The Fermionic Fock-Space Reconstruction in VERSF

▲ Programme Milestone — Matter-Sector Series

Deriving the Full CAR Algebra, Local Matter Fields, and the Leading QED-Form Matter–Gauge Vertex from Spinorial Commitment Loops

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Successor to *From Spinorial Source-Carriers to Fermionic Quantization in VERSF* and *The Microscopic Origin of the Record Current in VERSF*.

Headline result — a major conditional closure. Given the inherited spinorial-loop antisymmetry, one-particle Hilbert *positivity* together with *adjointness* force the full canonical anticommutation relations with *positive* normalization — and hence the fermionic Fock space.

This is a programme-marker paper, not another parameter derivation. It does not fit a number; it derives a structural layer of quantum field theory itself — the passage from spinorial source-carriers to fermionic Fock-space operators. The formal CAR reconstruction is standard exterior algebra; the programme claim is that the spinorial-loop sector supplies a physically motivated route to that setting, and that positivity and adjoint removal select the ordinary positive CAR over an indefinite or non-Fock alternative. The closure is *conditional*: exact given the named premises (alternating-exchange confinement FF-2a, exterior-sector surjectivity FF-2b, positivity FF-1, adjointness FF-4), where FF-4 is the load-bearing hinge and close in content to the conclusion. The QED-coupling bridge is the leading QED-form matter–gauge vertex only, and additionally carries the current-matching debt.

General Reader Summary

Most of physics is written down by assuming its rules and then checking them against experiment. The VERSF programme tries something more ambitious: it starts from a single plain idea — a medium of pure constraint in which distinctions get committed one at a time — and asks how much of ordinary physics is forced to follow. If the programme works, things normally taken as given, such as electric charge or the behaviour of electrons, should appear as consequences rather than as assumptions.

Ordinary matter is built from a family of particles physicists call fermions: electrons, quarks, and the rest. Two features make a fermion a fermion. First, no two identical fermions can sit in the same state — this "no crowding" rule is why matter takes up space, why chemistry has structure, and why you do not fall through your chair. Second, physics keeps track of fermions with a precise piece of bookkeeping: a complete set of rules for how particles are added and removed. That bookkeeping is what makes an electron field an electron field.

Earlier VERSF papers had already come surprisingly far. One showed that tiny, stubborn loops in the substrate — persistent knots of committed structure — behave, when you stand back from them, like a flowing electric current. Another showed that these loops carry the peculiar "turn it around twice to get back to the start" character of spin, and that swapping two identical loops flips a sign, which is exactly what delivers the no-crowding rule.

But one important piece was still missing. Getting a minus sign on a swap is not the same as having the full bookkeeping. The complete system also has to guarantee, for example, that removing exactly the loop you just added returns you cleanly to where you started, and that operations on different loops never quietly interfere with each other. Physicists bundle all of this into what they call the fermionic field structure. The earlier work had only half of it.

This paper supplies the rest, and the surprise is how little has to be added. Once single loops form a proper space of states with sensible probabilities, and once identical loops are already forced to swap with a minus sign, the full bookkeeping is no longer a choice — it is the only consistent option. It does not get put in by hand; it falls out. Put differently, "add a loop" and "remove a loop" are not two independent operations one defines separately: removing is taken to be the exact mirror image of adding, and once that is granted, the entire fermionic structure is fixed.

The paper then follows the consequence one step further. The electric current you can build out of these loop-particles turns out to match the current derived in the earlier work. Because of that match, the way this matter couples to light comes out looking like the standard interaction of ordinary electrodynamics — the basic process by which electrons and light influence each other — at leading order.

That is the milestone. VERSF no longer merely has loops that carry a few fermion-like features. It has loops that assemble into genuine fermion fields, sharing the core structure used for real electrons and quarks.

The claim is kept deliberately narrow. This paper does not reproduce the full list of known particles, the strong "colour" force, the fine corrections of mature quantum electrodynamics, or the richer forces beyond electromagnetism. It builds one structural layer — the fermion-field layer — for a single kind of loop, and shows that its current lines up with what the programme had already derived. It also rests on several clearly named assumptions, stated plainly in the body, any of which could fail and would sharpen or redirect the result.

Contents

Abstract

The earlier VERSF matter-sector papers establish three ingredients of a fermionic matter programme. First, the persistent cohomological sector couples uniquely to a conserved substrate record current J^μ through the admissible interaction −J^μ A_μ; the microscopic-origin paper identifies J^μ with the coarse-grained transport of primitive commitment loops, with hydrodynamic limit J^μ = ρ_pers u^μ, conservation from commitment continuity and projection–transport intertwining, and integer winding as a route to charge quantisation. Second, the spinorial-source-carrier sequence shows that persistent loops split into spinorial and trivial-holonomy sectors, that Finkelstein–Rubinstein exchange topology forces antisymmetric exchange on the spinorial sector, and that Pauli exclusion follows from the anticommuting creation–creation relation — while explicitly leaving open the CAR cross-relation $\{a_i, a_j†\} = \delta_{ij}$, the Fock construction, and one-particle positivity. Third, the VERSF Born-rule programme supplies the probability and Hilbert-space background for a positive inner-product state space.

This paper supplies the deferred Fock-space construction. Let $\mathscr{H}_1$ be the positive-definite one-particle Hilbert completion of single spinorial commitment-loop states. The antisymmetry

theorem forces the n-loop spinorial sector *into* $\Lambda^n\mathcal{H}_1$ (containment, FF-2a) and identifies it with the *full* antisymmetric power under the surjectivity premise (FF-2b), so the fermionic Fock space is

$$\mathcal{F}\_A(\mathcal{H}_1) = \bigoplus_{n=0}^{\infty} \Lambda^n\mathcal{H}_1.$$

Creation is the unique admissible antisymmetric insertion map $a^\dagger(f)\eta = f \wedge \eta$, and annihilation is its Hilbert adjoint — equivalently, interior contraction. The exterior-algebra identities then give the full CAR theorem,

$$\{a^\dagger(f), a^\dagger(g)\} = 0, \quad \{a(f), a(g)\} = 0, \quad \{a(f), a^\dagger(g)\} = \langle f, g\rangle \mathbb{1},$$

which in an orthonormal basis reads $\{a_i, a_j^\dagger\} = \delta_{ij}$. The cross-relation is not an independent postulate given adjointness (FF-4): the antiderivation algebra fixes the matching relation for any creation-type operator, and the specific yield of a *positive* one-particle inner product (FF-1) is the selection of the positive normalization $\{a, a^\dagger\} = +\langle f, g\rangle \mathbb{1}$ over an indefinite Krein form. FF-4 is load-bearing and close in content to the conclusion. A boundedness result ($\|a(f)\| = \|a^\dagger(f)\| = \|f\|$) upgrades these from dense-domain identities to bounded-operator identities on all of $\mathcal{F}$_A.

The paper then reconstructs a local spinorial field operator $\psi(x)$ from a local mode basis of $\mathcal{H}_1$. The equal-record CAR becomes the kernel of the persistent spinorial one-particle projector, reducing to the standard local delta relation in the continuum local-completeness limit. The normal-ordered bilinear current $j^\mu = q :\bar\psi\gamma^\mu \psi:$ coarse-grains to the loop current $J^\mu = \rho_{pers} u^\mu$ in the single-species hydrodynamic regime, and to $J^\mu = \Sigma_a \rho_a u_a^\mu$ after species decomposition. Substituting this operator current into the already-forced coupling $-J^\mu A_\mu$ yields the leading QED-form matter–gauge vertex $-q :\bar\psi\gamma^\mu \psi: A_\mu$.

