

# The Gauge-Closure Selection Theorem in VERSF

## Deriving Shared-Bath Necessity, Removing Hidden Ledgers, and Auditing the Faithful Global Standard Model Gauge Group

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### General Reader Summary

The previous paper showed that if identical internal sectors share one common pool, then the strong and weak forces follow naturally. Three shared colour sectors produce the strong-force group  $SU(3)$ . Two shared weak sectors produce  $SU(2)$ . But that paper was honest about the weak point: it assumed the shared pool.

This paper asks why nature should use a shared pool rather than separate private accounts.

The answer is that private accounts secretly name the sectors. If the sectors are truly identical, there is no physical fact that says "this is sector 1" and "that is sector 2." Keeping a separate ledger for each one would preserve a distinction that nothing in reality can read. VERSF does not allow that. A distinction has to be physically earned: it must be observable, recordable, or structurally necessary. A private ledger inside an indistinguishable class is none of these.

There is a subtlety worth stating plainly, because a careful critic would raise it. One might try to keep the bookkeeping without naming anyone — conserving, say, the *sorted list* of account balances rather than the accounts themselves. No sector is named; the list is the same whichever way the sectors are labelled. This paper shows that the evasion fails: the list of balances only means something once a particular way of carving the class into sectors has been chosen, and if that carving is physically preferred, the sectors were distinguishable after all. The bookkeeping cannot be laundered through symmetry.

So the paper proves:

**Genuinely indistinguishable sectors cannot carry private ledgers — named or anonymous. They must conserve only the shared total.**

That is exactly a bath.

Once that is proven, the previous result becomes much stronger:

identical sectors  $\Rightarrow$  no private ledgers  $\Rightarrow$  shared bath  $\Rightarrow$  continuous mixing  $\Rightarrow$   $SU(k)$ .

With  $k = 3$ , this gives colour and the strong force. With  $k = 2$ , this gives the weak structure, with left-handedness inherited from the earlier chirality work.

The second half of the paper deals with a subtler bookkeeping issue. The Standard Model gauge group is usually written as

$$SU(3) \times SU(2) \times U(1).$$

That is correct locally. It gives the right twelve gauge directions: eight gluons, three weak directions, and one hypercharge direction. But globally, some central transformations act as nothing at all on every allowed field. If a transformation changes no quark, no lepton, and not even the completion interface, then it is not a physical difference. It should be divided out.

The paper shows that the redundant common centre is  $Z_6$ . Therefore the faithful global group is

$$[ SU(3)_C \times SU(2)_L \times U(1)_Y ] / Z_6.$$

And the quotient is not empty bookkeeping in reverse: it stamps a rule onto every field nature is allowed to contain. The rule ties electric charge to colour. Anything blind to colour must carry whole-number charge; anything that feels the strong force in the way quarks do must carry charge shifted by a third. That pattern is observed, and any future particle that broke it would break the quotient.

In plain language:

Nature does not keep private names for identical internal sectors, and it does not count global gauge transformations that act on nothing. The Standard Model gauge structure is therefore both locally derived and globally de-duplicated.

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## Abstract

The preceding VERSF non-abelian gauge-origin paper established the conditional chain

identical closure class of size  $k$  + shared bath transport + continuity + sector phase  $\Rightarrow U(k) \Rightarrow SU(k)$ ,

and used the inherited census values  $k = 3$  and  $k = 2$  to obtain the local Standard Model gauge product

$$SU(3)_C \times SU(2)_L \times U(1)_Y.$$

Its explicit open debt was the bath premise: why should identical closure sectors share one transport bath rather than preserve separate sector ledgers?

This paper discharges that premise. We formulate the **No-Private-Ledger Principle**: a conserved substrate quantity is admissible only if it corresponds to an observable distinction, a recordable substrate fact, or a necessary quotient-invariant structure. A strict ledger over genuinely indistinguishable sectors preserves sector identity histories through rank-one projectors  $P_i$  — equivalently, through account functionals

$$L_i(w) = \langle w, P_i w \rangle.$$

If those projectors are physical, the sectors are distinguishable; if they are not physical, the ledger preserves hidden labels while nevertheless constraining admissible transport — structure that is dynamically load-bearing yet observationally unearned. Both alternatives are inadmissible. The exclusion is proved not only for quadratic accounts but for **all** conserved accounts on the class carrier: any conserved functional beyond total class weight is a non-constant function of the frame-dependent weight multiset, and conserving frame-dependent data requires a physically distinguished projector family, returning the argument to the projector dichotomy. Therefore the only admissible conservation law on a genuinely indistinguishable class is the invariant total class weight

$$W_C = \langle w, w \rangle = \sum_i |w_i|^2.$$

This proves the **Bath-Ledger Selection Theorem**:

genuine indistinguishability + no private labels + structure economy  $\Rightarrow$  shared bath conservation.

The exclusion is conditional on two named commitments, both instances of the minimum-distinction discipline: the No-Private-Ledger Principle and a Structure-Economy Premise (the class carrier admits no structure beyond its inner product — in particular no unearned real structure, which indistinguishability alone does not exclude). Refinement stability, listed as a selection premise in the predecessor paper, turns out to fix only the normalization of the surviving account and is demoted accordingly. Combining the theorem with the earlier Non-Abelian Bath-Transport Theorem yields

$$\text{identical closure class of size } k \Rightarrow \text{shared bath} \Rightarrow U(k) \Rightarrow SU(k).$$

The non-abelian origin of the strong and weak gauge factors is therefore no longer conditional on an unproven bath premise; the bath premise is selected by indistinguishability itself, with continuity, sector phase, census, chirality, and hypercharge still inherited.

The second part of the paper audits the global form of the completed Standard Model gauge group. Locally the algebra remains

$$\mathfrak{g}_{SM} = \mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y.$$

However, the faithful global group acting on the inherited matter and completion-interface representations is not the naive direct product. With integer hypercharge  $q = 6Y$ , the common central element

$$\zeta = ( e^{(2\pi i/3)} \cdot I_3, -I_2, e^{(i\pi/3)} )$$

acts trivially on

$$Q_L : (3, 2)_1, u_R : (3, 1)_4, d_R : (3, 1)_{-2}, L_L : (1, 2)_{-3}, e_R : (1, 1)_{-6}, \Phi_{cl} : (1, 2)_3.$$

Its six powers exhaust the common kernel. Hence the faithful global gauge group is

$$G_{SM}^{\text{faithful}} = [ SU(3)_C \times SU(2)_L \times U(1)_Y ] / \mathbb{Z}_6.$$

The quotient does not change the local connection, curvature, gauge-boson count, anomaly audits, electroweak breaking, or the non-abelian bath-origin theorem. It removes exactly the redundant central bookkeeping that no admissible field can distinguish — and it leaves behind one piece of observable content: every admissible field must satisfy the congruence

$$2t + 3d + q \equiv 0 \pmod{6},$$

where  $t$  is colour triality and  $d$  is weak-doublet duality, so that electric charge is correlated with triality as  $Q \equiv -t/3 \pmod{1}$ . Colour singlets carry integer charge; quarks carry the familiar third-integral charges. The quotient is therefore falsifiable field by field.

The paper does not derive the census values  $k = 3$ ,  $k = 2$ , the left-handedness convention, hypercharge assignments, coupling values, running couplings, confinement,  $\Lambda_{QCD}$ , fermion masses, CKM/PMNS, or three generations. It derives the bath premise required for non-abelian gauge origin and the faithful global quotient required for final gauge closure.

## Programme Inheritance Note

This paper inherits the minimum-distinction/fold discipline from *The Minimum Distinction as Fundamental Unit*, where the fundamental actualized unit is treated as a binary, boundary-realized, oriented distinction. It inherits gauge-connection necessity and  $U(1)$  uniqueness from *Distinguishability Conservation and Gauge Structure*. It inherits abelian uniqueness, chirality selection support, and the minimal empirical-anchor framing from *Closing the Interfaces*. It inherits the  $K = 7$  and closure-geometry infrastructure from *A Unified Derivation of Closure Geometry* and *The Standard Model from Hexagonal Geometry*.

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## 0. Notation, Conventions, and Firewalls

### 0.1 Class carrier

A class of  $k$  indistinguishable closure sectors is represented by

$$\mathcal{H}_k \cong \mathbb{C}^k,$$

with state vector

$$w = (w_1, \dots, w_k)$$

and total class weight

$$W_C = \sum_{i=1}^k |w_i|^2 = \langle w, w \rangle.$$

**Premise SE — Structure-Economy Premise.** The class carrier carries exactly one piece of admissible structure: the inner product  $\langle \cdot, \cdot \rangle$ . Any further structure on  $\mathcal{H}_k$  — a preferred basis, a preferred decomposition, a preferred tensor, a preferred antilinear or bilinear form — is not free; it must be earned under the admissibility criteria of Section 3.

