The Gauge-Response and Coupling Evaluation Theorem in VERSF

▲ Programme Milestone — Standard Model Gauge-Strength Evaluation and Boundary-Data Completion Series

Gate GRCE-1 / The Explicit Holonomy-Response Functional, the Blind Coupling-Evaluation Protocol, the Executed Benchmark GRCE-1C(0), and the Wheel-Profile Branch GRCE-1(1)

Keith Taylor VERSF Theoretical Physics Programme Tier A conditional microscopic-response evaluation theorem with constructed response functional, executed blind benchmark, and executed conditional support-harmonic derivation — July 2026

Primary predecessors: *The Gauge Action and Coupling Theorem in VERSF*; *The Quantum Gauge Completion of the VERSF Standard Model*; *The Gauge-Census Closure Theorem in VERSF*; *The Gauge-Closure Selection Theorem in VERSF*; *The Microscopic Origin of the Record Current*; *The Fermionic Fock-Space Reconstruction in VERSF*; *The Origin of the Fourteen-Generator Census*; *The Electroweak Representation and Connection Closure in VERSF*; *The Non-Abelian Bath-Transport Theorem*; *QCD Running and the Conditional Triality-Gap Confinement Criterion in VERSF*; the MCHY conditional charged-mass audit branch; the colour-participation, projected weak-doublet and CPF-1 response-map results; the Planck-certification and coherence-transport theorems; and the VERSF closure, refinement, record, holonomy and Standard Model completion programmes.

Principal Claim

The Gauge Action and Coupling Theorem proves what a VERSF gauge coupling is. This paper fixes how it must be calculated, constructs one general gauge-response evaluation framework, and executes two explicit constitutive branches — blind: a stationary-metric matter-census benchmark and a conditional Dirichlet-susceptibility branch built on a mutually exclusive wheel metric.

Each physical inverse squared coupling is the factorwise trace of the frozen holonomy-response Hessian after exact Schur relaxation of every same-order closure, record, bath, refinement and completion mode. The functional is the stationary-record cost of the vacuum-relative plaquette holonomy on the K = 7 commitment foam,

$\Gamma\_GRCE^{(n)}[\rho, \eta; a] = \Gamma\_sub^{(n)}[\rho, \eta] + (C\_hol/2)\ \Sigma\_\{p \in P_n\}\ |p|\ |p\bigstar|\ \langle \mathscr{R}\_p\ ,\ W\_rec\ \mathscr{R}\_p \rangle$,

with no independently adjustable colour, weak or Abelian coefficient, and conversion $g_s = Z_3^{(-1/2)}$, $g = Z_2^{(-1/2)}$, $g' = 6\, Z_q^{(-1/2)}$.

Two branches are executed without any measured coupling. The matter-census benchmark GRCE-1C(0) returns, exactly,

$$(Z_3, Z_2, Z_q) = (6 , 13/2 , 378)\, C_{hol} , \sin^2\theta_W = 13/34 , \alpha^{-1} = 68\pi ,$$

and proves its own insufficiency: factorisation makes the couplings blind to the wheel. The support-harmonic matter-census susceptibility branch GRCE-1C(1) — under a declared, falsifiable premise stack: census factorisation, susceptibility reading, arity–harmonic assignment for the colour and weak sectors, Abelian relative-phase selection, and Dirichlet pricing — returns

$$(Z_3, Z_2, Z_q) = (6 , 65/8 , 837/2)\, C_{hol} , \sin^2\theta_W = 65/158 , \alpha^{-1} = 79\pi ,$$

with $g_s = 1/\sqrt{6}$ identical on both readings. Under the declared Planck anchoring, the band-saturated matching surface $\mu\bigstar = \ell_{ref}^{-1} \min[1, (1 - 4v_c^{\infty})/(4v_c^{\infty})]$ equals 6.89×10^{18} GeV throughout the band-separated regime, where the transport velocity drops out entirely — a value conditional on the full stack PSA-1, $\kappa = \pi/2$, $\eta = 1/2$ and the sharp certification boundary.

The physical gate is not passed. What remains is finite and named: the constitutive premises behind each branch, the susceptibility–link bridge, the universal scalar C_hol, the microscopic γ_n equation and the quantum subtraction prescription — the transport velocity and the finite bridge constants are *proved* non-identifiable from the present axioms — the identification of ℓ_{ref}, and the numerical threshold ledger. Choosing any of these to reproduce the known couplings would make the calculation true by construction and would fail the gate. The remaining work is a set of corrections to two executed returns.

General Reader Summary

The Standard Model contains three interaction strengths: one for the strong force, one for the weak force, and one for hypercharge, which combines with the weak interaction to produce electromagnetism. Physicists normally measure these three numbers and insert them into the theory by hand.

VERSF proposes they can be calculated. A gauge field is the rule for comparing internal quantum states at neighbouring points; curvature is the mismatch left when that comparison is carried around a small loop. The underlying substrate resists this mismatch the way a sheet of material resists bending, and each interaction strength is the reading of a substrate stiffness: the stiffer the substrate against one kind of curvature, the weaker the corresponding force.

This paper builds the exact object whose stiffness must be measured — with no adjustable pieces — and fixes the complete rules for evaluating and publishing the result without ever looking at the measured values. Every loophole through which an answer could be smuggled in — rescaling a current, picking a convenient energy scale, choosing a flattering scheme, forgetting a heavy particle — is closed in advance.

Two versions of the calculation are then run — built on two different, mutually exclusive readings of how the substrate charges for mismatch. The simplest version combines the known particle content with the substrate's seven-state wheel in the most independent way possible, and returns exact stiffnesses of 6, 6½ and 378 times one universal constant — and proves that, combined this way, the wheel cancels out entirely, so this cannot be the whole story. The second version lets each force grip the wheel in the harmonic matching its own structure — colour threefold, the weak force in the alternating pattern, hypercharge in the hub-versus-boundary contrast — priced by the wheel's natural mismatch energy. The relative stiffnesses become 1, 5/4 and 31/28, giving new definite couplings and a boundary weak-mixing value of 65/158, with the strong coupling identical in both versions.

The dimensionless matching rule is now calculated as well: across the safely separated regime, the unknown transport speed cancels out completely, and the rule says the scale is simply the substrate's shortest length converted into an energy. Converting that into an actual energy remains conditional on identifying which length that is — the deepest open identification: the Planck floor gives about 10^{19} GeV; a mesoscopic record scale gives a few TeV.

The paper also proves two honest impossibilities: the present axioms cannot fix the transport speed, and cannot fix the fine constants bridging the substrate's accounting to standard conventions. Each requires one specific new equation, now precisely named.

As Paper 4 of the final eight in the VERSF Standard Model programme, this changes what the gauge couplings are. They are no longer three unconstrained parameters. They are two executed reference calculations and a finite list of named substrate bridges — including two quantities proved to require genuinely new equations: the microscopic transport normalisation and the quantum subtraction prescription.

Headline Results and Conditionality# Headline Results and Conditionality

Claim	GRCE-1 status
Gauge kinetic operator is the auxiliary-reduced curvature Hessian	Inherited exactly from GAC-1
Gauge response must be evaluated on Bianchi-admissible curvature directions	Closed as typing

Claim	GRCE-1 status
Explicit holonomy-response functional constructed	Closed conditionally on Premise HRF-1
HRF-1 is the minimal canonical candidate selected by the inherited stationary metric (unique within that class)	Closed as minimality; commutant freedom acknowledged
Every Hessian block displayed as a derivative of the constructed functional	Closed
Exact screening form $\mathcal{K}\_g = C\_hol\ J\_F^\dagger\ \mathbb{W}^{(1/2)}(1 - S\_\eta)\mathbb{W}^{(1/2)}\ J\_F$, $0 \preceq S\_\eta \preceq 1$	Closed; S_η an orthogonal projector exactly in the pure-metric case
Auxiliary enlargement can only decrease or preserve Z_A	Closed
Incomplete auxiliary census gives an upper bound, not the physical coupling	Closed
Factorwise cross terms vanish in the minimal SU(3)×SU(2)×U(1) census	Inherited
Simple-factor response is isotropic	Tested against the frozen embeddings; Haar quotient conditional on gauge-orbit reduction (F21 guards concealment)
Exact factorwise extraction of Z_3, Z_2, Z_q	Closed
Direct weights w_A are projections of one stationary metric, not free parameters	Closed
Fourteen-to-twelve response descent requires a derived equivariant map	Inherited and strengthened
Explicit Abelian reduction through the $\mathbb{Z}_7$ closure connection	Closed
Flat Gate-3 holonomy separated from local curvature energy	Closed
Primitive hypercharge lattice $q = 6Y$	Inherited
$g\_s = Z_3^{(-1/2)}$, $g = Z_2^{(-1/2)}$, $g' = 6\ Z\_q^{(-1/2)}$	Closed
Matter and gauge self-couplings share one canonical coupling	Structurally enforced by the shared-link construction; quantum preservation supplied by QGC-1
Absolute record-to-Fock current normalisation	Shared-link value returned exactly; independent record-to-Fock descent open

Claim	GRCE-1 status
Matching scale μ★ selected from substrate structure	Closed as the band-edge, saturation-corrected rule: $\mu^\star = \ell_{ref}^{-1} \min[1, (1 - 4v_c^\infty)/(4v_c^\infty)]$; absolute value open
Scale non-identifiability without a substrate selector	Closed
Finite VERSF-to-standard-scheme bridge	Typed; numerical bridge open
Complete threshold ledger	Protocol closed; numerical ledger depends on Papers 1–3 and the auxiliary spectrum
Conversion to e, α, $\sin^2\theta_W$	Closed as exact algebra
One-loop running coefficients	Inherited from QGC-1
Running supplies boundary values	Disproved
Blind benchmark branch GRCE-1C(0) (Premises MCR-0, AGI-0)	Executed exactly; matter-census induced response, not the fourteen-channel functional; no measured coupling used
Product-factor intertwining on the census carrier	Closed; exact by tensor structure. Physical fourteen-channel intertwining open
Auxiliary Schur correction on the factorised branch	Exactly zero: $H_F\eta = 0$, $b_A = 0$
Benchmark census triple	Returned: $(Z_3, Z_2, Z_q) = (6, 13/2, 378)\ C_{hol}$
Scale-independent ratios $g_s/g = \sqrt{13/12}$, $g'/g = \sqrt{13/21}$	Returned; independent of C_{hol}
Universal action-normalisation constant C_{hol}	Open; localised to the holonomy–Wilson bridge (Section 34.2); unit-action HRF-1 sets $C_{hol} = 1$ conditionally
Shared-link tree-level current matching $\nu_A^{(\text{shared-link})}$	Returned exactly: (1, 1, 1); full physical matching open
Native background-field MOM scheme (VBF)	Closed by definition: $\Delta^{V\to VBF} = 0$
Finite VBF→M$\bar{\text{S}}$ bridge	Open
One-loop beta census from the direct embeddings	Reproduced exactly: (41/6, −19/6, −7)
Wheel-gap ledger {1, 1, 2, 2, 31/14, 5/2}	Returned as raw local gaps; superseded as thresholds by band broadening and saturation (Section 25)
Factorisation obstruction	Closed; physical hierarchy requires nonfactorising embedding or factor-dependent relaxation
Wheel-profile embedding $\chi_A \otimes \rho_A$: general equivariant separable extension	Closed as typing (Premise WPE-1); one profile per simple factor (Theorem 16)
Exact wheel-mode ledger: modes, D_6 types, gaps, screening factors	Closed; all entries exact fractions

Claim	GRCE-1 status
Coherent floor $c_{A0}^2 \le N_A \le \|\chi_A\|^2_\pi$	Closed unconditionally (Theorem 18)
Audit-locked normalisation $c_{A0} = 1$ on the stationary reading	Closed conditional on wheel-coherent matter (Theorem 19)
Exact deformation laws for g_s/g, g'/g, α^{-1}, $\sin^2\theta_W$	Closed
Factor-Profile Non-Identifiability Certificate	Proved (Theorem 20); missing constitutive map named
Support-harmonic assignment: colour → $k = 2,4$; weak → $k = 3$ (AHP-1); q → radial (ARP-1)	Closed under the declared premises; sectors unique (Theorem 21)
Dirichlet profile stiffnesses $(s_3, s_2, s_q) = (1, 5/4, 31/28)$	Closed under Premise DRF-1; exact
Support-harmonic triple $(Z_3, Z_2, Z_q) = (6, 65/8, 837/2)$ C_{hol} on GRCE-1C(1)	Returned conditionally on MCR-1, SHR-1, AHP-1, ARP-1, DRF-1 (Theorem 22)
Unit-action returns (0.408248, 0.350823, 0.293294); $\alpha^{-1} = 79\pi$; $\sin^2\theta_W = 65/158$	Returned conditionally at $C_{hol} = 1$
Ratios $g_s/g = \sqrt{65/48}$, $g'/g = \sqrt{65/93}$	Returned; independent of C_{hol}
$g_s = 1/\sqrt{6}$ identical on the two displayed readings	Closed as a two-branch invariance; not yet full-theory robust
Wheel-metric fork: stationary norm vs Dirichlet form	Declared; constitutively unresolved
Susceptibility–link bridge theorem (why ϱ_A^{resp} may differ from the link profile)	Open; the branch's decisive obligation
Substrate unit $\ell_{sub} = \sqrt{\kappa/\eta}\, \ell_P$; sharp value $\sqrt{\pi}\, \ell_P = 2.865 \times 10^{-35}$ m	Closed conditionally on Premise PSA-1 and the certification theorems
Matching surface $\mu^\star = E_{sub} \min[1, (1 - 4v_c^\infty)/(4v_c^\infty)]$	Sharp PSA-1 value 6.89×10^{18} GeV for $v_c^\infty \le 1/8$; v_c^∞-independent under full band separation
Transport velocity v_c^∞	Proved non-identifiable from the present axioms (Proposition 27); admissible range $0 < v_c^\infty < 1/4$; five-band separation needs $v_c^\infty < 3/56$
Critical-locality candidate $v_c^\infty = 1/8$	Declared (Premise CLP-1); sharp candidate branch, not a result
Finite bridge $\Delta_A^{VBF \to M\bar{S}} = k_A^{VBF}/8\pi^2$	Proved non-identifiable (Proposition 28); three regulator constants owed to the quantum subtraction prescription

Claim	GRCE-1 status
Structural threshold ledger (EW block, charged fermions with removal increments, neutral, colour, substrate bands, κ)	Assembled conditionally (Section 27.1); common-scheme numerical ledger open
Anchoring fork: Planck ($\mu\bigstar \simeq 6.89 \times 10^{18}$ GeV) vs record-coherence ($\mu\bigstar \simeq 5.8$ TeV) on the critical branch	Open; decided entirely by the identification of ℓ_ref
Numerical Z_3, Z_2, Z_q	Returned on GRCE-1C(0) up to one universal constant; physical branch open
Numerical g_s, g, g′	Returned conditionally at C_hol = 1: (0.408248, 0.392232, 0.308607)
Boundary α^{-1} and $\sin^2\theta$_W	Returned conditionally: 213.628 and 13/34; low-energy transport open
Holonomy-response non-identifiability if the functional is left free	Closed
Blind publication and non-insertion protocol	Closed
Physical gauge-coupling gate	Not yet passed; open: derivations of MCR-1/SHR-1/AHP-1/ARP-1/DRF-1/PSA-1, the susceptibility–link and holonomy–Wilson bridges, the wheel-metric fork, the γ_n equation, the quantum subtraction, ℓ_ref, deeper relaxation and thresholds

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Abstract

The Gauge Action and Coupling Theorem identifies the leading Standard Model gauge kinetic operator in VERSF as the auxiliary-reduced curvature-response Hessian

$\mathcal{K}$_g(q) = H_FF(q) − H_Fη(q) H_ηη(q)$^{-1}$ H_ηF(q) ,

and proves that its zero-momentum factorwise coefficients determine the physical connection couplings. In the primitive charge convention q = 6Y,

g_s = Z_3^(−1/2) , g = Z_2^(−1/2) , g′ = 6 Z_q^(−1/2) .

That theorem fixes the architecture but neither evaluates the response coefficients nor supplies the functional they respond to. The present paper does both, to the extent the frozen corpus permits, and certifies exactly where the remaining numerical debt lies.

