y_f^c = \partial\mathcal{M}_f(\varphi)/\partial\varphi \,|_{\{\varphi = \varphi_0\}},$$

and the central construction of this gate is

$$Y_f^c = \mathcal{E}_{fL}^\dagger\, \mathcal{C}_Y\, \mathcal{E}_{fR} = \Sigma_\alpha\, \lambda_\alpha\, \mathcal{E}_{fL}^\dagger\, \Pi_\alpha\, \mathcal{E}_{fR}.$$

$H_{fL} = Y_f^c Y_f^{c\dagger}$, $H_{fR} = Y_f^{c\dagger} Y_f^c$; $M_f = (v_{cl}/\sqrt{2})\cdot Y_f^c$; singular values $y_{\{f,i\}} \geq 0$ decreasing; $m_{\{f,i\}} = (v_{cl}/\sqrt{2})\cdot y_{\{f,i\}}$.

Hierarchy truncations are $Y_f^{(k)}$ with **tail remainder $T_f^{(k)} = Y_f^c - Y_f^{(k)}$** (the symbol R is reserved for representations and right-embedding contexts).

Operator norms are spectral: $\|\cdot\|_2$ is the largest singular value; $\sigma_k(\cdot)$ the k-th largest. G_{exact} denotes the complete exact symmetry structure: the local Standard Model gauge group together with every exact global VERSF charge, exact discrete closure symmetry and orientation grading; local and global factors are distinguished wherever the distinction is load-bearing.

Declared-order convention (binding). "Full" and "complete" mean: complete at leading local, single-radial-insertion, renormalisable closure-Hessian order on the linear primitive-channel sector, within the declared carrier content and matching scheme. Composite channels, repeated insertions and nonlinear closure corrections are typed separately (Section 15).

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Part II — The Closure-Operator Yukawa Construction

Part I — Substrate Hessian, Closure Operator and Carrier Embeddings

1. Aim, Inheritance and Novelty

1.1 The unexplained Yukawa problem

The Standard Model permits the gauge-invariant interactions

$$-\bar{Q}_L Y_u \tilde{\Phi} u_R,\ -\bar{Q}_L Y_d \Phi d_R,\ -\bar{L}_L Y_e \Phi e_R,\ -\bar{L}_L Y_\nu \tilde{\Phi} N_R,$$

the last present only if N_R exists. Gauge symmetry fixes the operator types but not the ranks, hierarchies, relative frames, phases, zeros, cross-sector relations or strengths. These are the objects VERSF must derive.

1.2 Five smuggling channels, all closed

A derivation of the Yukawa matrices fails whenever the observed answer can be smuggled into the construction. Five channels exist, and this gate closes each with a named result:

1. **Coefficient smuggling** — free mode weights $\gamma_a^{(f)}$. Closed: the weights are the closure eigenvalues (Theorem 8, Section 9).
2. **Functional-form smuggling** — assuming the response is a function of the generator. Closed: the Spectral-Isotropy Theorem derives the functional form from commutant invariance, and proves mere commutation insufficient (Theorem 6).
3. **Embedding smuggling** — free embedding matrices. Closed: the Response-Form Insufficiency Theorem proves the factorised form alone is vacuous (Theorem 5), and the embedding-gauge theorem removes physical status from individual entries (Theorem 2); predictivity requires derived partial isometries (Section 4.4; falsifier F44).
4. **Channel smuggling** — silently identifying the one-mode operator with the complete effective interaction. Closed: composites are typed on $\mathscr{H}_{eff}^{(N)}$ and the leading theorem is restricted to the linear sector (Section 15).
5. **Phase smuggling** — inserting phases or misattributing them to basis artefacts. Closed: the four-entry provenance ledger, the general even-cycle invariants, and the Canonical CP-Equivalence Theorem (Sections 16–17).

1.3 What is genuinely established

The mediator is identified: $\mathcal{C}_Y = P_Y K_{cl}^{-1/2} H_{cl} K_{cl}^{-1/2} P_Y$, a derived projection of the inherited substrate Hessian — the operator previously denoted $\hat{H}_c$ in the closure-depth overlap formulation. Nothing new is postulated. The linearity of the completion in this operator is proved — the Spectral-Isotropy Theorem for the functional form, the Leading-Uniqueness Theorem for the identity function — and the entire remaining freedom is located, exactly, in three calculable objects — H_{cl}, P_Y, $\mathscr{E}_{fL/R}$ — plus, where the QN unit inheritance is unverified, the vertex normalisation $\mathcal{N}_Y$ (Section 8.8).

1.4 Inherited results

CMSY-1 inherits conditionally:

From the substrate and closure-dynamics programme: $\mathcal{F}_{sub}$; the admissibility and stability of q_0; P_{cl}; K_{cl}.

From the weak-doublet and mass-readout programme: the scalar-readout projector P_Y; the closure-depth overlap formulation in which $\mathcal{C}_Y$ first appeared as $\hat{H}_c$.

From the Standard Model census and occupancy gates: the chiral carriers, generation addresses and gauge labels; the raw embeddings ι_{fL}, ι_{fR}; address-before-ordering discipline.

From the electroweak representation-closure programme: the doublet and singlet representations; the allowed interface contractions; the conditional right-handed neutral singlet.

From the Higgs-radial and closure-interface programme: the closure-radial direction φ, its canonical normalisation, and the existence but not the value of v_{cl}.

From the spectral-frame programme: positive kinetic-metric discipline; canonical normalisation before spectral extraction; singular values as physical invariants; the frame-kernel/Yukawa type distinction.

From the charged-lepton magnitude programme: hierarchy/absolute-scale separation; the exponent-export prohibition; the non-insertion architecture.

From the neutrino completion gate: the conditional Y_ν; the Majorana firewall; the N_R requirement; the relative-frame definition of lepton mixing.

1.5 What is not inherited

Any numerical Yukawa matrix; the numerical Hessian, spectrum, projectors or readout projector; numerical embeddings; the composite generator $\mathcal{A}_{eff}^{(N)}$; the sterile census; the physical fermion ordering; any measured mass, angle or phase as an input.

1.6 Central aim

To derive every gauge-admissible Dirac Yukawa operator, at leading linear-sector single-radial-insertion renormalisable order, as the compression $Y_f^c = \mathcal{E}_{fL}^\dagger \, \mathcal{C}_Y \, \mathcal{E}_{fR}$ of the inherited scalar-readout closure Hessian between independently derived canonical chiral-carrier embeddings; to derive rather than assume the functional form through spectral isotropy; to prove the insufficiency of the factorised form without independently fixed maps; and to prohibit any observed fermion datum — through any of the five smuggling channels — from entering the construction.

1.7 Core novelty

1. Identification of the Yukawa mediator with the inherited closure Hessian through the scalar-readout projection;

2. the exact radial-derivative definition of the Yukawa operator;
3. **the Spectral-Isotropy Theorem**, deriving $\mathcal{G} = F(\mathcal{C}_Y)$ from commutant invariance under the closure-side factorisation FR, and proving mere commutation insufficient under degeneracy;
4. the Leading Closure-Functional Uniqueness Theorem, $F_{Yuk}(x) = x$;
5. **the Response-Form Insufficiency Theorem** and the **embedding-gauge theorem**;
6. the central compression formula with $\gamma_\alpha^{(f)} = \lambda_\alpha$ and the honest converse: F values need not be reconstructible from Y_f^c;
7. the declared-order effective channel carrier $\mathscr{H}_{eff}^{(N)}$ typing composite channels;
8. the closure-projector texture-zero theorem with a three-way zero taxonomy;
9. rank bounds counting surviving projections;
10. **the Hierarchy-Transfer Theorem**, with the dynamical shape of the spectrum ledgered separately;
11. the extended phase-provenance ledger with **the Canonical CP-Equivalence Theorem** and general even-cycle invariants;
12. valid Grassmann-even mediator constructions for resolvent-type responses (Appendix F);
13. pre-breaking versus post-breaking assembly;
14. the reduction of the Yukawa problem to H_{cl}, P_Y, $\mathscr{E}_{fL/R}$.

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2. Typed Chiral Carriers and Closure Interface

2.1 Chiral carriers

For each sector f, $\mathscr{H}_{fL}$ and $\mathscr{H}_{fR}$ are independently derived finite-dimensional chiral carriers; for three observed generations, $\dim \mathscr{H}_{uL} = \dim \mathscr{H}_{dL} = \dim \mathscr{H}_{eL} = 3$, with right-handed dimensions three whenever all conventional carriers are present. The left-handed up- and down-type quarks are components of one electroweak doublet carrier Q_L with Yukawa maps terminating in inequivalent right-handed singlets; likewise the lepton doublet. This shared-doublet origin is load-bearing for relative mixing (Section 20) and for the assembly statement (Section 19).

2.2 Neutral-sector qualification

$Y_\nu^c : \mathscr{H}_{NR}^c \to \mathscr{H}_{\nu L}^c$ exists only if $\mathscr{H}_{NR}$ is independently admitted; its dimensions are $3 \times n_R$ with n_R fixed by the sterile census, not the active census.

2.3 Closure interface and the vacuum convention

Vacuum-magnitude convention (binding). Φ_0 is unit-normalised in the $\mathscr{H}_\Phi$ metric; the closure-radial amplitude φ is canonically normalised with $\varphi_0 = v_{cl}/\sqrt{2}$; v_{cl} enters physics only through $M_f = (v_{cl}/\sqrt{2})\cdot Y_f^c$. Neither $\mathcal{C}_Y$ nor the embeddings contains v_{cl} (falsifier F35). The interface contractions differ by sector: $\tilde{\Phi}$ for up-type and neutral, Φ for down-type and charged-lepton.

2.4 Typed-interaction firewall

A nonzero canonical Yukawa element requires **all** of: a left carrier; a right carrier; the closure interface; a scalar-readout closure direction; a gauge-singlet contraction; neutrality under every exact VERSF charge; and nonzero projected pairing $\langle \mathcal{E}_{fL} e_{L,i}, \Pi_\alpha \mathcal{E}_{fR} e_{R,j}\rangle$ for some α. The conjunction is strict: a large closure eigenvalue cannot rescue a vanishing chiral projection, and a large overlap cannot rescue a charge-violating contraction.

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3. From the Substrate Hessian to the Scalar-Readout Closure Operator

3.1 The inherited generator

$G_{sub} = \delta^2\mathcal{F}_{sub}/\delta q\delta q \,|_{q_0}$, $H_{cl} = P_{cl}\, G_{sub}\, P_{cl}$, $\hat{H}_{cl} = K_{cl}^{-1/2}\, H_{cl}\, K_{cl}^{-1/2}$.

3.2 Premise C0 — Stability

q_0 is a stable admissible configuration, so $\hat{H}_{cl} \geq 0$ on the retained physical closure carrier. A negative eigenvalue signals an instability of q_0 — not a suppressed Yukawa channel — and falsifies the construction at that configuration (falsifier F41).

3.3 Theorem 1 — Hermiticity, Positivity and Spectral Resolution

Statement. $\mathcal{C}_Y = P_Y\, \hat{H}_{cl}\, P_Y$ satisfies $\mathcal{C}_Y^\dagger = \mathcal{C}_Y \geq 0$ and admits

$\mathcal{C}_Y = \Sigma_\alpha \lambda_\alpha \Pi_\alpha$, $\lambda_\alpha \geq 0$, $\Pi_\alpha \Pi_\beta = \delta_{\alpha\beta} \Pi_\alpha$, $\Sigma_\alpha \Pi_\alpha = P_Y$.

Prerequisites (mathematical only). $K_{cl} > 0$, $G_{sub} = G_{sub}^\dagger$, C0, and finite dimensionality of the readout channel. No channel-completeness or derivational premise enters this theorem.

Proof. G_{sub} is the second variation of a real functional, hence Hermitian; H_{cl} and $\hat{H}_{cl}$ inherit Hermiticity; C0 gives positivity; compression by the orthogonal projector P_Y preserves both; the spectral theorem on the finite readout channel gives the resolution. ■

3.4 Typing note

$\mathcal{C}_Y$ is not a new free mediator; it is a derived projection of the inherited closure Hessian. Its spectrum and projectors are calculable in principle from substrate microphysics (debts D1–D4) and are not adjustable.

3.5 Degeneracy discipline

Individual eigenvectors inside a degenerate level are basis dependent; the projectors Π_α are not. Every construction in this gate consumes spectral data through Π_α; no physical label attaches to an arbitrarily chosen vector inside a degenerate level (falsifier F34).

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4. Canonical Chiral-Carrier Embeddings and the Embedding Gauge

4.1 Raw and canonical embeddings

The occupancy and chiral-carrier maps supply $\iota_{fL} : \mathcal{H}_{fL} \to \mathcal{H}_{cl}$, $\iota_{fR} : \mathcal{H}_{fR} \to \mathcal{H}_{cl}$. With positive chiral kinetic metrics,

$$\boxed{\mathcal{E}_{fL} = K_{cl}^{1/2}\, \iota_{fL}\, K_{fL}^{-1/2} \qquad \mathcal{E}_{fR} = K_{cl}^{1/2}\, \iota_{fR}\, K_{fR}^{-1/2}}$$

both mapping fermion carriers **into** the canonical closure space, $\mathcal{E}_{fL} : \mathcal{H}_{fL}^c \to \mathcal{H}_{cl}^c$, $\mathcal{E}_{fR} : \mathcal{H}_{fR}^c \to \mathcal{H}_{cl}^c$. Kinetic normalisation — fermionic and closure — is built into the embeddings once; every downstream statement is automatically canonical.

4.2 Content of the embeddings

Each canonical embedding comprises: the generation-depth embedding; the gauge-admissible scalar-interface contraction; the chiral carrier typing; every exact VERSF charge projector; canonical kinetic normalisation.

4.3 Theorem 2 — Embedding Gauge

Statement. Extend $\mathcal{C}_Y$ — and the resolved response $\mathcal{G}$ — by zero on the P_Y-complement of $\mathcal{H}_{cl}^c$, so that the compression formula $Y_f^c = \mathcal{E}_{fL}^\dagger\, \mathcal{G}\, \mathcal{E}_{fR}$ is well typed on the full canonical closure space. The commutant of the extended $\mathcal{C}_Y$ on $\mathcal{H}_{cl}^c$ is

$\bigoplus_{\lambda_\alpha > 0} U(d_\alpha) \oplus U(d_0 + n_\perp)$,

where $d_0 = \dim \ker(\mathcal{C}\_Y|_{\mathrm{im}\, P\_Y})$ and $n_\perp = \dim (\mathrm{im}\, P\_Y)^\perp$: block unitaries on each positive level, together with arbitrary unitaries on the **joint zero eigenspace**, which mixes the readout kernel with the P_Y-complement. For any U in this commutant, the replacement

$\mathcal{E}\_fL \to U\, \mathcal{E}\_fL$, $\mathcal{E}\_fR \to U\, \mathcal{E}\_fR$

leaves $Y\_f^c = \mathcal{E}\_fL^\dagger\, \mathcal{G}\, \mathcal{E}\_fR$ invariant. Consequently, individual embedding components are defined only modulo this commutant and are **not physical observables** unless an upstream construction fixes the freedom. Embedding components in the joint zero eigenspace are annihilated by $\mathcal{G}$ and do not enter Y_f^c at all — they are even less constrained than the positive-level components.

