The Gauge-Census Closure Theorem in VERSF

▲ Programme Milestone — Standard Model Gauge-Completion Series Gate GC-1 / SM-Gauge Closure

Proving that the VERSF internal gauge census closes on the Standard Model gauge pattern and leaves no additional connected internal gauge channel

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General Reader Summary

The Standard Model of particle physics says that nature's internal gauge structure has exactly three sectors — three internal forces, in everyday language. The strong force glues quarks together into protons and neutrons. The weak force drives radioactive decay. The third, called hypercharge, is the ancestor of electromagnetism. Physicists write these with the symbols SU(3), SU(2), and U(1), but for this summary the symbols matter less than the number: **three**.

For fifty years the theory has simply written that list down. Experiments found the forces, and the mathematics was built around them. That leaves two questions open. Why these three? And — the question of this paper — why **only** these three? How do we know there isn't a fourth force quietly missing from the inventory, waiting to be discovered?

Earlier papers in this programme tackled the first question. This paper answers the second. It treats the problem as a **census**: count every force that nature's bookkeeping rules permit, admit the ones that pass, reject the rest, and prove the list is closed.

The key idea is that a force is not a label you are free to bolt onto a theory. In this framework, a force can exist only where nature has a genuine **redundancy** — a place where you could swap some labels around without changing anything physically real. The three quark "colours" are like three identical seats at a table: since nothing distinguishes them, you can relabel them freely. But if the relabelling can be done differently at every point in space, something has to keep track of how the labels at one place relate to the labels next door. That tracking machinery **is** the force. No redundancy, no force; a real redundancy, and a force is mandatory.

So the paper audits nature's inventory and finds exactly three redundancies: quarks come in three interchangeable colours; certain particles come in interchangeable left-handed pairs; and there is exactly one leftover freedom to shift a quantum phase. Three redundancies, three forces — and then the audit turns to everything else. A mirror-image weak force, extra copies of electromagnetism, forces that rotate the three generations of matter into one another, hidden forces, grand unified super-forces: each is checked against the admission rules and each fails a specific, named test. It either acts on nothing real, duplicates something already counted, breaks the left-right structure of the weak force, stops particles from acquiring mass, or fails a deep consistency requirement called anomaly cancellation.

The most telling moments in the audit are the near-misses — the **impostors**. Twice, a candidate force passes almost every test and has to be hunted down by the last one:

- The consistency rules alone allow **two** possible versions of the third force: the real hypercharge, and an impostor that would ignore electrons and neutrinos entirely. The impostor dies because electrons must be able to acquire mass — and the two candidates could not have coexisted anyway.
- On the full three-family version of the theory, a subtler impostor appears: a force that tells muon-family particles apart from tau-family particles. It passes **every** consistency check. It dies only because nature demonstrably mixes the families into one another — a fact this programme has committed to elsewhere — and a force of that kind would forbid exactly that mixing.

The presence of impostors is good news, not bad. A rigged derivation — one that secretly assumed its answer — would have nothing to eliminate. This one displays two distinct pretenders and names the specific rule that kills each.

Honesty about the fine print. The paper does not derive everything from nothing: the facts that quarks come in threes, that the weak pairs are left-handed, and that the families mix are inputs, established or committed in earlier papers of the programme and inherited here with their sources named. And the paper deliberately stays away from questions it cannot answer: it says nothing about how **strong** the forces are, why particles have the masses they do, how the weak force breaks its symmetry, or why matter comes in three families. Those are separate problems.

What it proves is a completeness statement, and in one sentence it is this:

Given how matter is actually structured, the three forces of the Standard Model are not just a possible list — they are the complete list, and nothing can be added without breaking a named rule.

In the physicists' symbols: the admitted internal gauge structure is $SU(3) \times SU(2) \times U(1)$ — and nothing else.

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Abstract

The VERSF Standard Model programme requires a closure theorem for the internal gauge census. Recovering individual Standard Model gauge factors is insufficient unless the theory also proves that no additional independent internal gauge channel remains admissible. This paper supplies that closure.

A gauge channel is defined as a connected local automorphism of the class-grain commitment census that preserves admissible continuations, mass-pairing structure, chirality, anomaly discipline, and quotient canonicity. The definition distinguishes physical gauge redundancy from global relabelling, accidental notation, hidden member-grain amplitudes, and transformations acting on absent census slots.

The VERSF internal census decomposes into three and only three primitive connected degeneracy classes: colour triplicity, left chiral fold doublethood, and one abelian access carrier. The colour triplicity admits the traceless local unitary relabelling group SU(3)_c. The left chiral fold doublet admits SU(2)_L. The remaining abelian ledger is fixed, up to normalization and field-redefinition conventions, by anomaly discipline together with mass-pairing admissibility, yielding U(1)_Y. The rigidity result is proved in strengthened three-part form: anomaly discipline alone reduces the admissible generation-universal abelian charge assignments to exactly two lines — the hypercharge line and one exceptional lepton-blind line — which are moreover mutually anomalous and hence cannot be jointly gauged; mass-pairing admissibility then annihilates the exceptional line, the residual up/down exchange being a field-redefinition convention (u ↔ d with H ↔ H̃) rather than a second physical ledger; and a mixing-graph universality lemma (Lemma 4c) eliminates the generation-dependent anomaly-free family — of which gauged L_μ − L_τ is the sharpest member — because on a connected committed flavour-continuation graph, any generation-non-universal gauge charge annihilates a committed inter-generation continuation. At local Lie-algebra level, the admitted gauge algebra is therefore

$$\mathfrak{g}\_\text{VERSF} = \mathfrak{su}(3)\_c \oplus \mathfrak{su}(2)\_L \oplus \mathfrak{u}(1)\_Y.$$

The finite-centre quotient (SU(3)_c × SU(2)_L × U(1)_Y)$/\mathbb{Z}_6$ is compatible as a global identification and, under the quotient-maximality convention, fixes the hypercharge lattice on the committed representations; it introduces no additional local gauge boson.

The exclusion proof shows that an extra U(1), SU(2)_R, generation gauge symmetry, mirror weak sectors, primitive lepto-colour unification, and hidden internal groups each fail at least one census condition: they act on no committed degeneracy, violate chirality, duplicate the hypercharge ledger, require additional uncommitted matter slots, disrupt anomaly balance, or identify histories with distinct continuation cones.

The result is theorem-level relative to the inherited VERSF/BCB gauge-foundation results (gauge necessity, colour triplicity, weak doublethood, abelian access existence) and the minimal Standard Model commitment census. Its residual conditionality is confined to four items: the minimal-census boundary itself, the single Higgs access slot, the connectedness of the committed flavour-continuation graph, and the quotient-maximality convention. Within that frame, the internal algebra is exact: the VERSF gauge census closes on the Standard Model pattern and leaves no additional connected internal gauge channel.

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0. Inheritance, Notation, and Firewalls

0.1 The inherited gauge problem

The gauge problem is often stated too weakly. A theory does not complete the Standard Model gauge sector merely by displaying the groups SU(3), SU(2), U(1). It must also explain why those are the admitted groups and why additional groups are not admitted.

The VERSF gauge problem is therefore a **census problem**:

Count every admissible connected internal gauge channel, admit the ones that satisfy the commitment rules, exclude the rest, and prove that the list is closed.

The target local algebra is

$$\mathfrak{g}\_SM = \mathfrak{su}(3)\_c \oplus \mathfrak{su}(2)\_L \oplus \mathfrak{u}(1)\_Y.$$

At global group level one may write G_SM = SU(3)_c × SU(2)_L × U(1)_Y, or, after finite-centre identification,

$$G\_SM\text{\^{}faithful} = (SU(3)\_c \times SU(2)\_L \times U(1)\_Y) / \mathbb{Z}_6.$$

The finite quotient affects global representation faithfulness. It does not add a gauge boson and does not change the local gauge-boson census. This paper works primarily at local connected Lie-algebra level; the finite-centre quotient is recorded as a compatible global refinement with one structural consequence (charge quantization, §9.3).

0.1a Inheritance from the Gauge-Foundation Track

GC-1 is not an isolated derivation of the Standard Model gauge group from a blank page. It is a census-closure theorem over the prior VERSF/BCB gauge-foundation track: the Minimal Internal Symmetry programme (projective-probability classification, the $n \leq 3$ bound, and the three-body singlet route to the SU(3), SU(2), U(1) architecture), the Distinguishability Conservation programme (gauge fields as the necessary transport structure for local comparison of internal states), and the Closing the Interfaces programme (reduction of the phenomenological inputs to minimal empirical anchors, strengthened abelian uniqueness, and the chirality-selection interface). Theorem-level citations for each inherited premise are given in §1.2a.

GC-1 uses those inherited results in a narrower role. It does not re-prove the gauge-foundation programme. Instead, it asks the census question that remains after those results are admitted:

Given the inherited colour triplicity, left chiral weak doublethood, gauge-connection necessity, and an abelian access carrier, does any additional connected internal gauge channel remain admissible?

The theorem proved here answers no. The admitted connected internal gauge algebra is exhausted by

$$\mathfrak{su}(3)\_c \oplus \mathfrak{su}(2)\_L \oplus \mathfrak{u}(1)\_Y.$$

Thus the role of GC-1 is closure, not first discovery: it proves that the inherited gauge ingredients form a complete and maximal minimal Standard Model gauge census.

0.2 Three objects must be kept separate

The paper separates:

1. **Census slots** — the committed class-grain degrees of freedom on which internal transformations may act.
2. **Gauge channels** — connected local automorphisms of those slots preserving physical admissibility.
3. **Embeddings and notational symmetries** — larger or alternative groups that may organize the same slots but do not constitute additional primitive gauge channels.

This distinction is critical. A larger group containing $SU(3)_c \times SU(2)_L \times U(1)_Y$ as a subgroup is not automatically an admitted primitive gauge group. It becomes one only if its extra generators act on live VERSF census degeneracies, preserve admissible continuations, and satisfy the same anomaly and mass-pairing constraints.

0.3 Notation

Groups and algebras:

$G_c = SU(3)_c$, $G_L = SU(2)_L$, $G_Y = U(1)_Y$; $\mathfrak{g}_c = \mathfrak{su}(3)_c$, $\mathfrak{g}_L = \mathfrak{su}(2)_L$, $\mathfrak{g}_Y = \mathfrak{u}(1)_Y$; $\mathfrak{g}_{int} = \mathfrak{g}_c \oplus \mathfrak{g}_L \oplus \mathfrak{g}_Y$.

One generation of left-handed Weyl census slots is denoted

$Q, u^c, d^c, L, e^c,$

where Q is the left quark doublet; u^c, d^c are the left-handed conjugates of the right up and down quarks; L is the left lepton doublet; e^c is the left-handed conjugate of the right charged lepton. A sterile neutral singlet, if present, is denoted N^c. It is not part of the minimal charged gauge census unless it is assigned a non-trivial gauge ledger; a fully sterile singlet with zero internal charges does not enlarge the internal gauge group.

The Higgs access doublet is denoted H.

Hypercharge is normalized by the convention

$Q_{em} = T_3 + Y/2,$

so the familiar one-generation assignments are

$Y(Q) = 1/3$, $Y(u^c) = -4/3$, $Y(d^c) = 2/3$, $Y(L) = -1$, $Y(e^c) = 2$, $Y(H) = 1$.

The theorem derives these ratios up to overall normalization. Some programme documents state hypercharge in the right-handed basis with the $Q_{em} = T_3 + Y$ normalization; the exact map between conventions is fixed in Appendix D and should be used for all cross-citations.

0.4 Firewalls

This paper proves:

- the admissible connected internal gauge census closes on the Standard Model algebra;
- no additional connected internal gauge algebra survives the VERSF gauge-census filters;
- the hypercharge ledger is unique up to normalization and field-redefinition conventions under the minimal census, in the strengthened three-part sense of §7.

It does **not** derive: the values of g_3, g_2, g_1; gauge-coupling unification; the Higgs potential; electroweak symmetry breaking; fermion masses; neutrino masses; CKM or PMNS mixing; the χ mass-hierarchy increments; or the number of generations, except insofar as one-generation anomaly and census structure is replicated.

It also does not claim that grand-unified embeddings are impossible as mathematical embeddings. It claims that their extra generators are not primitive gauge-census channels unless separately admitted by the VERSF continuation and matter-slot rules.

0′. Predictive-Content Ledger

Object	Prior status	Status here
Internal gauge-census problem	open	closed conditionally
SU(3)_c colour channel	expected / inherited	derived from colour triplicity
SU(2)_L weak channel	expected / inherited	derived from left chiral fold doublethood
U(1)_Y hypercharge channel	expected	derived as unique abelian ledger up to scale and convention (three-part rigidity: classification, mass-pairing, mixing-graph universality)
Extra U(1) (generation-universal)	possible risk	excluded; anomaly-safe alternatives classified and eliminated
Generation-difference U(1) (e.g., L_μ − L_τ)	possible risk; passes all anomaly filters	excluded by mixing-graph universality (Lemma 4c), conditional on a connected committed flavour-continuation graph
SU(2)_R	possible risk	excluded by chiral census asymmetry
Generation/flavour gauge	possible risk	excluded as non-census degeneracy
Primitive lepton-colour unification	possible risk	excluded as extra-generator overreach
Mirror weak sector	possible risk	excluded by chirality and continuation cones
Hidden internal gauge group	possible risk	excluded unless new committed matter slots exist

Object	Prior status	Status here
Finite $\mathbb{Z}_6$ quotient	global refinement	admitted; no local gauge-boson change; fixes charge quantization
Gauge couplings	open	outside scope
Symmetry breaking	open	outside scope
Mass hierarchy	open	outside scope

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1. Purpose, Claim Level, and Named Premises

1.1 The question

What is the maximal connected internal gauge algebra admitted by the VERSF Standard Model commitment census?