Construction. On the closure-compatible $K = 7$ simplicial commitment foam $\mathfrak{S}_n = (V_n, E_n, P_n)$, each oriented edge carries the frozen substrate transport $U_{e,\bigstar}(\rho, \eta)$ and the gauge-perturbed transport

$$U_e(\rho, \eta; a) = U_{e,\bigstar}(\rho, \eta) \cdot \exp[i\, \mathscr{E}_{SM}(a_e)] ,$$

where $\mathscr{E}_{SM} = \mathscr{E}_3 \oplus \mathscr{E}_2 \oplus \mathscr{E}_q : \mathfrak{g}_{SM} \to \mathfrak{u}(\mathscr{H}_{rec})$ is the frozen equivariant embedding into the fourteen-dimensional closure-response carrier, normalised by $\mathrm{tr}(T_a T_b) = \tfrac{1}{2} \delta_{ab}$ and the primitive lattice $q = 6Y$, with no factorwise coefficient. The curvature-sensitive object is the vacuum-relative plaquette holonomy $Q_p = H_{p,\bigstar}^{-1} H_p$ and its logarithmic residual

$$\mathscr{R}_p(\rho, \eta; a) = (-i/|p|)\, \mathrm{Log}_0\, Q_p(\rho, \eta; a) = \mathscr{E}_{SM}(F_p) + O(\varepsilon_n) .$$

The response metric is the gauge-invariant completion W_{rec} of the canonical stationary transport metric $W_\pi = \mathrm{diag}(\pi) \otimes I_{or}$ — isotropy is tested against the frozen embeddings first, and Haar averaging is applied only as a gauge-orbit quotient where that reduction is licensed (a failed test triggers F21, not a repair) — with factorwise unscaled direct weights

$$\bar{w}_A = (1/d_A)\, \mathrm{Tr}(\mathscr{E}_A^\dagger\, W_\pi\, \mathscr{E}_A) , \quad w_A = C_{hol}\, \bar{w}_A .$$

The holonomy-response functional is

$$\Gamma_{GRCE}^{(n)}[\rho, \eta; a] = \Gamma_{sub}^{(n)}[\rho, \eta] + (C_{hol}/2) \Sigma_{p\in P_n} |p|\, |p\bigstar|\, \langle \mathscr{R}_p , W_{rec}\, \mathscr{R}_p \rangle ,$$

with C_{hol} the universal stationary-norm-to-action conversion of Premise GR9.

Its sole new constitutive premise, HRF-1, states that the leading local energetic cost of an irreducible vacuum-relative plaquette holonomy is its squared norm in the canonical stationary-record metric, up to one universal action-conversion scalar C_{hol}. HRF-1 is falsifiable, and it is the minimal canonical candidate selected by the inherited stationary metric — analytic, positive, reversal-even and free of factorwise insertions; unique within that class, since on a reducible carrier the representation commutant admits other invariant forms.

Reduction. The second variation of the displayed functional returns every Hessian block explicitly:

$$H_{FF} = C_{hol}\, J_F^\dagger\, \mathbb{W}_n\, J_F , \quad H_{F\eta} = C_{hol}\, J_F^\dagger\, \mathbb{W}_n\, J_\eta , \quad H_{\eta\eta} = H_{\eta\eta}^{sub} + C_{hol}\, J_\eta^\dagger\, \mathbb{W}_n\, J_\eta ,$$

with $\mathbb{W}_n = \oplus_p |p||p\bigstar|\, W_{rec}$, $J_F = \oplus_p \mathscr{E}_{SM}$ at leading plaquette order and $J_\eta = \oplus_p\, \partial\mathscr{R}_p/\partial\eta|_\bigstar$. Assuming positivity and invertibility of $H_{\eta\eta}$ on the physical auxiliary support, exact relaxation yields

$$\mathcal{K}_g = C_{hol}\, J_F^\dagger\, \mathbb{W}_n^{1/2} (1 - S_\eta)\, \mathbb{W}_n^{1/2} J_F , \quad 0 \preceq S_\eta \preceq 1 ,$$

with S_η the auxiliary screening operator — an orthogonal projector onto the relaxable auxiliary image exactly when the auxiliary cost is purely metric, a strict positive contraction otherwise. This representation proves the auxiliary-census monotonicity theorem: enlarging the physical auxiliary carrier can only decrease or preserve the reduced gauge stiffness, so an incomplete census returns an upper bound on Z_A and a lower bound on g_A.

Locality, product-factor orthogonality, gauge covariance and factorwise adjoint isotropy reduce the leading response density to

$$\mathcal{K}_g^{(0)} = Z_3\, \mathbb{I}_{\Lambda^2} \otimes \mathbb{I}_8 \oplus Z_2\, \mathbb{I}_{\Lambda^2} \otimes \mathbb{I}_3 \oplus Z_q\, \mathbb{I}_{\Lambda^2} ,$$

with exact extraction

$$Z_A = (1/(6\, d_A))\, \mathrm{Tr}_{\Lambda^2\otimes g_A}(P_A\, \mathcal{K}_g^{(0)}\, P_A) ,\ (d_3, d_2, d_q) = (8, 3, 1) ,$$

and factorwise normal form, in the action-scaled convention ($w_A = C_{hol}\, \bar{w}_A$, $b_A = C_{hol}\, \bar{b}_A$, $C_A = H_A^{sub} + C_{hol}\, \bar{C}_A$),

$$Z_A = w_A - b_A^\dagger\, C_A^{-1}\, b_A ,$$

where b_A and C_A are now derivatives of the displayed functional rather than freely named matrices.

Normalisation, scale and matching. The shared-link construction closes the current loophole structurally: the same primitive edge link u_e appears in the curvature response and in the canonical matter transport $\Gamma_\Psi^{(n)} = \frac{1}{2} \Sigma_e (|e\bigstar|/|e|)\, |\Psi_y - u_e(b)\, \Psi_x|^2$, so the Abelian matter coupling is $q\, g_q$ with no independent vertex coefficient, and coupling universality requires the audited return $\nu_A = 1$ for every factor. The matching scale is the substrate spectral-gap rule

$$\mu\bigstar = \min_{r\in\eta} \ell_{ref}^{-1} \min(1 , \delta_r / 2v_c^\infty) ,$$

with the unsaturated Combes–Thomas relation $\xi_r\, \delta_r = 2v_c^\infty$ valid only for $\delta_r < 2v_c^\infty$; if $\delta_{min} = 0$ no local single-scale coupling exists and the mode must remain explicit. The VERSF response is connected to a standard scheme by an explicitly finite bridge $1/g_{\{A,\mathcal{S}\}}^2(\mu\bigstar) = Z_A^V(\mu\bigstar) + \Delta_A^{\{V\to\mathcal{S}\}}$, followed by the frozen threshold ledger and QGC-1 transport. The broken-phase conversion chain is

$$Z_Y = Z_q/36 ,\ e = g\, g'/\sqrt{(g^2 + g'^2)} ,\ \alpha = e^2/4\pi ,\ \sin^2\theta_W = 36\, Z_2/(Z_q + 36\, Z_2) .$$

Benchmark execution. The first blind numerical branch, GRCE-1C(0), is executed on the canonical factorised carrier $L^2(\pi) \otimes \mathcal{H}_{SM}$: the seven-state wheel with stationary measure $\pi = (7/31, 4/31, \ldots, 4/31)$ tensored with the frozen 47-dimensional three-generation-plus-Higgs census. Product-factor gauge–substrate intertwining is exact by construction, $[\hat{T} \otimes \mathbb{I}_{47}, \mathbb{I}_7 \otimes \rho_A] = 0$ (physical fourteen-channel intertwining remains open); the curvature image lies on the coherent wheel line while every wheel auxiliary is π-orthogonal to it, so $H_{F\eta} = 0$ identically and the Schur correction vanishes. The census traces return $Z_3 = 6C_{hol}$, $Z_2 = (13/2)C_{hol}$, Z_q

$= 378C\_hol$, with C_hol the universal stationary-norm-to-action conversion. At the literal unit-action reading $C\_hol = 1$: $(g\_s, g, g') = (0.408248, 0.392232, 0.308607)$, $\alpha^{-1} = 213.628$ and $\sin^2\theta\_W = 13/34$ at $\mu\bigstar$ in the native background-field MOM scheme; the C_hol-independent ratios are $g\_s/g = \sqrt{(13/12)}$ and $g'/g = \sqrt{(13/21)}$. Shared-link tree-level current matching returns $\nu\_A^{(\text{shared-link})} = (1, 1, 1)$, with the independent physical current audit open; the direct embeddings reproduce $(b\_Y, b_2, b_3) = (41/6, -19/6, -7)$ exactly; under Premise AGI-0 the wheel gaps give the raw ratio ledger $\{1, 1, 2, 2, 31/14, 5/2\}$, superseded as thresholds by the band-edge, saturation-corrected rule $\mu\bigstar = \ell\_ref^{-1} \min[1, (1 - 4v\_c^\infty)/(4v\_c^\infty)]$. The branch also proves its own insufficiency: factorisation makes the couplings blind to the wheel spectrum and to factor-dependent relaxation, and the absolute normalisation rests on the underived scalar C_hol.

Profile branch and support-harmonic execution. Branch GRCE-1(1) replaces $\mathbb{I}_7 \otimes \rho\_A$ by the general equivariant profile embedding $\chi\_A \otimes \rho\_A$ and derives everything symmetry allows: the six π-orthonormal wheel modes with D_6 types, gaps and screening factors; the coherent floor $c\_\{A0\}^2 \le N\_A \le \|\chi\_A\|^2\_\pi$; the audit-locked normalisation $c\_\{A0\} = 1$ on the stationary-norm reading; the exact deformation laws $\sin^2\theta\_W = 13N_2/(13N_2 + 21N\_q)$ and $\alpha^{-1} = 2\pi(13N_2 + 21N\_q)$; and a Factor-Profile Non-Identifiability Certificate proving census and symmetry leave the profiles free. The declared premise stack MCR-1, SHR-1, AHP-1, ARP-1 and DRF-1 then fixes them: AHP-1 assigns colour the period-three harmonics and weak isospin the alternating harmonic, ARP-1 selects the hub–boundary contrast for primitive hypercharge, and DRF-1 prices them in the reversible Dirichlet form $\mathbb{I} - \hat{T}$, returning stiffnesses (1, 5/4, 31/28) and $(Z_3, Z_2, Z\_q) = (6, 65/8, 837/2)$ C_hol — at $C\_hol = 1$: $g = 0.350823$, $g' = 0.293294$, $\alpha^{-1} = 79\pi = 248.186$, $\sin^2\theta\_W = 65/158$, with $g\_s = 1/\sqrt{6}$ invariant across the wheel-metric fork and both ratios C_hol-independent. Under the declared Planck anchoring, $\ell\_sub = \sqrt{(\kappa/\eta)}\ \ell\_P$ and $\mu\bigstar = E\_sub \min[1, (1 - 4v\_c^\infty)/(4v\_c^\infty)]$ — equal to 6.89×10^{18} GeV throughout the band-separated regime (conditional on PSA-1, $\kappa = \pi/2$, $\eta = 1/2$ and the sharp certification boundary), where v_c^∞ drops out; v_c^∞ itself and the three finite bridge constants $k\_A^{\{VBF\}}$ are proved non-identifiable from the present axioms. The branch's decisive open tests are internal and named: the susceptibility–link bridge theorem — why the sector registering curvature energy may differ from the sector transporting the gauge phase — and the constitutive fork between the stationary-norm and Dirichlet readings of the wheel factor.

Non-closure certificate. The paper proves matching-scale, response-map, current-normalisation, auxiliary-census and factor-profile non-identifiability theorems for any calculation that leaves the corresponding object free, and certifies exactly what the physical result still requires: substrate derivations of the branch premise stack (MCR-1, SHR-1, AHP-1, ARP-1, DRF-1), the susceptibility–link bridge theorem, resolution of the wheel-metric fork, the Planck-anchoring premise PSA-1, the fourteen-channel map with resolved multiplicity directions, the auxiliary derivatives beyond the coherent-orthogonal class, the universal constant C_hol, the microscopic γ_n equation behind v_c^∞ and the quantum subtraction behind the $k\_A^{\{VBF\}}$ (both proved non-identifiable from the present data), the identification of ℓ_ref against the record-coherence alternative, the independent record-to-Fock descent certificate, and the threshold masses awaiting Papers 1–3. The physical values of Z_3, Z_2, Z_q are therefore not yet derivable beyond the two executed returns — but the remaining calculation is a finite, named correction, with every freedom already spent, locked or declared.

GRCE-1 closes the evaluation framework, the explicit stationary-metric functional, the factorwise reduction, the normalisation, matching and blind-publication protocol, and executes two constitutive branches: the exact matter-census benchmark and the conditional support-harmonic susceptibility branch GRCE-1C(1) under Premises MCR-1, SHR-1, AHP-1, ARP-1 and DRF-1. It does not claim physical gauge couplings.

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Part V — The Blind Benchmark: Branch GRCE-1C(0)

Part VI — The Wheel-Profile Branch: GRCE-1(1)

Part VII — Blind Evaluation and Non-Identifiability

Part I — Frozen Gauge-Response Problem

1. Aim and Programme Position

The VERSF gauge programme has already derived:

local distinguishability ⟶ connection ⟶ curvature ⟶ $\mathfrak{su}(3)\_C \oplus \mathfrak{su}(2)\_L \oplus \mathfrak{u}(1)\_Y$.

The Gauge Action and Coupling Theorem then established

curvature ⟶ substrate response ⟶ $\mathcal{K}\_g$ ⟶ Z_3, Z_2, Z_q ⟶ g_s, g, g' .

Its purpose was structural. It proved why the leading local gauge action must be Yang–Mills, why its kinetic metric is positive, why the response is isotropic inside each simple gauge factor, why factor cross terms vanish and why one canonical coupling controls matter vertices and gauge self-interactions.

It did not evaluate

$Z_3(\mu\star)$, $Z_2(\mu\star)$, $Z\_q(\mu\star)$,

and it did not display the functional whose second variation defines them. The present paper carries both debts as far as the frozen corpus permits, and certifies the exact residue.

The correct order is:

frozen substrate ⟶ Γ_GRCE ⟶ $\mathbb{H}$_response ⟶ $\mathcal{K}$_g ⟶ Z_A(μ★) ⟶ g_A(μ★) ⟶ scheme and threshold matching ⟶ RG transport.

The order may not be reversed.

In particular, low-energy values of α_s, α, $\sin^2\theta_W$, m_W, m_Z or any grand-unification crossing point may not be used to define the response functional, its metric, its embeddings or its scale.

1.1 Role among the final eight papers

GRCE-1 is Paper 4 of the final eight-paper Standard Model completion programme.

It can proceed independently of the final Yukawa values at the boundary-response stage. Its complete threshold and broken-phase transport, however, ultimately consumes:

- the Higgs and completion spectrum from Paper 1;
- the neutral-carrier census from Paper 2;
- the fermion masses and flavour operators from Paper 3.

Paper 5 consumes GRCE-1's colour boundary value in the derivation of the QCD scale and confinement-sector matching.

Paper 7 consumes the completed coupling triple as part of the one common Standard Model parameter set.

1.2 The gate question

Does one frozen VERSF substrate return the three Standard Model gauge strengths as independently evaluated curvature-response invariants of one displayed functional, with their matching scale, current normalisation, scheme and thresholds fixed without measured-coupling input?

A positive answer closes the gauge-coupling sector numerically.

A negative answer identifies the exact substrate response, embedding, metric or matching surface at which the programme fails.

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2. Inherited Gauge Architecture

2.1 Gauge algebra

The inherited local gauge algebra is

$\mathfrak{g}_{SM} = \mathfrak{g}_3 \oplus \mathfrak{g}_2 \oplus \mathfrak{g}_q = \mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_q$.

The Abelian factor is written in the primitive integer basis $q = 6Y$.

The factor dimensions are

$(d_3, d_2, d_q) = (8, 3, 1)$.

2.2 Generator conventions

The non-Abelian generators are Hermitian:

$[T_a, T_b] = i f_{ab}{}^c T_c$,

with

$\mathrm{tr}(T_a T_b) = \frac{1}{2} \delta_{ab}$

in the fundamental representation. Thus colour uses $T_a = \lambda_a/2$, and weak isospin uses $T_i = \sigma_i/2$.

No coupling is inserted into the primitive connection:

$\mathcal{A}_\mu = A_{3\mu}^a T_a + A_{2\mu}^i T_i + B_\mu q$.

The primitive covariant derivative is

$D_\mu = \partial_\mu + i \mathcal{A}_\mu$.

2.3 Curvature

The curvature is

$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu + i [\mathcal{A}_\mu, \mathcal{A}_\nu]$.

Gauge-curvature perturbations lie in the Bianchi-admissible image of the linearised connection-to-curvature map.

2.4 Gauge action

At the declared boundary scale,

$S_{gauge} = -\frac{1}{4} \int d^4x \sqrt{|g|} \, [Z_3 G_{\mu\nu}^a G^{a\mu\nu} + Z_2 W_{\mu\nu}^i W^{i\mu\nu} + Z_q F_{\mu\nu}^q F_q^{\mu\nu}] + \cdots$.

In conventional hypercharge variables,

$Z_Y = Z_q / 36$.

2.5 Coupling identity

Canonical normalisation yields

$g_s = Z_3^{-1/2}$, $g = Z_2^{-1/2}$, $g_q = Z_q^{-1/2}$,

and

$g' = 6\, g_q = 6\, Z_q^{-1/2}$.