Proof. $\mathcal{E}\_fL^\dagger U^\dagger \mathcal{G}\, U\, \mathcal{E}\_fR = \mathcal{E}\_fL^\dagger\, \mathcal{G}\, \mathcal{E}\_fR$ since U commutes with the extended $\mathcal{G}$. Given isotropy, commuting with the extended $\mathcal{C}\_Y$ suffices on the positive levels; on the joint zero eigenspace the equivalence requires one condition: a unitary mixing the readout kernel with the P_Y-complement commutes with the extended $\mathcal{C}\_Y$ yet fails to commute with the extended $\mathcal{G}$ whenever $F(0) \neq 0$. The joint-zero mixing sector of the commutant therefore acts as gauge precisely when $F(0) = 0$ — which holds at leading order by $c_0 = 0$. The commutant computation itself is the standard block decomposition of unitaries commuting with a Hermitian operator, applied to the extended spectrum in which $\lambda = 0$ carries multiplicity $d_0 + n\_\perp$. ■

Discipline. The embedding gauge is the exact analogue of the coefficient gauge of the channel expansion: physical claims attach to gauge-invariant combinations — the operator Y_f^c, the projected pairings summed within each level, and the invariants built from them — never to individual entries of $\mathcal{E}\_fL$ or $\mathcal{E}\_fR$ (falsifier F47).

4.4 Embedding-relocation firewall (binding)

For a parameter-free construction, the embeddings must be **derived partial isometries or derived contractions**: entries follow from the occupancy map and kinetic metrics and may not be selected from fermion masses, mixing angles or phases. If the embeddings were arbitrary complex matrices, the Yukawa problem would merely be relocated into them (Theorem 5 makes this precise). Exercising entry-level freedom in $\mathcal{E}\_fL$ or $\mathcal{E}\_fR$ is a fit (falsifier F44).

Contraction convention. Where the embeddings are contractions, $\|\mathcal{E}\_fL\|_2 \leq 1$ and $\|\mathcal{E}\_fR\|_2 \leq 1$, sharpening the hierarchy-transfer bound of Section 13 to a pure spectral statement.

4.5 Shared-doublet consistency

ι_uL and ι_dL descend from one doublet embedding ι_QL, and K_uL, K_dL from one doublet metric K_QL; the two canonical left embeddings are the two weak components of one canonical doublet embedding. Unrelated left embeddings erase relative mixing (falsifier F19; Section 20.2).

5. Gauge-Admissible Contractions and the Selection Theorem

5.1 Scope conditions and symmetry typing

Admissibility statements hold under: (1) every participating object transforms in a defined linear representation; (2) the completion is local and multilinear at declared order; (3) admissibility is tested in the unbroken theory. **Symmetry typing:** G_{exact} contains a *local* gauge factor ($SU(3)\times SU(2)\times U(1)$) and *global* exact factors (VERSF charges, discrete closure symmetries, orientation gradings). The selection theorem below uses group averaging, valid for the compact local factor and for compact global factors alike; the two are distinguished because local invariance also constrains derivative couplings and current structure, while global invariance constrains only the representation content. Statements in this paper invoke representation-level selection and hold for both, with the local/global distinction flagged wherever derivative couplings enter (Section 15).

5.2 Theorem 3 — Gauge-Selection Theorem

Statement. A closure direction $\chi \in \mathrm{im}\, P_Y$ contributes to sector f only if $R_{fL}^* \otimes R_\Phi \otimes R_{fR} \otimes R_\chi$ contains a trivial representation of G_{exact}; otherwise the projected pairing vanishes identically, independently of overlap magnitudes.

Proof. Haar averaging over each compact factor projects onto the trivial isotypic component, zero when no singlet occurs; exact Abelian charges require zero total vertex charge; exact discrete symmetries impose the fixed-subspace condition. The embeddings carry the exact charge projectors, so forbidden pairings are annihilated inside $\mathscr{E}_{fL}^\dagger \Pi_\alpha \mathscr{E}_{fR}$. ■

5.3 Admissibility firewall

Admissibility is binary, tested pre-breaking, before any overlap is computed (falsifier F4).

6. Closure-Spectrum Insufficiency and Response-Form Insufficiency

6.1 Theorem 4 — Spectrum-to-Yukawa No-Go

Statement. The eigenvalue multiset $\{\lambda_\alpha\}$ of $\mathcal{C}_Y$ does not determine any sector Yukawa operator, its texture, its rank or its strength.

Proof. Hold the embeddings fixed and conjugate the readout operator by a unitary not commuting with them: the spectrum is unchanged, the projectors rotate, the compressions change. Isospectral readout operators therefore yield inequivalent Yukawa operators. ■

6.2 Theorem 5 — Response-Form Insufficiency Theorem

Statement. The existence of a factorisation

$$Y = \mathcal{E}_L^\dagger\, \mathcal{G}\, \mathcal{E}_R$$

with Hermitian positive $\mathcal{G}$ and contractive maps $\mathcal{E}_L$, $\mathcal{E}_R$ does not by itself constrain Y in any way: **every** matrix admits such a factorisation. Predictive content arises only when $\mathcal{G}$, $\mathcal{E}_L$ and $\mathcal{E}_R$ — including their dimensions, normalisation, support and phase content — are fixed independently of the target Yukawa operator.

Proof. Let $Y : \mathcal{H}_R \to \mathcal{H}_L$ be arbitrary — rectangular in general, as for Y_ν^c with dimensions $3 \times n_R$ — with singular-value decomposition $Y = U \Sigma V^\dagger$, Σ the $\dim \mathcal{H}_L \times \dim \mathcal{H}_R$ rectangular diagonal matrix of singular values. Choose a readout space of dimension $N \geq \max(\dim \mathcal{H}_L, \dim \mathcal{H}_R)$, with **separate** block inclusions $\iota_L : \mathbb{C}^{\dim \mathcal{H}_L} \to \mathbb{C}^N$ and $\iota_R : \mathbb{C}^{\dim \mathcal{H}_R} \to \mathbb{C}^N$, both onto the leading coordinates, and set $\mathcal{G} = \mathrm{diag}(\sigma_1, \ldots, \sigma_r, 0, \ldots) \geq 0$ in the standard readout basis, so that

$$\iota_L^\dagger\, \mathcal{G}\, \iota_R = \Sigma$$

as a rectangular matrix. Define

$$\mathcal{E}_L = \iota_L U^\dagger, \quad \mathcal{E}_R = \iota_R V^\dagger.$$

Then

$$\mathcal{E}_L^\dagger\, \mathcal{G}\, \mathcal{E}_R = U\, \iota_L^\dagger\, \mathcal{G}\, \iota_R V^\dagger = U \Sigma V^\dagger = Y,$$

and $\mathcal{E}_L^\dagger \mathcal{E}_L = U\, \iota_L^\dagger \iota_L U^\dagger = \mathbb{1}$ (likewise on the right): both maps are isometries, $\|\mathcal{E}_L\|_2 = \|\mathcal{E}_R\|_2 = 1$, with $\mathcal{G} \geq 0$. Hence the form is universal — square and rectangular cases alike — and vacuous absent independent fixing. ■

Consequence. The substance of this gate resides not in the compression formula but in the derivational addresses of its three ingredients: $\mathcal{C}_Y$ from the substrate Hessian and readout programme, and $\mathcal{E}_{fL/R}$ from the occupancy map under the partial-isometry discipline. This theorem is the formal charter of the embedding-relocation firewall.

6.3 The completion data

$\mathfrak{D}_f = (H_{cl}, K_{cl}, P_Y ; \iota_{fL}, \iota_{fR}, K_{fL}, K_{fR} ; \Phi_0, v_{cl}).$

Every element is inherited, derived here, or ledgered; none may be filled from data. The no-go of 6.1 names what the spectrum lacks; the insufficiency of 6.2 names what the form lacks; between them they define exactly what a substantive derivation must supply.

Part II — The Closure-Operator Yukawa Construction

7. Exact Definition Through the Radial Closure Mode

7.1 The completion operator

Let φ be the canonically normalised closure-radial amplitude with vacuum value $\varphi_0 = v_{cl}/\sqrt{2}$, and $\mathcal{M}_f(\varphi) : \mathcal{H}_{fR}^c \to \mathcal{H}_{fL}^c$ the left–right quadratic completion operator in radial background φ.

7.2 Definition — the Yukawa operator

$$Y_f^c = \partial\mathcal{M}_f(\varphi)/\partial\varphi \,\big|_{\varphi = \varphi_0}$$

For a completion linear in the radial field, $\mathcal{M}_f(\varphi) = \varphi\, Y_f^c$ and $M_f^c = \mathcal{M}_f(\varphi_0) = (v_{cl}/\sqrt{2})\cdot Y_f^c$, recovering the mass relation with no independent normalisation.

7.3 Three distinct objects

- **H_{cl}** — the parent closure Hessian, on the full retained closure carrier;
- **$\mathcal{C}_Y = P_Y \hat{H}_{cl} P_Y$** — its scalar Yukawa-readout compression, on the readout channel;
- **$Y_f^c = \mathcal{E}_{fL}^\dagger \mathcal{C}_Y \mathcal{E}_{fR}$** — the compression of $\mathcal{C}_Y$ between sector chiral embeddings, on the fermion carriers.

Each arrow — restriction, readout projection, chiral compression — is a derived operation on the one inherited object.

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8. The Spectral-Isotropy Theorem and Leading Closure-Functional Uniqueness

8.1 The resolved response operator, prior to structure

Let $\mathcal{G} : \mathrm{im}\, P_Y \to \mathrm{im}\, P_Y$ be the **resolved closure response operator**: the operator that the resolved closure response actually produces on the readout channel at the declared order, before any structural assumption, extended by zero on the P_Y-complement wherever it acts on $\mathscr{H}_{cl}^c$. It carries no sector index (Premise FR, Section 8.3). The most general construction from the closure system and the embeddings is

$$Y_f^c = \mathscr{E}_{fL}^\dagger \mathcal{G} \mathscr{E}_{fR},$$

and the question a derivation must answer — rather than assume — is why $\mathcal{G}$ should be a scalar function of $\mathcal{C}_Y$ at all.

8.2 Commutation is not enough

Suppose only $[\mathcal{G}, \mathcal{C}_Y] = 0$. On a nondegenerate spectrum this forces $\mathcal{G}$ diagonal in the eigenbasis, hence a function of $\mathcal{C}_Y$. But on a degenerate level of multiplicity d_α, commutation constrains $\mathcal{G}$ not at all inside the eigenspace: any $d_\alpha \times d_\alpha$ block is admissible. A degenerate closure level could then carry an arbitrary hidden matrix — functional-form smuggling in its degenerate-level subspecies (channel 2 of Section 1.2). Commutation must therefore be strengthened to invariance.

8.3 Premises FR and SI — factorisation and isotropy

Premise FR — Closure-side factorisation. The response is resolved on the closure side **prior to and independently of** the fermion compression: sector data enter the construction only through the embeddings $\mathscr{E}_{fL}$, $\mathscr{E}_{fR}$, and the resolved response $\mathcal{G}$ carries no sector index. This premise is load-bearing: it is exactly what prevents sector-dependent dressing of the response — for example fermion-loop dressing, which would renormalise the readout channel differently for different sectors — from breaking isotropy sector-by-sector. Sector-dependent response effects are typed as higher-order corrections (Section 15), never absorbed into $\mathcal{G}$ (falsifier F51).

Premise SI — Response isotropy. The resolved response $\mathcal{G}$ inherits every symmetry of the closure dynamics that preserves the generator: for every unitary U on the readout channel with $U \mathcal{C}_Y U^\dagger = \mathcal{C}_Y$,

$$U \mathcal{G} U^\dagger = \mathcal{G}.$$

Caveat (binding). If the closure dynamics distinguishes directions inside a degenerate eigenspace, the distinguishing structure is itself an additional derived operator that must be declared; the response is then a function of the joint commuting set, and the theorem below applies to that enlarged set. SI may not be silently weakened to bare commutation (falsifier F48).

8.4 Theorem 6 — Spectral-Isotropy Theorem

Statement. Let $\mathcal{C}_Y = \Sigma_\alpha \lambda_\alpha \Pi_\alpha$ on the finite-dimensional readout channel, and let $\mathcal{G}$ satisfy Premises FR and SI. Then

$$\Pi_\alpha \mathcal{G} \Pi_\beta = 0 \ (\alpha \neq \beta), \quad \Pi_\alpha \mathcal{G} \Pi_\alpha = F(\lambda_\alpha) \Pi_\alpha$$

on every eigenspace, degenerate or not, and therefore $\mathcal{G}$ admits the unique functional-calculus form

$$\boxed{\mathcal{G} = F(\mathcal{C}_Y) = \Sigma_\alpha F(\lambda_\alpha) \Pi_\alpha}$$

for a unique scalar function F on the spectrum.

Proof. The unitaries preserving $\mathcal{C}_Y$ on the readout channel are exactly the block unitaries $U = \bigoplus_\alpha U_\alpha$ with $U_\alpha \in U(d_\alpha)$ on each eigenspace. (i) *Off-diagonal blocks vanish.* Take $U = \bigoplus_\alpha e^{i\theta_\alpha} \mathbb{1}_\alpha$ with distinct phases; invariance gives $\Pi_\alpha \mathcal{G} \Pi_\beta = e^{i(\theta_\alpha - \theta_\beta)} \Pi_\alpha \mathcal{G} \Pi_\beta$ for all phase choices, forcing $\Pi_\alpha \mathcal{G} \Pi_\beta = 0$ for $\alpha \neq \beta$. (ii) *Diagonal blocks are scalar.* Within a level, invariance under every $U_\alpha \in U(d_\alpha)$ means the block $\Pi_\alpha \mathcal{G} \Pi_\alpha$ commutes with the full unitary group of the eigenspace; by Schur's lemma its commutant is the scalars, so $\Pi_\alpha \mathcal{G} \Pi_\alpha = g_\alpha \Pi_\alpha$. Setting $F(\lambda_\alpha) = g_\alpha$ gives the display; uniqueness of F on the spectrum is immediate. ■

Remark. This theorem is what licenses "one response value per closure eigenvalue." Without SI, the response could hide arbitrary structure inside degenerate levels; with SI, the hiding place is closed by Schur's lemma.

