The Coloured Participation Fraction Beyond 3/12 in VERSF

▲ Programme Milestone — Coloured Participation and Sector-Weighting Series Gate CPF-1 / Positive Participation Functional, Census-to-Dynamics Lift, Colour-Representation Compression, Sector-Contrast Uniqueness, Confinement Separation, and Numerical Non-Insertion Closure

Keith Taylor VERSF Theoretical Physics Programme Coloured Participation and Sector-Weighting Series Tier A conditional gate-theorem paper, CPF-1 July 2026

General Reader Summary

An earlier VERSF result counts the internal channels through which the theory's basic response can flow. There are twelve of them. Three of the twelve carry colour — the property the strong nuclear force acts on. The other nine do not.

The tempting next step is a simple fraction: three out of twelve, one quarter. This paper is about why that fraction is exactly right and, at the same time, seriously incomplete.

It is right as a count. Three of the twelve channels really are coloured, and nothing in this paper disturbs that. But a count answers only one question: how many of the channels are coloured? It does not answer the question a physicist usually wants: how much of what actually happens is carried by those channels?

The difference matters everywhere. Three of a committee's twelve seats belonging to one group tells you the seat share, not the influence — that depends on how much weight each seat carries in the decisions being made. Three of an ensemble's twelve instruments being brass does not tell you what fraction of the sound is brass — that depends on how loudly each instrument plays, and on whether you are measuring volume, melody, or time spent playing. A count gives every channel the same weight. A real physical process need not. The coloured channels might contribute more than their share, less, or exactly in proportion — and the answer can differ depending on what is being measured: energy, interaction strength, response to a particular probe.

With that distinction in place, the paper proves five things, stated here without the machinery.

First, any honest way of weighing the channels — any audit that never assigns a negative weight and combines consistently when situations are mixed — is forced into a single mathematical form. There is no exotic alternative scheme; the form of the question is settled before any physics is put in.

Second, the symmetry of colour forces the three coloured channels to share one common weight. No consistent audit can favour "red" over "blue" or "green"; the three move together as a unit.

Third — and this is the paper's sharpest result — the final fraction depends on exactly one number: how the common weight of the coloured trio compares with the average weight of the other nine. Everything else about the twelve channels, however intricate, is invisible to this particular fraction. Two completely different physical situations that happen to share this one comparison give exactly the same coloured share.

Fourth, if the coloured channels carry the same average weight as the rest, the answer is exactly one quarter — the count survives. If they carry more, the fraction rises; less, it falls. So "beyond 3/12" means going beyond the assumption of equal weights. It does not mean the true answer must be bigger than a quarter. It might be smaller.

Fifth, counting and symmetry alone can justify any answer at all, from zero to one hundred per cent. The paper constructs an explicit example for every possible value. The consequence is a discipline the whole VERSF programme enforces on itself: no number beyond the count is a prediction until the theory independently derives that one comparison number — without peeking at the answer it hopes to obtain. A resemblance between one quarter, or any adjusted fraction, and some percentage measured elsewhere in physics proves nothing by itself.

The paper also fences off the natural misreadings. This fraction is not a count of free coloured particles — confinement forbids those from appearing in isolation, yet coloured channels can still do real work inside composite objects, the way individually charged parts sit inside a neutral atom. It is not a share of a proton's mass, not a share of the matter in the universe, and not yet a probability of anything. Each of those is a different question, needing its own separately built bridge.

In one sentence:

Three out of twelve is an exact count; turning it into a physical share requires deriving exactly one number — how heavily the coloured channels weigh against the rest — and this paper proves that nothing less will do, nothing more is needed, and no such number has been derived yet.

Scope and Closure Box

Claim	CPF-1 status
A twelve-direction VERSF response carrier exists	Inherited premise
It contains one rank-three colour triplet and a rank-nine colour-singlet complement	Inherited premise
A positive carrier metric has been physically derived from VERSF primitives	Open dependency

Claim	CPF-1 status
All CPF-1 positive-functional results are conditional on that metric	Closed conditionally
3/12 = 1/4 is the exact raw census fraction	Closed
A rank ratio is not automatically a dynamical fraction	Closed
A physical participation audit must be explicitly typed	Closed
Audits satisfying the positive-linear-extension premise P6 admit a positive trace representation	Closed
The coloured fraction is a positive weighted projector expectation	Closed
The fraction lies in [0, 1]	Closed
The coloured projector must be defined by representation content	Closed
Finite-carrier SU(3)_c twirling leaves the aggregate fraction unchanged	Closed
One irreducible triplet carries one invariant mean weight	Closed
Colour symmetry does not fix the triplet-to-singlet relative weight	Closed
The aggregate fraction compresses to one ratio r_X	Closed
1/4 is recovered exactly when the two sector means agree	Closed
Full uniformity is sufficient but not necessary	Closed
The correction is the expectation of one centred sector-contrast operator	Closed
All operator anisotropy orthogonal to that contrast is invisible to this audit	Closed
Every value in [0, 1] can otherwise be constructed	Closed
A factorisation W_X = A_X† G_X A_X guarantees positivity	Closed
Factorisation alone does not constitute a physical derivation	Closed
Basis covariance and auxiliary partial-trace consistency hold	Closed
Universal multiplicative running cancels	Closed
Differential sector running obeys an exact log-odds identity	Closed
Small operator perturbations produce bounded fractional changes	Closed
Internal coloured participation is distinct from free asymptotic colour	Closed
Confinement does not force internal coloured participation to vanish	Closed
The canonical VERSF participation operator W⋆	Open
The numerical sector ratio r⋆	Open
A universal context-independent coloured fraction	Open
A parton momentum, hadron-mass, vacuum, or cosmological fraction	Not derived
Empirical confirmation	Not established

Abstract

A rank-three colour-triplet sector inside a twelve-direction VERSF carrier yields the exact raw census fraction 3/12 = 1/4. That ratio is a representation count, not automatically a probability,

energy share, interaction share, spectral contribution, occupation fraction, or observable abundance.

This paper establishes the Coloured Participation Fraction Beyond 3/12 theorem. The gate-level carrier is treated as a finite positive-metric internal representation and response space,

$\mathcal{H}_{12} = \mathcal{H}_{col} \oplus \mathcal{H}_0$, dim $\mathcal{H}_{col} = 3$, dim $\mathcal{H}_0 = 9$,

where $\mathcal{H}_{col}$ carries one irreducible $SU(3)_c$ triplet and $\mathcal{H}_0$ contains nine colour singlets. It is not identified with the complete gauge-invariant physical Hilbert space of local QCD.

For a declared participation audit X assumed under premise P6 to extend to a nonzero positive linear functional on the finite carrier, finite-dimensional trace duality gives a unique representation by a nonzero positive operator W_X. The aggregate coloured share is

$f_{col}^X = \mathrm{Tr}(\Pi_{col} W_X) / \mathrm{Tr}\, W_X$,

where Π_{col} is the representation projector onto the triplet.

Finite-representation $SU(3)_c$ twirling preserves both numerator and denominator. Schur reduction consequently gives

$\mathcal{T}_c(W_X) = w_{col}^X \cdot I_3 \oplus W_0^X$,

so that all triplet-component freedom collapses to one invariant weight. Defining the mean complement weight and the relative sector ratio

$r_X = \bar{w}_{col}^X / \bar{w}_0^X$,

the exact fraction is

$f_{col}^X = r_X / (r_X + 3)$.

The census value is recovered if and only if the two sector means agree. Full identity weighting is sufficient but not necessary.

The paper introduces the centred contrast operator

$Q_{col} = \Pi_{col} - \frac{1}{4} \cdot I_{12}$

and proves

$f_{col}^X - 1/4 = \mathrm{Tr}(Q_{col} W_X) / \mathrm{Tr}\, W_X$.

This is the exact one-direction theorem: on the trace-one affine space of normalised participation operators $\rho_X = W_X / \mathrm{Tr}\, W_X$, the aggregate coloured fraction detects exactly one traceless operator direction, Q_col. All Hilbert–Schmidt-orthogonal anisotropy is invisible, and two audits yield the same coloured fraction if and only if their normalised operators share one Q_col projection (Theorem 8′).

A constructive underdetermination theorem then proves that every value in [0, 1] is realised by a positive colour-invariant operator on the same $3 \oplus 9$ carrier, with the two boundary values attained exactly by the rank-supported extreme operators. The census, positivity, and colour symmetry therefore determine no numerical physical fraction.

The paper further derives the response-map architecture

$$W_X = A_X^\dagger G_X A_X, \quad G_X \geq 0,$$

the exact relative-running identity

$$d f_{col}^X / d \ln \mu = f_{col}^X (1 - f_{col}^X)(\gamma_{col}^X - \gamma_0^X),$$

its integrated log-odds crossing criterion, basis and coarse-graining consistency, sharp perturbative stability bounds with a sign-preservation corollary, and a two-way separation between internal coloured participation and asymptotic colour confinement.

CPF-1 closes the mathematical census-to-participation lift and identifies the exact single dynamical datum required for a numerical result. Numerical closure remains open until VERSF independently derives the normalised sector contrast — via a canonical participation operator or directly as r_X or κ_X — together with the scale prescription, local-gauge embedding, and observable bridge, without using the intended fraction as input.

Contents

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1. Aim, Inheritance, and Novelty

1.1 Inherited census structure

CPF-1 begins from a preceding VERSF census result: the twelve-direction internal channel carrier established by the Standard Model Census Series, anchored provisionally at gate SC-1 (generation/species census theorem) [17], with the possible role of SC-2 (world-to-framework occupancy map) to be settled at Master Ledger verification (Appendix E, D14). Its SU(3)_c colour identification is supplied through the Gauge-Completion Series gate GC-1 (gauge-census closure theorem) [18]. Under the declared channel-counting convention inherited from the census series, the admissible finite carrier contains twelve internal directions, $\mathscr{H}_{12}$. Its colour-representation content is

$$\mathscr{H}_{12} = \mathscr{H}_{\mathrm{col}} \oplus \mathscr{H}_0, \quad \dim \mathscr{H}_{\mathrm{col}} = 3, \quad \dim \mathscr{H}_0 = 9.$$

The coloured block carries one irreducible triplet representation of SU(3)_c. The complement is colour singlet under this gate-level bridge.

The present paper does not rederive that representation census. It makes its dependency explicit.

If the inherited carrier or its colour decomposition fails, CPF-1 does not survive.

1.2 Nature of the carrier

The notation $\mathscr{H}_{12}$ does not assert that every carrier vector is a freely preparable physical state of local QCD.

It denotes a finite positive-metric internal space used to organise:

- admissible channel directions;
- representation content;
- linearised response maps;
- positive participation audits.

The complete physical state space of a gauge theory must additionally satisfy local gauge constraints, Gauss-law or BRST conditions, confinement structure, and observable dressing requirements.

The colour-triplet block is therefore a representation sector of the internal audit carrier, not a claim that isolated colour-triplet vectors are asymptotic particles.

1.3 The census statement

The unweighted coloured census is

$f_{col}^{census} = \dim \mathscr{H}_{col} / \dim \mathscr{H}_{12} = 3/12 = 1/4.$

This is exact under the inherited counting convention.

It contains no information about amplitudes, couplings, residues, kinetic norms, response strengths, energies, state occupations, cross sections, parton distributions, or cosmological abundances.

1.4 The new problem

The new question is:

Given the inherited $3 \oplus 9$ representation carrier, what is the strongest invariant statement that can be made about coloured physical participation before a detailed dynamical operator has been derived?

