The Electroweak Representation and Connection Closure in VERSF

▲ Programme Milestone — Standard Model Gauge-Representation Series

Deriving Left-Handed Weak Action, Hypercharge Admissibility, and the Local W^i_μ / B_μ Connection Form from Spinorial Commitment Geometry

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Successor to *The Chiral Matter Representation Theorem in VERSF*.

Headline result — from the matter table to the electroweak representation operator.

The predecessor paper derives the one-generation chiral matter skeleton

$$L_L = (\nu_L, e_L),\ e_R,\ Q_L = (u_L, d_L),\ u_R,\ d_R,$$

with the anomaly-rigid hypercharge ledger

$$L_L : (1, 2, -\tfrac{1}{2}),\ e_R : (1, 1, -1),\ Q_L : (3, 2, \tfrac{1}{6}),\ u_R : (3, 1, \tfrac{2}{3}),\ d_R : (3, 1, -\tfrac{1}{3}).$$

The present paper advances from that table to the electroweak representation architecture acting on it.

The central claim is that weak unresolved commitment is not merely a label. It carries a two-state unitary operator algebra on the left-chiral sector, while the fully committed right-chiral sector admits only the trivial weak action. Under orientation-admissible spinorial transport this gives the chiral-locking identity

$$P_W = P_L,$$

and the admissible weak generators close as

$$[T_i, T_j] = i\, \varepsilon_{ijk}\, T_k,$$

acting nontrivially on (L_L, Q_L) and trivially on (e_R, u_R, d_R). The hypercharge generator Y is the commuting source-grade generator that completes the charge ledger and satisfies

$[T_i, Y] = 0$, $Q = T_3 + Y$.

The representation-level electroweak algebra is therefore

$$\mathfrak{g}_{EW}^{rep} = \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y.$$

This is not a derivation of electroweak symmetry breaking, W/Z masses, the Higgs vev, the Weinberg angle, Yukawa couplings, CKM, PMNS, or fermion masses. It is the prior representation result: why the chiral matter skeleton carries the electroweak operator structure it does.

General Reader Summary

Every particle of ordinary matter has a kind of handedness. Just as a corkscrew can twist one way or the other, an electron or a quark can be "left-handed" or "right-handed" — a property physicists call chirality. For most of physics, nature doesn't care which is which. Gravity treats both the same. So does electromagnetism. So does the force that binds atomic nuclei.

But one force is different. The weak force — the force behind radioactive decay, the force that lets the Sun burn — only acts on left-handed particles. Right-handed particles are completely invisible to it. This is one of the strangest facts in all of physics. It means the universe can tell the difference between itself and its own mirror image.

The Standard Model of particle physics describes this left-handedness perfectly — but it does not explain it. The asymmetry is simply written into the equations by hand, because that is what experiments show. Ask the Standard Model *why* the weak force is left-handed and it has no answer.

This paper proposes an answer.

The previous paper in this series derived the cast of characters: why one generation of matter contains an electron, a neutrino, and two kinds of quark, and why they carry the electric charges they do. This paper asks the next question: what does the weak force actually *do* to those characters, and why does it only do it to the left-handed ones?

In VERSF, reality is built from commitment events — moments where the underlying substrate settles an open question and writes the answer into permanent record. The proposal here is that the weak force is what an *unfinished* commitment looks like. Some particles carry an open two-way choice that has not yet been settled: neutrino-or-electron, up-quark-or-down-quark. The weak force is simply the operation that shuffles between the two options of that open choice. That is why the weak force always transforms particles in pairs — it is rotating an unresolved either/or.

And here is the key move. In VERSF, carrying an open choice through the geometry is a delicate business, and it turns out to be orientation-sensitive: only left-handed particles can hold the choice open. Right-handed particles have already committed — their either/or is settled — so there is nothing left for the weak force to rotate. They are deaf to it, not because a rule forbids the interaction, but because a finished record cannot be revised.

That single idea — *weak force = unfinished commitment, and only the left-handed side can stay unfinished* — is written in the paper as a compact equation, P_W = P_L. Once it is in place, the rest follows with almost no freedom. The mathematics of rotating a two-way choice is completely standard and completely rigid: it produces exactly the structure physicists call SU(2), the known shape of the weak force. Two options force this shape; the weak force could not have been bigger or smaller. Alongside it sits a second, quieter charge called hypercharge, which doesn't shuffle the pair but stamps a shared value on both members, and combining the two reproduces the ordinary electric charges of every particle in the table.

The result is the full operating manual of the electroweak force acting on one generation of matter — the structure textbooks write as SU(2)_L × U(1)_Y — derived rather than postulated, with the mysterious little subscript L now carrying an explanation. The paper then takes one further step. Knowing what the weak force does at a single point is not quite enough, because particles move: comparing a particle's unfinished choice *here* with its unfinished choice *over there* requires a messenger structure connecting the two points. The paper shows that the shape of that messenger structure is completely fixed by everything already derived — and it turns out to be exactly the shape of the fields physicists call W and B, the raw ingredients from which the W and Z bosons and the photon are later assembled. What is derived is their shape, not their strength or their masses; those remain for later papers.

To be clear about what this paper does *not* do: it does not explain why the W and Z particles are heavy, what the Higgs is doing, or the numerical strength of the weak force. Those questions come later in the programme, and other papers address them. This paper answers the question that comes before all of those: why the weak force has the left-handed, pair-shuffling shape it has in the first place. And it is honest about its one open debt — the claim that only left-handed particles can carry an unfinished commitment is stated here as a precise condition still awaiting its microscopic proof. That proof is now the sharpest single target in this part of the programme.

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Abstract

The preceding VERSF matter-representation paper derives a one-generation chiral Standard-Model-like matter skeleton from the spinorial Fock sector, persistent winding charge, weak-commitment chirality, and threefold confined support. It obtains the representation slots

$L_L, e_R, Q_L, u_R, d_R,$

and shows that, once the lepton charge normalization and perturbative anomaly freedom are imposed, the quark hypercharges are the unique real anomaly-free completion

$Y_Q = \tfrac{1}{6}, Y_u = \tfrac{2}{3}, Y_d = -\tfrac{1}{3}.$

That paper leaves a principal hinge: the chiral-locking condition

$P_W = P_L,$

namely that weak unresolved commitment coincides with left spinorial chirality.

The present paper addresses that hinge while moving the programme forward from a matter table to an electroweak representation operator. It introduces the weak unresolved-commitment

module $\mathcal{W}$, proves that a nontrivial weak doublet action requires a two-dimensional complex branch space with norm-preserving admissible transformations, and identifies its infinitesimal traceless operators as an $\mathfrak{su}(2)$ representation — with the two-branch structure of weak commitment forcing rank one and dimension three, so no larger faithful nontrivial weak-branch algebra acting solely on the two-branch module is admissible. The spinorial orientation-admissibility condition then selects the left-chiral sector as the only sector capable of retaining this unresolved two-branch access under closure transport. Right-chiral modes are fully branch-committed and therefore admit only the trivial weak action.

The resulting weak generators T_i act as Pauli generators on the left doublets L_L, Q_L and as zero on the right singlets e_R, u_R, d_R:

$$T_i|_{\{L_L,\, Q_L\}} = \sigma_i/2, \quad T_i|_{\{e_R,\, u_R,\, d_R\}} = 0.$$

The hypercharge generator Y is identified as the commuting source-grade operator compatible with weak doublets,

$$[T_i, Y] = 0,$$

and the charge relation is

$$Q = T_3 + Y.$$

Together these results produce the representation-level electroweak algebra

$$\mathfrak{g}_{EW}^{rep} = \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$$

acting on the chiral matter skeleton.

The paper also strengthens the anomaly status of the predecessor result in two ways. First, perturbative anomaly freedom is interpreted not as phenomenological fitting but as representation admissibility: an anomalous chiral ledger cannot preserve the substrate record-current conservation required of a persistent gauge/source representation. Second, the hypercharge audit is run across four physically distinct anomaly channels constraining three quark hypercharges; one linear condition becomes algebraically redundant once the others are imposed, but the same rational solution passes all channels, so the ledger clears a genuine multi-channel consistency test rather than a parameter fit. A full substrate derivation of anomaly inadmissibility remains owed, but the paper identifies the precise obstruction.

The paper then takes one disciplined step beyond the pointwise representation result. Because matter fields are local, comparing weak branch states at neighbouring record-layer points requires a connection, and the representation closure fixes its form: the only local comparison structure preserving weak branch coherence, chiral locking, the hypercharge grade, source-current admissibility, and anomaly admissibility of the gauged comparison algebra is

$$\mathcal{A}_\mu^{EW} = g\, W^i_\mu T_i + g'\, B_\mu Y,$$

with covariant derivative $D_\mu = \partial_\mu + i\mathcal{A}_\mu^{EW}$. For the minimal one-generation skeleton this connection form is unique; if the optional sterile ν_R is included, uniqueness holds up to the anomaly-admissible gauged B−L grade, whose low-energy absence is recorded as owed rather than derived. This derives the shape of the electroweak connection — not its dynamics, kinetic terms, or coupling values, which remain owed.

The result is conditional and sharply scoped. It does not derive dynamical gauge field equations, gauge kinetic terms, electroweak symmetry breaking, the Weinberg angle, W/Z masses, the Higgs vev, Yukawa couplings, fermion masses, CKM, PMNS, or three generations. It derives the electroweak representation action on the already-derived chiral matter skeleton.

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0. Notation, Conventions, and Firewalls

Notation

$\mathcal{H}_1$ denotes the one-particle spinorial source-carrier Hilbert space inherited from the fermionic Fock-space reconstruction.

$\mathcal{F}_A(\mathcal{H}_1)$ denotes the antisymmetric fermionic Fock space.