Kernel remark. The $\lambda = 0$ eigenspace participates on equal footing: once $\mathcal{C}_Y$ is extended by zero, its joint zero eigenspace comprises the readout kernel together with the P_Y-complement, the commutant includes arbitrary unitaries there (Theorem 2), and the isotropy argument forces the kernel block to a scalar F(0) — equal to zero at leading order by $c_0 = 0$. The zero level is not special.

8.5 Premises CY1–CY5, CY3′ and CU

CY1 — Unique scalar mediator. No independent scalar left–right completion exists outside the closure-radial channel (falsifier F49). **CY2 — Leading local completion.** The retained interaction is the leading local, renormalisable, single-radial-insertion completion on the linear primitive-channel sector. **CY2A — Linear-sector insertion-algebra closure.** On the linear primitive-channel sector, repeated insertions of the scalar-readout closure channel compose through ordinary operator multiplication, with no independent inter-insertion propagator, ordering kernel, contraction operator or additional support projection. Consequently the pure n-insertion contribution has the form $\mathcal{G}^{(n)} = c_n \mathcal{C}_Y^n$. If an intermediate operator occurs — $\mathcal{C}_Y K_{int} \mathcal{C}_Y$ rather than $\mathcal{C}_Y^2$ — the contribution is not a pure repeated insertion on $\mathscr{H}_{cl}$; it belongs to the declared-order effective carrier $\mathscr{H}_{eff}^{(N)}$ and must be separately derived (falsifier F52; debt D20). **CY3 — Canonical radial normalisation.** The radial mode's normalisation is fixed by its kinetic term; no free rescaling of φ remains. (CY3 normalises the field φ; it does **not** by itself fix the strength of the fermion–closure vertex.) **CY3′ — Completion–free-energy matching identity.** The left–right completion operator $\mathscr{M}_f(\varphi)$ is identified with the φ-derivative structure of the closure free energy restricted to the fermion-dressed configuration, in the **same canonical units** as $\hat{H}_{cl}$: the fermion-bilinear block of the dressed second variation is, by this identification, the chiral compression of the same Hessian that defines $\mathcal{C}_Y$. Under CY3′, the coefficient of the single insertion is fixed to unity as a **matching identity**, not inferred. CY3′ is the central physical bridge of the entire construction — without it, the closure Hessian is a plausible response operator but not yet the Yukawa operator — and its microscopic derivation from the fermion-dressed substrate free energy is ledgered as debt D21. **CU — Canonical unit inheritance.** The QN-series (Quantitative Normalisation) canonical conventions render $\hat{H}_{cl}$ dimensionless in substrate units, so the identity of CY3′ equates dimensionless objects and no residual scale freedom survives. Where CU is not yet verified, the vertex normalisation $\mathcal{N}_Y$ is a separate calculable object (debt D19; falsifier F50), and the closing count of concrete calculations rises from three to four. **CY4 — Complete scalar-readout projection.** $P_Y \hat{H}_{cl} P_Y$ contains the complete leading closure contribution to the left–right scalar channel. (CY1 and CY4 are complementary, not redundant: CY1 excludes scalar mediators *outside* the readout channel; CY4 certifies completeness *within* it.) **CY5 — Independent carrier maps.** $\mathscr{E}_{fL}$, $\mathscr{E}_{fR}$ are derived without target masses, mixing or phases, under the partial-isometry discipline.

8.6 Theorem 7 — Leading Closure-Functional Uniqueness

Statement. Under FR, SI, CY1–CY2, CY2A, CY3, CY3′/CU and CY4–CY5, organise the resolved response by its readout-channel **insertion-order expansion** — the dynamical expansion of the underlying response in the number of readout-channel insertions,

$$\mathcal{G} = c_0 P_Y + c_1 \mathcal{C}_Y + c_2 \mathcal{C}_Y^2 + \cdots \text{ (composition by CY2A)},$$

whose coefficients c_n are operator-order data of the dynamics, **not** coefficients of a function on the finite spectrum. Then

$$c_0 = 0,\ c_1 = 1,\ c_n = 0\ (n \geq 2),$$

so the leading-order resolved response equals the generator,

$$\boxed{\mathcal{G} = \mathcal{C}_Y}$$

with $F_{Yuk}(x) = x$ as the induced spectral function.

Typing note (binding). A function on a finite spectrum has no unique polynomial decomposition: on a three-point spectrum, "$c_n = 0$ for $n \geq 2$" is not a well-posed statement about F itself. What CY2 constrains is the insertion-order expansion of the dynamics — the number of readout-channel insertions in the underlying response — which then induces values on the spectrum. The theorem quantifies over insertion orders; the identity function is the spectral shadow of the leading operator, not the object of the uniqueness claim. CY2A supplies the composition law that identifies the pure n-insertion contribution with $c_n \mathcal{C}_Y^n$.

Proof. FR resolves the response on the closure side with no sector index. SI (Theorem 6) makes each declared insertion-order contribution scalar on every closure eigenspace. CY2A identifies the pure n-insertion contribution on the linear primitive carrier with $c_n \mathcal{C}_Y^n$: repeated insertions compose by ordinary operator multiplication, with no inter-insertion propagator, ordering kernel, contraction or additional support projection — any intermediate operator removes the contribution from the pure repeated-insertion class and consigns it to $\mathscr{H}_{eff}^{(N)}$. CY1 removes the zero-insertion (mediator-independent) term: $c_0 = 0$. CY2 retains only the single-insertion term at the declared order: $c_n = 0$ ($n \geq 2$).

For c_1: a universal dimensionless factor g multiplying the single insertion is **not** excluded by CY1–CY3 — it introduces no second mediator (CY1 untouched), remains a single insertion (CY2 untouched), and does not rescale φ (CY3 normalises the field, not the vertex). The coefficient is instead fixed by CY3′ as a matching identity: Y_f^c is *defined* as $\partial\mathscr{M}_f/\partial\varphi|_{\varphi_0}$, and CY3′ identifies $\mathscr{M}_f$ with the fermion-dressed restriction of the very free energy whose Hessian defines $\mathcal{C}_Y$, in the canonical units of CU. Differentiating that identification yields the compression $\mathscr{E}_{fL}^\dagger \mathcal{C}_Y \mathscr{E}_{fR}$ with coefficient exactly one — the two sides are the same object read twice, so no residual g exists to fix. Under CU the identity is dimensionally consistent (Section 8.8); absent CU, c_1 is fixed only up to the vertex normalisation $\mathcal{N}_Y$ (debt D19). CY4 certifies no leading contribution outside $\mathcal{C}_Y$; CY5 excludes compensating embedding freedom. ∎

8.7 Significance

The chain is: FR and SI ⟹ functional form (Theorem 6); CY1–CY2 ⟹ removal of the constant and the higher insertions; CY3′ with CU ⟹ $c_1 = 1$ as a matching identity (Theorem 7). Nothing about the response is assumed that is not either derived (the form), fixed by identification (the coefficient), or named as a premise with its own falsifier (the isotropy, the order, the units). All sector dependence resides in the embeddings; all spectral weight in the common operator; no separate $\gamma_a^{(f)}$ — and no sector index on the response — exists.

8.8 Dimensional consistency and the unit inheritance

$Y_f^c = \partial\mathscr{M}_f/\partial\varphi|_{\varphi_0}$ is dimensionless: $\mathscr{M}_f$ carries mass dimension and φ carries mass dimension. But a Hessian of a free-energy functional is a mass-matrix-type object, not a coupling-type object: the second variation with respect to canonically normalised configuration variables carries whatever dimension the substrate units assign. The identity $Y_f^c = \mathscr{E}_{fL}^\dagger \mathcal{C}_Y$

$\mathcal{E}_{fR}$ with partial-isometric embeddings therefore equates a dimensionless object with $\mathcal{C}_Y$, and is consistent only under one of two dispositions:

1. **CU holds:** the QN-series canonical conventions render $\hat{H}_{cl}$ dimensionless by construction, the upstream unit-fixing gate is cited, and no residual scale freedom survives into c_1. The identity is then exact and the calculable objects number three: H_{cl}, P_Y, $\mathcal{E}_{fL/R}$.
2. **CU open:** the vertex normalisation $\mathcal{N}_Y$ — the conversion between substrate Hessian units and the dimensionless completion — is a fourth calculable object, ledgered as debt D19, and $c_1 = 1$ is downgraded to "c_1 fixed up to $\mathcal{N}_Y$."

Silently absorbing $\mathcal{N}_Y$ is exactly the failure F35 exists to catch, extended here to the vertex (falsifier F50). This gate adopts disposition 1 conditionally, inherits CU from the QN-series, and carries D19 as the audit trail.

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9. The Closure-Operator Yukawa Construction Theorem

9.1 Theorem 8 — Closure-Operator Yukawa Construction (central theorem)

Statement. Let H_{cl} be the inherited closure Hessian, $K_{cl} > 0$ its kinetic metric, P_Y the independently derived scalar mass-readout projector, and $\mathcal{E}_{fL}$, $\mathcal{E}_{fR}$ the independently derived canonical chiral-carrier embeddings. Under C0, FR, SI, CY1–CY5 and the CY3′/CU matching identity, the complete leading Yukawa operator on the linear primitive-channel sector is

$$\boxed{Y_f^c = \mathcal{E}_{fL}^\dagger P_Y K_{cl}^{-1/2} H_{cl} K_{cl}^{-1/2} P_Y \mathcal{E}_{fR}}$$

and, with $\mathcal{C}_Y = \Sigma_\alpha \lambda_\alpha \Pi_\alpha$,

$$\boxed{Y_f^c = \Sigma_\alpha \lambda_\alpha \mathcal{E}_{fL}^\dagger \Pi_\alpha \mathcal{E}_{fR}}$$

The mode coefficients are the derived closure eigenvalues; rank, texture, hierarchy and phases are fixed jointly by the spectral projectors and the embeddings; no independent Yukawa entry or mode coefficient is admitted.

Conditioning note. CY2A is deliberately absent from this theorem's premises: at leading order, CY2 alone discards every multi-insertion contribution regardless of how the discarded terms compose. CY2A is load-bearing only for *typing* those discarded terms — the classification of the higher-order series in Section 15.3 — not for eliminating them here.

Proof. Theorems 6 (isotropy, under FR) and 7 — with $c_1 = 1$ secured by the CY3′/CU matching identity, and with only the CY2 single-insertion clause of Theorem 7 consumed — give $\mathcal{G} = \mathcal{C}_Y$ at leading order; Theorem 1 supplies the spectral resolution; substitution gives both displays. Uniqueness relative to $\mathfrak{D}_f$ follows because every object is inherited or derived, with residual freedom removed by CY5 and reduced to gauge by Theorem 2. ■

9.2 Uniqueness and gauge clauses

Y_f^c is unique relative to $\mathfrak{D}_f$, up to: biunitary canonical-frame rotations of the fermion carriers (preserving all singular values), and the embedding gauge of Theorem 2 (acting trivially on Y_f^c). Its matrix representation is basis dependent; its singular values, rank, determinant magnitude, and relative-sector invariants are not.

9.3 Reconstructibility disclaimer (binding)

The values $F(\lambda_\alpha) = \lambda_\alpha$ are uniquely specified by the derivation. They are **not** thereby reconstructible from Y_f^c, and they may not be described as independently observable coefficient values: the projected channels $\mathscr{E}_{fL}^\dagger \, \Pi_\alpha \, \mathscr{E}_{fR}$ can vanish (no sector support) or become linearly dependent after compression into the fermion space, in which case distinct spectral data yield the same Y_f^c. Orthogonality of the Π_α on the readout channel does **not** imply linear independence of their compressions. Forward determination is exact; backward inference is not guaranteed (falsifier F43).

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10. Exact Spectral-Projector Construction

10.1 Degeneracy-safe form

$$Y_f^c = \Sigma_\alpha \, \lambda_\alpha \, \mathscr{E}_{fL}^\dagger \, \Pi_\alpha \, \mathscr{E}_{fR},$$

invariant under unitary changes of basis within any degenerate eigenspace: contributions attach to Π_α, never to chosen eigenvectors.

10.2 Nondegenerate rank-one form

For a nondegenerate spectrum, $\Pi_n = |h_n\rangle\langle h_n|$; with projected profiles $a_n^{(f)} = \mathscr{E}_{fL}^\dagger|h_n\rangle$, $b_n^{(f)} = \mathscr{E}_{fR}^\dagger|h_n\rangle$,

$$Y_f^c = \Sigma_n \lambda_n \, a_n^{(f)} b_n^{(f)\dagger}$$

recovering the rank-one expansion with $\gamma_n^{(f)} = \lambda_n$ — subject to the reconstructibility disclaimer of Section 9.3 and the embedding gauge of Theorem 2 (the profiles rotate under the commutant; only gauge-invariant combinations are physical).

10.3 Degenerate form

For multiplicity d_α, any orthonormal eigenbasis gives $\mathcal{E}_{fL}^\dagger \Pi_\alpha \mathcal{E}_{fR} = \Sigma_s a_{\alpha s}^{(f)} b_{\alpha s}^{(f)\dagger}$; the individual vectors are basis dependent, the summed operator is not, and only the sum may carry physical claims.

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11. Texture Zeros: Theorem and Taxonomy

11.1 Theorem 9 — Closure-Projector Texture-Zero Theorem

Statement. In canonical carrier bases,

$$(Y_f^c)_{ij} = \Sigma_\alpha \lambda_\alpha \langle \mathcal{E}_{fL} e_{L,i}, \Pi_\alpha \mathcal{E}_{fR} e_{R,j} \rangle,$$

and if the pairing vanishes for every Π_α, then $(Y_f^c)_{ij} = 0$ exactly. Admissible mechanisms: disjoint closure support; gauge selection (Theorem 3); chiral-support orthogonality; an exact closure symmetry; an exact projector-level cancellation. No zero is inserted independently.

Proof. Term-level vanishing of the displayed sum, mechanism by mechanism as in the listed cases; a symmetry mapping the pairing to $\omega \neq 1$ times itself forces it to zero. ■

11.2 Zero taxonomy (binding)

Three inequivalent kinds of zero must be distinguished in every texture claim:

1. **Coordinate zero** — a vanishing entry in one chosen matrix representation. Not physical by itself: allowed canonical unitary rotations move it. A coordinate projector such as "row 1" is not physically meaningful merely because it commutes with a metric.
2. **Canonical-frame-preserved zero** — a raw block zero $P_L Y_f P_R = 0$ that survives the positive normalisation map because $[P_L, K_{fL}] = 0$ and $[P_R, K_{fR}] = 0$. This condition is **sufficient, not necessary**: zeros can survive normalisation without it, and its satisfaction does not by itself make the projectors meaningful.

