The Gauge Action and Coupling Theorem in VERSF

▲ Programme Milestone — Standard Model Gauge-Dynamics and Coupling-Normalisation Series

Gate GAC-1 / Connection Necessity, Persistent-Holonomy Curvature, Substrate Curvature-Response Hessian, Auxiliary-Mode Schur Reduction, Local Momentum-Space Typing, Positive Gauge-Kinetic Metric, Factorwise Adjoint Isotropy, Canonical Connection Normalisation, Universal Matter and Self-Coupling Identity, Primitive Hypercharge Normalisation, Wilsonian Running, Fourteen-to-Twelve Response Firewall, and Numerical Non-Insertion Closure

Keith Taylor VERSF Theoretical Physics Programme — Standard Model Gauge-Dynamics and Coupling-Normalisation Series Tier A conditional gate-theorem paper — working manuscript

Primary predecessors: *Distinguishability Conservation and Gauge Structure*; *Closing the Interfaces*; *The Minimal Internal Symmetry Theorem*; *The Electroweak Representation and Connection Closure in VERSF*; *The Electroweak Gauge-Curvature Dynamics in VERSF*; *The Non-Abelian Bath-Transport Theorem, Continuous Colour Gauge Dynamics, and Full Gauge-Product Closure in VERSF*; *The Gauge-Closure Selection Theorem in VERSF*; *The Gauge-Census Closure Theorem in VERSF*; *The Persistent Sector Carries the Gauge Theory*; *The Microscopic Origin of the Record Current*; *The Fermionic Fock-Space Reconstruction in VERSF*; *The Origin of the Fourteen-Generator Census*; and the Yang–Mills entropy–convexity companion programme.

General Reader Summary

Earlier VERSF papers answer two questions that conventional particle physics takes as starting assumptions.

The first is **why gauge fields exist at all**. If internal quantum states at two neighbouring points of the emergent continuum are to be compared, the theory needs a rule stating how the internal frame at one point relates to the frame at the next. That rule is a connection. If the comparison depends on the route taken, the connection has curvature. In ordinary language, the force field is the local residue left when comparison around a small closed loop fails to return exactly to its starting frame.

The second is **why the Standard Model has the gauge pattern it does**. The VERSF gauge-census programme identifies the admitted local internal algebra as

$$\mathfrak{g}_{SM} = \mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y,$$

with eight colour directions, three weak directions, and one Abelian direction. Separate papers derive the corresponding colour, weak, and hypercharge connections and their curvatures.

But a connection and a curvature are not yet a dynamical theory. A theory must state what a given amount of curvature **costs**. Nor is it enough to write the familiar Yang–Mills expression by inspection. If VERSF is to derive the Standard Model rather than merely reproduce its notation, it must explain why the leading curvature cost is quadratic, why it is positive, why it is local, why it is isotropic inside each gauge factor, and why its three factorwise coefficients become the physical strong, weak, and hypercharge couplings.

This paper supplies that missing architecture.

The central idea is simple. Near a flat connection, a small curvature perturbation changes the local closure and transport state of the substrate. The substrate may also rearrange auxiliary closure modes in response. The full second variation of the local closure cost therefore has a block form: one block for gauge curvature, one for auxiliary response, and off-diagonal blocks describing their coupling. The auxiliary modes are not discarded by assumption; they are minimised out exactly. The result is a Schur-complement operator on the gauge-curvature carrier.

That operator is the **gauge-kinetic metric**.

Eliminating auxiliary modes has a price that this paper accounts for explicitly: the inverse of the auxiliary block can, in general, generate long-range structure. The reduction is therefore typed in momentum space, and a declared analyticity condition on the auxiliary response guarantees that the leading reduced operator is a genuinely local curvature stiffness rather than a disguised long-range interaction.

If the substrate response is stable, the gauge-kinetic metric is positive. If the response is gauge covariant, it is invariant under the adjoint action of the gauge group. On a simple gauge algebra such as $\mathfrak{su}(3)$ or $\mathfrak{su}(2)$, an invariant positive bilinear form is unique up to one overall coefficient. Thus the eight colour directions inherit one common stiffness Z_3, the three weak directions inherit one common stiffness Z_2, and the single Abelian direction inherits a third stiffness, quoted on the primitive charge lattice as Z_q. If, in addition, the reference substrate state is homogeneous, these stiffnesses are three constants rather than three functions of position.

The leading gauge action is therefore forced into the form

$$S_{gauge} = -\tfrac{1}{4} \int d^4x \sqrt{(-g)} \left[Z_3 G^a{}_{\mu\nu} G^{a\mu\nu} + Z_2 W^i{}_{\mu\nu} W^{i\mu\nu} + Z_Y B_{\mu\nu} B^{\mu\nu} \right],$$

with $Z_Y = Z_q/36$ the conventional-hypercharge form of the primitive Abelian stiffness, up to topological terms and explicitly ordered higher-derivative corrections.

The crucial step is then canonical normalisation. A stiff connection field has a large kinetic coefficient and therefore a small physical coupling; a soft connection field has a small kinetic coefficient and therefore a larger coupling. After rescaling each field to canonical kinetic norm,

$$(g_s, g, g') = (Z_3^{-1/2}, Z_2^{-1/2}, 6\, Z_q^{-1/2}),$$

subject to a fixed generator convention and the fixed primitive charge lattice.

The same rescaling also proves an important universality result. The number that appears in the matter covariant derivative is automatically the same number that appears in the non-Abelian three-gauge-boson and four-gauge-boson vertices. These are not separate couplings that happen to agree. They are one connection normalisation seen in three different places.

The paper is deliberately strict about what has and has not been calculated. It derives the exact operator that must be evaluated and the exact typed projection formula that returns the inverse squared couplings. It does not insert the measured values of α_s, α_{em}, the weak angle, the W or Z masses, or any grand-unification normalisation. The numerical problem is reduced to constructing and evaluating three factorwise response invariants at a declared matching scale and then running them through a declared renormalisation scheme.

One further firewall is essential. VERSF also contains a fourteen-element closure-response architecture associated with $K = 7$. That is not the twelve-dimensional Standard Model gauge algebra. The fourteen may underlie the microscopic response, but only through an explicitly derived equivariant map from the twelve gauge-curvature directions into the closure-response carrier. Without that map, no trace over fourteen closure channels may be presented as a gauge-coupling prediction.

In one sentence:

The Standard Model gauge couplings are the inverse square roots of the three positive factorwise eigenvalues of the canonically reduced, momentum-typed VERSF holonomy-response Hessian, once the gauge generators, matter currents, Abelian charge lattice, and matching scale have been independently fixed.

Scope and Closure Box

Claim	GAC-1 status
Gauge connection is required for local internal-state comparison	Inherited structural theorem
Curvature is infinitesimal distinguishability holonomy	Inherited structural theorem

Claim	GAC-1 status
Standard Model local gauge algebra is $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$	Inherited gauge-census closure
Microscopic gauge-kinetic operator is an auxiliary-reduced substrate Hessian	Derived here
Reduced operator is local at leading order	Derived conditionally (auxiliary analyticity premise P6)
Positivity follows from positive substrate stability and Schur reduction	Derived here
Leading local parity-even action is curvature quadratic	Derived conditionally from P1–P11
Cross-factor kinetic terms vanish in the minimal single-U(1) census	Derived here
Simple-factor kinetic metrics are isotropic	Derived here
Coefficients are constants on the emergent continuum	Derived conditionally (vacuum homogeneity premise P10)
Projection formula fully typed over $\Lambda^2 \otimes \mathfrak{g}_A$, well-posed on admissible data	Derived here (Lemma 6A)
Canonical normalisation gives $g_A = Z_A^{(-½)}$	Derived here
Matter and non-Abelian self-couplings are the same coupling	Derived here
Primary Abelian output is the primitive-lattice stiffness Z_q	Derived here
Coupling ratios are response-eigenvalue ratios	Derived here
Wilsonian blocking preserves the leading kinetic channel and generates higher operators	Derived structurally, in Ward-identity-preserving schemes
Numerical g_s, g, g'	Open — requires construction and evaluation of the response
Numerical weak angle and electromagnetic coupling	Downstream — require breaking and matching
Full nonperturbative BRST/Slavnov–Taylor construction	Open downstream quantum gate
Fourteen-to-twelve closure-response descent	Conditional unless an explicit equivariant map is supplied
Empirical confirmation	Not established

Abstract

The VERSF Standard Model programme derives the necessity of gauge connection structure from local distinguishability conservation, identifies curvature as path-dependent internal-state comparison, closes the local internal gauge census on

$$\mathfrak{g}\_{SM} = \mathfrak{su}(3)\_C \oplus \mathfrak{su}(2)\_L \oplus \mathfrak{u}(1)\_Y,$$

and constructs the corresponding colour and electroweak curvatures. The remaining foundational gauge-dynamics debt is not the existence of the connection or the algebraic form of its curvature, but the microscopic origin and normalisation of the curvature action and the physical gauge couplings.

This paper establishes the Gauge Action and Coupling Theorem in VERSF. Let $\mathcal{V}\_F = \Lambda^2 T^*M \otimes \mathfrak{g}\_{SM}$ be the local gauge-curvature carrier, restricted to Bianchi-admissible perturbations in the image of the linearised connection-to-curvature map, and let $\mathcal{V}\_\eta$ collect all non-gauge closure, bath, record, and completion modes that respond at the same derivative order. The closure-cost functional $\mathcal{C}\_{\mu\star}[F, \eta]$ is declared as an already coarse-grained Wilsonian effective functional at the matching scale $\mu\star$, so that the reduction performed here is a constitutive Gaussian reduction among retained response variables, not a fresh quantum integration. Around a flat, source-free, homogeneous reference configuration, the Euclidean closure cost has Hessian

$$\mathbb{H} = \begin{pmatrix} H\_{FF} & H\_{F\eta} \\ H\_{\eta F} & H\_{\eta\eta} \end{pmatrix}.$$

When $H\_{\eta\eta} > 0$, exact minimisation over the auxiliary response yields the reduced gauge-curvature operator, typed in momentum space:

$$\boldsymbol{\mathcal{K}\_g(q) = H\_{FF}(q) - H\_{F\eta}(q)\, H\_{\eta\eta}(q)^{-1}\, H\_{\eta F}(q).}$$

Under the declared auxiliary analyticity premise — $H\_{\eta\eta}(q)$ invertible and analytic in a neighbourhood of $q = 0$, with no unresolved gapless pole below $\mu\star$ — the reduced operator admits the local expansion

$$\mathcal{K}\_g(q) = \mathcal{K}\_g^{(0)} + q^2\, \mathcal{K}\_g^{(2)} + O(q^4),$$

and only $\mathcal{K}\_g^{(0)}$ determines the leading F^2 coefficient. If the full substrate Hessian is positive on physical perturbations, then $\mathcal{K}\_g \geq 0$ by Schur-complement positivity. Locality, emergent Lorentz compatibility, gauge covariance, and derivative minimality reduce the leading parity-even continuum action to

$$S\_E^{(2)} = \tfrac{1}{4} \int d^4x\, \sqrt{g\_E} \cdot \Gamma\_{AB}\, F^A\_{\mu\nu}\, F^{B\,\mu\nu},$$

with the leading tensor structure

$$\mathcal{K}\_g^{(0)}{}\_{\{A\mu\nu,B\rho\sigma\}} = \Gamma\_{AB}\, \mathbb{I}\_{\mu\nu,\rho\sigma}, \quad \mathbb{I}\_{\mu\nu,\rho\sigma} = \tfrac{1}{2} (g\_{\mu\rho}\, g\_{\nu\sigma} - g\_{\mu\sigma}\, g\_{\nu\rho}),$$

where $\mathbb{I}$ is the identity on two-forms and $\dim \Lambda^2 = 6$ in four Euclidean dimensions. Adjoint invariance, the inherited factor decomposition, and vacuum homogeneity imply

$$\Gamma = Z_3\, \mathbb{1}_8 \oplus Z_2\, \mathbb{1}_3 \oplus Z\_Y, \quad Z\_A > 0 \text{ constant},$$

and the fully typed projection is

$Z_A = (1/(6 d_A)) \mathrm{Tr}_{\Lambda^2 \otimes \mathfrak{g}_A}(P_A \mathcal{K}_g^{(0)} P_A)$, $(d_3, d_2, d_1) = (8, 3, 1)$.