The answer must identify:

1. the admissible mathematical form of a participation fraction;
2. the consequences of colour symmetry;
3. the exact condition for recovering 1/4;
4. the unique operator direction that can move the result away from 1/4;
5. the information still missing before a number can be predicted.

1.5 Standard mathematical ingredients

CPF-1 uses established finite-dimensional tools: positive linear functionals and trace representation; representation projectors; compact-group averaging; Schur's lemma; Hilbert–Schmidt operator decomposition; quotient differentiation; trace-norm perturbation bounds.

These mathematical tools are not claimed as new.

1.6 CPF-1-specific contribution

The CPF-1 contribution is their assembly into a single VERSF closure theorem for the inherited $3 \oplus 9$ carrier.

Specifically, CPF-1:

1. separates the exact coloured census from all dynamical interpretations;
2. proves the unique positive weighted-projector form of the aggregate audit;
3. compresses the coloured block by representation symmetry;
4. reduces the aggregate lift to one relative sector weight;
5. identifies the unique centred contrast operator seen by the fraction;
6. proves that all orthogonal operator anisotropy is invisible;
7. proves constructively that every numerical value is otherwise admissible;
8. establishes the exact requirements for a future numerical derivation;
9. prevents confusion with free colour, hadron mass, parton momentum, or cosmological abundance.

1.7 Core aim

The core aim is:

To prove the exact census-to-participation lift for the inherited $3 \oplus 9$ VERSF carrier, isolate the one operator-space contrast that controls the aggregate coloured fraction, and demonstrate that no numerical fraction beyond 3/12 is derivable without an independently fixed normalised sector contrast, for which a canonical participation operator is one sufficient route.

1.8 Four levels of closure

CPF-1 distinguishes four levels of closure for the coloured fraction:

1. **Census closure.** 3/12 = 1/4. Closed by inheritance and rank counting.
2. **Functional closure.** $f_{col}^X = \mathrm{Tr}(\Pi_{col}\, \rho_X)$ is the unique admissible form of any participation audit satisfying the positive-linear-extension premise P6. Closed by this paper.
3. **Dynamical closure.** VERSF derives $\mathrm{Tr}(Q_{col}\, \rho_X)$, equivalently r_X, from independent dynamics. Open; the exact target of the successor gate.
4. **Observable closure.** The functional is matched to a gauge-invariant measurement. Open.

CPF-1 closes levels 1 and 2 and identifies the single missing datum at level 3.

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2. Typed Carrier and Premises

2.1 P1 — Finite response-carrier premise

There exists a twelve-dimensional complex vector space $\mathscr{H}_{12}$ equipped with a positive physical response metric. The carrier and its counting convention are provisionally inherited from gate SC-1 (§1.1, D14).

It is an internal representation and audit carrier, not automatically the complete gauge-invariant Hilbert space.

Failure condition: the twelve directions mix incompatible counting conventions, include null or negative-norm directions, or are not defined on one common positive carrier.

2.2 P2 — Orthogonal sector-decomposition premise

The carrier decomposes as

$\mathscr{H}_{12} = \mathscr{H}\_col \oplus \mathscr{H}_0$

with orthogonal dimensions 3 + 9 = 12. The colour-sector identification is inherited through gate GC-1.

Failure condition: the sectors overlap, the ranks do not sum to twelve, or the inherited SC-1/GC-1 decomposition fails upstream.

2.3 P3 — Colour-representation premise

The action of the finite internal colour representation is

$U\_c(g) = U_3(g) \oplus I_9$, $g \in SU(3)\_c$,

where U_3 is one irreducible triplet.

Failure condition: the three selected directions are merely labelled as colours but do not transform as a declared triplet.

2.4 P4 — Representation-projector premise

There exists an orthogonal projector $\Pi\_col : \mathscr{H}_{12} \to \mathscr{H}\_col$ satisfying

$\Pi\_col^2 = \Pi\_col$, $\Pi\_col^\dagger = \Pi\_col$, $\mathrm{Tr}\ \Pi\_col = 3$.

It is defined by representation content, not by a preferred red–green–blue basis.

Failure condition: the proposed projector fails idempotence, self-adjointness, or rank three, or is defined only through basis labels.

2.5 P5 — Participation-typing premise

Every physical fraction carries a label X specifying what is being measured.

Where relevant, X includes the response class; the state or ensemble; the scale; the renormalisation scheme; the kinematic region; the boundary conditions; the normalisation convention.

A bare symbol f_col is incomplete unless only the census fraction is meant.

Failure condition: a numerical fraction is asserted without a declared participation type.

2.6 P6 — Positive affine-audit premise

A participation audit assigns nonnegative weights to positive effects and composes affinely under probabilistic mixtures.

The premise asserts, as an explicit assumption rather than a derived equivalence, that the declared audit extends to a nonzero positive complex-linear functional Ω_X on End($\mathscr{H}_{12}$) — equivalently, to a positive real-linear functional on the real vector space of Hermitian operators, together with its unique complex-linear extension. Affine behaviour under probabilistic mixtures motivates this extension; it does not by itself construct it, and the extension is what Theorem 1 consumes.

Failure condition: the proposed "participation" assignment is nonlinear under mixtures, negative on a positive effect, undefined on the carrier algebra, or admits no linear extension to that algebra.

2.7 P7 — Common-denominator premise

The coloured numerator and total denominator must refer to the same participation type; the same state or ensemble; the same scale and scheme; the same kinematic region; the same carrier; the same normalisation.

Failure condition: numerator and denominator are evaluated on different operators, scales, states, or conventions.

2.8 P8 — Twirl-representative interpretation premise

The twirled operator $\mathcal{T}_c(W_X)$ is a physically faithful representative of the declared audit for aggregate colour-sector questions: no colour-frame-dependent quantity is smuggled in as "the" audit.

This premise is purely interpretive. Theorem 2 proves unconditionally, from P1–P4 and P6 alone, that the finite twirl preserves the aggregate fraction for every positive operator; P8 asserts only that the twirled representative answers the physically intended question, not that the preservation holds. It is likewise not asserted to be the full projection over local gauge transformations of continuum QCD.

Failure condition: the declared physical audit is essentially colour-frame-dependent, so that substituting the twirled representative changes the physical question being asked.

2.9 P9 — Dynamical-origin premise

Where an operator-level physical derivation is claimed, the participation operator must arise from an independently specified response construction such as

$$W_X = A_X^\dagger G_X A_X, \quad G_X \geq 0,$$

with the map A_X and metric G_X derived independently of the desired value of f_{col}^X.

Where only the aggregate fraction is claimed, VERSF may instead derive r_X or κ_X directly, provided that derivation is independently specified and does not use the target fraction.

Failure condition: any parameter of A_X or G_X is calibrated from the target fraction, or a directly derived r_X or κ_X is calibrated from the target fraction.

2.10 P10 — No-double-counting premise

The triplet multiplicity may be represented by the rank-three trace or by one explicitly per-component weight multiplied by three.

It may not be included twice.

Failure condition: colour multiplicity appears both inside the trace and as a separate enhancement factor.

2.11 P11 — Local-gauge and confinement premise

Internal triplet participation is not identified with freely observable asymptotic colour.

Any local gauge-theory bridge must use gauge-invariant composites, dressing, Wilson-line completion, BRST-compatible operators, or an equivalent admissible construction.

Failure condition: bare separated colour fields are treated as gauge-invariant observables, or internal participation is read as free-colour abundance.

2.12 P12 — Observable-bridge premise

An empirical claim requires an explicit map from the internal participation audit to a gauge-invariant observable.

Without such a bridge, the result remains an internal structural statement.

Failure condition: an empirical percentage is claimed without a declared measurement functional.

2.13 P13 — No-retrodictive-weight premise

No parameter in the participation operator may be calibrated from the target coloured fraction and then presented as its derivation.

Failure condition: the inverse formula $r_X = 3f_{col}^X / (1 - f_{col}^X)$ is used as input rather than as audit.

2.14 P14 — No-universalisation premise

A result obtained for one participation type X may not be promoted to a universal coloured value unless VERSF proves that all relevant normalised participation operators share one Q_{col} projection. A universal participation functional requires the stronger condition that the corresponding normalised functionals agree on the relevant operator algebra.

Failure condition: one context-specific fraction is announced as a universal coloured constant.

2.15 Premise-dependency map

Theorem	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
T1 — Positive linear-functional representation	●				●	●								
T2 — Aggregate twirl invariance	●	●	●	●		●								
T3 — Schur-compressed coloured block	●	●	●	●		●								
T4 — Raw census fraction	●	●		●						●				
T5 — Exact one-ratio participation	●	●		●	●	●	●			●				
T6 — Census-recovery criterion	●	●		●	●	●	●							
T7 — Exact sector contrast	●	●		●	●	●	●							
T8 — One visible operator direction	●	●		●	●	●	●							

Theorem	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
T9 — Constructive underdetermination	●	●	●	●		●								
T10 — No independently fixed contrast, no numerical prediction	●	●	●	●	●	●	●		●				●	●
T11 — Positive response factorisation	●					●			●					
T12 — Basis covariance	●	●		●		●								
T13 — Auxiliary partial-trace consistency	●	●		●		●	●							
T14 — Exact relative running	●	●		●	●	●	●							
T15 — Trace-norm stability	●	●		●		●								
T16 — Internal/asymptotic colour separation	●	●	●	●							●	●		

If a premise in a theorem's row fails, that theorem's CPF-1 status reverts to open.

P8 is deliberately absent from the rows of Theorems 2, 3, 9, and 10: the twirl-invariance identity, the Schur compression of the twirled operator, the constructive realisation of arbitrary fractions by colour-invariant operators, and the non-insertion argument are purely mathematical (§4.5); none substitutes a twirled operator for a physical audit. P8 governs only the physical substitution of the twirled operator for the original audit, and it is load-bearing exactly where that substitution is treated as answering the declared physical question.

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3. Positive Participation-Functional Representation

3.1 Definition

Let

$\Omega\_X : \mathrm{End}(\mathscr{H}_{12}) \to \mathbb{C}$

be a nonzero positive linear functional representing the participation audit X. Positivity means

$A \geq 0 \Longrightarrow \Omega\_X(A) \geq 0.$

Normalise it by

$\omega\_X(A) = \Omega\_X(A) / \Omega\_X(I_{12}),$

so that $\omega\_X(I_{12}) = 1$.

The word "state" may be used for ω_X in the finite operator-algebra sense. It does not imply that ω_X is the density operator of a freely preparable asymptotic quantum system.

3.2 Theorem 1 — Positive Linear-Functional Representation Theorem

Statement. Every nonzero positive linear participation functional on the finite carrier has a unique representation

$$\Omega_X(A) = \mathrm{Tr}(W_X A)$$

for a unique nonzero positive operator $W_X \geq 0$. Consequently,

$$\omega_X(A) = \mathrm{Tr}(W_X A) / \mathrm{Tr}\, W_X.$$

Proof. The Hilbert–Schmidt pairing

$$\langle B, A\rangle_{HS} = \mathrm{Tr}(B^\dagger A)$$

is nondegenerate on the finite-dimensional operator algebra [15]. Therefore there exists a unique operator W_X such that $\Omega_X(A) = \mathrm{Tr}(W_X A)$ for every A.

For a rank-one projector $P_\psi = |\psi\rangle\langle\psi|$, positivity of Ω_X gives

$$\langle\psi| W_X |\psi\rangle = \Omega_X(P_\psi) \geq 0$$

for every $|\psi\rangle$. Hence $W_X \geq 0$.

