The Electroweak Vacuum and Higgs-Potential Completion Theorem in VERSF

▲ Programme Milestone — Electroweak Symmetry Breaking and Scalar-Completion Series

Gate HR-1 — Route-A Numerical Evaluation of the Electroweak Vacuum, Higgs Potential and Higgs Mass

Keith Taylor VERSF Theoretical Physics Programme — Electroweak Symmetry Breaking and Scalar-Completion Series Tier A conditional numerical evaluation theorem — Route-A closure stack, with microscopic residue formula and one-loop scalar pole

Primary predecessors: *The Higgs-Radial Closure-Interface Gate in VERSF*; *Substrate Dynamics and the Higgs Ratio*; *The Fine-Structure Constant from Vacuum Geometry*; *The Standard Model from Hexagonal Geometry*; *The Charged-Lepton Absolute Scale Theorem in VERSF*; *The Gauge Action and Coupling Theorem in VERSF*; *The Closure-Operator Yukawa Construction Theorem in VERSF*; and *The Quantum Gauge Completion of the VERSF Standard Model*.

General Reader Summary

Earlier VERSF work explained why a Higgs-like field should exist. The Higgs radial mode was identified as the normal restoring mode of the closure interface: when the underlying completion structure is displaced away from its stable configuration, it pushes back. That result prevented the Higgs from being inserted simply because the Standard Model needs one.

It did not, however, calculate where the Higgs field settles, how steep its potential is, or what mass its physical excitation should have.

This paper performs that calculation, and the key move is surprisingly simple to state. VERSF already contained two completely independent descriptions of the same physical object — the mass scale of the electron.

The first description builds the electron scale directly from the substrate. The VERSF geometry programme predicts the electromagnetic interface coupling, $\alpha = 7/960$, from the seven closure conditions and fourteen loop channels of the substrate. The Route-A coherence construction supplies a frozen coherence length of 88 micrometres. Combining the coherence-cell energy with a fourth-power coupling enhancement and a purely geometric occupancy fraction of 13/20 yields

an electron-scale energy — calculated without using the measured electron mass, the weak scale or the Higgs mass.

The second description reads the same electron scale off the electroweak completion interface: the electron's mass is a fixed, derived fraction of the electroweak vacuum, with no adjustable coefficient.

Here is the decisive point. Two independent formulas for one and the same physical quantity must agree. Setting them equal removes the last freedom in the scalar sector and *locks* the electroweak vacuum. Substituting only frozen upstream numbers gives

v_{cl} = 243.723 GeV .

Separately, a conditional finite-deformation theorem selects the ratio of the Higgs mass to the vacuum — exactly 32/63, a little over one half — subject to its stated assumptions of closure-quartic democracy, uniform traceless weighting and linear finite-curvature action. That fixes the self-coupling of the Higgs field, $\lambda = 512/3969 \approx 0.129$, and with the vacuum already locked, the whole dimension-four Higgs potential is now numerical — its depth, its steepness and its minimum — with the paper stating exactly which residual substrate contributions must be proved small for these numbers to be the complete microscopic answer. The resulting leading Higgs mass is

m_H = 123.796 GeV .

The potential has exactly the required electroweak-breaking shape: the symmetric point is unstable, the potential is bounded, the nonzero minimum is stable, electromagnetism survives unbroken, and the same vacuum feeds the W boson, the Z boson and every fermion mass through relations derived in companion papers.

The quantum statement is made carefully. A gauge-fixed off-shell potential is not itself a physical observable, so the final Higgs mass is defined by the pole of the complete broken-phase propagator — the resonance the particle actually produces — and the Nielsen identity guarantees that this pole does not depend on calculational bookkeeping. The quoted 123.796 GeV is the leading pole-matched value under the declared matching premises; higher-order corrections are a calculable shift, not a licence to refit. The paper takes the first quantum step itself: the pure scalar-loop correction is evaluated exactly and nudges the mass by about nine MeV — less than one part in ten thousand — to 123.805 GeV, while the larger gauge and top-quark corrections await the numerical boundary data of their own gate papers.

The paper also confronts the deepest remaining conditionality head-on. The conversion factor that turns a substrate displacement into a physical field — the kinetic residue — is here fixed by two named premises rather than assumed silently, and the paper writes down the exact microscopic formula a future calculation must evaluate to replace those premises with a proof.

The reach of the calculation is then extended in two conditional directions. Borrowing one more closure relation — that the squared Higgs mass is exactly 15/14 of the combined squared W and

Z masses, the same 15/14 fraction that appears in the fine-structure formula — pins down the electroweak mixing angle to exactly two possible solutions. A principled tie-breaker then selects between them, and it does not peek at which solution resembles our world. Of everything in the gauge sector, only the electromagnetic field survives the substrate's refinement process unchanged; the selection principle says the physical photon must stay as close as possible to that one persistent carrier. Measured in the substrate's own geometry, one solution keeps 78% of the photon on the persistent carrier and the other only 12%, so the choice is immediate — giving a W boson of 79.3 GeV and a Z boson of 89.6 GeV. The principle itself remains a stated assumption rather than a proved law, and the paper reduces its future confirmation to a single inequality that one direct calculation will settle. And the quark-mass grid sets a numerical target for the top quark's coupling strength that a future direct calculation must hit or refute; taken at face value it sits in sharp tension with the running of the Higgs self-coupling, and the paper says so plainly rather than hiding it.

A methodological correction is folded in as well. The thousands of microscopic directions orthogonal to the Higgs direction are not thrown away — they are allowed to settle exactly, and the field's self-interaction strengths are read off from that fully settled landscape. An earlier shortcut differentiated only the simplest term of the substrate energy; the corrected treatment states precisely which additional contributions must be proved small for the quoted numbers to be the complete answer, and turns any deviation into a detectable fingerprint rather than a hidden error.

At no point are the observed electroweak vacuum (about 246 GeV) or the observed Higgs mass (about 125 GeV) used upstream. Every number above is produced by the frozen stack and only then may be compared with experiment. When that comparison is finally made — in a clearly separated section, after everything is frozen — the pattern is telling. The theory's own electron mass comes out 0.9% high and its vacuum 1% low, misses that partially cancel rather than compound, pointing at two specific components to refine. And the sharpest agreement is the pure ratio itself: 32/63 = 0.50794 against an observed ratio of about 0.508 — parts in a thousand or better — while the absolute numbers miss at the percent level. The group theory is performing an order of magnitude better than the scale-setting, which tells the programme exactly where to dig.

In plain terms, the paper claims:

VERSF can use one independently derived microscopic mass anchor, one independently derived interface coupling, and one independently derived Higgs-curvature ratio to calculate the location and shape of the electroweak vacuum without starting from the known Higgs answers.

The claim is conditional, and the paper says so precisely: it holds on a named stack of previously published VERSF results and bridge premises, each of which carries its own falsifier. If any named input fails, the numerical result fails with it — visibly, not quietly.

Headline Results and Conditionality

This paper is an evaluation paper, not another architecture paper.

It takes a frozen, previously published VERSF input stack and calculates

v_{cl} , λ , μ_H^2 , m_H

without using the observed electroweak vacuum or observed Higgs mass anywhere upstream.

The evaluated results, on the **Route-A / first-order interface-coupling branch**, are

$v_{cl} = 243.723$ GeV ,

$\lambda = 512/3969 = 0.128999748$,

$\mu_H^2 = 7.66269 \times 10^3$ GeV² , $\mu_H = 87.5368$ GeV ,

and

$m_H^{HR1} = 123.796$ GeV

at the declared leading finite-closure matching order, together with the exact one-loop scalar-only pole displacement

$M_{pole,scalar} = 123.804601005$ GeV (shift +8.889 MeV, residue $R_H = 1.000312804$) ,

and, under the additional 15/14 saturation relation, a conditional electroweak coupling and mass-branch reconstruction $(g, g', m_W, m_Z)_\pm$. Under the Persistent-Abelian Minimal-Lift Premise (P11), the lower branch is uniquely selected, giving

$\sin^2\theta_W = 0.21656$, $m_W = 79.268$ GeV , $m_Z = 89.556$ GeV ,

with the unconditional confirmation reduced to the single response inequality $Z_Y > Z_2$,

together with a conditional top-compression target $y_t^{grid} = 1.674$ — a grid quantity, not a derived top Yukawa.

The result is **conditional**, not unconditional. It depends on a named stack of existing VERSF results and bridge premises, including the Route-A coherence scale, the level-four electron scale law, the two-gate colourless interface readout, the $1/(8\pi)$ projection kernel, the closure-normalised Higgs embedding, the inertness of the relaxed vertex remainder ΔU, and the finite-curvature-to-pole matching condition. None of those inputs is selected using $v \simeq 246$ GeV or $m_H \simeq 125$ GeV.

Accordingly:

The numerical closure test passes for the frozen Route-A stack.

But:

An unconditional substrate-native closure remains dependent on proving every inherited bridge directly from the master substrate action.

One conditionality is elevated here because it is not minor: the calculation uses the first-order geometric coupling $\alpha^{-1} = 960/7 = 137.142857$, while the wider corpus contains a second-order candidate near $\alpha^{-1} = 137.034014$. Because $v_{cl} \propto \alpha^{-6}$, the coupling order moves the outputs to approximately $v_{cl} = 242.565$ GeV and $m_H = 123.207$ GeV. Until the coupling order is resolved, there are at least two frozen coupling branches; the primary output above is the first-order branch, and Section 12 quantifies the displacement.

This distinction is maintained throughout.

Scope and Closure Box

Claim	HR-1 status
Higgs-like closure-normal radial mode exists	Inherited
Scalar transforms as an electroweak doublet	Inherited conditionally
Canonical effective scalar coefficient	Unity by field definition and conditional closure-metric matching (Lemma 1)
Microscopic transport residue Z_H^{micro}	Exact Schur formula supplied; numerical evaluation open
Field-dependent kinetic corrections	Higher-order EFT ledger; not evaluated numerically
Geometric interface coupling	$\alpha = 7/960$, inherited conditionally
Route-A coherence scale	$\xi_A = 88$ µm, inherited conditional branch
Interface-coupling order	First-order branch (960/7) primary; second-order candidate open, shifts m_H by −0.59 GeV
Independent substrate electron scale	Evaluated
Colourless interface electron scale	Evaluated conditionally
Electroweak vacuum v_{cl}	Evaluated: 243.723 GeV
Higgs ratio m_H/v_{cl}	Evaluated: 32/63
Quartic λ	Evaluated: 512/3969
Quadratic coefficient μ_H^2	Evaluated

Claim	HR-1 status
Classical radial curvature	Evaluated
Leading finite-closure pole-matched Higgs mass	Evaluated: 123.796 GeV
Exact microscopic Z_H formula (Schur momentum expansion)	Supplied; numerical evaluation open
Complete relaxed quadratic, cubic and quartic vertex formulae	Closed (Theorem 4)
Field-dependent canonical vertex geometry (Z_1, Z_2 corrections)	Closed (Theorem 5)
Isolated-quartic vertex ratios	Conditional on remainder inertness (F20)
Exact top-Yukawa input address	Supplied; numerical evaluation open
Numerical effective transport kernel and doublet Hessian	Evaluated
Mixed radial–Goldstone vertices	Evaluated
Conditional electroweak coupling and mass-branch reconstruction (g, g′, m_W, m_Z)_±	Conditional numerical pass
Persistent-Abelian branch selection	Conditional theorem under P11; unconditional test Z_Y > Z_2 open
Conditional top-compression grid target (not a derived Yukawa)	1.67355737679; compression test pending
Grid target vs β_λ stability	Acute tension recorded; running bridge decisive
Post-output empirical orientation	Recorded downstream of firewall (Section 12.2); electron +0.90%, kernel +1.93%, ratio 32/63 at 10^{-3}–10^{-4}
Interface-to-electroweak Abelian running	Declared open axis (Section 10.4); shifts s_- from 0.2166 toward ≈ 0.235
One-loop scalar-only pole displacement	Evaluated exactly: +8.889 MeV
One-loop pole residue R_H	Evaluated: 1.000312804
Gauge, top, Goldstone/ghost and threshold pole shifts Δ_H	Defined; await numerical boundary data
Full all-order complex pole	Defined; higher-order displacement open
Local quartic stability	Passed
Complete global/RG vacuum stability	Open
Electromagnetic stabiliser	Derived
Broken-phase W, Z, γ pattern	Derived structurally
Fermion mass transmission	Derived structurally
Measured v or m_H used upstream	No
Route-A versus alternative coherence branches	Audited explicitly

Claim	HR-1 status
Unconditional master-action derivation of every bridge	Not yet established
Empirical confirmation	Not claimed

Abstract

The earlier VERSF Higgs-radial gate identifies the Standard Model scalar radial degree of freedom with the normal return-stiffness mode of the closure interface. It does not determine the canonical kinetic normalisation, electroweak vacuum, scalar potential, quadratic coefficient, quartic coupling or physical Higgs mass.

This paper closes those quantities numerically on a frozen Route-A VERSF input stack.

The derivation combines four independent upstream constructions.

The vacuum-geometry programme gives

$\alpha^{-1} = 2^7 \cdot 15/14 = 960/7$, $\alpha = 7/960$.

The Route-A coherence and level-four defect construction gives

$m_e^{sub} = (\hbar c/\xi_A)\, \alpha^{-4} \cdot 13/20$, $\xi_A = 88\ \mu m$.

The colourless completion-interface construction gives

$m_e^{CI} = (\alpha_{int}^2/8\pi)\, v_{cl}$.

At the frozen low-current interface matching point, $\alpha_{int} = \alpha$. Equating the independently obtained substrate and interface bilinears yields the scale-locking identity

$\mathbf{v_{cl} = (26\pi/5)\, (\hbar c/\xi_A)\, \alpha^{-6}}$ **.**

With $\hbar c = 197.3269804$ eV·nm, this gives

$\mathbf{v_{cl} = 243.7228076}$ **GeV .**

The conditional finite-deformation theorem selects

$m_H/v_{cl} = \frac{1}{2}\,(1 + 1/63) = 32/63$,

subject to closure-quartic democracy, uniform traceless weighting and the declared linear finite-curvature action.

Consequently,

$$\boldsymbol{\lambda = \tfrac{1}{2}\,(32/63)^2 = 512/3969 = 0.1289997480\ ,}$$

$$\boldsymbol{\mu\_H^2 = \lambda\, v\_cl^2 = 7.6626891 \times 10^3\ \mathrm{GeV}^2\ ,}$$

and

$$\boldsymbol{m\_H\text{^}HR1 = \sqrt{(2\lambda)}\ v\_cl = (32/63)\ v\_cl = 123.7957118\ \mathrm{GeV}\ .}$$

The closure-normalised Higgs embedding is pulled back through the SU(8)-invariant substrate metric. At the declared matching order this fixes the canonical *effective* coefficient to unity — by field definition and conditional closure-metric matching, not by numerical microscopic derivation:

$$Z\_H\text{^}eff(\mu\star) = 1\ ,$$

while the microscopic transport residue remains an exact formula with open numerical evaluation, and field-dependent and higher-derivative corrections enter the EFT remainder.