The result is a milestone, not a final Standard Model derivation. It closes the CAR/Fock-space gap for the spinorial source-carrier sector, conditional on one-particle positivity and exact antisymmetric-sector identification. It does not yet derive renormalised QED, anomalies, non-abelian colour, species decomposition, or the Standard Model fermion spectrum.

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0. Notation, Conventions, and Firewall

Notation. $\mathcal{H}_1$ denotes the one-particle spinorial source-carrier Hilbert space. $\Lambda^n\mathcal{H}_1$ is its n-th antisymmetric power; $\mathcal{F}$_A($\mathcal{H}_1$) the antisymmetric Fock space; $|0\rangle$ the no-loop vacuum. Inner products are conjugate-linear in the first argument. $\{A, B\} = AB + BA$ denotes the anticommutator. The symbol $\mathbb{1}$ is the identity on $\mathcal{F}$_A. Four-vector (Greek) indices μ, ν are written with caret/underscore (J^μ, A_μ, ∂_μ, γ^μ) since Unicode carries no superscript Greek forms; Latin mode indices and integers use true sub/superscripts. A caret over a factor ($\hat{g}_k$) marks its omission from a wedge product.

Inner-product convention. Each $\Lambda^n\mathcal{H}_1$ is taken as the *intrinsic* antisymmetric power of $\mathcal{H}_1$, equipped with the determinant inner product

$$\langle f_1 \wedge \cdots \wedge f_n, g_1 \wedge \cdots \wedge g_n\rangle = \det(\langle f_i, g_j\rangle)$$

as its definition. Under this definition, for any orthonormal system $\{e_i\} \subset \mathscr{H}_1$ the vectors $e_{i_1} \wedge \cdots \wedge e_{i_n}$ with $i_1 < \cdots < i_n$ form an orthonormal basis of $\Lambda^n\mathscr{H}_1$, creation $a^\dagger(f)\eta = f \wedge \eta$ and its adjoint (interior contraction, §7) satisfy the CAR with a unit-normalized cross-relation, and no separate normalization of the wedge need be fixed. (If one instead realizes $\Lambda^n\mathscr{H}_1$ as a subspace of the tensor power $\mathscr{H}_1^{\otimes n}$ via the antisymmetrizer, the embedding reproducing the determinant inner product carries the prefactor $1/\sqrt{(n!)}$, not $1/n!$. This paper does not use the tensor embedding; the intrinsic determinant inner product is primitive.)

Firewall. No observed fermionic algebra, no measured CAR relation, and no QED matrix element enters this construction as input data. The load-bearing inputs are the inherited substrate topology of §2–§3 (in particular the antisymmetric exchange, FF-2a) and the one-particle Hilbert structure (positivity FF-1, adjointness FF-4), together with the completeness premises FF-2b and FF-3. The CAR relations, the local field operator, and the QED-form coupling are *outputs* in the sense that no such relation is fitted to data. The honest content of the closure is narrow and stated plainly: given inherited antisymmetry, one-particle *positivity* (FF-1) together with *adjointness* (FF-4) force the canonical anticommutation relations with a *positive* normalization, $\{a(f), a^\dagger(g)\} = +\langle f, g\rangle \mathbb{1}$, rather than an indefinite Krein form. FF-4 — that substrate removal is the Hilbert adjoint of insertion — is close in content to the conclusion and is doing much of the work; §7 records that it is presently asserted from the microscopic-origin dynamics rather than independently derived. What FF-1 and FF-4 jointly *buy*, beyond the standard antiderivation algebra, is exactly the selection of the positive cross-relation over the indefinite one.

Grade vocabulary. *Inherited* — taken from a prior VERSF paper. *Exact* — proven with no free residue given the named premises. *Conditional* — holds given explicitly named premises that are not yet independently established. *Audit condition* — a finite pass/fail check whose satisfaction is required, not assumed. *Owed* — a proof debt named here and left open.

0′. Predictive-Content Ledger

Object	Grade	Status
Primitive commitment loops	Inherited	Record-current paper
$J^\mu = \rho_{pers} u^\mu$	Inherited	Hydrodynamic current
$\partial_\mu J^\mu = 0$	Inherited	Conservation theorem
Spinorial sector $U(2\pi) = -\mathbb{1}$	Inherited	Spinorial-carrier paper
Antisymmetric exchange	Inherited	Finkelstein–Rubinstein construction
$\{a^\dagger, a^\dagger\} = 0$, $\{a, a\} = 0$	Inherited / Exact	Inherited; rederived exactly here
One-particle space $\mathscr{H}_1$ positive-definite	Owed	Assumed (FF-1); microscopic proof owed
Antisymmetric Fock space $\mathcal{F}_A$	Exact	Identified: confinement FF-2a; full-power surjectivity FF-2b (premised)

Object	Grade	Status
Cross-relation $\{a_i, a_j†\} = \delta_{ij}$ (positive normalization)	Exact	Derived here, given FF-1–FF-4
Boundedness $\|a†(f)\| = \|f\|$	Exact	Corollary 2.1
$N_i^2 = N_i$ (occupancy 0 or 1)	Exact	§10
Local field operator ψ(x)	Conditional	On local completeness (FF-5)
Sector independence {ψ_α(x), ψ_β(y)} = 0	Owed	Consistency; tied to missing C operator (§11)
Microcausality {ψ_α(x), ψ̄_β(y)} = 0 at separation	Owed	Locality certificate; constrains u/v assembly (§11)
Opposite winding = antiparticle sector	Candidate	Charge-conjugation reading; tied to species decomposition
Local CAR → δ_{αβ} δ³(x−y)	Audit condition	Continuum local-completeness limit
Bilinear current j^μ = q :ψ̄ γ^μ ψ:	Exact (construction)	§13
Matching j^μ → J^μ	Conditional	Owed in sharp form; needs magnetization-current cancellation (§13)
Coupling −q :ψ̄ γ^μ ψ: A_μ	Conditional	Leading vertex, given matching + inherited coupling
Renormalised QED	Owed	Not derived
Species decomposition	Owed	Not derived
Non-abelian gauge sectors	Owed	Not derived
Standard Model fermion spectrum	Owed	Not derived

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1. Purpose and Claim Level

The spinorial-source-carrier paper ends at the exact frontier this paper addresses. It establishes antisymmetric exchange and the anticommuting creation–creation and annihilation–annihilation relations, but explicitly defers the CAR cross-relation, the Fock-space construction, and one-particle positivity. The purpose here is not to reprove spinorial exchange but to complete the algebraic quantisation layer.

On novelty. The formal CAR theorem is not novel as a theorem of exterior algebra; that annihilation, defined as the adjoint of antisymmetric creation on a positive Hilbert space, satisfies the canonical anticommutation relations is textbook. The programme claim is narrower and physical: the VERSF spinorial-loop sector supplies a physically motivated route to the exterior-algebra setting, and one-particle positivity together with adjoint removal *select the ordinary positive CAR* rather than an indefinite or non-Fock alternative. What is asserted new here is the identification of the bridges — the exact premises under which the standard reconstruction becomes available to this substrate — not the reconstruction theorem itself.

The target chain is

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spinorial source-carriers → fermionic Fock space → full CAR
    → local matter field → leading QED-form vertex.
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Claim. Given the spinorial source-carrier sector already derived in VERSF, and given the named premises below, the full fermionic Fock-space and positive CAR structure is realized. The resulting field-current bilinear coarse-grains to the previously derived record current, lifting the inherited source term to the leading QED-form matter–gauge vertex.

The load-bearing premises are named, each with the falsifier that would retire it (§17).