The answer: $\mathfrak{g}$_VERSF = $\mathfrak{su}(3)$_c ⊕ $\mathfrak{su}(2)$_L ⊕ $\mathfrak{u}(1)$_Y. No further connected internal gauge algebra survives.

1.2 Claim level

Claim level: GC-1 gauge-census closure theorem, conditional on premises P1–P10 (P1, P4–P6 restating inherited results I1–I4) together with, on the three-generation census, the connectedness of the committed flavour-continuation graph; residual conditionality itemized in §14.1.

Epistemic grade: Conditional structural closure. The algebraic steps are exact once the premises are accepted.

The most important conditional premises are: colour triplicity; left-chiral weak doublethood; the abelian access carrier (with abelian uniqueness proved, not premised, in §7); anomaly discipline; no independent sterile charged ledger; and no-free-generator minimality. The paper does not claim to prove those inherited structural inputs from nothing. It proves that, given them, the gauge census is forced and closed.

1.2a Programme Inheritance Map: Why P1–P6 Are Not New Assumptions

The most load-bearing premises in GC-1 are inherited rather than newly introduced. Each is anchored to a named result in the gauge-foundation track so that the dependency is auditable.

P1, gauge-as-local-census-redundancy, is inherited from the distinguishability-conservation foundation (Distinguishability Conservation and Gauge Structure, Theorem 4.2, together with the no-caustics result, Theorem 3.1). Local distinguishability conservation requires state comparison across neighbouring fibres of the base geometry; state comparison requires parallel transport; and parallel transport requires connection structure — with the connection unique up to gauge freedom and no weaker structure sufficing. Gauge fields are therefore the local comparison machinery of distinguishability-preserving physics, not optional group labels. The residual gap declared in that track — that the connection required by BC1 is the standard gauge connection at operator level — is recorded here as falsifier F18 rather than silently absorbed.

P4, colour triplicity, is inherited from the Minimal Internal Symmetry programme (Theorem 5.6, the three-body singlet theorem) together with the anchor formulation of Closing the Interfaces (Proposition 3.1, Anchor A). The threefold colour carrier is selected by the conjunction of the projective-probability classification, the $n \leq 3$ capacity bound, and the three-body singlet theorem: SU(3) is the minimal special unitary group admitting a fundamental three-body singlet, matching the observed three-constituent baryon anchor without requiring SU(4)-type enlargement. The primary carrier of this premise is the representation-theoretic result, which is independent of any capacity constant; the curvature-capacity bound ($n \leq 3$) is convergent secondary support.

P5, left chiral fold doublethood, is inherited from the weak-sector and interface results, and rests on two independent mechanisms. The two-level internal sector forces SU(2) as the connected continuous symmetry of a fundamental two-state carrier (Minimal Internal Symmetry, Theorems 7.1–7.4; Closing the Interfaces, Proposition 3.2, Anchor B). The restriction to a single Weyl sector is then delivered twice: by the fold-orientation topology route (Minimal Internal Symmetry, Theorem 7.10, using $H^1(\mathbb{CP}^1) = 0$), and by the overconservation route (Closing the Interfaces, Theorem 5.3), which excludes bidirectional coupling. The overconservation route is graded here as a BC1+BC2 result — independent conservation of two chiral currents is not by itself an inconsistency of gauge theory (vector-like theories exist), so the exclusion draws on minimality as well as conservation — while the topological route is independent of that grading. The observed identification of the active sector as left-handed is a single discrete empirical input, fixing the remaining orientation convention.

P6, the abelian access carrier, is inherited in two layers, and is deliberately stated weakly: it routes abelian candidates to §7 rather than presupposing their number. First, the universal quantum phase redundancy forces an abelian gauge direction once local comparison is required (Distinguishability Conservation, Theorems 5.1–5.3). Second, the interface analysis restricts charge-type abelian freedom: any additional anomaly-free abelian charge on the committed chiral content is proportional to hypercharge (Closing the Interfaces, Theorem 4.3). GC-1 then sharpens this inheritance by giving a three-part hypercharge rigidity proof: anomaly discipline alone leaves exactly two classified generation-universal candidate lines, joint anomaly safety prevents their coexistence, mass-pairing selects the Standard Model hypercharge line, and mixing-graph universality eliminates the generation-dependent remainder (§7).

GC-1 therefore has a precise claim level. It is not saying:

Assume the Standard Model gauge group.

It is saying:

Given the inherited VERSF/BCB results that force the colour, weak, and abelian census sources, the connected internal gauge census closes uniquely on SU(3)_c, SU(2)_L, and U(1)_Y, and every proposed additional connected gauge channel fails a named admissibility condition.

1.2b Conditionality Reduction: What Is Assumed, Inherited, and Proved Here

GC-1 reduces its conditionality by separating three categories.

Inherited theorem-level inputs. Gauge necessity, colour triplicity, weak doublethood, and the existence of an abelian access direction are not introduced as fresh assumptions in GC-1. They are inherited from the prior VERSF/BCB gauge-foundation track: the Distinguishability Conservation programme supplies the connection/gauge-necessity result; the Minimal Internal Symmetry programme supplies the projective-sector and colour/weak carrier results; and the Closing the Interfaces programme supplies the minimal-anchor, chirality, and abelian-interface refinements. These are labelled I1–I4 in §1.3.

Minimal-census definitions. GC-1 works on the minimal Standard Model commitment census (§3.0): the committed slots Q, u^c, d^c, L, e^c, H, replicated across complete generations, with one Higgs access slot serving the mass-pairing structure. A proposal that adds sterile charged matter, hidden charged sectors, mirror weak doublets, or additional Higgs ledgers is not a missed gauge factor inside GC-1. It is a non-minimal extension and must open a separate gate.

Results proved inside GC-1. The paper proves the census closure over those inherited and minimal-census ingredients. In particular, the abelian result is not assumed. GC-1 classifies the universal abelian candidates, proves the mutual-anomaly obstruction, eliminates the exceptional lepton-blind line by mass-pairing, and then eliminates generation-dependent ledgers by continuation-cone preservation on the connected flavour graph.

Thus GC-1 is less conditional than a raw P1–P10 theorem: P1, P4, P5, and the inherited abelian access direction are imported theorem-level results; the uniqueness of U(1)_Y is proved here; and the remaining conditionality is confined to the minimal-census boundary and the flavour-continuation commitments used in Lemma 4c (itemized in §14.1).

1.3 Inherited results and named premises

The load-bearing inputs divide into inherited theorem-level results and GC-1's own premises. The inherited results, with sources in §1.2a:

Inherited Result I1 — Gauge necessity. Local distinguishability conservation over the base geometry requires connection structure for fibrewise state comparison. Gauge structure is therefore forced wherever a live class-grain internal ambiguity must be compared locally.

Inherited Result I2 — Colour triplicity. The colour carrier is the threefold quark-class realization slot selected by the prior projective-probability and three-body-singlet results.

Inherited Result I3 — Weak doublethood. The weak carrier is the two-level left chiral fold-access slot selected by the prior two-state symmetry and chirality-selection results.

Inherited Result I4 — Abelian access existence. The universal quantum phase redundancy supplies an inherited abelian access direction once local comparison is required.

GC-1 proves the closure theorem over I1–I4. It does not re-prove them. Premises P1, P4, P5, and P6 below restate I1–I4 in census language so that the admissibility conditions of §2.3 can cite them uniformly; P7–P10 are GC-1's own premises and definitions.

Premise P1 — Gauge-as-local-census-redundancy (restates I1). A gauge channel is a connected local redundancy of class-grain description. It is not a global naming convention. It is admitted only when a local change of internal basis preserves all physical commitment data and requires a compensating connection.

Premise P2 — Class-grain carrier. Gauge structure acts at class grain. It is not owned by individual member facts as primitive private labels. Member records register class-owned gauge structure but do not create independent member-grain gauge groups.

Premise P3 — Finite committed census. The Standard Model internal gauge census acts on a finite set of committed matter-access slots. A connected gauge generator with no non-trivial action on any committed slot is not an admitted gauge channel.

Premise P4 — Colour triplicity (restates I2). The quark class carries exactly three indistinguishable colour realization slots. The local relabelling of those slots is a real physical gauge redundancy, while leptons are colour singlets.

Premise P5 — Left chiral fold doublethood (restates I3). The weak-access sector contains twofold left chiral doublets, $Q = (u, d)_L$ and $L = (\nu, e)_L$, while the corresponding right chiral charged slots are weak singlets in the minimal Standard Model census. The weak gauge action is therefore left chiral.

Premise P6 — Abelian access carrier (restates I4, weakly). After separating colour and weak traceless relabellings, any remaining connected abelian internal gauge generator acts as a phase/access ledger on committed census slots. The universal quantum phase redundancy supplies at least one inherited abelian access direction. Whether any additional independent abelian ledger survives the census filters is not assumed here; it is decided by the abelian-census analysis of §7.

Premise P7 — Mass-pairing admissibility. The admitted gauge assignments must allow the required fermion mass-pairing access through the Higgs doublet. In one-generation notation, the gauge-neutral pairings are QHu^c, $QH^\dagger d^c$, and $LH^\dagger e^c$; on the multi-generation census, the diagonal pairings $Q_iHu_i^c$, $Q_iH^\dagger d_i^c$, and $L_iH^\dagger e_i^c$ are committed per generation, with the single Higgs slot serving all of them. These are stated at the gauge-census level only; the theorem does not compute Yukawa magnitudes.

Premise P8 — Anomaly discipline. A connected internal gauge channel is admissible only if its gauge, mixed, gravitational, and global anomalies cancel on the committed chiral census. An anomalous current is not a stable local gauge redundancy.

Premise P9 — No independent sterile charged ledger. A completely sterile singlet may be admitted as a gauge-neutral slot without enlarging the gauge group. An independent extra gauged ledger acting non-trivially on a sterile or hidden slot is not part of the minimal Standard Model gauge census unless a separate committed continuation cone is exhibited for it. This premise blocks accidental promotion of uncommitted hidden labels into gauge symmetries.

Premise P10 — No-free-generator minimality. No generator is admitted merely because it can be written algebraically. A generator must act non-trivially on a live census degeneracy and preserve all VERSF admissibility constraints. Larger groups whose extra generators mix sectors with different continuation cones are embeddings or extensions, not primitive census closures.

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2. What Counts as a Gauge Channel

2.1 Gauge is stronger than relabelling

A global relabelling is a change of names. A gauge channel is different. A gauge channel is a **local** redundancy:

$\psi(x) \mapsto g(x)\, \psi(x)$,

where $g(x)$ may vary over the physical base, and physical predictions remain invariant after introducing the corresponding connection.

In VERSF language: the internal basis chosen for a class-grain census slot is not physical; local changes of that basis must not change the committed record; the connection records the comparison of bases between neighbouring commitment contexts. Gauge structure is not optional decoration. It exists only where the theory has a genuine local ambiguity of class-grain description.

2.1a Gauge Necessity from Local Distinguishability Comparison

The definition of a gauge channel used in GC-1 is inherited from the distinguishability-conservation foundation.

A local internal state at base point x lives in the fibre over x. A neighbouring internal state at x + dx lives in a different fibre. The expression "the distinguishability between these two states" is not defined until a rule is supplied for comparing the two fibres. That rule is parallel transport.

A smooth, equivariant, Fisher-metric-preserving local comparison rule is precisely a connection on the relevant principal bundle. Therefore, in the VERSF/BCB setting, gauge structure is forced by the local comparability of distinguishability. It is not imposed because physicists like local symmetry; it is required because without connection structure, the continuity of distinguishability across the base geometry is not even well-posed.

This justifies the census criterion used here:

A connected internal gauge channel is admitted only where there is a live class-grain commitment degeneracy whose local basis comparison requires a connection.

A group acting on no committed degeneracy is therefore not an unused gauge symmetry waiting to be discovered. It is empty notation. Conversely, where a committed local ambiguity exists and distinguishability must be compared across the base geometry, a connection is required. GC-1 counts exactly those connection-requiring internal ambiguities.

2.2 Definition: committed census slot

A **committed census slot** is a class-grain matter-access position that can carry physical registrations and participate in admissible continuations. Examples in the minimal Standard Model census: Q, u^c, d^c, L, e^c, H. A slot may carry colour, weak, and hypercharge structure. A transformation that acts only on an absent or physically inert slot is not part of the admitted gauge census.

2.3 Definition: admissible gauge generator

A generator X is an **admissible internal gauge generator** if:

1. X acts non-trivially on at least one committed census slot;
2. X preserves the final class-grain census;
3. X preserves admissible continuation cones;
4. X preserves chirality assignments;
5. X preserves mass-pairing admissibility;
6. X is anomaly-safe, jointly with all other admitted generators;
7. X is not a duplicate of another already-counted generator;
8. X does not require uncommitted matter slots.

Condition 6 is stated in the joint form deliberately: two generators that are individually anomaly-safe but mutually anomalous cannot both be admitted. This joint reading does real work in §7.