Since the functional is nonzero,

$$\mathrm{Tr}\, W_X = \Omega_X(I_{12}) > 0.$$

Normalisation gives the stated form. ■

3.3 Coloured participation fraction

The coloured participation fraction is the expectation of the representation projector:

$$f_{col}^X = \omega_X(\Pi_{col}) = \mathrm{Tr}(\Pi_{col} W_X) / \mathrm{Tr}\, W_X.$$

Because $0 \leq \Pi_{col} \leq I_{12}$ and $W_X \geq 0$,

$$0 \leq f_{col}^X \leq 1.$$

Define the normalised participation operator

$\rho_X = W_X / \mathrm{Tr}\, W_X$, $\rho_X \geq 0$, $\mathrm{Tr}\, \rho_X = 1$.

Then the foundational identity takes its simplest form:

$f_{col}^X = \mathrm{Tr}(\Pi_{col}\, \rho_X)$.

All subsequent aggregate statements are statements about ρ_X; the overall scale of W_X is discarded by normalisation once and for all.

3.4 Affinity and extreme values

The map $\omega \mapsto \omega(\Pi_{col})$ is affine on the convex set of normalised positive functionals. Its range is exactly [0, 1], with

$f_{col}^X = 0 \Leftrightarrow \mathrm{supp}\, W_X \subseteq \mathscr{H}_0$, $f_{col}^X = 1 \Leftrightarrow \mathrm{supp}\, W_X \subseteq \mathscr{H}_{col}$.

The boundary values are therefore attained precisely by nonzero audits supported entirely inside one sector. Every interior value satisfies $C_X > 0$ and $N_X > 0$, so the audit has nonzero support in both sectors. This does not require the participation operator to be an incoherent statistical mixture: it may also contain admissible inter-sector coherence — for example, a rank-one operator built from a superposition across $\mathscr{H}_{col}$ and $\mathscr{H}_0$ — before representation twirling, and Theorem 2 subsequently removes such off-diagonal blocks for the purpose of this aggregate audit. This support characterisation is used in Theorem 9.

3.5 Interpretation

The weighted-trace form is not an arbitrary convention once the audit is assumed to satisfy the positive-linear-extension premise P6.

What remains open is not the mathematical representation. What remains open is which positive functional — or equivalently which W_X — the VERSF dynamics supplies.

3.6 Scale invariance under common normalisation

For any positive scalar a, the rescaling $W_X \mapsto a \cdot W_X$ leaves the fraction unchanged:

$f_{col}^X[a \cdot W_X] = f_{col}^X[W_X]$.

Only relative weights can change the fraction.

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4. Colour-Representation Projector and Finite-Carrier Twirling

4.1 Representation-defined projector

The coloured projector may be expressed in a basis where

$\Pi\_col = I_3 \oplus 0_9$.

More invariantly, for an irreducible representation R of a compact group, the corresponding isotypic projector is

$\Pi\_R = d\_R \int d\mu(g)\, \chi\_R(g)^*\, U\_c(g)$.

For the inherited multiplicity-one triplet, Π_col = Π_**3** (character-projector construction: [8], [13]).

4.2 Finite colour-representation twirl

Define

$\mathcal{T}\_c(W) = \int\_\{SU(3)\}\, d\mu(g)\, U\_c(g)\, W\, U\_c(g)^\dagger$.

This averages over the finite internal colour representation carried by $\mathscr{H}_{12}$.

It is not asserted to perform the complete local-gauge projection of continuum QCD.

4.3 Theorem 2 — Aggregate Twirl-Invariance Theorem

Statement. For every positive operator W,

$\mathcal{T}\_c(W) \geq 0$, Tr $\mathcal{T}\_c(W)$ = Tr W, $[\mathcal{T}\_c(W), U\_c(g)] = 0$ for every g, Tr[Π_col $\mathcal{T}\_c(W)$] = Tr(Π_col W).

Therefore

$f\_col[\mathcal{T}\_c(W)] = f\_col[W]$.

Proof. Each conjugate U_c(g) W U_c(g)† is positive if W ≥ 0. Their Haar average is positive.

Unitary invariance of the trace gives Tr[U_c(g) W U_c(g)†] = Tr W, so the total trace is unchanged.

Haar invariance gives

$U\_c(h)\ \mathcal{T}\_c(W)\ U\_c(h)^\dagger = \mathcal{T}\_c(W)$,

which is the stated commutation.

Finally, because Π_col commutes with the colour representation,

$U\_c(g)^\dagger\ \Pi\_col\ U\_c(g) = \Pi\_col$.

Hence

$\mathrm{Tr}[\Pi\_col\ \mathcal{T}\_c(W)] = \int d\mu(g)\ \mathrm{Tr}[\Pi\_col\ U\_c(g)\ W\ U\_c(g)^\dagger] = \int d\mu(g)\ \mathrm{Tr}[U\_c(g)^\dagger\ \Pi\_col\ U\_c(g)\ W] = \mathrm{Tr}(\Pi\_col\ W)$.

Both numerator and denominator are unchanged. ■

4.4 Exact scope of the result

The theorem does not say that the original W is a gauge-invariant local observable.

It says that the value of this aggregate finite-carrier trace ratio depends only on the colour-invariant component obtained by the representation twirl. Without loss of generality, every subsequent aggregate statement may therefore be evaluated on $\mathcal{T}\_c(W\_X)$.

4.5 Separation of mathematical and physical content

Theorems 2 and 3 are purely mathematical statements: because Π_col commutes with the representation and the trace is unitarily invariant, twirling preserves this aggregate ratio automatically, and Schur's lemma compresses the twirled triplet block, with no covariance premise required for either. Premise P8 enters only at the interpretive level: treating $\mathcal{T}\_c(W\_X)$ as a physically equivalent representative of the declared audit — rather than merely a computational device for this one ratio — is what the twirl-representative premise supplies. The premise-dependency map of §2.15 records this by omitting P8 from the rows of Theorems 2 and 3 and attaching it instead to any physical claim that substitutes the twirled operator for the original.

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5. Schur Compression of the Coloured Block

5.1 Block form

Relative to $\mathscr{H}_{12} = \mathscr{H}\_col \oplus \mathscr{H}_0$, write

$W = (A\ B ;\ B^\dagger\ D)$.

After representation twirling, $\mathcal{T}_c(W)$ commutes with $U_3(g) \oplus I_9$.

5.2 Theorem 3 — Schur-Compressed Coloured Block Theorem

Statement. For one irreducible triplet occurring with multiplicity one,

$$\mathcal{T}_c(W_X) = w_{col}^X \cdot I_3 \oplus W_0^X,$$

where $w_{col}^X \geq 0$ and $W_0^X \geq 0$. No invariant off-diagonal operator connects the triplet to the singlet complement.

Proof. The triplet block of the twirled operator commutes with the irreducible representation $U_3(g)$. Schur's lemma [8, 13] therefore gives

$$A_{twirled} = w_{col}^X \cdot I_3,$$

and positivity of the twirled operator gives $w_{col}^X \geq 0$.

An invariant off-diagonal block would define an intertwiner between the nontrivial triplet and the trivial representation. Since the representations are inequivalent, that intertwiner must vanish.

The nine-dimensional singlet multiplicity block is unconstrained by colour symmetry except for positivity, so it remains a general positive matrix W_0^X. ■

5.3 Meaning of the compression

Colour symmetry removes any physically invariant distinction among the three triplet components.

It does not require $W_0^X \propto I_9$. Nor does it require

$$w_{col}^X = (1/9) \operatorname{Tr} W_0^X.$$

The remaining triplet-to-complement contrast is dynamical.

5.4 Aggregate fraction after compression

The coloured contribution is

$$C_X = \operatorname{Tr}(\Pi_{col} W_X) = 3\, w_{col}^X.$$

The complement contribution is

$N_X = \mathrm{Tr}[(I - \Pi_{col}) W_X] = \mathrm{Tr}\, W_0^X$.

Thus

$f_{col}^X = 3 w_{col}^X / (3 w_{col}^X + \mathrm{Tr}\, W_0^X)$.

---

6. Raw Census and Exact One-Ratio Lift

6.1 Theorem 4 — Raw Census Fraction Theorem

Statement. Under the unweighted counting audit $W_{census} = I_{12}$, the coloured fraction is

$f_{col}^{census} = \mathrm{Tr}\, \Pi_{col} / \mathrm{Tr}\, I_{12} = 3/12 = 1/4$.

Proof. Immediate from $\mathrm{Tr}\, \Pi_{col} = 3$ and $\mathrm{Tr}\, I_{12} = 12$. This is a rank statement and uses no dynamics. ■

6.2 Aggregate sector means

Define

$C_X = \mathrm{Tr}(\Pi_{col} W_X)$, $N_X = \mathrm{Tr}[(I - \Pi_{col}) W_X]$,

so that $\mathrm{Tr}\, W_X = C_X + N_X$. Define the per-direction sector means

$\bar{w}_{col}^X = C_X / 3$, $\bar{w}_0^X = N_X / 9$.

When $\bar{w}_0^X > 0$, define

$r_X = \bar{w}_{col}^X / \bar{w}_0^X$.

Use the limiting convention $r_X = \infty$ when $N_X = 0$ and $C_X > 0$.

By Theorem 2, C_X and N_X are unchanged by twirling, so the mean $\bar{w}_{col}^X$ computed from the untwirled operator coincides with the Schur invariant weight w_{col}^X of §5.2:

$\bar{w}_{col}^X = w_{col}^X$.

The two symbols denote one quantity reached by two routes.

6.3 Theorem 5 — Exact One-Ratio Participation Theorem

Statement.

$f_{col}^X = r_X / (r_X + 3)$.

The map $r \mapsto r/(r + 3)$ is a strictly increasing bijection from $[0, \infty]$ to $[0, 1]$.

Proof. Using the sector means,

$f_{col}^X = 3\, \bar{w}_{col}^X / (3\, \bar{w}_{col}^X + 9\, \bar{w}_0^X)$.

Dividing numerator and denominator by $3\, \bar{w}_0^X$ gives

$f_{col}^X = r_X / (r_X + 3)$.

The derivative is

$df/dr = 3 / (r + 3)^2 > 0$,

so the map is strictly increasing. Its endpoint values are $f(0) = 0$ and $\lim_{r\to\infty} f(r) = 1$. Hence it is a bijection onto $[0, 1]$. ■

6.4 Inverse relation

For $0 < f_{col}^X < 1$, the required relative sector weight is

$r_X = 3\, f_{col}^X / (1 - f_{col}^X)$.

This formula is useful for inference and audit. It is not a derivation if the target fraction is first chosen and then used to define r_X (P13).

6.5 Theorem 6 — Census-Recovery Criterion

Statement.

$f_{col}^X = 1/4 \Leftrightarrow \bar{w}_{col}^X = \bar{w}_0^X$.

Full uniformity $W_X \propto I_{12}$ is sufficient but not necessary.

Proof. From Theorem 5, $f_{col}^X = 1/4$ if and only if $r_X / (r_X + 3) = 1/4$. Cross-multiplication gives $4\, r_X = r_X + 3$, hence $r_X = 1$, that is $\bar{w}_{col}^X = \bar{w}_0^X$. The converse is immediate.

A nonuniform W_0^X can have mean equal to the triplet mean, so equality of means does not require $W_X \propto I_{12}$. An explicit witness is given in Appendix B.4. ■

6.6 Exact departure formula

The departure from the census value is

$f_{col}^X - 1/4 = 3(r_X - 1) / [4(r_X + 3)]$,

equivalently

$f_{col}^X - 1/4 = 3(\bar{w}_{col}^X - \bar{w}_0^X) / [4(\bar{w}_{col}^X + 3\,\bar{w}_0^X)]$.