The resulting scalar effective action is

$$\boldsymbol{\Gamma\_scalar\text{^}HR1 = \int d^4x\ [\ (D\_\mu H)^\dagger(D\text{^}\mu H) + \mu\_H^2\ H^\dagger H - \lambda\ (H^\dagger H)^2 + \cdots\ ]}$$

or, equivalently,

$$V\_CI(H) = \lambda\ (\ H^\dagger H - v\_cl^2/2\ )^2$$

up to the irrelevant additive vacuum constant and explicitly ordered higher operators.

The vacuum satisfies

$$H\_vac = (1/\sqrt{2})\ (0,\ v\_cl)^T\ ,$$

with stabiliser U(1)_em. The leading broken-phase relations are

$$m\_W^2 = \tfrac{1}{4}\, g^2\, v\_cl^2\ ,\ m\_Z^2 = \tfrac{1}{4}\,(g^2 + g'^2)\ v\_cl^2\ ,\ m\_\gamma^2 = 0\ ,$$

and

$$M\_f = (v\_cl/\sqrt{2})\ Y\_f\ .$$

The quantum completion is Nielsen-consistent. The physical Higgs pole is defined by

$$\det \Gamma^{(2)}\_broken(s\_H) = 0\ ,\ s\_H = M\_H^2 - i\ M\_H\ \Gamma\_H\ .$$

At the declared finite-closure, local two-derivative matching order, the pole is

$$s\_H^{(0)} = (123.7957118\ \mathrm{GeV})^2 .$$

The Nielsen identity guarantees gauge-parameter independence of the isolated pole when the Slavnov–Taylor identities are maintained. Higher-order self-energy, threshold and higher-derivative contributions shift the pole through a separately calculable Δ_H; they may not be chosen to force agreement.

The paper additionally supplies the exact microscopic definition that a future unconditional proof of the residue must evaluate. With the substrate Hessian blocks expanded as $H\_HH = A^{(0)} + p^2A^{(2)}$, $H\_H\eta = B^{(0)} + p^2B^{(2)}$, $H\_\eta\eta = C^{(0)} + p^2C^{(2)}$, the relaxed two-point kernel gives

$$Z\_H = E\_H^\dagger\ [\ A^{(2)} - B^{(2)}C^{(0)-1}B^{(0)\dagger} - B^{(0)}C^{(0)-1}B^{(2)\dagger} + B^{(0)}C^{(0)-1}C^{(2)}C^{(0)-1}B^{(0)\dagger}\ ]\ E\_H ,$$

and the frozen substrate potential contributes exactly zero to it: the residue lives entirely in the transport/coarse-graining sector, whose coefficients the corpus does not yet supply. Lemma 1 therefore remains a conditional matching bridge, and this formula is its outstanding invoice.

The static vertices are constructed to the same standard. With the orthogonal modes relaxed exactly along $\eta\star(\varrho)$ and the relaxed derivative $\mathcal{D}\_\varrho = \partial\varrho - (C^{-1})\text{^}\{ab\}U\{b\varrho\}\partial\_\{\eta\text{^}a\}$, the complete declared-order vertices are

$$K_2 = U\_\varrho\varrho - U\_\varrho a(C^{-1})\text{^}\{ab\}U\_b\varrho\ ,\ K_3 = \mathcal{D}\_\varrho K_2\ ,\ K_4 = \mathcal{D}\_\varrho^2 K_2\ ,$$

correcting an earlier shortcut in which only the isolated substrate quartic was differentiated. For field-dependent kinetic residue, the exact canonical geometry gives $g_3 = K_3/Z_0\text{^}\{3/2\} - 3K_2Z_1/(2Z_0\text{^}\{5/2\})$ and the corresponding fourth-order expression; the simple $K\_n/Z_0\text{^}\{n/2\}$ rules hold only at constant-residue order.

Finally, the first quantum correction is evaluated exactly. In the zero-momentum curvature-and-residue matching scheme, the pure radial scalar loop shifts the pole by

$$M^2\_\mathrm{pole,scalar} = m\_c^2\ [\ 1 + (9\lambda/16\pi^2)(11/6 - \pi/\sqrt{3})\ ]\ ,$$

giving $M\_\mathrm{pole,scalar}$ = 123.804601005 GeV, a displacement of +8.889 MeV, with pole residue R_H = 1.000312804 and vanishing scalar-only width below threshold. The gauge, top-Yukawa, Goldstone/ghost and threshold displacements are defined but await the numerical boundary data of the gauge and Yukawa gates.

Two conditional reconstructions extend the numerical reach without new insertion. Combining $e^2 = 4\pi\alpha$ with the leading-saturation relation $m\_H^2 = (15/14)(m\_W^2 + m\_Z^2)$ pins $\sin^2\theta\_W$ to two exact branches; under the Persistent-Abelian Minimal-Lift Premise — the photon is the completion of minimum operational displacement from the unique refinement-persistent Maxwell carrier — the lower branch is uniquely selected, giving $\sin^2\theta\_W$ = 0.2165573823, m_W = 79.268 GeV and m_Z = 89.556 GeV, with unconditional confirmation reduced to the response inequality $Z\_Y > Z_2$. Separately, the one-anchor quark grid supplies a conditional top-compression target $y_t^{grid} = 1.674$ (a grid quantity, not a derived top Yukawa), whose confrontation with the eventual direct σ_{max} calculation — and, under explicit scheme-identification conditions, with the $-6y_t^4$ term of β_λ — is recorded as the stack's sharpest open tension.

No observed electroweak vacuum or Higgs mass appears upstream. The Route-A numerical gate therefore passes its declared non-circularity test.

Contents

---

1. The Hard Closure Question

The previous Higgs-radial gate answered:

Why must VERSF contain a Higgs-like scalar radial mode?

It identified the mode as the normal return-stiffness excitation of the closure interface.

The present gate asks the harder numerical question:

What scalar action does that mode possess, where is its vacuum, and what physical radial mass follows?

A valid answer must provide

$$\Gamma\_scalar = \int d^4x \, [\, \tfrac{1}{2} Z\_H(\varphi) \, (\partial\varphi)^2 - V\_CI(\varphi) + \cdots \,]$$

and evaluate

v_cl , λ , μ_H^2 , m_H .

Definitions alone do not pass. A list of future calculations does not pass. Inserting the known answers does not pass.

This paper uses a scale-locking route in which two independently derived descriptions of the same electron bilinear determine the canonical electroweak vacuum. The scalar curvature then determines the potential coefficients. The overdetermination is the engine of the derivation: one physical quantity, two frozen formulas, zero remaining freedom.

1.1 Meaning of complete declared order

In this paper, **complete declared order** means: leading local scalar effective action; all retained substrate modes relaxed through the exact Schur complement; two-derivative kinetic order; dimension-four scalar-potential order; one radial insertion in the leading fermion bridge; one-loop reduced radial self-energy; zero-momentum curvature-and-residue matching; and no claim that higher-derivative, higher-dimensional, composite-channel, or heavy-threshold terms vanish. The word *complete* is exact within a declared truncation, not a claim that every order of the full VERSF master action has been calculated.

1.2 Proof standard

Every quantity in this paper carries one of four grades:

1. **Exact:** follows from operator algebra and the stated functional.
2. **Conditional theorem:** follows uniquely from a named VERSF premise stack.
3. **Conditional numerical result:** all numbers are fixed once that premise stack is frozen.
4. **Open numerical object:** formula and data address are known, but the required substrate matrix elements are absent.

Observed electroweak and Higgs values are not permitted upstream at any grade.

---

2. Inherited Frozen Inputs

2.1 Input I — Closure cardinality and loop count

The VERSF closure architecture supplies

$K = 7$, $N_{loop} = 2K = 14$.

These numbers are fixed upstream and are not selected in this paper.

2.2 Input II — Geometric interface coupling

The vacuum-geometry construction gives

$\alpha^{-1} = 2^K \cdot (N_{loop} + 1)/N_{loop}$.

Therefore

$\alpha^{-1} = 2^7 \cdot 15/14 = 960/7$,

and

$\boldsymbol{\alpha = 7/960}$ **.**

This is the coupling used throughout the blind calculation. The measured low-energy value of α is not substituted.

2.3 Input III — Route-A coherence scale

The Route-A coherence construction gives

$\boldsymbol{\xi_A = 88\ \mu m}$ **.**

This is a named upstream branch. It is frozen before any Higgs calculation. No choice between coherence branches is made by comparing their resulting weak scales with observation.

2.4 Input IV — Level-four electron scale law

The level-four defect construction gives

$\boldsymbol{m_e^{sub} = (\hbar c/\xi_A)\, \alpha^{-4} \cdot 13/20}$ **.**

The factors have distinct upstream meanings:

$\hbar c/\xi_A$

is the coherence-cell energy,

α^{-4}

is the level-four enhancement, and

$13/20 = [(K-1)/(K+1)] \cdot [(2K-1)/(2K+1)]$

is the geometric occupancy factor.

No electron mass is used to select these factors in this paper.

2.5 Input V — Colourless completion-interface readout

The charged-lepton interface construction gives

$$\mathbf{m_e^{CI} = (\alpha_{int}^2/8\pi)\, v_{cl}\,.}$$

The factor α_{int}^2 comes from two source-visible current gates. The factor

$1/(8\pi) = (1/2\pi) \cdot ½ \cdot ½$

is the candidate colourless phase–chiral–weak projection kernel.

At the low-current matching point used by the electron-scale construction, this paper freezes

$$\boldsymbol{\alpha_{int} = \alpha}\,.$$

This is a bridge premise, not a fitted equality.

2.6 Input VI — Finite Higgs curvature ratio

The SU(8) deformation calculation gives

$(m_H/v_{cl})_0 = ½$

from closure-quartic democracy, and the uniform traceless deformation correction gives

$1 + 1/\dim \mathfrak{su}(8) = 1 + 1/63\,.$

Thus

$$\mathbf{m_H/v_{cl} = ½\,(1 + 1/63) = 32/63\,.}$$

This input is frozen before the absolute vacuum scale is calculated.

2.7 Input VII — Frozen substrate functional

The strongest explicit VERSF substrate free energy is

$$\mathscr{F}[\rho] = \int d\mu \cdot (\lambda_{sub}/4) \left(\|\rho\|^2 - \rho_c^2 \right)^2 + \mathcal{A}_C[\rho] + \mathcal{A}_T[\rho] + \mathcal{A}_R[\rho] .$$

The transport term is declared as

$$\mathcal{A}_T[\rho] = \int d\mu \cdot \| \nabla\cdot J_\rho + \partial_\sigma \rho \|^2 ,$$

with J_ρ linear in ρ, but the linear operator, its absolute coefficient and its continuum momentum normalisation are not specified. This functional supplies the reduced radial vertices of Section 7.1 and defines the microscopic residue target of Section 4.4. Its unspecified transport coefficients are exactly where the conditionality of Lemma 1 resides.

A complementary coarse-grained closure-norm functional also appears in the corpus,

$$F[\varrho] = \int d^3x \left[\tfrac{1}{2}(\nabla\varrho)^2 + V(\varrho) + E_{R4}(\varrho) + E_{bath}(\varrho) \right] , \quad V(\varrho) = -(\alpha_\varrho/2)\varrho^2 + (\beta_\varrho/4)\varrho^4 , \quad \alpha_\varrho, \beta_\varrho > 0 ,$$

whose signs and minimal analytic structure are motivated by closure instability and finite commitment capacity. Its numerical coefficients and the higher radial derivatives of E_{R4} and E_{bath} are not independently evaluated; they enter the vertex remainder ΔU of Section 7.1.

---

3. Premises, Grades and Firewalls

P1 — Frozen input stack. The formulae and discrete factors listed in Section 2 are frozen before the scalar outputs are evaluated.

P2 — Route-A branch. The coherence scale is the named Route-A value $\xi_A = 88$ μm, selected by its upstream construction rather than Higgs agreement.

P3 — Electron identity. The substrate electron bilinear and the interface electron bilinear describe the same canonically normalised charged-lepton branch at the same declared matching point.

P4 — Interface coupling match. The source-readable interface coupling equals the geometric low-current coupling: $\alpha_{int} = \alpha$.

P5 — Canonical interface readout. The $1/(8\pi)$ kernel acts on canonically normalised scalar and fermion carriers.

P6 — Closure-unit scalar embedding. The Higgs direction is a unit closure-normalised direction in the SU(8)-invariant substrate metric.

P7 — Local two-derivative dominance. At the matching order,

$$\Gamma_H^{(2)}(p^2) = p^2 - m^2_{H,0} + O(p^4/M_\star^2) .$$

P8 — Finite curvature matching. The ratio 32/63 is the finite closure-renormalised radial curvature ratio at the declared matching order.

P9 — Quantum gauge consistency. The Slavnov–Taylor and Nielsen identities are maintained at the order used for physical pole matching.

P10 — No numerical insertion. No observed Higgs, weak-scale, gauge-boson or fermion mass is used to select any upstream factor, branch or normalisation.

P11 — Persistent-Abelian minimal lift. When the substrate equations return more than one isolated massless neutral completion satisfying all algebraic, positivity, anomaly and stabiliser conditions, admissibility selects the completion of minimum operational-geodesic displacement from the inherited persistent Abelian carrier. This is a conditional admissibility input — the neutral-gauge analogue of the minimal-Hermitian-lift principle of the C_3 branch analysis — not a computed Hessian output.

3.1 Kinetic-normalisation firewall

Writing $Z_H = 1$ is not by itself a derivation.

Here it follows from two linked statements:

1. the retained Higgs carrier is a unit direction of the SU(8)-invariant closure metric;
2. the interface scale-locking equation is written in the canonically normalised scalar field consumed by the Yukawa operator.

The raw microscopic coordinate may possess a nontrivial coordinate metric. The physical field is its canonical pullback. The residue claim therefore rests on premises P5–P6, and its falsifier F2 remains live: this is a conditional discharge of the kinetic debt, not its erasure.

3.2 Curvature-versus-pole firewall

The zero-momentum curvature and physical propagator pole are not identical in general. Their equality here is only the declared leading local matching result under P7–P9. The full pole remains the zero of the complete two-point matrix.

3.3 Route-selection firewall

The 88 μm branch may not be defended because it gives the closest Higgs result. Its legitimacy must come entirely from its upstream derivation.

3.4 Cross-sector scale-lock firewall

Using the electron branch is not a calibration if both its substrate mass and its interface coupling are derived independently of the measured electron mass. It becomes calibration immediately if an observed electron mass is substituted.

3.5 Gauge-dependence firewall

The shape of a gauge-fixed potential away from a stationary point is not a physical prediction. The physical claims are made through the gauge-invariant orbit radius, the stationary branch and the propagator pole.

---

4. Scalar Carrier and Kinetic Normalisation

4.1 Closure-normalised scalar direction

Let

$$\rho(x) \in \mathbb{C}^8$$

be the closure-normalised substrate carrier. Let E_H embed the electroweak Higgs doublet into the retained scalar direction:

$$\delta\rho(x) = E_H H(x) + \delta\rho_\perp(x) .$$

Closure-unit normalisation gives

$$E_H^\dagger E_H = 1 .$$

The SU(8)-invariant quadratic metric on the retained direction is therefore

$$\delta\rho^\dagger\delta\rho = H^\dagger H$$

after orthogonal modes are removed.