- **FF-1 — Positive one-particle Hilbert space.** Single spinorial commitment-loop states complete to a positive-definite Hilbert space $\mathscr{H}_1$. *Falsifier:* an indefinite substrate metric (F1).
- **FF-2a — Alternating exchange confinement.** Odd exchange of identical spinorial loops acts by −1, confining n-loop states to the alternating representation, i.e. *inside* $\Lambda^n\mathscr{H}_1$. This is topological, inherited from the Finkelstein–Rubinstein construction. *Falsifier:* a residual symmetric or parastatistical component (F2).
- **FF-2b — Exterior-sector surjectivity.** Every wedge state in $\Lambda^n\mathscr{H}_1$ is physically reachable, so the n-loop sector is *all* of $\Lambda^n\mathscr{H}_1$ rather than a dynamically closed proper subspace. This is a dynamical/completeness claim, not a topological one. *Falsifier:* an unreachable exterior subspace (F2b).
- **FF-3 — Fock density.** Finite wedge products of one-particle states are dense in the physical finite-loop sector. *Falsifier:* a physical finite-loop subspace not reached by finite wedges (F3).
- **FF-4 — Adjoint admissibility (the hinge).** Removing a loop is the Hilbert adjoint of inserting that loop, with respect to the Born/Hilbert inner product. This is the load-bearing premise: it is close in content to the conclusion, since annihilation-as-adjoint-of-creation is almost the whole of what forces the cross-relation. It is presently *asserted* from the microscopic-origin dynamics, not derived. Its discharge is named the **Adjoint-Removal Theorem** and is the natural next paper (§7, §16). *Falsifier:* a substrate number-lowering operation that is not the adjoint of creation (F4).

Claim hierarchy. The results separate into three tiers, and the paper should be read against this stratification. (Throughout, "FF-1–FF-4" abbreviates the full premise set {FF-1, FF-2a, FF-2b, FF-3, FF-4}.)

- **Tier 1 — Exact mathematical reconstruction.** Given a positive one-particle space $\mathscr{H}_1$, its exterior antisymmetric Fock space, and annihilation defined as the Hilbert adjoint of wedge insertion, the full positive CAR follows. This tier is *Exact* and is standard exterior algebra (§6–§10).
- **Tier 2 — Conditional VERSF physical reconstruction.** Given spinorial-loop antisymmetry (FF-2a), one-particle positivity (FF-1), exterior-sector surjectivity (FF-2b), and substrate adjoint removal (FF-4), the spinorial commitment-loop sector *realizes* Tier 1. This tier is *Conditional* and is the substantive VERSF claim.

- **Tier 3 — Local field / QED bridge.** Given local completeness (FF-5), sector independence and microcausality (§11), opposite-winding charge-conjugation structure, and current matching including magnetization cancellation (§13), the field current lifts the inherited source coupling to the leading QED-form vertex. This tier is *Conditional* and remains labelled as such (§11–§14).

Tier 1 is not the achievement; Tier 2 is. Tier 3 is the most fragile layer and is flagged accordingly throughout.

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2. Inherited Source-Carrier Ontology

The record-current paper establishes that the source current is not an arbitrary external object but is carried by primitive commitment loops: non-recombinable cycles of transported committed distinguishability satisfying irreversible closure, persistence, refinement stability, and finite primitive support. Those loops are the unique currently identified substrate structures satisfying source-admissibility.

At the continuum level, in the single-species hydrodynamic regime, the microscopic loop current coarse-grains to

$$J^\mu(x) = \rho_{pers}(x)\, u^\mu(x), \quad \text{with } u^\mu u_\mu = 1,$$

and in the generic multi-species regime to

$$J^\mu(x) = \Sigma_a\, \rho_a(x)\, u_a^\mu(x).$$

Conservation follows from projecting the full commitment current C^μ onto the persistent, topologically protected sub-complex,

$$J^\mu = \Pi_{pers}^{(1)}\, C^\mu,$$

with projection–transport intertwining giving $\partial_\mu J^\mu = 0$. This already renders the current source-admissible for the inherited coupling $-J^\mu A_\mu$. The present paper uses these facts as the current-side target for the fermionic field construction, and does not rederive them.

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3. Inherited Spinorial and Exchange Structure

The spinorial-source-carrier paper supplies the topological structure required for fermionic matter.

The one-loop sector carries spinorial closure transport,

$$U(2\pi) = -\mathbb{1}, \quad U(4\pi) = +\mathbb{1}.$$

For two identical spinorial loops in three spatial dimensions, Finkelstein–Rubinstein topology identifies the exchange path with a 2π rotation of one loop about the other. The exchange path therefore carries the same −1 holonomy, so the spinorial two-loop state satisfies

$$\Psi(\mathcal{C}_2, \mathcal{C}_1) = -\Psi(\mathcal{C}_1, \mathcal{C}_2).$$

The symmetric alternative is not merely disfavoured; it is *incompatible* with the spinorial exchange holonomy. This is stronger than "fermionic statistics are allowed."

At the operator level, that paper already gives the anticommuting half of the CAR,

$$\{a_i^\dagger, a_j^\dagger\} = 0, \quad \{a_i, a_j\} = 0,$$

conditional on Fock density. What remains open is the cross-relation $\{a_i, a_j^\dagger\} = \delta_{ij}$, which is derived below.

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4. The One-Particle Spinorial Loop Hilbert Space

Let $\mathcal{S}$_spin denote the set of admissible single spinorial commitment-loop states. A basis element is schematically

$$|\mathcal{C}; \alpha, w\rangle,$$

where $\mathcal{C}$ is the primitive commitment-loop support, α the spinorial frame index, $w \in \mathbb{Z}$ the loop winding (integer cohomology class, which carries the conserved topological charge of the state), and the state is supported in the persistent, refinement-stable sector. Let $\mathcal{D}_1$ be the finite linear span of such states. The symbol q is reserved throughout for the gauge coupling constant (§13–§14) and is *not* used as a state label; the per-state integer charge is w.

The one-particle inner product ⟨f, g⟩ is required to satisfy:

1. **linearity / conjugate-linearity** in the standard Hilbert convention;
2. **positive definiteness:** $\langle f, f\rangle > 0$ for $f \neq 0$;
3. **orthogonality of incompatible supports:** closure-incompatible loop states have zero inner product;
4. **Born compatibility:** $p(f \to g) = |\langle g, f\rangle|^2$ for unit-normalised states.

The VERSF Born-rule programme supports the use of the Hilbert inner product and squared-modulus probability, while being explicit that in some routes the Hilbert structure is reconstructed internally and in others imported. Define

$$\mathcal{H}_1 = \text{closure of } \mathcal{D}_1 \text{ under } \langle\cdot, \cdot\rangle.$$

This is the one-particle spinorial source-carrier Hilbert space.

Status. This definition discharges the *formal* requirement left open by the prior paper, but positive-definiteness (FF-1) remains a load-bearing physical condition, graded *Owed*. If the substrate instead supplies an indefinite metric, the construction becomes a Krein-space quantisation rather than ordinary CAR Fock quantisation (F1).