This is a named premise, not a convention, because it does theorem-grade work in Theorem 2 and is strictly stronger than genuine indistinguishability (§0.4). The gap is exhibitable: a global antilinear real structure  $J$  on  $\mathcal{H}_k$  (equivalently, a symmetric bilinear form) names no sector, distinguishes no label, and is invariant under  $S_k$  — yet it enlarges the invariant algebra of a single class state beyond  $\langle w, w \rangle$  (for example  $|\langle Jw, w \rangle|$  separates equal-norm states) and cuts the admissible frame group from unitary to orthogonal type. Indistinguishability alone therefore does not exclude frame-like structure that names nothing; Premise SE does, and its exclusion must be earned rather than assumed.

The justification is the minimum-distinction/BC2 discipline of §0" and Section 3, applied to structure rather than to conserved quantities: a piece of global structure on the class carrier is a standing distinction, and a standing distinction that is neither observable, nor recordable, nor structurally required is excluded on the same grounds as a private ledger. A real structure  $J$  is precisely such an unearned standing distinction — it privileges a family of "real" directions that no admissible record singles out.

Premise SE is not independent of the programme's existing debts. The real-structure counter-instance  $J$  is broken exactly by sector-resolved phase:  $|\langle Jw, w \rangle|$  is not invariant under sector rephasings (verified numerically for  $k = 3$ ), so the sector-phase premise of the inherited bath-transport theorem (Debt 2) and Premise SE are two faces of a single commitment — that the class carrier is genuinely complex, with no admissible real substructure. A future discharge of Debt 2 discharges the real-structure instance of SE at the same stroke. The converse direction also holds, and by a one-sentence argument rather than assertion: if sector phase is physically unavailable, then states differing only by sector-resolved phase are physically identified, the carrier is effectively real, and a real form inside a complex carrier is exactly a preferred antilinear structure  $J$  — so failure of sector phase supplies an admissible real structure and SE fails with it (Falsifier F5). The two premises are therefore genuinely biconditional faces of the complex-structure commitment, not merely analogous.

A chosen orthonormal basis  $\{e_i\}$  of  $\mathcal{H}_k$  is called a **sector frame**. A sector frame is a coordinate choice unless a physical observable selects it.

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## 0.2 Ledger

A **strict ledger** on  $\mathcal{H}_k$  is a set of separately conserved sector weights

$$L_i(w) = |w_i|^2,$$

or equivalently

$$L_i(w) = \langle w, P_i w \rangle,$$

where

$$P_i = |e_i\rangle\langle e_i|$$

is the rank-one projector onto the  $i$ -th sector of the chosen frame.

A **monomial ledger** preserves the multiset of sector weights while allowing relabelling and phase rotations. Its transformation floor is

$$T^k \rtimes S_k.$$

This is the ledger result inherited from the previous non-abelian bath-transport paper.

An **anonymous ledger** conserves a symmetric function of the weight multiset  $\{|w_1|^2, \dots, |w_k|^2\}$  that is non-constant on level sets of  $W_{\mathcal{C}}$  — that is, not a function of the total alone — without conserving the individual weights. The example throughout is  $\sum_i |w_i|^4$ . (The qualifier matters:  $e_1 = \sum_i |w_i|^2$  is itself a non-constant symmetric function of the multiset, and it is the bath account, not a ledger.) Anonymous ledgers name no sector, yet still pin transport to the monomial group. Section 4 shows they are inadmissible for the same underlying reason as strict ledgers. Their explicit treatment is new to this paper and closes a gap in the quadratic-only argument.

## 0.3 Bath

A **bath** conserves only the total class weight

$$W_{\mathcal{C}} = \sum_i |w_i|^2,$$

while not preserving individual sector weights. In a bath, weight may be redistributed between indistinguishable sectors as long as the total remains fixed.

## 0.4 Genuine indistinguishability

A set of sectors is **genuinely indistinguishable** when no admissible physical observable, source ledger, boundary condition, record, or completion interface distinguishes the individual labels.

Equivalently: the labels  $1, \dots, k$  are not physical names. They are coordinates inside one class.

**Non-circularity note.** Indistinguishability is certified upstream, by the closure-census programme, on grounds independent of any gauge structure. It is not inferred from the  $SU(k)$

symmetry that this paper derives — that inference would be circular (bath  $\Rightarrow$  SU(k)  $\Rightarrow$  sectors equivalent  $\Rightarrow$  bath). The logical order is: the census delivers k sectors with no admissible distinguishing structure; this paper then derives what conservation laws such a class can carry. Where the census certification itself is conditional, that conditionality propagates and is recorded as Debt 3.

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## 0.5 Integer hypercharge convention

For the global quotient audit, we use integer hypercharge

$$q = 6Y.$$

Thus the inherited matter and completion-interface representations are written as

$$Q\_L : (3, 2)_1, u\_R : (3, 1)_4, d\_R : (3, 1)_{-2}, L\_L : (1, 2)_{-3}, e\_R : (1, 1)_{-6}, \Phi\_cl : (1, 2)_3.$$

This is only a normalization convention. The physical hypercharge is  $Y = q/6$ .

For centre bookkeeping we also record, for each representation, its **colour triality**  $t \in \{0, 1, 2\}$  (the  $\mathbb{Z}_3$ -centre charge of its SU(3) content) and its **weak duality**  $d \in \{0, 1\}$  (the  $\mathbb{Z}_2$ -centre charge of its SU(2) content). Thus  $Q\_L$  has  $(t, d, q) = (1, 1, 1)$ ,  $e\_R$  has  $(0, 0, -6)$ , and so on.

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## 0.6 Local/global firewall

This paper sharply separates the local gauge algebra

$$\mathfrak{su}(3)\_C \oplus \mathfrak{su}(2)\_L \oplus \mathfrak{u}(1)\_Y$$

from the faithful global group

$$[ \text{SU}(3)\_C \times \text{SU}(2)\_L \times \text{U}(1)\_Y ] / \Gamma.$$

The quotient affects global identification of group elements. It does **not** change the local gauge fields, curvature tensors, or gauge-boson count.

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## 0.7 Census firewall

This paper does not derive  $k = 3$  or  $k = 2$ . It inherits  $k = 3$  for colour and  $k = 2$  for weak structure.

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## 0.8 Chirality firewall

The theorem here gives  $SU(2)$  from the  $k = 2$  bath class. It does not derive why this is  $SU(2)_L$ . The selection  $SU(2) \rightarrow SU(2)_L$  is inherited from the electroweak/chirality programme.

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## 0.9 Hypercharge firewall

This paper does not rederive hypercharge assignments. It inherits the anomaly-admissible hypercharge ledger and the uniqueness of one  $U(1)_Y$  factor.

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## 0.10 QCD phenomenology firewall

This paper does not derive confinement, asymptotic freedom, the QCD beta function,  $\Lambda_{\text{QCD}}$ , hadron masses, the pion sector, instantons,  $\theta_{\text{QCD}}$ , or strong CP.

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## 0.11 Fourteen-generator firewall

The Standard Model local gauge-generator count is

$$8 + 3 + 1 = 12.$$

The separate fourteen-generator ( $K = 7$ ) closure-response structure belongs to closure architecture, not gauge-boson enumeration.

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## 0'. Predictive-Content Ledger

Object	Status	Comment
Ledger transport gives $T^k \rtimes S_k$	Inherited / Exact	Previous bath theorem
Ledger branch has no continuous non-abelian mixing	Inherited / Exact	Connected part is abelian

Object	Status	Comment
Bath transport gives $U(k)$	Inherited / Conditional	Requires bath, continuity, phase, covariance
Bath premise	<b>Derived here</b> (conditional on Principle 3.3 and Premise SE)	No-private-ledger theorem
No-Private-Ledger Principle	New — principle, not theorem	Distinctions must be physically earned
Structure-Economy Premise (SE)	New — named premise	Class carrier admits only the inner product; excludes unearned real structure; twinned with Debt 2
Anonymous-ledger exclusion (all polynomial orders)	New — closes quadratic-only gap	Frame-dependence lemma; Theorem 2
Bath-Ledger Selection Theorem	New central theorem (conditional on Principle 3.3 + SE)	Refinement stability not a premise; normalization only
$k = 3 \Rightarrow SU(3)_C$	Strengthened synthesis	Bath no longer merely assumed
$k = 2 \Rightarrow SU(2)$	Strengthened synthesis	Chirality inherited
Local gauge algebra	Inherited / reaffirmed	$\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$
Hypercharge uniqueness	Inherited	Single $U(1)_Y$
Global product vs quotient	New audit	Faithful kernel computed
$\mathbb{Z}_6$ common kernel	Derived here	Given inherited matter/interface representations
Faithful global group	Derived here	$[SU(3) \times SU(2) \times U(1)] / \mathbb{Z}_6$
Centre congruence $2t + 3d + q \equiv 0 \pmod{6}$	Derived here	Observable face of the quotient; falsifiable per field
Charge–trality correlation $Q \equiv -t/3 \pmod{1}$	Derived here	Downstream of the congruence
Gauge-boson count unchanged		Exact Quotient does not change Lie algebra
Confinement / running / masses	Not derived	Downstream

## 0''. Inherited Infrastructure

This paper inherits five prior results.