3. **Physical block zero** — a vanishing block between **independently fixed invariant subspaces**: projectors associated with exact charges, representation sectors or upstream physical addresses, derived before and independently of the texture claim.

Only kind 3 is a physical texture statement. Kinds 1 and 2 are representation facts. Theorem 9 produces kind-3 zeros exactly when its vanishing mechanism is anchored in derived projectors (charge projectors inside the embeddings, representation selection, derived support structure); the basis-labelling rule stands: every zero claim names its kind (falsifier F42).

11.3 Cancellation and small entries

A pairing sum vanishing only at one numerical point is an accidental cancellation, unstable unless symmetry protected (falsifier F11); approximate zeros require derived bounds, never post-hoc truncation (falsifiers F10, F12).

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12. Rank Theorem in Closure-Operator Form

12.1 Theorem 10 — Closure-Operator Rank Bound

Statement. With $\mathcal{S}_f = \mathrm{span}\{\Pi_\alpha \ \mathrm{im}\ \mathcal{E}_{fR} : \lambda_\alpha > 0\}$,

$$\boxed{\mathrm{rank}(Y_f^c) \le \min\{ \dim \mathcal{S}_f,\ \mathrm{rank}\ \mathcal{C}_Y,\ \mathrm{rank}\ \mathcal{E}_{fL},\ \mathrm{rank}\ \mathcal{E}_{fR} \}}$$

Proof. $\mathrm{im}\ Y_f^c \subseteq \mathcal{E}_{fL}^\dagger \mathcal{S}_f$ gives the first bound; the remaining three are the product-rank inequalities, with $\mathrm{rank}\ \mathcal{C}_Y$ counting only $\lambda_\alpha > 0$. ■

12.2 Full-rank requirements

A full-rank three-generation matrix requires: at least three closure directions with $\lambda_\alpha > 0$; three independent left projections; three independent right projections; and nonvanishing pairing between them. This counts only closure directions that survive both chiral projections: three large eigenvalues with collinear projections give rank one; a large eigenvalue with no sector support gives nothing.

12.3 Nullity

$\text{nullity}(Y_f^c) = n_f - \text{rank}(Y_f^c)$ for a square sector. An exact zero singular value may arise from insufficient surviving directions, an exact chiral symmetry, gauge selection, or protected cancellation; every massless-mode claim names its mechanism — and remains **Dirac-level only** (Section 24.2): subsequent Majorana blocks can alter the physical massless count.

13. The Hierarchy-Transfer Theorem

13.1 Naming discipline

The theorem below **transfers** a hierarchy from the closure spectrum to the fermion spectrum. It does not derive the spectral hierarchy itself: whether $\{\lambda_\alpha\}$ is hierarchical is a dynamical property of the substrate Hessian, ledgered as debt D6. Calling the fermion hierarchy "derived" before D6 is discharged commits falsifier F14.

13.2 Ordered truncation

Order the positive eigenvalues $\lambda_1 \ge \lambda_2 \ge \cdots \ge 0$ — a derived ordering, never chosen from observed masses (falsifier F15). In the nondegenerate rank-one case,

$$Y_f^{(k)} = \Sigma_{n=1}^{k} \lambda_n a_n^{(f)} b_n^{(f)\dagger}, \quad T_f^{(k)} = Y_f^c - Y_f^{(k)},$$

with $\text{rank}\, Y_f^{(k)} \le k$. (Degenerate case: truncate by level through full projectors, $\text{rank}\, Y_f^{(k)} \le \Sigma_{\alpha \le k} \text{rank}(\mathscr{E}_{fL}^\dagger \Pi_\alpha \mathscr{E}_{fR})$.)

13.3 Theorem 11 — Hierarchy-Transfer Theorem

Statement.

$$\sigma_{k+1}(Y_f^c) \le \|T_f^{(k)}\|_2,$$

and since $T_f^{(k)} = \mathscr{E}_{fL}^\dagger (\Sigma_{n>k} \lambda_n \Pi_n) \mathscr{E}_{fR}$,

$$\sigma_{k+1}(Y_f^c) \le \|\mathscr{E}_{fL}\|_2 \|\mathscr{E}_{fR}\|_2 \lambda_{k+1}$$

reducing for contractive embeddings to

$$\sigma_{k+1}(Y_f^c) \le \lambda_{k+1}$$

Moreover, by Weyl's inequality, $|\sigma_i(Y_f^c) - \sigma_i(Y_f^{(k)})| \le \|T_f^{(k)}\|_2$ for every i, so

$\sigma_i(Y_f^c) \ge \sigma_i(Y_f^{(k)}) - \|\mathscr{E}_{fL}\|_2 \|\mathscr{E}_{fR}\|_2 \lambda_{k+1}$ for $i \le k$:

the tail suppresses the light magnitudes from above **and** the partial sums protect the heavy magnitudes from below.

Proof. Eckart–Young–Mirsky for the first line; submultiplicativity and the spectral norm of the positive tail (= λ_{k+1}) for the second; Weyl for the two-sided bound. ■

13.4 Transfer requirements

A hierarchical fermion spectrum follows from a hierarchical closure spectrum **only when** the leading modes carry nonvanishing, suitably independent chiral projections. A closure eigenmode with no sector support produces no fermion mass however large its eigenvalue (Example G.3); collinear projections collapse rank however spread the eigenvalues (Example G.4). Both the spectral shape (D6) and the projection structure (D5) are load-bearing.

---

14. Determinants, Products and Hierarchy Invariants

$|\det Y_f^c| = \Pi_i\, y_{f,i}$; $|\det M_f| = (v_{cl}/\sqrt{2})^{n_f}\, \Pi_i\, y_{f,i}$; rank deficiency of surviving directions forces a Dirac-level massless direction. The invariants $I_{f,1} = \mathrm{Tr}\, H_{fL}$, $I_{f,2} = ½[(\mathrm{Tr}\, H_{fL})^2 - \mathrm{Tr}\, H_{fL}^2]$, $I_{f,3} = \det H_{fL}$ equal the elementary symmetric polynomials in $\{y_{f,i}^2\}$ and audit any implementation internally. Entry inspection never substitutes for singular-value analysis (falsifier F8).

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15. Composite Channels, Higher Orders and the Meaning of "Full"

15.1 Where composites live

The effective interaction may contain composite channels — mode products $\chi_a \chi_b$, gradients $\nabla\chi_a$, interface dressings $\chi_a \Phi$, and higher combinations. **These do not live in the one-mode space $\mathscr{H}_{cl}$:** products belong to tensor-product spaces, gradients to a derivative-extended space, dressings to a closure–interface product space. A scalar function of the one-mode operator,

$F(\mathcal{C}\_Y)$, resolves only the linear response on $\mathcal{H}\_{cl}$ and cannot produce composite contributions. Identifying the two silently is channel smuggling (falsifier F5).

15.2 The declared-order effective carrier

Define, at declared order N,

$$\mathcal{H}\_{eff}^{(N)} = \mathcal{H}\_{cl} \oplus \mathcal{H}\_{cl}^{\otimes 2} \oplus \cdots \oplus \mathcal{H}\_{cl}^{\otimes N} \oplus \mathcal{H}\_\nabla \oplus \mathcal{H}\_{\Phi\text{-dress}},$$

with a **separately derived** effective generator $\mathcal{A}\_{eff}^{(N)}$ and extended embeddings $\mathcal{E}\_{fL/R}^{(N)} : \mathcal{H}\_f^c \to \mathcal{H}\_{eff}^{(N)}$. The declared-order generalisation of the central theorem is

$$Y\_f^c = \mathcal{E}\_{fL}^{(N)\dagger} F\_f(\mathcal{A}\_{eff}^{(N)}) \mathcal{E}\_{fR}^{(N)},$$

with the isotropy and uniqueness arguments rerun on the effective carrier. Equivalently, genuinely multilinear response kernels

$$Y\_f^c = \Sigma\_{n=1}^{N} \mathcal{E}\_{fL}^{(n)\dagger} F\_f^{(n)}(\mathcal{A}\_{cl}^{(1)}, \ldots, \mathcal{A}\_{cl}^{(n)}) \mathcal{E}\_{fR}^{(n)}$$

may be used. Deriving $\mathcal{A}\_{eff}^{(N)}$ and the extended embeddings is debt D7; until it is discharged, **the central theorem of this gate is explicitly the linear primitive-channel result**.

15.3 The repeated-insertion series

On the linear sector, higher-order closure effects generate

$$Y\_f^{full} = \mathcal{E}\_{fL}^\dagger [\mathcal{C}\_Y + c\_2 \mathcal{C}\_Y^2 + c\_3 \mathcal{C}\_Y^3 + \cdots] \mathcal{E}\_{fR},$$

each c_n requiring independent derivation from the nonlinear closure action (debt D7), the pure-power form itself holding only under CY2A — contributions with inter-insertion kernels belong to $\mathcal{H}\_{eff}^{(N)}$ (falsifier F52; debt D20) — and no c_n may be tuned to data (falsifier F43). Valid mediator realisations of resolvent-type resummations are constructed in Appendix F with correct Grassmann and representation typing.

15.4 The binding scope of "full"

"Full" in CMSY-1 means: **complete at leading local, single-radial-insertion, renormalisable closure-Hessian order on the linear primitive-channel sector.**

Without this convention no finite paper could satisfy F33; with it, the fullness claim is exact and auditable.

Part III — Phases, Frames and Physical Masses

16. Phase Provenance and the Canonical CP-Equivalence Theorem

16.1 The provenance ledger

$(Y\_f^c)ij = \Sigma\_\alpha\ \lambda\_\alpha\ \langle\mathscr{E}\_fL\ e$\{L,i\}, Π_α ℰ_fR e_\{R,j\}⟩,

with every λ_α real and nonnegative. All complex structure in the canonical representatives therefore arises from:

1. complex closure spectral projectors Π_α;
2. oriented or holonomic carrier embeddings;
3. complex chiral-interface contractions;
4. complex kinetic normalisation **transported** into the canonical representatives through $K\_cl^{\{\pm1/2\}}$, $K\_fL^{\{-1/2\}}$, $K\_fR^{\{-1/2\}}$ inside the embeddings.

There is no independent complex mode coefficient. Every complex canonical entry cites its ledger source (falsifier F16).

Transport, not injection (binding). Entry 4 is transport of relative complex structure between representations, not creation of physics: canonical normalisation is an invertible field redefinition. The metrics are part of the complete phase-provenance data and must not be omitted from the joint CP audit (falsifier F46) — but a complex-looking metric may be a pure basis artefact, and metric phases are never, by themselves, a physical CP source.

CP convention (binding, used throughout this gate). Flavour-space generalised CP acts as

$\psi\_fL \rightarrow W\_fL\ \psi\_fL^*$, $\psi\_fR \rightarrow W\_fR\ \psi\_fR^*$,

with unitary W_fL, W_fR (spinor structure factored out; the flavour analysis concerns flavour space only). Invariance of the complete system requires

$W\_fL^\dagger\ K\_fL\ W\_fL = K\_fL^*$, $W\_fR^\dagger\ K\_fR\ W\_fR = K\_fR^*$, $W\_fL^\dagger\ Y\_f\ W\_fR = Y\_f^*$.

The Jarlskog invariant is fixed as $J = \mathrm{Im}(V\_us\ V\_cb\ V\_ub^*\ V\_cs^*)$. Every CP statement in this gate uses these conventions; no theorem below is convention-hedged.

16.2 Theorem 12 — Canonical CP-Equivalence Theorem

Statement. Canonical normalisation preserves the existence or nonexistence of a generalised CP symmetry. Precisely: the raw system (K_{fL}, K_{fR}, Y_f; sector set as required) admits a generalised CP transformation in the convention above if and only if the canonically normalised system ($\mathbb{1}$, $\mathbb{1}$, Y_f^c) admits one. Canonical normalisation can redistribute phase structure between kinetic terms, interaction operators and carrier frames; it cannot create a physical CP obstruction absent from the complete raw theory, nor destroy one present in it.

Proof. Given raw W's satisfying the convention's invariance conditions, define

$$\tilde{W}_{fL} = K_{fL}^{1/2} W_{fL} (K_{fL}^*)^{-1/2}, \quad \tilde{W}_{fR} = K_{fR}^{1/2} W_{fR} (K_{fR}^*)^{-1/2}.$$

Then $\tilde{W}_{fL}^\dagger \tilde{W}_{fL} = (K_{fL}^*)^{-1/2} W_{fL}^\dagger K_{fL} W_{fL} (K_{fL}^*)^{-1/2} = (K_{fL}^*)^{-1/2} K_{fL}^* (K_{fL}^*)^{-1/2} = \mathbb{1}$, and

$$\tilde{W}_{fL}^\dagger Y_f^c \tilde{W}_{fR} = (K_{fL}^*)^{-1/2} W_{fL}^\dagger Y_f W_{fR} (K_{fR}^*)^{-1/2} = (K_{fL}^*)^{-1/2} Y_f^* (K_{fR}^*)^{-1/2} = (Y_f^c)^*,$$

so the canonical system admits the generalised CP with the $\tilde{W}$'s and identity metrics. The construction is invertible, so existence is preserved in both directions; full computation in Appendix E. ■

Consequence. Physical CP violation is a property of the **joint** kinetic-and-interaction system: it occurs precisely when no generalised CP symmetry of the complete system exists. Sector reality in some basis is sufficient for sector CP invariance; apparent complexity is never sufficient for violation.

16.3 Theorem 13 — Real-Representative Corollary

Statement. If $\mathcal{C}_Y$, $\mathscr{E}_{fL}$ and $\mathscr{E}_{fR}$ admit simultaneous real representatives in a compatible carrier basis, then Y_f^c is real in that basis and the isolated sector admits a generalised CP symmetry (trivial W's in the real basis): no intrinsic complex phase.

Proof. Real symmetric $\mathcal{C}_Y$ has real eigenprojectors; real embeddings give real pairings; real λ_α give a real sum; the identity CP transformation in the real basis is a generalised CP symmetry. ■

16.4 Rephasing invariants: general even cycles

Diagonal carrier rephasings act as $(Y_f^c)_{ij} \to e^{-i\alpha_i}(Y_f^c)_{ij} e^{i\beta_j}$. For an even support cycle

$$L_{i_1} - R_{j_1} - L_{i_2} - R_{j_2} - \cdots - L_{i_n} - R_{j_n} - L_{i_1},$$

define the cycle product

$$\Pi_C = \Pi_{a=1}^{n} (Y^c)_{i_a j_a} (Y^c)_{i_{a+1} j_a}^* \quad (i_{n+1} = i_1)$$

Its phase is invariant under all diagonal rephasings (Appendix E). **Four-edge plaquettes are the smallest special case, $n = 2$.** They generate the cycle invariants only for support graphs whose cycle spaces admit a four-cycle decomposition; a sparse graph may contain a chordless six-, eight- or longer even cycle with no four-cycle at all, and its invariant must be taken over the full cycle. The forest theorem stands: an acyclic canonical support graph carries no irreducible diagonal-rephasing phase (one matrix, one fixed basis, diagonal transformations only — not a CP theorem).