Radial–angular bridge and notation

Two objects must not share one symbol: the full substrate carrier and the gauge-invariant radial closure amplitude. Throughout this paper, **$\rho(x) \in \mathbb{C}^8$** denotes the full substrate carrier, and **$\varrho(x)$** denotes the gauge-invariant radial closure amplitude. The explicit bridge to the electroweak doublet is

$$\mathbf{H(x) = [\ (v\_cl + \varrho(x)) / \sqrt{2}\ ] \cdot U\_EW(x)\ (0, 1)^T\ ,}$$

where ϱ is the gauge-singlet radial closure amplitude, U_EW(x) carries the angular/interface orientation (the Goldstone directions), and the full H transforms as $(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$. There is therefore no contradiction between "the closure norm is gauge invariant" and "the scalar carrier transforms as a weak doublet": the invariant norm is the radial factor, and the doublet character lives entirely in the orientation factor.

4.2 Continuum kinetic pullback

The leading admissible continuum derivative functional has the form

$$\Gamma\_kin = \int d^4x\ (D\_\mu\delta\rho)^\dagger\ (D^\wedge\mu\delta\rho)\ .$$

Pulling this functional back along E_H gives

$$\Gamma\_kin^\wedge H = \int d^4x\ (D\_\mu H)^\dagger\ E\_H^\dagger E\_H\ (D^\wedge\mu H)\ .$$

Hence

$$\mathbf{Z\_H^\wedge eff(\mu\star) = 1\ ,}$$

as a matter of field definition on the canonical carrier — see the grading in Lemma 1 below.

Lemma 1 — Canonical Closure-Pullback Matching Lemma

Under P5 and P6, the leading two-derivative kinetic coefficient of the physical closure-interface Higgs carrier is unity at the matching point.

This is a *matching lemma*, not a numerical microscopic transport derivation. A unit-normalised embedding fixes the form of the pullback metric, but not its absolute microscopic coefficient unless the substrate metric's overall normalisation has already been independently fixed; likewise, writing $\frac{1}{2}(\partial\varrho)^2$ in a coarse-grained action defines a canonical coordinate — it does not calculate the raw substrate residue. The honest two-line status is therefore:

Canonical effective scalar coefficient: unity by field definition and conditional closure-metric matching.

Microscopic transport residue: exact Schur-complement formula supplied (Section 4.4); numerical evaluation open.

Proof

The substrate metric is SU(8)-invariant and the retained Higgs embedding is unit normalised. Its pullback is therefore the identity on the retained irreducible carrier. The colourless interface readout is defined on this same canonical carrier, so no independent field-rescaling freedom remains after the electron scale is locked. ■

Relation to the prior kinetic debt

The earlier blind evaluation located the programme's central obstruction in exactly this residue: the closure-unit metric had not been identified with the record-layer kinetic metric, and "set $Z_H = 1$" was rejected as renaming. Lemma 1 discharges that debt *conditionally*: P6 asserts the metric identification and P5 asserts that the interface readout consumes the same canonical carrier, so the two normalisation questions collapse into one, answered by SU(8) invariance. The debt has been converted from an uncalculated number into two named, falsifiable bridge premises. That is genuine progress, and its remaining conditionality is recorded in Section 16.2 rather than hidden.

4.3 Higher-order kinetic structure

The complete derivative expansion may contain

$$Z_H(H^\dagger H) = 1 + c_H (H^\dagger H/M_\star^2) + \cdots$$

and operators such as

$$(c_{HD}/M_\star^2)\,(H^\dagger H)\,(D_\mu H)^\dagger(D^\mu H)\,.$$

These terms do not alter the leading HR-1 evaluation unless their coefficients or $v_{cl}/M_\star$ invalidate P7.

4.4 Exact microscopic residue formula

Lemma 1 fixes the canonical effective coefficient conditionally. This subsection records exactly what an unconditional proof must evaluate, so that the premise can eventually be discharged by calculation rather than restated.

Let E_H be the Higgs-normal embedding and η the complete orthogonal substrate carrier. The quadratic substrate Hessian in momentum space is

$$\mathbb{H}(p^2) = \begin{pmatrix} H_{HH}(p^2) & H_{H\eta}(p^2) \\ H_{\eta H}(p^2) & H_{\eta\eta}(p^2) \end{pmatrix}.$$

After exact relaxation of η, the reduced scalar kernel is

$$\boldsymbol{\mathcal{K}_H(p^2) = E_H^\dagger \left[H_{HH}(p^2) - H_{H\eta}(p^2)\, H_{\eta\eta}(p^2)^{-1}\, H_{\eta H}(p^2) \right] E_H\,.}$$

Expand the blocks in momentum (parenthesised superscripts avoid collision with the loop function B_0 used in Section 11):

$H\_HH = A^{(0)} + p^2 A^{(2)} + O(p^4)$, $H\_H\eta = B^{(0)} + p^2 B^{(2)} + O(p^4)$, $H\_\eta\eta = C^{(0)} + p^2 C^{(2)} + O(p^4)$.

Then $\mathcal{K}\_H(p^2) = K\_H + Z\_H\, p^2 + O(p^4)$, with

$$\mathbf{K\_H = E\_H^\dagger \left(A^{(0)} - B^{(0)} C^{(0)-1} B^{(0)\dagger} \right) E\_H}$$

and

$$\mathbf{Z\_H = E\_H^\dagger \left[A^{(2)} - B^{(2)} C^{(0)-1} B^{(0)\dagger} - B^{(0)} C^{(0)-1} B^{(2)\dagger} + B^{(0)} C^{(0)-1} C^{(2)} C^{(0)-1} B^{(0)\dagger} \right] E\_H \,.}$$

The four terms have transparent meanings: the direct momentum response of the retained mode, two mixing-slope corrections, and the relaxed momentum response of the orthogonal sector fed back through the static mixing. Every one is a frozen-substrate object; none is a convention.

4.5 What evaluates from the current functional, and the hard limitation

The local substrate potential $(\lambda\_sub/4)(\|\rho\|^2 - \rho\_c^2)^2$ contains no continuum momentum. Its direct contribution is therefore

$$\mathbf{Z\_H^\wedge(V) = 0 \,.}$$

The entire positive propagating residue must be generated by the transport/coarse-graining sector, including $\mathcal{A}\_T$, and by the momentum-dependent Schur-complement response: Z_H lives in $A^{(2)}$, $B^{(2)}$ and $C^{(2)}$, and nowhere else. The current corpus does not supply those coefficients. Consequently:

The linearised record-current operator, its coefficient and the momentum-dependent auxiliary Hessian are not yet specified, so the microscopic scalar residue cannot yet be assigned a numerical value without the bridge premises P5–P6.

A related record-field functional sometimes used in the corpus,

$$\mathcal{C}[\rho] = \int d^4x \left[\alpha|\partial\rho|^2 + \beta\rho^2 + \gamma\rho^4 + \eta(\nabla\cdot J\_\rho)^2 + \lambda\mathcal{A}[\rho]^2 \right] ,$$

would give, on a unit radial embedding with no derivative mixing, the direct residue $Z\_H^\wedge direct = 2\alpha$ — but α is itself recorded as an unevaluated functional of substrate data, so this form provides a symbolic residue, not a numerical one.

Two further numerical facts sharpen the status of the residue. First, the coarse-grained closure-norm functional of Input VII carries the canonical kinetic term $\frac{1}{2}(\partial\_\mu\varrho)(\partial^\wedge\mu\varrho)$ for the radial amplitude, so at the declared effective order the *effective* transport residue is numerically established:

$$Z_H^{eff} = 1 \text{ (conditional pass at canonical effective order) },$$

while Z_H^{micro} remains an open numerical object — the two must not be conflated, and the grade difference is exactly the content of premises P5–P6. Second, the substrate paper's currently supplied closure Hessian H_{cl} is the *traceless* $\mathfrak{su}(8)$ restriction, whereas the Higgs radial mode lies in the excluded trace direction. The microscopic residue calculation therefore requires not merely numerical values for the existing blocks but an extension of H_{cl} to the trace sector. This is recorded as a named open debt in Section 16.2.

Lemma 1 and this subsection are therefore two halves of one honest statement: the residue is unity *given* the canonical-metric premises, and the formula above is the outstanding invoice that a substrate-native proof must pay to remove the word "given".

---

5. Independent Substrate Electron Scale

The coherence energy is

$$E_{\xi_A} = \hbar c/\xi_A .$$

Using

$$\hbar c = 197.3269804 \text{ eV·nm}$$

and

$$\xi_A = 88\ \mu\text{m} = 88\,000 \text{ nm},$$

gives

$$E_{\xi_A} = 197.3269804/88\,000 \text{ eV} = 2.24235205 \times 10^{-3} \text{ eV} .$$

The geometric coupling is $\alpha = 7/960$. Therefore

$$m_e^{sub} = E_{\xi_A} \cdot (960/7)^4 \cdot 13/20 .$$

Numerically,

$$\mathbf{m_e^{sub} = 5.15596460 \times 10^5 \text{ eV} = 0.515596460 \text{ MeV} .}$$

This value is an upstream output of the frozen Route-A stack. It is not replaced by the measured electron mass.

---

6. Electroweak Vacuum Scale-Locking Theorem

The interface expression is

$m_e^{CI} = (\alpha^2/8\pi)\, v_{cl}$.

The substrate expression is

$m_e^{sub} = (\hbar c/\xi_A)\, \alpha^{-4} \cdot 13/20$.

P3 requires

$m_e^{CI} = m_e^{sub}$.

Therefore

$(\alpha^2/8\pi)\, v_{cl} = (\hbar c/\xi_A)\, \alpha^{-4} \cdot 13/20$.

Solving,

$v_{cl} = 8\pi \cdot (13/20) \cdot (\hbar c/\xi_A) \cdot \alpha^{-6}$.

Thus

$\boldsymbol{v_{cl} = (26\pi/5)\, (\hbar c/\xi_A)\, \alpha^{-6}}$ **.**

Substituting $\alpha = 7/960$,

$\boldsymbol{v_{cl} = (26\pi/5)\, (\hbar c/\xi_A)\, (960/7)^6}$ **.**

Theorem 2 — Cross-Sector Electroweak Scale-Locking Theorem

Under P1–P6, the common electroweak vacuum is uniquely fixed by equality of the independently derived substrate and colourless-interface electron bilinears. No continuous rescaling freedom remains — a statement that holds once, and only once, the coherence branch, interface-coupling order, phase kernel and finite-curvature theorem are all frozen; the residual freedom is discrete branch choice, not continuous adjustment.

Proof

The substrate route returns a nonzero dimension-one electron bilinear. The interface route returns the same bilinear as a fixed dimensionless projection coefficient times v_{cl}. Since the projection coefficient is nonzero and fixed, the equality has one and only one solution for v_{cl}. ■

Structural remark

The closed form exposes the anatomy of the weak scale in this route: one geometric fraction ($26\pi/5$), one coherence energy ($\hbar c/\xi_A$), and six inverse powers of the interface coupling. The sixth power arises as four powers from the level-four enhancement plus two from the interface gates, and it is the origin of the amplified coupling sensitivity quantified in Section 12.1. Nothing else enters.

6.1 Numerical evaluation

Using the frozen Route-A values,

$v_{cl} = 243.722807638$ GeV .

Rounded at the precision justified by the upstream scale branch,

$v_{cl} = 243.723$ GeV .

---

7. Finite SU(8) Curvature and Quartic Evaluation

The dimension-four scalar potential is written

$V_{CI}(H) = -\mu_H^2 H^\dagger H + \lambda (H^\dagger H)^2$.

For

$H = (1/\sqrt{2})(0, v_{cl} + h)^T$,

the tree-level canonical radial relation is

$m_H^2 = 2 \lambda v_{cl}^2$.

Therefore

$m_H/v_{cl} = \sqrt{2\lambda}$.

The finite closure result gives

$\sqrt{2\lambda} = 32/63$.

Squaring,

$2\lambda = 1024/3969$,

and hence

$\boldsymbol{\lambda = 512/3969}$ **.**

Numerically,

$\boldsymbol{\lambda = 0.128999748047}$ **.**

Theorem 3 — Finite Closure Quartic Theorem

Under the closure-quartic democracy baseline, traceless SU(8) selection, Killing-form uniformity and linear finite-curvature action inherited from *Substrate Dynamics and the Higgs Ratio*,

$\boldsymbol{\lambda = 512/3969}$

at the HR-1 matching order. The result contains no fitted continuous parameter — every ingredient is a group-theoretic integer: 2^K, $K+1$, dim $\mathfrak{su}(8)$.

7.1 Complete relaxed vertices from one reduced functional

The vertices of the scalar potential are not obtained by differentiating one isolated substrate term. They are obtained by differentiating the *relaxed* reduced functional, in which every orthogonal mode has settled. This subsection supplies the exact machinery; the isolated-quartic special case then follows with its correct conditional grading.

Relaxed branch and relaxed derivative

Let the retained substrate fields be $q^I = (\varrho, \eta^a)$, where ϱ is the invariant radial closure amplitude and η^a contains every orthogonal closure, transport, record, angular and auxiliary mode retained at the declared order. Write the constant-background 1PI potential as $U(\varrho, \eta)$. For each admissible radial value, the orthogonal modes satisfy the stationary condition

$U_a(\varrho, \eta^\star(\varrho)) = 0$, and the auxiliary Hessian $C_{ab} = U_{ab}$ is assumed invertible and positive on the relaxed physical support (falsifier F19). The reduced scalar potential is

$$U_{red}(\varrho) = U(\varrho, \eta^\star(\varrho)) .$$

Implicit differentiation gives $d\eta^{\star a}/d\varrho = -(C^{-1})^{ab} U_{b\varrho}$. Define the derivative along the relaxed branch,

$$\boldsymbol{\mathcal{D}_\varrho = \partial_\varrho - (C^{-1})^{ab} U_{b\varrho}\, \partial_{\eta^a}} ,$$

so that $(d/d\varrho)\, X(\varrho, \eta^\star(\varrho)) = \mathcal{D}_\varrho X$ for any X.

Theorem 4 — Relaxed Vertex Theorem

The complete declared-order static vertices are generated by relaxed differentiation of one reduced functional:

$$\boldsymbol{K_2 = U_{\varrho\varrho} - U_{\varrho a} (C^{-1})^{ab} U_{b\varrho} ,\quad K_3 = \mathcal{D}_\varrho K_2 ,\quad K_4 = \mathcal{D}_\varrho^2 K_2} ,$$

all evaluated at $(\bar\varrho, \eta^\star(\bar\varrho))$. No individual substrate term may be called the complete vertex unless every other term is proved inert through fourth order. The static K_2 is the $p^2 = 0$ value of the Schur kernel of Section 4.4; the vertex tower and the residue therefore descend from one and the same reduced object.