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5. Antisymmetric Multi-Loop Sector

Given $\mathscr{H}_1$, let $\Lambda^n\mathscr{H}_1$ be its intrinsic antisymmetric n-loop power, spanned by decomposable elements $f_1 \wedge \cdots \wedge f_n$ that are totally antisymmetric and multilinear in their arguments ($f \wedge f = 0$; interchange of any two factors reverses sign). No embedding into a tensor power is invoked; the space is equipped directly with the determinant inner product (§0),

$$\langle f_1 \wedge \cdots \wedge f_n, g_1 \wedge \cdots \wedge g_n\rangle = \det(\langle f_i, g_j\rangle),$$

taken as primitive. Each $\Lambda^n\mathscr{H}_1$ is then positive-definite whenever $\mathscr{H}_1$ is positive-definite: for an orthonormal system the Gram determinant of any n distinct basis wedges equals 1, and it is strictly positive on independent families and zero on dependent ones. Define the fermionic Fock space

$$\mathcal{F}\_A(\mathcal{H}_1) = \bigoplus_{n=0}^\infty \Lambda^n\mathcal{H}_1, \quad \Lambda^0\mathcal{H}_1 = \mathbb{C}|0\rangle,$$

with $\mathbb{C}|0\rangle$ the no-loop vacuum sector.

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6. Fock Reconstruction Theorem

Theorem 1 — The antisymmetric Fock space is identified, given FF-2a and FF-2b

Given FF-1, FF-2a, and FF-2b, the finite spinorial multi-loop state space is the exterior algebra over $\mathscr{H}_1$:

$$\mathcal{F}\_spin^{finite} = \bigoplus_{n=0}^N \Lambda^n\mathcal{H}_1.$$

In the limit $N \to \infty$, with Fock density (FF-3), the full spinorial finite-excitation space is $\mathcal{F}\_A(\mathscr{H}_1) = \bigoplus_{n=0}^\infty \Lambda^n\mathscr{H}_1$.

Proof. The inherited exchange theorem (§3) makes odd permutations of identical spinorial loops act by −1, so the n-loop wavefunction transforms in the alternating representation of S_n:

$$\Psi(\mathcal{C}_{\sigma(1)}, \ldots, \mathcal{C}_{\sigma(n)}) = \mathrm{sgn}(\sigma)\ \Psi(\mathcal{C}_1, \ldots, \mathcal{C}_n).$$

Antisymmetric exchange (FF-2a) therefore *confines* the physical n-loop states to the alternating representation, i.e. inside $\Lambda^n\mathcal{H}_1$; the universal object carrying exactly this transformation law is the antisymmetric power, and any construction realising the alternating representation on n copies of $\mathcal{H}_1$ factors through it. That the physical n-loop space is *all* of $\Lambda^n\mathcal{H}_1$ — rather than a dynamically closed proper subspace — is the surjectivity content supplied separately by FF-2b, which is premised, not derived here. Under both premises, the direct sum over particle number is the antisymmetric Fock space. ■

Structural reading. The containment in the exterior algebra is forced by the inherited topology (FF-2a): odd exchange carries a minus sign, so no symmetric or mixed component survives. The identification with the *full* exterior power is the logically independent step that FF-2b grants. The direction of dependence is topology → algebra, not algebra → fit, but exterior-sector surjectivity is a named dynamical premise (F2b), not a theorem.

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7. Creation and Annihilation Operators

For $f \in \mathcal{H}_1$, define the creation operator $a^\dagger(f)$: $\Lambda^n\mathcal{H}_1 \to \Lambda^{n+1}\mathcal{H}_1$ by

$$a^\dagger(f)\ \eta = f \wedge \eta, \quad a^\dagger(f)|0\rangle = f.$$

Define the annihilation operator $a(f)$ as the Hilbert adjoint of $a^\dagger(f)$:

$$\langle a^\dagger(f)\ \eta, \chi\rangle = \langle \eta, a(f)\ \chi\rangle \quad \text{for all } \eta, \chi.$$

Lemma 0 — The adjoint is interior contraction

On decomposable n-particle vectors, the adjoint of wedge-insertion is interior contraction:

$$a(f)(g_1 \wedge \cdots \wedge g_n) = \Sigma_{k=1}^{n} (-1)^{k-1} \langle f, g_k\rangle\ g_1 \wedge \cdots \wedge \hat{g}_k \wedge \cdots \wedge g_n,$$
$$a(f)|0\rangle = 0.$$

Proof. For decomposables, expand both sides of $\langle f \wedge \eta, \chi\rangle = \langle \eta, \iota_f \chi\rangle$ using the determinant inner product and cofactor (Laplace) expansion along the row carrying f; the alternating signs $(-1)^{k-1}$ are exactly the cofactor signs. The identity extends to all of $\Lambda^n\mathcal{H}_1$ by sesquilinearity and to $\mathcal{F}_A$ by density (FF-3). ■

Thus, once FF-4 is granted, annihilation is not freely chosen: it is the unique number-lowering operation adjoint to admissible loop insertion with respect to the physical Hilbert inner product. The weight of the construction rests here. FF-4 — that removing a loop is the Hilbert adjoint of inserting it — is close in content to the CAR itself, and is presently *asserted* from the microscopic-origin dynamics of the record-current sector rather than *derived* from them.

The Adjoint-Removal Theorem (owed; the natural next paper). The next required microscopic result is a theorem establishing FF-4 from substrate transport alone: that the physical loop-removal operation is exactly the Born/Hilbert adjoint of loop insertion, with (i) no phase defect, (ii) no normalization defect, and (iii) no leakage into inaccessible sectors. A natural sufficient condition would discharge more than FF-4 alone: if the substrate transport map acts *isometrically* on $\mathscr{H}_1$, then the adjoint of insertion is forced to be the metric-adjoint (removal), and one-particle positivity comes along automatically — so a single isometry statement would collapse FF-1 and FF-4 into one fact, reducing four premises to three. Establishing transport isometry is therefore the sharpest target for the next paper. Only such a theorem converts the present *conditional* algebraic closure into an unconditional one for the spinorial-loop sector. The present paper does not attempt it; it discharges the algebraic layer *modulo* FF-4, and does not claim FF-4 as a weaker assumption than positive CAR — only that FF-4 together with positivity (FF-1) fixes the CAR *and its sign*. Its failure is falsifier F4.

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8. Full CAR Theorem

Theorem 2 — The full CAR algebra

For all f, g ∈ $\mathscr{H}_1$,

$$\{a^\dagger(f),\ a^\dagger(g)\} = 0, \quad \{a(f),\ a(g)\} = 0, \quad \{a(f),\ a^\dagger(g)\} = \langle f,\ g\rangle\, \mathbb{1}.$$

In an orthonormal basis $\{e_i\}$, with $a_i \coloneqq a(e_i)$,

$$\{a_i^\dagger,\ a_j^\dagger\} = 0, \quad \{a_i,\ a_j\} = 0, \quad \{a_i,\ a_j^\dagger\} = \delta_{ij}.$$

Proof. For the creation–creation relation, on any $\eta \in \Lambda^n \mathscr{H}_1$,

$$a^\dagger(f)\ a^\dagger(g)\ \eta = f \wedge g \wedge \eta,$$
$$a^\dagger(g)\ a^\dagger(f)\ \eta = g \wedge f \wedge \eta = -\ f \wedge g \wedge \eta,$$

so $\{a^\dagger(f), a^\dagger(g)\} = 0$. Taking Hilbert adjoints gives $\{a(f), a(g)\} = 0$.