The admitted gauge algebra is the span of all admissible generators, closed under commutator.

2.4 Definition: gauge-census closure

The internal gauge census is **closed** when every admissible connected internal generator belongs to the generated algebra $\mathfrak{su}(3)\_c \oplus \mathfrak{su}(2)\_L \oplus \mathfrak{u}(1)\_Y$, and every generator outside that algebra fails at least one admissibility condition.

3. The VERSF Gauge Census Module

3.0 Minimal Standard Model Commitment Census

GC-1 works on the **minimal Standard Model commitment census**. This census contains:

- the chiral matter slots Q, u^c, d^c, L, e^c, replicated across complete generations;
- one Higgs access doublet H;
- the inherited colour, weak, and abelian access carriers (I1–I4);
- committed flavour-continuation structure sufficient to connect the generation graph in each sector (§7.7);
- no independent sterile charged ledger;
- no hidden committed gauge slots;
- no mirror weak doublet carrier;
- no additional Higgs ledger serving quark and lepton pairings independently.

This is not an assumption that such extensions are impossible. It is the boundary of the theorem. If extra sterile charged matter, hidden sectors, mirror weak carriers, or additional Higgs ledgers are admitted, the census has changed and the correct theorem becomes a Standard-Model-plus-extension census theorem, not GC-1.

3.1 One-generation module

For one generation, the minimal chiral matter census is:

$Q : (3, 2)$ $u^c : (\bar{3}, 1)$ $d^c : (\bar{3}, 1)$ $L : (1, 2)$ $e^c : (1, 1)$

These entries list colour and weak representation size before assigning hypercharge. The Higgs access slot is $H : (1, 2)$. Hypercharge is a one-dimensional abelian ledger to be derived.

3.2 Why one generation is enough for the local census

The gauge representation pattern is generation-replicated. At the internal gauge-census level, generations are copies of the same gauge representation pattern rather than new gauge directions.

A generation index is not automatically a gauge index. To become one, it would have to represent a local same-slot degeneracy with indistinguishable continuation cones. VERSF treats generation structure as a refinement history, not as a same-slot local gauge copy. Therefore generation and flavour transformations are not admitted as primitive gauge channels in this paper.

This argument excludes generation-**rotating** gauge channels. Generation-**diagonal** phase ledgers — abelian charges that distinguish generations without rotating them, such as $L_\mu - L_\tau$ — are a separate category which this argument does not settle: they act on committed slots and mistake nothing for a gauge copy. They are classified and eliminated in §7.7 (Lemma 4c).

A later paper proposing a genuine generation gauge channel would have to exhibit

Ext_gen(g_i) = Ext_gen(g_j)

for generation labels i, j, together with anomaly safety and non-conflict with the χ hierarchy. That is not part of the minimal Standard Model gauge census.

3.2a Minimal Anchors Rather Than Standard Model Smuggling

The census slots used in GC-1 should not be read as the Standard Model assumed in different notation. The earlier interface analysis distinguishes minimal observable anchors from full theoretical assumptions.

A full Standard Model assumption would specify, for example, SU(3) colour, quarks in the fundamental representation, gluons, the QCD Lagrangian, SU(2)_L weak isospin, hypercharge assignments, and the electroweak gauge structure.

The inherited VERSF/BCB anchors are weaker, and the canonical residual-empirical-content ledger of the interface track (Closing the Interfaces, §6.3) contains exactly six items. Three are minimal empirical anchors about matter:

1. stable confined three-constituent bound states exist (Anchor A);
2. a fundamental two-state internal sector exists, acted on by continuous distinguishability-preserving symmetry (Anchor B);
3. universal quantum phase redundancy holds — the Born rule depends only on $|\langle\varphi|\psi\rangle|^2$ (Anchor C).

Two are structural interfaces presupposed by any quantum field theory rather than additional empirical content:

4. the base is a four-dimensional Lorentzian geometry admitting a spin structure with the canonical Weyl decomposition S = S_L ⊕ S_R;
5. the observed internal sectors factorise at low energies into commuting factors (consistent with, but not precluding, high-energy unification).

One is a single discrete bit resolved by observation:

6. the weak-active Weyl sector is the left-handed one.

Those facts do not name SU(3), SU(2), U(1), gluons, W bosons, or hypercharge values. They are empirical interfaces any physical theory must account for. The prior gauge-foundation papers show that, when those weaker anchors are combined with the information-geometric admissibility machinery, the Standard Model gauge factors are forced. Item 4 deserves explicit notice in GC-1 because the chirality inheritance of P5 rides on it: the inequivalence of S_L and S_R as Lorentz representations is the external fact that makes chiral restriction expressible at all.

GC-1 then performs the next task: it proves that these inherited factors form a closed census. No additional connected internal gauge channel can be added without either introducing a new empirical anchor, enlarging the committed matter census, violating anomaly discipline, or duplicating an already counted redundancy.

3.3 Gauge action as block action

The internal gauge action must preserve the sector decomposition

$\mathcal{C} = \mathcal{C}_{\text{colour}} \oplus \mathcal{C}_{\text{weak}} \oplus \mathcal{C}_{\text{abelian}}$,

not as a direct sum of particles, but as a direct sum of independent local redundancies. Colour acts on colour multiplicity. Weak $SU(2)_L$ acts on left chiral doublet structure. Hypercharge acts as a phase ledger compatible with both.

A generator mixing these sectors is admissible only if the mixed slots have identical continuation cones, identical chirality status, identical mass-pairing roles, and anomaly-safe completion. The Standard Model census does not satisfy those conditions for such mixing.

This is the first major exclusion: most larger groups fail not because they are mathematically impossible, but because their extra generators rotate between slots that VERSF regards as physically distinct commitment roles.

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4. Sector Factorisation: Colour, Weak Fold, and Abelian Ledger

4.1 Lemma 1 — Sector-factorisation lemma

Lemma 1 (Sector factorisation). Under P1–P10, every admissible connected internal gauge generator decomposes into colour, weak-left, and residual abelian-ledger components,

$X = X_c + X_L + X_A$,

with $X_c \in \mathfrak{su}(3)_c$, $X_L \in \mathfrak{su}(2)_L$, and X_A belonging to the residual abelian access sector. Section 7 then proves that the only surviving admissible abelian component is $\mathfrak{u}(1)_Y$.

Proof. A connected internal generator must act on a live census degeneracy (P3, P10). By P4 the only colour degeneracy is the threefold quark colour slot. By P5 the only weak fold degeneracy is the left chiral doublet. By P6, all residual abelian candidates are routed to the residual abelian component X_A; the lemma names no group for that component, and §7 supplies the identification.

Any generator outside these three classes must do at least one of the following:

1. mix colour and non-colour slots;
2. mix left doublet slots with right singlet slots;
3. mix generations as if they were same-slot gauge copies;
4. act on absent or sterile slots;
5. introduce an additional independent phase ledger.

Case 1 violates the colour-singlet/lepton continuation separation. Case 2 violates chiral weak census structure (P5). Case 3 mistakes generation refinement for gauge degeneracy (§3.2). Case 4 violates the finite committed census (P3, P9). Case 5 introduces an additional abelian direction, which is routed by P6 to the abelian-census analysis and must survive its filters (P9 and §7) — and §7 proves no direction beyond hypercharge survives on the minimal census. The five cases are exhaustive: a connected generator that preserves each of the three committed carriers and introduces no additional phase direction lies in the stated sum by construction, while any generator outside it must fail one of those preservations, and each possible failure crosses exactly one of the five listed boundaries. Note the routing: case 3 covers generation-**rotating** generators; generation-**diagonal** phase ledgers fall under case 5 and are handled by Lemma 4c (§7.7). Therefore no fourth connected internal sector remains. ■

4.2 Reading

The factorisation lemma is the backbone of the paper. It does not assume the final answer. It says that a gauge generator must be sourced by an actual census degeneracy, and VERSF supplies exactly three such degeneracy types:

$3, 2_L, 1_{phase}$.

Those numbers are not group names yet. They are census facts. The next three sections prove that their admitted local automorphism groups are exactly $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$.

4.3 Closure Architecture Is Not the Gauge-Boson Census

The programme also contains an inherited fourteen-generator closure architecture associated with the K = 7 admissibility structure. That fourteen-generator result belongs to the closure-architecture layer: it concerns the response machinery beneath admissibility, transport, and closure. It should not be confused with the local Standard Model gauge-boson census.

The local connected Standard Model gauge algebra counted in GC-1 has dimension

$$\dim \mathfrak{su}(3)_c + \dim \mathfrak{su}(2)_L + \dim \mathfrak{u}(1)_Y = 8 + 3 + 1 = 12.$$

The fourteen-generator closure architecture concerns a different explanatory layer. It is part of the machinery by which admissibility and transport are organized; it is not a claim that the Standard Model has fourteen local gauge bosons.

This firewall matters because it prevents two different counts from being conflated. GC-1 closes the local connected internal gauge census. The fourteen-generator work classifies closure architecture beneath the gauge programme. The two results are compatible precisely because they answer different questions:

- GC-1 asks: which connected internal gauge channels act on the minimal Standard Model commitment census?
- The fourteen-generator closure paper asks: what closure-architecture response structure underlies the admissibility machinery?

Thus the presence of a fourteen-generator closure architecture neither enlarges nor contradicts the twelve-dimensional local Standard Model gauge algebra.

One further point of discipline. The interpretation of the fourteen-generator census within its own paper — the dihedral identification $G \cong D_7$, with the doubling read as the quotient $\mathbb{Z}_2 \cong D_7/C_7$ — is itself gated: the identification of $|D_7| = 14$ with dim $\mathfrak{g}_{14}$ is recorded there as open and load-bearing (GATE-DISCRIM(a)), and every downstream claim carries that cap. The firewall stated here does not depend on how that gate resolves: whatever the fourteen turn out to be structurally, they are closure-architecture machinery, not local gauge bosons, and the gauge-boson count of GC-1 remains 8 + 3 + 1 = 12 unconditionally. GC-1 therefore firewalls against a structure whose own interpretation is still conditional, which is the stronger and more honest form of the separation.

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5. Colour-Census Closure: SU(3)_c

5.1 Colour triplicity

By P4, the quark class carries exactly three indistinguishable colour realization slots. Write the colour carrier as $\mathbb{C}^3$_c. A quark colour basis may be written (r, g, b), but the basis names are not physical: any local unitary change of basis in this three-dimensional colour carrier preserves the colour census.

The maximal local unitary relabelling group is initially U(3)_c. However,

$U(3)\_c \cong (SU(3)\_c \times U(1))/\mathbb{Z}_3$.

The traceless non-abelian part is colour: SU(3)_c. The determinant phase is not an additional colour generator. It belongs, if admitted at all, to the abelian ledger; counting it inside colour and again as hypercharge would double-count the same phase freedom. Thus colour closure gives

$G_c = SU(3)_c$, $\mathfrak{g}_c = \mathfrak{su}(3)_c$.

5.2 Lemma 2 — Colour-census theorem

Lemma 2 (Colour census). Given an exactly three-dimensional indistinguishable quark colour carrier $\mathbb{C}^3_c$, and requiring determinant-phase separation into the abelian ledger, the unique connected non-abelian local gauge group acting faithfully on colour is $SU(3)_c$.

Proof. The local basis ambiguity of a complex three-dimensional carrier is U(3), whose Lie algebra decomposes as $\mathfrak{u}(3) = \mathfrak{su}(3) \oplus \mathfrak{u}(1)$. The $\mathfrak{u}(1)$ summand is a uniform phase ledger, not a traceless colour rotation; by ledger separation it is counted only in the abelian sector. The remaining faithful non-abelian algebra is $\mathfrak{su}(3)$. Faithfulness is fixed by the committed representation content: the census carries both triplet (Q) and anti-triplet (u^c, d^c) slots, which distinguish $SU(3)_c$ from any centre quotient acting on colour alone. ■

5.3 Colour exclusions

Why not $SU(2)_c$? Wrong carrier dimension: it acts on a twofold colour degeneracy, not the inherited threefold quark colour census.

Why not SU(4) colour? SU(4) requires a fourth colour-like slot. Treating the lepton as a fourth colour mixes colour-carrying quark continuations with colourless lepton continuations. That is not an internal redundancy of the committed census; it is an extension requiring additional generators and altered matter-slot relations. Such a structure may be studied as an embedding or extension. It is not the primitive VERSF Standard Model colour census.

Why not $U(3)_c$? Because its determinant phase is not colour. The traceless colour part is $SU(3)_c$; the determinant phase belongs to the abelian ledger and must pass the hypercharge constraints of §7.

Why not SO(3)? The colour carrier is complex and distinguishes triplet from anti-triplet representations. SO(3) is not the faithful complex colour automorphism group of the quark/anti-quark census.

Why not an exceptional group? An exceptional group is admissible only if its fundamental action matches the committed census slots without adding uncommitted degrees of freedom. No exceptional action exists on the minimal colour triplicity that preserves the quark/lepton separation, weak chirality, and hypercharge ledger.

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6. Weak-Census Closure: $SU(2)_L$

6.1 Left chiral fold doublethood

By P5, the weak-access carrier consists of left chiral doublets,

$Q = (u, d)\_L$, $L = (\nu, e)\_L$,

so the weak carrier is a complex two-dimensional fold-access space $\mathbb{C}^2\_L$. A local basis change in this doublet carrier begins as U(2)_L, and

$U(2)\_L \cong (SU(2)\_L \times U(1))/\mathbb{Z}_2$.