Closure-norm decomposition

Write the reduced potential as

$$U_{red}(\varrho) = -(\alpha_\varrho/2)\varrho^2 + (\beta_\varrho/4)\varrho^4 + \Delta U(\varrho) ,$$

where $\Delta U = E_{R4}^{red} + E_{bath}^{red} + V_{relax}$ contains the complete reduced stabilisation, bath, closure, transport, record and relaxation remainder. The stationary condition and vertices are then

$$-\alpha_\varrho \bar\varrho + \beta_\varrho \bar\varrho^3 + \Delta U'(\bar\varrho) = 0 ,$$

$$\boldsymbol{K_2 = -\alpha_\varrho + 3\beta_\varrho \bar\varrho^2 + \Delta U''(\bar\varrho) ,\quad K_3 = 6\beta_\varrho \bar\varrho + \Delta U'''(\bar\varrho) ,\quad K_4 = 6\beta_\varrho + \Delta U''''(\bar\varrho)} .$$

This replaces the over-strong statement that the isolated quartic automatically supplies the complete scalar vertices.

Isolated-quartic special case

If ΔU is inert through fourth order — the closure, transport and record remainders contribute negligibly to $\Delta U''$, $\Delta U'''$, $\Delta U''''$ on the homogeneous admissible branch — the reduced potential is the radial restriction of the single substrate quartic of Input VII:

$$V_{red}(u) = (M_\star^4 \lambda_{sub}/4) (u^2 - u_0^2)^2 ,$$

and, setting $u = u_0 + \chi$,

$$\mathbf{\Gamma^{(2)}_{\chi\chi}(0) = 2 M_\star^4 \lambda_{sub} u_0^2 , \Gamma^{(3)}_{\chi\chi\chi}(0) = 6 M_\star^4 \lambda_{sub} u_0 , \Gamma^{(4)}_{\chi\chi\chi\chi}(0) = 6 M_\star^4 \lambda_{sub} .}$$

The fixed ratios

$$\Gamma^{(3)}/\Gamma^{(2)} = 3/u_0 , \Gamma^{(4)}/\Gamma^{(3)} = 1/u_0$$

are then parameter-free audit conditions — but their status is *conditional on the remainder inertness*, not exact (falsifier F20). Any measured violation of the ratios is a direct fingerprint of ΔU.

7.2 Canonical field geometry

The reduced action is

$$\Gamma_{scalar} = \int d^4x \left[\tfrac{1}{2} Z_H(\varrho) (\partial\varrho)^2 - U_{red}(\varrho) + O(\partial^4) \right] .$$

Define the canonical coordinate $\varphi(\varrho) = \int_0^\varrho \sqrt{Z_H(s)}\, ds$, the canonical vacuum $v_{cl} = \varphi(\bar{\varrho})$, and the physical fluctuation $h = \varphi - v_{cl}$. Let

$$Z_0 = Z_H(\bar{\varrho}) , Z_1 = Z_H'(\bar{\varrho}) , Z_2 = Z_H''(\bar{\varrho}) .$$

Theorem 5 — Canonical Scalar Geometry Theorem

Because $U_{red}'(\bar{\varrho}) = 0$, the exact canonical derivatives through fourth order are

$$\mathbf{m^2_{curv} = K_2/Z_0 ,}$$

$$\mathbf{g_3 = K_3/Z_0^{3/2} - 3K_2Z_1/(2Z_0^{5/2}) ,}$$

$$\mathbf{g_4 = K_4/Z_0^2 - 3K_3Z_1/Z_0^3 - 2K_2Z_2/Z_0^3 + 19K_2Z_1^2/(4Z_0^4) .}$$

Field-dependent kinetic geometry therefore feeds the cubic and quartic vertices even when the vacuum curvature is known. At leading constant-residue declared order ($Z_1 = Z_2 = 0$),

$$\mathbf{m^2_{curv} = K_2/Z_0 , g_3 = K_3/Z_0^{3/2} , g_4 = K_4/Z_0^2 ,}$$

and in the isolated-quartic special case these reduce, in canonical variables $h = M_\star\sqrt{Z_H}\,\chi$ and $v = M_\star\sqrt{Z_H}\,u_0$, to

$$\mathbf{m_c^2 = 2\,\lambda_{sub}\,M_\star^2\,u_0^2 / Z_H\;,\; g_3 = 6\,\lambda_{sub}\,M_\star\,u_0 / Z_H^{3/2}\;,\; g_4 = 6\,\lambda_{sub} / Z_H^2\;,}$$

with doublet map

$$\mathbf{\lambda_H = \lambda_{sub} / Z_H^2\;,\; \mu_H^2 = \lambda_{sub}\,M_\star^2\,u_0^2 / Z_H\;,\; m_c^2 = 2\mu_H^2 = 2\lambda_H\,v^2\;,}$$

exactly, in the quartic truncation. These reproduce the Section 8–9 relations and expose the Z_H bookkeeping explicitly.

7.3 Two readings of the 32/63 theorem

The finite closure claim $m_c/v = 32/63$ requires

$$\sqrt{2\lambda_{sub}} / Z_H = 32/63\;,$$

so the deformation argument by itself fixes only the combination

$$\mathbf{\lambda_{sub} / Z_H^2 = 512/3969 = \lambda_H\;.}$$

Two readings of the upstream theorem must therefore be kept apart.

Reading A — canonical claim. If the SU(8) finite-deformation theorem is a statement about the *canonically normalised* curvature ratio, it directly asserts $\lambda_H = 512/3969$, and no separate knowledge of Z_H is needed for the ratio m_c/v. The residue question then concentrates entirely in the absolute normalisation $v = M_\star\sqrt{Z_H}\,u_0$ — precisely where the scale-locking route of Section 6 operates, since the interface readout is defined on the canonical carrier (P5).

Reading B — closure-coordinate claim. If the theorem fixes only the closure-coordinate quartic λ_{sub}, the physical ratio $32/(63Z_H)$ is undetermined until Z_H is evaluated, and the residue premises carry the full weight of the numerical result.

This paper adopts Reading A, which is the natural companion of P5–P6 and of Lemma 1: on the canonical carrier the two normalisation questions collapse into one. The upstream deformation paper should state Reading A explicitly; if it can only defend Reading B, falsifier F2 sharpens accordingly.

---

8. Quadratic Coefficient and Complete Scalar Potential

The nonzero stationary condition is

$\partial V\_CI/\partial(H^\dagger H) |\_\{H^\dagger H = v\_cl^2/2\} = - \mu\_H^2 + 2\lambda \cdot v\_cl^2/2 = 0$.

Therefore

$\boldsymbol{\mu\_H^2 = \lambda\, v\_cl^2}$ **.**

Substituting the derived values,

$\boldsymbol{\mu\_H^2 = 7\,662.689132\ \mathrm{GeV}^2}$ **.**

The positive mass scale associated with the quadratic coefficient is

$\boldsymbol{\mu\_H = 87.5367873\ \mathrm{GeV}}$ **.**

The exact ratio is

$\mu\_H/v\_cl = \sqrt{\lambda} = 16\sqrt{2}/63$.

8.1 Complete dimension-four action

The evaluated scalar action is

$\boldsymbol{\Gamma\_scalar\hat{}HR1 = \int d^4x\, [\, (D\_\mu H)^\dagger(D\hat{}\mu H) + (7\,662.689132\ \mathrm{GeV}^2)\, H^\dagger H - 0.128999748047\, (H^\dagger H)^2 + \cdots\,]}$

in the Lagrangian sign convention $\mathscr{L} = \mathscr{L}\_kin - V$.

Equivalently, the potential is

$\boldsymbol{V\_CI(H) = - (7\,662.689132\ \mathrm{GeV}^2)\, H^\dagger H + 0.128999748047\, (H^\dagger H)^2}$ **.**

Adding an irrelevant constant, it can be written

$\boldsymbol{V\_CI(H) = (512/3969)\, (\, H^\dagger H - (243.722807638\ \mathrm{GeV})^2/2\,)^2}$ **.**

The unique dimension-four canonical potential reconstructed from the frozen scale lock and finite-curvature ratio is fully numerical. Its identification with the complete relaxed microscopic potential is conditional on the declared remainder-inertness ($\Delta U'' = \Delta U''' = \Delta U'''' = 0$ at the stated accuracy) and constant-residue ($Z_1 = Z_2 = 0$) premises of Theorems 4–5.

---

9. Vacuum Stability and Scalar Self-Interactions

9.1 Symmetric-point instability

At $H = 0$, the quadratic curvature is negative:

$\partial V\_CI/\partial(H^\dagger H)\,|_0 = -\,\mu\_H^2 < 0$.

Thus the symmetric point is unstable.

9.2 Quartic boundedness

Because

$\lambda = 512/3969 > 0$,

the potential is bounded below within the dimension-four truncation.

9.3 Nonzero minimum

The vacuum orbit satisfies

$H^\dagger H = v\_cl^2/2$.

A representative is

$H\_vac = (1/\sqrt{2})\,(0, v\_cl)^T$.

9.4 Radial curvature

The physical radial curvature at the minimum is

$m^2\_H{,}curv = 2\,\lambda\,v\_cl^2$.

Hence

$m\_H{,}curv = (32/63)\,v\_cl$.

Numerically,

$m\_H{,}curv = 123.795711816$ GeV .

9.5 Cubic and quartic radial vertices

Writing

$H = (1/\sqrt{2})\,(0, v_{cl} + h)^T$,

the potential has the expansion

$V = ½\, m_H^2\, h^2 + (g_{hhh}/3!)\, h^3 + (g_{hhhh}/4!)\, h^4$.

At the declared order,

$g_{hhh} = 6\,\lambda\, v_{cl} = 3\, m_H^2/v_{cl}$,

$g_{hhhh} = 6\,\lambda = 3\, m_H^2/v_{cl}^2$.

Numerically,

g_{hhh} = 188.641085 GeV ,

g_{hhhh} = 0.773998488 .

These self-couplings are downstream predictions with no residual freedom: any future VERSF or experimental determination of the Higgs self-interaction tests the same two numbers, v_{cl} and λ, that fix the mass. Their grade, per Section 1.2, is *conditional numerical*: by Theorems 4–5 they are the constant-residue, remainder-inert values, so a non-negligible $\Delta U'''$ or $\Delta U''''$, or a field-dependent kinetic slope Z_1, would shift g_3 and g_4 through the exact corrections of Theorem 5 (falsifier F20).

9.6 Numerical canonical vertices and mixed radial–Goldstone variations

In canonical Taylor form $V(h) = (K_2^{can}/2!)h^2 + (K_3^{can}/3!)h^3 + (K_4^{can}/4!)h^4$, the evaluated dimension-four values are

$K_2^{can} = 2\lambda\, v_{cl}^2$ = 15 325.3782641 GeV² , $K_3^{can} = g_3$ = 188.641084672 GeV , $K_4^{can} = g_4$ = 0.773998488284 ,

with $K_n^{can} = 0$ for $n \geq 5$ within the quartic truncation. (The superscript distinguishes these canonical-field coefficients from the unnormalised relaxed vertices K_n of Theorem 4; at $Z_H^{eff} = 1$ on the remainder-inert branch they coincide numerically.)

Keeping the Goldstone vector π, the doublet potential $V = \lambda(H^\dagger H - v_{cl}^2/2)^2$ gives

$V(h, \pi) = \lambda\, v_{cl}^2\, h^2 + \lambda\, v_{cl}\, h\, (h^2 + \pi^2) + (\lambda/4)(h^2 + \pi^2)^2$, so the mixed variations at the vacuum are

$$\partial^3V/\partial h\partial\pi_i\partial\pi_j\,|_0 = 2\lambda\, v_{cl}\, \delta_{ij} = 62.8803615574\ \mathrm{GeV} \cdot \delta_{ij}\ ,$$

$$\partial^4V/\partial h^2\partial\pi_i\partial\pi_j\,|_0 = 2\lambda\, \delta_{ij} = 0.257999496095 \cdot \delta_{ij}\ .$$

These mixed vertices feed the Goldstone contributions to the eventual complete pole (Section 11.8) and are supplied here so that no piece of the dimension-four scalar sector remains symbolic.

Per the full-microscopic qualification of Theorem 4, the complete vertices are $K_n^{full} = K_n^{can} + \Delta U^{(n)}(v_{cl})$; the condensation paper supplies the existence and qualitative form of E_{R4} and E_{bath} but treats their coefficients schematically, so the numbers above are the complete *dimension-four* radial variations, not yet the complete microscopic variations of every substrate sector.

Theorem 6 — Local Vacuum-Stability Theorem

The evaluated HR-1 scalar action has

$$Z_H > 0\ ,\ \mu_H^2 > 0\ ,\ \lambda > 0\ ,\ m^2_{H,curv} > 0\ .$$

Therefore the symmetric configuration is unstable and the selected nonzero electroweak orbit is a locally stable radial minimum at the declared matching order.

Global-stability limitation

This theorem does not establish the all-scale global stability of the electroweak vacuum. That requires:

- RG evolution of λ;
- all derived Yukawa and gauge boundary values;
- heavy thresholds;
- higher-dimensional scalar operators;
- competing closure branches;
- a gauge-consistent tunnelling calculation.

The one-loop running diagnostic that will govern that audit is schematically

$$16\pi^2\, \beta_\lambda = 24\lambda^2 - 6y_t^4 + (12y_t^2 - 9g^2 - 3g'^2)\lambda + (9/8)g^4 + (3/4)g^2g'^2 + (3/8)g'^4 + \cdots\ ,$$

which makes explicit why the numerical top Yukawa is the decisive missing input: the $-6y_t^4$ term controls whether $\lambda = 512/3969$ survives transport to high scales.

---

10. Broken-Phase Gauge and Fermion Reconstruction

10.1 Electromagnetic stabiliser

The vacuum representative obeys

$(T_3 + Y)\, H_{vac} = 0$.

Thus the unbroken generator is

$Q = T_3 + Y$,

and the stabiliser is $U(1)_{em}$.

10.2 Gauge-boson masses

The scalar kinetic term gives

$$\mathbf{m_W^2 = \tfrac{1}{4}\, g^2\, v_{cl}^2} \,,$$

$$\mathbf{m_Z^2 = \tfrac{1}{4}\, (g^2 + g'^2)\, v_{cl}^2} \,,$$

and

$$\mathbf{m_\gamma^2 = 0} \,.$$

No numerical W or Z mass is used to establish v_{cl}. Their eventual numerical evaluation must use the independent VERSF gauge-response boundary values and quantum matching.

10.3 Fermion masses

For a canonical Yukawa operator Y_f,

$$\mathbf{M_f = (v_{cl}/\sqrt{2})\, Y_f} \,.$$

In particular, the electron branch gives

$y_e = \sqrt{2}\, \alpha^2/(8\pi)$,

so

$(v_{cl}/\sqrt{2})\, y_e = (\alpha^2/8\pi)\, v_{cl} = m_e^{sub}$.

This equality is the scale-locking condition used in Section 6, now read in the opposite direction: the derivation is internally self-consistent by construction on the electron branch, so the genuine tests lie in every *other* channel the same vacuum must serve.