For the cross-relation, let $\eta \in \Lambda^n \mathscr{H}_1$. Then $a(f)\ a^\dagger(g)\ \eta = a(f)(g \wedge \eta)$. Interior contraction is an antiderivation over the wedge, so by Lemma 0,

$$a(f)(g \wedge \eta) = \langle f,\ g\rangle\ \eta - g \wedge a(f)\ \eta.$$

Since $g \wedge a(f)\ \eta = a^\dagger(g)\ a(f)\ \eta$,

$$a(f)\ a^\dagger(g)\ \eta + a^\dagger(g)\ a(f)\ \eta = \langle f,\ g\rangle\ \eta.$$

This holds as an operator identity on the dense finite-particle domain $\mathcal{D}$:

$$\{a(f),\ a^\dagger(g)\} = \langle f,\ g\rangle\ \mathbb{1} \quad \text{on } \mathcal{D}.$$

In an orthonormal basis this reads $\{a_i, a_j^\dagger\} = \delta_{ij}$. ■

The cross-relation is thus not an independent postulate *given FF-4*: it is the antiderivation identity for contraction against wedge-insertion once annihilation is fixed to be the Hilbert adjoint of antisymmetric creation. This framing should not be overstated. For *any* creation-type operator on a Hilbert space, the adjoint automatically satisfies the matching cross-relation — the bosonic CCR arise the same way. The non-trivial inputs here are therefore exactly two: the inherited antisymmetric statistics (FF-2a, genuinely upstream from the Finkelstein–Rubinstein topology) and the adjointness premise FF-4. What the antiderivation algebra alone does *not* decide, and what FF-1 together with FF-4 do decide, is the sign and positivity of the right-hand side: a positive one-particle inner product forces $\{a(f), a^\dagger(g)\} = +\langle f, g\rangle\ \mathbb{1}$, selecting the positive CAR over an indefinite Krein form. That positivity selection — not the antiderivation step, which is standard — is the substantive yield of this section.

Corollary 2.1 — Boundedness and extension to Fock space

Each $a^\dagger(f)$ and $a(f)$ extends to a bounded operator on $\mathcal{F}$_A with

$$\|a^\dagger(f)\| = \|a(f)\| = \|f\|,$$

and every CAR identity of Theorem 2 holds as an identity of bounded operators on all of $\mathcal{F}$_A.

Proof. Setting f = g in the cross-relation of Theorem 2 gives, on $\mathcal{D}$, $a(f)\, a^\dagger(f) + a^\dagger(f)\, a(f) = \|f\|^2\ \mathbb{1}$. Since $a^\dagger(f)\, a(f) \geq 0$, for $\xi \in \mathcal{D}$,

$$\|a^\dagger(f)\ \xi\|^2 = \langle \xi,\ a(f)\ a^\dagger(f)\ \xi\rangle \leq \langle \xi,\ (a(f)\ a^\dagger(f) + a^\dagger(f)\ a(f))\ \xi\rangle = \|f\|^2\ \|\xi\|^2,$$

so $\|a^\dagger(f)\| \leq \|f\|$ on $\mathcal{D}$, with equality already on the vacuum ($a^\dagger(f)|0\rangle = f$). A densely defined bounded operator extends uniquely to $\mathcal{F}$_A, and $a(f) = a^\dagger(f)^*$ inherits the same norm. The CAR identities, being bounded on both sides and valid on the dense domain $\mathcal{D}$, extend by continuity to all of $\mathcal{F}$_A. ■

Theorem 2 is established algebraically on $\mathcal{D}$ with no appeal to boundedness, so Corollary 2.1 introduces no circularity: boundedness is a *consequence* of the cross-relation, and it upgrades the dense-domain identities to bounded-operator identities on the full Fock space.

---

9. What Has Actually Been Closed

The prior paper had already derived $\{a_i^\dagger, a_j^\dagger\} = 0$ and $\{a_i, a_j\} = 0$. This paper derives

$$\{a_i,\ a_j^\dagger\} = \delta_{ij},$$

closing the canonical anticommutation relations at the algebraic Fock-space level, conditional on FF-1–FF-4. The closure is meaningful because the cross-relation is precisely what yields ordinary fermion number, orthogonal-mode occupancy, and the one-particle extraction rule

$$a_i\, a_j^\dagger\, |0\rangle = \delta_{ij}\, |0\rangle.$$

Without it, Pauli exclusion exists but the full fermionic field algebra does not.

---

10. Number and Charge Operators

In an orthonormal basis $\{e_i\}$, define the mode number operators $N_i = a_i^\dagger a_i$.

Occupancy is 0 or 1

$$N_i^2 = a_i^\dagger a_i a_i^\dagger a_i = a_i^\dagger (\mathbb{1} - a_i^\dagger a_i) a_i = a_i^\dagger a_i - a_i^\dagger (a_i^\dagger a_i) a_i = a_i^\dagger a_i = N_i,$$

using $\{a_i, a_i^\dagger\} = \mathbb{1}$ and $a_i^\dagger a_i^\dagger = 0$. Thus each N_i is a projector, its spectrum is $\{0, 1\}$, and Pauli exclusion is recovered in number-operator form. The total number operator is $N = \Sigma_i a_i^\dagger a_i$.

If the one-particle space decomposes into winding sectors indexed by the integer winding w,

$$\mathcal{H}_1 = \oplus_{w\in\mathbb{Z}} \mathcal{H}_{1,w},$$

define the charge operator $Q = \Sigma_i w_i a_i^\dagger a_i$, where $w_i \in \mathbb{Z}$ is the winding of mode i. This is the Fock-space lift of the topological winding charge of the loop ontology: the record-current paper supplies the integer-winding route to conserved topological charge, and this paper promotes that charge to the fermionic number-operator layer. The physical electric charge of a mode is $q\cdot w_i$, the gauge coupling q (§13–§14) times the integer winding; Q counts winding, and q enters only through the coupling.

---

11. Local Field Operator

Let $\{u_n(x)\}$ be an orthonormal local mode basis for the positive-winding sector and $\{v_n(x)\}$ an orthonormal local mode basis for the opposite-winding sector; the spinor index is α. Define the local field operator

$$\psi_\alpha(x) = \Sigma_n [u_{n\alpha}(x)\, b_n + v_{n\alpha}(x)\, d_n^\dagger],$$

where b_n annihilates a loop excitation in the positive-winding sector and $d_n^\dagger$ creates a loop excitation in the opposite-winding sector. The adjoint field is $\bar{\psi}(x) = \psi^\dagger(x)\, \gamma^0$.

Caveat (charge-conjugation reading). Reading the opposite-winding sector as the antiparticle sector is a *candidate* charge-conjugation identification, not a derived one. It is graded *Candidate* and remains tied to species decomposition: no charge-conjugation operator C is constructed here, and no CPT relation is established. The field is written in the two-sector form only to exhibit the standard mode structure; the substantive antiparticle content — a conjugation operator intertwining the winding sectors, and its consistency with the Dirac structure of §12 — is an owed step. Until that step is discharged, $d_n†$ should be read as "opposite-winding creation," with "antiparticle" as its provisional interpretation.

ψ_α(x) is an operator-valued distribution; sharp identities below hold after smearing against test functions and are stated pointwise for brevity. Using the CAR,

```
{ψ_α(x), ψ_β†(y)} = Π_αβ(x, y),
```

where Π(x, y) is the kernel of the persistent spinorial one-particle projector. In the local-completeness limit,

```
Π_αβ(x, y) → δ_αβ δ³(x − y)   (on a common commitment slice),
```

recovering the standard equal-record local CAR

```
{ψ_α(x), ψ_β†(y)} = δ_αβ δ³(x − y).
```

Status note. The exact delta relation requires local completeness of the chosen persistent spinorial mode basis (FF-5). Without local completeness the right-hand side is the persistent-sector projector kernel, not the full delta distribution. This is not a failure but the correct substrate-level refinement: the field lives only on the persistent spinorial source-carrier sector. The delta limit is therefore graded *Audit condition* — a check to be passed in the continuum limit, not an assumption.

Sector independence and microcausality (two owed relations). The dagger delta relation above is only part of what a local fermion field must satisfy, and two structurally distinct conditions are involved. It is important not to conflate them.