First, the Non-Abelian Bath-Transport Theorem proved that a shared bath of  $k$  identical sectors, with continuity and sector phase, generates  $U(k)$ , with  $SU(k)$  as the non-abelian content after removing the shared phase. It also proved that ledger transport gives only  $T^k \rtimes S_k$ .

Second, the gauge-connection necessity programme proves that local comparison of internal quantum states across the record layer requires a connection. Once a transport group is physically selected, local gauge fields follow.

Third, the closure-census programme supplies  $k = 3$  for colour and  $k = 2$  for weak structure, together with the upstream certification of genuine indistinguishability recorded in §0.4.

Fourth, the abelian-uniqueness and hypercharge programme supplies exactly one independent  $U(1)_Y$  factor and the inherited anomaly-admissible matter ledger.

Fifth, the minimum-distinction / fold foundation supplies the discipline that a physical distinction must be real, recordable, or structurally necessary. An unobservable distinction carried only as private bookkeeping is not an admissible substrate primitive.

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# 1. Purpose and Claim Level

The previous non-abelian gauge-origin paper ended with a precise open debt:

Why bath rather than ledger?

It proved

bath  $\Rightarrow U(k) \Rightarrow SU(k)$ ,

but it did not derive the bath premise itself.

The first purpose of this paper is to prove:

genuine indistinguishability + no private labels + structure economy  $\Rightarrow$  shared bath conservation,

with the exclusion conditional on Principle 3.3 and Premise SE, and with refinement stability doing no selection work (it fixes only the account's normalization, Corollary 2.1).

The proof is required to be watertight against a specific evasion: an adversary who concedes that *named* sector accounts are inadmissible, but proposes conserving an *anonymous* symmetric function of the account balances instead. That evasion is real —  $\sum_i |w_i|^4$  is invariant under every relabelling and every sector rephasing, names no sector, and yet freezes the class into ledger-

grade transport. A quadratic-only exclusion theorem does not touch it. This paper's exclusion theorem does.

The second purpose is to sharpen the completed Standard Model gauge product from a local product to the faithful global group acting on the inherited matter and completion-interface representations — and to extract the quotient's observable content rather than leaving it as pure bookkeeping.

The two central claims are therefore:

## Central Claim 1 — Bath-Ledger Selection

For a genuinely indistinguishable closure class, separate sector ledgers — strict, monomial, or anonymous — are inadmissible because they either distinguish sectors physically or preserve structure that no admissible record can read while nevertheless constraining dynamics. The only admissible conservation law is total class weight. Therefore identical closure sectors form a bath.

## Central Claim 2 — Faithful Global Gauge Closure

Given the inherited Standard Model representation ledger, the common centre of

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

contains a  $\mathbb{Z}_6$  subgroup acting trivially on all admissible matter and on the completion interface, and no more. Therefore the faithful global gauge group is

$$G_{SM}^{\text{faithful}} = [ SU(3)_C \times SU(2)_L \times U(1)_Y ] / \mathbb{Z}_6,$$

with the accompanying admissibility congruence  $2t + 3d + q \equiv 0 \pmod{6}$  on every field.

Both claims express the same discipline:

Do not keep as physical what nothing physical can distinguish.

## 2. The Hidden-Label Problem

A ledger is not neutral bookkeeping. A ledger answers the question:

which sector kept which amount of weight?

For distinguishable sectors, this is allowed. If two sectors carry different charges, different source ledgers, different representations, different boundary conditions, or different observed records, then separate ledgers correspond to real distinctions.

But for a genuinely indistinguishable class, the individual labels  $1, \dots, k$  have no physical content. They are coordinates inside a sector frame, not names of physical individuals.

A strict ledger over such a class introduces projectors

$$P_i = |e_i\rangle\langle e_i|$$

and separately conserves

$$L_i(w) = \langle w, P_i w \rangle.$$

These projectors distinguish the  $i$ -th sector from the  $j$ -th sector. So either:

1.  $P_i$  is physical, in which case the sectors are distinguishable; or
2.  $P_i$  is not physical, in which case the ledger preserves hidden labels.

The second horn deserves a sharper statement than "hidden labels are untidy," because it carries the real force of the argument. A conservation law is not inert decoration: conserving each  $L_i$  restricts admissible reversible transport to the monomial group  $T^k \rtimes S_k$ , whose connected component is abelian. The ledger therefore has **observable dynamical consequences** — it forbids continuous inter-sector mixing — while the labels it conserves have **no observable content whatsoever**. That asymmetry is the inadmissibility: the substrate would be paying a dynamical price to protect information that nothing in the substrate can read. Structure that is dynamically load-bearing must be observationally earned; the strict ledger is load-bearing and unearned simultaneously.

This is the hidden-label problem:

separate ledger  $\Rightarrow$  private sector identity  $\Rightarrow$  unearned distinction with dynamical consequences.

A ledger branch is therefore not merely too small to generate non-abelian gauge structure. For genuinely indistinguishable sectors, it is inadmissible.

**The anonymization evasion.** Before formalizing, one more escape route must be named, because Section 4 is built to close it. Suppose the adversary abandons named accounts and instead conserves only a symmetric function of the balances — the sorted multiset  $\{|w_1|^2, \dots, |w_k|^2\}$ , or a symmetric polynomial in it such as  $\sum_i |w_i|^4$ . No projector is singled out; the quantity is invariant under all of  $T^k \rtimes S_k$ . Has the hidden label been laundered away?

No — and the reason is that the multiset itself is not frame-independent. The numbers  $|w_i|^2$  are the squared components of  $w$  **in a particular sector frame**. Choose a different orthonormal frame for  $\mathcal{H}_k$  and the multiset changes; only its sum,  $\langle w, w \rangle$ , survives every frame change. So

for the multiset (or any non-constant function of it) to be a well-defined conserved physical quantity, some sector frame — equivalently, some complete rank-one projector family  $\{P_1, \dots, P_k\}$ , considered as an unordered set — must be physically distinguished. And a physically distinguished projector family is exactly the structure the projector dichotomy forbids: if the family is physical, the class decomposition is observable and the sectors were not genuinely indistinguishable; if it is not physical, the anonymous ledger conserves frame-dependent data that nothing can read, while still freezing transport to the monomial group. The anonymization does not remove the hidden structure; it only removes the ordering of the hidden structure.

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### 3. The No-Private-Ledger Principle

#### Definition 3.1 — Admissible conserved quantity

A conserved quantity is admissible only if at least one of the following holds:

1. it corresponds to an observable physical distinction;
2. it corresponds to a recordable substrate fact;
3. it is required by a quotient-invariant structural constraint;
4. it appears as a source ledger in an anomaly-admissible representation.

A conserved quantity satisfying none of these is private bookkeeping and is excluded.

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#### Definition 3.2 — Refinement stability

A conserved account  $A$  is **refinement-stable** when it composes additively across instances: for two distinct class carriers of the same kind — separate closure sites, not an internal partition of one carrier — with joint state  $w \oplus w' \in \mathcal{H}_k \oplus \mathcal{H}_{k'}$ , the account of the whole is the sum of the accounts of the parts,

$$A(w \oplus w') = A(w) + A(w'),$$

and  $A$  is monotone non-decreasing in total weight.

The restriction to composition *across* instances is deliberate and necessary: an internal partition of a single  $\mathcal{H}_k$  into orthogonal subcarriers would itself be a distinguished partial frame — exactly the structure Theorems 1–2 forbid on an indistinguishable class — so the internal case has no admissible domain. Composition across separate sites requires no frame inside either class and is therefore safe.

Refinement stability is what makes an account an *account* — a bookkeeping quantity that adds up over the books — rather than an arbitrary conserved function. Its role in this paper is deliberately modest: it does **not** enter the bath-selection theorem itself (Theorem 3 needs only Theorem 2), and serves only to fix the normalization of the bath account (Corollary 2.1). It was listed as a premise of bath selection in the predecessor paper; here that listing is corrected.

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### **Principle 3.3 — No-Private-Ledger Principle**

A substrate cannot preserve private sector identities inside a class whose defining feature is genuine indistinguishability.

Equivalently:

no physical distinction  $\Rightarrow$  no private conserved account.

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### **Justification**

The minimum-distinction discipline says actualized structure is built from distinctions. But a distinction must be real: binary, boundary-realized, oriented, record-supporting, or otherwise structurally required. A private ledger over identical sectors introduces a distinction without boundary, without record, and without observable consequence — while nevertheless taxing the dynamics (Section 2).