2.6 Quantum transport

The Quantum Gauge Completion theorem supplies the perturbative one-loop flow:

$\mu\, dg'/d\mu = (41/6)\, g'^3/(16\pi^2) + \cdots$,

$\mu\, dg/d\mu = -(19/6)\, g^3/(16\pi^2) + \cdots$,

$\mu\, dg_s/d\mu = -7\, g_s^3/(16\pi^2) + \cdots$.

These equations require boundary values. They do not generate them.

---

3. Binding Conventions and Firewalls

3.1 Euclidean response convention

Positivity is evaluated in Euclidean signature. The two-form inner product is

$\langle F, G\rangle = \tfrac{1}{2} F_{\mu\nu} G^{\mu\nu}$.

Therefore

$\mathrm{Tr}_{\Lambda^2}\, \mathbb{I}_{\Lambda^2} = 6$

in four dimensions. The factor 1/6 in every response trace is binding.

3.2 Connection-versus-coupling firewall

The primitive connection contains no coupling. A coupling appears only after the kinetic response is calculated and the field is canonically normalised. Writing gA_μ at the primitive layer is a convention, not a derivation.

3.3 Bare-versus-reduced firewall

The physical response is not generally H_{FF}. It is

$$\mathcal{K}_g = H_{FF} - H_{F\eta} H_{\eta\eta}^{-1} H_{\eta F} .$$

Using H_{FF} is admissible only if $H_{F\eta} = 0$ or a separate theorem proves the omitted auxiliary response is beyond the declared order.

3.4 Boundary-versus-running firewall

The beta functions transport $g_A(\mu\star)$. They cannot be integrated backwards from measured values and then presented as a VERSF boundary derivation.

3.5 Scheme firewall

A VERSF blocked response, a gradient-flow coupling, a background-field coupling, a momentum-subtraction coupling and an $\overline{\text{MS}}$ coupling are different scheme coordinates. They may be compared only through an explicit finite bridge.

3.6 Threshold firewall

States integrated out between two scales modify the running through finite and logarithmic matching terms. Ignoring a threshold is not a harmless simplification if it materially affects the claimed result.

3.7 Hypercharge firewall

The numerical value of an Abelian coupling is meaningless until the generator lattice is fixed. The primitive output is Z_q, not a convention-dependent Z_Y. No 5/3 grand-unification factor is admissible unless a unifying embedding is independently derived.

3.8 Current firewall

A coupling is not derived if the associated current retains an arbitrary multiplicative normalisation. The record current, canonical Fock current, generator norm and primitive charge unit must all be fixed independently. In the constructed functional this firewall is realised structurally by the shared-link rule of Section 23.

3.9 Fourteen-versus-twelve firewall

The fourteen-element closure-response architecture is not the twelve-dimensional Standard Model gauge algebra. No trace over fourteen directions becomes a gauge coupling without a derived equivariant map.

3.10 Metric firewall

The response metric W_{rec} is not a free positive matrix. It is the gauge-invariant completion of the stationary transport metric W_π, and its three factorwise weights w_A are projections of that one metric through the frozen embeddings. Rescaling W_{rec}, or any factor block of it, after output inspection is a fit.

3.11 Broken-phase firewall

The boundary response is evaluated on the unbroken gauge carrier. The photon direction, weak angle, W/Z masses and electromagnetic coupling are downstream consequences after the Higgs vacuum and scheme have been supplied.

3.12 Numerical non-insertion firewall

The following may not enter upstream:

- measured α_s;
- measured α;
- measured $\sin^2\theta_W$;
- measured m_W, m_Z;
- a unification crossing inferred from observed running;
- a fitted matching scale;
- an experimentally normalised current;
- an embedding rotation or metric rescaling chosen from coupling ratios.

---

4. Claim Levels and Named Premises

GRCE-1 separates four levels.

Level S — Exact structural mathematics

This includes: Schur reduction; the residual-projector form; positivity; auxiliary-census monotonicity; factorwise isotropy; trace extraction; canonical coupling conversion; scale non-identifiability; the current-normalisation audit; and the admissibility proofs for the constructed functional.

Level V — Conditional VERSF evaluation theorem

This requires the frozen functional Γ_GRCE, its embeddings, metric, auxiliary census and scale selector — all now displayed, conditional on Premise HRF-1 and the completeness premises below.

Level N — Numerical closure

This requires actual values for $Z_3(\mu\star)$, $Z_2(\mu\star)$, $Z\_q(\mu\star)$ with certified errors, obtained by inserting the frozen numerical matrices into the displayed functional.

Level O — Observable comparison

This requires finite scheme matching, threshold matching, running, electroweak breaking and only then comparison with experiment.

Premise GR1 — Frozen admissible substrate

There exists a stable gauge-neutral homogeneous substrate background $\rho\star$ on the closure-compatible $K = 7$ commitment foam.

Premise GR2 — Holonomy-normalised connection

The primitive connection normalisation is fixed by the holonomy $\exp(i \oint \mathcal{A})$ and the primitive generator lattice.

Premise HRF-1 — Stationary-Record Quadratic Cost (constitutive)

The leading local energetic cost of an irreducible vacuum-relative plaquette holonomy is its squared norm in the canonical stationary-record metric inherited from the $K = 7$ transport operator.

HRF-1 is the single new constitutive statement of this paper. It is falsifiable: the physical substrate could return a different local convex functional, or a different element of the representation commutant. It is, however, the minimal canonical candidate — the functional selected by the already inherited holonomy and stationary transport metric, analytic, positive, reversal-even, with no factorwise coupling constants inserted, and unique within that inherited-metric class (Section 12.7).

Premise GR3 — Wilsonian locality

At the substrate-derived matching scale $\mu\star$, the constructed functional $\Gamma\_\mathrm{GRCE}^{(n)}$ is local or admits a controlled quasi-local derivative expansion after auxiliary reduction.

Premise GR4 — Complete curvature embedding

The frozen equivariant embedding $\mathscr{E}\_\mathrm{SM} = \mathscr{E}_3 \oplus \mathscr{E}_2 \oplus \mathscr{E}\_q : \mathfrak{g}\_\mathrm{SM} \to \mathfrak{u}(\mathscr{H}\_\mathrm{rec})$ has ranks (8, 3, 1), mutually orthogonal factor images, and every multiplicity-space direction resolved from the substrate operator.

Premise GR5 — Complete auxiliary census

The carrier $\mathscr{H}\_\eta$ contains every same-order physical mode able to relax in response to curvature.

Premise GR6 — Auxiliary stability and locality

After gauge and exact null modes are removed, $H\_\eta\eta(q) > 0$ and is analytic near $q = 0$, with no unresolved light pole below $\mu\star$.

Premise GR7 — Gauge covariance

The response functional is invariant under the inherited gauge transformations. For the constructed functional this is a theorem (Section 12.1), not an assumption.

Premise GR8 — Vacuum homogeneity

The factorwise coefficients are constants on the declared background.

Premise GR9 — Relative stationary-record metric

The normalised stationary measure fixes the *relative* response metric W_rec uniquely: no factorwise or directional weight remains free. The conversion of that dimensionless metric norm into the canonically normalised Euclidean action is one universal scalar $C\_\mathrm{hol} > 0$, common to all gauge factors. Its value must be derived independently and may not be fitted from a measured coupling. The literal unit-action reading of HRF-1 asserts $C\_\mathrm{hol} = 1$; that assertion is a separate conditional statement, not a consequence of normalising π.

Premise GR10 — Current matching

The persistent record current is matched to the canonical Fock current by the shared-link construction, without free scale.

Premise GR11 — Primitive Abelian lattice

The integer charge lattice $q = 6Y$ is the frozen Abelian convention.

Premise GR12 — Matching-scale selector

The matching scale is returned by the auxiliary spectral-gap rule of Section 25 before any output inspection.

Premise GR13 — Scheme bridge

The finite conversion from the VERSF response scheme to the declared perturbative scheme is calculable.

Premise GR14 — Threshold completeness

Every state active between $\mu\bigstar$ and a comparison scale appears in the threshold ledger.

Premise GR15 — Quantum Ward closure

The scheme used for running preserves the relevant background Ward or Slavnov–Taylor identities.

Premise GR16 — Blind protocol

All upstream inputs are frozen before measured couplings are consulted.

---

5. Frozen Background and Response Carrier

Let the frozen background be

$$\Xi\bigstar = (\rho\bigstar, \eta\bigstar, \mathcal{A}\bigstar = 0) ,$$

with

$$\delta\Gamma\_GRCE/\delta\rho \,|\bigstar = 0 , \delta\Gamma\_GRCE/\delta\eta \,|\bigstar = 0 .$$

The local gauge-curvature carrier is

$$\mathcal{V}\_F = \Lambda^2 T^*M \otimes \mathfrak{g}\_SM ,$$

restricted to Bianchi-admissible perturbations. The complete retained response carrier is

$$\mathcal{V}\_response = \mathcal{V}\_F \oplus \mathcal{V}\_\eta .$$

The auxiliary carrier may include:

- closure-profile deformations;
- record-metric relaxations;
- bath-density redistributions;
- refinement-interface shifts;
- gauge-inert trace modes;
- completion-interface modes;
- heavy response channels retained at the matching surface;
- multiplicity-space directions in the fourteen-channel carrier;
- any physical scalar or tensor mode contributing at the same derivative order.

It excludes:

- gauge redundancies;
- pure coordinate changes;
- exact null modes already quotiented;
- physical light degrees of freedom that must remain explicit in the low-energy theory.

The auxiliary census is part of the result, not bookkeeping.

Part II — The Explicit Holonomy-Response Functional

The existing VERSF corpus supplies: a closure-compatible K = 7 simplicial commitment foam; an oriented edge transport operator U_ij; path and plaquette holonomies as ordered products of edge transports; a flat admissible vacuum on elementary plaquettes; a seven-state closure catalogue with stationary measure π; a canonical transport inner product $L^2(\pi \otimes \mu\_X)$; vacuum-subtracted and refinement-stable holonomy observables; a $\mathbb{Z}_7$ closure connection and its embedding into a U(1) link variable; the inherited decomposition of the gauge-response carrier into $\mathfrak{su}(3)\_C \oplus \mathfrak{su}(2)\_L \oplus \mathfrak{u}(1)\_q$; and the exact auxiliary Schur-reduction rule.

These ingredients suffice to define one explicit, gauge-covariant and refinement-compatible holonomy-response functional. The only genuinely new constitutive premise is HRF-1.

6. The Frozen Substrate Complex

At refinement order n, let

$\mathfrak{S}_n = (V_n, E_n, P_n)$

be the closure-compatible commitment foam, where:

- V_n is the set of commitment vertices;
- E_n is the set of oriented transport edges;
- P_n is the set of elementary admissible plaquettes or minimal simplicial loops;
- ε_n is the characteristic edge length;
- $|p|$ is the physical area of plaquette p;
- $|p\bigstar|$ is the area of its dual two-cell.

In the emergent four-dimensional continuum, $|p|\,|p\bigstar|$ is the dual four-volume associated with the plaquette. The discrete exterior-calculus weight is

$$\omega_p = |p\bigstar| / |p| .$$

This weight is not a new coupling. It is fixed entirely by the substrate geometry.

Each oriented edge $e : i \to j$ carries a frozen substrate transport operator

$$U_{e,\bigstar} = U_{ij,\bigstar} , \quad U_{\bar{e},\bigstar} = U_{e,\bigstar}^{-1} .$$

For an oriented plaquette boundary $\partial p = (e_1, e_2, \ldots, e_m)$, the frozen substrate holonomy is

$$H_{p,\bigstar} = U_{e_m,\bigstar}^{\sigma(p,e_m)} \cdots U_{e_2,\bigstar}^{\sigma(p,e_2)} U_{e_1,\bigstar}^{\sigma(p,e_1)} ,$$

where $\sigma(p,e) = +1$ if e agrees with the boundary orientation and -1 if e is traversed oppositely.

On the admissible vacuum plaquettes,

$$H_{p,\bigstar} = \mathbb{I}$$

after the orientation twist has been removed and the local closure connection has passed its flatness condition.

Nontrivial holonomy on non-bounding cycles is not removed by this condition. It remains as the flat topological sector and carries no local F^2 energy (Section 21).

---

7. The Common Gauge-to-Record Embedding

Let $\mathscr{H}_{rec} \simeq \mathbb{C}^{14}$ be the fourteen oriented closure-response directions of the $K = 7$ architecture.

Introduce the frozen equivariant embedding

$\mathscr{E}$_SM : $\mathfrak{g}$_SM ⟶ $\mathfrak{u}$($\mathscr{H}$_rec) , $\mathscr{E}$_SM = $\mathscr{E}_3 \oplus \mathscr{E}_2 \oplus \mathscr{E}$_q .

The embeddings satisfy

$\mathscr{E}$_A† $\mathscr{E}$_B = 0 (A ≠ B)

in the canonical response metric, with ranks

rank $\mathscr{E}_3$ = 8 , rank $\mathscr{E}_2$ = 3 , rank $\mathscr{E}$_q = 1 .

The remaining two directions of the fourteen-dimensional carrier are gauge-inert auxiliary or closure-completion directions.

No numerical factor multiplying $\mathscr{E}_3$, $\mathscr{E}_2$ or $\mathscr{E}$_q may be chosen from observed coupling ratios. Their normalisations are fixed by tr(T_a T_b) = ½ δ_ab for the non-Abelian factors and by the primitive integer lattice q = 6Y for the Abelian factor.

If an irreducible gauge representation appears more than once inside $\mathscr{H}$_rec, equivariance alone does not choose the embedding direction in the multiplicity space. That direction must be derived from the substrate operator. Choosing it to improve Z_3 : Z_2 : Z_q is a fit (Section 20).

---

8. Gauge-Perturbed Edge Transport

Let the primitive gauge connection integrated along edge e be

a_e := ∫_e A = $a_{3,e}^{a}$ T_a + $a_{2,e}^{i}$ T_i + b_e q .

No coupling constant appears in a_e. The convention is binding throughout the paper: a_e is the connection already integrated along the edge, so no additional refinement factor ε_n multiplies it inside the transport rule. For smooth fields on the level-n complex, a_e = O(ε_n) automatically, which is what drives every refinement expansion below.

Define the gauge-perturbed edge transport by the ordered rule

U_e(ρ, η; a) = U_{e,★}(ρ, η) · exp[i $\mathscr{E}$_SM(a_e)] .

The ordering is binding:

1. the local substrate fibre or commitment transport acts;
2. the gauge comparison is then made across the edge.

This is the gauge-response analogue of the composite local transport rule already used in the transport-geometry programme, where one local fibre step is followed by one spatial comparison step.

The fields η contain every same-order response mode capable of altering the substrate transport:

$\eta = (\eta_{cl}, \eta_{rec}, \eta_{bath}, \eta_{ref}, \eta_{comp}, \eta_{\perp})$.

The dependence $U_{e,\bigstar} = U_{e,\bigstar}(\rho, \eta)$ is the microscopic origin of the mixed blocks $H_{F\eta}$. This is worth stating plainly: the curvature–auxiliary mixing that the Schur complement relaxes is not an assumption of the reduction. It is generated by the same edge operators that carry the vacuum.

---

9. Vacuum-Relative Plaquette Holonomy

The full plaquette holonomy is

$H_p(\rho, \eta; a) = \overleftarrow{\prod}_{e\in\partial p} U_e(\rho, \eta; a)^{\sigma(p,e)}$,

the arrow denoting the ordered product around ∂p.

The curvature-sensitive object is not the bare holonomy but the vacuum-relative holonomy

$$\mathbf{Q_p(\rho, \eta; a) = H_{p,\bigstar}^{-1} H_p(\rho, \eta; a)} \text{ .}$$

At the frozen vacuum, $Q_p(\rho\bigstar, 0; 0) = \mathbb{I}$.

Define the Lie-algebra-valued plaquette residual by

$$\boldsymbol{\mathscr{R}_p(\rho, \eta; a) = (-i/|p|)\, \mathrm{Log}_0\, Q_p(\rho, \eta; a)} \text{ ,}$$

where Log_0 is the logarithm branch continuously connected to the identity, frozen before output inspection.

For a sufficiently small elementary plaquette,

$Q_p = \exp[\, i\, |p|\, \mathscr{E}_{SM}(F_p) + O(\varepsilon_n^3)\,]$,

and therefore

$\mathscr{R}_p = \mathscr{E}_{SM}(F_p) + O(\varepsilon_n)$.

This supplies the exact discrete-to-continuum curvature normalisation. No independent plaquette coefficient is required — the area division in $\mathscr{R}_p$ and the dual-volume weight in the action are both fixed by the foam geometry.

---

10. The Canonical Stationary-Record Metric

The seven-state transport operator $\hat{T}$ has stationary measure

$\pi = (\pi_h, \pi_{b_1}, \ldots, \pi_{b_6})$, $\Sigma_\kappa \pi_\kappa = 1$.

The transport programme already uses the inner product

$\langle x, y\rangle_\pi = \Sigma_\kappa \pi_\kappa x_\kappa^* y_\kappa$.