Therefore

$f_{col}^X > 1/4 \Leftrightarrow \bar{w}_{col}^X > \bar{w}_0^X$, $f_{col}^X < 1/4 \Leftrightarrow \bar{w}_{col}^X < \bar{w}_0^X$.

6.7 Small-contrast expansion

Write $r_X = 1 + \eta_X$. Then

$f_{col}^X = (1 + \eta_X) / (4 + \eta_X)$.

For $|\eta_X| \ll 1$,

$f_{col}^X = 1/4 + (3/16)\,\eta_X - (3/64)\,\eta_X^2 + (3/256)\,\eta_X^3 + O(\eta_X^4)$.

The coefficient 3/16 is fixed by the $3 \oplus 9$ census. The value of η_X is not.

---

7. The Centred Sector-Contrast Operator

7.1 Definition

Define

$Q_{col} = \Pi_{col} - \frac{1}{4} \cdot I_{12}$.

Its eigenvalues are +3/4 on the triplet and −1/4 on the nine-dimensional complement. It is traceless:

$\mathrm{Tr}\, Q_{col} = 3 \cdot (3/4) + 9 \cdot (-1/4) = 0$.

Its squared Hilbert–Schmidt norm is

$\mathrm{Tr}(Q_{col}^2) = 3 \cdot (9/16) + 9 \cdot (1/16) = 9/4$.

7.2 Theorem 7 — Exact Sector-Contrast Theorem

Statement.

$f_{col}^X - 1/4 = \mathrm{Tr}(Q_{col} W_X) / \mathrm{Tr}\, W_X$.

Proof. By definition,

$\mathrm{Tr}(Q_{col} W_X) = \mathrm{Tr}(\Pi_{col} W_X) - \frac{1}{4} \mathrm{Tr}\, W_X$.

Dividing by Tr W_X gives the stated identity. ■

7.3 Exact operator-space decomposition on the trace-one affine space

Consider the real vector space of Hermitian operators on $\mathscr{H}_{12}$, equipped with the Hilbert–Schmidt inner product

$\langle A, B\rangle_{HS} = \mathrm{Tr}(AB)$,

and within it the affine space of trace-one Hermitian operators, on which every normalised participation operator ρ_X lives.

Every normalised participation operator admits the orthogonal decomposition

$\rho_X = (1/12)\cdot I_{12} + \tilde{\beta}_X \cdot Q_{col} + \rho_X^\perp$,

where

$\mathrm{Tr}\, \rho_X^\perp = 0$ and $\mathrm{Tr}(Q_{col}\, \rho_X^\perp) = 0$.

The identity coefficient is fixed at 1/12 by the trace-one normalisation; it is no longer a free component. The contrast coefficient is

$\tilde{\beta}_X = \mathrm{Tr}(Q_{col}\, \rho_X) / \mathrm{Tr}(Q_{col}^2) = (4/9)\, \mathrm{Tr}(Q_{col}\, \rho_X)$.

7.4 Theorem 8 — One Visible Operator-Direction Theorem

Statement. On the trace-one affine space of normalised participation operators, the aggregate coloured fraction detects exactly one traceless operator direction, Q_{col}:

$f_{col}^X = 1/4 + (9/4)\cdot\tilde{\beta}_X = 1/4 + \mathrm{Tr}(Q_{col}\, \rho_X)$.

The component $\rho_X^\perp$ is exactly invisible, and the identity coefficient is fixed by normalisation.

Remark (rescaling versus additive identity dilution). An overall positive rescaling $W_X \mapsto a\cdot W_X$ cannot move the fraction (§3.6). An additive identity contribution $W_X \mapsto W_X + c\cdot I_{12}$, $c > 0$, generally does move the fraction: $\mathrm{Tr}(Q_{col}(W_X + c\cdot I_{12})) = \mathrm{Tr}(Q_{col} W_X)$ by tracelessness of Q_{col}, while $\mathrm{Tr}(W_X + c\cdot I_{12}) = \mathrm{Tr}\, W_X + 12c$, so after normalisation an existing contrast is diluted,

$$\tilde{\beta}_X \mapsto (4/9)\cdot\mathrm{Tr}(Q_{col} W_X) / (\mathrm{Tr}\, W_X + 12c),$$

driving the fraction toward 1/4 without introducing any new visible direction. An explicit witness: $W = 2\cdot I_3 \oplus I_9$ has $f_{col} = 2/5$, while $W + I_{12} = 3\cdot I_3 \oplus 2\cdot I_9$ has $f_{col} = 9/(9 + 18) = 1/3$. When the contrast vanishes, $\kappa_X = 0$, the addition is inert, consistent with F41.

Proof. Using the decomposition of §7.3,

$$\mathrm{Tr}(Q_{col}\, \rho_X) = (1/12)\cdot\mathrm{Tr}\, Q_{col} + \tilde{\beta}_X\cdot\mathrm{Tr}(Q_{col}^2) + \mathrm{Tr}(Q_{col}\, \rho_X^{\perp}).$$

The first term vanishes because Q_{col} is traceless. The last vanishes by construction. Therefore

$$\mathrm{Tr}(Q_{col}\, \rho_X) = (9/4)\cdot\tilde{\beta}_X,$$

and substitution into Theorem 7 gives the stated formula. ■

7.5 Identification with the sector-mean difference

For the unnormalised operator define $\beta_X = (4/9)\cdot\mathrm{Tr}(Q_{col} W_X)$, so that $\tilde{\beta}_X = \beta_X / \mathrm{Tr}\, W_X$. For the representation-twirled operator,

$$\beta_X = \bar{w}_{col}^X - \bar{w}_0^X.$$

To see this, note that the mean-sector part may be written $a_X\cdot I_{12} + \beta_X\cdot Q_{col}$. Its triplet mean is $a_X + (3/4)\beta_X$, while its complement mean is $a_X - (1/4)\beta_X$. Their difference is therefore β_X.

Thus the contrast coefficient is not an arbitrary new parameter. It is exactly the difference between the two sector means, and its normalised form

$$\tilde{\beta}_X = (\bar{w}_{col}^X - \bar{w}_0^X) / \mathrm{Tr}\, W_X$$

is the quantity the aggregate fraction actually sees.

7.6 Why this theorem is stronger than "only anisotropy matters"

An overall positive rescaling of the complete participation operator cannot move the fraction. An additive identity component can dilute an existing contrast; after trace normalisation the identity coefficient is fixed and all remaining variation of the aggregate fraction lies in the Q_{col} direction.

But not every nonuniform contribution can move the fraction either.

A participation operator may contain large traceless structure: within the nine-dimensional singlet complement; among additional internal labels; in off-diagonal directions removed by the representation twirl; in Hilbert–Schmidt directions orthogonal to Q_{col}.

None of that changes the aggregate coloured fraction unless it has a nonzero projection onto Q_{col}.

The exact statement is therefore:

Only the coloured-versus-complement contrast component can move the aggregate fraction beyond 3/12.

7.7 Minimal dynamical target

A successor VERSF dynamics paper need not derive every matrix element of W_X before it can determine the aggregate fraction.

It must derive the normalised contrast

$\kappa_X \coloneqq \mathrm{Tr}(Q_{col}\, \rho_X) = \mathrm{Tr}(Q_{col}\, W_X) / \mathrm{Tr}\, W_X$.

Then

$f_{col}^X = 1/4 + \kappa_X$.

Equivalently, it may derive r_X. These contain the same aggregate information, related by

$\kappa_X = 3(r_X - 1) / [4(r_X + 3)]$.

Positivity of W_X bounds the target:

$-1/4 \le \kappa_X \le 3/4$,

with the endpoints attained only by the sector-supported extreme operators of §3.4.

7.8 Theorem 8′ — Exact Fraction-Equivalence Theorem

Statement. For two positive participation operators W_X and W_Y,

$f_{col}^X = f_{col}^Y \Leftrightarrow \mathrm{Tr}(Q_{col}\, \rho_X) = \mathrm{Tr}(Q_{col}\, \rho_Y)$, that is, the aggregate coloured fractions agree if and only if the normalised operators have equal Hilbert–Schmidt projection onto Q_{col}.

Proof. Immediate from $f_{col} = 1/4 + \mathrm{Tr}(Q_{col}\,\rho)$, which is injective in the Q_{col} projection. ■

All other operator information — every component of $\rho_X^\perp$ and $\rho_Y^\perp$, and both overall scales — is irrelevant to agreement of this particular audit. This is the exact converse face of Theorem 8: not only is one direction sufficient to determine the fraction, it is the only direction the fraction can distinguish.

---

8. Constructive Underdetermination and Numerical Non-Insertion

8.1 Theorem 9 — Constructive Underdetermination Theorem

Statement. The inherited $3 \oplus 9$ census, positivity, and finite colour-representation invariance do not select a unique numerical coloured participation fraction.

For every target $f^\star \in [0, 1]$, there exists a positive colour-invariant operator $W^\star$ on the same carrier such that

$f_{col}[W^\star] = f^\star$.

Proof. For $0 < f^\star < 1$, define

$r^\star = 3 f^\star / (1 - f^\star)$,

and choose

$W^\star = r^\star \cdot I_3 \oplus I_9$.

This operator is positive and commutes with the colour representation. Its coloured fraction is

$f_{col}[W^\star] = 3 r^\star / (3 r^\star + 9) = r^\star / (r^\star + 3) = f^\star$.

For $f^\star = 0$, choose $W^\star = 0_3 \oplus I_9$. For $f^\star = 1$, choose $W^\star = I_3 \oplus 0_9$. These are convenient representatives of the endpoint classes; by §3.4, the endpoint values are realised precisely by nonzero positive audits supported entirely in the corresponding sector.

Thus every value in [0, 1] is constructible. ■

8.2 Corollary — No numerical value follows from symmetry alone

Colour symmetry determines equality within the triplet. It does not determine the triplet-to-complement contrast.

Therefore symmetry alone cannot determine whether the fraction is below one quarter, equal to one quarter, or above one quarter.

8.3 Corollary — No numerical coincidence has evidential force

A resemblance between 1/4, or any selected weighted fraction, and an empirical percentage in another domain is not a derivation.

Without an operator bridge, numerical similarity is physically untyped.

8.4 Theorem 10 — No Independently Fixed Contrast, No Numerical Prediction Theorem

Statement. A numerical value of f_{col}^X is VERSF-derived only if the theory independently fixes the participation type and its normalised coloured-versus-complement contrast — equivalently r_X or $\kappa_X = \mathrm{Tr}(Q_{col} \rho_X)$ — without using the intended fraction as input. Derivation of a canonical positive operator W_X up to an overall positive scalar is sufficient for this purpose but is not necessary: a dynamical theory may derive the ratio C_X/N_X directly without fixing every component of the operator.

Proof. Theorem 9 shows that the structural assumptions admit every value in [0, 1]. Therefore those assumptions alone cannot select one value.

By Theorem 8, the fraction is determined by the single normalised datum $\kappa_X = \mathrm{Tr}(Q_{col} \rho_X)$. Independently fixing that datum is therefore necessary and sufficient for a numerical result. Fixing the full normalised functional ω_X — equivalently W_X up to an overall positive scalar — is sufficient because it determines κ_X, but no other component of ω_X enters the aggregate fraction.