Equivalently, defining the internal operator $\mathcal{K}_g = (1/6) \mathrm{Tr}_{\Lambda^2} \mathcal{K}_g^{(0)}$, one has $Z_A = (1/d_A) \mathrm{Tr}_{\mathfrak{g}_A}(P_A \mathcal{K}_g P_A)$.

Using a connection convention in which the unnormalised covariant derivative is $D_\mu = \partial_\mu + i A^A_\mu T_A$, canonical field rescaling $\hat{A}^A_\mu = Z_A^{1/2} A^A_\mu$ gives

$g_A = Z_A^{-1/2}$.

The same g_A then appears in the matter covariant derivative, the non-Abelian cubic vertex, the non-Abelian quartic vertex, and the source-coupled field equations. Coupling universality is therefore a canonical-connection identity, not an additional equality assumption.

The Abelian output is primary in the primitive basis. With integer charge $q = 6Y$ on the inherited Standard Model ledger, the primitive stiffness Z_q and coupling $g_q = Z_q^{-1/2}$ are the fundamental quantities; the conventional-hypercharge quantities are derived from them by

$Z_Y = Z_q/36$, $g' = Z_Y^{-1/2} = 6 g_q$.

No numerical Abelian coupling is meaningful before this lattice convention is fixed. The headline output is therefore

$(g_s, g, g') = (Z_3^{-1/2}, Z_2^{-1/2}, 6 Z_q^{-1/2})$.

The paper also proves a gauge scale–shape separation theorem: if $Z_A = \Lambda_G r_A$, a common unknown stiffness Λ_G cancels from coupling ratios,

$g_A^2 / g_B^2 = r_B / r_A$.

Wilsonian blocking promotes Z_A to running coefficients $Z_A(k)$ and generates higher-dimension gauge invariants; in a gauge-invariant background-field or equivalent Ward-identity-preserving scheme, the running coupling is $g_A(k) = Z_A(k)^{-1/2}$. The leading dimension-six quadratic channel is represented in the action by $\mathrm{Tr}[(D_\mu F^{\mu\nu})^2]$; $\Box \mathrm{Tr}(F^2)$ is only a momentum-space projection diagnostic because its integrated bulk contribution is a total derivative.

Finally, the paper installs a fourteen-to-twelve firewall. The $K = 7$, fourteen-element closure-response architecture is not the twelve-dimensional Standard Model gauge algebra. It contributes to the gauge-kinetic metric only if an independently derived equivariant internal embedding $E_{int} : \mathfrak{g}_{SM} \to \mathscr{H}_{14}$ exists, extended to curvature by $E_F = \mathbb{1}_{\Lambda^2} \otimes E_{int}$, in which case

$\mathcal{K}_g = E_{int}^\dagger \mathscr{H}_{14}^{red} E_{int}$.

Absent this map, no fourteen-channel trace is a gauge-coupling calculation.

GAC-1 therefore closes the structural action and coupling-normalisation theorem while leaving the construction of the response functional, the curvature embedding, the auxiliary spectrum, the matching scale, the renormalisation scheme, thresholds, and the low-energy comparison as explicit open debts. It converts the Standard Model gauge couplings from unconstrained symbols into three formally specified VERSF substrate-response invariants, whose numerical calculability depends on constructing and evaluating the declared substrate response.

Contents

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1. Notation, Conventions, and Firewalls

1.1 Gauge algebra and factor labels

The inherited local Standard Model gauge algebra is

$\mathfrak{g}_{SM} = \mathfrak{g}_3 \oplus \mathfrak{g}_2 \oplus \mathfrak{g}_1 = \mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$.

Factor labels are $A \in \{3, 2, 1\}$. Within a factor, adjoint indices are

$a, b = 1, \ldots, 8$ for colour, $i, j = 1, 2, 3$ for weak isospin,

and the single Abelian index is suppressed. The factor dimensions are

$(d_3, d_2, d_1) = (8, 3, 1)$.

1.2 Generator conventions

For the non-Abelian factors, generators are Hermitian and fixed by

$[T_a, T_b] = i f_{ab}{}^c T_c$,

with fundamental-representation normalisation

$\mathrm{tr}(T_a T_b) = \frac{1}{2} \delta_{ab}$.

Thus colour uses $T_a = \lambda_a/2$ and weak isospin uses $T_i = \sigma_i/2$.

No coupling is placed into the primitive connection definition:

$\mathcal{A}_\mu = A^A{}_\mu T_A$, $D_\mu = \partial_\mu + i \mathcal{A}_\mu$.

The curvature is

$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu + i [\mathcal{A}_\mu, \mathcal{A}_\nu]$.

The physical couplings appear only after canonical gauge-field normalisation.

1.3 Engineering dimensions and absolute connection normalisation

In four continuum dimensions,

$[A_\mu] = 1$, $[F_{\mu\nu}] = 2$, $[Z_A] = 0$.

The absolute normalisation of the connection is not an open rescaling freedom: it is fixed by the holonomy

$$\exp\left(i \oint A_\mu \, dx^\mu \right),$$

once the generator lattice and the continuum length normalisation are fixed. In particular, no overall connection rescaling can be hidden inside the definition of the curvature embedding J_F of §5: J_F is evaluated on the holonomy-normalised connection.

1.4 Euclidean and Lorentzian conventions

Positivity is stated for the Euclidean closure cost. The leading Euclidean gauge action is

$$S_E = \tfrac{1}{4} \int d^4x \sqrt{g_E} \cdot \Gamma_{AB} F^A{}_{\mu\nu} F^{B\,\mu\nu}.$$

After Lorentzian continuation, the parity-even kinetic term is

$$S_L = -\tfrac{1}{4} \int d^4x \sqrt{(-g)}\, \Gamma_{AB} F^A{}_{\mu\nu} F^{B\,\mu\nu}.$$

A positive Euclidean kinetic metric corresponds to positive physical Hamiltonian energy; the Lorentzian minus sign is not a negative-energy statement.

The inner product on the two-form carrier is declared once and used throughout:

$$\langle F, G\rangle = \tfrac{1}{2} F_{\mu\nu} G^{\mu\nu},$$

so that each independent component pair is counted once and $\mathrm{Tr}_{\{\Lambda^2\}} \mathbb{1} = 6$. The wedge expansion of §5, $F = F^A{}_{\mu\nu}\, dx^\mu \wedge dx^\nu \otimes T_A$, is deliberately written without the conventional ½; the compensating ½ resides in the inner product above, not in the expansion, and all typed traces in §11 are evaluated in this convention. Mixing the two placements would double-count and is registered in the non-insertion audit.

1.5 Hypercharge convention

The programme convention is

$$Q_{em} = T_3 + Y.$$

The inherited one-generation ledger is

$$Q_L : (3, 2, +\tfrac{1}{6}),\ L_L = (\nu_L, e_L) : (1, 2, -\tfrac{1}{2}),\ u_R : (3, 1, +\tfrac{2}{3}),\ d_R : (3, 1, -\tfrac{1}{3}),\ e_R : (1, 1, -1),$$

with the completion interface

$\Phi_{cl} : (1, 2, +\frac{1}{2})$.

For global and primitive-charge bookkeeping define

$q = 6Y$.

Then all admitted hypercharges are integers and the primitive charge lattice is fixed up to an overall sign. Throughout this paper the primitive basis (Z_3, Z_2, Z_q) is primary; the conventional-hypercharge quantities $Z_Y = Z_q/36$ and $g' = 6g_q$ are derived, per §16 and Appendix D.

1.6 Connection-versus-coupling firewall

A connection is an algebra-valued comparison field. A coupling is the inverse square root of its canonically reduced kinetic coefficient. Writing gA_μ inside the connection before the kinetic metric has been derived is a convention, not a coupling derivation.

1.7 Gauge-action-versus-higher-operator firewall

The leading kinetic coefficient multiplies F^2. A generated coefficient of

$\mathrm{Tr}[(D_\mu F^{\mu\nu})^2]$

is a higher-derivative coupling, not the leading gauge coupling. Likewise,

$\Box\, \mathrm{Tr}(F^2)$

may diagnose the momentum-space q^4 channel, but its integrated bulk action is a total derivative under ordinary boundary conditions.

1.8 Fourteen-versus-twelve firewall

The Standard Model local gauge algebra has twelve generators:

$8 + 3 + 1 = 12$.

The $K = 7$ closure-response architecture has a fourteen-element or fourteen-channel census:

$N_{loop} = 2K = 14$.

These are not the same object. No equality of counts, traces, or normalisations may be assumed between them.

1.9 Broken-phase firewall

This paper derives the unbroken gauge action and the couplings at a declared matching scale. It does not derive electroweak symmetry breaking, the Higgs vacuum, the photon, the W/Z masses, or the low-energy weak angle. Those enter only as downstream corollaries after the broken-phase module is supplied.

1.10 Topological-term firewall

Parity-even kinetic terms and parity-odd topological terms are distinct. The general local gauge action may include

$\theta_A F_A \tilde{F}_A$.

The θ-sector does not determine the canonical kinetic coupling and is not derived here.

1.11 Quantum-consistency firewall

Classical gauge covariance and anomaly-admissible currents are inherited. A complete nonperturbative BRST, ghost, and Slavnov–Taylor construction is not claimed. Background-field Ward identities are structurally compatible with the result; full quantum closure remains a named debt, and every quantum statement in this paper is conditioned on a Ward-identity-preserving scheme.

1.12 Numerical non-insertion firewall

No measured gauge coupling, weak angle, gauge-boson mass, fine-structure constant, QCD scale, or grand-unification normalisation may enter the substrate response operator, its projectors, its generator conventions, or its matching-scale selection.

1.13 Premise labels

The paper's sixteen named premises are labelled P1–P16. The label GAC-1 refers exclusively to this gate; it is never a premise label. Falsifiers cited inline use the master numbering of §23.

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2. Inherited Infrastructure

2.1 Gauge connection necessity

The local comparison of internal quantum states at distinct continuum points requires a connection on the relevant principal bundle. Distinguishability conservation makes the existence of connection structure mandatory; it does not select one physical connection configuration. Different connection configurations may have different curvature and are dynamical.

2.2 Curvature as distinguishability holonomy

The curvature

$$\mathscr{F} = d\mathcal{A} + i\, \mathcal{A} \wedge \mathcal{A}$$

measures the infinitesimal mismatch produced by transporting an internal state around a closed loop. It is therefore the local physical residue of path-dependent comparison.

2.3 Gauge-census closure

The inherited minimal connected local gauge algebra is

$$\mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y,$$

with no additional connected internal gauge channel inside the declared minimal census.

2.4 Non-Abelian transport origin

Colour and weak non-Abelian structure arise from continuous bath transport on threefold and twofold indistinguishable classes, with chirality inherited for the weak factor. The resulting local connections and non-Abelian curvatures are already available.

2.5 Existing gauge-curvature dynamics

The electroweak and colour gauge-dynamics papers establish, conditionally on curvature-norm admissibility, the familiar field strengths, Bianchi identities, absence of unbroken gauge masses, and source-coupled field equations. GAC-1 does not repeat those component derivations. It discharges the deeper debt: the microscopic origin and normalisation of the curvature norm.

2.6 Current carriers

The record-current programme supplies a microscopic persistent current, while the Fock-space reconstruction supplies the canonically normalised field current

$$J_A^\mu = \Sigma_f :\bar{\psi}_f \gamma^\mu \rho_f(T_A) \psi_f:.$$

The remaining task is to match their normalisations without absorbing the gauge coupling into the definition of the current.