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17. Quark CP and Relative-Sector Invariants

17.1 Theorem 14 — Relative-Sector CP Theorem

Scope premises. Three conventional quark generations; no vector-like admixture at matching order; canonical Standard Model charged currents; nondegenerate up- and down-type canonical spectra.

Statement. With $H_{uL} = Y_u^c Y_u^{c\dagger}$, $H_{dL} = Y_d^c Y_d^{c\dagger}$ on the common canonical left doublet carrier and

$$\mathcal{J}_q = \mathrm{Im}\,\mathrm{Tr}([H_{uL}, H_{dL}]^3),$$

$\mathcal{J}_q \neq 0$ implies quark-sector CP violation, and under the scope premises $\mathcal{J}_q = 0$ is equivalent to the absence of a physical CKM phase, via the standard three-generation identity

$$\mathrm{Tr}([H_{uL}, H_{dL}]^3) = 6i \cdot J \cdot \Pi_{i<j}(y_{u,i}^2 - y_{u,j}^2) \cdot \Pi_{k<l}(y_{d,k}^2 - y_{d,l}^2),$$

where $J = \mathrm{Im}(V_{us} V_{cb} V_{ub}^* V_{cs}^*)$ is the Jarlskog invariant of $V_{CKM} = U_{uL}^\dagger U_{dL}$ in the convention fixed in Section 16, and the overall sign is set by the chosen eigenvalue ordering and CKM convention (Jarlskog 1985; Bernabéu–Branco–Gronau 1986). The coefficient 6 arises because $\mathrm{Tr}(C^3) = 3 \det C$ for a traceless 3×3 matrix C, combined with the factor $2iJ$ from the Jarlskog determinant identity.

Proof sketch. Basis invariance under common left rotations; sign flip under CP transposition of both magnitude operators; the factorisation identity with nondegeneracy giving the equivalence. Outside the premises the equivalence (not the sufficiency) can fail. ■

17.2 Closure obligation

By Theorems 12–13, quark CP violation requires that the **joint** system — $\mathcal{C}_Y$, the four quark embeddings, and the transported kinetic structure — admit no generalised CP symmetry. The obstruction must be derived in ledger entries 1–3 (projectors, embeddings, contractions), with entry 4 audited jointly; it may never be inserted from the measured CKM phase (falsifiers F16, F38).

---

18. Physical Dirac Magnitudes and the Vacuum Convention

18.1 Theorem 15 — Physical Yukawa-Magnitude Theorem

The physical running Dirac Yukawa magnitudes are the singular values of Y_f^c, invariant under biunitary canonical-frame rotations. **Proof.** H_{fL} conjugates under left rotations; its spectrum is invariant. ■

18.2 Mass relation with single counting

$m_{f,i} = (v_{cl}/\sqrt{2})\cdot y_{f,i}$ at a declared scale and convention; v_{cl} enters only here (Sections 2.3, 7.2); double counting invalidates every mass claim (falsifier F35). Physical masses additionally require v_{cl}, running, thresholds, self-energies and a declared convention (debts D9–D11).

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19. Post-Breaking Four-Sector Assembly

19.1 Pre-breaking structure

$\mathcal{L}_Y^{pre} = -\bar{Q}_L Y_u \tilde{\Phi} u_R - \bar{Q}_L Y_d \Phi d_R - \bar{L}_L Y_e \Phi e_R - \bar{L}_L Y_\nu \tilde{\Phi} N_R + \text{h.c.},$

four differently contracted gauge-covariant operators sharing doublet carriers — not a direct sum (falsifier F45).

19.2 Theorem 16 — Post-Breaking Sector-Assembly Theorem

Statement. In the minimal representation content, with no gauge-admissible inter-sector conversion operators, vacuum insertion and weak-component projection decompose the broken Dirac mass operator as

$$\boxed{\mathbf{M}\_\text{Dirac} = M\_u \oplus M\_d \oplus M\_e \oplus M\_\nu}$$

with $M_f = (v_{cl}/\sqrt{2})\cdot Y_f^c$, the last summand omitted absent N_R.

Proof. The vacuum insertion selects from each contraction one definite weak component paired with its right-handed partner; the four mass bilinears then connect pairwise disjoint carrier pairs; cross-sector blocks would need conversion operators excluded by Theorem 3 and hypothesis. ■

19.3 Majorana firewall

$M_L = M_L^T$ and $M_R = M_R^T$ are separately typed complex-symmetric forms, never entries of Y_ν^c (falsifier F25). With M_R present, $M_D = (v_{cl}/\sqrt{2})\cdot Y_\nu^c$ enters

$$\mathcal{B}_\nu = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}$$

whose Takagi spectrum belongs to the neutral-completion gate — and **can alter the physical massless count established at the Dirac level** (Section 24.2).

---

20. CKM Relative-Frame Construction

20.1 Theorem 17 — CKM Relative-Frame Theorem

Premises. Canonical Standard Model charged current at matching order; nondegenerate canonical spectra.

Statement. $V_{CKM} = U_{uL}^\dagger U_{dL}$; neither sector alone determines it.

Proof. The canonical current couples the doublet components in their common interaction frame; rotating each to its mass frame leaves the relative product; either factor is absorbable only by undiagonalising the other sector. ■

Degeneracy caveat. Degenerate singular values leave the individual frames non-unique; V_{CKM} then contains convention-dependent rotations inside degenerate subspaces, while invariants remain well defined. **Current caveat.** Nonstandard charged-current operators or vector-like mixing void the unitary-product formula.

20.2 Closure-operator origin of mixing

Both quark sectors compress the same $\mathcal{C}_Y$ through the two weak components of one doublet embedding (Section 4.5). Mixing originates in the misalignment of the projected families

$\{a_n^{(u)}\}$, $\{a_n^{(d)}\}$ on the common doublet carrier; perfect alignment gives $V_{CKM} = \mathbb{1}$. Small observed mixing is a derivational target for near-alignment of dominant embedded access — a statement about embeddings, since the spectrum is shared. The two constructions must be frozen before the relative frame is evaluated (debt D13; falsifier F36).

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21. Charged-Lepton and Neutrino-Frame Construction

21.1 Dirac case

$H_{eL} = Y_e^c Y_e^{c\dagger}$ with frame U_{eL}; for a purely Dirac completion, $H_{\nu L}^D = Y_\nu^c Y_\nu^{c\dagger}$ with frame $U_{\nu L}^D$ and

$$U_{PMNS} = U_{eL}^\dagger U_{\nu L}^D,$$

under the canonical-current premise. **Projection discipline (binding):** additional right-handed Dirac singlets do **not** by themselves require an active projection — a purely Dirac $3 \times n_R$ system has a well-defined 3×3 left mixing matrix from the left singular frame. Active projection P_A is required precisely when **additional neutral left-handed or mixed states enter the physical neutral system** — i.e., when the physical mass operator extends beyond the active left carrier (Majorana or mixed completions, left-handed sterile admixture).

21.2 Majorana or mixed case

With a complex-symmetric neutral completion, the physical frame is the Takagi frame U_ν of the full neutral operator and

$$U_{CC} = U_{eL}^\dagger P_A U_\nu,$$

generally rectangular and nonunitary on the active subspace when neutral states beyond the active carrier mix in; its dimensions and unitarity must be qualified, and its nonunitarity is itself a derived observable.

21.3 Theorem 18 — Lepton Relative-Frame Theorem

Lepton mixing is a relative-frame property of the charged-lepton mass operator and the **physical** neutral mass operator; Y_ν^c determines the neutrino frame only in a purely Dirac completion or as one block of a larger construction whose Takagi frame supersedes it; Majorana phases belong to the Takagi structure (falsifier F26).

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22. Hermitian Frame Kernels and Yukawa Compatibility

22.1 Typing discipline (binding)

An inherited flavour-frame kernel $\mathscr{F}_{fL}$ must be **typed before use**: is it (a) a quadratic form (a metric-like bilinear), (b) a generalised generator (self-adjoint with respect to a nontrivial metric), or (c) an ordinary canonical Hermitian operator? Its transformation law to the canonical carrier depends on the type: quadratic forms transform as $K^{-1/2,\dagger}\,\mathscr{F}\,K^{-1/2}$; generalised generators by similarity $K^{1/2}\,\mathscr{F}\,K^{-1/2}$; canonical operators are already in place. Evaluating commutators before typing and transporting is meaningless (falsifier F21).

22.2 Theorem 19 — Frame–Yukawa Compatibility

If the correctly transported $\mathscr{F}_{fL}^c$ satisfies $[\mathscr{F}_{fL}^c, H_{fL}] = 0$ on the common canonical carrier at the common scale, and both are nondegenerate, they share eigenvectors up to phases and ordering; the kernel may fix the left singular frame but not the singular values, the right frame, the strength, or phases outside H_{fL}. Under $H_{fL} = y_{f,0}^2\, g_f(\mathscr{F}_{fL}^c)$ with g_f strictly increasing, ordering transfers; strictly decreasing g_f reverses it; neither transfers $y_{f,0}$. The relation must be derived from $\mathscr{E}_{fL}$ and $\mathcal{C}_Y$, never assumed for its consequences (debt D14).

---

23. Cross-Sector Relations and Non-Universality

23.1 Theorem 20 — Cross-Sector Non-Universality

All four sectors compress the same $\mathcal{C}_Y$, yet a common closure spectrum does not force universal Yukawa matrices: with common $\{\lambda_\alpha, \Pi_\alpha\}$, the compressions differ whenever the embeddings differ — by representation, contraction, charge projectors or generation-depth structure. One direction admissible in f and forbidden in f′ already separates them. **Proof.** Immediate from the compression formula and Theorem 3. ■

Valid cross-sector relations require a shared closure symmetry mapping $\mathfrak{D}_d$ into $\mathfrak{D}_e$ — relating the embeddings, since the spectrum is common; numerical resemblance is not a theorem (falsifier F23). Differential embedded access to one shared spectrum is how the top quark and the electron descend from the same operator.

Part IV — Sector Constructions and Audit

24. The Four Sector Formulae

24.1 Sector compressions

$$Y_u^c = \mathscr{E}_{uL}^\dagger \, \mathcal{C}_Y \, \mathscr{E}_{uR}$$
$$Y_d^c = \mathscr{E}_{dL}^\dagger \, \mathcal{C}_Y \, \mathscr{E}_{dR}$$
$$Y_e^c = \mathscr{E}_{eL}^\dagger \, \mathcal{C}_Y \, \mathscr{E}_{eR}$$
$$Y_\nu^c = \mathscr{E}_{\nu L}^\dagger \, \mathcal{C}_Y \, \mathscr{E}_{NR} \quad \text{(iff } \mathscr{H}_{NR} \text{ exists)}$$

with M_u, M_d, $M_e = (v_{cl}/\sqrt{2})\cdot Y_{\{u,d,e\}}^c$ and $M_D = (v_{cl}/\sqrt{2})\cdot Y_\nu^c$; M_L, M_R separately typed.

24.2 Theorem 21 — Neutral Dirac Domain Theorem

Statement. Relative to the declared conventional chiral completion: absent $\mathscr{H}_{NR}$, no neutrino Dirac Yukawa operator exists. If $\dim \mathscr{H}_{NR} = n_R$,

$$\mathrm{rank}(Y_\nu^c) \le \min(3, n_R, \mathrm{rank}\, \mathcal{C}_Y),$$

sharpened by the Theorem 10 form to $\mathrm{rank}(Y_\nu^c) \le \dim \mathcal{S}_\nu$, so $n_R = 1$ forces at least two, and $n_R = 2$ at least one, exactly massless direction **at the Dirac level**.

Majorana qualification (binding). The Dirac-level massless count is provisional: subsequent Majorana blocks M_L, M_R reorganise the neutral spectrum through the Takagi problem of $\mathcal{B}_\nu$, and can lift, split or otherwise alter the massless directions. No physical massless-neutrino claim may rest on the Dirac count alone when a Majorana completion is admitted.

Proof. Domain, product-rank and nullity arguments as before; the qualification follows because the physical spectrum is that of $\mathcal{B}_\nu$, of which M_D is one block. A completion with some other opposite-chirality neutral carrier is a different completion, typed separately. ■

24.3 Up-sector target and charged-lepton firewall

Top dominance is a rank-one leading compression $\lambda_1 a_1^{(u)} b_1^{(u)\dagger}$ of magnitude $\lambda_1\|a_1^{(u)}\|\|b_1^{(u)}\|$; the derivational target is one dominant level with strong up-sector projection, a suppressed second independent level, and a bounded tail (Theorem 11). Charged-lepton localisation exponents from prior gates must reappear as derived depth profiles of $\mathscr{E}_{eL}$, $\mathscr{E}_{eR}$ and may not be exported by analogy (falsifier F27).

---

25. Stability and Conditioning

First-order variation: $\|\delta Y_f^c\|_2 \le \|\delta\mathcal{C}_Y\|_2\|\mathcal{E}_{fL}\|_2\|\mathcal{E}_{fR}\|_2 + \|\mathcal{C}_Y\|_2(\|\delta\mathcal{E}_{fL}\|_2\|\mathcal{E}_{fR}\|_2 + \|\mathcal{E}_{fL}\|_2\|\delta\mathcal{E}_{fR}\|_2)$; Weyl gives $|\delta y_{f,i}| \le \|\delta Y_f^c\|_2$ — Lipschitz stability in Hessian and embeddings. Near-degenerate singular values permit large frame rotations at scale $\|\delta Y_f^c\|_2$ over the gap (Davis–Kahan / Wedin); mixing claims near degeneracy require gap analysis. The embeddings contain $K_{cl}^{1/2}$, $K_{fL}^{-1/2}$, $K_{fR}^{-1/2}$; nearly singular metrics amplify uncertainty by $\kappa(K)^{1/2}$, and every numerical construction reports smallest kinetic eigenvalues, condition numbers and metric sensitivity (falsifier F28). A light singular value from fine cancellation among large projected contributions is unstable unless symmetry protected; tail-generated hierarchy (Theorem 11) is structurally stronger (falsifier F29).