First — sector independence, {ψ_α(x), ψ_β(y)} = 0 (no dagger). In ordinary Dirac theory this relation holds identically, at all separations, and carries no light-cone content: expanding $\psi = \Sigma_n (u_n b_n + v_n d_n†)$, the no-dagger anticommutator only ever pairs b with b, b with d†, or d† with d†, and all three vanish when the positive- and opposite-winding sectors are mutually independent ($\{b_n, b_m\} = 0$, $\{d_n†, d_m†\} = 0$, $\{b_n, d_m†\} = 0$). Here that independence is *not* automatic: the opposite-winding sector is only a *Candidate* antiparticle sector, with no charge-conjugation operator C intertwining it to the b-sector (§11 caveat). Sector independence — hence {ψ, ψ} = 0 — is therefore an owed *consistency* condition tied directly to the missing C, not a causality statement.

Second — microcausality proper, {ψ_α(x), ψ̄_β(y)} = 0 for causally independent (geometrically separated) points. This is the genuine locality certificate: it is the vanishing of the dagger anticommutator function outside the causal region, and it is what reconstructs a propagator supported inside the cone. Whether it holds depends nontrivially on how the positive-winding modes $\{u_n\}$ and opposite-winding modes $\{v_n\}$ assemble — the relative phase and support structure of the two families — which the paper does not constrain. This relation is absent from the construction and is graded *Owed*.

A local, microcausal fermion field is recovered only once all three are passed: the dagger delta limit $\{\psi, \psi^\dagger\} \to \delta$ (Audit condition, above), sector independence $\{\psi, \psi\} = 0$ (owed, tied to C), and microcausal support $\{\psi, \bar{\psi}\} = 0$ at geometric separation (owed, the true causality certificate). Falsifiers F10 and F11 separate the last two; the conclusion's phrase "a local, microcausal fermion field" is conditional on both.

Safe characterisation. Accordingly, what §11 reconstructs is a *canonical local-mode field candidate* — a field operator carrying the correct global CAR and the correct equal-record mode structure — not yet a fully microcausal relativistic Dirac field. The upgrade from candidate to genuine local Dirac field requires the sector-independence and microcausal-support relations above, which are owed. This is the weakest layer of the construction (Tier 3), and it is labelled as such throughout.

---

12. Dirac Admissibility and Field Dynamics

The spinorial-source-carrier paper establishes that first-order admissible closure flow on the persistent sector forces Clifford-compatible spinorial structure and the standard Dirac-operator form, conditional on the named inheritances. The present paper therefore takes the one-particle spinorial modes to satisfy the inherited first-order admissible-flow equation

```
(i γ^μ ∂_μ − m) ψ = 0
```

in the free persistent sector, with free Lagrangian

```
ℒ_free = ψ̄ (i γ^μ ∂_μ − m) ψ.
```

This paper does not rederive the Dirac operator. It reconstructs the Fock and operator layer over the already-derived spinorial single-carrier structure.

---

13. Bilinear Current Reconstruction

Define the normal-ordered fermionic current

```
j^μ(x) = q :ψ̄(x) γ^μ ψ(x):.
```

This is the ordinary Dirac current built from the VERSF spinorial source-carrier field. Its coarse-grained matching to the record current is a *conditional* statement, not an exact theorem; the sharp version is graded *Owed*.

Proposition 3 (Conditional) — Bilinear current coarse-grains to the record current

In the single-species comoving regime, for a finite-density state whose one-particle density matrix Γ represents a persistence-filtered ensemble of localized spinorial loops, the expectation value of j^μ coarse-grains to

```
⟨j^μ(x)⟩ → q ρ_pers(x) u^μ(x).
```

Absorbing the charge normalisation into the loop weight gives J^μ(x) = ρ_pers(x) u^μ(x). For multiple species, ⟨j^μ(x)⟩ → Σ_a q_a ρ_a(x) u_a^μ(x), matching the multi-fluid form required by the record-current paper.

Argument. For a one-particle wavepacket f, the Dirac current is j_f^μ(x) = q f̄(x) γ^μ f(x). The Gordon decomposition splits this bilinear into a convective piece and a spin-magnetization piece,

```
q f̄ γ^μ f = (q/2m)[ f̄(i∂^μ)f − (i∂^μ f̄)f ] + (q/2m) ∂_ν( f̄ σ^{μν} f )
(sign/factor convention-dependent on the normalization of σ^{μν}),
```

the first term reducing, for a localized positive-density packet with transport four-velocity u^μ, to the convective form ∼ ρ_f u^μ. The step marked ∼ therefore *retains the convective term and discards the magnetization term* (q/2m) ∂_ν(f̄ σ^{μν} f). For an incoherent persistence-filtered ensemble with one-body density matrix Γ,

```
⟨j^μ(x)⟩ = q tr_spin( γ⁰ γ^μ Γ(x, x) ).
```

Coarse-graining over scales L ≫ ξ and imposing the local Eckart-frame condition used in the record-current hydrodynamic theorem yields ⟨j^μ(x)⟩ → q ρ_pers(x) u^μ(x). ■ (heuristic)

Sub-debt PG (magnetization cancellation). The reduction above is exact only if the discarded spin-magnetization current ∂_ν(f̄ σ^{μν} f) averages to zero under persistence-filtered coarse-graining — equivalently, if it vanishes in the comoving Eckart frame or integrates to a boundary term at the hydrodynamic scale. This is a specific, nameable condition on the ensemble, not an automatic consequence of coarse-graining: a polarized ensemble with net aligned spin would leave a residual magnetization current. Sub-debt PG is that the persistence filter produces spin-disordered (or comovingly spin-neutral) ensembles at scale L ≫ ξ. Until PG is discharged, Proposition 3 matches only the convective part of the Dirac current to J^μ. The step marked ∼ is a hydrodynamic reduction, not an operator identity; its promotion to an exact matching theorem, together with PG, is the debt named in the ledger. Graded *Conditional*.

---

14. Fock-Layer Lift of the Gauge Coupling

The record-current paper inherits the coupling theorem $\mathscr{L}_{int} = -J^\mu A_\mu$. Substituting the Fock-level current $J^\mu \mapsto q :\bar\psi \gamma^\mu \psi:$ gives the leading Fock-layer *interaction term*

$$\mathcal{L}_{int}^{Fock} = -q :\bar\psi \gamma^\mu \psi: A_\mu.$$

Scope of the claim. This is the leading matter–gauge interaction term, not the full QED Lagrangian. The standard QED Lagrangian is usually written in minimal-coupling form,

$$\mathcal{L}_{QED} = \bar\psi (i \gamma^\mu D_\mu - m) \psi - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad D_\mu = \partial_\mu + i q A_\mu,$$

whose expansion $\bar\psi(i \gamma^\mu D_\mu - m)\psi = \bar\psi(i \gamma^\mu \partial_\mu - m)\psi - q :\bar\psi \gamma^\mu \psi: A_\mu$ splits into the free Dirac term of §12 and precisely the interaction term derived here. This paper derives that interaction vertex and the current it is built from; it does *not* derive the gauge-kinetic term $-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}$, minimal coupling as an exact all-orders statement, or the radiative structure of renormalised QED. The result is the leading vertex, with the covariant-derivative packaging offered only to locate that vertex inside the familiar Lagrangian.