2.7 Fisher and product-factor structure

The colour, weak, and Abelian directions are inherited as commuting gauge factors acting on tensor-factor representation spaces. Their generator algebras are independent even when a matter state carries all three quantum numbers.

2.8 Anomaly admissibility

Only anomaly-admissible chiral source ledgers persist as consistent gauge-current sectors. This supports quantum current conservation but does not by itself determine the kinetic coefficients.

2.9 Persistent-sector support

The gauge connection acts on the refinement-persistent transport sector, and Wilson-loop or holonomy observables register its curvature. This identifies the physical carrier on which a curvature response must be evaluated.

2.10 Wilsonian blocking support

Gauge-invariant blocking generates a local derivative expansion. The leading marginal curvature term retains its form with a running coefficient, while dimension-six and higher operators are generated. The nonredundant action-level representative of the leading dimension-six quadratic channel is

$$\mathrm{Tr}[(D_\mu F^{\mu\nu})^2].$$

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3. Purpose and Claim Level

The purpose of GAC-1 is to answer four questions.

1. **What microscopic VERSF object becomes the gauge-kinetic form?**
2. **Why is that form positive, local, and factorwise isotropic?**
3. **How do its coefficients become the physical couplings?**
4. **What must still be constructed and computed before numerical couplings can be claimed?**

The target chain is

persistent holonomy curvature ⟶ substrate response Hessian ⟶ auxiliary Schur reduction ⟶ momentum-typed local gauge-kinetic metric ⟶ (Z_3, Z_2, Z_q) ⟶ (g_s, g, g').

The paper contains three claim levels.

Exact structural results. Schur-complement reduction as constitutive Gaussian reduction, positivity, invariant-form classification, canonical normalisation, and coupling universality. These hold as mathematics wherever their hypotheses hold.

Conditional VERSF theorems. These require the existence of the declared local, already-blocked substrate closure-cost functional, its stable and analytic auxiliary spectrum, its homogeneous reference state, and the independently derived curvature embedding.

Open construction and numerical closure. Construction of the response functional and its embeddings, evaluation of the three response coefficients at a declared matching scale, and their evolution to observables.

The central claim is:

Given the inherited Standard Model connection carrier, a local gauge-covariant VERSF closure-cost functional with positive, analytic, auxiliary-reduced curvature Hessian on a homogeneous reference state, and independently fixed generator, current, and Abelian-lattice conventions, the leading gauge action is the Standard Model Yang–Mills action and the three physical gauge couplings are the inverse square roots of the three factorwise curvature-response eigenvalues.

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4. Gauge-Action Premises

P1 — Gauge carrier inheritance

The local internal connection takes values in

$\mathfrak{g}\_SM = \mathfrak{su}(3)\_C \oplus \mathfrak{su}(2)\_L \oplus \mathfrak{u}(1)\_Y$.

Status: inherited. **Falsifier:** failure of the gauge-census closure or the connection theorem.

P2 — Curvature physicality and Bianchi admissibility

The physical local field strength is the curvature of the inherited connection and represents persistent infinitesimal distinguishability holonomy. Physical curvature perturbations lie in the image of the linearised connection-to-curvature map and obey the linearised Bianchi identity; positivity and projection statements are evaluated on this admissible carrier, not on the full algebraic two-form carrier.

Status: inherited, with admissibility restriction declared here. **Falsifier:** a physical gauge field strength not represented by connection curvature.

P3 — Local closure-cost functional, Wilsonian interpretation declared

At a declared matching scale $\mu\star$, the substrate admits a local Euclidean closure-cost density

$\mathcal{C}\_\mu\star[F, \eta]$

on the direct sum of the admissible gauge-curvature carrier and all same-order auxiliary response modes. $\mathcal{C}\_\mu\star$ is declared as an already coarse-grained Wilsonian effective closure functional at $\mu\star$: all fluctuation determinants and radiative contributions generated above $\mu\star$ — including contributions of the type ½ Tr log $H\_\eta\eta$ — are contained in its coefficients. The reduction performed in this paper is therefore a constitutive Gaussian reduction among the retained response variables, not a fresh quantum integration.

Status: new load-bearing premise. **Falsifier:** no local response functional exists, or the leading response is intrinsically nonlocal (master falsifier F1).

P4 — Quadratic stability expansion

Around a flat, source-free reference configuration, $\mathcal{C}\_\mu\star$ is twice differentiable and its leading nontrivial variation is quadratic.

Status: conditional, motivated by stable-response expansion. **Falsifier:** the leading curvature cost is nonanalytic, linear, or dominated by another order (master falsifier F2).

P5 — Auxiliary stability

The auxiliary Hessian block is positive and invertible on the physical auxiliary support after exact zero modes and gauge redundancies are quotiented.

Status: conditional. **Falsifier:** a physical negative mode, unresolved null direction, or noninvertible auxiliary block contaminates the reduction (master falsifiers F4, F5).

P6 — Auxiliary analyticity and gap

On the physical auxiliary support, $H\_\eta\eta(q)$ is invertible and analytic in a neighbourhood of $q = 0$, with no unresolved gapless pole below the declared matching scale $\mu\star$. Consequently $H\_\eta\eta(q)^{-1}$ admits a convergent derivative expansion at small q, and the Schur reduction preserves locality at leading order.

Status: new load-bearing premise. **Falsifier:** auxiliary elimination produces a nonanalytic or pole-dominated gauge response at the claimed matching scale (master falsifier F3).

P7 — Gauge covariance of the response

The local closure cost is invariant under the inherited local gauge transformations. Its curvature Hessian therefore intertwines the adjoint action.

Status: required by connection physicality. **Falsifier:** the response depends on a local gauge representative rather than curvature invariants (master falsifier F6).

P8 — Factorwise carrier independence

The colour, weak, and primitive Abelian curvature carriers are distinct commuting factors at the matching scale, with no additional same-order Abelian factor.

Status: inherited from gauge-census and factorisation results. **Falsifier:** an additional admitted gauge factor or unavoidable same-order kinetic-mixing channel (master falsifier F8).

P9 — Emergent metric compatibility

The record layer supplies a Lorentzian metric and volume form, with Euclidean continuation available for positivity analysis.

Status: inherited. **Falsifier:** no local metric-compatible curvature contraction exists.

P10 — Vacuum homogeneity

The reference substrate state at the matching scale is gauge neutral, parity even, and homogeneous under the emergent continuum symmetries, so the reduced factorwise response coefficients are constants on the emergent continuum rather than functions of position, and the small-q expansion of Theorem 2 contains no odd powers.

Status: new load-bearing premise. **Falsifier:** the reduced response coefficients vary with position, with no identified scalar background accounting for the variation (master falsifier F7).

P11 — Derivative hierarchy

At the matching scale, the leading parity-even local gauge term is the lowest nontrivial derivative-order curvature scalar; higher-derivative terms are ordered corrections.

Status: inherited refinement principle. **Falsifier:** higher-derivative terms dominate the leading dynamics (master falsifier F14).

P12 — Canonical matter normalisation

Matter kinetic metrics are reduced to canonical form before the gauge current is defined. Representation generators are fixed independently of gauge-coupling data.

Status: inherited normalisation discipline. **Falsifier:** a current coefficient remains freely absorbable after canonical normalisation (master falsifier F11).

P13 — Primitive Abelian lattice

The single Abelian generator is fixed on a primitive integer charge lattice, up to overall sign, before its coupling is quoted.

Status: conditional on the inherited faithful global-group and charge-lattice audit. **Falsifier:** no canonical primitive charge lattice is selected (master falsifier F12).

P14 — Closure-to-gauge map when fourteen-channel architecture is used

If the microscopic Hessian is evaluated on the fourteen-channel closure-response carrier, an independently derived equivariant internal map from the twelve-dimensional gauge algebra into that response carrier is required, extended to curvature as $\mathbb{1}_{\Lambda^2} \otimes E_{int}$.

Status: conditional / open gate. **Falsifier:** couplings are obtained from a fourteen-channel trace without an explicit map (master falsifier F13).

P15 — Anomaly-admissible quantum ledger

All sourced chiral gauge currents are anomaly-admissible on the inherited matter census.

Status: inherited. **Falsifier:** a nonzero anomaly obstructs current conservation (master falsifier F15).

P16 — Declared matching scale and scheme

The output Z_A is defined at a declared matching scale and in a declared normalisation scheme. No direct comparison is made with a running experimental coupling without RG evolution and threshold matching.

Status: mandatory audit premise. **Falsifier:** bare, blocked, and measured couplings are compared as though identical (master falsifier F16).

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5. The Local Curvature-Response Carrier

At a continuum point x, define the gauge-curvature carrier

$$\mathcal{V}_F(x) = \Lambda^2 T^*_x M \otimes \mathfrak{g}_{SM},$$

restricted, per P2, to the Bianchi-admissible subspace: perturbations in the image of the linearised connection-to-curvature map. A formally negative direction outside that image would not describe a physical instability, so all positivity and projection statements below are evaluated on the admissible carrier.

A curvature perturbation is written

$$F = F^A_{\mu\nu}\, dx^\mu \wedge dx^\nu \otimes T_A.$$

Let $\mathcal{V}_\eta(x)$ denote all non-gauge local modes that can respond at the same order: closure deformations, bath redistributions, record-layer relaxations, completion-interface adjustments, and any other admissible auxiliary response channel.

The full local perturbation carrier is

$$\mathcal{V}_{loc} = \mathcal{V}_F \oplus \mathcal{V}_\eta.$$

The central discipline is that $\mathcal{V}_\eta$ is not simply deleted. A physical curvature perturbation generally induces substrate relaxation. The physical gauge action is the cost remaining **after** those auxiliary modes have responded optimally.

5.1 Canonical residual-map realisation of the gauge-response functional

This subsection specifies the minimal positive VERSF-compatible realisation of $\mathcal{C}_\mu\star$. Its status becomes *derived* only when the holonomy residual map and record metric are independently constructed from the substrate; until then it is the canonical form the construction must take, and the physical coupling problem is concentrated in the three objects $\mathscr{R}_{hol}$, G_{rec}, and J_F.

Let

$$\mathscr{R}_{hol}(F, \eta) \in \mathcal{V}_{rec}$$

be the local persistent-record residual produced by gauge curvature F and auxiliary response η. The flat reference satisfies

$$\mathscr{R}_{hol}(0, 0) = 0.$$

Let $G_{rec} > 0$ be the canonical record/Fisher response metric on $\mathcal{V}_{rec}$. The minimal local closure cost is

$$\boldsymbol{\mathcal{C}[F, \eta] = \tfrac{1}{2} \langle \mathscr{R}\_hol(F, \eta), G\_rec\ \mathscr{R}\_hol(F, \eta)\rangle.}$$

Linearising the residual map at the flat reference gives

$$\mathscr{R}\_hol(F, \eta) = J\_F\ F + J\_\eta\ \eta + O(2),$$

where

$$J\_F = \delta\mathscr{R}\_hol/\delta F\ |_0,\ J\_\eta = \delta\mathscr{R}\_hol/\delta\eta\ |_0,$$

with J_F evaluated on the holonomy-normalised connection of §1.3, so that no overall connection rescaling can be absorbed into its definition.

The Hessian blocks are then not arbitrary:

$$H\_FF = J\_F^\dagger\ G\_rec\ J\_F,\ H\_F\eta = J\_F^\dagger\ G\_rec\ J\_\eta,\ H\_\eta\eta = J\_\eta^\dagger\ G\_rec\ J\_\eta.$$

Define the G_rec-orthogonal projector onto the auxiliary-response image by

$$\Pi\_\eta = J\_\eta\ (J\_\eta^\dagger\ G\_rec\ J\_\eta)^{-1}\ J\_\eta^\dagger\ G\_rec.$$

Then the reduced gauge-kinetic operator takes the explicit form

$$\boldsymbol{\mathcal{K}\_g = J\_F^\dagger\ G\_rec\ (\mathbb{1} - \Pi\_\eta)\ J\_F.}$$

Only the component of curvature-induced record mismatch that cannot be cancelled by admissible auxiliary relaxation contributes to the propagating gauge stiffness. It is the gauge-sector analogue of a closure projector.