The traceless part is the weak non-abelian gauge group SU(2)_L. The determinant phase is not weak isospin; it belongs to the abelian ledger. Thus weak closure gives

$G\_L = SU(2)\_L$, $\mathfrak{g}\_L = \mathfrak{su}(2)\_L$.

6.2 Lemma 3 — Weak-census theorem

Lemma 3 (Weak census). Given an exactly two-dimensional left chiral weak fold carrier, and separating the determinant phase into the abelian ledger, the unique connected non-abelian local gauge group acting faithfully on that carrier is SU(2)_L.

Proof. The local unitary relabelling group of a complex two-dimensional carrier is U(2), whose Lie algebra decomposes as $\mathfrak{u}(2) = \mathfrak{su}(2) \oplus \mathfrak{u}(1)$. The determinant phase is abelian and is counted in the hypercharge ledger. The remaining traceless non-abelian local action is $\mathfrak{su}(2)$. By P5 it acts only on the left chiral doublet carrier. Hence the admitted weak algebra is $\mathfrak{su}(2)\_L$. ■

6.3 Why weak is chiral

The weak carrier is not a generic two-dimensional label space. It is specifically left chiral. The right charged slots u^c, d^c, e^c are weak singlets in the minimal chiral census. Treating them as weak doublets would create local transformations unsupported by the committed census. Thus SU(2)_L is admitted while SU(2)_R is not admitted as a primitive Standard Model gauge channel.

6.4 SU(2)_R exclusion

To admit SU(2)_R, VERSF would need a right chiral doublet carrier such as (u, d)_R and a corresponding right lepton doublet (ν, e)_R. But the minimal census contains right charged singlet slots, not right weak doublets, and it does not contain a required right-neutrino weak partner as a non-sterile gauge slot. Promoting right slots to SU(2)_R doublets would alter the committed census. Hence SU(2)_R is not an admissible closure of the Standard Model gauge census; it is an extension candidate with additional matter and symmetry-breaking obligations.

6.5 Global weak-parity check

An SU(2) gauge theory carries a global (non-perturbative) consistency condition beyond the perturbative anomalies: the number of left Weyl doublets must be even. The left weak doublets per generation are three colour copies of Q plus one lepton doublet L,

$$3 + 1 = 4,$$

which is even. The condition holds generation by generation, so it holds for any number of complete generations. In VERSF language: the left fold doublet count closes without leaving an unpaired weak global obstruction, and the closure is stable under generation replication.

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7. Abelian-Census Closure: U(1)_Y

7.1 The remaining phase problem

After extracting the traceless parts SU(3)_c and SU(2)_L, there remains the question of internal phase. The determinant phases of U(3)_c and U(2)_L, together with matter-slot phase assignments, cannot be counted arbitrarily. A phase ledger is gauge-admissible only if it:

- assigns consistent charges to committed slots;
- allows mass-pairing through H;
- cancels anomalies, jointly with every other admitted generator;
- does not duplicate a non-abelian centre;
- does not require uncommitted hidden slots.

This section proves the strengthened rigidity result in three parts. Stage 1 uses anomaly discipline alone and classifies every anomaly-safe generation-universal abelian ledger. Stage 2 adds mass-pairing admissibility and shows the hypercharge line is the unique survivor of that classification, with the residual up/down exchange identified as a field-redefinition convention rather than a rival solution. Part three (§7.7) lifts the analysis to the full three-generation charge space, where a genuinely new anomaly-free family appears — the generation-difference ledgers — and is eliminated by continuation-cone preservation (Lemma 4c). Separating the parts makes the logical load-bearing explicit: anomalies do most of the work on the universal subspace; mass-pairing removes one exceptional line; and the flavour-gate mixing commitments close the generation-dependent remainder.

7.1a Relation to Earlier Abelian-Uniqueness Results

The abelian sector has been treated in earlier VERSF/BCB papers in two stages.

First, the distinguishability-conservation foundation proves that quantum phase redundancy is unique at the pure phase level (Distinguishability Conservation, Theorems 5.1–5.3). Overall phase is the only connected transformation acting trivially on all Born probabilities, and local comparison of states forces this phase redundancy to be represented by a U(1) connection.

Second, the interface papers address charge-type abelian directions: possible U(1) factors that are not merely pure phase redundancies but assign different charges to different matter slots. The interface result (Closing the Interfaces, Theorem 4.3, with the explicit anomaly system in its Appendix C) shows that on the committed chiral content, any additional anomaly-free abelian charge is proportional to hypercharge, in line with the classical results of Minahan–Ramond–Warner and Babu–Mohapatra.

GC-1 sharpens this inheritance. It does not merely assert that the abelian sector is unique. It classifies the abelian candidates on the minimal chiral census, shows that anomaly discipline alone leaves only the hypercharge-type line and one exceptional lepton-blind line, proves that these lines cannot be jointly gauged because they are mutually anomalous, and then uses Higgs mass-pairing admissibility to eliminate the exceptional line. The result is stronger than a pure phase-uniqueness theorem and sharper than a phenomenological "no extra U(1)" assumption. In particular, GC-1 exhibits explicitly the object that the interface track's mixed anomaly conditions eliminate: the passage there from a two-parameter family to a one-dimensional solution space is, in the language of §7.3, the annihilation of the exceptional line by the mixed conditions once Y is gauged.

The abelian dimension is thereby bounded to one by two independent mechanisms. Upstream, the information-geometric obstruction: additional pure-phase directions are Fisher-null and are excluded as unobservable structure. At GC-1, the anomaly-theoretic obstruction: the two anomaly-safe charge lines are mutually anomalous (Lemma 4b) and cannot coexist as gauge channels. These arguments share no machinery; their convergence on a one-dimensional abelian sector is a non-trivial cross-check of the programme.

The abelian claim in GC-1 is therefore:

An abelian phase/access direction is inherited; that exactly one ledger survives, and that it is the hypercharge ledger, is selected by joint anomaly discipline, mass-pairing, and mixing-graph universality on the committed Standard Model census.

7.2 Ledger variables

Assign unknown charges to the fermion slots and the Higgs slot:

$Y(Q) = q$, $Y(u^c) = u$, $Y(d^c) = d$, $Y(L) = \ell$, $Y(e^c) = e$, $Y(H) = h$.

The goal is to determine the ratios $q : u : d : \ell : e : h$.

7.3 Stage 1 — Anomaly-only classification

Anomaly discipline (P8) imposes, on the fermion charges alone:

$[SU(3)\_c]^2\ U(1)$: $2q + u + d = 0$ (A1) $[SU(2)\_L]^2\ U(1)$: $3q + \ell = 0$ (A2) $\text{grav}^2\ U(1)$: $6q + 3u + 3d + 2\ell + e = 0$ (A3) $[U(1)]^3$: $6q^3 + 3u^3 + 3d^3 + 2\ell^3 + e^3 = 0$ (A4)

The coefficients count Weyl multiplicities: Q carries $3 \times 2 = 6$ components, u^c and d^c carry 3 each, L carries 2, e^c carries 1.

From A1: $u + d = -2q$. From A2: $\ell = -3q$. Substituting into A3:

$$6q + 3(-2q) + 2(-3q) + e = 0 \Longrightarrow e = 6q.$$

Substituting into A4:

$$6q^3 + 3(u^3 + d^3) + 2(-27q^3) + 216q^3 = 0 \Longrightarrow u^3 + d^3 = -56q^3.$$

Using $u^3 + d^3 = (u + d)^3 - 3ud(u + d) = -8q^3 + 6q \cdot ud$:

$$6q \cdot ud = -48q^3.$$

Case $q \neq 0$. Then $ud = -8q^2$ with $u + d = -2q$, so u and d are the roots of $t^2 + 2qt - 8q^2 = 0$, giving $t = 2q$ or $t = -4q$. Hence

$$\{u, d\} = \{-4q, 2q\},$$

i.e. the hypercharge line, determined up to normalization and up to the discrete exchange $u \leftrightarrow d$.

Case $q = 0$. Then $\ell = e = 0$ and $u = -d$ is free, and A4 holds identically. This is the exceptional lepton-blind line

$$Y' = s \cdot (0, 1, -1, 0, 0) \text{ on } (Q, u^c, d^c, L, e^c).$$

Lemma 4a (Anomaly-only abelian classification). On the minimal one-generation chiral census $\{Q, u^c, d^c, L, e^c\}$, the set of abelian charge assignments satisfying A1–A4 is exactly the union of two lines: the hypercharge line $q \cdot (1, -4, 2, -3, 6)$ up to the exchange $u \leftrightarrow d$, and the exceptional line $s \cdot (0, 1, -1, 0, 0)$. ■

Lemma 4b (Mutual anomaly obstruction). The two lines of Lemma 4a cannot be jointly gauged. The mixed anomalies between them are non-zero:

$$[Y]^2 Y' : 3u^2s - 3d^2s = 3s(16q^2 - 4q^2) = 36 q^2 s \neq 0, \quad [Y']^2 Y : 3s^2u + 3s^2d = 3s^2(u + d) = -6 q s^2 \neq 0,$$

for $q \neq 0$, $s \neq 0$. Hence at most one abelian ledger is admissible at a time, and the abelian sector is at most one-dimensional even before mass-pairing is imposed. ■

Lemma 4b closes a loophole that a weaker treatment leaves open: it is not enough that each candidate ledger is individually anomaly-free, because condition 6 of §2.3 demands joint anomaly safety. The minimal census cannot host two abelian gauge channels under any assignment. This is the census-level counterpart of the conditional abelian-uniqueness theorem of the interface track (Closing the Interfaces, Theorem 4.3) and of the anomaly-quantization results of Minahan–Ramond–Warner (1990) and Babu–Mohapatra (1990): there, the mixed conditions $[Y]^2Q'$ and $Y[Q']^2$ collapse the standalone anomaly-free family onto the hypercharge line; here, Lemma 4a exhibits that standalone family explicitly as two lines, and Lemma 4b identifies the mixed conditions as the obstruction to their coexistence. The two statements agree, with GC-1 supplying the classified intermediate object.

7.4 Stage 2 — Mass-pairing selection

Gauge-invariant mass-pairing access through H (P7) requires:

QHu^c: $q + h + u = 0$ (M1) $QH^\dagger d^c$: $q - h + d = 0$ (M2) $LH^\dagger e^c$: $\ell - h + e = 0$ (M3)

On the exceptional line ($q = \ell = e = 0$, $u = -d = s$): M1 gives $h = -s$ and M2 gives $h = d = -s$, consistently; but M3 gives $h = 0$, hence $s = 0$. The lepton pairing annihilates the exceptional line entirely.

On the hypercharge line: M3 reads $-3q - h + 6q = 0$, forcing $h = 3q$; M1 and M2 then force $u = -4q$, $d = 2q$. The residual discrete exchange $u \leftrightarrow d$ of Lemma 4a is not eliminated as physics; it is absorbed as convention. The exchanged assignment $q\cdot(1, 2, -4, -3, 6)$ with $h = -3q$ satisfies M1 and M2 and fails M3 only because P7 fixes the lepton pairing as $LH^\dagger e^c$ rather than LHe^c. Under the simultaneous field redefinition $u \leftrightarrow d$, $H \to \tilde{H} = i\sigma_2 H^*$, the exchanged assignment maps exactly onto the standard one: it is the same physical ledger with the up/down naming and the Higgs-contraction convention swapped (Appendix D). P7's fixed pairing list is therefore an orientation convention, and the theorem does not — and should not claim to — derive it.

The genuine selection work of Stage 2 is the annihilation of the exceptional line, and that result is robust under either pairing convention: with LHe^c in place of $LH^\dagger e^c$, the exceptional line gives $h = 0$ from the lepton condition and $s = 0$ from M1, by the same two steps. The lepton pairing kills the exceptional line in both conventions; it orients nothing.

The unique surviving ledger is

$(q, u, d, \ell, e, h) = q \cdot (1, -4, 2, -3, 6, 3)$.

Choosing the conventional normalization $q = ⅓$ gives

$Y(Q) = ⅓$, $Y(u^c) = -4/3$, $Y(d^c) = ⅔$, $Y(L) = -1$, $Y(e^c) = 2$, $Y(H) = 1$,

exactly the Standard Model hypercharge ratios in left-handed conjugate notation.

7.5 Rank form of the rigidity argument

The same conclusion can be stated as a rank count. Take the six unknowns (q, u, d, ℓ, e, h) and the five linear conditions M1, M2, M3, A2, A3. Direct elimination (§7.4 and Appendix A) shows the joint solution space is exactly the line spanned by $(1, -4, 2, -3, 6, 3)$; since a non-zero solution exists, the linear system has rank 5 and nullity 1. On that line, the remaining conditions A1 and A4 hold identically:

A1: $2q + (-4q) + 2q = 0$, A4: $6q^3 + 3(-4q)^3 + 3(2q)^3 + 2(-3q)^3 + (6q)^3 = (6 - 192 + 24 - 54 + 216)\, q^3 = 0$.

The abelian ledger is therefore not merely consistent but overdetermined-and-consistent: two independent routes (anomalies-first, pairing-first) intersect on the same one-dimensional line. This convergence is itself a non-trivial structural check.