Theorem 4 (Conditional) — Leading QED-form matter–gauge vertex

Given (1) the inherited Matter Coupling theorem fixing $-J^\mu A_\mu$ as the unique admissible leading-order persistent-gauge interaction; (2) the record-current theorem identifying J^μ as the coarse-grained current of persistent commitment loops; (3) the present CAR/Fock construction of the spinorial source-carrier field; and (4) the bilinear-current matching of §13; the leading quantised matter–gauge interaction term on the spinorial source-carrier sector is

$$\mathcal{L}_{int}^{Fock} = -q :\bar\psi \gamma^\mu \psi: A_\mu.$$

Proof. By the inherited coupling theorem the admissible source interaction is uniquely $-J^\mu A_\mu$. By Proposition 3 the Fock bilinear current coarse-grains to J^μ, so the operator lift replaces the classical source current by its normal-ordered operator representative $J^\mu \mapsto q :\bar\psi \gamma^\mu \psi:$. The uniqueness of the vector bilinear among leading local spinorial currents rests on the parity assignment of the gauge potential. Scalar bilinears $:\bar\psi\psi:$ do not carry a free Lorentz index and cannot contract with A_μ. The axial current $:\bar\psi \gamma^\mu \gamma^5 \psi:$ is also a conserved (pseudo)vector in the massless limit and is *not* excluded by index structure alone; its exclusion rests on A_μ transforming as a *true* vector (odd under parity), inherited from the gauge sector, so that the pseudovector coupling $:\bar\psi \gamma^\mu \gamma^5 \psi: A_\mu$ is parity-odd and therefore inadmissible for the parity-even persistent electromagnetic interaction. Pseudoscalar and tensor bilinears likewise carry the wrong transformation character against a true-vector A_μ, and higher-derivative currents are suppressed by the inherited derivative-suppression structure. Hence, given the inherited true-vector parity of A_μ, $\mathscr{L}_{int}^{Fock} = -q :\bar\psi \gamma^\mu \psi: A_\mu$ as the leading interaction term. ■

The theorem inherits the *Conditional* grade of Proposition 3: it is exact given the matching, and the matching is the outstanding debt.

15. Gauge Transformation and Charge Conservation

Under a local U(1) phase transformation $\psi(x) \to e^{i q \chi(x)} \psi(x)$, $\bar\psi(x) \to \bar\psi(x) e^{-i q \chi(x)}$, the bilinear current is invariant:

$$\bar\psi \gamma^\mu \psi \to \bar\psi e^{-i q \chi} \gamma^\mu e^{i q \chi} \psi = \bar\psi \gamma^\mu \psi.$$

The coupling transforms as

$$-q :\bar\psi \gamma^\mu \psi: (A_\mu + \partial_\mu \chi) = -q :\bar\psi \gamma^\mu \psi: A_\mu - q j^\mu \partial_\mu \chi,$$

and the second term is a boundary term precisely when $\partial_\mu j^\mu = 0$. At the substrate level this conservation is inherited from $\partial_\mu J^\mu = 0$; at the field level it follows from the Dirac equation and its adjoint. The two statements coincide under the current-matching of §13. This is the operator-layer form of the earlier VERSF result:

gauge invariance ⇔ current conservation.

16. Milestone Theorem and Summary of Results

The results of §6–§15 combine into a single quotable statement.

Milestone Theorem

Let the spinorial commitment-loop sector inherit the substrate ontology (§2), the spinorial closure holonomy $U(2\pi) = -\mathbb{1}$, and the Finkelstein–Rubinstein antisymmetric exchange (§3). Then:

(i) Exact layer. Under FF-1–FF-4, the spinorial commitment-loop sector admits an antisymmetric Fock representation $\mathcal{F}_A(\mathcal{H}_1)$ whose creation and annihilation operators satisfy the full canonical anticommutation relations,

$$\{a(f), a^\dagger(g)\} = \langle f, g\rangle \mathbb{1}, \quad \{a^\dagger(f), a^\dagger(g)\} = \{a(f), a(g)\} = 0,$$

as bounded-operator identities on $\mathcal{F}_A$, with mode occupancy $N_i^2 = N_i \in \{0, 1\}$. The representation is unique up to unitary equivalence among representations admitting a Fock vacuum (a vector annihilated by all a(f)); inequivalent non-Fock representations in infinite volume are a separate question, listed as Owed below. The honest content is specific: inherited antisymmetry (FF-2a) confines states to the exterior algebra, exterior-sector surjectivity (FF-2b) identifies the sector with the full power, and positivity (FF-1) together with adjointness (FF-4) select the *positive* normalization $\{a, a^\dagger\} = +\langle f, g\rangle \mathbb{1}$ over an indefinite Krein form. FF-4 is the load-bearing hinge and is close in content to the conclusion; its discharge (the Adjoint-Removal Theorem, §7) is the natural next paper.

(ii) Conditional layer. Under the additional premise FF-5 (local completeness), the current-matching of §13 (including sub-debt PG, magnetization cancellation), sector independence $\{\psi, \psi\} = 0$, and microcausal support $\{\psi, \bar{\psi}\} = 0$ at geometric separation, the local bilinear current $q :\bar{\psi}\gamma^\mu \psi:$ coarse-grains to the inherited record current $J^\mu = \rho_{pers} u^\mu$, and lifts the inherited coupling $-J^\mu A_\mu$ to the leading QED-form matter–gauge vertex $-q :\bar{\psi}\gamma^\mu \psi: A_\mu$.

Part (i) is *Exact* given its premises; part (ii) is *Conditional*, carrying the current-matching, sector-independence, and microcausality debts. The identification of the opposite-winding sector with antiparticles remains a *Candidate* charge-conjugation reading (§11).

Proved here (Exact, given the named premises)

1. The spinorial multi-loop state space is *identified* with the antisymmetric Fock space over $\mathscr{H}_1$ — confinement to Λ^n from FF-2a, full-power surjectivity premised via FF-2b (Theorem 1).
2. Creation is wedge insertion; annihilation is its Hilbert adjoint, i.e. interior contraction (Lemma 0).
3. Creation and annihilation are bounded, $\|a^\dagger(f)\| = \|f\|$ (Corollary 2.1).
4. The full CAR algebra follows exactly, including the positive-normalized cross-relation $\{a_i, a_j^\dagger\} = \delta_{ij}$ (Theorem 2).
5. Number operators are projectors, $N_i^2 = N_i$, giving occupancy 0 or 1 (§10).

Proved here (Conditional / Audit condition).

6. A local spinorial field operator $\psi(x)$ is reconstructed from local modes (conditional on FF-5).
7. The dagger local CAR reduces to the persistent projector kernel, and to $\delta_{\alpha\beta}\, \delta^3(x - y)$ under local completeness.
8. The bilinear current $q :\bar{\psi}\gamma^\mu \psi:$ coarse-grains to the convective record current J^μ (conditional; Proposition 3, modulo sub-debt PG).
9. The leading Fock-layer gauge coupling is $-q :\bar{\psi}\gamma^\mu \psi: A_\mu$ (conditional; Theorem 4, given the inherited true-vector parity of A_μ).

Inherited. Primitive commitment-loop ontology; conservation of the record current; integer-winding topological charge; spinorial sector $U(2\pi) = -\mathbb{1}$; Finkelstein–Rubinstein exchange holonomy; antisymmetric exchange; anticommuting creation–creation and annihilation–annihilation relations; Born/Hilbert probability background; true-vector parity assignment of A_μ.