---

26. Running, Thresholds and Observable Matching

Every constructed Y_f^c attaches to a declared scale $\mu\star$, with Hessian, projector, embeddings and metrics in one scheme; the construction supplies boundary data for $\mu\, dY_f^c/d\mu = \beta_f(\ldots)$, not an exemption from running. Decoupling closure directions, sterile carriers and completion fields require calculated threshold matching (falsifier F31); data comparison requires a declared mass convention; a match obtained by adjusting H_{cl}, P_Y or embeddings toward measured masses is a fit (falsifiers F32, F44).

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27. Numerical Non-Insertion Firewall

27.1 Predictivity conditions

$Y_f^c = \mathcal{E}_{fL}^\dagger\, \mathcal{C}_Y\, \mathcal{E}_{fR}$ is predictive only if: (1) $\mathcal{C}_Y$ is calculated from the substrate Hessian; (2) P_Y is calculated, not selected from a desired texture; (3) the embeddings are inherited from the occupancy map; (4) their normalisations are fixed by kinetic metrics; (5) no mass or mixing data enter the embeddings; (6) no eigenvalue is selected or reordered using observed masses; (7) no projector is retained or discarded because it improves agreement; (8) no phase is inserted outside the provenance ledger. **If the embeddings are arbitrary complex matrices, the Yukawa problem has merely been relocated (Theorem 5).**

27.2 Forbidden target set and blind protocol

$\mathcal{T}$_Y = {m_f, y_{f,i}, V_CKM, U_PMNS, angles, phases, ordering}; $\mathcal{T}$_Y ∩ Inputs(H_cl, K_cl, P_Y, ι_fL/R, K_fL/R) = ∅ apart from declared quarantined calibrations.

Protocol: (1) derive G_sub, verify C0; (2) derive P_cl, K_cl, form Ĥ_cl; (3) derive P_Y, form $\mathcal{C}$_Y, compute {λ_α, Π_α}; (4) derive ι_fL/R and metrics, form $\mathcal{E}$_fL/R, certify the partial-isometry/contraction discipline and the FR and SI premises; (5) establish the completion–Hessian matching CY3′ from the dressed free energy (D21); (6) assemble the compressions; (7) certify zeros (Theorem 9 + taxonomy) and rank (Theorem 10); (8) publish singular values and frames; (9) run and match; (10) only then compare. Each step freezes before the next; after (10), no upstream revision without restart. Partial-grade predictions (rank, kind-3 zeros, ordering, frames, ratios) are stated exactly, never inflated.

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28. Premise-Dependency Map

28.1 Named premises, typed by class

Class **M** — mathematical prerequisite. Class **D** — VERSF derivational prerequisite. Class **P** — predictive-grade requirement.

Premise	Class	Content	Falsifier
C0	M/D	q_0 stable; Ĥ_cl ≥ 0 on the retained carrier	F41
FR	D	Closure-side factorisation: response resolved prior to and independently of the fermion compression; no sector index on $\mathcal{G}$	F51
SI	D	Resolved response invariant under the commutant of $\mathcal{C}$_Y; distinguishing structure inside degenerate levels must be declared	F48
CY1	D	Unique scalar mediator	F49
CY2	D	Leading local single-radial-insertion completion, linear sector	F33
CY2A	D	Linear-sector insertion-algebra closure: repeated readout insertions compose by operator multiplication; no inter-insertion kernel	F52
CY3	D	Canonical radial normalisation (of φ, not the vertex)	F35
CY3′	D	Completion–free-energy matching identity in canonical units	F50
CU	D	QN-series canonical unit inheritance renders Ĥ_cl dimensionless	F50
CY4	D	Complete scalar-readout projection at leading order	F5
CY5	D/P	Embeddings derived; partial-isometry discipline; no target data	F44, F47
C2	D	Carriers, metrics, addresses independently derived	F1
C3	M/D	Complete symmetry typing (local and global distinguished), tested pre-breaking	F4
C6	M	Positive kinetic metrics; conditioning reported	F7, F28

Premise Class		Content	Falsifier
C7	P	No target insertion anywhere	F2, F6, F15, F36
C8	D	Unit Φ_0, canonical φ; v_cl only in the mass relation	F35
C9	P	Common matching scale and scheme	F30
C10	D	N_R independently established if Y_ν is used	F24
C11	D	Frame kernel typed and transported before use	F21
C12	D/P	Every phase ledger-certified; joint generalised-CP audit	F16, F38, F46
C15	M	Declared-order scope convention	F33
C16	D	Canonical charged current at matching order	F18

28.2 Dependency table

Entries: **M** mathematical prerequisite; **D** derivational prerequisite; **P** predictive-grade requirement; blank = no dependency.

Result	C0	FR,SI	CY1–3′,2A,CU	CY4	CY5	C2	C3	C6	C7	C8	C9	C10	C12	C15	C16
Thm 1 spectral/positivity	M							M							
Thm 2 embedding gauge		D			D			M							
Thm 3 gauge selection						D	M/D			D					
Thm 4 spectrum no-go					D	D									
Thm 5 form insufficiency															
Thm 6 spectral isotropy		D													
Thm 7 functional uniqueness	M	D	D	D	D				P	D				M	
Thm 8 central construction	M	D	D	D	D	D	D	M	P	D	P	as used		M	
Thm 9 texture zeros	M	D		D	D	D	D	M	P						
Thm 10 rank	M	D		D	D	D		M	P						
Thm 11 hierarchy transfer	M	D		D	D	D		M	P					M	

Result	C0	FR,SI	CY1–3′,2A,CU	CY4	CY5	C2	C3	C6	C7	C8	C9	C10	C12	C15	C16
Thm 12 CP equivalence								M					D		
Thm 13 real representative	M				D			M					D		
Thm 14 relative CP						D		M	P	D	P		D		D
Thm 15 magnitudes	M					D		M	P	D	P				
Thm 16 assembly						D	D			D		as used			
Thm 17 CKM					D	D		M	P	D	P		D		D
Thm 18 lepton frames					D	D		M	P	D	P	D	D		D
Thm 19 frame kernel						D		M			P				
Thm 20 non-universality	M	D		D	D	D	D								
Thm 21 neutral domain	M					D						D			
Empirical prediction	M	D	D	D	D/P	D	D	M	P	D	P	as used	P	M	D

The closure spectral theorem (Thm 1) depends on **no** channel-completeness or derivational premise beyond C0 and the metric positivity: only K_cl > 0, G_sub Hermitian, C0, finite dimension.

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29. Falsification Conditions

#	Failure condition	Consequence
F1	A Yukawa matrix is written before its carrier domain and codomain are derived	Carrier insertion
F2	Three closure directions are assumed because three generations are observed	Rank-target insertion
F3	Closure eigenvalues alone are claimed to determine the Yukawa matrix	Violates Thm 4
F4	A gauge-forbidden pairing is retained for its overlap, or admissibility is read post-breaking	Admissibility failure

#	Failure condition	Consequence
F5	Composite channels are silently identified with functions of the one-mode operator, or a readout contribution is omitted without a bound	Channel smuggling
F6	Any derivational quantity is selected from an observed mass	Numerical insertion
F7	Raw-basis structure is asserted as physical outside the canonical construction	Normalisation error
F8	Matrix entries are identified with physical masses	Wrong invariant
F9	Ordinary eigenvalues of a nonsymmetric Yukawa matrix are called masses	Spectral error
F10	A texture zero is introduced because it improves a fit	Structural insertion
F11	An accidental cancellation is called a symmetry-protected zero	Stability overclaim
F12	A small entry is set to zero without a derived bound	Truncation laundering
F13	Full rank is claimed from directions lacking surviving independent projections	Rank contradiction
F14	The fermion hierarchy is called derived while the spectral hierarchy (D6) is open	Transfer/derivation conflation
F15	Eigenvalue ordering is selected from the observed mass ordering	Retrospective ordering
F16	A complex phase is inserted outside the four-entry provenance ledger	Phase insertion
F17	A removable carrier phase is called physical CP violation	Rephasing error
F18	A single-sector phase is called the CKM phase, or mixing formulae used outside the canonical-current premise	Relative-frame error
F19	The up and down left embeddings are not the two components of one doublet embedding	Doublet-frame inconsistency
F20	A frame kernel is identified with a Yukawa matrix	Type conflation
F21	A frame kernel is used without typing (form / generalised generator / canonical operator) and correct transport	Bridge laundering
F22	A common closure spectrum is said to force universal sector matrices	Violates Thm 20
F23	A cross-sector relation is inferred from numerical resemblance	Post-hoc unification
F24	Y_ν is written without an independently existing N_R carrier	Hidden sterile insertion
F25	Majorana blocks are absorbed into Y_ν^c	Type error
F26	Majorana phases are attributed to Y_ν^c alone	Neutral-phase error
F27	Charged-lepton embedding exponents are exported to other sectors	Sector-domain violation
F28	Nearly singular kinetic metrics are used without conditioning analysis	Numerical instability
F29	A cancellation-generated light mass is called natural without symmetry	Fine-tuning laundering

#	Failure condition	Consequence
F30	High-scale compressions are compared directly with low-energy masses	Running/matching error
F31	Threshold corrections are invoked without calculation	Residual laundering
F32	A fitted result is described as a prediction	Grade inflation
F33	Leading linear-sector results are called full beyond the declared order	Closure-grade overclaim
F34	Vectors inside a degenerate level are given physical labels without a resolving observable	Degeneracy ambiguity
F35	v_cl is inserted from target masses or double counted	Scale insertion
F36	Observed CKM/PMNS matrices shape embeddings, P_Y or the Hessian	Mixing insertion
F37	A support structure is inferred from the desired texture	Structural circularity
F38	Projector or embedding phases are chosen to reproduce CP data	Phase fitting
F39	The right singular frames are ignored in a completeness claim	Incomplete operator claim
F40	Numerical agreement is claimed without publishing spectrum, projectors and embeddings	Audit failure
F41	A negative Hessian eigenvalue is treated as a suppressed channel rather than an instability of q_0	Stability misreading
F42	A zero claim omits its taxonomy kind (coordinate / canonical-frame / physical block)	Basis laundering
F43	Higher-order c_n are tuned rather than derived; or F values are described as reconstructible/observable from Y_f^c	Correction insertion / inference overclaim
F44	Embedding entries, P_Y or any coefficient exercised as free parameters	Problem relocation (Thm 5)
F45	The pre-breaking interaction is presented as a carrier direct sum	Assembly-type error
F46	Kinetic data omitted from the joint generalised-CP audit, or metric phases asserted as physical CP sources	Ledger/CP-typing error
F47	Individual embedding components claimed physical despite the embedding gauge	Gauge overclaim
F48	SI weakened to bare commutation, leaving hidden structure inside degenerate levels	Isotropy smuggling
F49	An independent scalar left–right completion channel exists or is introduced outside the closure-radial channel without separate typing	Mediator smuggling
F50	$c_1 = 1$ is asserted without the CY3′ matching identity and CU unit inheritance, or a vertex normalisation is silently absorbed into the construction	Vertex-normalisation smuggling

#	Failure condition	Consequence
F51	Sector-dependent dressing of the resolved response is absorbed into $\mathcal{G}$ rather than typed as a higher-order correction — the FR factorisation is violated	Factorisation smuggling
F52	An n-insertion contribution is identified with c_n $\mathcal{C}$_Y^n without deriving the insertion-composition law, or despite an independent intermediate kernel	Insertion-algebra smuggling

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30. CMSY-1 Closure Certificate

CMSY-1 requirement	Result
Substrate Hessian inheritance typed	Closed structurally
Closure Hessian and canonical form defined	Closed as definition
Stability premise C0 separated from Hermiticity	Closed as premise
$\mathcal{C}$_Y Hermitian and positive (mathematical prerequisites only)	Closed
$\mathcal{C}$_Y identified with inherited Ĥ_c	Closed as typing
Canonical embeddings with built-in normalisation, correct arrow direction	Closed as definition
Embedding-gauge theorem	Closed
Response-Form Insufficiency Theorem	Closed
Embedding-relocation firewall (chartered by Thm 5)	Closed as discipline
Shared-doublet embedding consistency	Closed as requirement
Gauge-selection theorem, local/global symmetry typing distinguished	Closed
Spectrum-insufficiency theorem	Closed
Radial-derivative definition of Y_f^c	Closed as definition
Closure-side factorisation premise FR named and fenced	Closed as premise
Spectral-Isotropy Theorem; insufficiency of bare commutation	Closed
Insertion-algebra closure premise CY2A named and fenced	Closed as premise
Leading closure-functional uniqueness, $\mathcal{G}$ = $\mathcal{C}$_Y	Closed under FR, SI, CY1–CY5 and CY2A, with c_1 = 1 as the CY3′ matching identity and the c_n typed as insertion-order coefficients composing under CY2A

CMSY-1 requirement	**Result**
Completion–Hessian matching derived microscopically	Open — D21, central physical bridge
Dimensional consistency of the central identity	Closed conditionally on CU
Canonical unit inheritance / vertex normalisation 𝒩_Y	Open — D19 unless the QN-series inheritance discharges
Central construction Y_f^c = ℰ_fL† 𝒞_Y ℰ_fR (linear sector)	Closed conditionally
Elimination of independent coefficients, γ = λ	Closed
Reconstructibility disclaimer (forward exact, backward not guaranteed)	Closed as clause
Degeneracy-safe projector form	Closed
Composite channels typed on ℋ_eff^(N); leading theorem restricted to linear sector	Closed as typing
Closure-projector texture-zero theorem	Closed
Zero taxonomy (coordinate / canonical-frame / physical block); preservation sufficient-not-necessary	Closed as discipline
Rank theorem, min over four bounds	Closed
Hierarchy-Transfer Theorem (two-sided; contraction case)	Closed
Spectral hierarchy itself	Open — dynamical (D6)
Phase ledger with transport typing	Closed
Canonical CP-Equivalence Theorem	Closed
Real-representative corollary as generalised-CP statement	Closed
General even-cycle rephasing invariants; four-plaquettes as the smallest case	Closed
Relative-sector CP theorem with convention declaration	Closed
Post-breaking assembly; pre-breaking shared-carrier statement	Closed
Majorana firewall; Dirac-level massless count qualified	Closed
CKM and lepton frame theorems with projection discipline	Closed
Frame-kernel typing discipline	Closed
Cross-sector non-universality	Closed

CMSY-1 requirement	Result
Valid Grassmann-even mediator constructions (Appendix F)	Closed as construction
Vacuum single-counting and declared-order conventions	Closed as conventions
Numerical H_cl, P_Y, spectrum, embeddings	Open — load-bearing
Effective generator 𝒜_eff^(N) and composite embeddings	Open
Higher-order c_n	Open
Sterile census, v_cl, RG, thresholds, mass conventions	Open
Fermion masses, CKM, PMNS, CP phase	Open numerically
Empirical confirmation	Open

Closure grade

CMSY-1 closes the canonical operator form and the complete conditional theorem for constructing the leading Dirac Yukawa matrices as sector-specific compressions of the scalar-readout closure Hessian on the linear primitive-channel sector. Its remaining central physical bridge is CY3′: the microscopic derivation that the radial derivative of the fermion-dressed completion operator is exactly the chiral compression of the inherited closure Hessian (debt D21). The functional form of the response is derived by the Spectral-Isotropy Theorem rather than assumed; the leading functional is proved to be the identity; the mode weights are the closure eigenvalues; the factorised form is proved insufficient without independently fixed maps; individual embedding components are gauged; texture zeros carry a three-way taxonomy; the hierarchy result is typed as a transfer theorem; and the phase audit is governed by the Canonical CP-Equivalence Theorem with general even-cycle invariants. The gate does not yet calculate the numerical matrices — the closure Hessian, the scalar-readout projector, the embeddings, the composite generator and the dynamical shape of the spectrum remain explicit bridge debts — and higher-order and composite-channel corrections remain outside the leading claim.