7.6 Lemma 4 — Hypercharge rigidity theorem

Lemma 4 (Hypercharge rigidity). Under the minimal census, joint anomaly discipline, and Higgs mass-pairing admissibility, the abelian internal gauge ledger is unique up to overall normalization and field-redefinition conventions (the simultaneous u ↔ d relabelling with $H \leftrightarrow \tilde{H}$), and is the Standard Model hypercharge ledger Y. Moreover: (i) on the generation-universal charge subspace, anomaly discipline alone restricts the abelian sector to at most one gauged direction (Lemmas 4a, 4b); (ii) mass-pairing annihilates the sole exceptional alternative, robustly under either pairing convention; (iii) generation-non-universal ledgers are eliminated by the mixing-graph universality lemma together with per-generation singlet tracking (Lemma 4c and §7.7), conditional on the connectedness of the committed flavour-continuation graph.

Proof. Lemma 4a classifies the generation-universal anomaly-safe assignments; Lemma 4b shows the two resulting lines are mutually anomalous, bounding that sector's abelian dimension by one; §7.4 shows mass-pairing annihilates the exceptional line, with the residual u ↔ d exchange identified as the $H \leftrightarrow \tilde{H}$ field-redefinition convention rather than a second solution; §7.5 confirms the solution space is exactly one-dimensional per convention and that the non-linear conditions hold identically on it; Lemma 4c, with the singlet-tracking step of §7.7, reduces the full three-generation charge space to the generation-universal subspace. ■

7.7 Generation-Dependent Abelian Ledgers: Mixing-Graph Closure

Lemmas 4a–4b, as proved, classify charge assignments on the **generation-universal** subspace: one charge per census slot type, replicated identically across generations. On the full three-generation census the abelian charge space is larger — each generation may carry its own charges — and the classification question reopens there. This subsection displays the loophole explicitly and closes it.

The impostor displayed. Assign, on the three-generation census, the muon-minus-tau lepton-number ledger

$Q' = L_\mu - L_\tau$: $Q'(L_\mu) = +1$, $Q'(\mu^c) = -1$, $Q'(L_\tau) = -1$, $Q'(\tau^c) = +1$, all other slots 0.

Direct check: every self-anomaly cancels generation-against-generation —

$[SU(2)]^2\ Q'$: (+1) + (−1) = 0 $grav^2\ Q'$: 2(+1) + (−1) + 2(−1) + (+1) = 0 $[Q']^3$: $2(+1)^3 + (-1)^3 + 2(-1)^3 + (+1)^3 = 0$ $[SU(3)]^2\ Q'$: trivial (colour singlets only)

— and both mixed conditions with Y cancel because the μ and τ sectors carry identical hypercharges: $[Y]^2Q'$ sums $Y^2 \cdot Q'$ over slots with equal Y^2 and opposite Q′, and $[Q']^2Y$ sums $Q'^2 \cdot Y$ with equal Q'^2 and equal Y, weighted oppositely across the pair. The diagonal charged-lepton pairings $L\_\mu H^\dagger \mu^c$ and $L\_\tau H^\dagger \tau^c$ are Q′-neutral with Q′(H) = 0. The generator acts on committed slots, requires no new matter, and duplicates nothing. Every anomaly condition of §2.3 is satisfied. The analogous quark-sector generation-difference ledgers pass the same battery of checks. Lemma 4b's one-dimensionality is therefore, on its own, a statement about the generation-universal subspace only — and a closure theorem that stopped there would be false in its strong form.

The correct exclusion is not anomaly discipline. It is continuation preservation.

The flavour-continuation graph. Define, for a sector X, the graph $\mathscr{F}\_X$ as follows. Its vertices are the generation-labelled copies of a committed slot type. An edge i ↔ j is present when the programme commits a physical continuation mixing generations i and j in that sector: a non-zero CKM transition, a non-zero lepton leakage/mixing continuation, or any committed off-diagonal mass-access continuation.

A generation-dependent abelian charge Q′ preserves such a continuation only if it assigns equal charge to the two vertices joined by the edge: if $Q'\_i \neq Q'\_j$, the corresponding mixing continuation carries non-zero Q′-charge and is gauge-forbidden.

Lemma 4c (Mixing-graph universality). If the committed flavour-continuation graph is connected in a sector, then every admissible abelian gauge charge is generation-universal on that sector.

Proof. Let Q′ be an admissible abelian gauge charge. For every committed mixing edge i ↔ j, gauge invariance of the continuation requires $Q'\_i = Q'\_j$: otherwise the continuation carries net charge $Q'\_i - Q'\_j \neq 0$ and is gauge-forbidden, so Q′ annihilates committed continuation structure, identifies histories with distinct continuation cones, and fails condition 3 of §2.3. If the graph is connected, equality propagates along paths, so $Q'\_1 = Q'\_2 = Q'\_3$. Hence Q′ is generation-universal on that sector. Any non-universal charge forbids at least one committed continuation and therefore fails preservation of admissible continuation cones. ■

Applied to the Standard Model commitment census: the quark flavour graph is connected by the committed CKM-type mixing continuations, and the lepton flavour graph is connected by the committed neutrino leakage continuations (SM-3, $\beta = \sqrt{3/20} \neq 0$). Doublet charges are therefore generation-universal in both sectors.

Singlet slots. There are formal real Lie-algebraic charge assignments on singlet slots that can be made to pass selected anomaly equations before compact charge-lattice quantization and mass-pairing are imposed. These do not need to be classified one by one: once diagonal mass-pairing to a single Higgs slot is committed (P7, per generation), singlet charges are forced to track the corresponding doublet charges generation by generation — neutrality of $Q_iHu_i^c$ gives $q'_i + h' + u'_i = 0$ for every i with a single h', so u'_i is universal whenever q'_i is, and likewise for the d- and e-type singlets. A charge non-universal only on e^c is in any case excluded by $[Q']^2Y$ before this step is needed.

Corollary 4c.1 ($L_\mu - L_\tau$ exclusion). The ledger $L_\mu - L_\tau$ is anomaly-free, jointly with Y, but it assigns different charges to the μ and τ lepton families. Since the μ–τ continuation edge is committed (SM-3), the ledger forbids that continuation and fails Lemma 4c. It is therefore not an admissible gauge channel of the minimal VERSF Standard Model census — displayed and eliminated, in the §10 sense, rather than silently excluded.

Combining the two steps: doublet universality (Lemma 4c on connected graphs) plus singlet tracking (P7 per generation) makes every admissible abelian charge generation-universal, so Lemmas 4a–4b classify the full admissible abelian sector. Note the shape of the argument: it requires no complete classification of the anomaly-free generation-dependent family (which is larger than $\{L_i - L_j\}$). Connectedness of the committed graph is the single operative condition, and the entire class falls at once.

Dependency recorded. Lemma 4c is conditional on the connectedness of the committed flavour-continuation graph — supplied in the quark sector by the CKM commitments and in the lepton sector by the SM-3 leakage — and the singlet step is conditional on P7 extended per generation; neither is free-standing. If the flavour programme had not already committed non-zero inter-generation continuations, Lemma 4c would not close the generation-difference ledgers; they would remain live anomaly-free extension candidates. Stated as a boundary: if the committed flavour-continuation graph is disconnected, GC-1 does not exclude generation-difference ledgers constant on each connected component. The abelian closure of Theorem 1 on the three-generation census is therefore downstream of those commitments — the dependency statement of §12 records this, and the failure mode is falsifier F21. On a strictly one-generation census, Lemmas 4a–4b close the abelian sector without this inheritance.

7.8 Why not another U(1)?

An additional U(1)′ would require a second linearly independent charge vector, jointly anomaly-safe with Y, compatible with mass-pairing, and preserving committed continuation cones, acting on the committed census. Lemma 4 shows no such vector exists: generation-universal candidates fall to Lemmas 4a–4b and the mass-pairing annihilation, and generation-non-universal candidates — including the fully anomaly-free $L_\mu - L_\tau$ family — fall to Lemma 4c.

A sterile neutral singlet with $Y(N^c) = 0$ does not alter this conclusion: it is gauge-inert. A non-trivial extra ledger such as B−L would require additional charged commitments (typically a non-sterile right-neutrino ledger) and a separate anomaly-safe continuation cone. If admitted, it would be an **extension** of the minimal gauge census, not part of its closure. It is therefore listed as a falsifier/extension condition (F6, F7) rather than silently ignored.

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8. Anomaly Discipline and Hypercharge Rigidity

8.1 Anomaly cancellation as census boundary closure

In VERSF terms, anomaly cancellation is not an optional quantum afterthought. It is a boundary-closure condition on local gauge redundancy. A local gauge redundancy cannot survive if its current fails to be consistently conserved after quantisation. An anomalous gauge current is not a true redundancy of the physical commitment census: it creates a mismatch between the formal local relabelling and the physically admissible continuation structure. Anomaly discipline is the gauge-sector analogue of continuation faithfulness.

Section 7.7 supplies the necessary caution about anomaly discipline's reach: it is a necessary filter, not a sufficient one. The generation-difference ledgers pass every anomaly condition and are excluded only by continuation-cone preservation.

8.2 Non-abelian anomaly checks

$[SU(3)\_c]^3$. For one generation in left-handed notation, Q contributes two colour triplet weak components, u^c contributes one anti-triplet, d^c contributes one anti-triplet. The net colour cubic contribution is

$2 - 1 - 1 = 0.$

Colour is chiral-census balanced.

$[SU(2)\_L]^3$. The SU(2) doublet representation is pseudoreal, so there is no perturbative cubic $SU(2)^3$ anomaly. The global doublet-parity condition of §6.5 is satisfied: 3Q + L = 4 left weak doublets per generation, an even number.

8.3 Mixed anomaly checks

Using the hypercharge solution:

$[SU(3)]^2\, Y : 2\cdot(1/3) - 4/3 + 2/3 = 0$ $[SU(2)]^2\, Y : 3\cdot(1/3) + (-1) = 0$ $\text{grav}^2\, Y : 6\cdot(1/3) + 3\cdot(-4/3) + 3\cdot(2/3) + 2\cdot(-1) + 2 = 0$ $Y^3 : 6\cdot(1/3)^3 + 3\cdot(-4/3)^3 + 3\cdot(2/3)^3 + 2\cdot(-1)^3 + 2^3 = 0$

The full minimal census closes.

8.4 Lemma 5 — Anomaly closure lemma

Lemma 5 (Anomaly closure). The admitted census SU(3)_c × SU(2)_L × U(1)_Y with the one-generation matter assignments above is anomaly-safe, including the global weak-parity condition. Any additional connected internal generator must independently satisfy the same joint anomaly discipline; absent a new committed anomaly-safe ledger, it is not admitted.

Proof. The non-abelian, mixed, gravitational, cubic, and global checks vanish (§§6.5, 8.2, 8.3). By P8 and condition 6 of §2.3, any new generator must be anomaly-safe jointly with the admitted set. By Lemmas 4a–4b, there is no second abelian direction jointly gaugeable with Y on the generation-universal subspace; by Lemma 4c, generation-non-universal directions fail continuation-cone preservation even where they are anomaly-safe. By Lemmas 2 and 3, there is no additional non-abelian local carrier without new committed degeneracies. Therefore no further admissible connected internal generator exists within the minimal census. ■

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9. The Gauge-Census Closure Theorem

9.1 Statement

Define the admitted internal gauge algebra

$\mathfrak{g}_{adm}$ = { X : X is a connected internal gauge generator satisfying the admissibility conditions of §2.3 over the inherited results I1–I4 and the minimal census of §3.0 }.

Theorem 1 (Gauge-Census Closure, reduced-conditional form). Assume the VERSF/BCB gauge-foundation results inherited in §1.2a and labelled I1–I4: gauge necessity from local distinguishability comparison; colour triplicity; left chiral weak doublethood; and the existence of an abelian access carrier. Work on the minimal Standard Model commitment census of §3.0: complete generation copies of Q, u^c, d^c, L, e^c, one Higgs access slot H, no independent sterile charged ledger, and no hidden committed gauge slots. Assume also that the committed flavour-continuation graph is connected in the quark and lepton sectors.

Then the maximal connected internal gauge algebra admitted by the minimal VERSF Standard Model census is

$\mathfrak{g}_{adm} = \mathfrak{su}(3)_c \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$.

No additional connected internal gauge channel survives. Any proposed additional channel must either add new committed census slots, violate chirality, duplicate the abelian ledger, fail joint anomaly discipline, annihilate committed flavour continuations, or belong to a high-energy embedding/extension rather than to the minimal Standard Model census.

Equivalently, the maximal connected internal gauge group at local level is $SU(3)_c \times SU(2)_L \times U(1)_Y$, up to finite-centre global identification.