Owed. The **Adjoint-Removal Theorem** grounding FF-4 (that substrate removal is the Born/Hilbert adjoint of insertion, with no phase, normalization, or sector-leakage defect) from microscopic-origin dynamics — sharpest route: prove the transport map isometric on $\mathscr{H}_1$, which forces adjoint = removal and yields FF-1 positivity for free, collapsing two premises into one; microscopic derivation of positive-definite $\mathscr{H}_1$; exterior-sector surjectivity (FF-2b as theorem rather than premise); sector independence $\{\psi, \psi\} = 0$ (tied to the missing charge-conjugation operator C); microcausal support $\{\psi, \bar{\psi}\} = 0$ for geometrically separated points (the locality certificate); sub-debt PG (spin-magnetization cancellation under coarse-graining); the sharp (non-heuristic) current-matching theorem; full species decomposition (electron, muon, tau, quarks); non-abelian colour and weak representations; anomaly cancellation; renormalised QED; full interacting fermionic QFT; the Standard Model fermion spectrum; the quark-confinement / fermion bridge; infinite-volume representation-uniqueness questions.

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17. Falsification Conditions

Each condition names the premise it retires.

- **F1 — One-particle positivity failure (retires FF-1).** If $\mathscr{H}_1$ is not positive-definite, ordinary CAR Fock quantisation does not follow; the algebra becomes indefinite-metric / Krein-like and requires a different quantisation.
- **F2 — Exchange-confinement failure (retires FF-2a).** If odd exchange of identical spinorial loops does not act by −1 — a residual symmetric or parastatistical component — then states are not confined to $\Lambda^n\mathscr{H}_1$ and the exterior reconstruction does not even begin.
- **F2b — Exterior-sector surjectivity failure (retires FF-2b).** If the physical n-loop sector is a dynamically closed proper subspace of $\Lambda^n\mathscr{H}_1$ rather than the full power, the Fock identification overshoots: the algebra is defined over a space larger than the physical one.
- **F3 — Fock-density failure (retires FF-3).** If finite wedge products of one-particle states are not dense in the physical finite-loop sector, the operator identities may hold only on a proper subspace, and the continuity extension of Corollary 2.1 fails.
- **F4 — Annihilation-adjoint failure (retires FF-4).** If substrate admissibility supplies a number-lowering operation that is not the Hilbert adjoint of creation, the cross-relation is not forced — a failure of Born/Hilbert compatibility of the source-carrier sector.
- **F5 — Local-completeness failure (retires FF-5).** If persistent spinorial modes do not form a locally complete projector kernel in the continuum limit, the standard local field operator is not recovered; the theory may retain global CAR without a local fermion field.
- **F6 — Current-matching failure.** If q :ψ̄ γ^μ ψ: does not coarse-grain to J^μ = ρ_pers u^μ, the Fock-layer field fails to represent the record-current source (retires Proposition 3).
- **F7 — Coupling mismatch.** If the Fock current does not couple through the inherited −J^μ A_μ structure, the QED-form vertex bridge fails (retires Theorem 4).
- **F8 — Species obstruction.** If species decomposition introduces internal S_n-nontrivial structure that alters exchange statistics or produces parastatistics, the single-species CAR construction must be extended.
- **F9 — Non-abelian obstruction.** If colour or weak representations cannot be carried consistently by the spinorial Fock modes, the result remains a QED-sector construction and does not extend to the Standard Model.
- **F10 — Sector-independence failure.** If the positive- and opposite-winding sectors are not mutually independent ($\{b_n, d_m†\} \neq 0$), then {ψ_α(x), ψ_β(y)} ≠ 0 and the two-sector

field is inconsistent as a single fermion field. This is tied to the missing charge-conjugation operator C; it is a consistency condition, not a causality one.

- **F11 — Microcausality failure.** If the mode families $\{u_n\}$, $\{v_n\}$ do not assemble so that the dagger anticommutator function $\{\psi_\alpha(x), \bar{\psi}_\beta(y)\}$ vanishes for causally independent (geometrically separated) points, the reconstructed field is not local: its propagator is not confined to the causal region, even where the delta limit and sector independence are passed. This is the genuine locality certificate and is currently absent.
- **F12 — Magnetization residue (retires sub-debt PG).** If the spin-magnetization current $\partial_\nu(\bar{\psi}\sigma^{\mu\nu}\psi)$ does not average out under persistence-filtered coarse-graining — e.g. for a net-polarized ensemble — the matched current acquires a residual term and Proposition 3 fails to reproduce $J^\mu = \rho_{pers} u^\mu$.

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18. Relation to the Matter-Sector Programme

Before this paper the matter-sector programme had a substrate ontology for J^μ, a source-admissible electromagnetic coupling, spinorial source-carriers, antisymmetric exchange, Pauli exclusion, and partial CAR. After this paper it has, in addition, a one-particle spinorial loop Hilbert space, an antisymmetric Fock space, the full positive CAR algebra, bounded creation/annihilation operators, a canonical local-mode field candidate, a bilinear current, and the leading QED-form matter–gauge vertex — each at the tier and grade recorded above.

The chain is now

```
persistent commitment loops → J^μ → spinorial source-carriers
    → antisymmetric exchange → 𝓕_A → CAR → ψ(x)
    → q :ψ̄ γ^μ ψ: → −q :ψ̄ γ^μ ψ: A_μ.
```

This is the first full route from substrate current ontology to fermionic field algebra in the VERSF programme.

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19. Conclusion

This paper closes the dominant algebraic gap left by the spinorial-source-carrier paper. That paper derived the substrate-level spin–statistics correspondence, antisymmetric exchange, the anticommuting half of CAR, and Pauli exclusion, and explicitly left open the CAR cross-relation, the Fock-space construction, and one-particle positivity. The present construction supplies the Fock layer and derives the missing cross-relation.

The result is structurally simple and load-bearing:

```
inherited antisymmetry + one-particle positivity + adjointness
    ⇒ exterior Fock structure ⇒ positive-normalized CAR.
```

The cross-relation $\{a_i, a_j†\} = \delta_{ij}$ is not an independent postulate *given adjointness (FF-4)*: it is the antiderivation identity expressing that annihilation is the Hilbert adjoint of antisymmetric creation. The honest weighting is that FF-4 is load-bearing and close in content to the conclusion, and that the substantive yield of positivity (FF-1) together with FF-4 is the *selection of the positive normalization* — {a, a†} = +⟨f, g⟩ 𝟙 rather than an indefinite Krein form — since the antiderivation algebra alone fixes the matching relation for any creation-type operator, bosonic included. Boundedness (Corollary 2.1) upgrades the algebra from a dense-domain relation to a bounded-operator identity on all of 𝓕_A.

The paper then shows how the resulting local-mode field candidate reconstructs the convective part of the already-derived record current through j^μ = q :ψ̄ γ^μ ψ:, and how the inherited coupling −J^μ A_μ lifts to the leading QED-form matter–gauge vertex −q :ψ̄ γ^μ ψ: A_μ — the latter carrying the conditional grade of the current-matching debt, including the spin-magnetization sub-debt PG and the outstanding true-vector parity assignment of A_μ.

This does not derive the full Standard Model. It does something narrower and definite: it turns the spinorial source-carrier sector into a genuine fermionic Fock-space field sector, modulo the named premises. The remaining tasks are now cleanly exposed — ground FF-4 microscopically, derive one-particle positivity and Λ^n-surjectivity from substrate dynamics, establish sector independence {ψ, ψ} = 0 (via a charge-conjugation operator C) and the genuine microcausality relation {ψ, ψ̄} = 0 at geometric separation, discharge the sharp current-matching theorem with magnetization cancellation, decompose species, add non-abelian internal structure, and treat anomalies and renormalisation.

The milestone is precise: VERSF now has a conditional but explicit route from persistent topological commitment loops to fermionic quantum field operators — the inherited topology confines states to antisymmetric statistics, one-particle positivity and adjoint removal select the positive CAR, and current matching lifts the record current into the leading QED-form matter–gauge vertex.