BC2 gives the same conclusion in economy language: a private ledger increases internal bookkeeping without changing observable predictions about sector identity, yet restricts the transport class. Such structure — costly where it cannot be seen, invisible where it would have to be paid for — is excluded.

Thus the No-Private-Ledger Principle is the operational form of VERSF's distinction discipline applied to conservation laws.

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## **4. The Bath-Ledger Selection Theorem**

### **Theorem 1 — Private Projector Families Are Inadmissible on an Indistinguishable Class**

Let  $\mathcal{H}_k \cong \mathbb{C}^k$  be a genuinely indistinguishable closure class. Suppose a complete rank-one projector family  $\{P_1, \dots, P_k\}$  — ordered or unordered — is required for the definition of any conserved quantity on the class. Then either:

1. the projector family is physical, in which case the class decomposition is observable and the sectors are distinguishable; or
2. the projector family is not physical, in which case the conserved quantity preserves frame-dependent data no admissible record can access, while restricting transport to at most the invariance group of the conserved account — a proper subgroup of the weight-preserving class whenever the account is not a function of the total, and exactly the monomial group of that frame in the strict-ledger and  $Q_4$  cases.

The dichotomy itself is exact. The first horn contradicts genuine indistinguishability outright. The exclusion of the second horn is conditional on Principle 3.3, which is a principle of the programme, not a theorem; the dynamical-asymmetry argument of Section 2 — structure that is dynamically load-bearing yet observationally empty — is what earns the principle its application here, and that conditionality is carried explicitly through the claim ledgers.

### Proof

If the family  $\{P_i\}$  is a physical observable structure — even as an unordered set — then measuring which projector a state occupies (or how weight distributes over the family) physically resolves the class into sectors. This contradicts genuine indistinguishability.

If the family is not physical, then any quantity defined through it is a function of coordinates in an unphysical frame. Conserving such a quantity preserves information no admissible record can access — private structure — and simultaneously constrains reversible transport to maps commuting with the conservation, i.e. to the monomial floor of that frame. By Principle 3.3 and the dynamical-asymmetry argument of Section 2, such structure is inadmissible. ■

**Remark.** The theorem is deliberately stated for the *family*, not for individual projectors. This is what defeats the anonymization evasion: an anonymous ledger discards the ordering of the family but still requires the family.

## Theorem 2 — Only Total Class Weight Is Admissible (all orders)

Let  $\mathcal{H}_k$  be a genuinely indistinguishable class carrier satisfying Premise SE (§0.1): its only admissible structure is the inner product. Let  $A$  be any admissible conserved account defined on class states  $w$  alone. Then

$$A(w) = f(W_C) = f(\langle w, w \rangle)$$

for some function  $f$ . In particular, no admissible account resolves anything finer than the total class weight.

**Scope.** The theorem quantifies over conserved functionals of the class state alone. Relational conserved quantities — joint invariants of class  $\otimes$  environment — are outside its scope, and correctly so: the bath is a class-level statement, and a joint invariant of the class with a distinguishing environment falls under Corollary 3.2 (distinguishable modes), not under this theorem.

## Proof

By hypothesis, no sector frame of  $\mathcal{H}_k$  is physical (else Theorem 1, first horn). An admissible conserved quantity must therefore be well-defined without reference to any frame: its value on  $w$  must be computable from frame-independent data alone. By Premise SE, the only admissible structure on  $\mathcal{H}_k$  is the inner product — in particular there is no admissible real structure, bilinear form, or other frame-like structure that names no sector (the necessity of excluding these separately, rather than deriving their absence from indistinguishability, is exactly why SE is a named premise; see §0.1). The frame-independent data attached to a single class state  $w$  therefore reduce to the inner-product invariants of  $w$  — and for a single vector, these are exhausted by  $\langle w, w \rangle$ . (Any two vectors of equal norm are related by a change of orthonormal frame; a quantity taking different values on them would distinguish frames.) Hence  $A(w) = f(\langle w, w \rangle) = f(W_{\mathcal{C}})$ . ■

## Corollary 2.1 — Normalization of the bath account

If  $A$  is in addition refinement-stable (Definition 3.2) — additive across instances and monotone in total weight — then

$$A(w) = c \cdot W_{\mathcal{C}}, c > 0.$$

**Proof.** For composition across instances,  $W_{\mathcal{C}}(w \oplus w') = W_{\mathcal{C}}(w) + W_{\mathcal{C}}(w')$ . Refinement stability demands  $A(w \oplus w') = A(w) + A(w')$ , i.e.

$$f(W + W') = f(W) + f(W')$$

for all admissible weights  $W, W' \geq 0$ . Cauchy additivity on the non-negative reals, with the monotonicity required by Definition 3.2 as the regularity hypothesis, gives  $f(W) = c \cdot W$  with  $c > 0$ . ■

**Reading.** The normalization is cosmetic relative to bath selection: Theorem 2 alone already delivers the operative conclusion that nothing finer than the total is conserved, and Theorem 3 uses only that. Refinement stability enters nowhere in the selection argument — it merely fixes which function of the total plays the role of the account.

## Corollary 2.2 — Quadratic route (commutant form)

If attention is restricted to quadratic accounts  $Q(w) = \langle w, A w \rangle$  invariant under the monomial frame group  $T^k \rtimes S_k$ , then  $A = c \cdot I$ : commuting with  $T^k$  eliminates all off-diagonal entries of  $A$ , and commuting with  $S_k$  forces all diagonal entries to be equal. Hence  $Q(w) = c \sum_i |w_i|^2 = c \cdot W_{\mathcal{C}}$ .

## Reading — why the quadratic route alone is not enough

Corollary 2.2 was, in effect, the predecessor paper's implicit exclusion argument. It is correct but insufficient, and the insufficiency is exhibitable: the quartic functional

$$Q_4(w) = \sum_i |w_i|^4$$

is invariant under all of  $T^k \rtimes S_k$ , names no sector, and is **not** invariant under generic  $U(k)$  transport. (For  $k = 3$  this is a one-line numerical check: a random monomial transformation preserves  $Q_4$  to machine precision; a random unitary does not.) An adversary conserving  $Q_4$  obtains ledger-grade dynamics while evading any quadratic exclusion theorem. Theorem 2 closes this because  $Q_4$  is not a function of  $\langle w, w \rangle$ : its value depends on the sector frame, so by Theorem 2 it is inadmissible outright — its conservation would require a physical frame, returning to Theorem 1. The exclusion is thus complete at all polynomial orders, and indeed for all functionals, not merely quadratic ones.

## Theorem 3 — Bath-Ledger Selection Theorem

For a genuinely indistinguishable closure class, admissible reversible transport conserves the total class weight

$$W_{\mathcal{C}} = \sum_i |w_i|^2$$

and conserves no further account — strict, monomial, or anonymous. Therefore the class is bath-conserved, not ledger-conserved:

genuine indistinguishability + no private labels + structure economy  $\Rightarrow$  shared bath.

Refinement stability is **not** a premise of this theorem; it enters only in Corollary 2.1 to normalize the surviving account. The premise set here is deliberately minimal: indistinguishability (certified upstream, §0.4), the No-Private-Ledger Principle 3.3, and Premise SE.

### Proof

By Theorem 1, any conserved structure requiring a projector family — named or anonymous — is inadmissible. By Theorem 2, every admissible conserved account of the class state is a function of  $W_{\mathcal{C}}$  alone; in particular, no admissible conserved quantity resolves the distribution

of weight over sectors, at any order. A conservation structure that preserves the total class weight and nothing finer is precisely a bath. ■

---

### **Corollary 3.1 — Ledger Branch Excluded for Identical Closure Classes**

The ledger branch  $T^k \rtimes S_k$  is inadmissible as a fundamental conservation-selected transport class for genuinely indistinguishable closure sectors.

#### **Proof**

Monomial transport is the invariance class of the weight multiset  $\{|w_1|^2, \dots, |w_k|^2\}$ . For transport to be pinned to the monomial group by a conservation law, some function of that multiset that is non-constant on level sets of  $W_C$  — one not reducible to a function of the total — must be conserved. By Theorem 2 no such function is admissible: every admissible account is a function of the total alone. With no admissible conservation law finer than  $W_C$ , there is no admissible principle that restricts transport below the full weight-preserving class. The monomial floor survives only as a *subgroup* of the bath transport group, not as a selected transport class. ■

---

### **Corollary 3.2 — Bath Selection Does Not Apply to Distinguishable Modes**

If the sectors are physically distinguished by charge, location, representation, boundary condition, source ledger, or observed record, then the distinguishing structure supplies the physical projector family, Theorem 1's first horn is not a contradiction but a description, and separate ledgers may be admissible.

#### **Reading**

The theorem does not claim that all modes of physics must share one bath. It claims that genuinely indistinguishable closure sectors cannot secretly retain private ledgers — and cannot retain them anonymously either.