Let the local Euclidean closure cost near the reference state be

$$\mathcal{C}[F, \eta] = \mathcal{C}_0 + \tfrac{1}{2}\ (F, \eta)^\dagger\ \mathbb{H}\ (F, \eta) + O(3),$$

where

$$\mathbb{H} = \begin{pmatrix} H\_FF & H\_F\eta \\ H\_\eta F & H\_\eta\eta \end{pmatrix},\ H\_\eta F = H\_F\eta^\dagger.$$

The blocks are local operators with the continuum and internal indices suppressed.

---

6. Auxiliary-Mode Reduction and the Gauge-Kinetic Schur Complement

Theorem 1 — Exact Auxiliary-Response Reduction

Assume $H_{\eta\eta} > 0$ on the physical auxiliary support. For fixed gauge curvature F, the auxiliary minimiser is unique and equals

$$\eta^\star(F) = -H_{\eta\eta}^{-1} H_{\eta F} F.$$

Substituting this into the quadratic closure cost gives

$$\mathcal{C}_{red}[F] = \mathcal{C}_0 + \tfrac{1}{2} F^\dagger \mathcal{K}_g F + O(3),$$

with

$$\boldsymbol{\mathcal{K}_g = H_{FF} - H_{F\eta} H_{\eta\eta}^{-1} H_{\eta F}.}$$

Proof. Differentiate the quadratic form with respect to η:

$$\delta\mathcal{C}/\delta\eta = H_{\eta F} F + H_{\eta\eta}\, \eta.$$

Setting this to zero gives the stated minimiser; positivity of $H_{\eta\eta}$ makes the stationary point a strict minimum and hence unique. Direct substitution yields the Schur complement. ■

Interpretation of the reduction

Two distinct operations must not be conflated. The Schur complement above is exact for the classical minimisation of nondynamical response variables, for Gaussian elimination at quadratic order, and for the constitutive reduction of an already-blocked effective closure cost. It is not by itself a complete quantum integration of general fluctuating fields, which would additionally generate functional determinants of the type $\tfrac{1}{2}$ Tr log $H_{\eta\eta}$ and interaction-dependent contributions beyond the quadratic Hessian. Under P3, $\mathcal{C}_{\mu^\star}$ is already a coarse-grained Wilsonian functional at $\mu^\star$, so those quantum contributions reside inside its coefficients, and the reduction performed here is the remaining constitutive step among the retained variables.

A natural objection deserves pre-emption: if the auxiliary modes are gapped at $\mu^\star$, why are they still present to be minimised — and if they are light enough to respond, why were they not integrated out already? The answer is that retaining gapped η at $\mu^\star$ is deliberate tree-level effective-field-theory matching. The loop contributions of the auxiliary modes already reside in the coefficients of $\mathcal{C}_{\mu^\star}$ per P3; what remains at the matching scale is their tree-level constitutive response to background curvature. The Schur step exists precisely to make that auxiliary shift to Z_A explicit — as the correction $-H_{F\eta} H_{\eta\eta}^{-1} H_{\eta F}$ — rather than leaving it buried inside an opaque redefinition of H_{FF}. Deleting the step would not remove the physics; it would only hide it.

The physical gauge-kinetic metric is not generally the bare curvature block H_{FF}. Auxiliary relaxation lowers the cost by $H_{F\eta} H_{\eta\eta}^{-1} H_{\eta F}$, so the gauge coupling is sensitive not only to direct curvature stiffness but also to how efficiently the substrate can reorganise around curvature.

Proposition 1A — Closure-Projector Form

For the residual-map realisation of §5.1, the Schur complement equals

$\mathcal{K}\_g = J\_F^\dagger G\_{rec} (\mathbb{1} - \Pi\_\eta) J\_F.$

The operator $\mathbb{1} - \Pi\_\eta$ removes exactly the portion of the curvature-induced residual that lies in the relaxable auxiliary image. Therefore a gauge direction has zero leading stiffness if and only if its record residual is completely auxiliary-relaxable.

Proof. Substitute the residual-map Hessian blocks into Theorem 1 and use the definition of Π_η. ■

Corollary 1.1 — No auxiliary-deletion shortcut

Replacing $\mathcal{K}\_g$ by H_{FF} is valid only if $H\_{F\eta} = 0$, or if a separate theorem proves the omitted response is higher order.

---

7. Locality of the Reduced Operator and Momentum-Space Typing

Exact auxiliary elimination does not automatically preserve locality. If $H\_{\eta\eta}$ contains derivatives or gapless modes, its inverse can contain long-range poles such as $1/q^2$, and the Schur complement need not then be a local F^2 coefficient. The reduction is therefore typed in momentum space and conditioned on P6.

Theorem 2 — Local Reduction under Auxiliary Analyticity

On the homogeneous reference state (P10), write the Hessian blocks as momentum-space kernels and define

$$\boldsymbol{\mathcal{K}\_g(q) = H\_{FF}(q) - H\_{F\eta}(q)\, H\_{\eta\eta}(q)^{-1}\, H\_{\eta F}(q).}$$

Assume P6: $H\_{\eta\eta}(q)$ is invertible and analytic in a neighbourhood of $q = 0$ with no unresolved gapless pole below $\mu\star$. Then $\mathcal{K}\_g(q)$ is analytic at $q = 0$ and admits the expansion

$$\boldsymbol{\mathcal{K}\_g(q) = \mathcal{K}\_g^{(0)} + q^2\, \mathcal{K}\_g^{(2)} + O(q^4).}$$

Only $\mathcal{K}\_g^{(0)}$ determines the leading F^2 coefficient; $\mathcal{K}\_g^{(2)}$ contributes to the dimension-six channel of §19.

Proof. Analyticity of $H\_{\eta\eta}(q)^{-1}$ near $q = 0$ follows from invertibility and analyticity of $H\_{\eta\eta}(q)$ there. Products and sums of kernels analytic at $q = 0$ are analytic at $q = 0$, so $\mathcal{K}\_g(q)$ admits a convergent small-q expansion; odd powers are excluded by the parity evenness of the reference state declared in P10. Each power of q^2 corresponds to one additional derivative pair acting on F in position space, so the q^0 term is the unique derivative-free — hence local F^2 — contribution. ■

Remark — Schematic form of the $O(q^2)$ term

The notation $q^2 \mathcal{K}\_g^{(2)}$ is schematic. The $O(q^2)$ kernel generally carries $q\_\mu q\_\nu$ tensor structure in addition to a scalar q^2 piece, and it is precisely this tensor structure that seeds the independent dimension-six contractions of §19.1 at the matching scale. Only the $q \to 0$ block $\mathcal{K}\_g^{(0)}$ is structure-free in momentum, which is why it alone defines the leading F^2 coefficient.

Inline falsifier (master F3)

If auxiliary elimination produces a nonanalytic or pole-dominated gauge response at the claimed matching scale — for example, an unresolved gapless auxiliary channel contributing $1/q^2$ structure to $\mathcal{K}\_g(q)$ — master falsifier F3 fires and no local leading gauge action may be claimed.

Remark — Locality is inherited only conditionally

Without P6, the locality of the unreduced functional $\mathcal{C}\_\mu\star$ does not by itself imply locality of the reduced operator. Theorem 4 below therefore depends on Theorem 2, not merely on P3.

---

8. Positivity of the Reduced Kinetic Metric

Theorem 3 — Gauge-Kinetic Positivity

Assume the full Hessian $\mathbb{H}$ is positive semidefinite on physical perturbations and $H\_{\eta\eta} > 0$. Then

$\mathcal{K}\_g \geq 0$

on the Bianchi-admissible carrier. Moreover, $\mathcal{K}\_g F = 0$ for an admissible curvature perturbation F if and only if the pair $(F, \eta\star(F))$ is a zero-cost direction of $\mathbb{H}$. Hence if no nonzero admissible gauge-curvature perturbation generates a zero-cost pair,

$\mathcal{K}\_g > 0$.

Proof. For any admissible curvature perturbation F, evaluate the full quadratic form on the minimising pair (F, $\eta^\star(F)$):

$$(F, \eta^\star)^\dagger\, \mathbb{H}\, (F, \eta^\star) = F^\dagger\, \mathcal{K}_g\, F.$$

The left-hand side is nonnegative by positivity of $\mathbb{H}$, so $\mathcal{K}_g \geq 0$. If $F^\dagger \mathcal{K}_g F = 0$, the pair (F, $\eta^\star(F)$) is a null direction of $\mathbb{H}$; conversely a null pair of the stated form gives $\mathcal{K}_g F = 0$. Strict positivity follows when no such nonzero admissible pair exists. ■

Physical meaning

A negative eigenvalue of $\mathcal{K}_g$ would make the flat connection unstable to spontaneous growth of gauge curvature. A zero eigenvalue would describe a gauge-curvature direction with no leading local cost. Either outcome must be separately interpreted; neither is compatible with an ordinary propagating Yang–Mills mode without further structure. The admissibility restriction of P2 matters here: a formally negative direction outside the image of the curvature map is not a physical instability and does not fire a falsifier.

Inline falsifier (master F4/F5)

If any propagating Standard Model gauge direction has a negative reduced kinetic coefficient after all gauge redundancies are quotiented, master falsifier F4 fires. If an admissible gauge-curvature direction has zero leading cost with no independent physical explanation, master falsifier F5 fires. Either failure closes GAC-1 negatively.

---

9. Local Continuum Form and Yang–Mills Uniqueness

The reduced operator $\mathcal{K}_g^{(0)}$ still carries continuum two-form indices and internal gauge indices. Locality (Theorem 2), metric compatibility, gauge covariance, and derivative minimality constrain its leading form.

Theorem 4 — Leading Curvature-Action Form

Assume P7, P9, P11, and Theorem 2. Then the leading parity-even local quadratic gauge action is

$$\mathbf{S_E^{(2)} = \tfrac{1}{4} \int d^4x\, \sqrt{g_E} \cdot \Gamma_{AB}\, F^A{}_{\mu\nu}\, F^{B\,\mu\nu},}$$

with leading tensor structure

$$\mathcal{K}\_g^{(0)}\_\{A\mu\nu,B\rho\sigma\} = \Gamma\_{AB}\, \mathbb{I}\_{\mu\nu,\rho\sigma},\quad \mathbb{I}\_{\mu\nu,\rho\sigma} = \tfrac{1}{2}(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}),$$

where $\mathbb{I}$ is the identity on two-forms and $\dim \Lambda^2 = 6$ in four Euclidean dimensions. The only independent same-order local parity-odd term is

$$S_\theta = (1/32\pi^2) \int d^4x\, \Theta_{AB}\, F^A_{\mu\nu} \tilde{F}^{B\mu\nu},$$

subject to factor and global-topology constraints. All further local terms are higher derivative or higher order in curvature.

Proof. By Theorem 2 the leading reduced kernel is momentum independent. A local quadratic scalar built from two curvature tensors and no additional derivatives must contract the two-form indices with the metric or with the Levi-Civita tensor; these are the only invariant contractions available on the emergent 4-manifold. The metric contraction acts on the two-form carrier as $\mathbb{I}_{\mu\nu,\rho\sigma}$ and gives $F_{\mu\nu} F^{\mu\nu}$; the Levi-Civita contraction gives $F_{\mu\nu} \tilde{F}^{\mu\nu}$. Gauge covariance requires the internal coefficient to be an invariant bilinear form on the gauge algebra. Any other local quadratic term contains extra derivatives or violates the declared symmetries. ■

Corollary 4.1 — No bare gauge mass

A term $M^2_{AB} A^A_\mu A^{B\mu}$ is not gauge invariant on the unbroken connection carrier and is absent at this layer.