$\psi(x)$ denotes the local spinorial field candidate inherited from the Fock-space reconstruction.

P_L, P_R denote spinorial chirality projectors:

$P_L + P_R = 1$, $P_L P_R = 0$.

P_W denotes the weak-commitment projector selecting unresolved weak two-branch support.

$\mathcal{W}$ denotes the weak unresolved-commitment branch space. When nontrivial, $\mathcal{W} \cong \mathbb{C}^2$.

T_i denote weak generators acting on $\mathcal{W}$.

Y denotes hypercharge, treated here as the commuting source-grade generator.

Q denotes electric/source charge. The convention throughout is

$Q = T_3 + Y$.

Hypercharge Convention Note

This paper uses the convention $Q = T_3 + Y$, so the doublet hypercharges are $Y(L_L) = -\tfrac{1}{2}$ and $Y(Q_L) = \tfrac{1}{6}$. Some companion papers — including the closure-norm condensation module — use the convention $Q = T_3 + Y_{SM}/2$, in which $Y_{SM} = 2Y$, giving $Y_{SM}(L_L) = -1$ and

$Y_{SM}(Q_L) = \frac{1}{3}$. The two conventions are equivalent and differ only by this factor of two in the hypercharge normalization; all tables and anomaly sums in this paper use the $Q = T_3 + Y$ normalization. Any downstream module importing this paper's ledger must apply $Y_{SM} = 2Y$ before matching against $Q = T_3 + Y_{SM}/2$ expressions.

Representation labels use the standard shorthand

$(SU(3)_C, SU(2)_L, Y)$,

but the colour entry is inherited as a C_3-support skeleton, not yet as a full derivation of continuous SU(3) Yang–Mills dynamics.

Mass Firewall

This paper does not compute or revise any fermion mass. It does not derive Yukawa couplings. It does not use CKM or PMNS data. It does not address generation mass hierarchy. It does not derive the Higgs mass or the Higgs vev.

The paper is prior to mass. It derives the electroweak representation action on matter slots before any mass operator is attached.

Electroweak-Breaking Firewall

This paper does not derive electroweak symmetry breaking. It does not derive the physical photon, $W^\pm$, or Z as broken-phase fields. It does not derive the Weinberg angle. It does not derive W/Z masses.

The claim is representation-level:

$\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$

acts on the chiral matter skeleton.

Gauge-Dynamics Firewall

This paper does not derive gauge kinetic terms

$-\frac{1}{4} W^i_{\mu\nu} W^{i\,\mu\nu}, -\frac{1}{4} B_{\mu\nu} B^{\mu\nu}$.

It does not derive the values of g and g', nor the gauge field equations. It derives the representation operators and — in §14′ — the unique local connection *form* that a future gauge-dynamical module would supply with kinetic terms and dynamics.

Observational-Input Firewall

No observed electroweak datum — measured coupling, mixing angle, boson mass, or decay rate — enters any derivation in this paper as construction data. The only empirical touchpoints are the target pattern being explained (left doublets, right singlets, the hypercharge ledger) and the lepton charge normalization inherited from the predecessor paper.

0′. Predictive-Content Ledger

Object	Grade	Status
Spinorial Fock sector	Inherited	From fermionic Fock-space reconstruction
Chiral matter skeleton L_L, e_R, Q_L, u_R, d_R	Inherited / Conditional	From chiral matter representation paper
Hypercharge ledger (−½, −1, ⅙, ⅔, −⅓)	Inherited / Conditional	Rigid within anomaly audit
Weak projector P_W	Inherited / Conditional	From weak-commitment programme
Chirality projectors P_L, P_R	Inherited	Spinorial closure / Dirac-admissibility layer
Non-Abelian refinement admissibility (EW-0)	Inherited	Commutator defects refinement-irrelevant: A_C$^{(n)}$ ∼ 16^{-n} A_C$^{(0)}$
Relation to closure-norm condensation (EWSB)	Downstream module	Sequential, not overlapping — see §13′
Weak branch space $\mathcal{W} \cong \mathbb{C}^2$	Conditional	Derived from unresolved two-branch commitment
Rank restriction (no larger faithful weak-branch algebra)	Exact within EW-2/EW-7	Two-branch support forces dim $\mathcal{W} = 2$
Chiral locking P_W = P_L	Conditional identity (premise-organised)	Physical content in EW-4; packaged for downstream consumption
Right-singlet exclusion	Conditional theorem	Right modes lack unresolved weak branch
Weak generators T_i	Exact within $\mathcal{W}$	Norm-preserving traceless endomorphisms
$\mathfrak{su}(2)$_L representation	Conditional / Exact	Conditional on weak branch module
Hypercharge generator Y	Conditional	Commuting source-grade operator
$[T_i, Y] = 0$	Exact within representation	Hypercharge scalar on weak irreps
$Q = T_3 + Y$	Representation identity	Substrate derivation of unit normalization owed
$\mathfrak{su}(2)$_L ⊕ $\mathfrak{u}(1)$_Y	Conditional theorem	Electroweak representation closure

Object	Grade	Status
Weak charged-current operators $T_\pm$	Conditional	Act only on left doublets
Local connection form $\mathcal{A}_\mu^{EW} = g W^i_\mu T_i + g' B_\mu Y$	Conditional theorem	Unique under conditions 1–5, minimal skeleton; unique up to gauged B−L with ν_R (Theorem 5)
Formal curvature $F_{\mu\nu}^{EW}$	Preparatory notation	No dynamics claimed
Anomaly freedom as representation admissibility	Conditional	Full substrate proof owed
Four-channel anomaly consistency (one channel algebraically redundant)	Audit condition	Passed across all channels
Gauge boson dynamics	Owed	Not derived
Electroweak symmetry breaking	Owed	Not derived
Weinberg angle	Owed	Not derived
Higgs/Yukawa/mass sector	Firewalled	Not addressed
Three generations	Owed	Not derived

0″. Inherited Electroweak Infrastructure

The present paper sits between two already-developed layers of the VERSF programme.

The predecessor, *The Chiral Matter Representation Theorem in VERSF*, derives the one-generation chiral matter skeleton

$L_L, e_R, Q_L, u_R, d_R,$

with the anomaly-rigid hypercharge ledger

$L_L : (1, 2, -\tfrac{1}{2}),\ e_R : (1, 1, -1),\ Q_L : (3, 2, \tfrac{1}{6}),\ u_R : (3, 1, \tfrac{2}{3}),\ d_R : (3, 1, -\tfrac{1}{3}).$

The downstream closure-norm condensation papers address the later question of electroweak symmetry breaking: how the already-present electroweak representation structure is broken by the closure-norm condensate, why $W^\pm$ and Z acquire mass, why the photon remains massless, and why $SU(3)_C$ remains unbroken.

The present paper occupies the intervening layer. Its task is neither to rederive the matter table nor to derive vacuum condensation. Its task is to explain why the chiral matter table carries the electroweak representation operator

$\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$

before symmetry breaking occurs.

The present paper is the missing bridge between the chiral matter skeleton and the closure-norm electroweak-breaking module. It derives the unbroken electroweak representation algebra acting on matter; the later closure-norm condensation paper acts on this algebra to produce the broken-phase vacuum.

This gives the programme ladder:

fermionic Fock algebra → chiral matter skeleton → **electroweak representation closure** → closure-norm condensation / EWSB → mass and mixing attachment.

Inherited non-Abelian compatibility

The present paper does not need to prove from scratch that non-Abelian operator structure is admissible in the substrate. It inherits the non-Abelian refinement result from the substrate-dynamics programme.

For non-commuting closure-transport generators $H_1, H_2 \in \mathfrak{su}(8)$, the leading path-comparison defect is

$A\_C^{(0)} = \varepsilon^4 \|[H_1, H_2]\|^2 \rho\_c^4 + O(\varepsilon^5)$.

Under admissible edge-bisection refinement, with ε → ε/2, this defect decays as

$A\_C^{(n)} \sim 16^{-n} A\_C^{(0)}$.

Non-Abelian commutator content is therefore refinement-irrelevant: it is not forbidden by the substrate, but is driven toward admissibility under refinement. The structural conclusion matters for the present paper. A non-Abelian weak algebra such as $\mathfrak{su}(2)$ is compatible with admissible closure transport; the paper's remaining task is to determine why this non-Abelian action is restricted to the left-chiral weak branch.

Representation-before-condensation firewall

The electroweak representation algebra derived here is the unbroken representation structure. It is the operator architecture on matter before closure-norm condensation selects the broken vacuum phase.

The downstream closure-norm condensation paper shows that the closure-norm field is a gauge singlet under $SU(3)\_C \times SU(2)\_L \times U(1)\_Y$, that electroweak gauge transport couples to the closure-norm condensate through interface stiffness, and that the photon is the unique combination of B_μ and W^3_μ leaving the closure-norm vacuum invariant. Those results belong after the present paper in the logical order.

The present paper therefore does not derive

SU(2)_L × U(1)_Y → U(1)_EM,

nor W/Z masses, the Higgs vev, the Weinberg angle, or the Higgs radial mode. It derives the representation algebra that the closure-norm condensation paper later acts upon.

Layer firewall from the hierarchy paper

The hierarchy-problem paper distinguishes the closure layer from the record layer. The closure layer supplies admissibility conditions and substrate thresholds; the record layer supports field-theoretic structures such as gauge and matter fields. The present paper operates at the record-layer representation level. It does not invoke Planck-scale propagating degrees of freedom, does not compute scalar naturalness corrections, and does not derive the electroweak scale.

This matters because the paper's claim is deliberately representation-prior:

which operators act on matter?

not

why does the Higgs have its scale?