At the declared leading closure-Hessian order the Yukawa operators have the exact address

$$Y_f^c = \mathscr{E}_{fL}^\dagger P_Y K_{cl}^{-1/2} H_{cl} K_{cl}^{-1/2} P_Y \mathscr{E}_{fR} ,$$

so the top Yukawa required by the pole calculation of Section 11.8 is

$$\mathbf{y_t(\mu\star) = \sigma_{max}[\mathscr{E}_{uL}^\dagger P_Y K_{cl}^{-1/2} H_{cl} K_{cl}^{-1/2} P_Y \mathscr{E}_{uR}] .}$$

This is an exact input address, graded *open numerical*: H_{cl}, P_Y and the chiral embeddings have not been numerically evaluated by the present corpus.

Cross-sector closure statement

One derived scalar vacuum must simultaneously serve:

- the Higgs radial potential;
- the electroweak gauge mass matrix;
- every closure-derived Yukawa operator.

Any sector requiring a different vacuum falsifies common electroweak completion.

10.4 Conditional Electroweak Coupling and Mass-Branch Reconstruction

This subsection does not evaluate the gauge-response Hessian. A direct gauge-response calculation requires the holonomy residual map, record metric, curvature embedding and auxiliary Hessian,

$$K_g = J_F^\dagger G_{rec} (1 - \Pi_\eta) J_F ,$$

whose numerical evaluation the gauge theorem explicitly leaves open. What follows is instead an algebraic *conditional reconstruction* — the outputs are classified as $(g, g', m_W, m_Z)_\pm$ — obtained by combining three frozen VERSF relations at one common matching point:

$$e^2 = 4\pi\alpha , \quad m_H^2 = (15/14)(m_W^2 + m_Z^2) ,$$

together with the broken-phase relations of Section 10.2. The 15/14 relation is the leading-saturation gauge-mass relation of the closure-condensation paper, conditional on its substrate-isotropy premises. Structurally it is the same factor $(N_{loop} + 1)/N_{loop} = 15/14$ that appears in the geometric coupling formula of Input II — an internal echo the programme should eventually derive rather than observe.

Scale declaration. The relation $e^2 = 4\pi\alpha$ is imposed here with the frozen first-order geometric coupling at the same low-current matching point as the electron lock of Section 6. In any realised theory the Abelian coupling runs substantially between that point and the electroweak scale — in the Standard Model roughly from 1/137 to about 1/128 at m_Z, dominated by calculable lepton and hadron vacuum polarisation. This is not cosmetic: re-running the quadratic below with a coupling near 1/128 moves s_- from 0.2166 to ≈ 0.235, so the reconstruction's residual deviation from the observed mixing angle is plausibly dominated by this presently uncomputed running rather than by the 15/14 relation or the branch selection of Section 10.5. The interface-to-electroweak running of the Abelian coupling is therefore entered in the Section 16.2 ledger as its own item, parallel to Δ_H. It is a *different axis* from the coupling-order branch question of Section 12 — one concerns which frozen geometric value is correct at the interface, the other concerns transport of that value across ~five decades of scale — and the two must not be conflated.

With $s \equiv \sin^2\theta_W$, $g = e/\sqrt{s}$, $g' = e/\sqrt{(1-s)}$, and $m_H/v_{cl} = 32/63$, the combined relations give

$$(32/63)^2 = (15e^2/56)\left(2/s + 1/(1-s) \right) .$$

Defining

$$A = 56(32/63)^2/(15e^2) = 10.5118451504 ,$$

s satisfies the quadratic

$$\mathbf{A s^2 - (A + 1) s + 2 = 0 , \quad s_\pm = [(A+1) \pm \sqrt{((A+1)^2 - 8A)}] / (2A) ,}$$

which has two mathematically admissible branches:

Quantity	Lower branch	Upper branch
$\sin^2\theta_W$	0.2165573823	0.8785733950
g	0.6504768409	0.3229455552
g'	0.3419910489	0.8686831733
$Z_2 = 1/g^2$	2.3633950533	9.5882947665
$Z_Y = 1/g'^2$	8.5500867587	1.3251870455
$Z_q = 36\, Z_Y$ (hypercharge convention $q = 6Y$)	307.803123314	47.7067336371
$m_W = \frac{1}{2}\, g\, v_{cl}$	79.26802099 GeV	39.35459872 GeV
$m_Z = \frac{1}{2}\, v_{cl} \sqrt{(g^2+g'^2)}$	89.55594468 GeV	112.93760197 GeV

Both branches satisfy the combined equations exactly. The lower branch has $g > g'$ and $\sin^2\theta_W < \frac{1}{2}$; the upper branch has the reverse ordering. The direct substrate-response theorem does not yet derive a rule selecting between them, and the selection may not be made by comparing the m_W and m_Z columns with observation. A *conditional* selection theorem is supplied in Section 10.5, and its unconditional confirmation is reduced there to one clean response inequality (falsifier F22).

These numbers are *conditional coupling and mass reconstructions*, graded per Section 1.2 as conditional numerical results. The coefficients $Z_2 = 1/g^2$ and $Z_Y = 1/g'^2$ are reconstructed inverse-coupling coefficients; they are not yet demonstrated to equal the microscopic VERSF response coefficients bearing the same names, and the factor 36 in $Z_q = 36Z_Y$ depends on the hypercharge convention $q = 6Y$. None of this is yet a direct evaluation of the holonomy-response trace

$$(1/6d_A)\ \mathrm{Tr}[\ P_A\ (\ H_{FF} - H_{F\eta}\ H_{\eta\eta}^{-1}\ H_{\eta F}\ )\ P_A\ ]\ .$$

10.5 Persistent-Abelian branch selection: the Minimal-Lift Theorem

The two branches of Section 10.4 are algebraically symmetric; physically they are not, because the two neutral directions have differently typed substrate roles. Three inherited results make this precise.

Lemma 2 — Unique Abelian anchor. The refinement-persistent observable sector carries a Maxwell-form U(1) connection, and the electroweak algebra $\mathfrak{g}_{EW} = \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$ contains exactly one Abelian factor. Under the minimal-gauge-census condition (no hidden U(1)), the persistent Abelian carrier embeds as $\mathfrak{u}(1)_Y$, up to generator scale and sign; the primitive charge lattice $q = 6Y$ with unit greatest common divisor removes the continuous scaling, and the residual simultaneous sign flip $(q, g_q) \mapsto (-q, -g_q)$ has no physical effect. The canonically normalised hypercharge field B_μ is therefore the unique pre-completion persistent Abelian anchor: $L_{pers} = \mathrm{span}\{\hat{B}_\mu\}$.

Lemma 3 — The weak direction is differently typed. By the weak-orientation theorem, $SU(2)_L$ transport acts on the unresolved two-branch commitment carrier; it is not the persistent Abelian ledger carrier. In the neutral plane, $\hat{B}_\mu$ is persistent Abelian source transport while $\hat{W}^3_\mu$ is the neutral component of unresolved weak-branch transport.

Lemma 4 — Orthogonal neutral carrier. The gauge-action theorem forbids same-order kinetic mixing between the simple weak factor and the unique Abelian factor, so after canonical normalisation

$$N_{EW} = \mathrm{span}\{\hat{B}_\mu, \hat{W}^3_\mu\}\ ,\ \langle\hat{B}, \hat{W}^3\rangle_{op} = 0\ ,\ \|\hat{B}\|_{op} = \|\hat{W}^3\|_{op} = 1\ ,$$

in the Hilbert inner product supplied by the operational-geometry papers.

The persistent-displacement functional

With $s \equiv \sin^2\theta_W$, the canonically normalised massless neutral candidate is

$$\hat{A}(s) = \sqrt{1-s}\ \hat{B}_\mu + \sqrt{s}\ \hat{W}^3_\mu\ .$$

Let $P\_pers = |\hat{B}\rangle\langle\hat{B}|$ project onto the persistent Abelian line, and define the nonpersistent displacement $C\_np[\hat{A}] = \|(1 - P\_pers)\hat{A}\|^2\_op$. Then

$$\mathbf{C\_np(s) = s\ ,\ R\_pers(s) = \|P\_pers\ \hat{A}(s)\|^2\_op = 1 - s\ ,}$$

and the projective operational angle from the inherited anchor is

$$\mathbf{d\_op([\hat{B}], [\hat{A}(s)]) = \arccos\sqrt{(1-s)} = \arcsin\sqrt{s}\ ,}$$

strictly increasing on $0 < s < 1$. The abstract selection principle thereby becomes a one-line comparison of two numbers.

Theorem 7 — Persistent-Abelian Electroweak Branch Selection

Assume the orthonormal neutral carrier of Lemma 4, the unique persistent anchor of Lemma 2, the two algebraic roots s_- and s_+ of Section 10.4, and premise P11. Then the unique physical branch is

$$\mathbf{\sin^2\theta\_W = s_- = 0.2165573823\ .}$$

Proof

$C\_np(s) = s$ and $s_- < s_+$, so $C\_np(s_-) < C\_np(s_+)$; equivalently d_op is strictly increasing in s, giving

$\theta_- = \arcsin\sqrt{s_-} = 27.7333° < \theta_+ = \arcsin\sqrt{s_+} = 69.6067°$.

The lower branch is the unique minimum-displacement completion. ■

Robustness and the true location of P11's content

The theorem is stronger than its statement suggests: since $C\_np(s) = s$ exactly, *any* displacement functional strictly monotone in s selects s_-. The result therefore does not depend on the specific operational metric at all — only on Lemmas 2–4 and the direction "minimise displacement from $\hat{B}$". Conversely, the entire physical content of P11 is the *asymmetry* of that direction: why minimise displacement from the persistent carrier rather than, say, maximise the weak carrier's absorption into the massive sector. Lemma 3 — the typing of $\hat{W}^3$ as unresolved-branch transport, categorically unlike persistent ledger transport — is what licenses the asymmetry, and the sceptical reader should interrogate Lemma 3 directly rather than the metric.

Numerical consequences

The selected branch fixes

$g = 0.6504768409$, $g' = 0.3419910489$, $m\_W = 79.26802099$ GeV , $m\_Z = 89.55594468$ GeV ,

and the photon composition

$\hat{A}\_\mu = 0.88512294\, \hat{B}\_\mu + 0.46535726\, \hat{W}^3\_\mu$,

with 78.3443% of its support on the persistent Abelian carrier and 21.6557% on the weak carrier. The rejected upper branch would carry only 12.1427% persistent support against 87.8573% weak support — a photon built mostly from the unresolved weak carrier, which is exactly what P11 forbids.

Equivalent microscopic criterion

Since $g = Z_2\^\{-1/2\}$ and $g' = Z\_Y\^\{-1/2\}$,

$s = g'^2/(g^2 + g'^2) = Z_2/(Z\_Y + Z_2)$,

so the following are exactly equivalent:

$s < ½ \Leftrightarrow g > g' \Leftrightarrow Z\_Y > Z_2 \Leftrightarrow \|P\_pers\ \hat{A}\|^2 > \|(1-P\_pers)\hat{A}\|^2 \Leftrightarrow d\_op([\hat{B}],[\hat{A}]) < \pi/4$.

Because the two roots straddle ½ ($s_- < ½ < s_+$), a direct microscopic result

$Z\_Y > Z_2$

would select the lower branch with no appeal to the minimal-lift principle at all. That single inequality is the eventual unconditional test, and it is what a genuine evaluation of the holonomy-response trace must return.

Honest status

Derived: an exact branch-selection theorem, conditional on P11, together with the reduction of unconditional selection to one response inequality. Not derived: the general principle that completion must minimise operational displacement among candidate physical laws. The operational metric exists, the anchor is structurally identified and both distances are calculable — but broad admissibility alone does not force all reversible completions to choose the shortest lift, and the admissibility framework itself states that coupling values require additional dynamical selection. P11 is therefore graded as a conditional admissibility premise with its own falsifier, exactly like the bridge premises of the scalar sector.

10.6 Conditional top-Yukawa grid target

The exact microscopic top compression (Section 10.3) cannot yet be performed because H_cl, P_Y and the chiral embeddings are unevaluated. The current-mass projection construction nonetheless supplies a conditional numerical *target*. On its frozen premises,

$m_d^{(0)} = (9\alpha^2/8\pi)\, v_{cl} = 4.64036813821$ MeV , $y_d^{(0)} = (\sqrt{2}/v_{cl})\, m_d^{(0)} = 9\sqrt{2}\, \alpha^2/(8\pi) = 2.69259640452 \times 10^{-5}$,

and the one-anchor quark grid gives the top/down ratio

$R_t = (6/13) \cdot 5e^{16/3} \cdot (77/3)^{3/2} = 62\,154.0374183$.

Hence the conditional top singular-value target is

$y_t^{grid} = R_t\, y_d^{(0)} = 1.67355737679$,

with completion-boundary mass

$m_t^{grid} = (v_{cl}/\sqrt{2})\, y_t^{grid} = 288.417614897$ GeV .

The quark projection and grid papers themselves record that the interface coupling, scale choice, running map and several hierarchy factors remain conditional; this is a completion-boundary quantity, **not a pole-mass prediction**. Its correct use is as a future compression test:

$\sigma_{max}[\, \mathscr{E}_{uL}^\dagger\, P_Y\, K_{cl}^{-1/2}\, H_{cl}\, K_{cl}^{-1/2}\, P_Y\, \mathscr{E}_{uR} \,] \overset{?}{\doteq} 1.67355737679$.

If the independently calculated operator returns another value, the grid — not the matrix calculation — must yield (falsifier F23).

Note the anchor structure: $m_d^{(0)} = 9\alpha^2 v_{cl}/(8\pi) = 9\, m_e^{CI}$ exactly, by construction — the grid's down anchor is nine electron interface units, which makes the grid's dependence on the electron channel fully visible.

Two consequences of this target must be stated plainly rather than left implicit. First, the observed top pole mass is near 172.6 GeV; since 172.6/288.4 = 0.598, the open running and hierarchy bridge must compress the boundary value to ≈ 60% of itself — a ≈ 40% reduction — which is large but not excluded for a boundary-to-pole map spanning many decades of scale, so the bridge calculation is decisive, not cosmetic. Equivalently, since $v_{cl}/\sqrt{2} = 172.34$ GeV, the compression test asks whether the effective top Yukawa runs from the grid value 1.674 to ≈ 1.00 at the pole-matching point. Second, the target feeds into the stability diagnostic of Section 9, under an explicit conditionality: *if* y_t^{grid} is the canonically normalised Standard Model Yukawa at the same matching point at which $\lambda = 512/3969$ is imposed, and no threshold contribution modifies the one-loop beta function, *then* the term $-6y_t^4 \approx -47.1$ in $16\pi^2\beta_\lambda$ overwhelms $24\lambda^2 \approx 0.40$ by two orders of magnitude and the quartic would be driven negative almost immediately above the matching point. The two quantities are differently typed boundary objects — a completion-boundary grid target and a matching-scale quartic — so the comparison is a conditional diagnostic, not yet a demonstrated instability. Either the running bridge substantially reduces the effective Yukawa before it enters β_λ, or the scheme identification fails, or the vacuum-stability audit fails. This remains the sharpest internal tension the numerical stack has produced, and it is recorded as such rather than smoothed over.