---

## **5. From Bath Necessity to Non-Abelian Gauge Origin**

The previous non-abelian theorem proved:

bath + continuity + sector phase + relabelling covariance  $\Rightarrow U(k)$ .

This paper supplies the missing first term:

genuine indistinguishability  $\Rightarrow$  bath.

Therefore the combined result is:

genuinely indistinguishable closure class of size  $k$  + continuity + sector phase + relabelling covariance  $\Rightarrow U(k)$ .

Removing the shared central phase gives  $SU(k)$ .

---

## Theorem 4 — Non-Abelian Gauge-Origin Completion

Given:

1. a genuinely indistinguishable closure class of size  $k$ ;
2. no private ledgers (now a theorem conditional on Principle 3.3 and Premise SE, not a free premise);
3. continuous reversible transport;
4. sector-resolved phase;
5. relabelling covariance;
6. local distinguishability comparison requiring a connection;

then the local gauge structure of the class is  $SU(k)$ .

### Proof

By Theorem 3, the class is bath-conserved. By the inherited Non-Abelian Bath-Transport Theorem, bath conservation plus continuity, sector phase, and relabelling covariance generate  $U(k)$ . The shared central phase is not a new independent non-abelian factor and is removed by the inherited abelian-uniqueness discipline, leaving  $SU(k)$ . Local comparison of  $SU(k)$  frames then requires a gauge connection. ■

---

## Corollary 4.1 — Colour

For the inherited colour census  $k = 3$ :

$k = 3 \Rightarrow \text{SU}(3)_C$ .

The colour gauge sector is no longer conditional on an unproven bath premise; the bath premise has been selected by the inadmissibility of private colour ledgers. A private colour ledger would amount to nature conserving "how much red there is" — a quantity requiring an unphysical colour frame. Theorem 2 forbids it at every order.

---

## Corollary 4.2 — Weak Structure

For the inherited weak census  $k = 2$ :

$k = 2 \Rightarrow \text{SU}(2)$ .

With the inherited chirality selection:

$\text{SU}(2) \rightarrow \text{SU}(2)_L$ .

---

## Programme Reading

The old chain was:

if bath  $\Rightarrow \text{SU}(k)$ .

The new chain is:

indistinguishable closure class  $\Rightarrow$  no private ledger  $\Rightarrow$  shared bath  $\Rightarrow \text{SU}(k)$ .

That is the first major advance of this paper.

---

## 6. Local Standard Model Gauge Algebra

With the inherited census and abelian uniqueness:

$k = 3 \Rightarrow \text{SU}(3)_C$ ,  $k = 2 \Rightarrow \text{SU}(2)_L$ , single abelian sector  $\Rightarrow \text{U}(1)_Y$ .

Therefore the local Standard Model gauge algebra is

$\mathfrak{g}_{\text{SM}} = \mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$ .

The local gauge connection is

$$A_\mu^{\text{SM}} = g_s G_\mu^a T_a + g W_\mu^i \tau_i + g' B_\mu Y.$$

The local gauge-boson count is

$$\dim \mathfrak{su}(3) + \dim \mathfrak{su}(2) + \dim \mathfrak{u}(1) = 8 + 3 + 1 = 12.$$

Nothing in the bath-ledger selection theorem changes this local result. What remains is the global audit.

## 7. Global Centre Audit

The local algebra does not determine the faithful global group. Distinct global groups can share the same Lie algebra.

The candidate covering product is

$$\tilde{G}_{\text{SM}} = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y.$$

The faithful physical group is

$$G_{\text{faithful}} = \tilde{G}_{\text{SM}} / \Gamma,$$

where  $\Gamma$  is the subgroup of  $\tilde{G}_{\text{SM}}$  acting trivially on every admissible matter and interface representation.

The centre of the covering product is

$$Z(\tilde{G}_{\text{SM}}) = \mathbb{Z}_3 \times \mathbb{Z}_2 \times \text{U}(1).$$

Let  $\omega = e^{(2\pi i/3)}$ . A generic central element is

$$(\omega^a \cdot I_3, (-1)^b \cdot I_2, e^{(i\theta)}), \quad a \in \{0, 1, 2\}, \quad b \in \{0, 1\}, \quad \theta \in \mathbb{R}/2\pi\mathbb{Z}.$$

It acts on a representation  $(R_3, R_2)_q$  with centre charges  $(t, d, q)$  by the single phase

$$e^{(2\pi i \cdot a \cdot t/3)} \cdot (-1)^{(b \cdot d)} \cdot e^{(i \cdot q \cdot \theta)}.$$

The inherited representations, with their centre charges  $(t, d, q)$ :

$$\begin{aligned} Q_L : (3, 2)_1 &\rightarrow (1, 1, 1) & u_R : (3, 1)_4 &\rightarrow (1, 0, 4) & d_R : (3, 1)_{-2} &\rightarrow (1, 0, -2) & L_L : (1, 2)_{-3} &\rightarrow (0, \\ 1, -3) & e_R : (1, 1)_{-6} &\rightarrow (0, 0, -6) & \Phi_{cl} : (1, 2)_3 &\rightarrow (0, 1, 3) \end{aligned}$$

---

## 8. The Faithful Gauge-Group Theorem

### Theorem 5 — Common $\mathbb{Z}_6$ Kernel

The element

$$\zeta = ( \omega \cdot I_3 , -I_2 , e^{i\pi/3} ), \quad \omega = e^{2\pi i/3},$$

acts trivially on all inherited matter sectors and on the completion interface. Its powers form a  $\mathbb{Z}_6$  subgroup. This subgroup exhausts the common kernel of the covering-product action.

Therefore

$$G_{SM}^{\text{faithful}} = [ SU(3)_C \times SU(2)_L \times U(1)_Y ] / \mathbb{Z}_6.$$

#### Proof

**Triviality.** The action of  $\zeta$  on a representation with centre charges  $(t, d, q)$  is the phase

$$e^{i\pi \cdot (2t/3 + d + q/3)} = e^{i\pi(2t + 3d + q)/3},$$

which is trivial precisely when

$$2t + 3d + q \equiv 0 \pmod{6}.$$

Check each inherited representation:

$$\begin{aligned} Q_L : 2 \cdot 1 + 3 \cdot 1 + 1 = 6 \equiv 0 \quad \checkmark \quad u_R : 2 \cdot 1 + 3 \cdot 0 + 4 = 6 \equiv 0 \quad \checkmark \quad d_R : 2 \cdot 1 + 3 \cdot 0 - 2 = 0 \equiv 0 \quad \checkmark \\ L_L : 2 \cdot 0 + 3 \cdot 1 - 3 = 0 \equiv 0 \quad \checkmark \quad e_R : 2 \cdot 0 + 3 \cdot 0 - 6 = -6 \equiv 0 \quad \checkmark \quad \Phi_{cl} : 2 \cdot 0 + 3 \cdot 1 + 3 = 6 \equiv 0 \quad \checkmark \end{aligned}$$

Thus  $\zeta$  acts trivially on every admissible field. Since  $\omega$  has order 3,  $-1$  has order 2, and  $e^{i\pi/3}$  has order 6,  $\zeta$  has order 6, and  $\langle \zeta \rangle \cong \mathbb{Z}_6$ . Trivial action of  $\zeta$  extends to all its powers.

**Exhaustion.** Take a generic central element  $g = ( \omega^a, (-1)^b, e^{i\theta} )$  acting trivially on all admissible representations.

From  $e_R : (0, 0, -6)$ :

$$e^{-6i\theta} = 1 \implies \theta = n\pi/3, \quad n \in \mathbb{Z}_6.$$

From  $L_L : (0, 1, -3)$ :

$$(-1)^b e^{(-3i\theta)} = (-1)^b (-1)^n = 1 \implies b \equiv n \pmod{2}.$$

From  $u_R : (1, 0, 4)$ :

$$\omega^a e^{(4i\theta)} = \omega^a e^{(4in\pi/3)} = \omega^a \omega^{(2n)} = 1 \implies a \equiv -2n \equiv n \pmod{3}.$$

Therefore

$$g = ( \omega^n, (-1)^n, e^{(in\pi/3)} ) = \zeta^n.$$

The remaining representations  $Q_L, d_R, \Phi_{cl}$  impose no further condition: with  $(a, b, \theta) = (n \pmod{3}, n \pmod{2}, n\pi/3)$ , each of their triviality conditions reduces to  $2t + 3d + q \equiv 0 \pmod{6}$ , which holds by the census check above. Every central element acting trivially is a power of  $\zeta$ , so the common kernel is exactly

$$\Gamma = \langle \zeta \rangle \cong \mathbb{Z}_6,$$

and

$$G_{SM}^{\text{faithful}} = \tilde{G}_{SM} / \Gamma = [ SU(3)_C \times SU(2)_L \times U(1)_Y ] / \mathbb{Z}_6. \blacksquare$$

*(The triviality of all six powers of  $\zeta$  on all six representations, and the exhaustion of the kernel by  $\langle \zeta \rangle$  over the full central scan, have been verified by independent numerical computation.)*

## Corollary 5.1 — Centre Congruence as Admissibility Condition

In the faithful group, only representations of the covering product satisfying

$$2t + 3d + q \equiv 0 \pmod{6}$$

descend to representations of  $G_{SM}^{\text{faithful}}$ . Every field of the theory — present or future — must satisfy this congruence. A single admissible field violating it would falsify the quotient (Falsifier F8).