The result is therefore insulated from the hierarchy, Higgs-ratio, and mass-generation modules. Those modules become downstream attachments, not hidden assumptions.

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1. Purpose and Claim Level

The predecessor paper derives a chiral matter representation skeleton. That is a major step, but it still leaves a question:

What electroweak operator structure acts on that skeleton?

The Standard Model does not merely contain the matter slots

L_L, e_R, Q_L, u_R, d_R.

It contains a chiral electroweak representation action:

SU(2)_L × U(1)_Y.

The purpose of this paper is to derive that action at the representation level from VERSF spinorial commitment geometry.

The target chain is

spinorial Fock sector → chiral matter skeleton → weak unresolved branch → P_W = P_L → 𝔰𝔲(2)_L → 𝔰𝔲(2)_L ⊕ 𝔲(1)_Y.

Central Claim

Given the inherited chiral matter skeleton and the premises EW-0–EW-7 below, the VERSF matter sector carries a representation-level electroweak algebra

𝔤_EW^rep = 𝔰𝔲(2)_L ⊕ 𝔲(1)_Y,

where:

1. 𝔰𝔲(2)_L acts nontrivially only on L_L and Q_L;
2. e_R, u_R, d_R are weak singlets;
3. Y commutes with the weak action;
4. $Q = T_3 + Y$;
5. the inherited anomaly-rigid hypercharge ledger is admissible and unique under the no-exotics one-generation skeleton.

This is an electroweak representation theorem, not a full electroweak field theory.

Logical architecture of the argument

The derivation has exactly one load-bearing physical premise for the representation closure proper (Theorems 1–4): weak orientation admissibility (EW-4/EW-5). The connection lift of §14′ adds a second: anomaly admissibility as a substrate condition (Proposition 6, whose full substrate proof is Debt 3). Everything downstream of these is representation-theoretic and essentially forced:

- the inherited non-Abelian refinement result (EW-0) clears the structural path — non-commuting weak transport is refinement-admissible, so a non-Abelian weak algebra is not an alien insertion into the substrate;
- given a two-branch unresolved module and Born-norm preservation, the weak algebra is 𝔰𝔲(2) with no alternative (§6);
- given orientation admissibility, chiral locking P_W = P_L follows (§7), and right-singlet exclusion is its corollary (§8);
- given a commuting source-grade, Schur rigidity makes Y scalar on doublets (§11) and forces $Q = T_3 + Y$ up to normalization (§12).

The paper is therefore structured so that its falsifiability concentrates where the physics is. The representation closure (Theorems 1–4) is exposed in exactly one place: if EW-4 stands, it stands; if EW-4 falls, it falls with it, and nothing in the algebra can rescue it. The connection closure (Theorem 5) is exposed in exactly two: EW-4 and the anomaly-admissibility principle of Proposition 6.

2. Representation Premises

EW-0 — Non-Abelian refinement admissibility

The substrate admits non-Abelian closure-transport content under admissible refinement. In particular, for non-commuting generators H_1, H_2, the leading closure defect is proportional to $\|[H_1, H_2]\|^2$, but under admissible refinement the defect is irrelevant:

$A\_C^{(n)} \sim 16^{-n} A\_C^{(0)}$.

A non-Abelian weak branch algebra is therefore not excluded by closure admissibility. It may exist as a refinement-stable record-layer representation, provided the associated action respects weak-commitment, chirality, and source-grade constraints.

Status: Inherited from substrate dynamics.

Falsifier: If non-Abelian commutator defects fail to refine away for the specific weak generators T_i, the $\mathfrak{su}(2)\_L$ representation closure becomes substrate-unstable.

Role in this paper: EW-0 does not by itself derive weak SU(2). It clears the structural path: non-Abelian weak action is admissible; the paper then derives why that action is left-chiral and how it combines with hypercharge.

EW-1 — Fermionic and matter-skeleton inheritance

The paper inherits

$\mathscr{F}\_A(\mathscr{H}_1)$, $\psi(x)$,

and the chiral matter skeleton

L_L, e_R, Q_L, u_R, d_R.

Status: Inherited from the Fock-space and chiral-representation papers.

Falsifier: Failure of either predecessor result.

EW-2 — Weak unresolved two-branch commitment

Weak-active states are those retaining a nontrivial unresolved two-branch commitment support. The branch space is a complex two-dimensional module

$\mathcal{W} \cong \mathbb{C}^2$.

The dimension is not adjustable: "two-branch" means exactly two unresolved commitment alternatives, so dim $\mathcal{W} = 2$ is part of the premise content, and every rank consequence downstream (§6) inherits this rigidity.

Status: Conditional; inherited from the weak-commitment programme.

Falsifier: Weak support is not two-branch, or cannot be represented as a complex two-state module.

EW-3 — Born-norm preservation on unresolved weak branches

Admissible weak transformations preserve the Born/Hilbert norm on $\mathcal{W}$. Weak branch transformations are therefore unitary on the unresolved branch space.

Status: Inherited from the Born/Hilbert structure of the fermionic programme.

Falsifier: Weak branch transformations do not preserve transition probabilities.

EW-4 — Weak orientation admissibility

A nontrivial unresolved weak branch must be compatible with spinorial closure orientation. Under the VERSF orientation convention used here, the branch-preserving weak unresolved module is compatible with left chirality and incompatible with nontrivial right-chiral weak action:

$P_W P_L = P_W$, $P_W P_R = 0$.

The two displayed relations are not independent: the second follows from the first via $P_R = 1 - P_L$. Both are displayed because downstream sections consume them in different forms.

Status: Conditional; this is the load-bearing new physical premise and the principal theorem target for future substrate derivation.

Falsifier: Right-chiral source-carriers also retain nontrivial weak unresolved branch access.

EW-5 — Right-commitment closure

Right-chiral modes are fully committed with respect to the weak two-branch access. A nontrivial weak action on them would reintroduce unresolved branch structure and violate their committed-singlet status.

Status: Conditional; paired with EW-4. EW-4 states which sector *can* carry the branch; EW-5 states why the excluded sector *cannot* be dressed with one by hand.

Falsifier: Right-chiral modes carry weak doublet structure without violating commitment closure.

EW-6 — Hypercharge as commuting source-grade

There exists a source-grade generator Y that commutes with the weak branch action and is scalar on each weak irreducible representation:

$[T_i, Y] = 0.$

Status: Conditional; inherited from winding/source charge plus representation consistency.

Falsifier: Source charge does not decompose into a weak component plus a commuting grade.

EW-7 — No extra anomaly-carrying chiral exotics

At this representation layer the one-generation skeleton contains no additional anomaly-carrying chiral matter beyond

$L_L, e_R, Q_L, u_R, d_R,$

with the optional sterile $\nu_R : (1, 1, 0)$ permitted because it contributes zero to every anomaly sum — it is, in fact, the unique nontrivial chiral addition with that property (§16).

Status: Minimal-skeleton condition.

Falsifier: Additional unavoidable chiral sectors alter anomaly cancellation or hypercharge rigidity.

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3. Inherited Chiral Matter Skeleton

The predecessor paper derives the one-generation skeleton

L_L = (ν_L, e_L), e_R, Q_L = (u_L, d_L), u_R, d_R,

with representation ledger

L_L : (1, 2, −½), e_R : (1, 1, −1), Q_L : (3, 2, ⅙), u_R : (3, 1, ⅔), d_R : (3, 1, −⅓).

It also proves, within the anomaly audit and the no-exotics one-generation skeleton, that the quark hypercharges are not a free choice. Given lepton normalization and anomaly freedom, the unique real completion is

Y_Q = ⅙, Y_u = ⅔, Y_d = −⅓.

This paper does not rederive the matter skeleton. It derives the representation operator acting on it.

4. The Electroweak Representation Problem

A matter table alone is not yet an electroweak theory. It must be supplied with operators satisfying the following conditions:

1. the upper and lower members of L_L are connected by weak action;
2. the upper and lower members of Q_L are connected by the same weak action;
3. the right-handed sectors e_R, u_R, d_R are not weak doublets;
4. the weak action is norm-preserving;
5. the weak action has exactly three infinitesimal generators;
6. the source-grade generator Y commutes with the weak action;
7. electric/source charge decomposes as $Q = T_3 + Y$.

The problem is therefore to derive

𝔰𝔲(2)_L ⊕ 𝔲(1)_Y

as a representation algebra — not as a broken electroweak field theory.

5. Weak Unresolved Branch Space

The weak representation does not begin as an abstract gauge group. It begins as a substrate fact: some source-carrier states retain an unresolved two-branch commitment.

Let 𝒲 denote this unresolved weak branch space. A weak-active state carries a branch label

$\Psi = (\psi_+, \psi_-)^T \in \mathcal{W}$.

The two entries are not two unrelated particles. They are two admissible alternatives inside a single unresolved weak commitment structure. The weak interaction is the record-layer operation that moves within this branch space without destroying its Born/Hilbert norm.

For the lepton sector, the unresolved branch is

$L_L = (\nu_L, e_L)^T$.

For the quark sector, it is

$Q_L = (u_L, d_L)^T$.

The inherited non-Abelian refinement result (EW-0) matters here. A two-branch weak space will support non-commuting rotations. Such non-commutativity would be fatal if the substrate could not tolerate non-Abelian transport. But the substrate-dynamics programme shows that non-Abelian commutator defects are refinement-irrelevant under admissible edge-bisection:

$A_C^{(n)} \sim 16^{-n} A_C^{(0)}$.

The existence of a non-Abelian weak branch algebra is therefore structurally admissible. The remaining question is not whether non-Abelian weak action can exist, but where it is allowed to act. The answer developed below is

$P_W = P_L$:

weak unresolved branch structure is locked to left chirality, so the non-Abelian weak action becomes $\mathfrak{su}(2)_L$, not a vectorlike $\mathfrak{su}(2)$ acting equally on left and right.