On the fourteen oriented response directions, define

$W_\pi = \mathrm{diag}(\pi) \otimes \mathbb{I}_{or}$.

A physical gauge-response metric must be invariant under the adjoint action of each exact gauge factor. The binding order of operations is: first compute W_π; then compute the frozen embeddings $\mathscr{E}_A$; then *test*

$[W_\pi , \mathscr{E}_A(X)] = 0$ for all $X \in \mathfrak{g}_A$.

If the test passes, isotropy holds without intervention. If it fails, the correct conclusion may be that the embedding or the declared vacuum is wrong — not that the metric should be averaged until the anisotropy disappears. Haar projection is legitimate only where a gauge-orbit reduction theorem establishes that averaging is the physical quotient; absent that theorem, a failed test triggers Falsifier F21, not a repair. Subject to that discipline, define the canonical invariant completion by Haar averaging:

$$\mathbf{W_{rec} = \bigoplus_{A=3,2,q} \int_{G_A} Ad_{\mathscr{E}_A(g)}^\dagger\, W_\pi\, Ad_{\mathscr{E}_A(g)}\, d\mu_A(g) \oplus P_\perp W_\pi P_\perp} ,$$

with $d\mu_A$ the normalised Haar measure on G_A and $P_\perp$ the projector onto the two gauge-inert directions.

By Schur isotropy,

$\mathscr{E}_A^\dagger W_{rec} \mathscr{E}_A = \bar{w}_A \mathbb{I}_{d_A}$,

with

$$\mathbf{\bar{w}_A = (1/d_A)\, Tr(\mathscr{E}_A^\dagger W_\pi \mathscr{E}_A) , \quad w_A = C_{hol}\, \bar{w}_A} .$$

Three consequences are binding.

First, the three direct response weights are not introduced independently. They are projections of one stationary substrate metric through three already fixed gauge embeddings. The ratio $w_3 : w_2 : w\_q$ is an output of the wheel and the embeddings, never an input.

Second, factorwise isotropy is tested, not manufactured. The commutation test against the frozen embeddings comes first; Haar completion is a conditional quotient, licensed only by gauge-orbit reduction, and a genuine microscopic anisotropy inside a claimed exact factor is a falsification signal (F21), not noise to be averaged away.

Third, the metric is relatively rigid but not absolutely so. Normalising the stationary measure spends every factorwise and directional freedom — no weight can be moved between colour, weak and hypercharge blocks. One universal scalar survives: the conversion C_hol between the dimensionless stationary norm and the Euclidean action (Premise GR9). It multiplies all three responses together and cancels from the explicit common prefactor of every coupling ratio; the ratios can retain implicit C_hol dependence through factor-dependent screening, N_A = N_A(C_hol), vanishing exactly on the two executed branches, whose mixed blocks are zero. It is the single normalisation debt this paper carries to the end.

---

11. The Holonomy-Response Functional

The microscopic functional is

$$\boldsymbol{\Gamma\_GRCE^{(n)}[\rho, \eta; a] = \Gamma\_sub^{(n)}[\rho, \eta] + (C\_hol/2)\ \Sigma\_\{p\in P_n\}\ |p|\ |p\bigstar|\ \langle\ \mathscr{R}\_p(\rho, \eta; a)\ ,\ W\_rec\ \mathscr{R}\_p(\rho, \eta; a)\ \rangle\ .}$$

Equivalently, before dividing the logarithm by the plaquette area,

$$\Gamma\_GRCE^{(n)} = \Gamma\_sub^{(n)} + (C\_hol/2)\ \Sigma\_\{p\in P_n\}\ (|p\bigstar|/|p|)\ \|\ -i\ Log_0(\ H\_\{p,\bigstar\}^{-1}\ H\_p\ )\ \|^2\_W\_rec\ .$$

This is the **VERSF holonomy-response functional**.

It has no independently adjustable colour, weak or Abelian coefficient. Its inputs are:

- the frozen commitment foam;
- the frozen transport operators;
- the inherited gauge embeddings;
- the stationary measure of the canonical wheel;
- the substrate functional Γ_sub controlling auxiliary fluctuations;
- the dual-cell geometry.

Its only new constitutive statement is HRF-1: the leading local cost is the stationary-record squared norm of the vacuum-relative plaquette logarithm. The universal conversion $C_{hol} > 0$ (Premise GR9) multiplies every response block uniformly. Because the substrate block $H_{\eta\eta}^{sub}$ does not scale with C_{hol}, the constant does not in general cancel from the auxiliary minimiser or the relaxation fraction (Section 13); it factors out exactly when the mixed block vanishes — as on both executed branches — or when the factorwise relaxations are independently fixed, and only then do the coupling ratios become C_{hol}-free. It is introduced here, at the definition of the functional — not later — because it is a property of the action, not of any particular evaluation branch.

Parametric reading. The wheel-metric fork of Section 50 is accommodated by reading the functional parametrically in a declared, frozen, positive-semidefinite gauge-invariant response operator W_{phys} — with the curvature image certified off its kernel, since a reading whose kernel meets the curvature image returns $Z_A = 0$ and fires F4: $W_{phys} = W_{rec}$ is the HRF-1 stationary branch developed here; $W_{phys} = D_{cl} \otimes \mathbb{I}$ is the Dirichlet susceptibility branch of Part VI. Every structural theorem of Parts III–IV — reduction, positivity, monotonicity, extraction, the non-identifiability certificates — applies verbatim to any declared W_{phys}; only the constitutive selection of the reading differs, and it must be declared and frozen before evaluation.

Definition 11.1 — Physical response functional

A candidate functional is physical only if:

1. the curvature insertion is holonomy normalised;
2. the full response is gauge invariant;
3. all same-order responding modes are present;
4. the response metric is fixed up to the declared universal scalar C_{hol};
5. the background, logarithm branch and matching scale are frozen before output inspection;
6. the resulting Hessian is local or admits a controlled derivative expansion;
7. no measured coupling enters any coefficient.

$\Gamma_{GRCE}^{(n)}$ satisfies conditions 1, 2, 4, 5 and 7 by construction (Section 12). Conditions 3 and 6 are the completeness and locality premises GR5 and GR6, tested during evaluation.

---

12. Admissibility of the Constructed Functional

12.1 Gauge covariance

Under a vertex gauge transformation,

$U_e \longmapsto G_{t(e)}\, U_e\, G_{s(e)}^{-1}$,

the plaquette holonomy transforms by conjugation at its basepoint:

$H_p \longmapsto G_p\, H_p\, G_p^{-1}$,

hence

$Q_p \longmapsto G_p\, Q_p\, G_p^{-1}$ and $\mathscr{R}_p \longmapsto \mathrm{Ad}_{G_p}\, \mathscr{R}_p$.

The adjoint-invariance of W_{rec} makes $\langle \mathscr{R}_p, W_{rec}\, \mathscr{R}_p \rangle$ gauge invariant. Premise GR7 is a theorem for this functional.

12.2 Positivity

Every plaquette term is nonnegative:

$\langle \mathscr{R}_p, W_{rec}\, \mathscr{R}_p \rangle \geq 0$,

vanishing precisely when the vacuum-relative local holonomy is trivial on the physical response support.

12.3 Reversal invariance

Changing the plaquette orientation sends $Q_p \to Q_p^{-1}$ and $\mathscr{R}_p \to -\mathscr{R}_p$. The squared norm is unchanged.

12.4 Locality

The functional is a sum over elementary plaquettes and local auxiliary variables. Nonlocality can arise only after a genuinely light auxiliary mode is integrated out. That possibility is detected by the locality test on $H_{\eta\eta}(q)^{-1}$ (Section 16).

12.5 Refinement compatibility

The vacuum-relative holonomy removes the trivial accumulated transport factor. The plaquette-area division gives a finite curvature density, while the dual-volume factor converts the local density into a refinement-compatible action sum. Under the refinement-stability theorems,

$\Gamma_{GRCE}^{(n)} \longrightarrow \Gamma_{GRCE}^{(\infty)}$

provided the auxiliary spectrum and gauge embeddings also converge.

12.6 Continuum limit

Using $\mathscr{R}\_p = \mathscr{E}\_{SM}(F_{\mu\nu}) + O(\varepsilon_n)$, the response term tends to

$$(C\_{hol}/2) \int d^4x \sqrt{|g|} \langle \mathscr{E}\_{SM}(F_{\mu\nu}) , W\_{rec}\, \mathscr{E}\_{SM}(F^{\mu\nu}) \rangle .$$

Factorwise isotropy then gives

$$\tfrac{1}{4} \int d^4x \sqrt{|g|} [Z_3 G_{\mu\nu}^a G^{a\mu\nu} + Z_2 W_{\mu\nu}^i W^{i\mu\nu} + Z_q F_{\mu\nu}^q F_q^{\mu\nu}] ,$$

before auxiliary relaxation, with direct coefficients w_A.

12.7 Minimality and canonical selection at leading order

The correct claim is minimality, not unrestricted uniqueness. On a reducible carrier, every positive operator in the representation commutant defines a gauge-invariant quadratic form, so invariance alone does not single out one functional. What selects HRF-1 is the inherited stationary metric: among functionals built only from the inherited objects — the vacuum-relative holonomy, the stationary transport metric and the foam geometry — that are (i) analytic at the identity, (ii) positive, (iii) reversal-even, (iv) gauge covariant and (v) free of inserted factorwise coefficients, the quadratic stationary-record norm is the minimal canonical candidate: unique *within the class generated by the inherited metric*. Any competitor differs at higher order in $\mathscr{R}\_p$ (subleading by the derivative hierarchy), by a relative reweighting (excluded by GR9's rigidity of the normalised measure and by the metric firewall), by an overall scalar (which is exactly C_{hol}, carried explicitly), or by a different commutant element — which would require a substrate derivation of its own and would falsify HRF-1. HRF-1 adds a cost principle and one universal scalar debt, not a parameter set.

Part III — Exact Reduction of the Constructed Functional

13. The Full Response Hessian in Explicit Form

Expand about $(\rho, \eta, a) = (\rho\star, 0, 0)$. Because $\mathscr{R}\_p(\rho\star, 0; 0) = 0$, the linear term vanishes; gauge neutrality of the background independently excludes a term linear in F. The leading physical response is quadratic.

The first variation of the residual is

$$\delta\mathscr{R}\_p = J\_{pF}\, \delta F\_p + J\_{p\eta}\, \delta\eta ,$$

with

$$J_{pF} = \mathscr{E}_{SM}$$

at leading plaquette order, and

$$J_{p\eta} = \partial\mathscr{R}_p/\partial\eta\ |_\bigstar\ ,$$

generated by the auxiliary dependence of the frozen edge transports $U_{e,\bigstar}(\rho, \eta)$.

Assemble

$$\mathbb{W}_n = \bigoplus_{p\in P_n} |p|\ |p\bigstar|\ W_{rec}\ ,\quad J_F = \bigoplus_p J_{pF}\ ,\quad J_\eta = \bigoplus_p J_{p\eta}\ .$$

Then the gauge-dependent Hessian blocks are

$$\mathbf{H_{FF} = C_{hol}\ J_F^\dagger\ \mathbb{W}_n\ J_F\ ,}$$

$$\mathbf{H_{F\eta} = C_{hol}\ J_F^\dagger\ \mathbb{W}_n\ J_\eta\ ,}$$

$$\mathbf{H_{\eta\eta} = H_{\eta\eta}^{sub} + C_{hol}\ J_\eta^\dagger\ \mathbb{W}_n\ J_\eta\ ,}$$

where $H_{\eta\eta}^{sub}$ is the auxiliary Hessian of Γ_{sub} at the frozen background. The Schur complement is therefore

$$\mathcal{K}_g = C_{hol}\ J_F^\dagger\ \mathbb{W}_n\ J_F - C_{hol}^2\ J_F^\dagger\ \mathbb{W}_n\ J_\eta\ (\ H_{\eta\eta}^{sub} + C_{hol}\ J_\eta^\dagger\ \mathbb{W}_n\ J_\eta\ )^{-1}\ J_\eta^\dagger\ \mathbb{W}_n\ J_F\ .$$

Because $H_{\eta\eta}^{sub}$ does not scale with C_{hol}, the auxiliary minimiser and the relaxation fraction depend on C_{hol} in general; C_{hol} factors out exactly when $H_{F\eta} = 0$, or when the factorwise relaxations have been independently fixed.

Convention. Two equivalent bookkeepings exist. In the *unscaled microscopic convention*, barred blocks strip the action conversion — $\bar{w}_A = (1/d_A)\mathrm{Tr}(\mathscr{E}_A^\dagger W_{rec} \mathscr{E}_A)$, $\bar{b}_A$, $\bar{C}_A$ — and

$$Z_A = C_{hol}\ \bar{w}_A - C_{hol}^2\ \bar{b}_A^\dagger\ (\ H_A^{sub} + C_{hol}\ \bar{C}_A\ )^{-1}\ \bar{b}_A\ .$$

In the *action-scaled convention*, adopted throughout the remainder of this paper,

$$w_A := C_{hol}\ \bar{w}_A\ ,\quad b_A := C_{hol}\ \bar{b}_A\ ,\quad C_A := H_A^{sub} + C_{hol}\ \bar{C}_A\ ,$$

so that the normal form $Z_A = w_A - b_A^\dagger\ C_A^{-1}\ b_A$ holds with C_{hol} carried inside the blocks. Any display of unscaled weights is barred explicitly.

The full physical Hessian is

$\mathbb{H}$_response = (H_FF H_Fη ; H_ηF H_ηη) .

Every block is the derivative of one displayed functional. No block may be imported from a separate phenomenological model unless its descent to the same frozen substrate is proved.

The physical zero-momentum coupling response is extracted only after:

- gauge directions are quotiented;
- exact null modes are classified;
- H_ηη is restricted to its positive physical support;
- the small-momentum limit is demonstrated analytic.

---

14. Exact Auxiliary Relaxation

Theorem 1 — Gauge-Response Schur Reduction

Assume H_ηη > 0 on the physical auxiliary support. For fixed curvature F, the unique auxiliary minimiser is

$\eta^\bigstar(F) = -H\_\eta\eta^{-1}\ H\_\eta F\ F$.

The reduced response is

$\Gamma\_red[F] = \Gamma^\bigstar + \frac{1}{2} F^\dagger\ \mathcal{K}\_g\ F + O(3)$,

with

$\boldsymbol{\mathcal{K}\_g = H\_FF - H\_F\eta\ H\_\eta\eta^{-1}\ H\_\eta F}$ **.**

Proof

The stationary auxiliary equation is H_ηF F + H_ηη η = 0. Invertibility gives the minimiser. Substitution into the quadratic form gives the Schur complement. Positivity makes the stationary point the unique minimum. ■

For the constructed functional,

$\boldsymbol{\mathcal{K}\_g = C\_hol\ J\_F^\dagger\ \mathbb{W}_n\ J\_F - C\_hol^2\ J\_F^\dagger\ \mathbb{W}_n\ J\_\eta\ (\ H\_\eta\eta\text{\^{}sub} + C\_hol\ J\_\eta^\dagger\ \mathbb{W}_n\ J\_\eta\ )^{-1}\ J\_\eta^\dagger\ \mathbb{W}_n\ J\_F}$ **.**

This is no longer an unspecified Schur complement. Every matrix in it is displayed.

Corollary 14.1 — Bare response is not physical response

$\mathcal{K}_g \preccurlyeq H_{FF}$

in the positive-semidefinite ordering.

Corollary 14.2 — No deletion shortcut

The direct block equals the physical response only if $H_{F\eta} = 0$, i.e. only if the frozen edge transports are exactly insensitive to every auxiliary mode.

---

15. Residual-Projector Realisation

The constructed functional has, by construction, the positive residual form

$$\Gamma_{GRCE} - \Gamma_{sub} = (C_{hol}/2) \langle \mathscr{R}(F, \eta) , \mathbb{W}_n \mathscr{R}(F, \eta) \rangle , \mathscr{R}(0, 0) = 0 , \mathscr{R} = J_F F + J_\eta \eta + O(2) .$$

Define the auxiliary screening operator

$$S_\eta = \mathbb{W}_n^{1/2} J_\eta (H_{\eta\eta}^{sub}/C_{hol} + J_\eta^\dagger \mathbb{W}_n J_\eta)^{-1} J_\eta^\dagger \mathbb{W}_n^{1/2} ,$$

understood on the physical auxiliary support.

Theorem 2 — Irreducible Holonomy-Residual Theorem

The physical gauge response is

$$\boldsymbol{\mathcal{K}_g = C_{hol} J_F^\dagger \mathbb{W}_n^{1/2} (1 - S_\eta) \mathbb{W}_n^{1/2} J_F , 0 \preceq S_\eta \preceq 1 .}$$

When $H_{\eta\eta}^{sub}$ vanishes on the auxiliary support — the pure-metric case — S_η is the idempotent orthogonal projector, in the $\mathbb{W}_n$ geometry, onto the auxiliary-relaxable image,

$$S_\eta = \tilde{\Pi}_\eta = \mathbb{W}_n^{1/2} J_\eta (J_\eta^\dagger \mathbb{W}_n J_\eta)^{-1} J_\eta^\dagger \mathbb{W}_n^{1/2} , \tilde{\Pi}_\eta^2 = \tilde{\Pi}_\eta .$$

For $H_{\eta\eta}^{sub} \neq 0$, S_η is a positive contraction, not a projector: the substrate stiffness makes relaxation partial. The projector form is also exact whenever $H_{\eta\eta}^{sub}$ is represented by augmenting the residual space with the substrate modes.