## Corollary 5.2 — Charge–Triality Correlation

Electric charge is  $Q = T_3 + Y = T_3 + q/6$ . Within an  $SU(2)$  multiplet,  $T_3$  is integer or half-integer according to  $d$ , so  $Q \pmod{1}$  is fixed by  $(d, q)$ ; the congruence then ties it to  $t$ . Working mod 1:

$$Q \equiv T_3 + q/6 \equiv d/2 + q/6 \pmod{1},$$

and substituting  $q \equiv -2t - 3d \pmod{6}$ :

$$Q \equiv d/2 + (-2t - 3d)/6 \equiv d/2 - t/3 - d/2 \equiv -t/3 \pmod{1}.$$

Therefore:

$$Q \equiv -t/3 \pmod{1}.$$

Colour singlets ( $t = 0$ ) carry integer electric charge; fundamental-triplet fields ( $t = 1$ ) carry charge  $\equiv 2/3 \pmod{1}$  — the quark pattern ( $u : 2/3, d : -1/3$ ); anti-triplet fields the conjugate pattern. The observed correlation of fractional electric charge with colour is thus not an accident of the hypercharge table but the direct observable face of the  $\mathbb{Z}_6$  quotient. This is the quotient's empirical content: it is bookkeeping removal upstream and charge quantization pattern downstream.

---

## 9. Consequences of the $\mathbb{Z}_6$ Quotient

### 9.1 Local physics unchanged

The quotient by a finite central subgroup does not change the Lie algebra:

$$\text{Lie}([SU(3) \times SU(2) \times U(1)] / \mathbb{Z}_6) = \mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1).$$

Therefore the local gauge fields remain  $G_{\mu^a}, W_{\mu^i}, B_{\mu}$ , and the gauge-boson count remains  $8 + 3 + 1 = 12$ .

### 9.2 Curvatures unchanged

The local curvature forms remain  $G_{\mu\nu^a}, W_{\mu\nu^i}, B_{\mu\nu}$ . The quotient does not alter the Yang–Mills kinetic terms.

### 9.3 Electroweak breaking unchanged

The completion interface remains  $\Phi_{cl} : (1, 2)_3$ . The quotient acts trivially on  $\Phi_{cl}$ , so the electroweak breaking pattern remains the inherited one. The gluons remain unaffected because  $\Phi_{cl}$  is colourless.

### 9.4 Anomaly audits unchanged

Anomaly cancellation is a local representation and current-conservation question. The quotient does not alter the local chiral matter content or hypercharge assignments. Therefore the inherited anomaly-admissibility results remain valid.

## 9.5 Global structure constrained

The quotient affects genuinely global questions: which bundles exist, which line operators are admissible, which magnetic sectors are allowed, and the charge-quantization pattern of Corollary 5.2. Of these, only the charge–trality correlation is derived here; the bundle and line-operator spectrum of  $G_{SM}^{\text{faithful}}$  is recorded as owed (Debt 6). What can be said without further derivation is directional: the faithful quotient enlarges the allowed magnetic/line-operator spectrum relative to the covering product while restricting the allowed electric representations by the congruence of Corollary 5.1.

## 9.6 Programme reading

The quotient removes redundant central bookkeeping. This is the global analogue of the no-private-ledger theorem:

if no admissible field distinguishes it, quotient it out.

Local rule and global rule are one rule at two layers: unearned distinctions between sectors are forbidden locally (bath selection); unearned distinctions between group elements are forbidden globally (faithful quotient).

---

# 10. Relation to Hypercharge, Chirality, and Abelian Uniqueness

## 10.1 Hypercharge

The  $\mathbb{Z}_6$  quotient relies on the inherited hypercharge ledger. It does not derive the ledger.

The quotient audit says: given the inherited hypercharge assignments, a common central  $\mathbb{Z}_6$  acts trivially. It does not say: the quotient derives the hypercharges. That derivation belongs to the abelian-uniqueness and anomaly-inadmissibility programme.

One direction of dependence is worth flagging because it strengthens the programme's internal consistency: the congruence  $2t + 3d + q \equiv 0 \pmod{6}$  holds for every representation in the anomaly-admissible ledger. Had even one inherited assignment violated it, the kernel would have been smaller than  $\mathbb{Z}_6$ . The full  $\mathbb{Z}_6$  is therefore a non-trivial consistency check that the inherited ledger passes, not a convention imposed on it.

## 10.2 Chirality

The quotient uses the inherited chiral matter skeleton  $Q_L, L_L, u_R, d_R, e_R$ . It does not derive why the weak doublets are left-handed. The chirality selection remains inherited.

## 10.3 Abelian uniqueness

The quotient does not introduce a second  $U(1)$ . It removes a finite overlap between the centres of the already-derived factors. The independent abelian factor remains exactly one:  $U(1)_Y$ .

---

# 11. Relation to Closure Geometry and the Fourteen-Generator Firewall

The closure-geometry programme contains the recurring count

$$N_{\text{loop}} = 2K = 14$$

for  $K = 7$ . That count belongs to closure architecture: paired channels, dihedral transport, and response machinery.

This paper's gauge count remains

$$8 + 3 + 1 = 12.$$

The quotient by  $\mathbb{Z}_6$  does not change this count. It is a finite global identification, not a local generator subtraction.

Therefore

$$14 \neq 12$$

because they count different things at different layers. The fourteen-generator structure is closure-response architecture. The twelve-generator structure is the Standard Model local gauge algebra.

---

# 12. What Has Actually Been Derived

Conditioned on inherited infrastructure, this paper derives:

1. private projector families — ordered or unordered — are inadmissible on genuinely indistinguishable closure classes (exact dichotomy; exclusion conditional on Principle 3.3);
2. every admissible conserved account on such a class is a function of total class weight (Theorem 2, conditional on Principle 3.3 and Premise SE); refinement stability adds only the normalization  $A = c \cdot W_C$  (Corollary 2.1);
3. the exclusion holds at all orders — in particular, anonymous symmetric-function ledgers such as  $\sum_i |w_i|^4$  are excluded, closing the loophole a quadratic-only argument leaves open;
4. identical closure classes are therefore bath-conserved rather than ledger-conserved;
5. the bath premise of the previous non-abelian gauge-origin theorem is thereby discharged;
6. the route: identical class of size  $k \implies \text{SU}(k)$  is strengthened;
7.  $k = 3$  gives  $\text{SU}(3)_C$  with bath now selected rather than assumed;
8.  $k = 2$  gives  $\text{SU}(2)$ , with  $\text{SU}(2)_L$  inherited;
9. the local Standard Model gauge algebra remains  $\mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$ ;
10. the common central kernel acting trivially on all inherited matter/interface sectors is exactly  $\mathbb{Z}_6$ , with kernel exhaustion proved and numerically verified;
11. the faithful global gauge group is  $[\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y] / \mathbb{Z}_6$ ;
12. the quotient is equivalent to the field-by-field congruence  $2t + 3d + q \equiv 0 \pmod{6}$ , yielding the charge–trality correlation  $Q \equiv -t/3 \pmod{1}$ ;
13. the quotient leaves the local gauge-boson count, connection, curvature, anomaly audits, and electroweak breaking unchanged.

This paper does not derive:

1.  $k = 3$ ;
2.  $k = 2$ ;
3. chirality;
4. hypercharge assignments;
5. the Higgs completion interface;
6. gauge coupling values;
7. running couplings;
8. confinement;
9.  $\Lambda_{\text{QCD}}$ ;
10. hadron masses;
11. fermion masses;
12. CKM/PMNS;
13. three generations;
14. strong CP;
15. the bundle and line-operator spectrum of the faithful group beyond the quotient identification and the congruence.

---

## 13. Open Debts

## Debt 1 — Transport continuity

The paper derives bath conservation. It still inherits continuity of transport from the prior bath-transport framework. Bath conservation alone constrains the conserved accounts; it does not by itself force the transport group to be connected.

## Debt 2 — Sector phase

The derivation of complex  $U(k)$ , rather than a thinner real orthogonal mixing group  $O(k)$ , still requires sector-resolved phase. Theorem 2's frame-independence argument is insensitive to the real/complex distinction and does not discharge this debt. The debt is now explicitly twinned with Premise SE (§0.1): an admissible global real structure  $J$  is exactly the structure that would both break sector phase and enlarge the single-state invariant algebra beyond  $\langle w, w \rangle$ . Sector phase and structure economy are two faces of one complex-structure commitment, and a future derivation of either discharges the real-structure instance of both.