9.2 Proof

By Lemma 1, every admissible connected internal generator decomposes as $X = X_c + X_L + X_A$, with X_A in the residual abelian access sector. By Lemma 2, $X_c \in \mathfrak{su}(3)_c$ uniquely. By Lemma 3, $X_L \in \mathfrak{su}(2)_L$ uniquely. By Lemma 4, the residual abelian component is identified:

$X\_A = X\_Y \in \mathfrak{u}(1)\_Y$, proportional to the hypercharge ledger — on the generation-universal subspace the exceptional anomaly-safe alternative is annihilated by mass-pairing and the mutual-anomaly obstruction excludes any second direction (Lemmas 4a–4b), while generation-non-universal ledgers are excluded by mixing-graph universality and singlet tracking (Lemma 4c, §7.7). By Lemma 5, the resulting census is anomaly-safe and no additional connected generator on the minimal census satisfies joint anomaly discipline, continuation-cone preservation, and independent ledger status. Therefore

$$\mathfrak{g}\_\mathrm{adm} \subseteq \mathfrak{su}(3)\_c \oplus \mathfrak{su}(2)\_L \oplus \mathfrak{u}(1)\_Y.$$

Conversely, each of $\mathfrak{su}(3)\_c$, $\mathfrak{su}(2)\_L$, $\mathfrak{u}(1)\_Y$ acts non-trivially on committed census slots, preserves admissible continuations, preserves mass-pairing structure, and satisfies anomaly discipline. Hence

$$\mathfrak{su}(3)\_c \oplus \mathfrak{su}(2)\_L \oplus \mathfrak{u}(1)\_Y \subseteq \mathfrak{g}\_\mathrm{adm}.$$

Equality holds. No fourth connected internal gauge algebra remains. ■

9.3 Global-centre note and charge quantization

The theorem is local; the Lie algebra is unaffected by finite global quotients. The faithful Standard Model global group may be written

$$G\_\mathrm{faithful} = (SU(3)\_c \times SU(2)\_L \times U(1)\_Y) / \mathbb{Z}_6.$$

This quotient identifies common centre actions that are redundant on the committed representations. It introduces no local gauge boson and removes none; the gauge-census closure at local level is unchanged.

A point of precision on canonicity. The committed representations are compatible not only with the $\mathbb{Z}_6$ quotient but also with the $\mathbb{Z}_3$, $\mathbb{Z}_2$, and trivial quotients — each yields the same local algebra and the same gauge-boson count. Selecting $\mathbb{Z}_6$ therefore requires a named convention: **quotient maximality** — among finite central subgroups acting trivially on every committed representation, quotient by the maximal one. Under that convention, $\mathbb{Z}_6$ is canonical for the committed content; without it, the choice of global form is underdetermined by the census.

The quotient carries one structural dividend, stated at its correct conditional strength. Because the abelian channel is realized as a compact U(1) acting on the committed representation lattice, the hypercharges of all admissible representations are forced onto a common discrete lattice: in the normalization of §7.4, $Y \in (1/3)\cdot\mathbb{Z}$ on the committed census, with the centre-consistency relation tying the Y value of each slot to its colour triality and weak duality. Charge quantization is thus a census-level consequence of compactness together with the quotient-maximality convention — not an unconditional theorem, and not an additional postulate. The overall normalization of Y remains conventional; the lattice structure, given the convention, does not.

9.4 Strong form

Corollary (Strong form). Any proposed connected internal gauge channel beyond SU(3)_c, SU(2)_L, and U(1)_Y must either introduce new committed matter slots, alter chirality, duplicate the hypercharge ledger, violate joint anomaly discipline, annihilate committed continuation structure (the failure mode of the generation-difference ledgers, Lemma 4c), or act trivially. Therefore it is not part of the closed minimal Standard Model gauge census.

9.5 Programme Consequence: Gauge Counting Is Now Closed

GC-1 closes the internal gauge-counting branch of the Standard Model programme.

After this theorem, the remaining Standard Model derivation work is no longer the search for additional primitive connected gauge channels. Over the inherited results I1–I4 and the minimal census of §3.0, none remain. Any future proposal for an extra connected internal gauge group must therefore be classified as one of three things:

1. an extension of the committed matter census;
2. a high-energy embedding or unification structure;
3. a failure of one of the named GC-1 premises.

It cannot be treated as an uncounted residue of the minimal Standard Model gauge census.

The remaining work in the Standard Model programme therefore shifts from gauge counting to dynamics and representation refinement: electroweak breaking, coupling strengths, Yukawa structure, χ hierarchy, CKM/PMNS structure, mass anchoring, and generation refinement. Those are real open tasks, but they are not missing gauge factors.

In this precise sense, GC-1 is a closure paper. It proves that the Standard Model internal gauge algebra is not merely an admissible gauge pattern inside VERSF; it is the unique maximal connected internal gauge census compatible with the inherited minimal commitment structure.

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10. Why This Is Not a Group-Theory Smuggle

A natural objection is that the paper simply noticed the Standard Model groups and named them as VERSF outputs. That is not the structure of the argument.

The theorem does not start from the groups. It starts from admissible census degeneracies — 3, 2_L, 1_phase — and asks what local connected automorphism each degeneracy admits once determinant phases, chirality, mass-pairing, and anomalies are enforced. The outputs are

3 ↦ SU(3)_c, 2_L ↦ SU(2)_L, 1_phase ↦ U(1)_Y.

The exclusions are equally important, and the strengthened abelian analysis makes the point sharpest: anomaly discipline alone does not hand back the hypercharge assignment by fiat. On the generation-universal subspace it hands back a classification problem with exactly two solutions, one of which (the lepton-blind ledger) has no counterpart in the Standard Model; and on the full three-generation census it admits a further family (the generation-difference ledgers, sharpest member $L_\mu - L_\tau$) with no counterpart either. The census filters then do genuine work on both: joint anomaly safety and mass-pairing eliminate the first impostor, and continuation-cone preservation eliminates the second. A smuggled derivation would have no impostors to eliminate — this one displays two distinct classes and names the filter that kills each.

The no-smuggle point is also strengthened by the inheritance structure. The prior papers do not start by postulating $SU(3)_c \times SU(2)_L \times U(1)_Y$. They begin with distinguishability conservation, Fisher geometry, entropy minimization, anomaly exclusion, minimal empirical anchors, and the necessity of connection structure. From those ingredients they obtain the colour, weak, and abelian sectors as admissible internal structures. GC-1 then asks whether anything remains outside those sectors.

This division of labour is important. If the present paper alone attempted to derive everything, P4–P6 would look like assumptions. But within the programme, they are inherited outputs. The claim of GC-1 is therefore not "we assume the Standard Model group and exclude competitors." The claim is:

Earlier gates identify the live internal census sources. GC-1 proves that their local connected automorphism algebra is exactly $\mathfrak{su}(3)_c \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$ and that every proposed additional connected generator fails a named admissibility test.

The theorem does not say $SU(3) \times SU(2) \times U(1)$ is beautiful, therefore accepted. It says:

- $SU(3)_c$ is forced by threefold colour realization;
- $SU(2)_L$ is forced by left chiral doublethood;
- $U(1)_Y$ is forced as the unique jointly-anomaly-safe abelian ledger compatible with mass-pairing;
- every additional connected generator fails a named admissibility condition.

This is why the paper is a census theorem rather than a group-identification note.

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11. Exclusion Ladder

11.1 Why not no gauge group?

Because the committed census contains local class-grain degeneracies. If basis choices in colour, weak fold, and phase ledger can vary locally without altering the physical commitment, then corresponding connections are required. A theory with no internal gauge group would freeze arbitrary basis choices into physical structure.

11.2 Why not $U(3)_c$?

Because U(3)_c = (SU(3)_c × U(1))/$\mathbb{Z}_3$, and the determinant phase is an abelian ledger, not a ninth gluon. Counting U(3)_c as colour would double-count the phase sector, which §7 shows is one-dimensional in total.

11.3 Why not SU(N)_c with N ≠ 3?

Because the inherited colour census has exactly three indistinguishable quark colour slots. N ≠ 3 changes the committed census.

11.4 Why not SU(2)_R?

Because the weak census is left chiral. Right charged slots are weak singlets in the minimal census. An SU(2)_R gauge group requires additional right doublet structure and, generically, a non-sterile right-neutrino partner. That is an extension, not a closure of the minimal Standard Model census.

11.5 Why not an extra U(1)′?

Because Lemma 4a classifies the anomaly-safe generation-universal abelian directions on the committed census — there are exactly two lines — and Lemma 4b shows they are mutually anomalous, so at most one can be gauged; mass-pairing then eliminates the exceptional line. On the three-generation census, the anomaly-safe family is larger — it includes the generation-difference ledgers such as L_μ − L_τ, which pass every anomaly condition jointly with Y — and those are eliminated by Lemma 4c: on a connected committed flavour-continuation graph, a generation-non-universal charge annihilates a committed mixing continuation. The remaining loopholes are a new sterile or hidden charged ledger (F6/F7) and a failure of the flavour-gate mixing commitments (F21); neither is ignored.

11.6 Why not gauged B−L?

A gauged B−L can be considered only if the census admits the required additional anomaly-safe matter structure, in particular a non-trivial sterile-neutrino ledger or equivalent completion. In the minimal Standard Model census used here, no independent B−L gauge ledger is admitted. B−L is not part of GC-1 closure; it is an extension candidate with its own gate obligation.

11.7 Why not generation gauge symmetry?

Generations are not local same-slot degeneracies in this census. They are not colour-like copies or weak-basis copies. VERSF treats generation hierarchy as refinement/history structure, not as a freely rotatable internal gauge index. A generation gauge symmetry would also be immediately constrained by mass hierarchy and flavour structure; since those structures distinguish generations, local generation rotation is not a redundancy of the committed census. The generation-diagonal abelian case — charges that distinguish generations without rotating them — is treated separately and eliminated in §7.7.

11.8 Why not flavour SU(3)_F?

For the same reason. A flavour group treats generations as equivalent basis states, but the census does not present generations as identical gauge-degenerate slots: their masses and mixings are physical structure, not gauge-redundant labels. Flavour symmetry may appear as an approximate global pattern or a model-building extension, but not as a primitive gauge-census channel.

11.9 Why not primitive SU(5), SO(10), or larger simple unification?

Larger simple groups may organize Standard Model representations. But their extra generators rotate between census slots that VERSF treats as physically distinct: quark/lepton, colour/weak, singlet/doublet, or different hypercharge roles. To admit such a group as primitive, VERSF would need to show that those extra rotations preserve admissible continuation cones and do not require uncommitted gauge bosons or matter transitions. The minimal Standard Model gauge census does not do that. Grand-unified groups are not ruled out as embeddings; they are ruled out as already-admitted primitive gauge channels at GC-1.

11.10 Why not a mirror weak sector?

A mirror weak sector requires a second weak doublet structure, usually right chiral or hidden. The minimal census contains no second fold-access carrier. A mirror weak group therefore acts either on absent slots or on slots whose continuation cones differ from the left weak doublet. It is not admitted.

11.11 Why not hidden gauge groups?

A hidden gauge group must act on hidden committed slots. If the slots are absent, the group is empty notation. If the slots are present, the Standard Model census was not complete and must be enlarged. Hidden gauge sectors are not part of this closure theorem; they are extension candidates.

11.12 Why not electromagnetic U(1)_em as the primitive abelian group?

Because U(1)_em is the unbroken post-electroweak combination $Q_{em} = T_3 + Y/2$. Before electroweak symmetry breaking, the primitive gauge ledger is hypercharge Y, not electric charge alone. Treating U(1)_em as primitive would miss the left weak doublet structure.

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12. Relation to χ, Mass Hierarchy, and Later Gates

This paper belongs to the gauge-completion track, not the mass-hierarchy track.

The χ-readout theorem answers a different question: how a non-zero up/down fold-access contrast can survive when the final standing record is symmetric. It proves that χ must read ordered commitment history rather than only the final record. The present paper answers: what are the admitted internal gauge channels of the Standard Model census?

The two tracks interact but do not replace each other. Gauge closure supplies SU(3)_c, SU(2)_L, U(1)_Y. Mass-hierarchy closure must later supply non-zero χ source existence, the χ halving or refinement law, Yukawa structure, and the mass anchor. This paper computes no χ magnitude; conversely, the χ-readout theorem does not close the gauge census. The dependency discipline is

gauge-census closure ∥ χ readout-domain closure,

followed later by Yukawa and mass-hierarchy construction — with one recorded asymmetry. The parallel-tracks statement holds for the χ readout question, but the abelian clause of Theorem 1 on the three-generation census inherits one-directionally from the flavour-gate commitments that connect the flavour-continuation graph: Lemma 4c invokes the committed CKM mixing in the quark sector and the SM-3 leakage amplitude $\beta = \sqrt{3}/20$ in the lepton sector to make the graph connected and eliminate the generation-difference ledgers. Gauge closure at three generations is therefore downstream of those flavour commitments; the flavour gates do not depend on GC-1 in return. On the one-generation census the closure is free of this inheritance.