Proof

Insert the residual-map blocks into the Schur complement of Section 13 and factor $\mathbb{W}_n^{(1/2)}$; positivity of $H_{\eta\eta}^{sub}$ gives $0 \preceq S_\eta \preceq 1$ and idempotency in the pure-metric case by direct computation. ■

Interpretation

The curvature perturbation produces the record mismatch $J_F F$. The auxiliary modes can cancel the component lying in im J_η. The propagating gauge stiffness is the norm of the residual component that cannot be relaxed away.

A zero stiffness direction means the entire curvature-induced mismatch is auxiliary-relaxable.

---

16. Positivity, Locality and Zero-Mode Classification

Theorem 3 — Gauge-Response Positivity

If the full physical Hessian is positive semidefinite and $H_{\eta\eta} > 0$, then

$$\mathcal{K}_g \succcurlyeq 0 .$$

It is strictly positive on a gauge factor if no nonzero admissible curvature in that factor is completely relaxable.

Proof

For every F,

$$F^\dagger \mathcal{K}_g F = \min_\eta \, (F, \eta)^\dagger \, \mathbb{H}_{response} \, (F, \eta) .$$

The minimum of a nonnegative quadratic form is nonnegative. ■

For the constructed functional, positivity of the plaquette form guarantees the gauge-dependent part of $\mathbb{H}_{response}$ is positive semidefinite; strict positivity of $\mathcal{K}_g$ on a factor is exactly the statement that the factor's curvature residual is not contained in the auxiliary image.

Zero-mode classification

A zero eigenvalue may indicate:

1. a remaining gauge redundancy;
2. a topological direction outside the local kinetic norm (Section 21);
3. an exact protected flat response;
4. an omitted physical mode;
5. complete auxiliary cancellation;
6. failure to generate a propagating Yang–Mills term.

The classification must be published before a coupling is quoted.

Locality

Write

$$\mathcal{K}_g(q) = H_{FF}(q) - H_{F\eta}(q)\, H_{\eta\eta}(q)^{-1}\, H_{\eta F}(q)\,.$$

If $H_{\eta\eta}(q)^{-1}$ is analytic near $q = 0$,

$$\mathcal{K}_g(q) = \mathcal{K}_g^{(0)} + q^2\, \mathcal{K}_g^{(2)} + O(q^4)\,.$$

Only $\mathcal{K}_g^{(0)}$ defines the leading F^2 coupling. A pole such as $1/q^2$ signals an unresolved light mode and invalidates the local extraction; by the spectral-gap rule of Section 25 it also drives $\mu\star \to 0$ and forces the mode into the retained low-energy theory.

---

17. Auxiliary-Census Monotonicity

Let $\mathscr{H}_{\eta,1} \subseteq \mathscr{H}_{\eta,2}$ be nested physical auxiliary carriers, with reduced responses $\mathcal{K}_g^{(1)}$ and $\mathcal{K}_g^{(2)}$.

Theorem 4 — Auxiliary-Enlargement Monotonicity

$$\boldsymbol{\mathcal{K}_g^{(2)} \preccurlyeq \mathcal{K}_g^{(1)}}\,,$$

hence $Z_A^{(2)} \le Z_A^{(1)}$ for each factor.

Proof

The response on the larger carrier is the minimum of the same quadratic form over a larger set of admissible auxiliary variations. Minimising over more directions cannot raise the minimum. The factorwise trace preserves the ordering. ■

Corollary 17.1 — Coupling bound

Since $g_A = Z_A^{(-1/2)}$, an incomplete auxiliary census gives

$$g_A^{incomplete} \leq g_A^{physical} .$$

It yields a lower bound on the physical coupling.

Corollary 17.2 — Convergence criterion

For an increasing regulator or auxiliary sequence $\mathscr{H}_\eta^{(1)} \subset \mathscr{H}_\eta^{(2)} \subset \cdots$, the extracted coefficients must converge monotonically from above:

$$Z_A^{(1)} \geq Z_A^{(2)} \geq \cdots \geq 0 .$$

Failure of convergence means the response is not numerically closed.

Significance

The auxiliary census is not a minor uncertainty. Every omitted mode systematically biases the stiffness upward and the coupling downward. Because the direction of the bias is fixed, auxiliary convergence is measurable rather than rhetorical.

---

18. Factorwise Isotropy and Coefficient Extraction

Gauge covariance and the product-factor algebra give

$$\mathcal{K}_g^{(0)} = \mathcal{K}_3 \oplus \mathcal{K}_2 \oplus \mathcal{K}_q .$$

For the simple factors, adjoint invariance — certified by the isotropy test against the frozen embeddings, with Haar completion only where gauge-orbit reduction licenses the quotient — implies

$$\mathcal{K}_3 = Z_3\, \mathbb{I}_{\Lambda^2} \otimes \mathbb{I}_8 , \mathcal{K}_2 = Z_2\, \mathbb{I}_{\Lambda^2} \otimes \mathbb{I}_3 , \mathcal{K}_q = Z_q\, \mathbb{I}_{\Lambda^2} .$$

Let $\mathcal{V}_n = \Sigma_c |c|$ be the physical four-volume of the evaluation region and define the response density

$$\mathcal{K}_g = \mathcal{K}_g / \mathcal{V}_n .$$

Theorem 5 — Factorwise Gauge-Response Extraction

The three physical response coefficients are

$$\mathbf{Z\_A = (1/(6\, d\_A))\, Tr\_\{\Lambda^2 \otimes g\_A\}(\, P\_A\, \mathcal{K}\_g^{(0)}\, P\_A\,)\, ,\ (d_3, d_2, d\_q) = (8, 3, 1)\ .}$$

Proof

The factorwise operator is a scalar multiple of the identity on a space of dimension 6 d_A. Dividing its trace by 6 d_A returns that scalar. ■

Isotropy residual

Define

$$\Delta\_A\text{\^{}iso} = P\_A\, \mathcal{K}\_g^{(0)}\, P\_A - Z\_A\, P\_A\ .$$

A valid unbroken-factor evaluation requires ‖Δ_A^iso‖ to vanish within certified numerical error. A nonzero residual indicates broken gauge symmetry, incomplete projection, regulator leakage, an incorrect embedding, or a nonphysical background.

---

19. Susceptibility–Relaxation Normal Form

Before auxiliary relaxation, the direct coefficients of the constructed functional are

$$\mathbf{Z\_A,direct = (1/d\_A)\, Tr(\, \mathscr{E}\_A^\dagger\, W\_rec\, \mathscr{E}\_A\,) = w\_A\ .}$$

After factorwise isotropy, write the curvature–auxiliary coupling as a vector b_A and the physical auxiliary response as C_A > 0, both derivatives of the displayed functional.

Theorem 6 — Factorwise Susceptibility–Relaxation Formula

$$\mathbf{Z\_A = w\_A - b\_A^\dagger\, C\_A^{-1}\, b\_A}$$

in the action-scaled convention of Section 13; in unscaled microscopic blocks, $Z\_A = C\_hol\ \bar{w}\_A - C\_hol^2\ \bar{b}\_A^\dagger(H\_A\text{\^{}sub} + C\_hol\ \bar{C}\_A)^{-1}\ \bar{b}\_A$.

Explicitly:

$$Z_3 = w_3 - b_3^\dagger\, C_3^{-1}\, b_3\ ,$$

$$Z_2 = w_2 - b_2^\dagger\, C_2^{-1}\, b_2\ ,$$

$$Z\_q = w\_q - b\_q^\dagger\, C\_q^{-1}\, b\_q\ .$$

Proof

Apply the factorwise trace to the Schur complement after reducing the adjoint-isotropic curvature block to one representative direction per factor. ■

Positivity constraints

A physical evaluation requires

$w_A > b_A^\dagger C_A^{-1} b_A$.

Equality gives a zero kinetic coefficient. Violation gives a negative physical gauge mode.

Interpretation

The response coefficient is the direct curvature susceptibility — one stationary metric read through three fixed embeddings — minus the amount of curvature cost the substrate can remove by reorganising auxiliary structure. The hierarchy of the three couplings is therefore carried by two computable structures: the spectral shape of the wheel measure π as seen by each embedding, and the differential relaxability of each factor's residual.

---

20. Fourteen-to-Twelve Response Descent

If the microscopic response is constructed on the fourteen-channel carrier $\mathscr{H}_{14}$, the required internal embedding is

$\mathscr{E}_{int} : \mathfrak{g}_{SM} \to \mathscr{H}_{14}$.

An admissible map must satisfy:

1. rank $\mathscr{E}_{int} = 12$;
2. gauge equivariance;
3. separate colour, weak and Abelian support;
4. metric compatibility;
5. no double counting of loop or orientation factors;
6. independent resolution of every multiplicity space;
7. no dependence on measured coupling ratios.

Theorem 7 — Evaluated Fourteen-to-Twelve Descent

Given a reduced fourteen-channel response $\bar{H}_{14}^{red}$, the internal gauge response is

$\mathcal{K}_g = \mathcal{E}_{int}^\dagger \bar{H}_{14}^{red} \mathcal{E}_{int}$,

with coefficients

$Z_A = (1/d_A)\, \mathrm{Tr}_{g_A}(P_A \mathcal{K}_g P_A)$.

Multiplicity firewall

If an irreducible gauge representation appears more than once inside $\mathscr{H}_{14}$, equivariance alone does not choose the embedding direction in the multiplicity space. That direction must be derived from the substrate operator. Choosing it to improve $Z_3 : Z_2 : Z_q$ is a fit. The two gauge-inert directions of $\mathscr{H}_{rec}$ must likewise be certified inert, not merely assumed so.

---

21. Explicit Abelian Reduction and the Flat Gate-3 Sector

The Abelian branch can be made completely explicit using the Gate-3 closure connection.

Each edge carries a closure offset $\rho_e \in \mathbb{Z}_7$, with phase-circle link

$u_{\{e,\bigstar\}} = \exp(2\pi i\, \rho_e / 7)$.

With the primitive Abelian gauge perturbation carried as the integrated link variable $b_e = \int_e B$ — the same binding convention as Section 8, so no refinement factor multiplies it —

$u_e(b) = \exp(2\pi i\, \rho_e / 7) \cdot \exp(i\, q\, b_e)$, $b_e = O(\varepsilon_n)$ under refinement.

The plaquette holonomy is

$H_p^q = \prod_{e\in\partial p} u_e(b)^{\sigma(p,e)} = \exp[(2\pi i/7)\, \Omega(p) + i\, |p|\, q\, F_p^q + O(\varepsilon_n^3)]$,

where

$\Omega(p) = \Sigma_{e\in\partial p}\, \sigma(p,e)\, \rho_e \pmod 7$.

On a locally flat vacuum plaquette, $\Omega(p) = 0$, and hence

$\mathscr{R}_p^q = q\, F_p^q + O(\varepsilon_n)$.

This gives the primitive Abelian curvature response directly, with the charge q entering exactly once, through the lattice.

A nonzero Gate-3 class

$\kappa \in H^1(\Gamma\_vac; \mathbb{Z}_7)$

on a non-bounding loop remains as flat holonomy. It is not counted as local curvature and does not contribute to the local $Z_q F^2$ coefficient. It instead labels a topological sector or boundary condition.

This cleanly separates

local Abelian coupling response

from

persistent flat commitment memory .

Counting the flat class as curvature energy is a falsifier (F23), not an option.

Part IV — From Response Coefficients to Physical Couplings

22. Primitive Connection Normalisation

The primitive connection is

$\mathcal{A}_\mu = A_{3\mu}^a T_a + A_{2\mu}^i T_i + B_\mu q$,

with non-Abelian curvature

$F_{A\mu\nu}^a = \partial_\mu A_{A\nu}^a - \partial_\nu A_{A\mu}^a - f^a{}_{bc} A_{A\mu}^b A_{A\nu}^c$.

Define canonical fields

$\hat{A}_{A\mu} = Z_A^{(1/2)} A_{A\mu}$, equivalently $A_{A\mu} = g_A \hat{A}_{A\mu}$, $g_A = Z_A^{(-1/2)}$.

Theorem 8 — Boundary Coupling Identity

At the frozen VERSF matching scale,

$g_s(\mu\star) = Z_3(\mu\star)^{(-1/2)}$, $g(\mu\star) = Z_2(\mu\star)^{(-1/2)}$, $g_q(\mu\star) = Z_q(\mu\star)^{(-1/2)}$.

These are the primitive connection couplings before finite scheme conversion.

---

23. Shared-Link Current Matching and the Universality Audit

23.1 The structural closure

The same primitive edge link used in the holonomy response must appear in the record and matter transport terms. For the Abelian committed-record amplitude

$\Psi_x = \sqrt{n_x} \cdot e^{i\theta_x}$,

the canonical link transport is

$\Gamma_\Psi^{(n)} = \frac{1}{2} \Sigma_{e: x\to y} (|e\bigstar|/|e|) \mid \Psi_y - u_e(b) \Psi_x \mid^2$.

Expanding at long wavelength gives $|D_\mu\Psi|^2$ with

$D_\mu = \partial_\mu - i\, q\, B_\mu$.

No independent coefficient multiplies $q\, B_\mu$. After canonical normalisation of B_μ, the matter coupling is therefore

$q\, g_q$, $g_q = Z_q^{(-1/2)}$,

and because $q = 6Y$ the conventional hypercharge coupling is $g' = 6\, g_q$.

The non-Abelian audit must use the analogous canonical transport terms for the already-derived CAR matter carriers. Any additional coefficient would violate the shared-link construction and fail coupling universality. The current firewall of Section 3.8 is thereby realised structurally: there is no place in the displayed action where a vertex normalisation could be inserted without visibly breaking the shared link.

23.2 The audit

Structural closure must still be audited, because the record-current programme supplies its own microscopic current $J_{A,rec}^\mu$, and the canonically normalised matter theory supplies

$J_{A,Fock}^\mu = \Sigma_f : \bar{\psi}_f \gamma^\mu \rho_f(T_A) \psi_f :$.

Define the matching coefficient by

$$J_{A,rec}^{\mu} = \nu_A J_{A,Fock}^{\mu} + J_{A,\perp}^{\mu} ,$$

with $J_{A,\perp}$ orthogonal to the retained Standard Model source carrier. The canonical matter interaction is then $g_A \nu_A \hat{A}_{\mu}^{A} J_{A,Fock}^{\mu}$, while the non-Abelian self-coupling remains g_A.

Theorem 9 — Current-Universality Closure Test

Exact coupling universality requires

$\nu_A = 1$ and $J_{A,\perp} = 0$

on the retained physical current support.

Proof

The same primitive connection appears in the curvature and source term. Canonical gauge-field normalisation supplies g_A. Any remaining factor multiplying the canonical matter current would make the matter vertex differ from the self-coupling. ■

Universality residual

Define $\varepsilon_A^{current} = |\nu_A - 1|$. A coupling calculation is not complete until this residual is evaluated. The shared-link construction predicts $\nu_A = 1$; the audit tests whether the record-current descent respects it. The Slavnov–Taylor identities can preserve equality once established. They cannot determine an unfixed microscopic current unit.

The full audit set

The following couplings must agree after canonical normalisation and quantum matching:

1. matter–gauge vertex;
2. three-gauge-boson vertex;
3. four-gauge-boson vertex;
4. source-coupled field equation;
5. background-field two-point coupling;
6. ghost–gauge vertex where the Slavnov–Taylor identity requires it.

Factorwise universality residuals:

$$\varepsilon_A^{mat} = |g_A^{mat} - g_A^{kin}| / \max(g_A^{mat}, g_A^{kin}) ,$$

$$\varepsilon_A^{(3)} = |g_A^{(3)} - g_A^{kin}| / \max(g_A^{(3)}, g_A^{kin}) ,$$

$$\varepsilon_A^{(4)} = |\sqrt{g_A^{(4)}} - g_A^{kin}| / \max(\sqrt{g_A^{(4)}}, g_A^{kin}) .$$

A closed calculation requires all residuals to vanish within the certified truncation and numerical error.

---

24. Primitive Abelian Lattice

The primitive charge is $q = 6Y$. On the inherited ledger:

$q(Q_L) = 1$, $q(u_R) = 4$, $q(d_R) = -2$,

$q(L_L) = -3$, $q(e_R) = -6$, $q(\Phi) = 3$.

The nonzero charges have greatest common divisor one. Therefore the continuous rescaling

$q \to cq$, $g_q \to g_q/c$

is removed up to an overall sign.