The weak branch inner product is inherited from the Born/Hilbert structure:

$\langle \Psi, \Phi \rangle_{\mathcal{W}} = \psi_+^\dagger \varphi_+ + \psi_-^\dagger \varphi_-$.

An admissible weak transformation must preserve this norm. Weak branch transformations therefore lie in U(2) at the branch level.

The canonical split

The Lie algebra $\mathfrak{u}(2)$ decomposes canonically as

$\mathfrak{u}(2) = \mathfrak{su}(2) \oplus \mathfrak{u}(1)$,

where the $\mathfrak{u}(1)$ factor is the centre — the generators proportional to the identity — and the $\mathfrak{su}(2)$ factor is the traceless part. This split is not a basis choice. The centre of $\mathfrak{u}(2)$ is unique, so the separation of the commuting scalar grade from the traceless branch rotations is canonical, and no alternative decomposition exists.

In the electroweak representation, the traceless branch rotations become $\mathfrak{su}(2)$_L, while the commuting scalar grade is absorbed into the source-grade structure that §11 identifies as hypercharge.

6. The Weak-Branch Operator Lemma

Lemma 1 — The nontrivial weak branch algebra is $\mathfrak{su}(2)$, and nothing larger

Let $\mathcal{W} \cong \mathbb{C}^2$ be a two-dimensional complex unresolved-commitment branch space with Born-norm-preserving transformations. Then:

(i) the admissible infinitesimal branch operators form $\mathfrak{u}(2)$;

(ii) the traceless admissible operators close as $\mathfrak{su}(2)$, with basis

$T_i = \sigma_i/2$, $[T_i, T_j] = i$ ε_ijk T_k,

where σ_i are the Pauli matrices;

(iii) no larger faithful nontrivial weak-branch algebra acting solely on the two-branch module is admissible: any compact Lie algebra acting faithfully and unitarily on $\mathbb{C}^2$, and intersecting the centre of $\mathfrak{u}(2)$ trivially, embeds in $\mathfrak{su}(2)$ (equivalently: after quotienting the central $\mathfrak{u}(1)$, what remains embeds in $\mathfrak{su}(2)$). Larger groups could act only nonfaithfully, trivially, or by introducing extra hidden branch degrees of freedom — the first two options carry no additional physical weak content, and the third is excluded by EW-2 (the branch space is exactly two-dimensional) and EW-7 (no hidden chiral exotics).

Proof. (i) Norm-preserving transformations on a two-dimensional complex Hilbert space form U(2); the generator algebra is $\mathfrak{u}(2)$. (ii) Splitting off the central scalar generator leaves the traceless anti-Hermitian generators, which close as $\mathfrak{su}(2)$; in the standard Hermitian convention they are represented by $\sigma_i/2$. (iii) A faithful unitary representation of a compact algebra on $\mathbb{C}^2$ is a subalgebra of $\mathfrak{u}(2)$; removing the centre leaves a subalgebra of $\mathfrak{su}(2)$, which has rank one. Hence $\mathfrak{su}(3)$, $\mathfrak{su}(2) \oplus \mathfrak{su}(2)$, or any higher-rank candidate cannot act faithfully and nontrivially on the two-branch module alone; it could enter only through nonfaithful action, trivial factors, or an enlarged branch space, and the last is excluded by EW-2 and EW-7. ■

Reading

Parts (i) and (ii) are mathematically standard. The VERSF content is not that a two-state Hilbert module has an $\mathfrak{su}(2)$ algebra. The VERSF content is threefold: weak unresolved commitment supplies precisely such a two-state branch module (EW-2); part (iii) shows the substrate could not have produced a larger weak group from this branch structure even in principle; and spinorial orientation admissibility restricts the module to the left-chiral sector (§7). The weak group is SU(2) because weak commitment is two-branch — the rank of the weak force is a counting fact about unresolved alternatives.

7. Chiral Locking

The predecessor paper used chiral locking as a premise:

P_W = P_L.

The present paper promotes it from premise to organised identity, conditional on weak orientation admissibility. To be candid about the weight of the result: the physical content of chiral locking lives in EW-4, and the theorem's work is to package that content, together with the definitional structure of the weak-active sector, into the clean projector identity that every downstream section consumes. That packaging is legitimate and load-bearing work — Corollary 1.1, Theorem 2, and Theorem 5 all consume the identity — but it is organisation, not new transport physics. The transport physics is precisely Debt 1.

Theorem 1 — Chiral-Locking Theorem

Given EW-2 through EW-5, the weak unresolved-commitment projector equals the left-chirality projector on the low-energy weak-active matter support:

P_W = P_L |_{ℋ_weak matter}.

Equivalently: unconditionally,

P_W P_R = 0,

and within the weak-active matter sector,

P_W P_L = P_L P_W = P_W = P_L.

Weak-active modes are left-chiral, and right-chiral modes are weak singlets. Note the asymmetry of the two clauses: the exclusion P_W P_R = 0 is sector-independent (EW-4 states it for all admissible transport), while the identification P_W = P_L holds on the weak-active matter sector, where the skeleton of §3 lives.

Proof

The identity is established as two inclusions.

Inclusion 1 (P_W $\mathscr{H}_1$ ⊆ P_L $\mathscr{H}_1$). Weak-active modes are, by EW-2, precisely those retaining nontrivial unresolved two-branch support. The nontrivial weak branch must be transported through spinorial closure without destroying branch coherence. By EW-4, orientation-admissible spinorial transport permits such branch coherence only on the left-chiral sector: P_W P_R = 0. Therefore all weak-active support lies in P_L $\mathscr{H}_1$.

Inclusion 2 (P_L $\mathscr{H}_1$ ⊆ P_W $\mathscr{H}_1$ on the weak-active sector — definitional). Restrict attention to the low-energy weak-active source-carrier sector — the sector in which the matter skeleton of §3 lives. On this sector P_W acts as the identity by the sector's defining property, so the inclusion is immediate. This inclusion is definitional, not derived transport content; the derived content of the theorem is Inclusion 1, which is EW-4 organised into projector form.

The two inclusions give P_W = P_L on the weak-active source-carrier sector. ■

Scope note

The sector restriction in the theorem statement is doing honest work, and it is stated rather than buried. EW-4 alone gives only the one-way inclusion P_W $\mathscr{H}_1$ ⊆ P_L $\mathscr{H}_1$; the reverse inclusion — and hence the equality — requires the weak-active matter condition, which is why the identity carries the restriction $\mathscr{H}$_weak matter explicitly. The theorem does not claim that every left-chiral state in every VERSF sector is weak-active; it claims that on the matter sector where the skeleton is defined, left chirality and weak unresolved commitment coincide. This is exactly the scope at which the Standard Model pattern is stated, and no stronger claim is made.

Corollary 1.1 — The L in SU(2)_L is derived at representation level

Given chiral locking,

P_W = P_L,

the weak branch algebra obtained from $\mathcal{W} \cong \mathbb{C}^2$ acts only on left-chiral source-carrier states. The weak algebra is therefore not merely

$\mathfrak{su}(2)$,

but specifically

$\mathfrak{su}(2)$_L.

Proof. Lemma 1 establishes that a two-dimensional norm-preserving complex branch module has traceless infinitesimal generators closing as $\mathfrak{su}(2)$. Theorem 1 establishes that the only source-carrier sector retaining such unresolved branch structure is P_L $\mathscr{H}_1$. The nontrivial $\mathfrak{su}(2)$ action therefore has support only on the left-chiral subspace. On P_R $\mathscr{H}_1$ the weak branch module is absent, and the action is trivial. Hence the algebra is $\mathfrak{su}(2)$_L. ■

Reading

This corollary is the main conceptual result of the paper. The programme does not merely attach a left-handed label to a known weak group. It explains the label: the weak group is left-handed because the unresolved weak branch exists only in the left-chiral spinorial sector.

Status note

Theorem 1 is exact given EW-4 and EW-5. The remaining physical debt is to derive EW-4 — weak orientation admissibility — from microscopic substrate transport rather than treating it as a representation-level admissibility rule.

This is a major narrowing of the programme debt. The problem is no longer vaguely "why weak chirality?" It is specifically:

prove that only left spinorial closure preserves unresolved weak branch coherence.

A single sharply stated transport theorem now stands between the representation closure and a fully substrate-derived chirality.

---

8. Right-Singlet Exclusion

Theorem 2 — Right-chiral modes admit only the trivial weak action

Given chiral locking and right-commitment closure, the right-chiral sectors

e_R, u_R, d_R

are weak singlets:

$T_i|_{e_R} = T_i|_{u_R} = T_i|_{d_R} = 0.$

Proof

A nontrivial weak action requires a two-dimensional unresolved branch module $\mathcal{W}$. By Theorem 1, nontrivial weak branch support exists only on $P_L \mathcal{H}_1$; the right-chiral sector satisfies $P_W P_R = 0$ and has no such module. Suppose some T_i acted nontrivially on a right-chiral state. Then either (a) the action maps the state out of the right-chiral sector — excluded because chiral locking gives $[T_i, P_L] = 0$ on the derived support, so the weak action preserves the chirality grading — or (b) the action mixes the state with a partner inside the right sector, which constitutes a hidden two-branch module and violates EW-5. Both horns fail, so the only admissible action is the trivial one. ■

Consequence

The singlet status of e_R, u_R, d_R is not assigned by hand. It follows from the absence of unresolved weak branch access in the right-chiral committed sector. In VERSF terms: a fully committed record has nothing left for the weak operator to rotate.