---

11. Nielsen Consistency and Physical Pole Definition

11.1 Effective-action identity

For gauge parameter ξ_g, the renormalised effective action obeys

$$\partial\Gamma/\partial\xi_g + \int d^4x\, C^I[\varphi, \xi_g] \cdot \delta\Gamma/\delta\varphi^I = 0 .$$

For a homogeneous scalar background,

$$\partial V_{eff}/\partial\xi_g + C(\varphi, \xi_g) \cdot \partial V_{eff}/\partial\varphi = 0 .$$

The off-shell potential is therefore gauge dependent in general. At a stationary configuration, its value is gauge independent order by order when the Slavnov–Taylor identity is restored.

11.2 Pole equation

Let $\mathbf{\Gamma}^{(2)}_{broken}(p^2)$ be the complete two-point matrix including Higgs, Goldstone and any admissible mixing sectors. The physical pole satisfies

$$\det \mathbf{\Gamma}^{(2)}_{broken}(s_H) = 0 .$$

For an unstable resonance,

$$s_H = M_H^2 - i\, M_H\, \Gamma_H .$$

11.3 Nielsen pole theorem

The two-point Nielsen identity has the form

$$\partial\mathbf{\Gamma}^{(2)}/\partial\xi_g = \mathbf{\Lambda}\, \mathbf{\Gamma}^{(2)} + \mathbf{\Gamma}^{(2)}\, \tilde{\mathbf{\Lambda}} .$$

At an isolated simple zero of the determinant, regularity of the Nielsen kernels implies

$$\partial s_H/\partial\xi_g = 0 .$$

Thus the complex pole is gauge-parameter independent even though individual self-energies are not.

11.4 HR-1 matching-order pole

Under P7 and P8,

$\mathbf{\Gamma}^{(2)}$_radial(p^2) = $p^2 - 2\lambda\, v\_cl^2 + O(p^4/M\star^2, \hbar)$.

The leading pole is therefore

$s\_H^{(0)} = 2\lambda\, v\_cl^2$.

Hence

$M\_H^{(0)}$ = 123.795711816 GeV .

The numerical effective radial two-point kernel is, in Euclidean signature,

$\Gamma^{(2)}$_hh(p_E^2) = p_E^2 + 15 325.3782641 GeV² + O(p_E^4) ,

and in Minkowski signature $\Gamma^{(2)}$_hh(p^2) = p^2 − 15 325.3782641 GeV² + O(p^4). Before gauge fixing, the full scalar-doublet Hessian in the basis (h, π_1, π_2, π_3) is

$\Gamma^{(2)}$_scalar(p_E^2) = diag(p_E^2 + 15 325.3782641 , p_E^2 , p_E^2 , p_E^2) [GeV²] .

The three zero-curvature modes are the Goldstone directions; their gauge-fixed entries depend on the chosen R_ξ gauge and enter the complete pole system of Section 11.8 through the mixed vertices of Section 9.6.

The full pole may be written

$s\_H = s\_H^{(0)} + \Delta\_H$^loop + Δ_H^threshold + Δ_H^higher .

Every term in Δ_H must be calculated from independently derived VERSF gauge, Yukawa, scalar and threshold data, and published before empirical comparison. No term may be adjusted to force the pole to a measured value.

11.5 Scalar-only one-loop self-energy

The first entry of Δ_H is calculable now, because it needs only the scalar sector this paper has already evaluated. Use the canonically normalised one-field radial theory

$V(h) = \frac{1}{2}\, m\_c^2\, h^2 + \lambda\, v\_cl\, h^3 + (\lambda/4)\, h^4$, so that

$g_3 = 6\,\lambda\, v\_cl$, $m\_c^2 = 2\,\lambda\, v\_cl^2$, $g_3^2 = 18\,\lambda\, m\_c^2$.

The only momentum-dependent one-loop radial diagram is the two-cubic bubble. With the full inverse propagator written as $D^{-1}(p^2) = p^2 - m_0^2 - \Pi(p^2)$, its momentum-dependent part is

$\Pi^{(1)}(p^2) - \Pi^{(1)}(0) = (g_3^2/32\pi^2)\,[\,B_0(p^2; m\_c^2, m\_c^2) - B_0(0; m\_c^2, m\_c^2)\,] = (9\lambda\, m\_c^2/16\pi^2)\,[\,B_0(p^2) - B_0(0)\,]$.

The quartic tadpole is momentum independent and is absorbed into the curvature matching.

11.6 Zero-momentum residue matching and the scalar-only pole

Because the microscopic scalar programme defines both the curvature and the residue at $p^2 = 0$ (Section 4.4), the appropriate finite nonlocal self-energy is

$\bar{\Pi}(p^2) = \Pi(p^2) - \Pi(0) - p^2\,\Pi'(0)$.

For equal internal masses (Appendix D),

$B_0(m\_c^2) - B_0(0) = 2 - \pi/\sqrt{3}$, $m\_c^2\, B_0'(0) = 1/6$.

Therefore

$\bar{\Pi}(m\_c^2) = (9\lambda\, m\_c^2/16\pi^2)\,(\,11/6 - \pi/\sqrt{3}\,)$,

and the scalar-only pole is

$M^2\_pole,scalar = m\_c^2\,[\,1 + (9\lambda/16\pi^2)(11/6 - \pi/\sqrt{3})\,] + O(\lambda^2)$.

Because $m\_c^2 < 4m\_c^2$, the one-radial-field bubble has no on-shell cut at this order:

$\Gamma^{(1)}\_scalar\text{-}only = 0$.

The physical width comes from gauge and fermion channels absent from this reduced one-field calculation.

The numerical smallness of the bracket, $11/6 - \pi/\sqrt{3} \approx 0.01953$, is not tuning: the near-cancellation between the threshold constant $2 - \pi/\sqrt{3}$ and the slope subtraction $1/6$ reflects how little the bubble curves between $p^2 = 0$ and $p^2 = m\_c^2$ when the pole sits far below the $4m\_c^2$ branch point. The scalar self-correction is therefore intrinsically at the 10^{-4} level in m^2, and no scheme choice within this class can inflate it.

11.7 Evaluated scalar-loop pole, residue and convention control

For the derived values $\lambda = 512/3969$ and $m_c = 123.795711816$ GeV,

$(M^2_{pole,scalar} - m_c^2)/m_c^2 = 1.436157729 \times 10^{-4}$,

so

$M_{pole,scalar} = 123.804601005$ GeV , $M_{pole,scalar} - m_c = +8.889$ MeV .

The direct scalar-loop correction to the zero-momentum inverse-propagator slope is

$\Pi'(0) = 3\lambda/32\pi^2 = 1.225350671 \times 10^{-3}$,

so $Z_{eff}(0) = Z_{micro} - 3\lambda/32\pi^2 + O(\lambda^2)$. This loop term is not the missing microscopic substrate residue; it is the quantum correction applied *after* Z_{micro} has been supplied. Confusing the two would repeat, at one loop, exactly the conflation the tree-level firewalls forbid.

After zero-momentum canonical matching, the pole residue is $R_H = [1 - \Pi'(M^2_{pole})]^{-1}$ with

$\Pi'(m_c^2) = (9\lambda/16\pi^2) \left(2\pi/(3\sqrt{3}) - 7/6 \right)$, $R_H = 1.000312804191$.

For reference, if the field is *not* recanonicalised at zero momentum and only the curvature subtraction is made, the scalar-only pole is 123.880418923 GeV — a shift of 84.707 MeV. That number uses a different field-normalisation convention and must not be mixed with a programme that defines Z_H at $p^2 = 0$. The 76 MeV spread between the conventions is an order of magnitude larger than the physical scalar-loop shift, so convention discipline is quantitatively mandatory, not stylistic.

11.8 What cannot be evaluated without new VERSF input

A full Nielsen-consistent Standard Model Higgs pole additionally requires, at one loop: the numerical weak and hypercharge couplings at one declared scale; the numerical top and other relevant Yukawa operators at that scale; the Goldstone and ghost sectors in one declared gauge; the complete scalar and gauge two-point mixing matrix; the heavy substrate threshold spectrum; and the regulator and finite matching prescription.

The Gauge Action and Coupling Theorem explicitly leaves the numerical gauge response coefficients open, and the Yukawa and charged-lepton gates leave the running-to-pole bridge open. Therefore

Δ_H^{gauge} , Δ_H^{top} , $\Delta_H^{Goldstone/ghost}$, $\Delta_H^{threshold}$

cannot presently be calculated without importing Standard Model data. For orientation only: in the Standard Model the top-quark contribution parametrically dominates such shifts, at a level far above the 9 MeV scalar term. The blind programme must therefore expect Δ_H to dominate the final pole displacement, and the leading value below is quoted with that expectation attached rather than presented as loop-stable.

Theorem 8 — Nielsen-Consistent Higgs Output

The HR-1 mass output is the isolated pole of the declared-order broken-phase radial propagator, not the curvature of an arbitrary gauge-fixed off-shell plot. At leading finite-closure order,

$m_H^{HR1} = 123.796$ GeV ,

and including the exactly evaluated scalar-only one-loop displacement,

$M_{pole,scalar} = 123.805$ GeV .

The gauge, Yukawa and threshold displacements are separate calculable objects, defined in Section 11.8 and awaiting their own boundary data.

---

12. Route and Sensitivity Audit

The principal result uses the upstream Route-A value

$\xi_A = 88$ μm .

Because

$v_{cl} \propto \xi^{-1}$, $m_H \propto \xi^{-1}$, $\mu_H \propto \xi^{-1}$, $\mu_H^2 \propto \xi^{-2}$,

alternative coherence branches give different absolute outputs while leaving λ and m_H/v unchanged.

Coherence scale	v_{cl}	$m_H^{(0)}$	μ_H
60 μm	357.460 GeV	181.567 GeV	128.387 GeV
70 μm	306.394 GeV	155.629 GeV	110.046 GeV
82 μm	261.556 GeV	132.854 GeV	93.942 GeV
88 μm, Route A	**243.723 GeV**	**123.796 GeV**	**87.537 GeV**

Coherence scale	v_{cl}	$m_H^{(0)}$	μ_H
110 μm	194.978 GeV	99.037 GeV	70.029 GeV
88 μm, second-order coupling candidate (α^{-1} = 137.034014)	242.565 GeV	123.207 GeV	87.121 GeV

The last row varies the coupling rather than the coherence branch, exploiting $v_{cl} \propto \alpha^{-6}$: a shift of the geometric coupling from its first-order value 960/7 = 137.142857 to a second-order candidate 137.034014 — a 0.08% change — moves the vacuum by 1.16 GeV and the curvature mass by 0.59 GeV, *away* from the observed resonance. The displayed decimal precision of the primary calculation must therefore not be mistaken for upstream branch precision.

This table is not used to select Route A. It shows exactly how strongly the scalar result depends on the upstream scale branch — and, by the same token, it separates the two grades of prediction in this paper: the dimensionless pair (λ, m_H/v) = (512/3969, 32/63) is branch-independent and stands or falls with the SU(8) calculation alone, while the absolute quadruple (v_{cl}, μ_H, m_H, μ_H^2) inherits the full conditionality of the Route-A branch.

12.1 Differential sensitivity

For

$$v_{cl} = (C_e/P_\ell)\,(\hbar c/\xi)\,\alpha^{-6}\,,$$

where

$$C_e = 13/20\,,\quad P_\ell = 1/(8\pi)\,,$$

the logarithmic sensitivity is

$$\boldsymbol{\delta v_{cl}/v_{cl} = -\,\delta\xi/\xi - 6\,\delta\alpha/\alpha + \delta C_e/C_e - \delta P_\ell/P_\ell\,.}$$

Therefore:

- a one-percent shift in ξ shifts v_{cl} and m_H by one percent;
- a one-percent shift in α shifts them by six percent;
- a one-percent shift in the projection kernel shifts them by one percent in the opposite direction.

This makes the next upstream precision targets explicit, and it ranks them: the α^{-6} amplification makes the exactness of the geometric coupling formula the single most leveraged assumption in the stack.

The coupling-order question also has a cross-observable discriminant, developed in Section 12.2: the second-order candidate moves the scalar outputs *away* from the observed resonance but moves the substrate electron mass *toward* its measured value. The two channels currently disagree about the preferred coupling order — itself a structural datum.

12.2 Post-output empirical orientation

Everything in this subsection sits strictly *downstream* of the non-insertion firewall: all outputs above were frozen before these comparisons were made, and nothing here may flow back upstream (audit items 1–12 of Section 13). The comparisons are recorded because omitting them would be selective — the electron channel is the stack's most directly testable output — and because their *structure* is diagnostic in a way single-number comparisons are not.

Electron-channel decomposition. Against the measured values ($m_e = 0.51099895$ MeV, $v_{obs} \approx 246.22$ GeV, G_F-derived):

- Substrate route: $m_e^{sub}/m_e^{obs} = 0.515596/0.510999 = 1.0090$ — a **+0.90%** excess.
- Interface kernel: $\alpha^2/8\pi = 2.1155 \times 10^{-6}$ against the observed ratio $m_e^{obs}/v_{obs} = 2.0754 \times 10^{-6}$ — a **+1.93%** excess.
- The scale lock, which divides the first by the second, therefore lands v_{cl} at **−1.0%** of v_{obs}.

The sign structure matters. Inserting the *measured* electron mass into the interface formula would give $v = 241.5$ GeV — further from 246, not closer. The ~1% deficits in v_{cl} and m_H are therefore not a single inherited miss from the electron channel: the substrate route and the interface kernel miss in ways that partially cancel. That is a structural fingerprint, not noise, and it points the precision effort at two named objects — the Route-A electron construction (F3) and the $1/(8\pi)$ kernel (Section 13 provenance list) — rather than diffusely at "the stack".

The strongest number. The branch separation of Section 12 buys a sharp empirical statement. The branch-independent dimensionless ratio,

$$32/63 = 0.507937 ,$$

stands against an observed pole-over-G_F ratio of ≈ 0.5081–0.5085 (depending on the Higgs-mass determination; the comparison also carries a scheme caveat, tree-matched ratio versus pole/G_F definitions). Agreement is at the few-$\times 10^{-4}$ to 10^{-3} level — an order of magnitude closer than the ~1% absolute misses. The SU(8) finite-curvature theorem, the one ingredient owing nothing to Route-A, the coherence branch or the interface kernel, is thus the component sitting closest on top of experiment, and the ~1% absolute discrepancy localises entirely in the ξ_A/α^{-6} scale sector. Where the programme stands is therefore precise: the group theory currently performs at the 10^{-3} level or better; the scale-setting sector at the 10^{-2} level.

Coupling-order discriminant. The second-order coupling candidate ($\alpha^{-1} = 137.034014$) moves m_H to 123.21 GeV — away from the observed resonance — but moves m_e^{sub} to 0.51396 MeV, a +0.58% excess — *toward* the measured electron mass. The scalar and electron channels disagree about the preferred coupling order. Since both descend from the same α, this disagreement is itself a datum: it suggests one resolution may lie not in switching branch wholesale but in the scale-dependence axis declared in Section 10.4, where the interface value and the electroweak-scale value need not coincide.

None of the above modifies any output, selects any branch, or licenses any repair. It orients.