Theorem 10 — Conventional Hypercharge Conversion

With $Y = q/6$, the conventional coupling and kinetic coefficient are

$\mathbf{g' = 6\, g_q = 6\, Z_q^{(-1/2)}\ ,\ Z_Y = Z_q / 36\ .}$

No additional normalisation factor is admissible without a separately derived embedding.

---

25. The Substrate-Gap Matching Scale

A coupling boundary value is meaningful only at a specified scale, and the scale must be returned by the substrate.

The matching scale is tied to the longest correlation length among the modes eliminated by the Schur complement. For each physical auxiliary response mode r, let:

- δ_r be its spectral distance from the retained transport band;
- v_c be the continuum coherence velocity;
- ξ_r be its Combes–Thomas response length.

The transport programme gives, in the unsaturated regime,

$\xi_r\, \delta_r = 2\, v_c$ (valid for $\delta_r < 2\, v_c$) .

A dimensional point is binding here. If δ_r and v_c are dimensionless substrate quantities, conversion to a physical scale requires the substrate refinement length ℓ_{ref}. The open dimensionful debt is therefore the identification of ℓ_{ref} — a substrate-to-energy conversion — alongside the dimensionless transport constant.

25.1 Global band structure

The gap entering the rule is not the raw local wheel gap. The global transport operator

$$T_{bulk} = \hat{T} \otimes \mathbb{I} + \gamma\, \mathbb{I} \otimes A_X$$

broadens every wheel eigenvalue into a band of half-width $v_c = \gamma\, \rho(A_X)$. The coherent Perron band is $B_1 = [1 - v_c, 1 + v_c]$; the first incoherent band, centred at 1/2, is $B_{1/2} = [1/2 - v_c, 1/2 + v_c]$. The actual edge-to-edge distances from the coherent band are therefore

$$\boldsymbol{\delta_1(v_c) = 1/2 - 2v_c\ ,\ \delta_0 = 1 - 2v_c\ ,\ \delta_{rad} = 31/28 - 2v_c\ ,\ \delta_{alt} = 5/4 - 2v_c\ ,}$$

and the first gap is positive only when

$$\boldsymbol{v_c < 1/4\ ,}$$

which is stronger than the basic weak-coupling condition $v_c < 1/2$. The 1/2-band pair is always the longest-range auxiliary sector.

25.2 Combes–Thomas saturation

The exact transport decay rate saturates:

$$\boldsymbol{\eta_r = \min(\,1\ ,\ \delta_r / 2v_c\,)\ ,\ M_r = \eta_r / \ell_{ref}\ .}$$

The unsaturated identity $\xi\delta = 2v_c$ applies only for $\delta < 2v_c$; the saturation $\eta = 1$ caps every correlation length at one refinement unit. For the longest-range pair, the binding physical boundary scale is

$$\boldsymbol{\mu\bigstar = \ell_{ref}^{-1} \min[\,1\ ,\ (1 - 4v_c^{\infty})/(4v_c^{\infty})\,]\ ,}$$

with the dimensionless substrate version $\hat{\mu}\bigstar = \min[1, (1 - 4v_c^{\infty})/(4v_c^{\infty})]$. This supersedes the provisional unsaturated expression $\mu\bigstar = 1/(4\, v_c^{\infty}\, \ell_{ref})$, which omitted both the band broadening and the saturation.

25.3 The two regimes

Local-cutoff regime, $0 < v_c^{\infty} \le 1/8$. Then $\delta_1/(2v_c^{\infty}) \ge 1$, every wheel auxiliary is localized within one refinement unit, and

$\mu^\star = \ell_{ref}^{-1}$.

Extended-first-auxiliary regime, $1/8 < v_c^\infty < 1/4$. The first incoherent pair becomes longer-ranged,

$\mu^\star = (1 - 4v_c^\infty) / (4v_c^\infty \ell_{ref})$,

while the remaining wheel modes stay at the refinement cutoff.

Full five-band separation. The distinct band centres $\{1, 1/2, 0, -3/28, -1/4\}$ have minimum spacing 3/28, so complete separation of all five transport bands requires $v_c^\infty < 3/56$. Since $3/56 < 1/8$, full band separation automatically lies in the local-cutoff regime:

$v_c^\infty < 3/56 \Longrightarrow \mu^\star = \ell_{ref}^{-1}$,

and the exact value of v_c^∞ drops out of the matching scale entirely.

If $\delta_{min} = 0$ — the first gap closes at $v_c^\infty = 1/4$ — then $\xi_{aux} = \infty$ and no local single-scale gauge coupling can be extracted: the supposedly auxiliary mode must remain in the low-energy theory. This is the same failure detected by the locality test of Section 16, now with its scale diagnosis attached.

The threshold ledger is then seeded by the ordered list $M_r = \eta_r/\ell_{ref}$.

Definition 25.1 — Canonical matching surface

A matching surface is canonical if:

1. it is determined solely by frozen substrate objects;
2. it is unique or has a pre-registered finite branch set;
3. its definition is invariant under field reparametrisation;
4. all modes above and below it are unambiguously classified;
5. it is selected before numerical coupling comparison.

The spectral-gap rule satisfies 1–4 by construction; 5 is a protocol obligation.

A selector may never be based on: a desired value of α; a desired weak angle; coupling unification; the W or Z masses; or minimisation of disagreement with experiment.

---

26. Finite Scheme Bridge

The finite part of this bridge is not merely uncalculated but non-identifiable from the present data: any finite gauge-invariant local F^2 term shifts it factor by factor while preserving the betas and the classical equations, so its strongest admissible form is $\Delta_A^{\mathrm{VBF}\to\overline{\mathrm{MS}}} = k_A^{\mathrm{VBF}}/8\pi^2$ with three regulator constants owed to the quantum subtraction prescription (Proposition 28, Section 62).

Let Z_A^V denote the inverse squared coupling in the VERSF response scheme, and $\mathcal{S}$ a standard gauge-consistent scheme. The finite bridge is

$$\mathbf{1/g_{A,\mathcal{S}}^2(\mu\bigstar) = Z_A^V(\mu\bigstar) + \Delta_A^{V\to\mathcal{S}}} \, ,$$

equivalently

$$g_{A,\mathcal{S}} = g_A^V \left[1 + c_{A,1} (g_A^V)^2 + c_{A,2} (g_A^V)^4 + \cdots \right] .$$

The coefficients $c_{A,n}$ depend on the regulator, field definition and coupling observable.

Preferred operational bridge

A gauge-invariant gradient-flow coupling may be used:

$$g_{A,GF}^2(\mu) = \mathcal{N}_A \, t^2 \langle E_A(t)\rangle \big|_{\mu = 1/\sqrt{8t}} \, ,$$

with $\mathcal{N}_A$ fixed by the generator convention. The VERSF response must then be matched to the flow observable by calculating the same background response in both definitions.

Scheme closure requirement

The bridge must be calculated. Declaring the VERSF output to be $\overline{\mathrm{MS}}$ by notation is not a scheme conversion.

---

27. Threshold Ledger and Decoupling

Let the ordered threshold set be $\mathcal{T} = \{ M_1, M_2, \ldots, M_n \}$. It may contain: quarks; charged leptons; heavy neutrino states; the Higgs and Goldstone completion sector; W and Z thresholds after breaking; heavy substrate response modes $M_r = \ell_{\mathrm{ref}}^{-1} \min(1, \delta_r/2v_c^\infty)$; completion-interface excitations; record or bath modes; and any additional VERSF states.

Across a threshold M_i,

$$1/g_{A,\mathrm{low}}^2(M_i) = 1/g_{A,\mathrm{high}}^2(M_i) + \Delta_{A,i}^{\mathrm{threshold}} \, ,$$

and the beta-function census changes below the threshold.

Theorem 11 — Threshold-Complete Transport

A low-energy coupling is determined only by the tuple

$\{ Z_A^V(\mu\bigstar) , \Delta_A^{scheme} , \mathcal{T} , \Delta_{A,i}^{threshold} , \beta_A^{(r)} \}$.

No subset is, by itself, the observable prediction.

27.1 Current structural ledger

A complete status and field-content ledger can now be assembled; a complete common-scheme physical numerical ledger cannot. All numerical entries below are conditional imports from their source branches, recorded for structure — none is a GRCE Hessian output, and none may be used as a response input.

Electroweak and scalar block.

Object	Threshold / result	Status
Photon	0	Exact vacuum-null direction
W±	79.26802098 GeV	Conditional branch
Z	89.55594468 GeV	Conditional branch
Higgs, leading	123.79571182 GeV	Conditional HR-1 result
Higgs, scalar-only pole	123.80460101 GeV	Scalar loop evaluated
Goldstones	no physical pole	Included with W/Z matching
Ghosts	no physical pole	Included with the gauge-fixed W/Z block

The W and Z values consume the conditional 15/14 saturation relation and the Persistent-Abelian branch selector. The full Higgs pole still lacks gauge, top, Goldstone/ghost and heavy-threshold contributions.

Charged-fermion block. The boundary outputs of the MCHY audit branch, with the exact one-loop removal increments below each threshold (exact for a Dirac charged fermion):

Carrier	Conditional output	One-loop removal below threshold
e	0.521691 MeV	$\Delta b\_em = 4/3$
u	2.16702 MeV	$\Delta b\_em = 16/9$, $\Delta b_3 = 2/3$
d	4.69522 MeV	$\Delta b\_em = 4/9$, $\Delta b_3 = 2/3$
s	93.9044 MeV	$\Delta b\_em = 4/9$, $\Delta b_3 = 2/3$

Carrier	Conditional output	One-loop removal below threshold
μ	108.056 MeV	$\Delta b_{em} = 4/3$
c	1.11241 GeV	$\Delta b_{em} = 16/9$, $\Delta b_3 = 2/3$
τ	1.86512 GeV	$\Delta b_{em} = 4/3$
b	4.86254 GeV	$\Delta b_{em} = 4/9$, $\Delta b_3 = 2/3$
t	291.827 GeV	$\Delta b_{em} = 16/9$, $\Delta b_3 = 2/3$

The masses are unmatched matching-point outputs, not common-scheme decoupling masses; the RG and pole conversion maps remain owed by their source paper.

Neutral block. No numerical values exist for M_D, M_R, M_L^{irr} or the Takagi spectrum of the complete neutral block; the corpus does not yet decide whether ν_R exists physically, nor whether the light branch is Dirac, Majorana or seesaw. Light-neutrino and heavy-neutral thresholds: open.

Colour block. The unbroken perturbative gluons are massless; a confinement crossover or hadronic matching scale is not calculated by the present corpus and is not equivalent to a gluon pole threshold.

Substrate auxiliary block. On the canonical coherent-band branch, using the band-edge distances and saturation of Section 25:

Wheel centre	Band distance from coherent band	$M_r \ell_{ref}$
1/2 (multiplicity 2)	$1/2 - 2v_c^\infty$	$\min(1, (1 - 4v_c^\infty)/(4v_c^\infty))$
0 (multiplicity 2)	$1 - 2v_c^\infty$	1 (for $v_c^\infty < 1/4$)
−3/28	$31/28 - 2v_c^\infty$	1
−1/4	$5/4 - 2v_c^\infty$	1

Actual defect and completion thresholds require solving the Birman–Schwinger condition $1 \in \mathrm{spec}[-V (T_{bulk} - \lambda\mathbb{I})^{-1}]$; the corpus supplies the equation but not the defect catalogue or the numerical coupling γ needed to enumerate its roots.

κ-field block. The proposed $m_\kappa = C_m \xi^{-1}$ carries a conjectural constant, and the corpus currently contains incompatible cubic and quartic coherence formulas, $\xi_3 = (\hbar c/\rho)^{1/3}$ versus $\xi_4 = (\hbar c/\rho)^{1/4}$, with one paper superseding the quartic form and later papers still using it. Until the density dimension and the interface-to-volume bridge are resolved, no absolute κ-field threshold may enter the certified ledger.

Ledger verdict. Structurally near-complete conditionally; numerically incomplete: common-scheme charged masses, neutrino thresholds, physical substrate modes, the κ threshold and the gauge-consistent finite electroweak matching block remain open.

28. Quantum Running Boundary

In a declared perturbative interval,

$\mu\, dg_A/d\mu = (b_A/16\pi^2)\, g_A^3 + \cdots$.

At one loop,

$1/g_A^2(\mu) = 1/g_A^2(\mu\bigstar) - (b_A/8\pi^2) \ln(\mu/\mu\bigstar) + \Delta_A^{threshold} + \Delta_A^{scheme} + \cdots$.

For the Standard Model census,

$(b_Y, b_2, b_3) = (41/6 , -19/6 , -7)$.

Theorem 12 — Boundary/Transport Separation

The beta function determines the map $g_A(\mu\bigstar) \mapsto g_A(\mu)$. It does not determine $g_A(\mu\bigstar)$.

Proof

A first-order RG equation requires one boundary condition. Distinct boundary values generate distinct solutions under the same beta function. ■

---

29. Broken-Phase Conversion Chain

After the scalar vacuum and electroweak mixing are independently supplied,

$\tan \theta_W = g'/g$,

therefore

$\mathbf{\sin^2\theta_W = g'^2 / (g^2 + g'^2) = Z_2/(Z_Y + Z_2) = 36\, Z_2 / (Z_q + 36\, Z_2)}$ **.**

The electromagnetic coupling is

$\mathbf{e = g\, g' / \sqrt{(g^2 + g'^2)} , \alpha = e^2/4\pi}$ **.**

The gauge-boson consistency relations are

$m_W^2 = \frac{1}{4} g^2 v_{cl}^2 , m_Z^2 = \frac{1}{4} (g^2 + g'^2) v_{cl}^2$.

These relations test the independent outputs of Papers 1 and 4. They may not be inverted using observed masses to define the couplings.

Part V — The Blind Benchmark: Branch GRCE-1C(0) — Factorised Matter-Census Induced-Response Benchmark

The evaluation theorem of Parts I–IV licenses a first numerical execution. The strongest calculation supported by the present frozen corpus is a blind, factorised, product-census branch, designated GRCE-1C(0). It produces explicit values for Z_3, Z_2, Z_q up to one universal constant, exact gauge–substrate intertwining and exact tree-level current matching — and it proves, from its own structure, why it cannot yet be promoted to the physical VERSF coupling result.

Everything in this Part is exact arithmetic on frozen corpus objects. No measured coupling enters at any stage.

30. Status and Admissibility of the Benchmark Branch

The fourteen-channel route cannot presently be executed. Section 20 requires an independently derived equivariant map $\mathfrak{g}_{SM} \to \mathscr{H}_{14}$ and forbids obtaining the couplings by merely distributing fourteen closure channels among twelve gauge directions. No such map is numerically supplied by the corpus.

The first executable alternative is a direct embedding into the already frozen Standard Model representation carrier. This must be stated precisely for what it is. The branch does *not* execute the fourteen-channel microscopic response functional: it replaces the record carrier $\mathscr{H}_{rec} \simeq \mathbb{C}^{14}$ by the product carrier $L^2(\pi) \otimes \mathscr{H}_{SM}$ and obtains the coefficients by tracing the gauge generators over the matter-plus-Higgs representation census. That is a legitimate benchmark of the evaluation machinery — every ingredient is a declared corpus object: the canonical wheel and its stationary measure, the representation census, the primitive charge lattice and the trace conventions — but it is an *induced-response* calculation, a matter-census index, not a derived substrate response. The branch is therefore labelled GRCE-1C(0), the C marking the census carrier, and it runs under one additional declared premise:

Premise MCR-0 — Matter-Census Response (benchmark-scoped)

On the benchmark branch only, the response carrier is taken to be $L^2(\pi) \otimes \mathscr{H}_{SM}$, with the gauge curvature acting through $\mathbb{I}_7 \otimes \rho_A$ and every canonically normalised matter or scalar carrier weighted by the identity metric. This is a frozen benchmark premise, not a derivation of the physical fourteen-channel gauge-to-record embedding, and nothing downstream of it may be quoted as a microscopic substrate result.

Its headline return is

$$\mathbf{Z_3 = 6\ C\_hol\ ,\ Z_2 = (13/2)\ C\_hol\ ,\ Z\_q = 378\ C\_hol\ ,}$$

equivalently, in conventional hypercharge normalisation,

$$\mathbf{Z\_Y = Z\_q\ /\ 36 = (21/2)\ C\_hol\ ,}$$

where C_hol is the universal conversion between the stationary-record norm and the dimensionless Euclidean action. The literal unit-normalised reading of HRF-1 sets C_hol = 1; the existing corpus does not independently derive that choice (Section 34).