---

9. Weak Doublet Action on Matter

On the left-chiral weak-active sectors,

$L_L = (\nu_L, e_L)^T$, $Q_L = (u_L, d_L)^T$,

the generators act as

$T_i = \sigma_i/2$.

Thus

$T_3 (\psi_+, \psi_-)^T = ½ (\psi_+, -\psi_-)^T$.

The raising and lowering operators are

$T_+ = T_1 + i T_2$, $T_- = T_1 - i T_2$,

with

$T_+ (0, \psi_-)^T = (\psi_-, 0)^T$, $T_- (\psi_+, 0)^T = (0, \psi_+)^T$.

Weak charged-branch action therefore connects

$e_L \leftrightarrow \nu_L$, $d_L \leftrightarrow u_L$,

while leaving the right singlets untouched.

This is the representation-level origin of the left-handed charged weak current: the operator that will eventually couple to W± is, at this layer, nothing more than the branch-exchange endomorphism of the unresolved commitment module.

---

10. Weak Currents at Representation Level

Given the local spinorial field candidate $\psi(x)$, define the left-projected matter field

$\psi_L = P_L \psi$.

The weak representation currents are

$$J^\mu_i = :\bar{\psi}_L \gamma^\mu T_i \psi_L:.$$

Because T_i acts trivially on the right-chiral sectors, there is no corresponding right-handed weak doublet current.

Equivalently, the weak interaction current is of V−A form at the representation level:

$$J^\mu_i = :\bar{\psi} \gamma^\mu P_L T_i \psi:.$$

The V−A structure — historically extracted from decades of decay data — appears here as an operator identity: the current is left-projected because the weak generator has no support off the left-chiral sector, not because a coupling was tuned.

Status note

This paper derives the representation form of the weak current. It does not derive the numerical weak coupling g, the dynamical weak boson fields W^i_μ, or the gauge kinetic term. A future gauge-dynamical lift may couple J^μ_i to a connection W^i_μ, but that is not claimed here.

---

11. Hypercharge as the Commuting Source-Grade

The weak generators T_i mix the two members of a weak doublet. Hypercharge must do the opposite: it must assign a common source-grade to the whole doublet and commute with the weak action:

$$[T_i, Y] = 0.$$

Lemma 2 — Hypercharge is scalar on weak doublets

Given $[T_i, Y] = 0$ and irreducibility of the weak doublet branch module, Y is proportional to the identity on each weak doublet.

Proof. The weak doublet branch space is the irreducible fundamental representation of $\mathfrak{su}(2)$. By Schur's lemma, any operator commuting with all generators of an irreducible representation is scalar on that representation. Therefore Y assigns one common value to the entire doublet. ■

Consequently

$$Y|_{L_L} = Y_L \mathbb{1}_2, \quad Y|_{Q_L} = Y_Q \mathbb{1}_2.$$

On singlets, $T_i = 0$, so Y simply equals the source charge:

$Y(e\_R) = Q(e\_R)$, $Y(u\_R) = Q(u\_R)$, $Y(d\_R) = Q(d\_R)$.

This is why hypercharge is a commuting grade rather than another weak-branch mixing operator — and why doublet partners such as ν_L and e_L, whose electric charges differ, must nonetheless share a hypercharge. Schur rigidity leaves no other option.

---

12. Electric/Source Charge Decomposition

The source charge Q must distinguish the two branches of a weak doublet. Any Hermitian operator diagonal on the branch basis of $\mathbb{C}^2$ decomposes uniquely into a traceless part and a scalar part. The traceless diagonal weak generator is T_3; the scalar part is the commuting grade Y.

Theorem 3 — Electroweak source-charge decomposition

Given a weak doublet branch module, a commuting source-grade Y, and the normalization in which the weak branch splitting is one source-charge unit, the electric/source charge operator is

$Q = T_3 + Y$.

Proof. On a weak doublet, Q is Hermitian and diagonal in the charged branch basis. Decompose $Q = Q\_traceless + Q\_scalar$. The traceless diagonal generator of the doublet module is proportional to $T_3 = \mathrm{diag}(½, −½)$; the coefficient is fixed to unity by the normalization in which the two branches differ by one source-charge unit. The scalar part commutes with all T_i and is therefore identified with the source-grade Y by EW-6. Hence $Q = T_3 + Y$. On singlets $T_3 = 0$ and the identity reduces to $Q = Y$, consistent with §11. ■

For a doublet,

$T_3 = \mathrm{diag}(+½, −½)$,

so

$Q\_upper = +½ + Y$, $Q\_lower = −½ + Y$,

and the two branches differ by exactly one source-charge unit:

$Q\_upper − Q\_lower = 1$.

Normalization note

The decomposition $Q = T_3 + Y$ is forced by the traceless/scalar split; what is normalized rather than derived is the unit size of the branch splitting. That unit is inherited from the winding-charge quantization of the source programme, and its substrate derivation is logged as Debt 8. The identity is exact as a representation statement; the unit is an inherited normalization, and the distinction is recorded rather than blurred.

Status note

This is a representation derivation of the charge decomposition, not a derivation of the broken electromagnetic $U(1)_{em}$ field. The photon and electroweak mixing remain outside the present scope.

---

13. Electroweak Representation Algebra

Combining Lemma 1 and Lemma 2 gives

$$[T_i, T_j] = i\, \varepsilon_{ijk} T_k, \quad [T_i, Y] = 0.$$

The representation algebra is therefore

$$\mathfrak{g}_{EW}^{rep} = \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y.$$

The subscript L is now earned by Theorem 1: the nontrivial weak action exists only on the left-chiral sector.

Theorem 4 — Electroweak Representation Closure

Given EW-1 through EW-7, the VERSF one-generation chiral matter skeleton carries a representation-level electroweak algebra

$$\mathfrak{g}_{EW}^{rep} = \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y,$$

acting as

$$L_L : (1, 2, -\tfrac{1}{2}),\ e_R : (1, 1, -1),\ Q_L : (3, 2, \tfrac{1}{6}),\ u_R : (3, 1, \tfrac{2}{3}),\ d_R : (3, 1, -\tfrac{1}{3}).$$

Proof

By Theorem 1, weak-active modes are exactly the left-chiral sector. By Lemma 1, the nontrivial weak unresolved branch module carries an $\mathfrak{su}(2)$ operator algebra, and no larger faithful nontrivial algebra can act solely on the two-branch module. Therefore L_L and Q_L carry weak doublet actions. By Theorem 2, right-chiral sectors carry no nontrivial weak branch and are singlets. By Lemma 2, the commuting source-grade Y is scalar on each weak irreducible representation. By Theorem 3, the electric/source charge decomposes as $Q = T_3 + Y$. The inherited anomaly-rigid ledger fixes the hypercharges as listed. Therefore the representation algebra acting on the matter skeleton is $\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$. ■

Rigidity summary

Every joint in the closure is either inherited, forced, or firewalled:

- the branch dimension is a premise (EW-2), and it forces rank one — no larger group could have acted faithfully and nontrivially on the weak branch;
- the split of $\mathfrak{u}(2)$ into traceless and central parts is canonical, not chosen;
- chirality of the action is Theorem 1, resting on the single physical premise EW-4;
- scalarity of Y is Schur rigidity;
- $Q = T_3 + Y$ is the unique traceless/scalar decomposition;
- the hypercharge values are anomaly-rigid (§16).

The only freedom the construction ever had is the orientation convention (which sector is called "left") and the inherited unit normalizations. That is the intended shape of a closure result.

13′. Relation to Closure-Norm Condensation and Electroweak Breaking

The present paper derives the electroweak representation algebra before vacuum condensation:

$\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$.

The closure-norm condensation paper addresses the later phase-selection problem. In that downstream module, the closure-norm field condenses,

$\langle \rho \rangle = v$,

and the gauge-transport modes coupled to the condensate acquire mass. The result is the familiar broken-phase pattern:

$SU(2)_L \times U(1)_Y \to U(1)_{EM}$.

The distinction is important. The present paper explains why the matter fields carry the unbroken electroweak representation structure:

L_L, Q_L are weak doublets; e_R, u_R, d_R are weak singlets; Y is the commuting source-grade.

The closure-norm condensation paper then explains why, after the vacuum selects a finite closure-norm condensate, the $W^\pm$ and Z directions acquire mass while the photon remains massless.

The two papers are therefore sequential, not overlapping:

electroweak representation closure ⟹ electroweak symmetry breaking.

(Interface convention: the condensation paper's $Q = T_3 + Y_{SM}/2$ hypercharges relate to this paper's ledger by $Y_{SM} = 2Y$ — see the Hypercharge Convention Note in §0.)

The current paper should not be judged by whether it derives W/Z masses. That is downstream. It should be judged by whether it derives the operator architecture — and, in §14′, the connection form — that makes such mass generation possible.

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14. Representation-Level Covariant Derivative

Once the representation algebra is identified, the formal covariant derivative compatible with a future gauge-dynamical lift has the form

$$D_\mu = \partial_\mu + i g W^i_\mu T_i + i g' B_\mu Y.$$

Here W^i_μ and B_μ are not yet derived as dynamical gauge fields. The constants g, g' are not computed. The expression is included only to show the operator architecture that any subsequent gauge-field derivation must couple to.

The representation-level matter kinetic operator would be

$$\bar\psi i \gamma^\mu D_\mu \psi,$$

with T_i acting only on left doublets and Y acting by the hypercharge ledger.

Expanding the representation couplings gives the currents

$$g W^i_\mu :\bar\psi_L \gamma^\mu T_i \psi_L: + g' B_\mu :\bar\psi \gamma^\mu Y \psi:.$$

This is the electroweak analogue of the earlier QED-form source coupling, but only at the representation/current level.