Corollary 4.2 — Curvature-square necessity is not coupling derivation

The theorem fixes the form of the action but not the values of Γ_{AB}. Those values are fixed by the reduced substrate response $\mathcal{K}\_g^{(0)}$.

---

10. Product-Factor Orthogonality

Let Γ be the internal bilinear form defined by Theorem 4 once the two-form identity $\mathbb{I}$ is factored out.

Theorem 5 — Minimal Product-Factor Orthogonality

For

$$\mathfrak{g}\_{SM} = \mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1),$$

any bilinear form invariant under the full adjoint action has no cross-term between distinct factors:

$\Gamma = \Gamma_3 \oplus \Gamma_2 \oplus \Gamma_1$.

Proof. Consider a putative invariant bilinear $B : \mathfrak{g}_3 \times \mathfrak{g}_2 \to \mathbb{R}$. Fixing the second entry defines a linear functional on the simple algebra $\mathfrak{g}_3$ invariant under its adjoint action. A simple algebra has no nonzero invariant linear functional, so $B = 0$. The same argument excludes non-Abelian–Abelian cross-terms, because an invariant linear functional on a simple adjoint representation vanishes. With only one Abelian factor, no Abelian kinetic-mixing matrix remains. ■

Important exception

If more than one independent U(1) factor were admitted, a non-diagonal Abelian kinetic matrix would generally be allowed. The single-Abelian gauge census is therefore load-bearing for the one-coefficient result.

---

11. Simple-Factor Spectral Isotropy and the Typed Projection

Theorem 6 — Adjoint-Isotropy Theorem

Let $\mathfrak{g}_A$ be a compact simple factor and let Γ_A be a positive bilinear form invariant under the adjoint action. Then

$\Gamma_A = Z_A \kappa_A$,

where κ_A is the chosen canonical invariant form and $Z_A > 0$ is a scalar.

With the generator conventions of §1.2,

$\Gamma_3 = Z_3 \delta_{ab}$, $\Gamma_2 = Z_2 \delta_{ij}$.

For the one-dimensional Abelian factor, quoted in the primitive basis,

$\Gamma_1 = Z_q$.

Proof. The centroid of a real simple Lie algebra is scalar, so every endomorphism commuting with all adjoint maps is proportional to the identity. Invariant symmetric bilinear forms therefore constitute a one-dimensional vector space generated by the canonical trace form, and positivity fixes the overall sign. See Appendix B. ■

The fully typed projection

The operator $\mathcal{K}\_g^{(0)}$ acts on $\Lambda^2T^*M \otimes \mathfrak{g}\_SM$, not on the internal algebra alone. Two equivalent typed extractions are declared, subject to the inner-product convention of §1.4.

Route 1 — full trace. Let P_A project onto factor $\mathfrak{g}\_A$. Then

$Z\_A = (1/(6\, d\_A))\, \mathrm{Tr}\_\{\Lambda^2\otimes\mathfrak{g}\_A\}(P\_A\, \mathcal{K}\_g^{(0)}\, P\_A)$, $(d_3, d_2, d_1) = (8, 3, 1)$,

where the factor 6 = dim Λ^2 removes the two-form multiplicity.

Route 2 — internal reduction first. Define the internal operator

$\mathcal{K}\_g = (1/6)\, \mathrm{Tr}\_\{\Lambda^2\}\, \mathcal{K}\_g^{(0)}$,

and then

$Z\_A = (1/d\_A)\, \mathrm{Tr}\_\{\mathfrak{g}\_A\}(P\_A\, \mathcal{K}\_g\, P\_A)$.

Both routes give the same Z_A by Theorem 4, since $\mathcal{K}\_g^{(0)} = \Gamma \otimes \mathbb{I}$ and $(1/6)\, \mathrm{Tr}\_\{\Lambda^2\}\, \mathbb{I} = 1$. Omitting the two-form normalisation would introduce a spurious factor of six into the numerical calculation; the typed formulas above are the calculation target of GAC-1.

Lemma 6A — Admissible-Data Sufficiency of the Typed Projection

The projection formulas above are stated on the full two-form carrier, while the physical data of P2 live only on the Bianchi-admissible subspace. This lemma closes that gap.

Under P7 and P9, any constant kernel on $\Lambda^2 \otimes \mathfrak{g}\_A$ invariant under the emergent rotation group decomposes as

$\mathcal{K}^{(0)}\_A = a\, \mathbb{I} + b\, \varepsilon$,

where $\mathbb{I}$ is the two-form identity of Theorem 4 and ε is the Levi-Civita (duality) pairing $\varepsilon\_\mu\nu,\rho\sigma = ½\, \varepsilon\_\mu\nu\rho\sigma$. The ε-component pairs curvatures as $F\,\tilde{F}$, which on exact curvatures — those in the Bianchi-admissible image of the connection-to-curvature map — is a total derivative and contributes only to the θ-sector of §1.10, not to the kinetic norm. Hence:

1. the Bianchi-admissible data determine the coefficient a = Z_A uniquely;
2. Routes 1 and 2 coincide for the invariantly extended kernel, since $(1/6)\, \mathrm{Tr}\_\{\Lambda^2\}\, \varepsilon = 0$.

Proof. Invariance under the emergent rotation group forces a constant kernel on two-forms to be a linear combination of the metric contraction $\mathbb{I}$ and the Levi-Civita contraction ε; by the argument of Theorem 4 these are the only invariant pairings available on the emergent 4-manifold. On an exact curvature, the ε-pairing integrates to the topological density $F\,\tilde{F}$, a total derivative under ordinary boundary conditions (§1.7), so it drops from the kinetic quadratic form on admissible perturbations, and the kinetic norm registers only a. The trace identities $\mathrm{Tr}_{\{\Lambda^2\}}\, \mathbb{I} = 6$ and $\mathrm{Tr}_{\{\Lambda^2\}}\, \varepsilon = 0$ then give Route 1 = Route 2 = a = Z_A. ■

Consequence for the blind protocol

The admissible data fix the invariant extension of the kernel up to its θ-component, which the typed projection annihilates. Concretely: the constructed $\mathcal{K}_g^{(0)}$ must be invariantly extended — or the trace restricted to the admissible subspace with the matching normalisation — before the $1/(6d_A)$ projection is applied. This requirement is registered as step 7 of the blind protocol and squares it with the double-counting item of the non-insertion audit.

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12. The Gauge Action Theorem

Theorem 7 — Gauge Action and Kinetic-Coefficient Theorem

Under P1–P11, the leading unbroken Standard Model gauge action at the declared matching scale $\mu\star$ is

$$\mathbf{S_{gauge} = -\tfrac{1}{4} \int d^4x \sqrt{(-g)}\, [\, Z_3\, G^a_{\mu\nu} G^{a\,\mu\nu} + Z_2\, W^i_{\mu\nu} W^{i\,\mu\nu} + Z_Y\, B_{\mu\nu} B^{\mu\nu}\,] + S_\theta + S_{\geq 6},}$$

with Z_3, Z_2, Z_Y constants on the emergent continuum,

$$Z_A = (1/(6\, d_A))\, \mathrm{Tr}_{\{\Lambda^2 \otimes g_A\}}[\, P_A\, (H_{FF} - H_{F\eta}\, H_{\eta\eta}^{-1}\, H_{\eta F})^{(0)}\, P_A\,],$$

$(d_3, d_2, d_1) = (8, 3, 1)$, $Z_Y = Z_q/36$ per §16, and $S_{\geq 6}$ containing explicitly ordered higher-dimension operators.

Proof. Theorem 1 gives the exact reduced response. Theorem 2 gives locality of its leading block. Theorem 3 gives positivity. Theorem 4 fixes the leading local contraction. Theorem 5 removes cross-factor terms. Theorem 6 reduces each simple-factor response to one scalar coefficient. Vacuum homogeneity (P10) promotes each coefficient from a function $Z_A(x)$ to a constant. ■

Remark — Role of the homogeneity premise

Gauge covariance forces isotropy inside each factor; it does not force constancy. Absent P10, the theorem delivers position-dependent coefficients $Z_A(x)$ and an action $-\tfrac{1}{4} Z_A(x) F_A^2$, describing position-dependent couplings or coupling to an additional scalar background. Constancy is a property of the declared reference state, not of gauge symmetry.

Corollary 7.1 — Gauge-action uniqueness at leading order

Within the declared premises, every leading parity-even local gauge action differs only through the three positive constants Z_3, Z_2, Z_q. All texture inside a simple factor is forbidden by adjoint invariance.

Corollary 7.2 — No gauge-direction texture

A proposal assigning different kinetic coefficients to different gluons or to different weak generators violates the adjoint-isotropy theorem unless the corresponding gauge factor is already broken.

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13. Canonical Connection Normalisation

The primitive connection convention contains no explicit coupling:

$$\mathcal{A}_\mu = A^a_{3\mu} T_a + A^i_{2\mu} T_i + B_\mu Y.$$

The unnormalised curvature of a non-Abelian factor is

$$F^a_{A\mu\nu} = \partial_\mu A^a_{A\nu} - \partial_\nu A^a_{A\mu} - f_A^{abc} A^b_{A\mu} A^c_{A\nu},$$

with the sign fixed by the convention $D = \partial + i\mathcal{A}$.

Define canonical fields

$$\hat{A}^a_{A\mu} = Z_A^{1/2} A^a_{A\mu}.$$

Equivalently,

$A^a_{A\mu} = g_A \hat{A}^a_{A\mu}$, $\mathbf{g_A = Z_A^{(-1/2)}}$.

Theorem 8 — Canonical Gauge-Coupling Identity

After the field redefinition above, the gauge action becomes

$$S_{gauge} = -\tfrac{1}{4} \Sigma_A \int d^4x \sqrt{(-g)}\, \hat{F}^a_{A\mu\nu} \hat{F}^a_A{}^{\mu\nu},$$

where

$$\hat{F}^a_{A\mu\nu} = \partial_\mu \hat{A}^a_{A\nu} - \partial_\nu \hat{A}^a_{A\mu} - g_A f_A^{abc} \hat{A}^b_{A\mu} \hat{A}^c_{A\nu}$$

for non-Abelian factors, and

$$\hat{B}_{\mu\nu} = \partial_\mu \hat{B}_\nu - \partial_\nu \hat{B}_\mu$$

for the Abelian factor.

Proof. Substitute $A_A = g_A \hat{A}_A$ into $Z_A F_A^2$ and use $Z_A g_A^2 = 1$. The derivative terms become canonical and the commutator term acquires one factor of g_A. ■

Physical reading

large Z_A ⇔ stiff curvature response ⇔ small g_A,

while

small Z_A ⇔ soft curvature response ⇔ large g_A.

This is the VERSF meaning of interaction strength.

---

14. Universal Matter and Self-Coupling

Let a canonically normalised matter field ψ_f transform in representation ρ_f of the gauge algebra. Before gauge-field normalisation,

$$D_\mu \psi_f = [\, \partial_\mu + i A^a_{3\mu}\, \rho_f(T_a) + i A^i_{2\mu}\, \rho_f(T_i) + i B_\mu\, \rho_f(Y) \,]\, \psi_f.$$

After canonical gauge-field normalisation,

$$D_\mu \psi_f = [\, \partial_\mu + i g_s \hat{G}^a_\mu\, \rho_f(T_a) + i g \hat{W}^i_\mu\, \rho_f(T_i) + i g' \hat{B}_\mu\, \rho_f(Y) \,]\, \psi_f.$$

Theorem 9 — Universal Connection-Coupling Theorem

For each simple gauge factor, the coefficient in the canonically normalised matter covariant derivative is identical to the coefficient in the cubic and quartic gauge self-interactions.

Thus one and the same g_A appears in:

1. the matter–gauge vertex;
2. the cubic gauge vertex;
3. the quartic gauge vertex;
4. the source-coupled field equation.