---

31. Direct Gauge Embeddings on the Matter-Census Carrier

Define the frozen Standard Model representation carrier

$$\mathscr{H}\_SM = \bigoplus\{g=1\}^{\wedge}\{3\}\ [\ (\mathbb{C}^3 \otimes \mathbb{C}^2)\{Q\_L\} \oplus \mathbb{C}^3\_\{u\_R\} \oplus \mathbb{C}^3\_\{d\_R\} \oplus \mathbb{C}^2\_\{L\_L\} \oplus \mathbb{C}\_\{e\_R\}\ ] \oplus \mathbb{C}^2\_\Phi\ ,$$

with dimension

$$\dim \mathscr{H}\_SM = 3(6 + 3 + 3 + 2 + 1) + 2 = 47\ .$$

A neutral singlet N_R, if present, contributes zero to all three gauge traces.

For $X \in \mathfrak{su}(3)$,

$$\rho_3(X) = \bigoplus\_\{g=1\}^{\wedge}\{3\}\ [\ X \otimes \mathbb{I}_2 \oplus X \oplus X \oplus 0_2 \oplus 0_1\ ] \oplus 0_2\ .$$

For $Y_2 \in \mathfrak{su}(2)$,

$$\rho_2(Y_2) = \bigoplus\_\{g=1\}^{\wedge}\{3\}\ [\ \mathbb{I}_3 \otimes Y_2 \oplus 0_3 \oplus 0_3 \oplus Y_2 \oplus 0_1\ ] \oplus Y_2\ .$$

The conventional hypercharge embedding is

$$\rho\_Y(1) = \bigoplus\_\{g=1\}^{\wedge}\{3\}\ \mathrm{diag}(\ (1/6)\ \mathbb{I}_6\ ,\ (2/3)\ \mathbb{I}_3\ ,\ -(1/3)\ \mathbb{I}_3\ ,\ -(1/2)\ \mathbb{I}_2\ ,\ -1\ ) \oplus (1/2)\ \mathbb{I}_2\ ,$$

and the primitive generator is

$\rho\_q = 6\,\rho\_Y$,

with integer charges (1, 4, −2, −3, −6; 3).

These are the already declared Standard Model representation and primitive-charge conventions of the gauge programme. No new normalisation is introduced at this step.

---

32. The Product Carrier and Exact Gauge–Substrate Intertwining

Use the canonical wheel carrier $\mathscr{H}_0 = L^2(\pi)$, where the wheel transition matrix, on the ordered basis $(h, b_1, \ldots, b_6)$, is

$$\hat{T} = \begin{pmatrix} 1/7 & 1/7 & 1/7 & 1/7 & 1/7 & 1/7 & 1/7 \\ 1/4 & 1/4 & 1/4 & 0 & 0 & 0 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 & 0 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 1/4 & 0 & 0 \\ 1/4 & 0 & 0 & 1/4 & 1/4 & 1/4 & 0 \\ 1/4 & 0 & 0 & 0 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 0 & 0 & 0 & 1/4 & 1/4 \end{pmatrix}$$

The hub state refreshes uniformly over all seven states; each boundary state distributes equally over the hub, itself and its two cyclic neighbours. The stationary measure is

$\pi = (7/31 , 4/31 , 4/31 , 4/31 , 4/31 , 4/31 , 4/31)$,

and the spectrum is

$\operatorname{spec} \hat{T} = \{ 1 , 1/2 , 1/2 , -1/4 , -3/28 , 0 , 0 \}$.

These are explicit corpus returns.

Define the complete benchmark carrier

$\mathscr{H}\_\mathrm{GRCE}^{(0)} = L^2(\pi) \otimes \mathscr{H}\_\mathrm{SM}$,

of dimension 7 × 47 = 329. The frozen substrate transport and gauge embeddings are

$T\_\mathrm{sub} = \hat{T} \otimes \mathbb{I}_{47}$, $\mathscr{E}\_A(X) = \mathbb{I}_7 \otimes \rho\_A(X)$.

Theorem 13 — Exact Gauge–Substrate Intertwining on the Product Carrier

For every $A \in \{3, 2, q\}$ and every X,

$$[\, T_{sub} , \mathscr{E}_A(X) \,] = 0 ,$$

and more generally

$$(T_e \otimes \mathbb{I})(\mathbb{I} \otimes e^{i a_e}) = (\mathbb{I} \otimes e^{i a_e})(T_e \otimes \mathbb{I}) .$$

Proof

The operators act on disjoint tensor factors. ■

The substrate transport therefore does not conjugate or distort the gauge generator during the plaquette product. The vacuum-relative plaquette logarithm expands as

$$\mathscr{R}_p = \mathbb{I}_7 \otimes \rho_A(F_p) + O(\varepsilon_n)$$

without extra transport commutators. A direct matrix calculation on the 329-dimensional carrier confirms

$$\|[T_{sub}, \mathscr{E}_3]\|_F = \|[T_{sub}, \mathscr{E}_2]\|_F = \|[T_{sub}, \mathscr{E}_Y]\|_F = 0 ,$$

exactly, algebraically — not as a numerical near-zero.

What this does and does not establish

It establishes a valid factorised census embedding satisfying the equivariance, rank and orthogonality requirements of Premise GR4 on this carrier. But the identity is exact *because* the two operators were deliberately placed on separate tensor factors: it proves the product carrier is internally consistent, nothing more.

It does not demonstrate that the physical VERSF edge transport intertwines the gauge algebra. The physical branch still requires

$$U_{e,\star}\, \mathscr{E}_{SM}(X)\, U_{e,\star}^{-1} = \mathscr{E}_{SM}(\mathrm{Ad}_{\tau_e} X) ,$$

or, on a gauge-neutral homogeneous branch, $[\, U_{e,\star} , \mathscr{E}_{SM}(X) \,] = 0$, proved for the frozen microscopic transports — and it does not derive the still-missing map $\mathfrak{g}_{SM} \to \mathscr{H}_{14}$. The correct statement is: exact product-factor intertwining is proved on GRCE-1C(0); gauge–substrate intertwining for the physical fourteen-channel embedding remains open, and with it the possibility — required by Section 38 — that the physical embedding is nonfactorising.

33. Exact Vanishing of the Auxiliary Schur Correction

The coherent wheel direction is $\mathbb{1} = (1, 1, \ldots, 1)$, and the incoherent carrier is

$\mathscr{H}_{inc} = \{ f : \langle f, \mathbb{1} \rangle_\pi = 0 \}$.

The canonical reversible-wheel quadratic form is generated by

$C_0 = \mathbb{I} - \hat{T}$

on $\mathscr{H}_{inc}$. Its physical eigenvalues are

$\{ 1/2 , 1/2 , 1 , 1 , 31/28 , 5/4 \}$,

all positive: Premise GR6 is satisfied exactly on this branch.

On the product carrier, the gauge-curvature residual is supported on the coherent wheel line,

$J_F F = \mathbb{1} \otimes \rho_A(F)$,

whereas every wheel auxiliary response has the form

$J_\eta \eta = \Sigma_r f_r \otimes M_r , f_r \in \mathscr{H}_{inc}$.

Theorem 14 — Exact Vanishing of the Auxiliary Schur Correction

Since $\langle \mathbb{1}, f_r \rangle_\pi = 0$ for every r, the mixed Hessian vanishes identically:

$\mathbf{H_{F\eta} = 0}$ **,**

hence

$\mathbf{b_3 = b_2 = b_q = 0}$

and the Schur complement equals the direct response exactly:

$\mathcal{K}_{g^{(0)}} = \mathbf{H_{FF}}$ **.**

Proof

Orthogonality of the coherent line and $\mathscr{H}_{inc}$ in the π inner product, applied blockwise through $\mathbb{W}_n$. ■

This is the cleanest possible benchmark: Corollary 14.2's condition $H_F\eta = 0$ holds not by truncation but by exact orthogonality. It also supplies the diagnostic developed in Section 38: any physical hierarchy generated by factor-dependent auxiliary relaxation requires a nonfactorising gauge-to-record embedding, or a substrate response that mixes the coherent gauge image with incoherent wheel directions. The factorised branch cannot generate such a relaxation hierarchy.

---

34. Benchmark Response Coefficients and the Universal Constant C_hol

The benchmark metric is

$$W_0 = \mathrm{diag}(\pi) \otimes \mathbb{I}_{47} .$$

Because $\Sigma_\kappa \pi_\kappa = 1$, the wheel factor contributes exactly one, leaving the representation traces. On this carrier the isotropy discipline of Section 10 is satisfied without averaging: $[W_0 , \mathbb{I}_7 \otimes \rho_A(X)] = 0$ identically, so the commutation test passes and no Haar quotient is invoked.

For any canonically normalised colour generator,

$$\mathrm{Tr}_{\mathscr{H}_{SM}}(\rho_3(T_a) \rho_3(T_b)) = 6 \delta_{ab} ,$$

from 3 generations × [2T(3) + T(3) + T(3)] = 3(1 + 1/2 + 1/2) = 6.

For weak isospin,

$$\mathrm{Tr}(\rho_2(T_i) \rho_2(T_j)) = (13/2) \delta_{ij} ,$$

from 3 generations × [3T(2) + T(2)] + $T(2)_\Phi$ = 3(2) + 1/2 = 13/2.

For conventional hypercharge,

$$\mathrm{Tr}\, \rho_Y^2 = 3 [6(1/6)^2 + 3(2/3)^2 + 3(1/3)^2 + 2(1/2)^2 + 1] + 2(1/2)^2 = 3(10/3) + 1/2 = 21/2 ,$$

and for the primitive generator $q = 6Y$,

$$\mathrm{Tr}\, \rho_q^2 = 36\, \mathrm{Tr}\, \rho_Y^2 = 378 .$$

The universal action conversion enters the functional exactly once:

$\Gamma\_response = (C\_hol / 2) \Sigma\_p |p| |p\bigstar| \langle \mathscr{R}\_p , W_0 \mathscr{R}\_p \rangle$.

Theorem 15 — Benchmark Census Values

$Z_3 = 6$ C_hol , $Z_2 = (13/2)$ C_hol , Z_Y = (21/2) C_hol , Z_q = 378 C_hol .

Proof

Theorem 14 reduces $\mathcal{K}\_g^{(0)}$ to H_FF. The factorised metric reduces each factor trace to the displayed representation index and charge sums. The extraction rule of Theorem 5 returns the coefficients. ■

34.1 The absolute-normalisation boundary

This is where the calculation reaches a hard boundary.

The stationary distribution fixes the *relative* response metric. It does not, by itself, prove that the conversion

dimensionless stationary norm ⟶ dimensionless Euclidean action

carries coefficient exactly one. The complete answer is therefore

Z_A = C_hol S_A ,

not merely Z_A = S_A, unless HRF-1 is elevated to the constitutive equality

C_hol = 1 .

The present corpus does not independently derive C_hol. Other VERSF sectors similarly retain an open closure-normalisation coefficient when converting substrate structures into physical action coefficients, which suggests one shared upstream principle rather than three sector-local conventions.

The status is exact:

- relative coupling normalisation: numerically closed on this branch;
- absolute coupling normalisation: conditional on C_hol = 1;
- upstream derivation of C_hol: open, but now localised (Section 34.2).

This is one universal scalar debt, rather than three independent coupling inputs.

34.2 Conditional holonomy–Wilson action-normalisation bridge

The Commitment Barrier and Universal Measure results supply a named route to the outstanding scalar. If the holonomy-curvature norm is proved to be the gauge-sector realisation of the same primitive Wilson/closure quadratic cost — the holonomy–Wilson equivalence — then C_hol equals the sector-commensurability ratio:

holonomy–Wilson equivalence ⟹ C_hol = r ,

and if the universal-measure principle fixes one action unit across commensurable sectors under the physical-entropy premise, then r = 1:

physical-entropy + universal-measure premises ⟹ C_hol = 1 .

Neither implication is yet proved for the gauge sector; both are substrate derivation tasks, not conventions. The bridge does not close C_hol — it reduces the debt from a free-standing constant to one named bridge theorem, whose failure would leave C_hol conventional and the absolute couplings underived (Falsifier F24).

---

35. Unit-Action Returns, Electroweak Conversion and Scale-Free Ratios

Taking HRF-1 literally as an equality in the dimensionless Euclidean action sets C_hol = 1. The returned values are then:

Quantity	GRCE-1C(0) return
Z_3	6
Z_2	13/2 = 6.5
Z_Y	21/2 = 10.5
Z_q	378
g_s	0.4082483
g	0.3922323
g_q	0.05143445
g′ = 6 g_q	0.3086067

The electroweak conversions at the matching surface are

$1/e^2 = Z_2 + Z\_Y = 17$, $e = 1/\sqrt{17} = 0.2425356$,

$\alpha = 1/(68\pi) = 0.00468103$, $\alpha^{-1} = 213.6283$,

and

$\sin^2\theta_W = Z_2 / (Z_2 + Z_Y) = 13/34 = 0.3823529$.

These are boundary values at $\mu\bigstar$, in the native scheme of Section 37, before threshold matching and running. They are not the low-energy observables and may not be compared with measurement until Stages 14–17 of the evaluation algorithm have been executed.

The scale-independent coupling ratios are

$g_s / g = \sqrt{13/12} = 1.040833$, $g' / g = \sqrt{13/21} = 0.786796$.

These ratios do not depend on C_{hol}. They are the sharpest fully unconditional numerical outputs of the benchmark.

---

36. Shared-Link Tree-Level Current Matching

Use the same edge link in the matter action:

$$S_{matter} = \tfrac{1}{2} \Sigma_{\{e: x\to y\}} (|e\bigstar|/|e|) \, \| \Psi_y - (\mathbb{I}_7 \otimes e^{i a_e^A \rho_A(T_A)}) \Psi_x \|^2 .$$

Differentiating with respect to a_e^A at $a = 0$ produces exactly the canonical representation current:

$$\partial S_{matter}/\partial a_e^A |_0 \propto \Psi^\dagger \rho_A(T_A) \Psi .$$

The link derivative and the plaquette derivative contain the same generator, with the same normalisation. Therefore the benchmark current-matching coefficients are

$(\nu_3, \nu_2, \nu_q) = (1.000000, 1.000000, 1.000000)$,

exactly. For primitive hypercharge, the smallest nonzero current unit is $q = 1$, so no continuous Abelian current rescaling remains.

This closes current matching at the shared-link, canonically normalised tree level — the value predicted structurally in Section 23.1, now returned numerically. It is a link-level result and must not be quoted as full current matching. Full matching additionally requires: canonical matter wavefunction normalisation; the record-to-Fock embedding; the absence of orthogonal current components; and quantum vertex renormalisation. None of these is discharged here.

In particular, it does not prove that the independently constructed microscopic persistent record current coarse-grains to exactly the same Fock current. The Fock reconstruction paper itself lists failure of

$q : \bar{\psi} \gamma^\mu \psi : \longrightarrow J_{rec}^\mu$

as a live falsifier. The correct status is therefore:

$\nu_A^{(\text{shared link})} = 1$, $\nu_A^{(\text{record}\to\text{Fock})}$ remains to be independently verified.

---

37. Native Scheme, One-Loop Transport and the Threshold Ledger

37.1 The VBF scheme

A complete direct bridge to $\overline{\text{MS}}$ cannot be calculated without the regulator-specific finite part of the VERSF two-point functional. A legitimate intermediate closure is available.

Define the native perturbative coupling by the background-field two-point coefficient at the matching surface:

$$\mathbf{1/g_{A,VBF}^2(\mu\bigstar) = Z_A^V(\mu\bigstar)} ,$$

so that, with this as the renormalisation condition,

$$\mathbf{\Delta_A^{V\to VBF}(\mu\bigstar) = 0}$$

by construction. This is not an assertion that the VERSF coefficient is automatically $\overline{\text{MS}}$. It defines a gauge-consistent background-field momentum-subtraction scheme in which the finite matching at the boundary is zero by definition.

37.2 One-loop transport

QGC-1 supplies

$$(b_Y, b_2, b_3) = (41/6 , -19/6 , -7)$$

and explicitly states that these transport boundary values rather than generating them. In the VBF scheme, between thresholds,

$$1/g_A^2(\mu) = Z_A(\mu\bigstar) - (b_A/8\pi^2) \ln(\mu/\mu\bigstar) .$$

For the unit-action benchmark, running from $\mu\bigstar$ to $2\mu\bigstar$ with no intervening threshold:

Coupling	At μ★	At 2μ★
g_s	0.408248	0.406174
g	0.392232	0.391396
g'	0.308607	0.309492

The finite bridge $\Delta_A^{VBF\to\overline{MS}}$ remains open. It requires the explicit regulator kernel and one-loop finite determinant, not merely the beta coefficients.

Thus: native background-field bridge closed by definition; one-loop running available and demonstrated; finite $\overline{MS}$ bridge open.

37.3 Substrate auxiliary thresholds

On the factorised branch, the canonical auxiliary Hessian is $C_0 = \mathbb{I} - \hat{T}$, with the six positive eigenvalues

$$\delta_r = \{ 1/2 , 1/2 , 1 , 1 , 31/28 , 5/4 \} .$$

Identifying these local wheel-Hessian eigenvalues with the spectral distances of physical auxiliary modes from the retained transport band is not automatic. It is declared as a second benchmark-scoped premise:

Premise AGI-0 — Auxiliary-Gap Identification (benchmark-scoped)

On GRCE-1C(0), the nonzero eigenvalues of $\mathbb{I} - \hat{T}$ are provisionally identified with the relevant dimensionless auxiliary spectral distances δ_r entering the Combes–Thomas relation $\xi_r \delta_r = 2v_c$.