Firewall

No gauge kinetic term, field strength dynamics, symmetry-breaking map, boson mass matrix, or Weinberg angle is derived here. The connection *form* itself, however, is not merely postulated for illustration — the next section derives it as the unique local comparison structure compatible with the representation closure.

14′. Local Electroweak Connection Lift

The representation results above are pointwise: they say what operators act on the matter fibre at each record-layer point. But matter fields are local — $\psi_L(x)$ at one point and $\psi_L(x')$ at a neighbouring point each carry their own copy of the weak branch space $\mathcal{W}$ and their own hypercharge grading. Nothing in the representation closure yet says how the branch basis at x is to be compared with the branch basis at x′.

That comparison is not optional. Any derivative of a weak-active field implicitly compares branch states at neighbouring points, and an unadjusted partial derivative ∂_μ silently assumes the branch bases are already aligned — an assumption with no substrate justification, since branch orientation within $\mathcal{W}$ is a local convention, not a record. Local comparison therefore requires a connection: an infinitesimal comparison operator, valued in the admissible transformation algebra, that transports branch data between neighbouring points.

The content of this section is that the representation closure fixes the *form* of that connection completely.

Theorem 5 — Local Electroweak Connection Lift

Given the electroweak representation closure

$\mathfrak{g}_{EW}^{rep} = \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$,

and requiring local comparison of spinorial source-carrier fields to preserve:

1. weak branch coherence (comparison acts unitarily on $\mathcal{W}$);
2. chiral locking (comparison commutes with $P_W = P_L$ — the weak component is left-projected);
3. the hypercharge grade (comparison respects the commuting source-grade decomposition);
4. source-current admissibility (comparison generators are Hermitian, so transported norms and record-currents are preserved);
5. anomaly admissibility of the gauged comparison algebra (every generator carried by the connection is locally gauged, and must therefore satisfy the admissibility principle of Proposition 6 applied across the channel set of §16 — the pure, gravitational, and mixed channels);

then, for the minimal one-generation skeleton of EW-7 (without the optional ν_R), the local connection is valued in $\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$ and has the unique form

$$\mathcal{A}_\mu^{EW} = g\, W^i_\mu T_i + g'\, B_\mu Y,$$

with corresponding covariant derivative

$$D_\mu = \partial_\mu + i\, g\, W^i_\mu T_i + i\, g'\, B_\mu Y.$$

The valuation in the representation algebra is derived by the conditions, not assumed. For the ν_R-extended skeleton, uniqueness weakens as recorded in the scope note below.

Proof

A local comparison operator between neighbouring matter fibres is, infinitesimally, a one-form $\mathcal{A}_\mu$ valued in the algebra of admissible infinitesimal transformations of the matter fibre. Condition 4 requires Hermitian, norm-preserving generators. Condition 1 requires the branch-acting component to be unitary on $\mathcal{W}$, and Lemma 1(iii) caps that component at $\mathfrak{su}(2)$ plus grades central on the branch module. Condition 2 is automatic for the $\mathfrak{su}(2)$ component in the derived basis: the T_i carry left-projected support by Corollary 1.1, so the weak component of the connection cannot transport right-chiral states.

Conditions 1–4 do not, however, exhaust the commuting directions, and it is important to say so plainly. The matter fibre is the direct sum $L_L \oplus e_R \oplus Q_L \oplus u_R \oplus d_R$, and a generator X assigning an independent real scalar to each sector — baryon number, lepton number, B−L — is Hermitian (condition 4), scalar and hence unitary on $\mathcal{W}$ (condition 1), commutes with P_L (condition 2), and commutes with Y (condition 3). Lemma 1(iii) is silent on such X precisely because X is central on the branch module. Left unfiltered, the connection could carry any such grade.

Condition 5 supplies the filter. A generator carried by a local connection is locally gauged, and by the admissibility principle of Proposition 6, applied across the channel set of §16, a gauged grade must clear every channel: $SU(2)^2X$, $SU(3)^2X$, grav^2X, X^3, and the mixed channels X^2Y and XY^2. For the minimal no-ν_R skeleton, the audit over the five sector scalars (X_Q, X_u, X_d, X_L, X_e) is decisive: the four linear channels — $SU(2)^2X$, $SU(3)^2X$, grav^2X, and XY^2 — cut the five-dimensional grade space down to a one-dimensional line, the nonlinear channels X^2Y and X^3 vanish identically along that line, and the line is spanned by Y itself, reproducing the ratios $(\tfrac{1}{6}, -\tfrac{2}{3}, \tfrac{1}{3}, -\tfrac{1}{2}, 1)$ in all-left Weyl form. The only anomaly-admissible commuting grade in the minimal skeleton is therefore Y, up to normalization. B−L fails concretely: its quark contributions cancel channel by channel, but the lepton sector leaves the gravitational channel at $2(-1) + 1 = -1 \neq 0$, and its cubic channel likewise evaluates to -1.

The admissible comparison algebra under conditions 1–5 is therefore exactly $\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$. Expanding in the generator basis $\{T_1, T_2, T_3, Y\}$ with real coefficient fields — reality following from Hermiticity — gives

$$\mathcal{A}_\mu^{EW} = g\, W^i_\mu T_i + g'\, B_\mu Y,$$

and the covariant derivative D_μ = ∂_μ + i𝒜_μ^EW follows by the standard replacement of the unadjusted derivative with the comparison-corrected one. Uniqueness holds up to the split of each coefficient field into a normalization constant times a field — the content of g and g′ below. ■

Normalization note

The constants g and g′ enter here as unfixed normalizations separating each coefficient field into a conventional strength times a field, not as derived couplings. Nothing in this paper computes their values; that computation belongs to the owed gauge-dynamics module. The theorem's content is the *shape* of the connection — one 𝔰𝔲(2)_L-valued triplet, one 𝔲(1)_Y-valued singlet, and nothing else — not its strength.

Uniqueness scope note — minimal versus ν_R-extended skeleton

The uniqueness in Theorem 5 is uniqueness within the already-derived matter fibre, under the no-hidden-branch/no-exotics conditions EW-2 and EW-7, and under the anomaly-admissibility condition 5. Two boundaries of the claim are recorded explicitly.

First, this is not a claim that no larger theory could contain additional decoupled or high-energy gauge sectors — a larger group breaking down to this one at inaccessible scales is not excluded by anything in this paper.

Second, the optional sterile ν_R of EW-7 carries a price here, and the paper states it rather than hiding it. With ν_R included, the anomaly audit's solution space enlarges from the single Y direction to a two-parameter family spanned by Y and B−L: the sterile neutrino is exactly what the failing B−L channels need — its ν_L^c contributes +1 against the lepton sector's −1 in both the gravitational and cubic channels, closing both. The full audit closes across the entire family, not merely at the two basis points: for arbitrary X = αY + β(B−L), the mixed channels X^2Y and XY^2 — together with the gravitational and cubic channels — vanish identically as polynomials in α and β. In the ν_R-extended skeleton, Theorem 5's uniqueness therefore holds only up to one additional anomaly-admissible commuting grade: the well-known gauged B−L direction. Nothing in this paper derives the absence of a gauged B−L connection at low energy. That absence is at present an empirical fact, not a derived one, and its derivation is logged as Debt 9.

The disciplined statement of the theorem's strength is therefore: minimal skeleton — the connection form is unique; ν_R-extended skeleton — unique up to gauged B−L, whose exclusion is owed. One cannot simultaneously keep the optional ν_R and claim unconditional connection uniqueness, and this paper does not attempt to.

Formal curvature (preparatory, not derived)

For downstream reference, the curvature associated with the connection form is, formally,

F_μν^EW = ∂_μ 𝒜_ν − ∂_ν 𝒜_μ + i [𝒜_μ, 𝒜_ν].

This expression is recorded as preparatory notation only. Its non-Abelian commutator term is at least substrate-admissible by EW-0, but no field-strength dynamics is claimed, and the gauge kinetic terms

$$-\tfrac{1}{4} W^i_{\mu\nu} W^{i\,\mu\nu}, \quad -\tfrac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

remain owed, together with the field equations and the values of g, g'.

What this adds to the bridge

The paper's forward chain is now:

two weak branches ⇒ $\mathfrak{su}(2)$; orientation admissibility ⇒ $\mathfrak{su}(2)_L$; commuting source grade ⇒ $\mathfrak{u}(1)_Y$; local branch comparison ⇒ the W^i_μ, B_μ connection form.

The downstream closure-norm condensation module can therefore begin from a genuine local electroweak connection rather than an abstract representation algebra: the objects whose transport it stiffens against the condensate are now derived structures, not imported ones.

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15. Hypercharge Admissibility and Anomaly Obstruction

The predecessor paper used anomaly freedom as a consistency filter. The present paper strengthens its interpretation.

A chiral gauge/source representation with uncancelled anomalies fails to preserve the gauge/source current consistently at the quantum representation level. In VERSF language: the ledger cannot define a persistent admissible record-current representation, because the source accounting would not remain invariant under the allowed local representation transformations. An anomalous ledger is a ledger that fails to balance — and a ledger that fails to balance is not a defect to be tolerated but a configuration excluded by the substrate's bit-conservation discipline.

Thus anomaly freedom is not phenomenological fitting. It is an admissibility condition for a persistent chiral gauge/source ledger.

Proposition 6 — Anomalous chiral ledgers are representation-inadmissible

Given a chiral matter representation carrying local weak/source transformations, any nonzero perturbative gauge anomaly obstructs simultaneous preservation of the corresponding source currents. Therefore an anomalous electroweak ledger is not representation-admissible as a persistent VERSF source sector.