Proof. All four structures arise from one unnormalised connection A_A. A single canonical rescaling $A_A = g_A \hat{A}_A$ acts simultaneously on the matter derivative and the curvature commutator. The cubic term contains one commutator and hence one factor of g_A; the quartic term arises by squaring the commutator contribution and hence contains g_A^2. Variation with respect to the canonical field gives a source coefficient g_A. No independent coefficients remain. ■

Corollary 9.1 — One coupling per connected simple factor

Species-dependent couplings to the same simple connection are inadmissible. Different matter species carry different representation matrices, not different gauge couplings.

Corollary 9.2 — Abelian charge universality

For the Abelian factor, different interaction strengths arise from different eigenvalues of the one generator Y. There is one factor coupling g', not a separate coupling for each hypercharge.

Corollary 9.3 — Source equation

Variation of the canonical action gives

$$(\hat{D}_\nu \hat{F}_A^{\nu\mu})^a = g_A J_A^{a\mu}$$

for non-Abelian factors, and

$$\partial_\nu \hat{B}^{\nu\mu} = g' J_Y^\mu$$

for hypercharge, before electroweak breaking.

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15. Record-Current and Fock-Current Normalisation

A coupling can always be hidden by rescaling a current unless the current has been independently normalised. GAC-1 therefore separates three layers.

15.1 Microscopic record current

The record-current programme supplies a persistent transport current schematically of the form

$$J_{rec}^\mu = \rho_{pers} u^\mu,$$

with charge grade fixed by the persistent winding or ledger structure.

15.2 Field current

The Fock-space theory supplies

$$J_A^\mu = \Sigma_f :\bar{\psi}_f \gamma^\mu \rho_f(T_A) \psi_f:.$$

15.3 Canonical matching

The field kinetic term must first be brought to canonical form. If

$$\mathcal{L}_{kin} = \bar{\psi} K_f i\gamma^\mu \partial_\mu \psi, K_f > 0,$$

then

$$\psi_c = K_f^{1/2} \psi$$

is required before the physical current is read off.

Theorem 10 — Current-Normalisation Firewall

Once the following are independently fixed:

1. the canonical matter kinetic norm;
2. the representation-generator norm;
3. the primitive charge or winding unit;
4. the record-to-Fock current matching;

no continuous current rescaling remains available to absorb g_A.

Proof. A current rescaling after these four choices would alter either the one-particle charge, the representation algebra, the canonical anticommutation normalisation, or the record-current unit. Each is independently fixed, so the rescaling would change physical content rather than convention. ■

Inline falsifier (master F11)

If the current used in the source term is defined only up to an arbitrary multiplicative constant, the corresponding gauge coupling has not been derived and master falsifier F11 fires.

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16. Primitive Abelian Normalisation

The Abelian sector carries a special convention freedom. If

$Y \mapsto cY$, $g' \mapsto g'/c$,

then the product $g'Y$ is unchanged. A numerical value of g' is therefore meaningless until a generator lattice is fixed. GAC-1 removes this ambiguity by making the primitive basis primary throughout: the fundamental Abelian outputs are Z_q and g_q, and all conventional-hypercharge quantities are derived from them.

The inherited matter and completion-interface ledger admits the integer normalisation

$q = 6Y$.

On the minimal one-generation ledger,

$q(Q_L) = 1$, $q(u_R) = 4$, $q(d_R) = -2$, $q(L_L) = -3$, $q(e_R) = -6$, $q(\Phi_{cl}) = 3$.

The greatest common divisor of the nonzero charges is one. This fixes the primitive lattice up to sign.

Theorem 11 — Primitive Hypercharge-Coupling Theorem

Let q be the primitive integer generator and let the canonically normalised Abelian covariant derivative be

$D_\mu = \partial_\mu + i\, g_q\, \hat{B}_\mu\, q$.

Then

$\mathbf{g_q = Z_q^{(-1/2)}}$

is invariant under all remaining continuous generator rescalings. The conventional-hypercharge quantities are derived:

$\mathbf{Z_Y = Z_q/36,\ g' = Z_Y^{(-1/2)} = 6\, g_q,}$

so that in the conventional generator $Y = q/6$,

$D_\mu = \partial_\mu + i\, g'\, \hat{B}_\mu\, Y$.

The overall sign of q is paired with the sign of g_q and has no physical effect.

Proof. Primitive integrality removes continuous rescaling freedom. Substituting $q = 6Y$ into $g_q\, q = g'Y$ gives $g' = 6g_q$; the kinetic identity $Z_Y = Z_q/36$ follows from $Zg^2 = 1$ in each convention. ■

Corollary 11.1 — Headline coupling triple

The primary output triple of GAC-1 is (Z_3, Z_2, Z_q), and

$$(g_s, g, g') = (Z_3^{-1/2}, Z_2^{-1/2}, 6\, Z_q^{-1/2}).$$

Corollary 11.2 — No automatic 5/3 factor

A grand-unification normalisation such as $g_1^2 = (5/3)\, g'^2$ is not part of GAC-1 unless a larger unifying embedding is independently derived. It may not be inserted, because it is conventional in a different framework.

---

17. Coupling-Ratio and Scale–Shape Separation

The response coefficients can be separated into a common scale and factorwise shape when the substrate Hessian admits

$$Z_A = \Lambda_G\, r_A,\quad r_A > 0.$$

Here Λ_G is a common overall gauge stiffness and r_A are dimensionless response-shape invariants.

Theorem 12 — Gauge Scale–Shape Separation

If $Z_A = \Lambda_G\, r_A$, then

$$g_A = \Lambda_G^{-1/2}\, r_A^{-1/2}$$

and

$$g_A^2 / g_B^2 = r_B / r_A.$$

Thus coupling ratios are derivable even when the common absolute stiffness remains unknown.

Proof. Immediate from $g_A = Z_A^{-1/2}$. ■

Projected response invariants

A natural definition is

$r_A = (1/(6\, d_A\, \Lambda_G))\, \mathrm{Tr}_{\Lambda^2\otimes \mathfrak{g}_A}(P_A\, \mathcal{K}_g^{(0)}\, P_A)$.

If no common factorisation exists, the three Z_A remain independently calculable and the ratio formula still holds as

$g_A^2 / g_B^2 = Z_B / Z_A$.

Corollary 12.1 — Weak-angle response formula

At a common scale, after converting the primitive Abelian coefficient to the conventional Y normalisation,

$\sin^2\theta_W = g'^2 / (g^2 + g'^2) = Z_2 / (Z_Y + Z_2)$, $Z_Y = Z_q/36$.

This is a downstream structural corollary, not yet a numerical prediction.

---

18. Fourteen-to-Twelve Closure-Response Descent

The closure-architecture programme contains a K = 7 response carrier with a fourteen-element or fourteen-channel structure. The gauge-curvature carrier is twelve-dimensional internally. To use the former in calculating the latter, a typed map is required.

Typing of the map

If the fourteen-channel carrier $\mathscr{H}_{14}$ is purely internal, the required object is the internal embedding

$E_{int} : \mathfrak{g}_{SM} \longrightarrow \mathscr{H}_{14}$,

extended to the curvature carrier by

$E_F = \mathbb{1}_{\Lambda^2} \otimes E_{int} : \Lambda^2 T M \otimes \mathfrak{g}_{SM} \longrightarrow \Lambda^2 T M \otimes \mathscr{H}_{14}$.

If instead $\mathscr{H}_{14}$ also carries orientation or loop-plane data of the emergent continuum, the domain must be the full curvature carrier,

$E_F : \Lambda^2 T^*M \otimes \mathfrak{g}_{SM} \longrightarrow \mathscr{H}_{14}^{full}$,

and the rank and equivariance conditions below apply to E_F directly. The rank-12 condition in Definition 18.1 presumes the internal typing; whichever typing is used must be declared before any trace is computed.

Definition 18.1 — Admissible gauge-response embedding

An internal embedding E_int is admissible only if:

1. **Rank:** rank E_int = 12 on the local gauge algebra.
2. **Equivariance:** E_int ∘ Ad_SM(g) = ρ_{14}(g) ∘ E_int.
3. **Factor typing:** colour, weak, and Abelian projectors are mapped without cross-identification.
4. **Gauge-null removal:** gauge redundancies and pure coordinate directions have zero physical norm before projection.
5. **No double counting:** the loop-doubling factor and boundary-frame factor are not counted twice.
6. **Metric compatibility:** the induced internal gauge-kinetic form is Γ = E_int† $\mathscr{H}_{14}$^red E_int.
7. **Multiplicity resolution:** if any SM-irreducible component of $\mathscr{H}_{14}$ appears with multiplicity greater than one, the embedding direction within the multiplicity space must be independently derived; an undetermined multiplicity angle is a hidden continuous parameter and fires the numerical non-insertion audit.

Theorem 13 — Closure-to-Gauge Response Descent

If an admissible embedding E_int exists, the internal gauge-kinetic operator may be computed from the fourteen-channel reduced response as

$\mathcal{K}$_g = E_int† $\mathscr{H}_{14}$^red E_int,

and consequently

Z_A = (1/d_A) Tr_{g_A}(P_A E_int† $\mathscr{H}_{14}$^red E_int P_A).

Proof. The pullback of a positive quadratic form through an equivariant embedding is a positive invariant quadratic form on the gauge algebra. Theorems 5 and 6 then give factor orthogonality and isotropy. ■

Corollary 13.1 — Structural prediction for the K = 7 carrier

Admissibility conditions 1–3 force the SM-decomposition of $\mathscr{H}_{14}$ to contain the adjoint pattern 8 ⊕ 3 ⊕ 1. In the multiplicity-one case this leaves a two-dimensional gauge-inert complement, so the fourteen-channel carrier must decompose under the gauge action as

$\mathscr{H}_{14}$ ≅ 8 ⊕ 3 ⊕ 1 ⊕ (gauge-inert 2).

This upgrades §18 from a pure firewall to a checkable structural prediction about the K = 7 architecture: if the independently derived closure-response carrier fails to contain 8 ⊕ 3 ⊕ 1, no admissible embedding exists and the fourteen-channel route to the couplings is closed. It also sharpens Debt 11 to a concrete target: derive the SM-decomposition of $\mathscr{H}_{14}$ and, wherever a multiplicity exceeds one, derive the embedding direction within the multiplicity space. (If the inert complement is itself gauge-trivial, the Abelian direction inherits a multiplicity space and clause 7 applies to it.)

Remark — A live arithmetic loophole

The multiplicity clause is not hypothetical caution. The count 8 + 3 + 3 = 14 is achievable, so a carrier could match the fourteen-count while doubling the weak triplet. Such a carrier would lack the required Abelian singlet — already fatal under conditions 1–3 — and, in the doubled triplet space, would leave a continuous embedding angle undetermined. Without clause 7, a fitted multiplicity angle could masquerade as a derived coupling ratio. The clause converts that failure mode from a silent fit into an audited violation.

Without E_int, the expression $Tr_{14}(\mathscr{H}_{14})$ contains no factor typing and cannot determine g_s, g, g′. A numerical use of 14, 7, 2K, seven rotations, or seven reflections in a coupling formula is inadmissible unless the map showing how those structures resolve onto the twelve gauge directions is explicitly supplied.

Status

The theorem is exact if E_int is derived. The derivation of E_int is an open programme gate and not silently assumed here.

---

19. Wilsonian Blocking, Running Couplings, and the Higher-Curvature Hierarchy

The coefficients Z_A are defined at a matching scale. Coarse-graining promotes them to scale-dependent effective coefficients.

Let $\mathcal{B}_{k\leftarrow\mu\star}$ be a local gauge-invariant blocking map. The blocked effective action has the derivative expansion

$$\Gamma_k = \int d^4x \sqrt{g_E} \left[\tfrac{1}{4} \Sigma_A Z_A(k) F_A^2 + \Sigma_r (c_{A,r}(k)/k^{d_r-4}) \mathcal{O}_{A,r} \right].$$

Theorem 14 — Gauge-Kinetic RG Compatibility

Under local gauge-invariant blocking:

1. the leading dimension-four kinetic channel remains factorwise of the form $Z_A(k) F_A^2$;

2. in a gauge-invariant background-field or equivalent scheme in which the relevant Ward or Slavnov–Taylor identities are maintained, the running connection coupling is defined by $g_A(k) = Z_A(k)^{-1/2}$;
3. higher-dimension operators do not become part of the definition of $g_A(k)$ unless the chosen matching scheme explicitly mixes them;
4. canonical normalisation may be performed at each scale without changing physical observables.