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31. Conclusion

The Yukawa matrices cannot remain unexplained inputs — and the explanation cannot be an accounting trick. Five channels existed through which the observed answer could have been smuggled into a purported derivation: free coefficients, an assumed functional form, free embeddings, silently absorbed composite channels, and phase misattribution. This gate closes each with a named theorem.

The mediator is the inherited closure Hessian, projected onto its scalar readout:

$$\mathcal{C}_Y = P_Y K_{cl}^{-1/2} H_{cl} K_{cl}^{-1/2} P_Y.$$

The Spectral-Isotropy Theorem derives — from commutant invariance, with bare commutation proved insufficient — that any resolved response is a scalar function of this operator, one value per eigenvalue, with Schur's lemma closing the degenerate hiding places. The uniqueness theorem collapses the function to the identity at leading order. The complete leading Yukawa operator on the linear primitive-channel sector is

$$Y_f^c = \mathscr{E}_{fL}^\dagger \mathcal{C}_Y \mathscr{E}_{fR} = \Sigma_\alpha \lambda_\alpha \mathscr{E}_{fL}^\dagger \Pi_\alpha \mathscr{E}_{fR}.$$

The Response-Form Insufficiency Theorem then locates the substance exactly: the factorised form is universal and vacuous by itself, so everything rests on the independent derivation of $\mathcal{C}_Y$ and the embeddings — with the embedding gauge stripping physical status from individual entries, and the reconstructibility disclaimer keeping forward determination distinct from backward inference.

The structural physics follows with matching honesty. Texture zeros are projector-level statements with a three-way taxonomy; only block zeros between independently fixed invariant subspaces are physical. Rank counts closure directions surviving both chiral projections. The hierarchy result is a transfer theorem — $\sigma_{k+1}(Y_f^c) \le \|\mathscr{E}_{fL}\|_2 \|\mathscr{E}_{fR}\|_2 \lambda_{k+1}$, with Weyl protection below — converting a hierarchical spectrum into a hierarchical fermion spectrum while leaving the spectrum's own shape as the open dynamical question it is. Phases obey a four-entry ledger closed by the reality of the spectrum, with the Canonical CP-Equivalence Theorem guaranteeing that normalisation transports but never manufactures CP structure, general even-cycle invariants with four-plaquettes as the smallest case, and quark CP violation remaining the relative invariant $\mathrm{Im}\,\mathrm{Tr}([H_{uL}, H_{dL}]^3)$. Composites live on their own declared-order carrier with their own generator; the leading theorem does not pretend to contain them.

The programme advance is

| substrate free-energy Hessian ⟶ closure Hessian ⟶ scalar-readout projection ⟶ chiral cross-compression ⟶ Y_u, Y_d, Y_e, Y_ν |

and the Yukawa problem is reduced to three concrete calculations:

| $H_{cl}, P_Y, \mathscr{E}_{fL/R}$ |

under the CU unit inheritance; absent it, the vertex normalisation $\mathcal{N}_Y$ joins the list (debt D19). The remaining central physical bridge is the completion–Hessian matching CY3′ (debt D21): the microscopic proof that the radial derivative of the fermion-bilinear block of the dressed free energy returns exactly $\mathscr{E}\_{fL}^\dagger \mathcal{C}\_Y \mathscr{E}\_{fR}$.

The strongest immediate successor paper is:

The Closure-Hessian Spectrum and First Blind Sector Evaluation in VERSF

Its burden: derive the mixed radial–fermion second variation of the dressed substrate free energy and prove the CY3′ matching —

$\mathcal{F}\_{sub}^{dressed} \longrightarrow \delta^2\mathcal{F}\_{sub}^{dressed} / \delta\bar{\psi}\_{eL}\, \delta\psi\_{eR} \longrightarrow \partial\varphi(\delta^2\mathcal{F}\_{sub}^{dressed} / \delta\bar{\psi}\_{eL}\, \delta\psi\_{eR})|\{\varphi_0\} = \mathscr{E}\_{eL}^\dagger \mathcal{C}\_Y \mathscr{E}\_{eR}$,

the real microscopic finish line (debt D21) — then compute $\{\lambda\_\alpha, \Pi\_\alpha\}$, derive P_Y and the charged-lepton embeddings, verify C0, FR, SI, CU and the partial-isometry discipline, and evaluate Y_e^c blindly — the first complete sector matrix with no observed fermion mass anywhere in the pipeline. Everything it derives must land in $\mathscr{E}\_{eL}^\dagger \mathcal{C}\_Y \mathscr{E}\_{eR}$, or explain structurally why it cannot.

Final Theorems

The Closure-Operator Yukawa Construction Theorem

Under C0, FR, SI, CY1–CY5 and the CY3′/CU matching identity, on the linear primitive-channel sector (CY2A types, but is not needed to eliminate, the discarded multi-insertion terms),

$Y\_f^c = \mathscr{E}\_{fL}^\dagger P\_Y K\_{cl}^{-1/2} H\_{cl} K\_{cl}^{-1/2} P\_Y \mathscr{E}\_{fR} = \Sigma\_\alpha \lambda\_\alpha \mathscr{E}\_{fL}^\dagger \Pi\_\alpha \mathscr{E}\_{fR}$,

with mode coefficients equal to the closure eigenvalues and no independent Yukawa entry admitted.

Spectral-Isotropy Theorem

Under the closure-side factorisation FR, if the resolved response is invariant under every unitary preserving $\mathcal{C}\_Y$, then it is scalar on every eigenspace and $\mathcal{G} = F(\mathcal{C}\_Y) = \Sigma\_\alpha F(\lambda\_\alpha) \Pi\_\alpha$ uniquely. Bare commutation does not suffice under degeneracy.

Leading Closure-Functional Uniqueness Theorem

Under FR, SI, CY1–CY5, CY2A and CY3′/CU, in the insertion-order expansion of the resolved response (composition by CY2A): $c_0 = 0$ (unique mediator), $c\_n = 0$ for $n \geq 2$ (single insertion), and $c_1 = 1$ as the CY3′ matching identity in inherited canonical units — not an inference from the mediator premises. The leading resolved response equals the generator, $\mathcal{G} = \mathcal{C}_Y$, with $F_{Yuk}(x) = x$ as the induced spectral function.

Response-Form Insufficiency Theorem

Every matrix admits a factorisation $\mathcal{E}_L^\dagger \, \mathcal{G} \, \mathcal{E}_R$ with positive $\mathcal{G}$ and contractive maps; the form alone constrains nothing. Predictive content requires $\mathcal{G}$, $\mathcal{E}_L$, $\mathcal{E}_R$ — dimensions, normalisation, support, phases — fixed independently of the target.

Embedding-Gauge Theorem

For any unitary U with $[U, \mathcal{C}_Y] = 0$, $\mathcal{E}_{fL} \to U\mathcal{E}_{fL}$, $\mathcal{E}_{fR} \to U\mathcal{E}_{fR}$ leaves Y_f^c invariant; individual embedding components are defined modulo the commutant.

Closure-Projector Texture-Zero Theorem and Taxonomy

$(Y_f^c)_{ij} = 0$ exactly when every projector-level pairing vanishes; zero claims are typed as coordinate, canonical-frame-preserved, or physical block zeros, and only the third kind is physics.

Closure-Operator Rank Theorem

$\operatorname{rank}(Y_f^c) \le \min\{\dim \mathcal{S}_f, \operatorname{rank} \mathcal{C}_Y, \operatorname{rank} \mathcal{E}_{fL}, \operatorname{rank} \mathcal{E}_{fR}\}$: only directions surviving both chiral projections count.

Hierarchy-Transfer Theorem

$\sigma_{k+1}(Y_f^c) \le \|\mathcal{E}_{fL}\|_2 \|\mathcal{E}_{fR}\|_2 \, \lambda_{k+1}$ ($\le \lambda_{k+1}$ for contractions), with Weyl protection $\sigma_i(Y_f^c) \ge \sigma_i(Y_f^{(k)}) - \|\mathcal{E}_{fL}\|_2 \|\mathcal{E}_{fR}\|_2 \, \lambda_{k+1}$ for $i \le k$. The spectral hierarchy itself remains a dynamical debt.

Canonical CP-Equivalence Theorem

Canonical normalisation preserves the existence or nonexistence of a generalised CP symmetry: it redistributes phase structure and cannot create or destroy a physical CP obstruction. Physical CP violation is the absence of a generalised CP symmetry of the joint kinetic-and-interaction system; quark CP violation is the relative invariant $\operatorname{Im} \operatorname{Tr}([H_{uL}, H_{dL}]^3)$.

Relative-Frame Corollary

Under canonical charged currents and nondegenerate spectra, $V_{CKM} = U_{uL}^\dagger U_{dL}$ and $U_{CC} = U_{eL}^\dagger P_A U_\nu$, with active projection required precisely when neutral states beyond the active left carrier enter the physical system.

Appendix A — Minimal Algebraic Skeleton

Substrate Hessian: $G_{sub} = \delta^2\mathcal{F}_{sub}/\delta q\delta q|_{q_0}$. *Closure Hessian:* $H_{cl} = P_{cl} G_{sub} P_{cl}$; $\hat{H}_{cl} = K_{cl}^{-1/2} H_{cl} K_{cl}^{-1/2} \geq 0$ *(C0). Scalar readout:* $\mathcal{C}_Y = P_Y \hat{H}_{cl} P_Y = \Sigma_\alpha \lambda_\alpha \Pi_\alpha$, $\lambda_\alpha \geq 0$. *Embeddings:* $\mathcal{E}_{fL} = K_{cl}^{1/2} \iota_{fL} K_{fL}^{-1/2}$, $\mathcal{E}_{fR} = K_{cl}^{1/2} \iota_{fR} K_{fR}^{-1/2}$ *(into the closure space). Embedding gauge:* $\mathcal{E} \to U\mathcal{E}$ *(both),* $[U, \mathcal{C}_Y] = 0 \Rightarrow Y_f^c$ *invariant. Isotropy: FR + SI* $\Rightarrow \mathcal{G} = F(\mathcal{C}_Y)$*; bare commutation insufficient. Uniqueness: leading resolved response equals the generator,* $\mathcal{G} = \mathcal{C}_Y$*, under FR, SI, CY1–CY5, CY2A and CY3′/CU;* $F_{Yuk}(x) = x$ *is the induced spectral function. Central construction:* $Y_f^c = \mathcal{E}_{fL}^\dagger \mathcal{C}_Y \mathcal{E}_{fR} = \Sigma_\alpha \lambda_\alpha \mathcal{E}_{fL}^\dagger \Pi_\alpha \mathcal{E}_{fR}$. *Rank-one form:* $Y_f^c = \Sigma_n \lambda_n a_n^{(f)} b_n^{(f)\dagger}$*; forward exact, backward inference not guaranteed. Texture zero: all projector pairings vanish* $\Rightarrow$ *entry zero; taxonomy: coordinate / canonical-frame / physical block. Rank:* $\mathrm{rank}(Y_f^c) \leq \min\{\dim \mathcal{S}_f, \mathrm{rank}\, \mathcal{C}_Y, \mathrm{rank}\, \mathcal{E}_{fL}, \mathrm{rank}\, \mathcal{E}_{fR}\}$. *Hierarchy transfer:* $\sigma_{k+1}(Y_f^c) \leq \|\mathcal{E}_{fL}\|_2 \|\mathcal{E}_{fR}\|_2 \lambda_{k+1}$; $|\sigma_i(Y_f^c) - \sigma_i(Y_f^{(k)})| \leq \|T_f^{(k)}\|_2$. CP: canonical normalisation preserves generalised CP; $\mathcal{J}_q = \mathrm{Im}\,\mathrm{Tr}([H_{uL}, H_{dL}]^3)$. Cycles: $\Pi_C = \Pi_a (Y^c)_{i_a j_a} (Y^c)^*_{i_{a+1} j_a}$, general even cycles. Masses: $M_f = (v_{cl}/\sqrt{2})\cdot Y_f^c$; $m_{f,i} = (v_{cl}/\sqrt{2})\cdot y_{f,i}$. Assembly: $\mathbf{M}_{Dirac} = M_u \oplus M_d \oplus M_e \oplus M_\nu$ (post-breaking). Composites: $\mathcal{H}_{eff}^{(N)} = \mathcal{H}_{cl} \oplus \mathcal{H}_{cl}^{\otimes 2} \oplus \cdots \oplus \mathcal{H}_\nabla \oplus \mathcal{H}_\Phi$-dress, generator $\mathcal{A}_{eff}^{(N)}$.