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13. Falsification Conditions

#	Failure	Consequence
F1	Colour census is not exactly threefold	SU(3)_c not forced
F2	Quark colour slots are physically distinguishable labels, not gauge-redundant copies	colour gauge interpretation fails
F3	Weak fold carrier is not exactly two-dimensional	SU(2)_L not forced
F4	Weak carrier is not left chiral	SU(2)_L placement fails; SU(2)_R or vector weak alternatives reopen
F5	Hypercharge ledger is not one-dimensional under joint anomaly and mass constraints	U(1)_Y uniqueness fails
F6	A second independent charge vector, jointly anomaly-free with Y, exists on committed slots	extra U(1)′ survives
F7	Sterile or hidden slots carry non-trivial committed gauge-access structure	hidden gauge extension required
F8	Anomaly discipline is not required for gauge admissibility	anomalous gauge candidates reopen
F9	Mass-pairing admissibility equations are not required, or the census admits multiple Higgs doublets serving the quark and lepton pairings separately	the exceptional lepton-blind ledger of Lemma 4a reopens; its annihilation assumes a single Higgs serving both sectors and must be re-audited in any multi-Higgs census

#	Failure	Consequence
F10	Generations are local same-slot gauge-degenerate copies	generation gauge channel reopens
F11	Larger simple-group extra generators preserve all VERSF continuation cones	primitive unification reopens
F12	Determinant phases of U(3) and U(2) are counted as independent non-abelian gauge channels	overcount; gauge census fails
F13	The finite-centre quotient changes the local gauge algebra	local/global separation fails
F14	A gauge generator may act on absent slots	no-free-generator premise fails
F15	VERSF admits member-owned primitive gauge labels outside class grain	class-grain carrier premise fails
F16	The global weak-parity condition fails on the committed doublet count	SU(2)_L global consistency fails
F17	The inherited Minimal Internal Symmetry results fail to select the projective sectors n = 3, n = 2, and the abelian phase/access sector	P4–P6 lose inherited support; GC-1 becomes a conditional census theorem over unproven inputs
F18	Distinguishability conservation does not force connection structure for local state comparison	P1 weakens; gauge-as-local-census-redundancy becomes an assumption rather than an inherited theorem
F19	The minimal empirical anchors are not weaker than Standard Model assumptions	no-smuggle firewall fails; the derivation risks circularity
F20	The fourteen-generator closure architecture is identified with the local gauge-boson census	category error; 14-generator closure machinery conflated with the 12-dimensional Standard Model gauge algebra
F21	The committed flavour-continuation graph is disconnected in some sector (a generation pair with no committed mixing continuation; e.g., a CKM pair unmixed or the SM-3 leakage β exactly zero)	Lemma 4c fails on that component; generation-difference ledgers constant on the components (e.g., L_μ − L_τ) reopen as admissible extra U(1)s, and Theorem 1's abelian clause fails on the three-generation census

F6, F7, and F21 are the most important extension warnings. If VERSF later admits a non-trivial sterile or hidden gauge ledger, the theorem must be revised from minimal Standard Model gauge closure to Standard-Model-plus-extension closure. F9 is now sharper than in a naive treatment: its failure modes are named — the exceptional line of Lemma 4a, and its single-Higgs annihilation assumption — rather than an unspecified weakening. F17–F19 make the inheritance structure honest: they record exactly which prior-gate results GC-1 depends on and what fails if those results fail. F21 does the same for the flavour-gate dependency introduced by Lemma 4c.

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14. GC-1 Closure Certificate

Condition	Result
Gauge-as-local-census-redundancy defined	closed
Committed census slot defined	closed
Admissible gauge generator defined (joint anomaly form)	closed
Sector factorisation proven	closed, Lemma 1
Colour triplicity to SU(3)_c	closed, conditional on P4
U(3)_c overcount removed	closed
Weak doublethood to SU(2)_L	closed, conditional on P5
SU(2)_R exclusion	closed under minimal chiral census
Global weak-parity condition	closed; 4 doublets per generation
abelian ledger variables introduced	closed
Anomaly-only abelian classification	closed, Lemma 4a
Mutual-anomaly obstruction between candidate ledgers	closed, Lemma 4b
Hypercharge ratios derived; exceptional line eliminated; u↔d exchange identified as H ↔ H̃ convention	closed, Lemma 4
Cubic anomaly checked	closed
Mixed anomalies checked	closed
Extra U(1) excluded (generation-universal)	closed under minimal census and P9
Generation-difference ledgers displayed and eliminated (L_μ − L_τ family)	closed conditionally, Lemma 4c and Corollary 4c.1; inherits connectedness of the committed flavour-continuation graph (F21)
Charge quantization from compactness and quotient maximality	closed conditional on the quotient-maximality convention (§9.3)
Hidden gauge sectors excluded	closed unless new committed slots are admitted
Generation-rotating gauge excluded	closed under generation-not-gauge premise
Generation-diagonal ledgers excluded	closed conditionally, Lemma 4c (mixing-graph universality) with singlet tracking via P7
Larger simple groups classified as embeddings/extensions	closed
Finite $\mathbb{Z}_6$ quotient handled	closed as global refinement

Condition	Result
Inheritance from Minimal Internal Symmetry programme	closed as prior-source inheritance; P4–P6 not introduced ad hoc
Gauge necessity from distinguishability conservation	inherited; supports P1
Minimal-anchor/no-smuggle firewall	inherited from Closing the Interfaces; strengthened here by census closure
Fourteen-generator architecture separated from local gauge-boson census	closed; no 14-vs-12 conflation; firewall independent of the open D_7 identification
Dual inheritance routes for chirality restriction recorded (topological and overconservation)	closed as inheritance bookkeeping; overconservation route graded BC1+BC2
Abelian one-dimensionality cross-checked by two independent mechanisms (Fisher degeneracy; mutual anomalies)	closed as convergence note, §7.1a
Hypercharge convention dictionary	closed, Appendix D
No further connected internal gauge channel remains under P1–P10 together with, on the three-generation census, a connected committed flavour-continuation graph	closed; central GC-1 result
Coupling constants	outside scope
Electroweak symmetry breaking	outside scope
Higgs potential	outside scope
Fermion masses	outside scope
Neutrino-mass mechanism	outside scope

14.1 Residual Conditionality After Reduction

After replacing P1, P4, P5, and abelian access existence with the inherited theorem-level inputs I1–I4, the remaining conditionality of GC-1 is confined to four items:

Residual condition	Why it remains
Minimal SM census boundary (§3.0)	Extra hidden, sterile charged, mirror, or multi-Higgs structure would define an extension, not the minimal census
Single Higgs access slot	Needed to annihilate the exceptional lepton-blind abelian ledger and to force singlet charges to track doublet charges; a multi-Higgs census must be separately audited (F9)
Connected flavour-continuation graph	Needed to eliminate generation-difference ledgers such as L_μ − L_τ (Lemma 4c; F21)
Quotient-maximality convention	Needed to select the global $\mathbb{Z}_6$ form and charge lattice; the local algebra is unaffected (§9.3; F13)

Everything else in GC-1 is either inherited from the prior gauge-foundation track or proved inside the present paper.

GC-1 closure grade, reduced-conditional form: closed at minimal connected internal gauge-census level. The closure is theorem-level relative to the inherited VERSF/BCB gauge-foundation results and the minimal Standard Model commitment census. The remaining conditions are not hidden assumptions inside the proof; they mark the boundary between the minimal Standard Model census and possible extensions: hidden charged sectors, sterile charged ledgers, mirror weak sectors, multi-Higgs ledgers, disconnected flavour continuations, or non-maximal global quotient choices.

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15. Conclusion

The VERSF Standard Model gauge problem is a census problem. It is not enough to recover the names SU(3), SU(2), U(1). One must prove that the admissible internal gauge list closes.

This paper proves that closure. A connected internal gauge channel is admitted only when it acts as a local redundancy of a live class-grain commitment degeneracy while preserving continuation cones, chirality, mass-pairing, joint anomaly discipline, and quotient canonicity. The minimal VERSF Standard Model census contains exactly three such sources — threefold colour realization degeneracy, twofold left chiral weak fold degeneracy, and one abelian phase ledger — and they give $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$.

The hypercharge ledger is not arbitrary, and its rigidity is stronger than mere consistency. On the generation-universal subspace, anomaly discipline alone reduces the abelian possibilities to exactly two lines; the two lines cannot coexist as gauge channels; and mass-pairing annihilates the impostor, the residual up/down exchange being a field-redefinition convention rather than a rival ledger. On the three-generation census, the anomaly-free generation-difference family — gauged $L_\mu - L_\tau$ at its sharpest — is eliminated by mixing-graph universality: on a connected committed flavour-continuation graph, a generation-non-universal charge forbids a committed continuation. The exclusion is conditional on that connectedness. The surviving ledger is:

$Y(Q) = 1/3$, $Y(u^c) = -4/3$, $Y(d^c) = 2/3$, $Y(L) = -1$, $Y(e^c) = 2$, $Y(H) = 1$.

The admitted local internal algebra is therefore

$$\mathfrak{g}_{VERSF} = \mathfrak{su}(3)_c \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$$

with global finite-centre refinement

$$G_{faithful} = (SU(3)_c \times SU(2)_L \times U(1)_Y) / \mathbb{Z}_6,$$

which, under the quotient-maximality convention of §9.3, additionally fixes the hypercharge lattice on the committed representations.

The significance of GC-1 is therefore not merely that VERSF can reproduce the Standard Model gauge algebra. It is that, once the prior gauge-foundation results are admitted, the Standard Model gauge algebra is the unique closed connected internal census.

Colour triplicity supplies SU(3)_c. Left chiral weak doublethood supplies SU(2)_L. The single jointly-anomaly-safe abelian access ledger supplies U(1)_Y. Gauge necessity follows from local distinguishability comparison. Extra generators are not free: they must act on live census slots, preserve continuation cones, respect chirality, allow mass-pairing, satisfy joint anomaly discipline, and avoid duplicating already counted structure.

Every proposed additional connected internal gauge channel fails that test unless it introduces new committed matter structure or belongs to an extension beyond the minimal Standard Model census.

Thus GC-1 turns the gauge programme from reconstruction into closure:

The Standard Model is not one admissible internal gauge census among many. Under the VERSF/BCB admissibility rules and the inherited minimal commitment structure, it is the maximal connected internal gauge census.

All remaining Standard Model work now concerns dynamics, breaking, representation refinement, and mass hierarchy — not the discovery of an omitted primitive connected gauge factor.

Appendix A — Hypercharge Rigidity: Full Two-Stage Calculation

This appendix covers Stages 1–2 (the generation-universal classification and mass-pairing selection). The third part of the rigidity argument — mixing-graph universality and singlet tracking — is Lemma 4c, §7.7.

Left-handed Weyl notation throughout: Q, u^c, d^c, L, e^c, with Higgs H. Assign

$Y(Q) = q$, $Y(u^c) = u$, $Y(d^c) = d$, $Y(L) = \ell$, $Y(e^c) = e$, $Y(H) = h$.

A.1 Stage 1 — anomalies only (fermion charges)

Conditions, with Weyl multiplicities 6, 3, 3, 2, 1:

A1: $2q + u + d = 0$ A2: $3q + \ell = 0$ A3: $6q + 3u + 3d + 2\ell + e = 0$ A4: $6q^3 + 3u^3 + 3d^3 + 2\ell^3 + e^3 = 0$

Elimination:

$u + d = -2q$, $\ell = -3q$, $e = -6q - 3(u + d) - 2\ell = -6q + 6q + 6q = 6q$.

Substituting into A4:

$6q^3 + 3(u^3 + d^3) - 54q^3 + 216q^3 = 0 \Longrightarrow u^3 + d^3 = -56q^3$.

With $u^3 + d^3 = (u + d)^3 - 3ud(u + d) = -8q^3 + 6q\cdot ud$:

$-8q^3 + 6q\cdot ud = -56q^3 \Longrightarrow 6q\cdot ud = -48q^3$.

$q \neq 0$: $ud = -8q^2$, $u + d = -2q \Longrightarrow t^2 + 2qt - 8q^2 = 0 \Longrightarrow t = (-2q \pm 6q)/2 \Longrightarrow \{u, d\} = \{2q, -4q\}$.

$q = 0$: $\ell = e = 0$, $u = -d \equiv s$ free; A4 gives $3s^3 + 3(-s)^3 = 0$ identically.

So the anomaly-safe set is exactly

$Y = q\cdot(1, -4, 2, -3, 6)$ (up to $u \leftrightarrow d$) $\cup$ $Y' = s\cdot(0, 1, -1, 0, 0)$.

A.2 Mutual anomalies between Y and Y′

$\mathrm{Tr}[Y^2 Y'] = 3u^2 s + 3d^2(-s) = 3s(u^2 - d^2) = 3s(16q^2 - 4q^2) = 36\, q^2\, s$ $\mathrm{Tr}[Y'^2 Y] = 3s^2 u + 3s^2 d = 3s^2(u + d) = -6\, q\, s^2$

Both are non-zero for $q \neq 0$, $s \neq 0$. Hence Y and Y′ cannot be jointly gauged: the abelian gauge sector on the minimal census has dimension at most one.

A.3 Stage 2 — mass-pairing selection

M1: $q + h + u = 0$ M2: $q - h + d = 0$ M3: $\ell - h + e = 0$

On Y′ ($q = \ell = e = 0$, $u = -d = s$): M1 $\Longrightarrow h = -s$; M2 $\Longrightarrow h = d = -s$ (consistent); M3 $\Longrightarrow h = 0 \Longrightarrow s = 0$. The exceptional line is annihilated by the lepton pairing.

On Y: M3 $\Longrightarrow -3q - h + 6q = 0 \Longrightarrow h = 3q$. M1 $\Longrightarrow u = -q - h = -4q$. M2 $\Longrightarrow d = -q + h = 2q$. The exchanged assignment $u = 2q$, $d = -4q$, $h = -3q$ satisfies M1–M2 and fails M3 under the P7 pairing list; it is not a second solution but the same ledger in the swapped naming convention, undone by $u \leftrightarrow d$ together with $H \leftrightarrow \tilde{H}$ (see §7.4 and Appendix D). Per convention, the solution is unique.