## Debt 3 — Census and indistinguishability certification

The values  $k = 3$  and  $k = 2$  remain inherited, and — as flagged in §0.4 — so does the upstream certification that the census sectors are *genuinely* indistinguishable in the sense of Definition 0.4. The present theorem converts that certification into bath conservation; it does not supply the certification. Any conditionality in the census propagates through Theorem 3 unchanged.

## Debt 4 — Chirality

The bath theorem gives  $SU(2)$ , not  $SU(2)_L$ . Left-handed selection remains inherited.

## Debt 5 — Hypercharge derivation

The quotient audit depends on the inherited hypercharge ledger. It does not replace the hypercharge derivation. (It does, however, subject the ledger to the mod-6 consistency check of §10.1, which the ledger passes.)

## Debt 6 — Global consequences

The quotient has consequences for bundles, line operators, monopole sectors, and  $\theta$ -angle periodicity in the faithful group. Beyond the congruence and the charge–trality correlation, these are not derived here.

## Debt 7 — QCD dynamics

The paper does not derive confinement, running,  $\Lambda_{\text{QCD}}$ , or hadron physics.

## Debt 8 — Non-conserved frame structure

Theorem 1 excludes physically distinguished projector families *as conserved structure*. It does not by itself exclude a distinguished frame arising dynamically and impermanently (spontaneous frame selection). The programme position is that such a frame would constitute a recordable substrate fact and hence fall under Corollary 3.2 (distinguishable modes), but the interaction between spontaneous structure formation and the class-level bath theorem is owed a dedicated treatment.

---

## 14. Falsification Conditions

### F1 — Physical private ledgers

If genuinely indistinguishable closure sectors are shown to possess physical, recordable, sector-resolved ledgers, the No-Private-Ledger Principle fails.

### F2 — Ledger-compatible non-abelian mixing

If a strict ledger preserving individual sector weights nevertheless generates continuous off-diagonal non-abelian transport, the inherited ledger theorem fails.

### F3 — Admissible anonymous account

If a frame-dependent conserved account — for example a non-constant symmetric function of the sector-weight multiset such as  $\sum_i |w_i|^4$  — is shown to be admissible without a physically distinguished projector family, Theorem 2 fails and with it Theorem 3.

### F4 — Non-bath indistinguishability

If an indistinguishable closure class admits a conservation structure that is neither ledger nor bath and that produces non-abelian transport without hidden frame structure, the dichotomy is incomplete.

### F5 — Failure of sector phase

If sector-resolved phase is unavailable, the route to complex  $SU(k)$  fails or collapses to a real subgroup. By the biconditional twinning of Debt 2 with Premise SE (argued in §0.1: an unphased carrier is effectively real, and a real form inside a complex carrier is a preferred J), this failure mode simultaneously admits a real structure on the class carrier, enlarging the invariant algebra beyond  $\langle w, w \rangle$  and breaching Theorem 2.

## **F6 — Wrong census**

If the colour or weak census values are not 3 and 2, the specialisations fail.

## **F7 — Hypercharge ledger failure**

If the inherited hypercharge assignments are wrong, the  $\mathbb{Z}_6$  quotient audit detaches. In particular, if any admissible field violates  $2t + 3d + q \equiv 0 \pmod{6}$ , the kernel is smaller than  $\mathbb{Z}_6$ .

## **F8 — Quotient kernel mismatch (too large)**

If an admissible Standard Model field exists on which  $\zeta$  acts nontrivially — equivalently, a field violating the congruence, e.g. a colour-triplet weak-singlet with  $q = 1$ , or any colour-singlet particle with electric charge  $\not\equiv 0 \pmod{1}$  — the  $\mathbb{Z}_6$  quotient is too large. The charge–triality correlation  $Q \equiv -t/3 \pmod{1}$  makes this falsifier empirical: a free particle with charge  $e/2$ , or a fractionally charged colour singlet, would break the quotient.

## **F9 — Larger kernel**

If a central element not generated by  $\zeta$  acts trivially on all admissible fields, the kernel is larger than  $\mathbb{Z}_6$ . Given the exhaustion proof this would require the admissible field content to be smaller than the inherited census — for example, if  $e_R$  with  $q = -6$  were not fundamental, the  $\theta$ -quantization step would relax.

## **F10 — Additional fundamental abelian factor**

If an independent fundamental  $U(1)'$  is required at the same level, abelian uniqueness fails.

## **F11 — Gauge-count conflation**

If the fourteen-generator closure-response census is identified with the twelve local gauge generators, the layer firewall fails.

---

# **15. Milestone Statement**

The previous milestone was:

if bath  $\Rightarrow$   $SU(k)$ .

This paper advances the programme to:

genuine indistinguishability  $\Rightarrow$  no private ledger (at any order)  $\Rightarrow$  shared bath  $\Rightarrow$   $SU(k)$ .

With inherited census and chirality:

$k = 3 \Rightarrow SU(3)_C$ ,  $k = 2 \Rightarrow SU(2)_L$ .

With inherited abelian uniqueness:

$U(1)_Y$ .

With global centre redundancy removed:

$G_{SM}^{\text{faithful}} = [ SU(3)_C \times SU(2)_L \times U(1)_Y ] / \mathbb{Z}_6$ ,

carrying the field-level congruence  $2t + 3d + q \equiv 0 \pmod{6}$  and the charge–trality correlation  $Q \equiv -t/3 \pmod{1}$ .

The paper therefore converts the previous conditional local gauge-product closure into:

**bath-selected non-abelian gauge origin plus faithful global Standard Model gauge closure, with the quotient's observable content made explicit.**

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## 16. Conclusion

The VERSF Standard Model programme had reached a precise bottleneck. It had shown that a shared bath of identical closure sectors generates non-abelian transport:

bath  $\Rightarrow$   $U(k) \Rightarrow SU(k)$ .

It had also shown that with inherited  $k = 3$  and  $k = 2$ , the local Standard Model gauge algebra closes as

$\mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$ .

But it still owed the bath premise.

This paper has argued that the bath premise is not optional. A ledger over genuinely indistinguishable sectors preserves hidden sector identity. If the ledger's projector family is physical, the sectors are distinguishable; if it is not physical, the ledger conserves structure nothing can read while forbidding continuous mixing — dynamically load-bearing, observationally empty. Either way, the ledger is inadmissible. And the exclusion cannot be evaded by anonymizing the accounts: a symmetric function of the sector-weight multiset is frame-dependent, and conserving frame-dependent data requires the very physical frame the

class lacks. The only admissible conservation law on a genuinely indistinguishable closure class — at every polynomial order — is total class weight. That is the bath.

Therefore non-abelian gauge structure is no longer derived from an assumed bath. The bath follows from the inadmissibility of hidden ledgers.

The paper then applied the same principle globally. The naive product

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

contains central transformations that act trivially on every inherited matter sector and on the completion interface. Those transformations are not physical distinctions. They form exactly  $\mathbb{Z}_6$ , and the faithful global gauge group is therefore

$$[ SU(3)_C \times SU(2)_L \times U(1)_Y ] / \mathbb{Z}_6.$$

The quotient is not silent bookkeeping. It is equivalent to a congruence every admissible field must satisfy, and that congruence surfaces observably as the correlation between electric charge and colour triality — the reason no fractionally charged colour singlet has ever been seen.

The result is a cleaner Standard Model gauge closure:

no hidden ledgers locally + no redundant centre globally.

The programme chain now reads:

$$\text{minimum distinction} \rightarrow \text{closure class} \rightarrow \text{no private ledger} \rightarrow \text{shared bath} \rightarrow SU(k) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow [ SU(3)_C \times SU(2)_L \times U(1)_Y ] / \mathbb{Z}_6.$$

That is the milestone.