If the contrast is instead fixed by the desired output, the construction merely encodes that output and is not predictive. ■

8.5 Canonicality requirements

A participation operator is canonical for the declared question X only if:

1. X is specified before the calculation;
2. the carrier domain is inherited or independently derived;
3. the response map is fixed by prior VERSF equations;
4. the response metric is fixed by prior physical structure;
5. gauge covariance or gauge invariance is demonstrated;
6. the operator is unique up to an overall positive scalar or an explicitly irrelevant response-space isometry;
7. all remaining parameters are fixed by non-target information;
8. scale, state, and boundary data are specified;
9. threshold and confinement matching are supplied where required;
10. no target fraction enters the calibration.

8.6 Universal-fraction criterion

Two levels of universality must be distinguished.

Universal functional. The condition $W_Y = a \cdot W_X$ with $a > 0$ is sufficient for equality of the entire normalised functional: every audit built on the carrier, not only the coloured one, then agrees.

Universal coloured value. By Theorem 8′, the exact criterion for agreement of the coloured fraction alone is

$$f_{col}^X = f_{col}^Y \Leftrightarrow \mathrm{Tr}(Q_{col}\, \rho_X) = \mathrm{Tr}(Q_{col}\, \rho_Y).$$

Equality of the coloured fractions therefore constrains only the Q_{col} projections and proves nothing about proportionality of the operators; the full $\rho^\perp$ freedom of both remains untouched, and the two audits may disagree on every other question.

A single context-independent coloured fraction may accordingly be claimed in either of two ways: by deriving one canonical functional shared by all relevant participation types, or — more weakly but sufficiently for this one audit — by proving that all relevant normalised participation operators share one Q_{col} projection.

---

9. Response-Map Origin and Canonicality Requirements

9.1 Positive response construction

Let

$A_X : \mathscr{H}_{12} \to \mathcal{K}_X$

be a linear response map into a response space $\mathcal{K}_X$, and let $G_X \geq 0$ be its positive response metric. Define

$W_X = A_X^\dagger G_X A_X$.

Then for every $|\psi\rangle$,

$\langle\psi| W_X |\psi\rangle = \langle A_X \psi| G_X |A_X \psi\rangle \geq 0$.

Therefore $W_X \geq 0$.

9.2 Theorem 11 — Positive Response-Factorisation Theorem

Statement. A quadratic participation audit is positive if it admits a factorisation

$W_X = A_X^\dagger G_X A_X, G_X \geq 0$.

Conversely, every positive finite-dimensional operator admits such a factorisation.

Proof. The forward implication follows from the preceding quadratic form.

For the converse, take $A_X = W_X^{1/2}$ and $G_X = I$. Then $W_X = A_X^\dagger G_X A_X$. ■

9.3 Factorisation is not derivation

The converse construction proves that factorisation by itself carries no explanatory content.

Every fitted positive operator can be written as a response square. The physical derivation lies in obtaining A_X and G_X from independently specified VERSF structure.

9.4 Colour-equivariant response map

Suppose

$A_X U_c(g) = V_X(g) A_X$ and $V_X(g)^\dagger G_X V_X(g) = G_X$

for every $g \in SU(3)_c$. Then

$U_c(g)^\dagger W_X U_c(g) = U_c(g)^\dagger A_X^\dagger G_X A_X U_c(g) = A_X^\dagger V_X(g)^\dagger G_X V_X(g) A_X = W_X$.

Thus an equivariant response map with invariant response metric produces a colour-invariant participation operator, and the Schur compression of Theorem 3 applies to it exactly rather than only after twirling.

9.5 Candidate participation classes

CPF-1 permits, but does not select, operators such as:

Closure-response participation. $W_{cl} = A_{cl}^\dagger G_{cl} A_{cl}$, with $G_{cl} \geq 0$.

Kinetic participation. $W_{kin} = Z_{phys}^{1/2} K Z_{phys}^{1/2}$, with $Z_{phys} \geq 0$ and $K \geq 0$.

Interaction participation. $W_{int}(\mu) = A_{int}(\mu)^\dagger G_{int}(\mu) A_{int}(\mu)$, with $G_{int}(\mu) \geq 0$.

Spectral participation. $W_{spec}(\Omega) = \int_\Omega K(s)\, d\rho(s)$, with $K(s) \geq 0$ and $d\rho$ a positive measure, so that the integral defines a positive operator.

Observable-response participation. $W_{\mathcal{O}} = M_{\mathcal{O}}^\dagger \Gamma_{\mathcal{O}} M_{\mathcal{O}}$, with $\Gamma_{\mathcal{O}} \geq 0$.

Each example is a participation operator only when its stated positivity condition holds. These operators answer different questions. No theorem in CPF-1 equates their fractions.

9.6 Parameter-identifiability requirement

Let θ_X denote all parameters entering A_X and G_X. A numerical prediction requires

$$\{f_{col}^{target}, r_X^{target}\} \cap \mathrm{Inputs}(\theta_X) = \emptyset.$$

If a parameter controls only the coloured-to-complement contrast and is inferred from the intended fraction, it is a hidden insertion.

9.7 Minimal successor-gate task

The next genuine dynamics gate should select one participation class and derive either r_X or

$$\kappa_X = \mathrm{Tr}(Q_{col}\, \rho_X).$$

Deriving unrelated matrix detail does not by itself close the aggregate fraction: by Theorem 8, everything except the Q_{col} component is invisible to it.

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10. Basis Covariance, Mixtures, and Auxiliary Coarse-Graining

10.1 Theorem 12 — Basis-Covariance Theorem

Statement. Under a unitary carrier-basis change

$W_X \mapsto S W_X S^\dagger$, $\Pi_{col} \mapsto S \Pi_{col} S^\dagger$,

the fraction is unchanged.

Proof. By cyclicity of the trace,

$\mathrm{Tr}[S \Pi_{col} S^\dagger S W_X S^\dagger] = \mathrm{Tr}(\Pi_{col} W_X)$,

and $\mathrm{Tr}(S W_X S^\dagger) = \mathrm{Tr}\, W_X$. ■

10.2 Inconsistent transformations

Transforming W_X without transforming the projector is not a basis change. It is a change of the physical assignment relative to the fixed representation decomposition.

10.3 Mixtures of contexts

Suppose

$W = \Sigma_k W_k$, $W_k \geq 0$.

Then

$f_{col}[W] = \Sigma_k [\mathrm{Tr}\, W_k / \Sigma_j \mathrm{Tr}\, W_j] \cdot f_{col}[W_k]$.

The combined fraction is a trace-weighted convex combination. It is not generally the unweighted arithmetic mean. In particular, the combined fraction lies in the closed interval between the smallest and largest component fractions.

10.4 Theorem 13 — Auxiliary Partial-Trace Consistency

Statement. Let a positive operator W_{tot} act on $\mathscr{H}_{12} \otimes \mathcal{K}_{aux}$. Extend the coloured projector as $\Pi_{col} \otimes I_{aux}$ and define the reduction

$W_{red} = \mathrm{Tr}_{aux}\, W_{tot}$.

Then

$\mathrm{Tr\_tot}[(\Pi\_\mathrm{col} \otimes I\_\mathrm{aux})\, W\_\mathrm{tot}] / \mathrm{Tr\_tot}\, W\_\mathrm{tot} = \mathrm{Tr}_{12}(\Pi\_\mathrm{col}\, W\_\mathrm{red}) / \mathrm{Tr}_{12}\, W\_\mathrm{red}$.

Proof. This follows directly from the defining property of the partial trace:

$\mathrm{Tr\_tot}[(A \otimes I)\, W\_\mathrm{tot}] = \mathrm{Tr}_{12}[A \cdot \mathrm{Tr\_aux}\, W\_\mathrm{tot}]$.

Apply this once with $A = \Pi\_\mathrm{col}$ and once with $A = I_{12}$. ■

10.5 Gauge-theory caveat

A literal tensor-product factorisation of spatial regions in gauge theory may require edge modes, dressing, or an algebraic replacement for the naive partial trace (constraint and BRST structure: [9]).

The theorem applies only after an admissible factorisation or reduced-algebra construction has been specified.

---

11. Relative Running and Threshold Discipline

11.1 Scale-labelled operator

For a scale-dependent audit,

$W\_X = W\_X(\mu; S)$,

write $f\_\mathrm{col}^X = f\_\mathrm{col}^X(\mu; S)$. Internal fractions may depend on renormalisation scale μ and scheme S.

A completely matched observable must be independent of unphysical conventions after all contributions are included.

11.2 Aggregate running quantities

Define

$C\_X(\mu) = \mathrm{Tr}(\Pi\_\mathrm{col}\, W\_X(\mu))$, $N\_X(\mu) = \mathrm{Tr}[(I - \Pi\_\mathrm{col})\, W\_X(\mu)]$.

When both are positive, define the aggregate logarithmic rates

$\gamma_{col}^X = d \ln C_X / d \ln \mu$, $\gamma_0^X = d \ln N_X / d \ln \mu$.

CPF-1 does not derive their QCD or VERSF values. In a QCD application these aggregate rates would have to be derived from the renormalisation-group evolution and operator mixing of the specific audit; the asymptotic-freedom results of [3, 4] provide foundational coupling behaviour but do not by themselves determine γ_{col}^X or γ_0^X.

11.3 Theorem 14 — Exact Relative-Running Identity

Statement. Let $t = \ln \mu$, and assume $W_X(\mu)$ is differentiable in t on the interval considered (in finite dimension, entrywise differentiability suffices and coincides with trace-class differentiability), with $\mathrm{Tr}\, W_X(\mu) > 0$ throughout. Then

$$d f_{col}^X / dt = \mathrm{Tr}[(\Pi_{col} - f_{col}^X \cdot I) \dot{W}_X] / \mathrm{Tr}\, W_X.$$

If $C_X, N_X > 0$, then

$$d f_{col}^X / dt = f_{col}^X (1 - f_{col}^X)(\gamma_{col}^X - \gamma_0^X),$$

equivalently

$$d/dt \ln[f_{col}^X / (1 - f_{col}^X)] = \gamma_{col}^X - \gamma_0^X.$$

Since $r_X = 3 C_X / N_X$,

$$d \ln r_X / dt = \gamma_{col}^X - \gamma_0^X.$$

Proof. Write $f = C/D$ with $D = C + N$. Then

$$\dot{f} = (\dot{C} D - C \dot{D}) / D^2.$$

Using $\dot{C} = \mathrm{Tr}(\Pi_{col} \dot{W})$ and $\dot{D} = \mathrm{Tr}\, \dot{W}$ gives the first identity.

Using $\dot{C} = \gamma_{col} C$ and $\dot{N} = \gamma_0 N$ gives

$$\dot{f} = [C N / (C + N)^2](\gamma_{col} - \gamma_0).$$

Since $C/(C+N) = f$ and $N/(C+N) = 1 - f$, the logistic form follows. The log-odds and r_X equations follow by direct logarithmic differentiation. ■

11.4 Kinematic identity, not anomalous-dimension prediction

Theorem 14 is an exact identity once the aggregate running rates are defined.

It does not derive a beta function, an anomalous dimension, a QCD evolution kernel, or a VERSF flow law. Those remain dynamical inputs.

11.5 Universal multiplicative running

If

$$W_X(\mu) = Z_X(\mu)\, W_X(\mu_0)$$

with one positive scalar Z_X common to all sectors, then

$$f_{col}^X(\mu) = f_{col}^X(\mu_0).$$

Only differential sector evolution can change the fraction. This is the running form of the scale invariance of §3.6.