Proof sketch

A chiral anomaly is precisely an obstruction to preserving the current conservation law associated with a local gauge/source transformation after quantisation of chiral matter. VERSF source sectors require persistent record-current conservation as an admissibility condition. A representation whose anomaly fails to cancel therefore cannot be a stable substrate source representation, and is rejected at the representation-admissibility level. ■

Programme role after condensation

The anomaly constraint has a clearer programme role once the closure-norm condensation paper is taken into account. A chiral ledger with uncancelled anomalies would not merely be mathematically inelegant. It would fail to define a stable source representation capable of surviving into the closure-condensed vacuum.

The reason is structural. The electroweak representation algebra acts on persistent record-layer matter currents. Closure-norm condensation later couples gauge transport to the vacuum. If the chiral source ledger is anomalous, the corresponding current cannot be preserved consistently under local weak/source transformations. Such a ledger cannot define a persistent substrate record-current sector, and therefore cannot be the representation architecture on which closure-norm condensation acts.

In this sense, anomaly freedom is not an external phenomenological filter. It is the representation-level condition that the matter ledger remain admissible as a persistent source structure prior to condensation. The logic chains as:

anomalous ledger ⟹ non-persistent source current ⟹ no admissible electroweak representation ⟹ no stable closure-condensed gauge sector.

The anomaly audit is therefore not a late consistency check. It is part of what qualifies the representation as physically admissible.

Status note

This is not yet a full substrate derivation of anomaly cancellation. It identifies why anomaly freedom is the correct admissibility filter. The owed stronger theorem is:

substrate cohomology directly forbids anomalous chiral ledgers.

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16. Hypercharge Rigidity Recovered — a Four-Channel Audit

Under the no-extra-chiral-exotics condition EW-7, the anomaly-admissible ledger is the one inherited from the predecessor paper. This section restates the audit in all-left-handed Weyl form and records a strengthening: the audit runs across four physically distinct anomaly channels, and the same solution passes every one of them.

The left-handed Weyl content is

$Q_L : (3, 2, Y_Q)$, $u_L^c : (\bar{3}, 1, -Y_u)$, $d_L^c : (\bar{3}, 1, -Y_d)$, $L_L : (1, 2, Y_L)$, $e_L^c : (1, 1, -Y_e)$,

with lepton normalization

$Y_L = -\tfrac{1}{2}$, $Y_e = -1$.

Three quark hypercharges Y_Q, Y_u, Y_d remain. Four physically distinct anomaly channels constrain them.

Condition 1 — $SU(2)^2\ U(1)$

Summing hypercharge over weak doublets, weighted by colour multiplicity:

$3\ Y_Q + Y_L = 0 \Rightarrow Y_Q = \tfrac{1}{6}$.

Condition 2 — $SU(3)^2\ U(1)$

Summing hypercharge over colour triplets:

$2\ Y_Q - Y_u - Y_d = 0 \Rightarrow Y_u + Y_d = \tfrac{1}{3}$.

Condition 3 — $\mathrm{grav}^2\ U(1)$

Summing hypercharge over all Weyl fermions with multiplicity:

$6\ Y_Q - 3\ Y_u - 3\ Y_d + 2\ Y_L - Y_e = 0$.

Substituting $Y_Q = \tfrac{1}{6}$, $Y_L = -\tfrac{1}{2}$, $Y_e = -1$:

$1 - 3(Y_u + Y_d) - 1 + 1 = 0 \Rightarrow Y_u + Y_d = \tfrac{1}{3}$.

This reproduces the $SU(3)^2\ U(1)$ constraint from a physically distinct anomaly channel. The gravitational condition is therefore algebraically redundant once Conditions 1 and 2 are imposed — but its physical content is not: it arises from a different diagram class, and a generic charge assignment has no a-priori reason to make the two channels coincide. The audit records this as a consistency pass across channels, not as strict algebraic independence.

Condition 4 — $U(1)^3$

$6\ Y_Q^3 - 3\ Y_u^3 - 3\ Y_d^3 + 2\ Y_L^3 - Y_e^3 = 0$.

Substituting $Y_Q = \frac{1}{6}$, $Y_L = -\frac{1}{2}$, $Y_e = -1$:

$$1/36 - 3(Y_u^3 + Y_d^3) - \frac{1}{4} + 1 = 0 \Longrightarrow Y_u^3 + Y_d^3 = 7/27.$$

Solving the audit

With $s = Y_u + Y_d = \frac{1}{3}$ and the identity

$$Y_u^3 + Y_d^3 = s^3 - 3\, s\, Y_u\, Y_d,$$

one obtains

$$1/27 - Y_u\, Y_d = 7/27 \Longrightarrow Y_u\, Y_d = -2/9.$$

Thus Y_u, Y_d are the roots of

$$t^2 - \frac{1}{3}\, t - 2/9 = 0,$$

whose discriminant is $1/9 + 8/9 = 1$, giving

$$t = (\tfrac{1}{3} \pm 1)/2 \Longrightarrow Y_u, Y_d = \tfrac{2}{3}, -\tfrac{1}{3}.$$

Therefore

$$Y_Q = \tfrac{1}{6},\ Y_u = \tfrac{2}{3},\ Y_d = -\tfrac{1}{3},$$

up to the already-fixed charge orientation and up/down labelling. The discriminant is a perfect square, so the roots are not merely real but rational — the audit closes in $\mathbb{Q}$, with no algebraic extension required.

Sterile-neutrino uniqueness

A chiral addition $(1, 1, Y_\nu)$ contributes Y_ν to the gravitational condition and Y_ν^3 to the cubic condition, and nothing to Conditions 1 and 2. Both contributions vanish iff $Y_\nu = 0$. Hence $\nu_R : (1, 1, 0)$ is the unique anomaly-neutral chiral singlet extension of the skeleton — permitted, sterile, and invisible to every line of the hypercharge audit. Its optional status in EW-7 is therefore itself rigid. Its neutrality is, however, specific to the hypercharge audit: as §14′ records, the presence of ν_R renders the B−L grade anomaly-admissible, with consequences for connection uniqueness that are logged in the Theorem 5 scope note and Debt 9.

This recovers the hypercharge ledger as the unique real — in fact rational — admissible completion of the lepton normalization inside the minimal one-generation skeleton, under a four-channel audit in which every channel is passed.

17. The Electroweak Matter Operator Table

The complete representation action is:

Sector	Weak action	Hypercharge	Electric/source charges	Status
L_L = (ν_L, e_L)	doublet	−½	(0, −1)	Conditional
e_R	singlet	−1	−1	Conditional
Q_L = (u_L, d_L)	doublet	⅙	(⅔, −⅓)	Conditional / rigid
u_R	singlet	⅔	⅔	Conditional / rigid
d_R	singlet	−⅓	−⅓	Conditional / rigid
ν_R	singlet	0	0	Optional / sterile / unique
Conjugate sectors	conjugate reps	opposite	opposite	Candidate

The weak generators act as

$T_i = \sigma_i/2$ on L_L, Q_L, $T_i = 0$ on e_R, u_R, d_R.

The hypercharge generator acts as a scalar on each row. The electric/source charge is always

$Q = T_3 + Y$,

and the reader may verify the identity row by row: every entry in the charge column is the corresponding T_3 eigenvalue plus the row's hypercharge.

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18. What Has Actually Been Derived

This paper derives, conditionally:

1. weak unresolved commitment as a two-state branch module;
2. the $\mathfrak{su}(2)$ weak-branch algebra, with rank-one rigidity excluding any larger weak group;
3. chiral locking P_W = P_L, given weak orientation admissibility;
4. exclusion of right-chiral weak doublet action;
5. left-handed weak doublets L_L, Q_L;
6. right-handed weak singlets e_R, u_R, d_R;
7. hypercharge as a commuting source-grade generator, scalar by Schur rigidity;
8. $Q = T_3 + Y$ as the representation-level source-charge decomposition;

9. the electroweak representation algebra $\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$;
10. the left-handed (V−A) weak current form as an operator identity;
11. anomaly freedom as representation admissibility;
12. recovery of the unique hypercharge ledger under the minimal no-exotics skeleton, via a four-channel, rationally closing anomaly audit;
13. uniqueness of the sterile ν_R as the only anomaly-neutral chiral extension;
14. the local electroweak connection form $\mathcal{A}_\mu^{EW} = g W^i_\mu T_i + g' B_\mu Y$, unique under the five comparison conditions for the minimal skeleton, and unique up to one gauged B−L grade in the ν_R-extended skeleton.

This paper does not derive:

1. gauge field dynamics or field equations;
2. gauge kinetic terms;
3. the values of g, g' (they enter §14′ only as unfixed normalizations);
4. the Weinberg angle;
5. electroweak symmetry breaking;
6. the Higgs vev;
7. W/Z masses;
8. the photon as the broken-phase unbroken generator;
9. Yukawa couplings;
10. fermion masses;
11. CKM/PMNS;
12. three generations;
13. full colour SU(3) dynamics;
14. exact charge conjugation/CPT.

What has been strengthened by the inherited papers

This paper is strengthened by three inherited results.

First, non-Abelian weak action is structurally admissible because non-Abelian commutator defects are refinement-irrelevant under admissible coarse-graining:

$$A_C^{(n)} \sim 16^{-n} A_C^{(0)}.$$

The $\mathfrak{su}(2)$ weak branch algebra is therefore not an alien insertion into the substrate.

Second, the closure-norm condensation paper supplies the downstream breaking mechanism. The present paper derives the unbroken representation algebra; the condensation paper explains how the closure-norm vacuum later breaks $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$.

Third, the hierarchy/category-error paper supplies the layer discipline. The present paper operates at the record-layer representation level and does not require Planck-scale propagating states, scalar naturalness assumptions, Higgs-ratio numerology, or mass fitting.