Proof sketch. Gauge invariance and power counting close the dimension-four quadratic channel on F_A^2. Blocking changes its coefficient and generates irrelevant operators. In a background-field-type scheme the relevant Ward identities tie vertex and field-strength renormalisation together, so a field rescaling at scale k defines the canonical effective field and therefore the running coupling. Scheme changes redistribute finite pieces but do not alter the need for a declared matching prescription. ■

Remark — Quantum scope of the identity

Classically, $g_A = Z_A^{-1/2}$ is exact in the chosen connection convention. Quantum mechanically, identifying the running coupling solely with the gauge-field kinetic coefficient is licensed only in schemes maintaining the appropriate Ward identities; in a general scheme, vertex and field-strength renormalisation need not preserve the classical identity. Full BRST and Slavnov–Taylor closure is Debt 12, so Theorem 14 claims no more than the background-field-conditioned statement above.

19.1 Leading dimension-six quadratic channel

The blocked two-point function has

$$\Gamma^{(2)}_{A,k}(q) = Z_A(k)\, q^2 + (\lambda_A(k)/k^2)\, q^4 + O(q^6/k^4),$$

with the q^4 channel seeded at the matching scale by the $O(q^2)$ kernel of Theorem 2, including its $q_\mu q_\nu$ tensor structure. The momentum-space q^4 coefficient may be diagnosed using a projection related to $\Box\, \mathrm{Tr}(F_A^2)$. The nonredundant action-level representative is

$$\boldsymbol{\mathcal{O}_{DFD,A} = \mathrm{Tr}[\, (D_\mu F_A^{\mu\nu})(D_\rho F_A{}^{\rho}{}_{\nu})\,].}$$

Thus

$$\Gamma_k \supset (\lambda_A(k)/k^2) \int d^4x\, \mathcal{O}_{DFD,A}.$$

19.2 Running does not retroactively derive the boundary value

The RG equation determines how Z_A changes once a boundary value is supplied. It does not by itself derive the VERSF matching value $Z_A(\mu\star)$. The substrate-response theorem supplies the boundary condition; Wilsonian running transports it.

19.3 General running relation

In a perturbative regime, with convention

$$\beta_A(g_A) = \mu\, dg_A/d\mu = -(b_A/16\pi^2)\, g_A^3 + \cdots,$$

one has

$$1/g_A^2(\mu) = 1/g_A^2(\mu\star) + (b_A/8\pi^2) \ln(\mu/\mu\star) + \Delta_A^{scheme} + \Delta_A^{threshold} + \cdots.$$

The coefficients depend on matter content, generator normalisation, and scheme. GAC-1 fixes the boundary response, not the entire downstream renormalisation calculation.

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20. Observable Matching and Renormalisation Firewall

A coupling quoted by experiment is a scheme- and scale-dependent quantity. A VERSF response coefficient is a matching-scale effective-action coefficient. They may be compared only through a declared bridge.

Definition 20.1 — Matching data

A complete comparison specifies:

1. matching scale $\mu\star$;
2. primitive generator convention;
3. gauge-field normalisation;
4. renormalisation scheme;
5. active matter content;
6. threshold spectrum;
7. loop order or nonperturbative running method;
8. observable definition.

Theorem 15 — Bare-to-Observable Matching Discipline

Let

$g\_A^V(\mu\star) = Z\_A(\mu\star)^{(-1/2)}$

be the GAC-1 output. A low-energy observable coupling is

$g\_A^{obs}(\mu) = \mathscr{M}\_A\mu, \mu\star$, scheme, thresholds, matter.

No equality

$g\_A^V(\mu\star) = g\_A^{obs}(\mu)$

is meaningful without specifying $\mathscr{M}\_A$.

Allowed matching routes

- background-field effective-action coupling;
- gradient-flow coupling;
- Schrödinger-functional coupling;
- lattice step scaling;
- perturbative $\overline{\text{MS}}$ matching where valid.

Preferred gauge-invariant route

A gradient-flow definition provides a gauge-invariant operational bridge:

$g^2\_\{A,\text{flow}\}(k) = \mathcal{N}\_A\, t^2 \langle E\_A(t)\rangle |\_\{t = 1/k^2\}$,

with $\mathcal{N}\_A$ fixed by the chosen generator convention. Matching this to the VERSF blocked coefficient remains a finite, explicit calculation.

---

21. Broken-Phase Corollaries

The following are not premises of GAC-1. They become available after the electroweak completion interface and broken-phase modules are supplied.

Corollary 21.1 — Weak mixing angle

At a common renormalisation scale,

$\tan \theta\_W = g'/g$.

Therefore

$\sin^2\theta_W = g'^2 / (g^2 + g'^2) = Z_2 / (Z_Y + Z_2)$, $Z_Y = Z_q/36$.

Corollary 21.2 — Electromagnetic coupling

After the unbroken photon direction is identified,

$e = g \sin \theta_W = g' \cos \theta_W = g g' / \sqrt{(g^2 + g'^2)}$.

Hence

$\alpha_{em} = e^2 / 4\pi$.

Corollary 21.3 — Gauge-boson mass consistency

Given an independently derived completion scale v_{cl}, the standard broken-phase relations become consistency tests:

$m_W^2 = \frac{1}{4} g^2 v_{cl}^2$, $m_Z^2 = \frac{1}{4} (g^2 + g'^2) v_{cl}^2$.

These equations may test a coupling derivation. They may not be inverted using measured masses and then presented as a VERSF prediction.

---

22. Derivation Summary and Open Debts

22.1 Derived here

GAC-1 derives the following. The microscopic gauge-kinetic operator is the Schur complement of the substrate closure-response Hessian after auxiliary modes are exactly minimised, and under auxiliary analyticity that reduction is local at leading order and typed in momentum space. Positive substrate stability implies a positive reduced gauge-kinetic metric on the Bianchi-admissible carrier. Locality, metric compatibility, gauge covariance, and derivative minimality force the leading parity-even curvature-square action; the minimal product gauge algebra forbids cross-factor kinetic terms; adjoint invariance forces one kinetic coefficient per simple factor; and vacuum homogeneity promotes the coefficients to constants. The three coefficients are the fully typed projected response invariants $Z_A = (1/(6d_A)) \mathrm{Tr}_{\{\Lambda^2\otimes g_A\}}(P_A \mathcal{K}_g^{(0)} P_A)$. Canonical normalisation gives $g_A = Z_A^{(-1/2)}$, and matter coupling, cubic self-coupling, quartic self-coupling, and source coupling are one canonical connection normalisation. The Abelian output is primary on the primitive lattice, with $(g_s, g, g') = (Z_3^{(-1/2)}, Z_2^{(-1/2)}, 6Z_q^{(-1/2)})$. Coupling ratios are inverse response ratios. Wilsonian blocking preserves the leading kinetic channel in Ward-identity-preserving schemes while generating higher-curvature operators. A fourteen-channel closure architecture contributes only through a typed equivariant internal embedding.

22.2 Not derived here

The numerical values of Z_3, Z_2, Z_q; the matching scale; the low-energy couplings, weak angle, and fine-structure constant; the QCD confinement scale; any grand-unification relation; the θ-angles; threshold spectra; full nonperturbative quantum gauge fixing; and every empirical comparison remain open. The precise register is the debt ledger below.

22.3 Open debts

1. **Explicit substrate closure functional** — construct $\mathcal{C}_{\mu\star}[F, \eta]$ directly from the VERSF substrate rather than postulating its differentiability and locality.
2. **Curvature embedding** — derive the map from persistent holonomy curvature into the primitive substrate perturbation carrier, on the holonomy-normalised connection.
3. **Auxiliary carrier census** — prove that all same-order response modes are included in η and that no omitted mode changes the Schur complement.
4. **Auxiliary positivity, gap, and null-space audit** — identify and quotient all exact zero modes, and establish the analyticity/gap condition of P6, before inverting $H_{\eta\eta}$.
5. **Numerical factorwise response** — evaluate Z_3, Z_2, Z_q without measured-coupling input.
6. **Primitive Abelian closure** — complete the faithful global-group and charge-lattice derivation at the same claim level as the local gauge algebra.
7. **Record-to-Fock current matching** — fix the absolute current unit by matching persistent record winding to the canonical field bilinear.
8. **Matching scale** — derive or independently declare the scale at which the substrate response is an effective gauge-kinetic coefficient.
9. **Scheme bridge** — compute finite matching between the VERSF blocked scheme and a standard gauge-invariant running-coupling scheme.
10. **Threshold spectrum** — supply all states active between $\mu\star$ and the comparison scale.
11. **Fourteen-to-twelve map** — derive the SM-decomposition of $\mathscr{H}_{14}$, verify that it contains $8 \oplus 3 \oplus 1$ per Corollary 13.1, resolve any multiplicity spaces per clause 7 of Definition 18.1, and derive the equivariant embedding E_{int} if the fourteen-channel closure response is used microscopically.
12. **Quantum Ward closure** — construct the gauge-fixed quantum theory, ghost sector, and Slavnov–Taylor identities without anomaly or regulator leakage.
13. **Higher-curvature control** — bound the generated dimension-six and higher operators sufficiently to ensure the leading coupling extraction is stable.
14. **Topological sector** — derive or classify the allowed θ-terms and their global-group periodicities.
15. **Empirical comparison** — only after Debts 1–13 are discharged may the derived couplings be compared with measured running couplings.

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23. Falsification Conditions

F1 — No local response. The substrate has no local continuum closure-cost functional for persistent curvature.

F2 — Nonquadratic leading order. The first nontrivial stable curvature response is not quadratic.

F3 — Nonlocal-reduction failure. Auxiliary elimination produces a nonanalytic or pole-dominated gauge response at the claimed matching scale.

F4 — Negative physical kinetic mode. The reduced response has a negative eigenvalue on a propagating, Bianchi-admissible gauge direction.

F5 — Unremoved null mode. A Standard Model gauge-curvature direction has zero leading cost with no independent physical explanation.

F6 — Gauge-covariance failure. The response depends on the local gauge representative rather than curvature invariants.

F7 — Vacuum-inhomogeneity failure. The reduced response coefficients vary with position, with no identified scalar background accounting for the variation.

F8 — Factor-mixing failure. An unavoidable same-order colour–weak, colour–hypercharge, or weak–hypercharge kinetic cross-term survives the minimal gauge census.

F9 — Isotropy failure (derived falsifier of P7). Different generators within an unbroken simple factor receive inequivalent kinetic coefficients. By Theorem 6 this cannot occur while P7 holds, so F9 is not an independent condition: its firing entails a gauge-covariance failure and therefore also fires F6. It is retained as a separately observable symptom.

F10 — Coupling non-universality. Matter and self-interaction vertices require independent couplings after canonical normalisation.

F11 — Current-normalisation failure. The current retains a free multiplicative normalisation.

F12 — Abelian-lattice failure. No primitive hypercharge generator can be fixed.

F13 — Fourteen-to-twelve failure. A proposed closure-response descent has wrong rank or typing, is non-equivariant, or double-counts a factor of two.

F14 — Derivative-hierarchy failure. Higher-dimension operators dominate at the claimed matching scale.

F15 — Quantum anomaly failure. The sourced ledger fails anomaly closure.

F16 — RG bridge failure. No consistent scheme matching connects the substrate coefficient to a measurable running coupling.

F17 — Numerical failure. Once evaluated blindly and run correctly, the three couplings disagree with observation beyond the declared uncertainty and threshold freedom.

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24. Numerical Non-Insertion Audit

The following operations are forbidden.