11.6 Integrated relative-running criterion

For scales over which $C_X, N_X > 0$, Theorem 14 integrates exactly:

$$\ln[f_{col}^X(\mu)/(1 - f_{col}^X(\mu))] = \ln[f_{col}^X(\mu_0)/(1 - f_{col}^X(\mu_0))] + \int_{\ln \mu_0}^{\ln \mu} (\gamma_{col}^X - \gamma_0^X)\, dt.$$

The census value $f = 1/4$ corresponds to log-odds $\ln(1/3) = -\ln 3$. Define the census displacement

$$\Delta_X(\mu) = \ln[f_{col}^X(\mu)/(1 - f_{col}^X(\mu))] + \ln 3,$$

which evolves at rate

$$d\Delta_X/d \ln \mu = \gamma_{col}^X - \gamma_0^X.$$

The fraction crosses its census value at scale μ exactly when the integrated rate cancels the initial displacement:

$$\int_{\ln \mu_0}^{\ln \mu} (\gamma_{col}^X - \gamma_0^X)\, dt = -\Delta_X(\mu_0).$$

A constant nonzero $\gamma_{col}^X - \gamma_0^X$ suffices for such a crossing, provided the running interval extends far enough for the integrated rate to reach $-\Delta_X(\mu_0)$; no change in its sign is required. Moreover, if $\gamma_{col}^X - \gamma_0^X$ is strictly positive or strictly negative almost everywhere on the interval, Δ_X is strictly monotone and the census value can be crossed at most once; with a merely weak fixed sign, Δ_X is only nondecreasing or nonincreasing and may sit at the census value on a subinterval.

If, additionally, the aggregate sector contributions evolve separately multiplicatively, $\dot{C}_X = \gamma_{col}^X C_X$ and $\dot{N}_X = \gamma_0^X N_X$ with no additive source or transfer terms, then $f = 0$ and $f =$

1 are invariant boundaries and an interior fraction remains interior under finite running. That boundary statement requires the no-source assumption; it does not follow from the general quotient identity alone, since an evolution containing sector-generation terms can create coloured support from $C_X = 0$.

11.7 Thresholds

If a threshold changes the active carrier, the representation content, the response operator, the operator mixing, or the matching scheme, then W_X and possibly the raw census must be rematched.

The number 3/12 may not be carried unchanged across a threshold that changes the relevant channel space.

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12. Stability Under Operator Corrections

12.1 Exact perturbation identity

Let $W \geq 0$ with $D = \mathrm{Tr}\, W > 0$. Let $E = E^\dagger$ satisfy $W + E \geq 0$ and $\mathrm{Tr}(W + E) > 0$, so that the perturbed fraction is defined. Then

$$f_{col}[W + E] - f_{col}[W] = \mathrm{Tr}[(\Pi_{col} - f \cdot I)\, E] / (D + \mathrm{Tr}\, E),$$

where $f = f_{col}[W]$. The trace-norm condition $\|E\|_1 < D$ of Theorem 15 automatically guarantees $\mathrm{Tr}(W + E) \geq D - \|E\|_1 > 0$.

12.2 Theorem 15 — Trace-Norm Stability Theorem

Statement. If $\|E\|_1 < D$, then

$$| f_{col}[W + E] - f_{col}[W] | \leq \max(f, 1 - f) \cdot \|E\|_1 / (D - \|E\|_1),$$

and therefore also

$$| f_{col}[W + E] - f_{col}[W] | \leq \|E\|_1 / (D - \|E\|_1).$$

Proof. Trace duality [15, 16] gives

$$| \mathrm{Tr}[(\Pi_{col} - f \cdot I)\, E] | \leq \|\Pi_{col} - f \cdot I\|_\infty \cdot \|E\|_1.$$

The eigenvalues of $\Pi_{col} - f \cdot I$ are $1 - f$ on the triplet and $-f$ on the complement, hence

$\|\Pi\_col - f \cdot I\|\_\infty = \max(f, 1 - f)$.

Also,

$D + \mathrm{Tr}\, E \geq D - |\mathrm{Tr}\, E| \geq D - \|E\|_1 > 0$.

Combining the bounds proves the theorem. ■

12.3 Sign stability relative to one quarter

Corollary. If

$| f\_col[W] - 1/4 | > \|E\|_1 / (D - \|E\|_1)$,

then the perturbation cannot reverse whether the fraction lies above or below one quarter. A derived sign of the sector contrast therefore survives all sufficiently small admissible corrections, quantified purely by the trace-norm budget of the correction.

12.4 Stability near uniform weighting

Let

$W = w \cdot I_{12} + E,\ w > 0,\ \mathrm{Tr}\, E = 0$.

Then the perturbation identity gives exactly

$f\_col[W] - 1/4 = \mathrm{Tr}(\Pi\_col\ E) / (12\ w)$.

Since $|\mathrm{Tr}(\Pi\_col\ E)| \leq 3\|E\|\_\infty$,

$| f\_col[W] - 1/4 | \leq \|E\|\_\infty / (4\ w)$.

By Theorem 8, only the Q_col component of E contributes to the numerator; the bound is saturated when E ∝ Q_col within its positivity window (Appendix B.6).

---

13. Confinement and Local-Gauge Separation

13.1 Two different questions

CPF-1 distinguishes:

Internal representation participation. How much of a declared positive response on $\mathscr{H}_{12}$ lies in the triplet representation sector?

Asymptotic non-singlet abundance. How many freely observable external states carry net colour?

These are not the same object.

13.2 Total colour versus constituent colour

A local mesonic composite may have the schematic form

$\bar{q}_i(x)\, q^i(x)$.

A baryonic composite may have the form

$\varepsilon_{ijk}\, q^i(x)\, q^j(x)\, q^k(x)$.

The total operators are colour singlets, even though their constituent fields transform nontrivially.

For separated fields, gauge links or equivalent dressing are required [5, 6, 12], schematically

$\bar{q}_i(x)\, [\mathcal{P} \exp(ig \int_x^y A_\mu dx^\mu)]^i{}_j\, q^j(y)$.

13.3 Distinct projectors on distinct spaces

Let $\Pi_{nonsing}^{asym}$ project onto non-singlet total representations in an asymptotic state space. It is not the same operator as Π_{col} on the finite VERSF channel carrier.

It is therefore possible that

$\Pi_{nonsing}^{asym} |\Psi_{phys}\rangle = 0$

for every admissible asymptotic state while

$\mathrm{Tr}(\Pi_{col} W_X) > 0$.

13.4 Theorem 16 — Internal/Asymptotic Colour Separation Theorem

Statement. Nonzero internal coloured participation is compatible with a purely colour-singlet asymptotic spectrum. Conversely, a singlet asymptotic spectrum does not force the internal coloured participation fraction to vanish.

Proof (by type separation and explicit composite witness). The internal projector and the total asymptotic representation projector act on different typed spaces and test different properties.

Gauge-singlet composite operators provide explicit witnesses in which colour-transforming constituents combine into total singlets. Therefore constituent representation support may be nonzero even when net asymptotic colour is absent.

Neither quantity determines the other without a further local-gauge, confinement, and observable bridge. ■

The proof establishes compatibility by category separation plus witness; it deliberately does not attempt a field-theoretic construction, which belongs to the D10–D11 bridge debts.

13.5 No free-colour interpretation

A hypothetical result $f_{col}^X = 0.30$ would mean that thirty per cent of the declared positive response X is assigned to the triplet sector of the declared internal carrier.

It would not mean: thirty per cent of detected particles are free quarks; thirty per cent of matter is coloured; thirty per cent of hadronic mass is carried by a specific constituent class; thirty per cent of the universe is colour matter.

13.6 Confinement does not set the fraction to zero

Confinement excludes free asymptotic net colour. It does not imply

$\mathrm{Tr}(\Pi_{col} W_X) = 0$

for every microscopic or internal response operator.

---

14. Observable-Class Firewalls

14.1 Census fraction

$f_{col}^{census} = \mathrm{rank}\, \Pi_{col} / 12$ is a channel count.

14.2 Closure-response fraction

A closure fraction requires a closure-response map and closure metric. It is not automatically an energy or interaction fraction.

14.3 Kinetic fraction

A kinetic fraction depends on the physical kinetic metric, field normalisation, constraints, and scale.

14.4 Interaction fraction

An interaction fraction depends on a process; a vertex or response kernel; a scale; external states; phase space; interference conventions. Different processes generally yield different operators.

14.5 Spectral fraction

A spectral fraction requires a positive spectral measure and declared integration region.

14.6 Probability

The normalised functional is probability-like on projectors. It becomes a literal probability only when the projectors represent mutually exclusive outcomes in a declared state or ensemble. As in §3.1, "state" here carries only its finite operator-algebra sense; it does not assert that ω_X is the density operator of a freely preparable asymptotic quantum system.

14.7 Parton momentum fraction

A parton momentum fraction requires a hadronic state; a factorisation theorem [14]; light-cone or equivalent operators; scale and scheme; evolution equations; observable matching. It is not derivable from 3/12.

14.8 Hadron mass fraction

A hadron mass decomposition requires the QCD energy-momentum tensor or Hamiltonian; a hadronic state; renormalised operator mixing; scale and scheme; a complete decomposition convention [11]. The generic W_X is not a mass operator.

14.9 Vacuum and cosmological fractions

A vacuum or cosmological fraction requires a renormalised stress-energy prescription; gravitational coupling; population or field evolution; cosmological history; an observational bridge. Numerical proximity to one quarter establishes no relation.

14.10 Observable contribution fraction

An observable fraction must have the form

$f\_col^{\mathcal{O}} = \mathcal{O}\_col / \mathcal{O}\_total$,

where both terms arise from the same gauge-invariant measurement functional. Interference terms must be assigned by an explicit physically justified convention.

---

15. Falsification Conditions

CPF-1 fails, or a claimed numerical extension fails, under any of the following conditions.

#	Failure condition	Consequence
F1	The twelve-direction carrier is not explicitly defined	No valid denominator
F2	Incompatible spin, chirality, generation, particle–antiparticle, or momentum multiplicities are mixed	Census ambiguity
F3	The rank-three block is not an invariant colour representation	No coloured projector theorem
F4	The positive carrier metric is absent or indefinite	No positive participation audit
F5	The coloured numerator is defined by preferred basis labels	Gauge-orientation failure
F6	The audit is negative on positive effects	No participation interpretation
F7	Affine composition under mixtures fails, or the positive-linear-extension premise fails	Trace-representation claim fails
F8	Tr W_X = 0	Fraction undefined
F9	The participation type X is not declared	Typing failure
F10	Numerator and denominator use different operators	Denominator mismatch
F11	Numerator and denominator use different scales or states	Inconsistent audit
F12	A finite colour rotation changes the aggregate result	Representation-invariance failure
F13	Red, green, and blue receive independent invariant weights	Schur-symmetry failure
F14	Colour multiplicity is counted in both the trace and a separate enhancement factor	Double counting
F15	3/12 is called an energy, interaction, or probability share without an operator bridge	Census/dynamics confusion
F16	"Beyond 3/12" is assumed to mean above one quarter	Sign insertion
F17	Equality with one quarter is claimed to imply $W\_X \propto I_{12}$	Converse overclaim
F18	Arbitrary nonuniformity is claimed to affect the fraction without projection onto Q_col	Operator-direction error
F19	A fitted diagonal matrix is renamed a derived participation operator	Hidden insertion