Appendix B — Spectral, Positivity and Isotropy Proofs

Hermiticity/positivity. G_{sub} is the second variation of a real functional, hence Hermitian; H_{cl}, $\hat{H}_{cl}$ inherit Hermiticity; C0 gives $\hat{H}_{cl} \geq 0$; compression by orthogonal P_Y preserves both; the spectral theorem on the finite readout channel gives $\mathcal{C}_Y = \Sigma_\alpha \lambda_\alpha \Pi_\alpha$ with $\lambda_\alpha \geq 0$. Prerequisites: $K_{cl} > 0$, $G_{sub} = G_{sub}^\dagger$, C0, finite dimension — nothing else. ■

Isotropy. The unitaries preserving $\mathcal{C}_Y$ are $\bigoplus_\alpha U(d_\alpha)$. Phase choices $U = \bigoplus e^{i\theta_\alpha} \mathbb{1}_\alpha$ kill off-diagonal blocks; full $U(d_\alpha)$ invariance within a level makes each diagonal block commute with the level's entire unitary group, hence scalar by Schur's lemma: $\Pi_\alpha \mathcal{G} \Pi_\alpha = F(\lambda_\alpha) \Pi_\alpha$. Counterexample to bare commutation: on a level of multiplicity 2, $\mathcal{G} = \Pi_\alpha M \Pi_\alpha$ with any non-scalar Hermitian M commutes with $\mathcal{C}_Y$ but is not a function of it. ■

Appendix C — Gauge-Selection Proof

For each compact factor of G_{exact} (local gauge group and compact global factors alike), Haar averaging projects the candidate contraction onto its trivial isotypic component, zero absent a singlet; exact Abelian charges require zero total vertex charge; exact discrete symmetries impose the fixed-subspace condition. The embeddings carry the exact charge projectors, so forbidden pairings are annihilated inside $\mathscr{E}_{fL}^\dagger \Pi_\alpha \mathscr{E}_{fR}$. Local invariance additionally constrains derivative couplings — relevant on $\mathscr{H}_\nabla$ within $\mathscr{H}_{eff}^{(N)}$ — and is flagged wherever those channels are derived. ■

Appendix D — Rank, Hierarchy and Preservation Proofs

Rank. $Y_f^c v = \Sigma_\alpha \lambda_\alpha \mathscr{E}_{fL}^\dagger(\Pi_\alpha \mathscr{E}_{fR} v) \in \mathscr{E}_{fL}^\dagger \mathcal{S}_f$; the three factor bounds are product-rank inequalities with rank $\mathcal{C}_Y$ counting $\lambda_\alpha > 0$. ■

Transfer. rank $Y_f^{(k)} \leq k$ gives $\sigma_{k+1}(Y_f^c) \leq \|T_f^{(k)}\|_2$ by Eckart–Young–Mirsky; submultiplicativity and $\|\Sigma_{n>k} \lambda_n \Pi_n\|_2 = \lambda_{k+1}$ give the spectral bound; Weyl gives the two-sided estimate. ■

Insufficiency (Thm 5). Given arbitrary $Y = U\Sigma V^\dagger$ (rectangular in general), take readout dimension ≥ the larger carrier dimension, $\mathcal{G} = \mathrm{diag}(\sigma_1, \ldots, \sigma_r, 0, \ldots) \geq 0$ in the standard readout basis, and separate leading-coordinate inclusions ι_L, ι_R of the two carrier dimensions, so that $\iota_L^\dagger \mathcal{G} \iota_R = \Sigma$ as a rectangular matrix. With $\mathscr{E}_L = \iota_L U^\dagger$, $\mathscr{E}_R = \iota_R V^\dagger$: $\mathscr{E}_L^\dagger \mathcal{G} \mathscr{E}_R = U \Sigma V^\dagger = Y$, and $\mathscr{E}_L^\dagger \mathscr{E}_L = \mathbb{1}$ confirms isometry. ■

Preservation (sufficiency only). $[P, K] = 0$ with $K > 0$ implies P commutes with every function of K, in particular $K^{-1/2}$; hence $P_L K_{fL}^{-1/2} Y_f K_{fR}^{-1/2} P_R = K_{fL}^{-1/2}(P_L Y_f P_R)K_{fR}^{-1/2} = 0$. The condition is not necessary: zeros can survive without it. Neither survival nor the commutation condition makes the projectors physically meaningful; kind-3 status requires independently fixed invariant subspaces (Section 11.2). ■

Appendix E — Phase, Cycle and CP-Equivalence Proofs

Reality of weights. $\lambda_\alpha \in \mathbb{R}_{\geq 0}$ by Appendix B; no phase resides in spectral weights.

Real representative. Real symmetric $\mathcal{C}_Y$ has real eigenprojectors; real embeddings give real pairings; the sum is real, and the identity transformation in the real basis is a generalised CP symmetry of the isolated sector. ■

General cycle invariant. Under $(Y^c)_{ij} \to e^{-i\alpha_i}(Y^c)_{ij} e^{i\beta_j}$, each factor pair $(Y^c)_{i_a j_a}(Y^c)^*_{i_{a+1} j_a}$ contributes $e^{-i\alpha_{i_a}} e^{i\alpha_{i_{a+1}}}$ (the β_{j_a} phases cancel within the pair); the product over the closed cycle telescopes the α phases to unity, so $\arg \Pi_C$ is invariant for every even cycle length $2n$. The four-plaquette is $n = 2$. Forest case: root each component, propagate outward making each new edge real nonnegative; no closed path, no obstruction. A support graph with a chordless $2n$-cycle ($n \geq 3$) and no four-cycle carries invariants only of the long-cycle type. ■

CP equivalence. In the convention of Section 16, the raw system admits generalised CP with unitary W's satisfying $W_{fL}^\dagger K_{fL} W_{fL} = K_{fL}^*$, $W_{fR}^\dagger K_{fR} W_{fR} = K_{fR}^*$, $W_{fL}^\dagger Y_f W_{fR} = Y_f^*$. Define

$$\tilde{W}_{fL} = K_{fL}^{1/2} W_{fL} (K_{fL}^*)^{-1/2}, \quad \tilde{W}_{fR} = K_{fR}^{1/2} W_{fR} (K_{fR}^*)^{-1/2}.$$

Unitarity: $\tilde{W}_{fL}^\dagger \tilde{W}_{fL} = (K_{fL}^*)^{-1/2} W_{fL}^\dagger K_{fL} W_{fL} (K_{fL}^*)^{-1/2} = (K_{fL}^*)^{-1/2} K_{fL}^* (K_{fL}^*)^{-1/2} = \mathbb{1}$, likewise on the right. Canonical interaction condition:

$$\tilde{W}_{fL}^\dagger Y_f^c \tilde{W}_{fR} = (K_{fL}^*)^{-1/2} W_{fL}^\dagger K_{fL}^{1/2} \cdot K_{fL}^{-1/2} Y_f K_{fR}^{-1/2} \cdot K_{fR}^{1/2} W_{fR} (K_{fR}^*)^{-1/2} = (K_{fL}^*)^{-1/2} Y_f^* (K_{fR}^*)^{-1/2} = (Y_f^c)^*.$$

The canonical system therefore admits generalised CP with the $\tilde{W}$'s and identity metrics; the construction is invertible by conjugating back with $K^{\mp 1/2}$. Canonical normalisation redistributes phase structure and cannot create or destroy a physical CP obstruction. ■

Appendix F — Valid Mediator Constructions for Resolvent Responses

Resolvent-type responses $F(\lambda) \propto 1/(m_0^2 + \alpha\lambda)$ arise from integrating out heavy mediators. Any such construction must be Grassmann even, Lorentz invariant and representation consistent: **a bosonic field cannot couple linearly in the action to a Grassmann-odd single-fermion source.** The following construction is valid.

Heavy-scalar mediator. Let X be a heavy scalar channel carrying the Higgs-like representation required by gauge invariance, with

$$\mathscr{L} \supset -X^\dagger M_X^2 X - \left(\bar{\psi}_{fL} \Lambda_f X \psi_{fR} + X^\dagger B_f \Phi + \text{h.c.} \right),$$

all couplings Grassmann even: X couples to the fermion **bilinear** $\bar{\psi}_{fL} \Lambda_f \psi_{fR}$ and to the scalar interface Φ. Integrating out X at tree level produces the cross-term

$$\bar{\psi}_{fL} \Lambda_f (M_X^2)^{-1} B_f \Phi \psi_{fR} + \text{h.c.}$$

If the heavy-mass operator is closure-dressed,

$M_X^2 = m_{0f}^2 \mathbb{1} + \alpha_f \mathcal{A}_{eff}$,

then the induced response is

$F_f(\lambda) = \alpha_f' / (m_{0f}^2 + \alpha_f \lambda)$, $\alpha_f' \equiv \Lambda_f B_f$,

with α_f' the cross-coupling combination — the product of the fermion-bilinear coupling Λ_f and the interface coupling B_f, in the normalisation of the mediator kinetic term — so that every constant in F_f is fixed by the mediator action: a valid resolvent respecting Lorentz structure, gauge representations and Grassmann parity. A heavy **vector-like fermion** mediator can likewise generate an inverse-operator response, but through a fermionic quadratic action (Grassmann-odd fields, Dirac-type kernel) — a different closure action requiring its own typing, not the bosonic quadratic form.

Positioning. At the leading single-insertion order of this gate no mediator is integrated out: the response is $\mathcal{C}_Y$ itself (Theorems 6–7). Resolvent structures belong to the higher-order series of Section 15.3, and any appearance of one must cite a mediator construction of the present valid type, never a bosonic field with fermionic linear sources (falsifier F43). ■

Appendix G — Worked Structural Examples

G.1 One surviving direction. One level λ_1 with rank-one surviving projection: $Y_f^c = \lambda_1 a_1 b_1^\dagger$, one massive fermion, magnitude $\lambda_1 \|a_1\| \|b_1\|$.

G.2 Sequential rank. $Y^{(2)} = \lambda_1 a_1 b_1^\dagger + \lambda_2 a_2 b_2^\dagger$ with independent profiles: rank two; $\sigma_3(Y^c) \leq \|\mathcal{E}_{fL}\|_2 \|\mathcal{E}_{fR}\|_2 \lambda_3$, with σ_1, σ_2 protected when $\sigma_2(Y^{(2)})$ exceeds the bound.

G.3 Large eigenvalue, no mass. $\Pi_1 \operatorname{im} \mathcal{E}_{fR} = 0$: the largest eigenvalue contributes nothing to sector f. Spectral size without surviving projection is inert.

G.4 Collinear projections. $\lambda_1 > \lambda_2 > \lambda_3$ with $a_1 \parallel a_2 \parallel a_3$: rank one, two exactly massless Dirac directions, regardless of eigenvalue spread.

G.5 Degenerate level with hidden structure (why SI matters). On a level of multiplicity 2, a response $\mathcal{G}$ commuting with $\mathcal{C}_Y$ may contain an arbitrary 2×2 block — e.g. a rotation that swaps two generation profiles — invisible to commutation but forbidden by SI. Without SI the "spectrum" could carry a hidden matrix; with SI, Schur's lemma reduces the block to a scalar.

G.6 Insufficiency in action (why Thm 5 matters). Take the observed Y_e mass matrix, SVD it, and build $\mathcal{G} = \Sigma$, $\mathcal{E}$'s from U, V: a perfect "closure-operator construction" of the data with zero content. Only the independent derivation of $\mathcal{C}\_Y$ and the embeddings distinguishes the present gate from this vacuity.

G.7 Coordinate zero versus physical zero. Y^c with $(Y^c)_{13} = 0$ in one basis: a canonical unitary rotation of the left frame generically fills it — kind-1, not physics. The same entry forced to zero by a charge projector inside $\mathscr{E}_{eL}$ that annihilates every pairing between the address-1 left subspace and the address-3 right subspace: kind-3, physical.

G.8 Long-cycle invariant. A 3×3 support pattern with nonzero entries (1,1),(1,2),(2,2),(2,3),(3,3),(3,1) is a chordless six-cycle with no four-cycle; its unique rephasing invariant is the six-edge Π_C, not any plaquette.

G.9 Misalignment and CKM. Both quark sectors compress the same $\mathcal{C}_Y$ through one doublet embedding; with dominant profiles at angle θ, the leading CKM structure is a rotation by θ in the heavy 2-plane. Small mixing = nearly aligned dominant embedded access to a shared spectrum.

G.10 One spectrum, four sectors. Common $\{\lambda_\alpha, \Pi_\alpha\}$, different $\mathscr{E}_f$: four different matrices sharing every spectral weight — non-universality in one line.

Appendix H — Open Bridge-Debt Ledger

Debt	Required content	Status
D1 — Substrate-Hessian debt	Derive G_{sub} and verify C0 stability	Open — load-bearing
D2 — Closure-restriction debt	Derive P_{cl} and K_{cl}; form $\hat{H}_{cl}$	Open — load-bearing
D3 — Readout-projector debt	Derive P_Y; verify CY4 completeness	Open — load-bearing
D4 — Spectrum debt	Compute $\{\lambda_\alpha, \Pi_\alpha\}$ of $\mathcal{C}_Y$	Open — load-bearing
D5 — Embedding debt	Derive $\iota_{fL/R}$; form $\mathscr{E}_{fL/R}$; certify partial-isometry/contraction discipline and fix the embedding gauge upstream	Open — load-bearing
D6 — Spectral-shape debt	Establish whether and why the closure spectrum is hierarchical	Open — load-bearing for hierarchy claims
D7 — Composite/nonlinear debt	Derive $\mathcal{A}_{eff}^{(N)}$, extended embeddings, and the c_n series, or bound them	Open
D8 — Factorisation and isotropy debt	Verify FR and SI for the resolved response — in particular that fermion-loop dressing induces no	Open — load-bearing

Debt	Required content	Status
	sector dependence — declaring any degeneracy-resolving structure	
D9 — RG debt	Run the constructed operators to comparison scales	Open
D10 — Threshold debt	Match closure and sterile thresholds	Open
D11 — Observable-mass debt	Convert running operators to declared mass observables	Open
D12 — Phase-audit debt	Certify the phase content of Π_α, $\mathscr{E}_{fL/R}$ and the transported kinetic structure; perform the joint generalised-CP audit	Open — load-bearing for CP
D13 — Relative-frame debt	Evaluate the CKM and lepton relative frames blindly	Open
D14 — Frame-bridge debt	Type, transport and derive any frame-kernel compatibility	Open
D15 — Sterile-census debt	Derive $\mathscr{H}_{NR}$ and n_R	Open — load-bearing for Y_ν
D16 — Blind-freeze debt	Publish spectrum, projectors and embeddings before data comparison	Open procedural requirement
D17 — First-sector debt	Evaluate one complete sector (recommended: Y_e^c) blindly	Open — load-bearing
D18 — Empirical debt	Compare completed predictions with data	Open
D19 — Vertex-normalisation debt	Verify the CU canonical-unit inheritance from the QN-series, or derive the vertex normalisation $\mathcal{N}_Y$ explicitly	Open — load-bearing for $c_1 = 1$
D20 — Insertion-algebra debt	Derive CY2A from the closure response action, or demonstrate that every nontrivial inter-insertion kernel belongs to $\mathscr{H}_{eff}^{(N)}$	Open — load-bearing for the higher-order classification
D21 — Completion–Hessian matching debt	Derive CY3′ directly from the fermion-dressed substrate free-energy functional: calculate the mixed radial–fermion second variation and prove that its canonical chiral block equals $\mathscr{E}_{fL}^\dagger \mathcal{C}_Y \mathscr{E}_{fR}$, with no additional operator or sector-dependent coefficient	Open — central physical bridge

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