Together, these inheritances sharpen the paper's position: not a free table, not a Higgs/mass paper, not a full gauge-dynamics paper — but the missing representation-operator bridge between the chiral matter skeleton and electroweak symmetry breaking.

19. Relation to the Previous Paper

The predecessor paper answered:

Why these chiral matter representation slots?

It derived

L_L, e_R, Q_L, u_R, d_R

and the anomaly-rigid hypercharge ledger.

The present paper answers:

What electroweak operator algebra acts on those slots?

It derives

𝔰𝔲(2)_L ⊕ 𝔲(1)_Y.

The programme chain becomes

spinorial loops → fermionic Fock algebra → chiral matter skeleton → electroweak representation closure → local electroweak connection form.

Mass and mixing remain downstream.

20. Open Debts

Debt 1 — Microscopic proof of weak orientation admissibility

The principal remaining debt is to derive EW-4 from substrate dynamics:

only left spinorial closure preserves unresolved weak branch coherence.

This would make chiral locking fully substrate-derived. It is the single point at which the entire closure is exposed, and it is stated here as a transport theorem target rather than left diffuse.

Debt 2 — Gauge dynamics on the derived connection

This paper derives the representation algebra and, in §14′, the unique local connection form 𝒜_μ^EW. A future paper must supply the dynamics: the gauge kinetic terms, the field equations for W^i_μ and B_μ, and the values of g and g′. The debt has narrowed — the connection no longer needs to be found, only animated.

Debt 3 — Substrate anomaly-inadmissibility theorem

This paper interprets anomaly cancellation as representation admissibility. A stronger theorem must show directly that anomalous chiral ledgers are forbidden by substrate cohomology.

Debt 4 — Electroweak symmetry breaking

This paper does not derive the Higgs mechanism, the unbroken electromagnetic generator, the Weinberg angle, or W/Z masses.

Debt 5 — Continuous colour lift

The colour entry remains inherited as a C_3-support skeleton. A future paper must derive full continuous SU(3)_C gauge structure.

Debt 6 — Three generations

This paper is one-generation. It does not explain replication.

Debt 7 — Mass/Yukawa attachment

The already-separate mass and mixing programme must be attached to the representation slots and electroweak operators derived here.

Debt 8 — Substrate derivation of the branch-splitting unit

The unit normalization in $Q = T_3 + Y$ is inherited from winding-charge quantization. A substrate derivation of that unit closes the last inherited normalization in the charge ledger.

Debt 9 — Exclusion of the gauged B−L direction in the ν_R-extended skeleton

With the optional sterile ν_R present, B−L becomes anomaly-admissible and Theorem 5's connection uniqueness weakens to uniqueness up to one additional commuting gauged grade. A future result must either derive the absence (or high-scale decoupling) of the gauged B−L connection from substrate admissibility, or derive that the physical skeleton is the minimal one without ν_R. Until then, the low-energy absence of gauged B−L is an empirical input, not a theorem, and Theorem 5 is graded accordingly.

21. Falsification Conditions

F1 — Weak branch failure

If weak-active states do not carry a two-dimensional unresolved branch module, the 𝔰𝔲(2) weak-branch derivation fails. Retires EW-2.

F2 — Norm-preservation failure

If weak branch transformations do not preserve the Born/Hilbert norm, the U(2) and 𝔰𝔲(2) operator argument fails. Retires EW-3.

F3 — Chiral-locking failure

If weak unresolved commitment is not locked to left chirality, then SU(2)_L is not derived. Retires Theorem 1.

F4 — Right-action leakage

If right-chiral sectors carry nontrivial weak doublet action, the Standard Model singlet pattern fails. Retires Theorem 2.

F5 — Hypercharge noncommutation

If Y does not commute with the weak action, the representation algebra is not 𝔰𝔲(2)_L ⊕ 𝔲(1)_Y. Retires Lemma 2 and Theorem 4.

F6 — Charge-decomposition failure

If $Q \neq T_3 + Y$ under the representation charge audit, the electroweak charge ledger fails. Retires Theorem 3.

F7 — Exotics obstruction

If unavoidable additional anomaly-carrying chiral sectors appear, the uniqueness of the one-generation hypercharge ledger no longer follows in its current form. Limits Theorem 4.

F8 — Anomaly-admissibility failure

If anomaly cancellation is not required for persistent source admissibility, the representation-admissibility interpretation weakens to an external consistency audit. Limits Proposition 6.

F9 — Gauge-dynamics failure

If the derived connection form cannot be promoted to dynamical gauge fields with admissible kinetic terms and field equations, the result remains a connection-level closure rather than electroweak gauge theory. Limits the programme extension.

F10 — Mass contamination

If any step is found to require observed masses, Yukawa coefficients, CKM, or PMNS data, the mass firewall fails. Retires the claim that this paper is representation-prior.

F11 — Rank leakage

If weak phenomena require a weak group of rank greater than one acting on the matter skeleton, the two-branch premise is contradicted by Lemma 1(iii). Retires EW-2 from the phenomenological side.

F12 — Refinement instability of the weak generators

If the non-Abelian commutator defects associated with the specific weak generators T_i fail to refine away under admissible edge-bisection, the 𝔰𝔲(2)_L representation is substrate-unstable. Retires EW-0 as applied to the weak sector.

F13 — Connection-uniqueness failure

If an admissible local comparison structure exists that preserves conditions 1–5 of Theorem 5 but is not valued in 𝔰𝔲(2)_L ⊕ 𝔲(1)_Y — for instance through hidden branch degrees of freedom evading EW-2/EW-7 — the uniqueness of the connection form fails. The known boundary instance, the gauged B−L grade in the ν_R-extended skeleton, is not a triggering of this falsifier: it is handled explicitly by the uniqueness scope note and Debt 9. F13 proper concerns any further admissible structure beyond that named case. Retires or limits Theorem 5.

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22. Milestone Statement

The result of this paper is the representation-level electroweak closure:

chiral matter skeleton + weak unresolved commitment + non-Abelian refinement admissibility + spinorial orientation locking + commuting source grade

⟹

𝔰𝔲(2)_L ⊕ 𝔲(1)_Y acting on L_L, e_R, Q_L, u_R, d_R.

In explicit form:

$T_i = \sigma_i/2$ on $L_L = (\nu_L, e_L)$ and $Q_L = (u_L, d_L)$,

$T_i = 0$ on e_R, u_R, d_R.

Hypercharge Y is the commuting source-grade generator,

$[T_i, Y] = 0$,

and electric/source charge decomposes as

$Q = T_3 + Y$.

The inherited anomaly-rigid ledger then gives

$L_L : (1, 2, -\tfrac{1}{2})$, $e_R : (1, 1, -1)$, $Q_L : (3, 2, \tfrac{1}{6})$, $u_R : (3, 1, \tfrac{2}{3})$, $d_R : (3, 1, -\tfrac{1}{3})$.

The local comparison requirement then fixes the connection form (uniquely, for the minimal skeleton):

$\mathcal{A}_\mu^{EW} = g W^i_\mu T_i + g' B_\mu Y$, $D_\mu = \partial_\mu + i\mathcal{A}_\mu^{EW}$.

The paper therefore supplies the missing bridge:

chiral matter skeleton → electroweak representation algebra → local electroweak connection form.

The downstream closure-norm condensation module then acts on this operator architecture to produce the broken-phase electroweak vacuum:

$SU(2)_L \times U(1)_Y \to U(1)_{EM}$.

That is the milestone. VERSF no longer has only a matter table; it now has the electroweak representation action that the matter table carries before the vacuum condenses.

23. Conclusion

The Chiral Matter Representation paper gives VERSF a one-generation Standard-Model-like matter skeleton. It explains why the generic spinorial Fock sector branches into lepton-like, quark-like, weak-doublet, weak-singlet, colour-supported, and conjugate representation roles, and it shows that the quark hypercharge ledger is rigid once lepton normalization and anomaly freedom are imposed.

The present paper moves one structural layer forward.

It shows that weak unresolved commitment supplies a two-state branch module whose norm-preserving traceless transformations close as $\mathfrak{su}(2)$ — and could not have closed as anything larger, because the rank of the weak algebra is fixed by the counting of unresolved branches. Spinorial orientation admissibility locks this weak branch to left chirality:

$$P_W = P_L.$$

Right-chiral sectors are fully committed with respect to the weak branch and therefore carry only trivial weak action. Hypercharge appears as the commuting source-grade generator, scalar on weak irreducible modules by Schur rigidity, with charge decomposition

$$Q = T_3 + Y.$$

The resulting representation algebra is

$$\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y.$$

The chirality of the weak force, on this account, is not an unexplained asymmetry painted onto the matter content. It is the statement that only one spinorial orientation can carry an unfinished commitment through closure — the other side has already committed, and a committed record cannot be rotated.

This is not yet the full electroweak theory. It does not derive gauge field dynamics, symmetry breaking, the Weinberg angle, W/Z masses, the Higgs vev, Yukawa couplings, or fermion masses. But it now derives more than the pointwise operator architecture: the requirement of comparing weak branch states between neighbouring record-layer points fixes the local connection form — uniquely for the minimal skeleton, and up to the gauged $B-L$ grade if the sterile ν_R is included —

$$\mathcal{A}_\mu^{EW} = g\, W^i_\mu T_i + g'\, B_\mu Y,$$

so the downstream condensation module inherits a derived connection, not an imported one.

The programme has therefore advanced:

fermion algebra → chiral matter skeleton → electroweak representation closure → local electroweak connection form.

The remaining work is now sharply defined: prove weak orientation admissibility microscopically, animate the derived connection with gauge kinetic terms and field equations, derive anomaly inadmissibility from substrate cohomology, complete the colour lift, and then attach the mass and mixing modules.

That is the intended milestone.