#	Failure condition	Consequence
F20	The target fraction is used to calibrate A_X, G_X, or r_X	Retrodiction
F21	The factorisation W_X = A_X† G_X A_X is presented as physical derivation by itself	Square-root fallacy
F22	Common normalisation is claimed to generate a sector contrast	Ratio-normalisation error
F23	A quotient identity is presented as a derived QCD beta function	Running overclaim
F24	Different scales are compared without matching	RG failure
F25	A threshold changes the carrier but 3/12 is retained without audit	Threshold failure
F26	Auxiliary degrees are removed differently in numerator and denominator	Coarse-graining failure
F27	A context-specific result is called a universal coloured constant	Universalisation failure
F28	A numerical fraction is claimed without an independently fixed normalised contrast or another sufficient operator-level derivation	Numerical overclaim
F29	A small correction is asserted without a norm or bound	Stability failure
F30	Internal coloured participation is interpreted as free asymptotic colour	Confinement category error
F31	Singlet asymptotic states are used to set all internal coloured participation to zero	Reverse confinement error
F32	Bare separated colour fields are treated as gauge-invariant observables	Local-gauge failure
F33	The fraction is identified with a quark count	Number/participation confusion
F34	The fraction is identified with a hadron mass share without an energy-momentum bridge	Mass-decomposition failure
F35	The fraction is identified with a parton momentum share without factorisation	Parton-bridge failure
F36	The fraction is identified with baryon, dark-matter, dark-energy, or vacuum abundance by numerical resemblance	Cosmological coincidence failure
F37	An empirical claim lacks a gauge-invariant observable bridge	Observable failure
F38	Passing CPF-1 is presented as experimental confirmation	Admissibility/truth confusion
F39	The representation content changes but the numerator rank remains fixed at three	Representation-count failure
F40	The finite representation twirl is presented as the complete local-gauge projection of QCD	Gauge-scope overclaim
F41	An additive identity contribution is claimed to leave a nonzero contrast unchanged, or is confused with an overall rescaling	Normalisation-dilution error

#	Failure condition	Consequence
F42	The declared physical audit is essentially colour-frame-dependent, so the twirled representative answers a different question	Twirl-representative failure

Absorption note on F12. For any genuine positive affine audit, Theorem 2 makes finite colour-rotation invariance of the aggregate fraction automatic. A violation of F12 therefore cannot occur while F5–F7 hold: its firing signals a prior failure of F5 (basis-defined numerator) or F6–F7 (non-functional audit) rather than an independent failure channel. F12 is retained as the diagnostic surface of those upstream failures. The interpretive twirl-representative failure of P8 is the separate condition F42.

---

16. CPF-1 Closure Certificate

CPF-1 requirement	Result
Internal twelve-direction carrier distinguished from the full QCD physical Hilbert space	Closed
Rank-three triplet and rank-nine singlet complement typed	Inherited/conditional
Raw 3/12 census proved	Closed
Census/dynamics distinction proved	Closed
Participation type required	Closed
Positive linear-extension premise (P6) installed	Closed
Positive trace representation proved	Closed
Weighted projector expectation defined	Closed
Fraction bounded in [0, 1] with extreme-support characterisation	Closed
Representation-defined projector supplied	Closed
Finite-carrier colour twirl defined	Closed
Aggregate twirl invariance proved	Closed
Local-gauge scope of twirl restricted	Closed
Schur compression of the triplet proved	Closed
Invariant triplet–singlet off-diagonal block excluded	Closed
Non-coloured block allowed to remain nonuniform	Closed
Sector means defined	Closed
Relative ratio r_X defined	Closed
Exact r_X/(r_X + 3) formula proved	Closed
Inverse relation proved	Closed

CPF-1 requirement	Result
Census-recovery criterion proved	Closed
Full uniformity shown sufficient but unnecessary	Closed
Centred contrast operator Q_col defined	Closed
Exact contrast identity proved	Closed
One visible operator-direction theorem proved on the trace-one affine space	Closed
Normalised-operator (trace-one) formulation installed	Closed
Multiplicative rescaling distinguished from additive identity dilution	Closed
Exact fraction-equivalence criterion (Theorem 8′) proved	Closed
Orthogonal anisotropy shown invisible to the aggregate fraction	Closed
Positivity window $-1/4 \leq \kappa_X \leq 3/4$ established	Closed
Constructive underdetermination proved for every value in [0, 1]	Closed
Canonicality requirements supplied	Closed at audit level
Response-map factorisation proved	Closed mathematically
Factorisation/derivation distinction installed	Closed
Equivariance route to exact colour invariance proved	Closed
Basis covariance proved	Closed
Mixture rule proved	Closed
Auxiliary partial-trace consistency proved	Closed
Exact relative-running identity proved	Closed
Running identity distinguished from dynamical beta functions	Closed
Universal multiplicative cancellation proved	Closed
Integrated log-odds crossing criterion proved; boundary invariance conditional on no-source evolution	Closed
Threshold discipline installed	Closed
Perturbation bound proved	Closed
Sign-preservation corollary proved	Closed
Internal/asymptotic colour separation proved	Closed
Free-colour interpretation blocked	Closed
Confinement-based erasure blocked	Closed
Parton, mass, vacuum, and cosmological firewalls installed	Closed
Premise-dependency map supplied, with mathematical/physical separation for the twirl	Closed

CPF-1 requirement	Result
Upstream inheritance anchored to named gates (SC-1 carrier, GC-1 colour identification)	Provisionally anchored; Master Ledger verification pending (D14)
Crossing criterion sharpened: displacement form with at-most-one-crossing under fixed-sign running	Closed
Four-level closure hierarchy installed	Closed
Canonical VERSF participation operator	Open
Numerical r⋆ or κ⋆	Open
Local-gauge field-theory embedding	Open
Observable extraction	Open
Empirical confirmation	Open

Closure grade

CPF-1 is mathematically closed as a conditional positive-functional, representation-compression, sector-contrast, and numerical non-insertion theorem on the inherited 3 ⊕ 9 carrier. It is not numerically closed because the canonical participation type, response operator, scale evolution, local-gauge embedding, and observable bridge are not derived here.

---

17. Conclusion

The inherited VERSF census contains one rank-three colour triplet and a rank-nine colour-singlet complement inside a twelve-direction internal response carrier.

The exact raw census fraction is

3/12 = 1/4.

That statement is correct. It is not yet a physical participation prediction.

A physical participation audit must specify what is being measured and must assign positive weights consistently under mixtures; under the positive-linear-extension premise P6, it is represented on the finite carrier by a nonzero positive operator W_X. The coloured share is therefore

$$f_{col}^X = Tr(\Pi_{col} W_X) / Tr\, W_X.$$

The coloured projector is defined by representation content, not by preferred component labels.

Finite-carrier colour twirling leaves this aggregate ratio unchanged. Schur reduction then forces the triplet block to carry one common invariant weight. The nine-dimensional singlet complement may remain internally nonuniform, but only its total trace affects this aggregate fraction.

The exact result compresses to

$$f_{col}^X = r_X / (r_X + 3), \quad r_X = \bar{w}_{col}^X / \bar{w}_0^X.$$

The raw value 1/4 is recovered if and only if the mean triplet weight equals the mean singlet-complement weight.

The decisive structural refinement is the centred sector-contrast operator

$$Q_{col} = \Pi_{col} - \tfrac{1}{4} \cdot I_{12},$$

which yields

$$f_{col}^X - 1/4 = \mathrm{Tr}(Q_{col} W_X) / \mathrm{Tr}\, W_X.$$

This proves that the aggregate fraction is sensitive to exactly one direction in the Hermitian operator space: the coloured-versus-complement contrast. Overall multiplicative normalisation cannot move the fraction. Once the participation operator is normalised to unit trace, its only component capable of moving the aggregate coloured fraction is its projection onto Q_{col}; a positive additive identity contribution acts purely by diluting that projection, and every traceless component orthogonal to Q_{col} is invisible. Internal nonuniformity matters only insofar as it changes the two sector means, and positivity confines the entire admissible correction to $-1/4 \leq \kappa_X \leq 3/4$.

CPF-1 then proves constructively that every number in [0, 1] can be produced by a positive colour-invariant operator on the same carrier. Therefore neither the $3 \oplus 9$ census nor colour symmetry selects a numerical physical fraction.

The route to numerical closure is an independent derivation of the normalised coloured-versus-complement contrast. This may arise from a canonical positive response operator, whose admissible architecture may be written

$$W_X = A_X^\dagger G_X A_X, \quad G_X \geq 0,$$

but a direct, independently specified derivation of r_X or κ_X is equally sufficient. In either case the factorisation is not itself a derivation. The response map or contrast dynamics, the response metric, state, scale, thresholds, local-gauge embedding, and observable bridge must all be supplied by the dynamics.

If the operator runs, the aggregate fraction obeys the exact relative-running identity

$d\, f\_col^X / d \ln \mu = f\_col^X (1 - f\_col^X)(\gamma\_col^X - \gamma_0^X)$.

This is a quotient identity, not a derived QCD or VERSF beta function. A common multiplicative factor cancels; only differential sector evolution changes the fraction.

Finally, internal coloured participation is not free asymptotic colour. Confinement may exclude non-singlet asymptotic states while permitting nonzero colour-transforming constituent support inside gauge-singlet observables. The reverse inference is equally invalid: singlet asymptotics do not require the internal triplet contribution to vanish.

The programme sequence is therefore:

representation census → positive participation functional → sector-contrast derivation → scale and confinement matching → observable extraction

CPF-1 closes the second step and identifies the exact target for the third.

In one sentence:

Three out of twelve is the exact coloured census; beyond three out of twelve, the aggregate physical fraction is the positive expectation of the colour-representation projector, its entire correction lies in one centred sector-contrast direction, and no numerical value is VERSF-derived until the normalised sector contrast is independently obtained, whether through a canonical participation operator or by direct derivation.

Appendix A — Minimal Algebraic Skeleton

Carrier:

$\mathscr{H}_{12} = \mathscr{H}\_col \oplus \mathscr{H}_0$, dim $\mathscr{H}\_col = 3$, dim $\mathscr{H}_0 = 9$.

Representation:

$U\_c(g) = U_3(g) \oplus I_9$.

Projector:

$\Pi\_col = I_3 \oplus 0_9$.

Raw census:

$f\_col^census = \mathrm{Tr}\, \Pi\_col / \mathrm{Tr}\, I_{12} = 1/4$.

Positive audit:

$W\_X \geq 0$, $\mathrm{Tr}\, W\_X > 0$.

Normalised operator:

$\rho\_X = W\_X / \mathrm{Tr}\, W\_X$, $\mathrm{Tr}\, \rho\_X = 1$.

Coloured fraction:

$f\_col\text{\^{}}X = \mathrm{Tr}(\Pi\_col\, W\_X) / \mathrm{Tr}\, W\_X = \mathrm{Tr}(\Pi\_col\, \rho\_X)$.

Aggregate weights:

$C\_X = \mathrm{Tr}(\Pi\_col\, W\_X)$, $N\_X = \mathrm{Tr}[(I - \Pi\_col)\, W\_X]$.

Sector means:

$\bar{w}\_col\text{\^{}}X = C\_X / 3$, $\bar{w}_0\text{\^{}}X = N\_X / 9$.

Relative sector ratio:

$r\_X = \bar{w}\_col\text{\^{}}X / \bar{w}_0\text{\^{}}X$.

One-ratio form:

$f\_col\text{\^{}}X = r\_X / (r\_X + 3)$.

Inverse relation:

$r\_X = 3\, f\_col\text{\^{}}X / (1 - f\_col\text{\^{}}X)$.

Census-recovery condition:

$f\_col\text{\^{}}X = 1/4 \Leftrightarrow r\_X = 1$.

Centred contrast operator:

$Q\_col = \Pi\_col - \frac{1}{4} \cdot I_{12}$, $\mathrm{Tr}\, Q\_col = 0$, $\mathrm{Tr}(Q\_col^2) = 9/4$.