Unique survivor:

$(q, u, d, \ell, e, h) = q\cdot(1, -4, 2, -3, 6, 3)$.

A.4 Rank verification

Unknowns (q, u, d, ℓ, e, h); linear conditions M1, M2, M3, A2, A3. A non-zero solution exists (the line above) and elimination shows it is unique up to scale, so the system has rank 5 and nullity 1. On the solution line, A1 and A4 hold identically:

A1: $2q - 4q + 2q = 0$ A4: $(6 - 192 + 24 - 54 + 216)\, q^3 = 0$

A.5 Normalization

Setting $q = ⅓$ gives

$Y(Q) = ⅓$, $Y(u^c) = -4/3$, $Y(d^c) = ⅔$, $Y(L) = -1$, $Y(e^c) = 2$, $Y(H) = 1$,

and $Q\_em = T_3 + Y/2$ reproduces the observed electric charges: $Q\_em(u\_L) = ½ + ⅙ = ⅔$, $Q\_em(d\_L) = -½ + ⅙ = -⅓$, $Q\_em(e\_L) = -½ - ½ = -1$, $Q\_em(\nu\_L) = ½ - ½ = 0$.

Appendix B — Candidate Gauge Channels and Their Status

Candidate	Status	Reason
SU(3)_c	admitted	traceless local relabelling of three colour slots
SU(2)_L	admitted	traceless local relabelling of left chiral doublets
U(1)_Y	admitted	unique jointly-anomaly-safe abelian ledger compatible with mass-pairing
U(1)′ = s·(0, 1, −1, 0, 0)	excluded	anomaly-safe alone but mutually anomalous with Y; annihilated by lepton mass-pairing (single-Higgs census; a multi-Higgs census reopens it — see F9)
U(3)_c	reduced	determinant phase belongs to abelian ledger
U(2)_L	reduced	determinant phase belongs to abelian ledger
SU(2)_R	excluded	no right chiral weak doublet in minimal census
generic extra U(1)″ (generation-universal)	excluded	no third anomaly-safe direction exists (Lemma 4a)
L_μ − L_τ and lepton generation-difference ledgers	excluded conditionally	jointly anomaly-free with Y on three generations; forbid the committed μ–τ leakage (SM-3, β = √3⁄20); fail continuation-cone preservation (Lemma 4c)
quark generation-difference ledgers	excluded conditionally	pass the anomaly battery; forbid committed CKM mixing continuations; fail continuation-cone preservation (Lemma 4c)
scalar-only ledger (charges H only, fermions neutral)	excluded	M1 alone forces h = 0

Candidate	Status	Reason
gauged B−L	extension candidate	requires separate sterile/hidden ledger gate
generation SU(3)_F	excluded	generations are not gauge-degenerate same-slot copies
mirror weak group	excluded	no second weak fold carrier
SU(4) colour	extension candidate	mixes lepton and colour roles; extra generators not census-redundant
SU(5)	embedding/extension	extra generators not primitive census redundancies
SO(10)	embedding/extension	requires enlarged matter-slot commitments
hidden gauge group	extension candidate	requires hidden committed slots
U(1)_em	post-breaking survivor	not the primitive unbroken census ledger
finite $\mathbb{Z}_6$ quotient	admitted refinement	global identification; no local algebra change; fixes charge lattice

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Appendix C — Referee Objections and Replies

Objection 1 — You are just assuming the Standard Model gauge group.

Reply. The paper assumes the VERSF census facts: colour triplicity, left chiral doublethood, and an abelian access carrier — deliberately not the number of abelian ledgers, which is proved in §7, not premised (P6). It then proves the corresponding local automorphism groups and the exclusion of additional channels. The output is not read off from the Standard Model group names; it is derived from the admitted census degeneracies and admissibility filters. The clearest evidence is the abelian sector: the anomaly-only analysis returns two candidate ledgers on the generation-universal subspace, and a further anomaly-free family (the generation-difference ledgers) on the three-generation census — and the census filters must do genuine work to eliminate both (Lemmas 4a, 4b, 4c; §7.4, §7.7). A derivation that merely assumed the answer would have nothing to eliminate.

Objection 2 — Why is colour SU(3) rather than U(3)?

Reply. The local basis ambiguity of a three-dimensional complex colour carrier is U(3), but its determinant phase is abelian. The traceless colour rotations are SU(3). The determinant phase must be counted in the abelian ledger or not at all; counting it as colour would double-count phase freedom, and §7 proves the total abelian sector is one-dimensional.

Objection 3 — Why is weak SU(2)_L rather than U(2)_L?

Reply. Same logic. The two-dimensional left weak carrier first gives U(2). Its traceless component is SU(2)_L; its determinant phase belongs to the abelian ledger.

Objection 4 — Why is the weak sector left chiral?

Reply. Because the committed weak census contains left chiral doublets and right chiral singlets. A right weak group would require right doublets and additional matter-slot commitments. It is an extension, not a closure of the minimal Standard Model census.

Objection 5 — Could there be an extra U(1)?

Reply. Only if a second charge vector exists that is jointly anomaly-safe with Y, compatible with mass-pairing, and continuation-cone-preserving on committed slots. On the generation-universal subspace, Lemma 4a shows the anomaly-safe set is exactly two lines; Lemma 4b shows they are mutually anomalous, so at most one is gaugeable; the lepton mass-pairing annihilates the non-hypercharge line. On the three-generation census, the generation-difference ledgers pass every anomaly condition and are eliminated by Lemma 4c. An extra U(1) therefore requires either new committed structure (F6, F7) or a failure of the flavour-gate mixing commitments (F21).

Objection 6 — Anomaly cancellation famously admits the B−L-like solution; have you swept it under the rug?

Reply. No — it is exhibited explicitly. On the minimal census without a charged right-neutrino slot, the second anomaly-safe direction is the lepton-blind ledger $s\cdot(0, 1, -1, 0, 0)$, not gauged B−L. It appears in Lemma 4a, is shown to be mutually anomalous with Y in Lemma 4b, and is annihilated by the lepton pairing in §7.4. Gauged B−L proper requires additional committed matter (a non-sterile right-neutrino ledger) and is classified as an extension gate, not part of GC-1 closure.

Objection 7 — What about grand unification?

Reply. Grand-unified groups may embed the Standard Model representations. This paper does not forbid such embeddings. It denies that the extra generators of a larger simple group are already primitive gauge-census redundancies of the minimal VERSF commitment structure. To admit them, one must prove they preserve VERSF continuation cones and supply the necessary extended matter and gauge commitments.

Objection 8 — Why not generation gauge symmetry?

Reply. Because generations are not local basis choices inside one indistinguishable census slot. They carry physical mass and hierarchy structure. A gauge symmetry cannot rotate physically distinguished refinement histories as if they were redundant labels. The generation-diagonal abelian case — which rotates nothing — is not covered by this reply; it is treated and eliminated separately in §7.7.

Objection 9 — Does this prove coupling constants?

Reply. No. It proves the gauge algebra and census closure. Coupling magnitudes are separate dynamical data.

Objection 10 — Does the $\mathbb{Z}_6$ quotient matter?

Reply. It matters for global representation faithfulness, and — under the quotient-maximality convention named in §9.3 — it fixes the hypercharge quantization lattice on the committed representations. The committed content is equally compatible with the $\mathbb{Z}_3$, $\mathbb{Z}_2$, and trivial quotients, so $\mathbb{Z}_6$ is canonical only given that convention. None of the choices changes the local Lie algebra or the number of gauge bosons; the quotient is a global refinement, not a separate gauge-census channel.

Objection 11 — Does anomaly cancellation alone derive the Standard Model gauge group?

Reply. No, and §7.7 makes the insufficiency concrete. The non-abelian factors are derived from the committed census degeneracies: colour triplicity and left chiral doublethood. Anomaly discipline constrains the generation-universal abelian ledger to two candidate lines and bounds that sector's dimension by one via the mutual-anomaly obstruction; mass-pairing completes the universal selection. But anomaly discipline alone admits the generation-difference ledgers — $L_\mu - L_\tau$ passes every anomaly condition — and only continuation-cone preservation excludes them. The theorem uses census structure, anomaly discipline, mass-pairing, and continuation preservation jointly; no proper subset suffices.

Objection 12 — What about gauged $L_\mu - L_\tau$? It passes every anomaly condition on three generations.

Reply. Correct, and the paper displays it rather than hiding it (§7.7). On the three-generation census, $L_\mu - L_\tau$ is anomaly-free jointly with Y, acts on committed slots, is neutral under the diagonal lepton pairings, and requires no new matter. It fails a different filter: condition 3, preservation of admissible continuation cones. Gauge invariance under $L_\mu - L_\tau$ forbids exactly the μ–τ leakage that SM-3 commits as physical continuation structure ($\beta = \sqrt{3/20} \neq 0$). Lemma 4c generalizes the point as a graph statement: on a connected committed flavour-continuation graph, equality of charges propagates along mixing edges, so every admissible abelian charge is generation-universal; singlet charges then track doublet charges through the per-generation diagonal pairings. No complete classification of the anomaly-free family is needed — connectedness is the single operative condition. The exclusion is conditional on that connectedness and carries falsifier F21.

Objection 13 — Is the theorem unconditional?

Reply. No. It is conditional on P1–P10 — with P1, P4, P5, and P6 restating the inherited results I1–I4 — together with, on the three-generation census, the connectedness of the committed flavour-continuation graph (Lemma 4c; F21). The residual conditionality is itemized in §14.1. Within that frame, the local gauge-census closure is exact. If any condition fails — a sterile or hidden slot carrying a committed charged ledger, or a disconnected flavour graph — the falsification table (§13) states the consequence.

Appendix D — Hypercharge Convention Dictionary

Programme documents state hypercharge in two conventions, differing in field basis (left-handed conjugates versus right-handed fields) and in normalization ($Q\_em = T_3 + Y/2$ versus $Q\_em = T_3 + Y$). This appendix fixes the map so that cross-citations, in particular to the interface track's abelian-uniqueness analysis, are unambiguous.

Convention I (this paper). Left-handed Weyl fields with conjugates: Q, u^c, d^c, L, e^c, H. Electric charge

$Q\_em = T_3 + Y/2$.

Derived assignments (§7.4):

$Y(Q) = 1/3$, $Y(u^c) = -4/3$, $Y(d^c) = 2/3$, $Y(L) = -1$, $Y(e^c) = 2$, $Y(H) = 1$.

Convention II (interface track, Closing the Interfaces Appendix C, and much of the literature). Right-handed fields for the singlets: Q_L, u_R, d_R, L_L, e_R, H. Electric charge

$Q\_em = T_3 + Y$.

Assignments:

$Y(Q\_L) = 1/6$, $Y(u\_R) = 2/3$, $Y(d\_R) = -1/3$, $Y(L\_L) = -1/2$, $Y(e\_R) = -1$, $Y(H) = 1/2$.

The dictionary. Two independent substitutions connect the conventions:

1. Conjugation flips the sign of every charge carried by a singlet field: $Y(f^c) = -Y(f\_R)$.
2. The normalization change doubles every charge: $Y\_I = 2 \cdot Y\_II$ for fields in the same basis.

Composed, for the three singlets:

$Y\_I(f^c) = -2 \cdot Y\_II(f\_R)$,

and for the doublets and Higgs:

$Y\_I = 2 \cdot Y\_II$.

Verification across the full census:

Slot (Conv. I)	Y (Conv. I)	Slot (Conv. II)	Y (Conv. II)	Check
Q	$1/3$	Q_L	$1/6$	$2 \times 1/6 = 1/3$ ✓
u^c	$-4/3$	u_R	$2/3$	$-2 \times 2/3 = -4/3$ ✓

Slot (Conv. I)	Y (Conv. I)	Slot (Conv. II)	Y (Conv. II)	Check
d^c	⅔	d_R	−1⁄3	−2 × (−1⁄3) = ⅔ ✓
L	−1	L_L	−1⁄2	2 × (−1⁄2) = −1 ✓
e^c	2	e_R	−1	−2 × (−1) = 2 ✓
H	1	H	1⁄2	2 × 1⁄2 = 1 ✓

Electric charges are convention-independent, as they must be: for example Q_em(u) = ½ + ⅙ = ⅔ in Convention I equals 0 + ⅔ for u_R in Convention II.

Solution-vector translation. The one-generation solution line of Lemma 4 reads, in Convention I on (Q, u^c, d^c, L, e^c, H),

q · (1, −4, 2, −3, 6, 3),

and in Convention II on (Q_L, u_R, d_R, L_L, e_R, H),

q′ · (1, 4, −2, −3, −6, 3), with q′ = q⁄2.

Both are the same physical ledger. Any apparent mismatch between programme documents in the u- and d-type entries — sign flips or an apparent u ↔ d exchange — is an artifact of mixing these conventions and dissolves under the dictionary above. All cross-citations from this paper to the interface track's anomaly system (Closing the Interfaces, Appendix C) should be read through this map.

One further entry completes the dictionary, this time within a single convention. The residual u ↔ d exchange of Lemma 4a is undone by the simultaneous field redefinition H ↔ $\tilde{H} = i\sigma_2 H^*$: the exchanged solution q·(1, 2, −4, −3, 6, −3) is the standard ledger with the up/down naming and the Higgs-contraction convention swapped, not a second physical assignment. This is why §7.4 treats the exchange as convention absorbed by P7's pairing list rather than as physics eliminated by it.