1. Choosing Z_3, Z_2, Z_q to reproduce measured couplings.
2. Using m_W, m_Z, or the Higgs vacuum to infer g, g′ before the response calculation.
3. Fixing hypercharge normalisation from the measured weak angle.
4. Inserting the grand-unified 5/3 factor without deriving a unifying embedding.
5. Setting the matching scale by searching for the best numerical fit.
6. Using measured α_s to normalise the substrate Hessian.
7. Treating λ_A, the dimension-six coefficient, as Z_A.
8. Replacing the Schur complement by the bare curvature block without proving decoupling.
9. Omitting the two-form trace normalisation 1/6, double-counting it, or double-placing the ½ of the two-form inner product (§1.4) in the Z_A projection.
10. Tracing over fourteen closure channels and dividing them among twelve gauge directions by arithmetic.
11. Counting the loop factor of two and the boundary-frame dimension twice.
12. Rescaling the matter current after seeing the coupling result.
13. Comparing a bare, blocked, gradient-flow, and $\overline{\text{MS}}$ coupling without finite matching.
14. Omitting heavy thresholds that materially affect running.
15. Selecting among substrate branches after consulting measured couplings.
16. Reporting a postdiction as a first-principles prediction.

Blind calculation protocol

A valid numerical calculation must proceed in this order:

1. freeze the gauge algebra, generators, primitive charge lattice, and holonomy normalisation;
2. freeze the substrate state and matching-scale rule, and verify homogeneity (P10);
3. derive the curvature and auxiliary embeddings on the admissible carrier;
4. build the full Hessian in momentum space;
5. remove exact redundancies and verify the auxiliary gap (P6);
6. compute the Schur complement $\mathcal{K}$_g(q) and extract $\mathcal{K}\_g^{(0)}$;
7. invariantly extend the constructed $\mathcal{K}\_g^{(0)}$ — or restrict the trace to the admissible subspace with the matching normalisation, per Lemma 6A — then project Z_3, Z_2, Z_q with the typed 1/(6d_A) normalisation;
8. convert g_q to the declared hypercharge convention;
9. publish the matching-scale output;

10. only then perform RG evolution and empirical comparison.

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25. GAC-1 Closure Certificate

Gate question

Has VERSF converted the gauge action and coupling symbols into calculable substrate objects without importing their measured values?

Closure answer

The reduction and normalisation architecture is closed conditionally. The VERSF dynamical derivation remains open until the response functional, curvature embedding, auxiliary spectrum, and matching prescription are independently supplied.

Certificate

GAC-1 is closed at the theorem-architecture level if the following objects are supplied:

$\{ \mathcal{C}[F, \eta], H_{FF}(q), H_{F\eta}(q), H_{\eta\eta}(q), P_3, P_2, P_1, q, \mu_\star \}$.

The physical outputs are then fixed by

$\mathcal{K}_g(q) = H_{FF}(q) - H_{F\eta}(q)\, H_{\eta\eta}(q)^{-1}\, H_{\eta F}(q)$, $\mathcal{K}_g(q) = \mathcal{K}_g^{(0)} + q^2\, \mathcal{K}_g^{(2)} + O(q^4)$,

$Z_A = (1/(6\, d_A))\, \mathrm{Tr}_{\Lambda^2 \otimes g_A}(P_A\, \mathcal{K}_g^{(0)}\, P_A)$,

$g_A = Z_A^{-1/2}$, $(g_s, g, g') = (Z_3^{-1/2}, Z_2^{-1/2}, 6\, Z_q^{-1/2})$.

No further dynamical choice enters the leading coupling extraction.

The gate remains open on the dynamical side until the listed inputs are independently constructed and evaluated.

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26. Conclusion

The VERSF gauge programme establishes the structural route internal distinguishability ⟶ local comparison ⟶ connection ⟶ curvature ⟶ $\mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$.

The missing question was why this curvature has the Yang–Mills action and why the three forces have the strengths they do.

GAC-1 answers the first question conditionally and converts the second into a formally specified calculation.

The gauge action is the local quadratic cost of persistent distinguishability holonomy after every same-order auxiliary substrate mode has been allowed to respond. Mathematically, that cost is the Schur complement of the complete closure-response Hessian, typed in momentum space so that its leading block is a genuinely local stiffness. Stability makes it positive. Gauge covariance makes it adjoint invariant. Product-factor independence makes it block diagonal. Simplicity of SU(3) and SU(2) makes it isotropic inside each factor. Homogeneity of the reference state makes its coefficients constants.

The entire leading gauge sector is therefore encoded in three numbers:

Z_3, Z_2, Z_q.

Canonical normalisation converts them into

$$\mathbf{(g_s, g, g') = (Z_3^{-1/2}, Z_2^{-1/2}, 6\, Z_q^{-1/2})}$$

in the declared generator convention. The same normalisation forces each coupling to appear universally in matter transport and non-Abelian self-interaction.

This paper does not pretend that the numerical couplings have already been obtained. Its advance is architectural: it identifies the unique typed substrate operator that must be calculated, proves the exact reduction, locality, and normalisation theorems, and removes the conventional freedoms that would otherwise let a fitted number masquerade as a prediction.

The next numerical gauge gate is therefore sharply defined:

Construct the VERSF holonomy-response Hessian on the admissible carrier, verify the auxiliary gap and vacuum homogeneity, derive any required fourteen-to-twelve embedding, evaluate the three typed factorwise Schur-complement traces blindly at the matching scale, and run them to observation without parameter insertion.

If that calculation succeeds, the gauge strengths of the Standard Model will no longer be external parameters of VERSF. They will be measurable consequences of the substrate's resistance to persistent curvature.

Appendix A — Schur-Complement Proof

Let

$$Q(F, \eta) = \tfrac{1}{2} F^\dagger H_{FF} F + F^\dagger H_{F\eta} \eta + \tfrac{1}{2} \eta^\dagger H_{\eta\eta} \eta.$$

Completing the square gives

$$Q(F, \eta) = \tfrac{1}{2} (\eta + H_{\eta\eta}^{-1} H_{\eta F} F)^\dagger H_{\eta\eta} (\eta + H_{\eta\eta}^{-1} H_{\eta F} F) + \tfrac{1}{2} F^\dagger (H_{FF} - H_{F\eta} H_{\eta\eta}^{-1} H_{\eta F}) F.$$

The first term is nonnegative and vanishes exactly at the minimiser. This proves both the reduction and the positivity of the Schur complement.

If $H_{\eta\eta}$ has exact null modes, replace the inverse by the Moore–Penrose inverse only after projecting to the physical support and proving that the mixed block has no component along unresolved null directions. Otherwise the reduction is ill typed. The analyticity condition of P6 must additionally be verified in momentum space; a Moore–Penrose inverse on the physical support does not by itself exclude a gapless pole in the retained channels.

Appendix B — Invariant-Form and Isotropy Proof

Let B be a symmetric bilinear form on a compact simple Lie algebra $\mathfrak{g}$ satisfying

$$B([X, Y], Z) + B(Y, [X, Z]) = 0.$$

Choose the canonical trace form κ. There exists an endomorphism L such that

$$B(X, Y) = \kappa(LX, Y).$$

Invariance implies

$$L \circ \mathrm{ad}_X = \mathrm{ad}_X \circ L$$

for all $X \in \mathfrak{g}$. The centroid of a real simple Lie algebra is scalar: every endomorphism commuting with all adjoint maps is proportional to the identity. Therefore

$$L = Z\,\mathbb{1},$$

and

$B = Z\,\kappa$.

Positivity gives $Z > 0$.

For distinct simple ideals $\mathfrak{g}_A$ and $\mathfrak{g}_B$, a cross-bilinear would define an invariant map between inequivalent adjoint representations. Such a map vanishes. A cross-term between a simple ideal and $\mathfrak{u}(1)$ would define an invariant vector in the simple adjoint, which also vanishes.

Appendix C — Canonical-Normalisation Identities

Begin with one simple factor:

$$\mathcal{L} = -\tfrac{1}{4} Z\, F^a_{\mu\nu} F^{a\,\mu\nu} + \bar{\psi}\, i\gamma^\mu (\partial_\mu + i A^a_\mu T_a)\, \psi.$$

Set

$$A^a_\mu = g\, \hat{A}^a_\mu,\quad g = Z^{-1/2}.$$

Then

$$F^a_{\mu\nu} = g\,[\, \partial_\mu \hat{A}^a_\nu - \partial_\nu \hat{A}^a_\mu - g\, f^{abc} \hat{A}^b_\mu \hat{A}^c_\nu\,] = g\, \hat{F}^a_{\mu\nu}.$$

Therefore

$$-\tfrac{1}{4} Z\, F^2 = -\tfrac{1}{4} Z\, g^2\, \hat{F}^2 = -\tfrac{1}{4} \hat{F}^2.$$

The matter term becomes

$$\bar{\psi}\, i\gamma^\mu (\partial_\mu + i\, g\, \hat{A}^a_\mu T_a)\, \psi.$$

Expanding $\hat{F}^2$ gives derivative, cubic, and quartic terms with coefficients 1, g, and g^2, respectively.

Appendix D — Primitive Hypercharge Dictionary

The primitive basis is primary; the two common conventional bases are derived from it.

Convention A

$Q = T_3 + Y$,

with

$Y(Q\_L) = +1/6$, $Y(L\_L) = -1/2$, $Y(\Phi) = +1/2$.

Convention B

$Q = T_3 + Y\_SM/2$,

with

$Y\_SM = 2Y$.

The primitive integer generator is

$q = 6Y = 3Y\_SM$.

If

$D = \partial + i\ g\_q\ B\ q$,

then

$g' = 6\ g\_q$, $Z\_Y = Z\_q/36$

in Convention A, and

$g\_\{Y\_SM\} = 3\ g\_q$, $Z\_\{Y\_SM\} = Z\_q/9$

in Convention B.

All physical predictions depend on the generator–coupling product, not on the isolated convention-dependent number.

Appendix E — Wilsonian Operator Hierarchy

At a scale k, the pure-gauge effective action may be organised as

$$\Gamma_k = \int d^4x \sqrt{g} \left[\tfrac{1}{4} Z F^2 + (\lambda_{DFD}/k^2) \mathrm{Tr}[(D_\mu F^{\mu\nu})^2] + (\lambda_{F^3}/k^2) \mathrm{Tr}(F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu) + O(k^{-4}) \right].$$

The roles are distinct:

- Z: leading inverse squared coupling;
- λ_{DFD}: quadratic q^4 response and higher-derivative coercivity, seeded by $\mathcal{K}_g^{(2)}$;
- λ_{F^3}: higher-point non-Abelian interaction;
- dimension-eight terms: further suppressed corrections.

Integration by parts, Bianchi identities, and field redefinitions must be handled at the action level. A total derivative may serve as a projection diagnostic but not as a bulk interaction.

Appendix F — Dependency and Falsifier Map

Result	Depends on	Principal falsifier
Connection carrier	Gauge census, connection theorem (P1)	P1 falsifier
Admissible curvature carrier	Bianchi admissibility (P2)	P2 falsifier
Curvature response	P2–P4	F1–F2
Schur reduction	P5	F5
Local reduction	Auxiliary analyticity (P6)	F3
Positivity	Full Hessian stability	F4
F^2 action	Locality, metric, derivative hierarchy (P7, P9, P11)	F14
Constant coefficients	Vacuum homogeneity (P10)	F7
Factor orthogonality	Product gauge algebra, one U(1) (P8)	F8
Isotropy	Adjoint invariance (P7)	F9 (derived; entails F6)
Typed projection	Two-form normalisation (§11)	Audit item 9
Canonical couplings	$Z_A > 0$	F10
Current universality	Canonical matter/current norm (P12)	F11
Primitive g′	Charge lattice (P13)	F12
Fourteen-channel descent	Equivariant E_{int} (P14)	F13
Running and matching	Ward-preserving scheme, RG, thresholds (P16)	F16
Empirical prediction	All previous rows	F17