Contrast identity:

$f\_col\text{\^{}}X - 1/4 = \mathrm{Tr}(Q\_col\, \rho\_X) = \kappa\_X$, $-1/4 \leq \kappa\_X \leq 3/4$.

Operator decomposition (trace-one affine space):

$\rho_X = (1/12)\cdot I_{12} + \tilde{\beta}_X \cdot Q_{col} + \rho_X^{\perp}$, $\mathrm{Tr}\, \rho_X^{\perp} = 0$, $\mathrm{Tr}(Q_{col}\, \rho_X^{\perp}) = 0$.

Contrast coefficient:

$\tilde{\beta}_X = (4/9)\, \mathrm{Tr}(Q_{col}\, \rho_X) = (\bar{w}_{col}^X - \bar{w}_0^X) / \mathrm{Tr}\, W_X$.

Fraction in contrast form:

$f_{col}^X = 1/4 + (9/4)\cdot \tilde{\beta}_X$.

Fraction equivalence:

$f_{col}^X = f_{col}^Y \Leftrightarrow \mathrm{Tr}(Q_{col}\, \rho_X) = \mathrm{Tr}(Q_{col}\, \rho_Y)$.

Finite colour twirl:

$\mathcal{T}_c(W) = \int d\mu(g)\, U_c(g)\, W\, U_c(g)^\dagger$, $f_{col}[\mathcal{T}_c(W)] = f_{col}[W]$.

Schur form:

$\mathcal{T}_c(W_X) = w_{col}^X \cdot I_3 \oplus W_0^X$.

Response form:

$W_X = A_X^\dagger\, G_X\, A_X$, $G_X \geq 0$.

Relative-running identity:

$d\, f_{col}^X / d \ln \mu = f_{col}^X (1 - f_{col}^X)(\gamma_{col}^X - \gamma_0^X)$.

Stability bound:

$|\, f_{col}[W + E] - f_{col}[W]\, | \leq \max(f, 1 - f) \cdot \|E\|_1 / (\mathrm{Tr}\, W - \|E\|_1)$.

Appendix B — Worked Models

These examples illustrate the algebra. They are not numerical VERSF predictions.

B.1 Uniform census model

Let $W_1 = I_{12}$. Then $r_1 = 1$ and

$f_{col}[W_1] = 1/4$.

B.2 Coloured enhancement

Let $W_2 = 2 \cdot I_3 \oplus I_9$. Then $r_2 = 2$ and

$f\_col[W_2] = 2/(2 + 3) = 2/5$.

The triplet is one quarter of the directions but two fifths of this declared response. The factor two has not been derived.

B.3 Coloured suppression

Let $W_3 = \frac{1}{2} \cdot I_3 \oplus I_9$. Then $r_3 = 1/2$ and

$f\_col[W_3] = (1/2)/(1/2 + 3) = 1/7$.

Going beyond the unweighted census may reduce the fraction.

B.4 Nonuniform complement with exact census recovery

Let

$W_4 = I_3 \oplus \mathrm{diag}(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, 3/2, 3/2, 3/2)$.

The complement weights sum to $3 \cdot (\frac{1}{2}) + 3 \cdot (1) + 3 \cdot (3/2) = 9$. Thus

$\bar{w}_0 = 1 = \bar{w}\_col$,

and $f\_col[W_4] = 1/4$. However, $W_4 \not\propto I_{12}$. This proves that full uniformity is unnecessary.

B.5 Anisotropy invisible to the aggregate fraction

Let D_0 be any traceless Hermitian operator acting wholly within the nine-dimensional singlet complement, $\mathrm{Tr}\, D_0 = 0$. Choose

$W_5 = I_3 \oplus (I_9 + \varepsilon \cdot D_0)$,

with ε small enough to preserve positivity. Then

$\mathrm{Tr}(Q\_col\ W_5) = 0$,

so $f\_col[W_5] = 1/4$. The complement may be highly nonuniform, but this audit cannot see traceless rearrangements confined within it.

B.6 Pure sector-contrast perturbation

Let

$W_6 = I_{12} + \varepsilon \cdot Q\_col.$

The triplet eigenvalue is $1 + (3/4)\varepsilon$ and the complement eigenvalue is $1 - (1/4)\varepsilon$. Positivity requires

$-4/3 \le \varepsilon \le 4.$

Because $\mathrm{Tr}\, Q\_col = 0$, $\mathrm{Tr}\, W_6 = 12$. Also,

$\mathrm{Tr}(Q\_col\, W_6) = \varepsilon \cdot \mathrm{Tr}(Q\_col^2) = (9/4)\varepsilon.$

Therefore

$f\_col[W_6] = 1/4 + (3/16)\varepsilon.$

This perturbation lies exactly along the one operator direction visible to the aggregate fraction, and at the positivity endpoints it attains the extreme values $f = 0$ ($\varepsilon = -4/3$) and $f = 1$ ($\varepsilon = 4$), saturating the κ_X window of §7.7.

B.7 Same carrier, different participation types

Let

$W\_closure = 2 \cdot I_3 \oplus I_9$, $W\_kinetic = I_{12}$.

Then

$f\_col^{closure} = 2/5$, $f\_col^{kinetic} = 1/4$.

There is no contradiction. The two operators answer different questions.

B.8 Constructing a target fraction

Suppose a target fraction is $f^\star = 0.30$. Then

$r^\star = 3 \cdot (0.30)/0.70 = 9/7.$

The operator

$W^\star = (9/7) \cdot I_3 \oplus I_9$

reproduces the target. This is not a prediction. It is an explicit demonstration of underdetermination.

Appendix C — General $g \oplus h$ Extension

Let a finite carrier decompose into a selected sector of rank g and complement of rank h.

Define the sector means $\bar{w}_s$ and $\bar{w}_0$. Then

$f_s = g \bar{w}_s / (g \bar{w}_s + h \bar{w}_0)$.

With $r = \bar{w}_s / \bar{w}_0$,

$f_s = g r / (g r + h)$.

The census value is

$f_s^{census} = g / (g + h)$.

The exact departure is

$f_s - f_s^{census} = g h (r - 1) / [(g r + h)(g + h)]$.

Therefore

$f_s > f_s^{census} \Leftrightarrow r > 1$.

The inverse relation is

$r = h f_s / [g (1 - f_s)]$.

The centred sector projector is

$Q_s = \Pi_s - [g/(g + h)]\cdot I$,

with

$\mathrm{Tr}\, Q_s = 0$, $\mathrm{Tr}(Q_s^2) = g h / (g + h)$,

and

$f_s - f_s^{census} = \mathrm{Tr}(Q_s W) / \mathrm{Tr}\, W$.

For CPF-1, $g = 3$ and $h = 9$, giving $\mathrm{Tr}(Q_s^2) = 27/12 = 9/4$, in agreement with §7.1.

C.1 Repeated triplets

If the colour triplet occurs with multiplicity m, then the coloured carrier is $\mathbb{C}^3 \otimes \mathbb{C}^m$. A colour-invariant operator has the form

$W_{col} = I_3 \otimes K_m,\ K_m \geq 0.$

Colour symmetry enforces equality across the three colour components but does not force equality among distinct triplet copies. The aggregate coloured rank is 3m.

C.2 Triplets and antitriplets

The **3** and **3̄** representations are inequivalent complex representations. A carrier containing both requires distinct projectors unless an additional symmetry proves equality of their participation weights. Equal dimension alone is insufficient.

Appendix D — Standard Mathematics and CPF-1-Specific Content

D.1 Standard mathematical machinery

The following ingredients are standard: positive-functional trace representation in finite dimensions; compact-group character projectors; Haar twirling; Schur reduction; cyclic trace invariance; Hilbert–Schmidt decomposition; quotient-rule flow identities; trace-norm perturbation estimates.

CPF-1 does not claim these results as new mathematics. Representative sources are cited in text: [8], [9], [11], [13], [14], [15], [16].

D.2 CPF-1-specific closure content

The programme-specific content is:

1. the application of positive-functional discipline to the inherited VERSF $3 \oplus 9$ carrier;
2. the exact separation of census and participation;
3. the compression of the physical aggregate to one relative sector ratio;
4. the identification of $Q_{col} = \Pi_{col} - \frac{1}{4} \cdot I$ as the unique visible contrast direction;
5. the proof that all orthogonal operator anisotropy is invisible to the aggregate fraction;
6. the positivity window $-1/4 \leq \kappa_X \leq 3/4$ with its extreme-support characterisation;
7. the constructive demonstration that every numerical value is otherwise admissible;
8. the resulting no-canonical-operator/no-prediction theorem;
9. the confinement and observable non-identification firewall;

10. the exact fraction-equivalence criterion on the trace-one affine space and the four-level closure hierarchy;
11. the explicit bridge-debt ledger for future numerical closure.

Appendix E — Open Bridge-Debt Ledger

Debt	Required content	Status
D1 — Carrier-origin debt	Derive the twelve-direction carrier from VERSF primitives	Provisionally assigned to SC-1 [17]; consumption conditional on D14
D2 — Representation debt	Derive the exact Standard Model colour identification	Provisionally assigned to GC-1 [18]; consumption conditional on D14
D3 — Physical-metric debt	Derive the positive carrier metric	Open dependency
D4 — Participation-type debt	Select the physical question X	Open
D5 — Response-map debt	Derive A_X	Open
D6 — Response-metric debt	Derive G_X	Open
D7 — Contrast debt	Derive r_X, β̃_X, or κ_X without target fitting	Open
D8 — Scale-evolution debt	Derive running and operator mixing	Open
D9 — Threshold debt	Supply matching across changing carriers	Open
D10 — Local-gauge debt	Embed the finite carrier in gauge-invariant local field theory	Open
D11 — Confinement debt	Connect internal participation to confined observables	Open
D12 — Observable debt	Derive the measurement functional	Open
D13 — Empirical debt	Compare with data after all conventions are fixed	Open
D14 — Ledger-verification debt	Verify the SC-1 and GC-1 consumer edges against the Master Ledger record, including whether the	Open

Debt	Required content	Status
	twelve-direction carrier belongs to SC-1, SC-2, or a combination of the census and occupancy-map gates	

CPF-1 does not discharge these debts by naming them. It identifies exactly why they are required.

Appendix F — References

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Final One-Sentence Theorem

Within the CPF-1 gate, let a finite positive-metric VERSF response carrier decompose under its declared internal SU(3)_c representation into one rank-three irreducible triplet and a rank-nine singlet complement, let the selected sector be defined by its representation projector, and let a declared participation question extend, under the positive-linear-extension premise, to a nonzero positive linear functional fixed independently of its intended result; then the aggregate coloured share is necessarily the positive weighted projector expectation $f_{col}^X = \mathrm{Tr}(\Pi_{col} W_X)/\mathrm{Tr}\, W_X = r_X/(r_X + 3)$, the raw value 3/12 is recovered exactly when the two sector means agree, the entire departure from one quarter is the normalised expectation of the unique centred contrast operator $Q_{col} = \Pi_{col} - I_{12}/4$ and is confined by positivity to the window $-1/4 \leq \kappa_X \leq 3/4$, every component of the trace-normalised audit orthogonal to that contrast is invisible to this aggregate audit while a positive additive uniform contribution acts only by diluting it, every numerical fraction in [0, 1] is otherwise constructible on the same carrier, and no value beyond one quarter is therefore VERSF-derived until an independently fixed normalised sector contrast — supplied by a canonical response operator or derived directly — together with a scale prescription, local-gauge embedding, confinement bridge, and observable map, is obtained without using that value as input.