---

13. Numerical Non-Insertion Audit

The following values were not used upstream:

v_{obs} , $m_{H,obs}$, m_W^{obs} , m_Z^{obs} , G_F , $\lambda_{inferred}^{obs}$.

The upstream numerical stack is:

$K = 7$,

$N_{loop} = 14$,

$\alpha = 7/960$,

$\xi_A = 88\ \mu m$,

$C_e = 13/20$,

$P_\ell = 1/(8\pi)$,

$m_H/v_{cl} = 32/63$,

and, for the conditional reconstructions of Sections 10.4–10.5 only:

$m_H^2 = (15/14)(m_W^2 + m_Z^2)$, $m_d^{(0)} = (9\alpha^2/8\pi)v_{cl}$, $R_t = (6/13)\cdot 5e^{16/3}\cdot(77/3)^{3/2}$.

No measured electron, W, Z, top or Higgs mass enters any of these; the gauge reconstruction in particular derives m_W and m_Z rather than consuming them.

The order of calculation is:

1. freeze K, N_{loop};
2. calculate α;
3. freeze Route-A ξ_A;
4. calculate m_e^{sub};
5. freeze the colourless interface projection;
6. solve for v_{cl};
7. calculate λ;

8. calculate μ_H^2;
9. calculate the leading Higgs pole;
10. only then compare with observation.

Forbidden repairs

The following would invalidate the result:

1. replacing m_e^sub with the measured electron mass;
2. choosing ξ to reproduce the weak scale;
3. replacing $\alpha = 7/960$ with a measured value upstream;
4. altering $1/(8\pi)$ to improve agreement;
5. changing 32/63 after seeing the Higgs result;
6. adding a finite Higgs counterterm chosen from observation;
7. reporting a gauge-dependent off-shell potential coordinate as the physical mass;
8. hiding the Route-A dependence;
9. dropping higher operators after they are shown to be non-negligible;
10. calling the result unconditional while its named bridge premises remain conditional;
11. selecting the electroweak response branch of Section 10.4 by comparing its m_W, m_Z columns with observation;
12. adjusting the grid hierarchy factors of Section 10.6 to move m_t^grid toward the observed top mass.

What non-insertion does not prove

The absence of direct insertion in this paper does not establish that every upstream number was selected independently of observation. A full provenance audit must still demonstrate that:

- the Route-A branch was selected before weak-scale comparison;
- the level-four exponent and occupancy factor were not chosen from the electron mass;
- the two-gate law and $1/(8\pi)$ kernel were not selected from the charged-lepton scale;
- the 32/63 relation was not selected from the Higgs ratio;
- the coupling order and readout scale were frozen before scalar evaluation.

This paper certifies its own chain; the provenance of the frozen inputs is certified, or not, by their own papers.

Continuous identifiability

Under the frozen premise stack, no continuous parameter remains inside the dimension-four numerical evaluation. The remaining uncertainty is discrete and structural: branch selection, kernel theorems, matching identities, and the missing microscopic matrices. This is a materially stronger position than a fitted model, whose uncertainty is continuous, and it is what makes the falsification ledger finite.

14. Falsification Conditions

F1 — Higgs-axis failure. The closure-normalised scalar carrier is not an electroweak (**2**, ½) doublet.

F2 — Kinetic-pullback failure. The physical retained scalar direction does not possess positive unit canonical residue after interface matching.

F3 — Electron-scale failure. The independent level-four substrate calculation does not yield the declared electron bilinear. The post-output orientation of Section 12.2 records the current +0.90% excess of m_e^{sub} over the measured electron mass; whether that excess lies within the (currently undeclared) upstream branch precision or is a genuine F3 signal is decided by the Route-A provenance and precision audit — not by adjustment.

F4 — Interface-law failure. The colourless electron bilinear is not $\alpha_{int}^2 v_{cl}/(8\pi)$.

F5 — Coupling-match failure. The relevant interface coupling is not the geometric α at the declared readout point.

F6 — Coherence-branch failure. The upstream Route-A construction does not independently select $\xi_A = 88$ μm.

F7 — Quartic-democracy failure. The closure baseline is not $1/(K+1)$.

F8 — SU(8) deformation failure. The finite correction is not $1 + 1/63$.

F9 — Vacuum-instability failure. The reduced scalar quadratic coefficient is not negative in the potential.

F10 — Quartic-stability failure. The quartic is nonpositive or higher operators destabilise the branch below the cutoff.

F11 — Wrong stabiliser. The selected vacuum breaks electromagnetism or colour.

F12 — Competing-vacuum failure. A lower admissible closure branch exists.

F13 — Pole-matching failure. The finite closure curvature cannot be matched to the physical radial pole at the declared order.

F14 — Nielsen failure. The claimed pole changes with gauge parameter after complete order-by-order matching.

F15 — Threshold failure. A heavy substrate field gives an unaccounted shift large enough to invalidate the output.

F16 — Non-insertion failure. Any observed Higgs or weak-scale value enters before the outputs are frozen.

F17 — Cross-sector inconsistency. The same v_{cl} fails to close the gauge and Yukawa sectors.

F18 — Empirical failure. After all derived running and matching corrections are included without fitting, the physical outputs disagree with experiment beyond declared theoretical uncertainty. This falsifier is *inoperable* until the Δ_H corrections and their uncertainties are published: no uncertainty has yet been declared, so F18 cannot fire before that publication, and any earlier claim of empirical failure (or success) is out of order.

F19 — Auxiliary-Hessian failure. The orthogonal-mode Hessian C_{ab} is singular or indefinite on the claimed relaxed branch without a separate zero-mode treatment.

F20 — Remainder-vertex failure. The closure/transport/record remainder ΔU changes the second, third or fourth relaxed derivative in a way incompatible with the dimension-four conditional matching, breaking the isolated-quartic vertex ratios.

F21 — Residue/scale-lock inconsistency. The eventual microscopic residue calculation returns a Z_H incompatible with the canonical scale lock already used, so the electron-bilinear identity and the kinetic normalisation cannot hold simultaneously.

F22 — Gauge-branch failure. The minimal-lift premise P11 fails as an admissibility principle; or the direct evaluation of the gauge-response Hessian returns $Z_Y < Z_2$, contradicting the minimal-lift selection; or the 15/14 saturation relation fails its own isotropy premises.

F23 — Grid-compression failure. The directly compressed top singular value $\sigma_{max}[\mathscr{E}_{uL}^\dagger P_Y K_{cl}^{-1/2} H_{cl} K_{cl}^{-1/2} P_Y \mathscr{E}_{uR}]$ differs from the grid target 1.67355737679; the grid construction then yields to the matrix calculation.

---

15. The Electroweak Vacuum and Higgs-Potential Completion Theorem

Theorem 9 — Route-A Electroweak Vacuum and Higgs-Potential Completion

Assume P1–P10.

Then the VERSF closure-interface scalar has the leading effective action

$$\Gamma_{scalar}^{HR1} = \int d^4x \, [(D_\mu H)^\dagger (D^\mu H) - V_{CI}(H) + O(\mathcal{O}_6/M_\star^2, \hbar)] ,$$

where

$$V_{CI}(H) = - \mu_H^2 H^\dagger H + \lambda (H^\dagger H)^2$$

with

$$\mathbf{v_{cl} = (26\pi/5) (\hbar c/\xi_A) (960/7)^6 = 243.722807638\ GeV ,}$$

$$\mathbf{\lambda = 512/3969 = 0.128999748047 ,}$$

$$\mathbf{\mu_H^2 = \lambda\, v_{cl}^2 = 7\,662.689132\ GeV^2 ,}$$

and

$$\mathbf{m_H^{HR1} = \sqrt{2\lambda}\, v_{cl} = (32/63)\, v_{cl} = 123.795711816\ GeV .}$$

The vacuum orbit is

$$H^\dagger H = v_{cl}^2/2 ,$$

is locally stable at dimension four, and has electromagnetic stabiliser $U(1)_{em}$. The leading broken-phase relations are

$$m_W^2 = \tfrac{1}{4} g^2 v_{cl}^2 , \quad m_Z^2 = \tfrac{1}{4} (g^2 + g'^2) v_{cl}^2 , \quad m_\gamma^2 = 0 ,$$

and

$$M_f = (v_{cl}/\sqrt{2})\, Y_f .$$

The complete declared-order static vertices are generated by relaxed differentiation of the single reduced functional (K_2, $K_3 = \mathcal{D}_\varrho K_2$, $K_4 = \mathcal{D}_\varrho^2 K_2$), and the quoted g_3 and g_4 are their constant-residue, remainder-inert canonical values.

The physical Higgs pole is the isolated zero of the complete broken-phase two-point matrix. At the declared finite-closure local matching order it equals the mass above and is gauge-parameter independent under the Nielsen identity.

Proof

The canonical effective kinetic coefficient follows from Lemma 1. The independent electron bilinears and scale-locking identity give v_{cl} by Theorem 2. The finite SU(8) curvature gives λ by Theorem 3. The stationary condition gives $\mu_H^2 = \lambda\, v_{cl}^2$. The radial Hessian gives $m_H^2 =$

$2\lambda\, v\_cl^2$. The complete declared-order vertices follow from Theorems 4–5. The vacuum-stability conditions follow from Theorem 6. The gauge and fermion relations follow from the canonical electroweak covariant derivative and Yukawa action. The conditional branch selection follows from Theorem 7 under P11. The pole statement follows from Theorem 8. ■

---

16. What Has Been Calculated and What Remains Conditional

16.1 Calculated in this paper

This paper calculates:

1. the canonical effective kinetic coefficient Z_H^eff = 1, by field definition and conditional closure-metric matching;
2. the independent Route-A substrate electron scale;
3. the electroweak vacuum v_cl;
4. the quartic λ;
5. the quadratic coefficient μ_H^2;
6. the radial curvature;
7. the leading finite-closure pole-matched Higgs mass;
8. the cubic and quartic radial self-couplings;
9. the local vacuum-stability conditions;
10. the electromagnetic stabiliser;
11. the broken-phase gauge and fermion mass relations;
12. the exact microscopic Schur-complement formula for Z_H, and the exact vanishing of its potential-sector contribution;
13. the complete relaxed vertex formulae K_2, $K_3 = \mathcal{D}\_\varrho K_2$, $K_4 = \mathcal{D}\_\varrho^2 K_2$, with the closure-norm decomposition isolating the remainder ΔU;
14. the exact field-dependent canonical vertex geometry, including the Z_1 and Z_2 corrections to g_3 and g_4;
15. the isolated-quartic reduced vertices with their conditional ratio audit;
16. the exact input address of the top Yukawa;
17. the exact scalar-only one-loop self-energy, pole displacement (+8.889 MeV) and pole residue in the zero-momentum matching scheme;
18. the numerical effective radial transport kernel and the full pre-gauge-fixing scalar-doublet Hessian;
19. the numerical mixed radial–Goldstone third and fourth variations;
20. the conditional electroweak coupling and mass-branch reconstruction (g, g′, m_W, m_Z)_±, with both branches evaluated exactly;
21. the conditional Persistent-Abelian branch-selection theorem, its exact displacement functional C_np(s) = s, and the reduction of unconditional selection to the response inequality $Z\_Y > Z_2$;

22. the conditional top-compression grid target and its completion-boundary mass;
23. the route and coupling sensitivity;
24. the post-output empirical orientation, including the electron-channel decomposition and the branch-independent ratio comparison;
25. the complete non-insertion and provenance-boundary audit.

16.2 Still conditional upstream

The following inputs remain conditional on their predecessor papers:

1. uniqueness of $K = 7$;
2. exactness of the geometric α formula beyond leading channel correction;
3. unique selection of the Route-A coherence scale;
4. the α^{-4} level-four electron enhancement;
5. the 13/20 occupancy factor;
6. the two-gate interface readout;
7. the $1/(8\pi)$ projection kernel;
8. equality $\alpha_{int} = \alpha$ at the readout point;
9. the closure-normalised Higgs-axis identification;
10. the linear finite action of the SU(8) deformation correction;
11. the local curvature-to-pole matching premise;
12. the transport-sector Hessian coefficients $A^{(2)}$, $B^{(2)}$, $C^{(2)}$ that must eventually evaluate Z_H microscopically;
13. inertness of the vertex remainder $\Delta U''$, $\Delta U'''$, $\Delta U''''$ from the stabilisation, bath, closure, transport, record and relaxation sectors;
14. Reading A of the SU(8) deformation theorem (Section 7.3);
15. the numerical top Yukawa $y_t = \sigma_{max}[\mathcal{E}_{uL}^\dagger P_Y K_{cl}^{-1/2} H_{cl} K_{cl}^{-1/2} P_Y \mathcal{E}_{uR}]$, for which the grid target of Section 10.6 is a conditional stand-in;
16. extension of the supplied closure Hessian H_{cl} from its traceless $\mathfrak{su}(8)$ restriction to the trace direction in which the Higgs radial mode lies;
17. the general admissibility principle behind P11 — that completion minimises operational displacement among candidate physical laws — together with the direct microscopic confirmation $Z_Y > Z_2$, and the identification of the reconstructed $1/g^2$, $1/g'^2$ with the microscopic response coefficients;
18. the 15/14 leading-saturation gauge-mass relation and its isotropy premises;
19. the quark-grid hierarchy factors and the completion-boundary-to-pole running map;
20. the interface-to-electroweak running of the Abelian coupling entering the Section 10.4 reconstruction — a distinct axis from the coupling-order branch of Section 12;
21. complete higher-order threshold and RG matching, including Δ_H^{gauge}, Δ_H^{top}, $\Delta_H^{Goldstone/ghost}$ and $\Delta_H^{threshold}$.

Three of these carry disproportionate weight and are named as the priority targets: item 2, because the α^{-6} dependence amplifies any imprecision in the geometric coupling sixfold; items 8–9 jointly, because they are the premises through which the earlier evaluation's kinetic-residue debt was conditionally discharged; and item 12, because Section 4.4 now supplies the exact formula those coefficients must feed — the debt has an address, and paying it converts Lemma 1 from matching bridge to proof.

16.3 Meaning of "completion"

The paper completes the numerical scalar action **conditional on the frozen upstream stack**. It does not claim that every member of that stack has already been derived from one unique microscopic master action. That stronger statement is the remaining programme-level unification target.

---

17. HR-1 Closure Certificate

Required outputs

Z_H , V_{CI} , v_{cl} , λ , μ_H^2 , m_H .

Returned outputs

$Z_H^{eff}(\mu\star) = 1$ (field definition and conditional closure-metric matching; Z_H^{micro} open) ,

$V_{CI}(H) = -(7\,662.689132\ \mathrm{GeV}^2)\, H^\dagger H + 0.128999748047\, (H^\dagger H)^2$,

$v_{cl} = 243.722807638$ GeV ,

$\lambda = 0.128999748047$,

$\mu_H^2 = 7\,662.689132\ \mathrm{GeV}^2$,

$g_3 = 188.641084672$ GeV , $g_4 = 0.773998488284$ (constant-residue, remainder-inert) ,

$m_H^{HR1} = 123.795711816$ GeV ,

$M_{pole,scalar} = 123.804601005$ GeV (scalar-only one loop, zero-momentum matching) ,

$R_H = 1.000312804191$,

$(\sin^2\theta_W, m_W, m_Z) = (0.21656, 79.268\ \mathrm{GeV}, 89.556\ \mathrm{GeV})$ (lower branch selected conditionally under P11; upper branch (0.87857, 39.355, 112.938) excluded by minimal lift; unconditional test $Z_Y > Z_2$ open) ,

$y_t^{grid} = 1.67355737679$, $m_t^{grid} = 288.418$ GeV (compression target, not a pole prediction) .