## Appendix A — Explicit $\mathbb{Z}_6$ Kernel Calculation

Let

$$\tilde{G} = SU(3) \times SU(2) \times U(1), Z(\tilde{G}) = \mathbb{Z}_3 \times \mathbb{Z}_2 \times U(1), \omega = e^{(2\pi i/3)}.$$

A generic central element is

$$g = ( \omega^a, (-1)^b, e^{(i\theta)} ), a = 0, 1, 2; b = 0, 1.$$

The inherited representations with centre charges (t, d, q):

$$Q\_L : (3, 2)_1 \rightarrow (1, 1, 1) \quad u\_R : (3, 1)_4 \rightarrow (1, 0, 4) \quad d\_R : (3, 1)_{-2} \rightarrow (1, 0, -2) \quad L\_L : (1, 2)_{-3} \rightarrow (0, 1, -3) \\ e\_R : (1, 1)_{-6} \rightarrow (0, 0, -6) \quad \Phi : (1, 2)_3 \rightarrow (0, 1, 3)$$

Triviality conditions:

$$Q\_L : \omega^a (-1)^b e^{i\theta} = 1 \quad u\_R : \omega^a e^{4i\theta} = 1 \quad d\_R : \omega^a e^{-2i\theta} = 1 \quad L\_L : (-1)^b e^{-3i\theta} = 1 \quad e\_R : e^{-6i\theta} = 1 \quad \Phi : (-1)^b e^{3i\theta} = 1$$

From  $e\_R$ :

$$\theta = n\pi/3, \quad n \in \mathbb{Z}_6.$$

From  $L\_L$ :

$$b \equiv n \pmod{2}.$$

From  $u\_R$ , using  $e^{4in\pi/3} = \omega^{2n}$ :

$$\omega^{a+2n} = 1 \implies a \equiv -2n \equiv n \pmod{3}.$$

Consistency of the remaining three conditions: with  $(a, b, \theta) = (n, n, n\pi/3)$ ,

$$Q\_L : \omega^n (-1)^n e^{in\pi/3} = e^{i(n(2\pi/3 + \pi + \pi/3))} = e^{2\pi in} = 1 \quad \checkmark \quad d\_R : \omega^n e^{-2in\pi/3} = e^{i(n(2\pi/3 - 2\pi/3))} = 1 \quad \checkmark \quad \Phi : (-1)^n e^{in\pi} = e^{i(n\pi)} e^{i(n\pi)} = e^{2\pi in} = 1 \quad \checkmark$$

Therefore

$$g = (\omega^n, (-1)^n, e^{in\pi/3}) = \zeta^n, \quad \zeta = (\omega, -1, e^{i\pi/3}),$$

and

$$\Gamma = \langle \zeta \rangle \cong \mathbb{Z}_6.$$

**Numerical audit.** A brute-force scan over all central elements  $(a, b, n)$  with  $a \in \mathbb{Z}_3$ ,  $b \in \mathbb{Z}_2$ ,  $\theta = n\pi/3$ ,  $n \in \mathbb{Z}_6$  — the full set permitted by the  $e\_R$  condition — confirms that exactly six elements act trivially on all six representations, and that they are precisely  $\{\zeta^n : n = 0, \dots, 5\}$ , i.e.  $(a, b, n) \in \{(0,0,0), (1,1,1), (2,0,2), (0,1,3), (1,0,4), (2,1,5)\}$ . All six powers of  $\zeta$  act trivially on all six representations to machine precision.

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## Appendix B — Ledger Inadmissibility as a Projector Theorem

Let  $\mathcal{H}_k \cong \mathbb{C}^k$ . A strict ledger selects a complete rank-one projector family

$$\{P_1, \dots, P_k\}, P_i P_j = \delta_{ij} P_i, \sum_i P_i = I.$$

This is more than total Hilbert-space structure. It is a decomposition into private accounts.

If  $\{P_i\}$  is physical, then the sectors are physically distinguished. If  $\{P_i\}$  is not physical, then conserving the values  $\langle w, P_i w \rangle$  preserves hidden decomposition data. Therefore a physical theory of genuinely indistinguishable sectors cannot contain such a conserved projector family.

**The unordered-family extension.** An anonymous ledger conserves a symmetric function

$$F(\langle w, P_1 w \rangle, \dots, \langle w, P_k w \rangle)$$

for symmetric  $F$ . Symmetry of  $F$  removes the ordering of the family but not the family: distinct orthonormal frames  $\{P_i\}$  and  $\{P'_i\}$  generically assign different multisets  $\{\langle w, P_i w \rangle\}$  and  $\{\langle w, P'_i w \rangle\}$  to the same state  $w$ , so  $F$ 's value is frame-dependent whenever  $F$  is non-constant on multisets of fixed total. A well-defined conserved  $F$  therefore requires a distinguished unordered frame, and the dichotomy applies to that frame verbatim.

**Explicit witness.** For  $F = \sum_i x_i^2$ , i.e. the quartic account  $\sum_i |w_i|^4$ : it is invariant under every monomial transformation (permutation composed with sector rephasing) and non-invariant under generic unitary frame change — verified numerically for  $k = 3$ , where a random monomial map preserves the account to machine precision and a random unitary changes it at order unity. The quartic account is thus a live counterexample to any claim that quadratic exclusion suffices, and a worked example of the frame-dependence that Theorem 2 uses to exclude all such accounts.

The only projector-valued structure invariant under all frame changes is  $I$  itself. Therefore the only admissible conserved account is built from  $\langle w, I w \rangle = W_{\mathcal{C}}$ . This is the bath account.

## Appendix C — Alternative Global Groups and Failure Modes

The local algebra

$$\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$$

does not by itself choose among global groups. Candidate global forms include:

$$SU(3) \times SU(2) \times U(1), [SU(3) \times SU(2) \times U(1)] / \mathbb{Z}_2, [SU(3) \times SU(2) \times U(1)] / \mathbb{Z}_3, [SU(3) \times SU(2) \times U(1)] / \mathbb{Z}_6.$$

The faithful group is selected by the kernel of the actual representation ledger.

Failure modes of the alternatives, given the inherited matter and interface representations:

- **The unquotiented product** retains a  $\mathbb{Z}_6$  of transformations that act on nothing admissible — global private bookkeeping, excluded by the same discipline that excludes local private ledgers.
- **The  $\mathbb{Z}_2$  quotient** (generated by  $\zeta^3 = (I_3, -I_2, -1)$ , which acts on a representation with centre charges  $(t, d, q)$  as  $(-1)^{(d+q)}$  — trivial on all admissible fields since  $d + q$  is even throughout the census) removes only the order-2 part of the redundancy, leaving a residual  $\mathbb{Z}_3$  of trivially-acting elements.
- **The  $\mathbb{Z}_3$  quotient** (generated by  $\zeta^2 = (\omega^2, I_2, e^{(2i\pi/3)})$ ) symmetrically leaves a residual  $\mathbb{Z}_2$ .
- **Any quotient strictly larger than  $\mathbb{Z}_6$**  would identify central elements acting nontrivially on at least one admissible field — for example, quotienting by the full  $\mathbb{Z}_3$  centre of  $SU(3)$  alone would trivialize triality and force the charge–triality correlation of Corollary 5.2 to fail for quarks.

Given the inherited ledger, the kernel is exactly  $\mathbb{Z}_6$ : smaller quotients under-remove redundancy; larger quotients over-identify physical transformations.

## Appendix D — Claim-Status Ledger

Claim	Status	Comment
Strict ledger uses private projectors	Exact	Definition
Private projector family: dichotomy (physical $\Rightarrow$ distinguishable; unphysical $\Rightarrow$ frame-dependent conserved data)	Exact	dichotomy; Theorem 1; §2 dynamical-exclusion conditional asymmetry argument carries on Principle 3.3 the justification
Frame-independent single-state invariants reduce to $\langle w, w \rangle$	Exact given Premise SE	Theorem 2; SE excludes real-structure counter-instances
Refinement stability + monotonicity fix normalization $A = c \cdot W_{\mathcal{C}}$	Exact; normalization only, no selection work	Corollary 2.1; domain is composition across instances
Quadratic commutant route: monomial-invariant Hermitian forms are $c \cdot I$	Exact	Corollary 2.2; verified numerically for $k = 3$
Quartic account $\Sigma$	$w_i$	<sup>4</sup> is monomial-invariant but not $U(k)$ -invariant
Anonymous ledgers require a distinguished unordered frame	Exact	Appendix B extension
Bath-Ledger Selection	Derived here (conditional on Principle 3.3 + SE)	Theorem 3; refinement stability not a premise

Claim	Status	Comment
Bath + continuity + phase gives $U(k)$	Inherited	Previous theorem
$U(k) \rightarrow SU(k)$ after shared phase removal	Inherited / Exact	Abelian firewall
$k = 3 \Rightarrow SU(3)_C$	Conditional synthesis	Census inherited
$k = 2 \Rightarrow SU(2)_L$	Conditional synthesis	Census/chirality inherited
Local gauge algebra	Inherited	$\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$
$\zeta$ acts trivially on all six inherited representations	Exact	Verified numerically
Kernel exhaustion: $\Gamma = \langle \zeta \rangle \cong \mathbb{Z}_6$	Derived here	Theorem 5; verified numerically
Faithful global group	Derived here	Product quotient by $\mathbb{Z}_6$
Congruence $2t + 3d + q \equiv 0 \pmod{6}$	Derived here	Corollary 5.1
$Q \equiv -t/3 \pmod{1}$	Derived here	Corollary 5.2
Gauge-boson count unchanged	Exact	Finite quotient
Hypercharge assignments	Inherited	Not derived here; pass the mod-6 consistency check
Genuine-indistinguishability certification	Inherited	Debt 3; non-circularity note §0.4
Confinement / QCD running	Owed	Not derived here
Fermion masses / CKM / PMNS	Owed	Not derived here
Bundle / line-operator spectrum of faithful group	Owed	Debt 6