Stability certificate

$Z_H > 0$, $\lambda > 0$, $\mu_H^2 > 0$, $m_H^2 > 0$.

Non-insertion certificate

Neither

$v \simeq 246$ GeV

nor

$m_H \simeq 125$ GeV

appears anywhere upstream of the output.

Gate verdict

Route-A conditional numerical closure: PASS.

Unconditional one-master-action closure: OPEN on the named upstream bridge proofs.

---

18. Conclusion

The earlier Higgs-radial theorem established that VERSF contains a scalar completion mode. This paper calculates its leading physical vacuum and potential.

The decisive move is scale locking.

VERSF already possessed one construction for an absolute electron scale:

$m_e^{sub} = (\hbar c/\xi_A)\, \alpha^{-4} \cdot 13/20$.

It also possessed an independent completion-interface expression:

$m_e^{CI} = (\alpha^2/8\pi)\, v_{cl}$.

These are not two adjustable parametrisations. Under the frozen VERSF stack, they are two descriptions of one physical bilinear. Their equality fixes the vacuum:

$$v_{cl} = (26\pi/5)\,(\hbar c/\xi_A)\,\alpha^{-6} .$$

The Route-A numerical result is

$$v_{cl} = 243.723 \text{ GeV} .$$

The finite SU(8) closure curvature then fixes the shape of the scalar potential:

$$m_H/v_{cl} = 32/63 \,,\ \lambda = 512/3969 \,,\ \mu_H^2 = \lambda\, v_{cl}^2 .$$

The resulting leading Higgs mass is

$$m_H^{HR1} = 123.796 \text{ GeV} .$$

The complete evaluated potential is

$$V_{CI}(H) = -\,\mu_H^2\, H^\dagger H + \lambda\, (H^\dagger H)^2 .$$

It has the correct qualitative and quantitative architecture:

- the origin is unstable;
- the quartic is positive;
- the nonzero radial minimum is stable;
- electromagnetism remains unbroken;
- the vacuum transmits one common mass scale to the gauge and fermion sectors.

The quantum statement is made at the level required by the earlier quantum-gauge completion paper. The physical Higgs mass is defined by a pole, not by an arbitrary off-shell plot. The Nielsen identity protects that pole from gauge-parameter dependence. The leading finite-closure result supplies the declared-order pole; higher-order shifts must be calculated rather than fitted — and the paper calculates the first of them itself. The pure radial scalar loop, evaluated exactly in the zero-momentum matching scheme, moves the pole by +8.889 MeV to 123.805 GeV, with residue 1.000313 and vanishing scalar-only width. The gauge, top and threshold displacements are defined, ordered and awaiting their gate papers.

The kinetic residue receives the same treatment. Lemma 1 fixes the canonical effective coefficient to unity on the named canonical-metric premises, and Section 4.4 writes down the exact Schur-complement momentum formula — with the potential sector proved to contribute zero — that a substrate-native calculation must evaluate to discharge those premises. The residue debt is no longer diffuse: it has one formula, one sector (transport), and three named coefficients.

The vertex tower is held to the same standard. The complete declared-order vertices descend from one reduced functional by relaxed differentiation, the isolated-quartic values are graded as the remainder-inert special case, and the exact field-dependent canonical geometry states how any kinetic slope would deform the self-couplings. Nothing in the scalar sector is now derived from a term in isolation; everything descends from the one relaxed object that also defines the residue.

The numerical reach now extends conditionally beyond the scalar sector. One further closure relation pins the electroweak mixing angle to two exact branches, and the Persistent-Abelian Minimal-Lift Theorem then selects between them on typed substrate grounds: the photon is the completion of minimum operational displacement from the unique refinement-persistent Maxwell carrier, which uniquely picks the lower branch — m_W = 79.268 GeV and m_Z = 89.556 GeV from the same vacuum — while the selection principle itself is carried as premise P11 with its own falsifier, and its unconditional confirmation is reduced to the single response inequality $Z_Y > Z_2$. The quark grid supplies a top-compression target of 1.674 whose eventual confrontation with the direct matrix calculation is a designed, binding test: if the matrix wins, the grid yields. And the stack's sharpest tension is stated in the open: at face value that target would destabilise the quartic through β_λ almost immediately, so the completion-boundary-to-pole running bridge is now the single most consequential unbuilt component of the programme.

The post-output orientation sharpens the verdict without touching it: the branch-independent ratio 32/63 sits within parts in a thousand of experiment while the absolute scales miss at the percent level, and the electron channel's decomposed misses partially cancel through the lock. The discrepancy has an address — the scale sector, not the group theory — and the coupling channels even disagree productively about where in that sector it lives.

Assigning unique microscopic transport blocks, holonomy-response traces or chiral embedding matrices beyond this point would not be derivation — it would be fabrication. The paper stops exactly where the frozen corpus stops.

This is therefore no longer merely a statement of what VERSF should calculate. It is an explicit calculation.

Its strength is also its vulnerability, and deliberately so. The result depends strongly on the coherence-scale branch and on the colourless interface kernel, with the interface coupling entering to the sixth power. Those dependencies are exposed, tabulated and differentiated, not hidden. If the Route-A coherence scale, the electron scale law, the two-gate readout, the $1/(8\pi)$ kernel or the 32/63 finite-curvature theorem fails, the numerical Higgs completion fails with it — and Section 14 says exactly how.

That is the correct standard. A genuine derivation must produce numbers, preserve non-circularity and identify exactly what would kill it.

On its declared Route-A premises, HR-1 now does all three.

Appendix A — Exact Arithmetic

A.1 Coupling

$\alpha^{-1} = 2^7 \cdot 15/14 = 128 \cdot 15/14 = 960/7$.

Hence $\alpha = 7/960$.

A.2 Coherence energy

E_{ξ_A} = 197.3269804 eV·nm / 88 000 nm = 0.00224235205 eV .

A.3 Electron scale

m_e^sub = $0.00224235205 \cdot (960/7)^4 \cdot 13/20$ eV = 515 596.459802 eV .

A.4 Vacuum

v_cl = $(8\pi/\alpha^2)$ m_e^sub = 243.722807638 GeV .

A.5 Quartic

$\lambda = ½ (32/63)^2 = 512/3969 = 0.128999748047$.

A.6 Quadratic coefficient

μ_H² = λ v_cl² = 7 662.689132 GeV² .

A.7 Higgs mass

m_H = (32/63) v_cl = 123.795711816 GeV .

A.8 Scalar-only one-loop pole

Bracket constant: $11/6 - \pi/\sqrt{3} = 0.019533969$.

Prefactor: $9\lambda/16\pi^2 = 0.007352104$.

Fractional m^2 shift: $1.436157729 \times 10^{-4}$.

M_pole,scalar = $123.795711816 \times \sqrt{(1 + 1.436157729\times10^{-4})} = 123.804601005$ GeV .

$\Pi'(0) = 3\lambda/32\pi^2 = 1.225350671 \times 10^{-3}$.

$\Pi\Gamma'(m\_c^2) = 0.007352104 \times 0.042532909 = 3.12706 \times 10^{-4}$; $R\_H = 1.000312804191$.

A.9 Two-branch gauge reconstruction

$e^2 = 4\pi\alpha = 0.0916297857$; $A = 56(32/63)^2/(15e^2) = 10.5118451504$.

Discriminant: $(A+1)^2 - 8A = 48.4275$; $\sqrt{} = 6.95899$.

$s_- = 0.2165573823$; $s_+ = 0.8785733950$.

Lower branch: $g = 0.6504768409$, $g' = 0.3419910489$, $m\_W = 79.26802099$ GeV, $m\_Z = 89.55594468$ GeV.

Upper branch: $g = 0.3229455552$, $g' = 0.8686831733$, $m\_W = 39.35459872$ GeV, $m\_Z = 112.93760197$ GeV.

Check (both branches): $\sqrt{[(15/14)(m\_W^2 + m\_Z^2)]} = 123.795711816$ GeV = m_curv. ✓

A.10 Top grid target

$y\_d^{(0)} = 9\sqrt{2}\, \alpha^2/(8\pi) = 2.69259640452 \times 10^{-5}$; $m\_d^{(0)} = (9\alpha^2/8\pi)v\_cl = 4.64036813821$ MeV.

$R\_t = (6/13)\cdot 5e^{16/3}\cdot(77/3)^{3/2} = 62\ 154.0374183$.

$y\_t^{grid} = R\_t\, y\_d^{(0)} = 1.67355737679$; $m\_t^{grid} = (v\_cl/\sqrt{2})\, y\_t^{grid} = 288.417614897$ GeV.

Stability cross-check: $6(y\_t^{grid})^4 = 47.067$ versus $24\lambda^2 = 0.399$.

A.11 Minimal-lift branch selection

Displacement: $C\_np(s) = s$; persistent support $R\_pers(s) = 1 - s$; angle $d\_op = \arcsin\sqrt{s}$.

$\theta_- = \arcsin\sqrt{0.2165573823} = 27.7333°$; $\theta_+ = \arcsin\sqrt{0.8785733950} = 69.6067°$.

Photon composition (lower branch): $\sqrt{(1-s_-)} = 0.88512294$; $\sqrt{s_-} = 0.46535726$.

Persistent support: lower 78.3443% ; upper 12.1427%.

Equivalence check: $Z_2/(Z\_Y + Z_2) = 2.3633950533/10.9134818120 = 0.2165573823 = s_-$. ✓

Appendix B — Route Sensitivity

For fixed α, C_e, P_ℓ,

$v_{cl}(\xi) = v_{cl}(\xi_A) \cdot \xi_A/\xi$.

Therefore

$m_H(\xi) = m_H(\xi_A) \cdot \xi_A/\xi$,

and

$\mu_H^2(\xi) = \mu_H^2(\xi_A) \cdot (\xi_A/\xi)^2$.

The quartic is route independent within the finite SU(8) calculation:

$\lambda(\xi) = 512/3969$.

Appendix C — Dependency Map

Output	Direct dependencies
α	$K = 7$, $N_{loop} = 14$, channel correction
m_e^{sub}	ξ_A, α^{-4}, 13/20
v_{cl}	m_e^{sub}, $\alpha^2/(8\pi)$ interface projection
λ	closure baseline ½, dim $\mathfrak{su}(8) = 63$, linear action
μ_H^2	v_{cl}, λ
$m_H^{(0)}$	v_{cl}, λ, local two-derivative matching
m_W, m_Z	v_{cl}, independent g, g'
M_f	v_{cl}, independent Y_f
Z_H (microscopic)	transport-sector Hessian coefficients $A^{(2)}$, $B^{(2)}$, $C^{(2)}$ via the Section 4.4 Schur formula
K_2, K_3, K_4	full reduced potential U_{red} and relaxed branch (Theorem 4); remainder ΔU
g_3, g_4	K_2, K_3, K_4 and kinetic geometry Z_0, Z_1, Z_2 (Theorem 5)
y_t	closure Hessian H_{cl}, projector P_Y, chiral embeddings (open); grid target 1.674 as conditional stand-in
$\sin^2\theta_W$, g, g', m_W, m_Z (reconstruction)	$e^2 = 4\pi\alpha$, 15/14 saturation relation, 32/63, v_{cl}; branch selected by P11 minimal lift; unconditional test $Z_Y > Z_2$
Mixed h–π vertices	λ, v_{cl} (doublet potential)
reduced vertices $\Gamma^{(2)}$, $\Gamma^{(3)}$, $\Gamma^{(4)}$	single substrate quartic (Input VII)
$M_{pole,scalar}$	m_c, λ, equal-mass bubble constants (Appendix D)

Output	Direct dependencies
full complex pole	all above plus Δ_H^gauge, Δ_H^top, Δ_H^Goldstone/ghost, Δ_H^threshold

---

Appendix D — One-Loop Bubble Constants

For the equal-mass bubble, $B_0(p^2; m^2, m^2) - B_0(0; m^2, m^2) = -\int_0^1 dx \ln[(m^2 - x(1-x)p^2)/m^2]$.

At $p^2 = m^2$, using $\int_0^1 \ln(x^2 - x + 1)\, dx = -2 + \pi/\sqrt{3}$,

$\mathbf{B_0(m^2) - B_0(0) = 2 - \pi/\sqrt{3}}$ **.**

The slope at zero momentum is $B_0'(0) = \int_0^1 dx\, x(1-x)/m^2 = 1/(6m^2)$, so

$\mathbf{m^2 B_0'(0) = 1/6}$ **.**

The slope at $p^2 = m^2$ uses $\int_0^1 dx\, x(1-x)/(x^2 - x + 1) = 2\pi/(3\sqrt{3}) - 1$, giving

$\mathbf{m^2 B_0'(m^2) = 2\pi/(3\sqrt{3}) - 1}$ **,**

and hence the subtracted slope constant

$m^2 [B_0'(m^2) - B_0'(0)] = 2\pi/(3\sqrt{3}) - 7/6$,

used in the pole-residue evaluation of Section 11.7.

Appendix E — Internal and External References

Internal VERSF programme

1. *The Higgs-Radial Closure-Interface Gate in VERSF.*
2. *Substrate Dynamics and the Higgs Ratio.*
3. *The Fine-Structure Constant from Vacuum Geometry.*
4. *The Standard Model from Hexagonal Geometry.*
5. *The Charged-Lepton Absolute Scale Theorem in VERSF.*
6. *The Gauge Action and Coupling Theorem in VERSF.*
7. *The Closure-Operator Yukawa Construction Theorem in VERSF.*
8. *The Quantum Gauge Completion of the VERSF Standard Model.*

9. *The Variational–Closure Suppression Identity in VERSF.*
10. *The Saturated-Maintenance Projection and Tau-Maintenance-Suppression Theorem in VERSF.*

Standard quantum-field foundations

1. P. W. Higgs, "Broken Symmetries and the Masses of Gauge Bosons," *Physical Review Letters* **13**, 508–509 (1964).
2. F. Englert and R. Brout, "Broken Symmetry and the Mass of Gauge Vector Mesons," *Physical Review Letters* **13**, 321–323 (1964).
3. N. K. Nielsen, "On the Gauge Dependence of Spontaneous Symmetry Breaking in Gauge Theories," *Nuclear Physics B* **101**, 173–188 (1975).
4. R. Fukuda and T. Kugo, "Gauge Invariance in the Effective Action and Potential," *Physical Review D* **13**, 3469–3494 (1976).
5. P. Gambino and P. A. Grassi, "The Nielsen Identities of the Standard Model and the Definition of Mass," *Physical Review D* **62**, 076002 (2000).
6. A. Andreassen, W. Frost and M. D. Schwartz, "Consistent Use of the Standard Model Effective Potential," *Physical Review Letters* **113**, 241801 (2014).