Under AGI-0, with the unsaturated relation $M_r = \delta_r/(2 v_c \ell_{ref})$, the lowest substrate matching surface on this branch was provisionally

$$\mu\star^{(unsaturated)} = 1/(4 v_c \ell_{ref}) ,$$

where ℓ_{ref} is the substrate refinement length converting dimensionless substrate quantities into a physical scale. This expression is superseded by the band-edge, saturation-corrected rule of Section 25, $\mu\star = \ell_{ref}^{-1} \min[1, (1 - 4v_c^\infty)/(4v_c^\infty)]$; it is retained here only as the benchmark's original provisional return. The threshold ratios are independent of both unknowns:

$M_r / \mu\star = \{ 1 , 1 , 2 , 2 , 31/14 , 5/2 \} = \{ 1, 1, 2, 2, 2.214286, 2.5 \}$.

These are the raw local wheel-Hessian gaps. The global transport analysis of Section 25 supersedes their direct threshold reading: band broadening replaces δ_r by the edge distances $1 - \lambda_r - 2v_c^\infty$, and Combes–Thomas saturation caps every M_r at the refinement cutoff, so for $v_c^\infty < 1/4$ all wheel auxiliaries except (at most) the 1/2-pair sit exactly at ℓ_{ref}^{-1}, and throughout the band-separated regime $v_c^\infty < 3/56$ the entire ledger collapses to the cutoff. The raw ratio list {1, 1, 2, 2, 31/14, 5/2} is retained as the wheel-gap ledger only; conversion to GeV requires ℓ_{ref} and, in the extended regime, v_c^∞.

37.4 One-loop field-content census check

The direct embeddings reproduce the QGC beta census. For three fermion generations (Weyl sums):

$\Sigma T_3 = 6$, $\Sigma T_2 = 6$, $\Sigma Y^2 = 10$,

and one Higgs doublet gives $T_2(\Phi) = 1/2$, $\Sigma_\Phi Y^2 = 1/2$. Therefore

$b_3 = -(11/3)(3) + (2/3)(6) = -7$,

$b_2 = -(11/3)(2) + (2/3)(6) + (1/3)(1/2) = -19/6$,

$b_Y = (2/3)(10) + (1/3)(1/2) = 41/6$.

The unbroken-phase contribution ledger is:

Sector	Δb_Y	Δb_2	Δb_3
One fermion generation	20/9	4/3	4/3
Three generations	20/3	4	4
One Higgs doublet	1/6	1/6	0
Gauge and ghosts	0	−22/3	−11
Total	**41/6**	**−19/6**	**−7**

This exactly reproduces the QGC-1 result from the same representation embedding. It is a strong consistency check on the embedding and generator normalisation: the objects that produced the boundary traces also produce the correct quantum transport census.

37.5 Provisional charged-particle thresholds

The MCHY conditional audit branch supplies the following unmatched tree-level masses:

Carrier	Provisional threshold
e	0.521691 MeV
u	2.16702 MeV
d	4.69522 MeV
s	93.9044 MeV
μ	108.056 MeV

Carrier	Provisional threshold
c	1.11241 GeV
τ	1.86512 GeV
b	4.86254 GeV
t	291.827 GeV

These values are explicitly identified in their source paper as unmatched matching-point outputs, not pole masses or common-scale running masses. They can seed a provisional threshold ledger, but they cannot yet be used for precision running.

Still missing are: the Higgs pole threshold; the W and Z thresholds; the Goldstone and broken-phase matching; heavy neutral thresholds; possible substrate completion modes; and conversion of the charged values to one declared scheme. The physical threshold ledger is therefore not complete.

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38. The Factorisation Obstruction: What the Benchmark Teaches

The calculation produces an important negative constraint.

Because the gauge embedding factorises as $\mathbb{I}_7 \otimes \rho_A$, the wheel dynamics cancel out of the factor ratios: $\Sigma_\kappa \pi_\kappa = 1$. Because the curvature image occupies the coherent wheel line, the auxiliary Schur correction vanishes: $b_A = 0$. The entire coupling pattern then comes only from the representation census:

$Z_3 : Z_2 : Z_Y = 6 : 13/2 : 21/2$.

Therefore:

The factorised coherent-wheel embedding cannot make the interaction strengths sensitive to the nontrivial wheel spectrum or to factor-dependent closure relaxation.

A successful physical calculation must return at least one of:

1. a nonfactorising gauge-to-record embedding;
2. nonzero factor-dependent curvature–auxiliary vectors b_A;
3. different physical auxiliary Hessians C_A per factor;
4. a nontrivial common response scale — a substrate derivation of C_{hol} from the absolute action normalisation;
5. a genuine fourteen-to-twelve substrate map whose pullback differs from the matter-census trace.

This gives the missing microscopic structures sharply defined jobs.

Two properties make the benchmark more than a placeholder. First, it is a *class* floor, and only that: by Theorem 4, any branch that retains the direct embedding, the direct metric and the value of C_{hol}, and only enlarges the auxiliary response, satisfies $Z_A^{phys} \leq Z_A^{benchmark}$, hence $g_A^{phys} \geq g_A^{benchmark}$. The bound does not extend beyond that class: a nonfactorising embedding changes the direct weights w_A themselves and can move the coefficients in either direction, so the benchmark is not a global floor for all VERSF realisations. Second, it is exact and unfitted: nothing in it was chosen, so any physical branch must either reproduce its census skeleton or exhibit, explicitly, the substrate structure that deforms it.

The coupling calculation now has its analogue of the Yukawa programme's blind baseline — and the benchmark identifies precisely why the simplest microscopic realisation is insufficient.

Part VI — The Wheel-Profile Branch: GRCE-1(1)

39. Aim, Position and the Category Firewall

The executed benchmark GRCE-1C(0) proved a negative theorem: on the factorised carrier, the wheel dynamics cancel from the coupling pattern and the auxiliary relaxation vanishes identically. The factorisation obstruction of Section 38 therefore assigns the physical branch a specific job: make the response wheel-sensitive.

This branch performs the minimal calculation with that property, and performs it completely.

One firewall is binding from the outset. The fourteen closure directions and the twelve Standard Model gauge directions belong to different explanatory layers; the gauge-census result forbids identifying them directly, and the fourteen-to-twelve descent of Section 20 requires an independently derived equivariant map, not an assignment. Nothing in this branch identifies closure channels with gauge generators. The wheel profile χ_A is a response weighting on the record carrier — a statement about *where on the wheel* factor A's curvature is registered — not an embedding of gauge directions into closure channels. The genuine map $\mathfrak{g}_{SM} \to \mathscr{H}_{14}$ remains a separate, open derivation.

A second point of honesty concerns the word "nonfactorising." The profile embedding $\chi_A \otimes \rho_A$ is still a separable operator; what it removes is the *trivial* wheel action $\mathbb{1} \otimes \rho_A$. It is the complete equivariant extension inside the separable class under one declared homogeneity assumption (Premise WPE-1 below). Block-resolved profiles — one wheel profile per irreducible matter block — and genuinely nonseparable embeddings (sums of products) are strictly larger classes, noted where they matter.

Everything below is exact arithmetic on frozen corpus objects. No measured coupling enters at any stage.

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40. The Wheel-Profile Embedding

Premise WPE-1 — Wheel-Profile Link Embedding (branch-scoped)

WPE-1 defines the **link-profile branch**: χ_A enters the actual shared-link gauge embedding, so it acts on the matter transport and is subject to the vertex audit. (The susceptibility-profile reading, in which a wheel density selects where curvature energy is *measured* without entering the link, is a distinct branch declared in Section 49.) On the link-profile branch, the gauge-to-record embedding is

$$\mathscr{E}_A(X) = \chi_A \otimes \rho_A(X) \ , \ A \in \{3, 2, q\} \ ,$$

with χ_A a real vector on the wheel carrier $L^2(\pi)$, acting multiplicatively, and ρ_A the frozen census representations on $\mathscr{H}_{SM}$. One profile is assigned per gauge factor across all matter blocks (carrier homogeneity). The auxiliary carrier remains the incoherent wheel class of the benchmark: every auxiliary response is π-orthogonal to the coherent line.

Falsifier for WPE-1. The substrate returns block-dependent profiles, coherent auxiliary components, or a nonseparable embedding at the same derivative order.

Theorem 16 — One Profile per Simple Factor

Gauge equivariance,

$$(\mathbb{I} \otimes \rho_A(g)) \, \mathscr{E}_A(X) \, (\mathbb{I} \otimes \rho_A(g))^{-1} = \mathscr{E}_A(\mathrm{Ad}_g X) \ ,$$

forces the profile to be constant over each adjoint orbit. For the simple factors the adjoint representation is irreducible, so a single χ_A serves every generator of that factor; for the Abelian factor there is one generator. Equivariance does not relate χ_3, χ_2 and χ_q to one another.

Proof

A generator-dependent profile $\chi_A(X) \otimes \rho_A(X)$ already fails linearity of $\mathscr{E}_A$ in X unless χ_A is constant on linear spans, so linearity alone forces constancy on lines; equivariance then extends it to adjoint orbits. Writing $\mathscr{E}_A(X) = \chi_A(X) \otimes \rho_A(X)$, the equivariance condition gives $\chi_A(\mathrm{Ad}_g X) = \chi_A(X)$ for all g. Irreducibility of the adjoint action makes every generator reachable from any other, so χ_A is constant on the factor. No condition connects distinct factors. ■

Expand each profile in the coherent direction and the π-orthonormal wheel modes of Section 41:

$\chi_A = c_{A0}\, \mathbb{1} + \Sigma_{r=1}^{6} c_{Ar}\, \hat{f}_r$, $\|\chi_A\|^2_\pi = c_{A0}^2 + \Sigma_r c_{Ar}^2$.

The benchmark is $c_{A0} = 1$, $c_{Ar} = 0$.

---

41. The Exact Wheel-Mode Ledger

The wheel transition matrix $\hat{T}$ is reversible with respect to π, so $C_0 = \mathbb{I} - \hat{T}$ is self-adjoint in $\langle\cdot,\cdot\rangle_\pi$ and its eigenvectors are π-orthogonal. On the ordered basis $(h, b_1, \ldots, b_6)$, the six incoherent modes are exact:

Mode	Components (hub; boundary j = 1…6)	D_6 type	Gap λ_r	π-norm²	Screening $\lambda_r/(1+\lambda_r)$
f_1	(0 ; $\cos(\pi j/3)$)	E_1	1/2	12/31	1/3
f_2	(0 ; $\sin(\pi j/3)$)	E_1	1/2	12/31	1/3
f_3	(0 ; $\cos(2\pi j/3)$)	E_2	1	12/31	1/2
f_4	(0 ; $\sin(2\pi j/3)$)	E_2	1	12/31	1/2
f_5	(0 ; $(-1)^j$)	B	5/4	24/31	5/9
f_6	(−24/7 ; 1, 1, 1, 1, 1, 1)	A (trivial)	31/28	24/7	31/59

Here D_6 is the wheel automorphism group (rotations of the boundary cycle and reflections, fixing the hub); A is the trivial representation, B the rotation-sign representation, E_1 and E_2 the two-dimensional representations. The gaps λ_r are the eigenvalues of C_0 — the auxiliary spectrum of Section 33 {1/2, 1/2, 1, 1, 31/28, 5/4} — now resolved by symmetry type. π-orthonormalised modes $\hat{f}_r = f_r / \|f_r\|_\pi$ are used throughout.

Every entry in this table is an exact fraction, verified by direct diagonalisation: eigenvector residuals, π-orthogonality to $\mathbb{1}$, and mutual π-orthonormality all vanish identically.

The two hub-sensitive facts worth noting: only f_6 carries hub weight, so hub-versus-boundary contrast is the unique D_6-invariant deformation; and f_5 is the unique mode odd under single-step rotation, the wheel's alternation channel.

---

42. Symmetry Classification of Admissible Profiles

The role projectors the corpus must eventually supply — colour-support, weak-role and primitive-phase — act on the wheel by selecting which states participate in each factor's response. Whatever they are, they determine a residual symmetry subgroup $H_A \subseteq D_6$, and the profile must lie in the H_A-invariant subspace. The classification is exact:

Residual symmetry H_A	Invariant profile space	Dimension
D_6 (full wheel symmetry)	$\mathrm{span}\{ \mathbb{1}, \hat{f}_6 \}$	2
$\mathbb{Z}_6$ (rotations only)	$\mathrm{span}\{ \mathbb{1}, \hat{f}_6 \}$	2
$\mathbb{Z}_3$ (rotation by two steps)	$\mathrm{span}\{ \mathbb{1}, \hat{f}_6, \hat{f}_5 \}$	3
$\mathbb{Z}_2$ (rotation by three steps)	$\mathrm{span}\{ \mathbb{1}, \hat{f}_6, \hat{f}_3, \hat{f}_4 \}$	4
trivial	all seven directions	7

This table is the exact interface between the missing role projectors and the response calculation: supplying the projectors selects a row; the surviving coefficients are the branch's only remaining freedom. A colour assignment that tiles the six boundary states with period two lands in the $\mathbb{Z}_3$ row and opens the alternation mode $\hat{f}_5$; a period-three tiling lands in the $\mathbb{Z}_2$ row and opens E_2; a fully symmetric role keeps only the hub-contrast mode $\hat{f}_6$.

---

43. The Profile Response Formula

Theorem 17 — Profile Response

Under WPE-1 and the evaluation theorem, for each factor

$$\mathbf{Z_A = C_{hol}\, I_A \,\|\chi_A\|^2_\pi - b_A^\dagger C_A^{-1} b_A \equiv C_{hol}\, I_A\, N_A(C_{hol})\ ,\quad N_A(C_{hol}) = \|\chi_A\|^2_\pi - b_A^\dagger C_A^{-1} b_A / (C_{hol}\, I_A)\ ,}$$

in the action-scaled convention; fully unscaled, $Z_A = C_{hol} I_A \|\chi_A\|^2_\pi - C_{hol}^2 \bar{b}_A^\dagger (H_A^{sub} + C_{hol} \bar{C}_A)^{-1} \bar{b}_A$. Here $I_3 = 6$, $I_2 = 13/2$, $I_q = 378$ are the unchanged representation indices. The explicit prefactor cancels from all coupling ratios, but the screening term generally carries implicit C_{hol} dependence, since H_A^{sub} does not scale with it: $N_A = N_A(C_{hol})$ on the general profiled link branch. The dependence vanishes exactly where the mixed block does — on the benchmark ($H_{F\eta} = 0$) and on the support-harmonic branch (the retained harmonics are orthogonal to the eliminated wheel pair) — and only there are the ratios unconditionally C_{hol}-free. The remaining objects are

$$w_A = C_{hol}\, I_A \,\|\chi_A\|^2_\pi\ ,\quad b_{A,r} \propto c_{Ar}$$

the direct weight and mixed vector. The benchmark is the point $\chi_A = \mathbb{1}$: $N_A = 1$, $Z_A = C_{hol} I_A$.

Proof

The metric factorises as $\mathrm{diag}(\pi) \otimes \mathbb{I}_{47}$, so the matter trace returns I_A exactly as in the benchmark, while the wheel factor now returns $\|\chi_A\|^2_\pi$ in place of $\Sigma\pi_\kappa = 1$. The curvature image $\chi_A \otimes \rho_A(F)$ has incoherent components $c_{Ar}\, \hat{f}_r \otimes \rho_A(F)$, which overlap the incoherent auxiliary class; each mixed entry is proportional to c_{Ar} with a coefficient supplied by the auxiliary derivatives of the frozen edge transports. Schur reduction gives the displayed normal form. ■

The proportionality constants in b_A — and the physical auxiliary Hessians C_A — are exactly the open object $\partial U_{e,\bigstar}/\partial\eta$ of the corpus ledger, now localised: they are needed only on the incoherent components that the profile switches on.

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44. The Coherent Floor

Theorem 18 — Coherent Floor and Two-Sided Bound

On any branch whose auxiliaries are wheel-incoherent (WPE-1), and for any positive auxiliary dynamics whatever,

$$c_{A0}^2 \le N_A \le \|\chi_A\|^2_\pi = c_{A0}^2 + \Sigma_r\, c_{Ar}^2 \,.$$

The upper bound is attained when relaxation vanishes; the lower bound is the unremovable coherent weight.

Proof

N_A is the infimum, over auxiliary displacements, of a sum of the record norm of the total residual and a nonnegative substrate stiffness. Every auxiliary direction is π-orthogonal to the coherent line, so the coherent component $c_{A0}\, \mathbb{1} \otimes \rho_A(F)$ of the curvature image is invariant under relaxation; the record norm of the total is therefore at least c_{A0}^2 times the normalised coherent weight, giving the floor. Setting the displacement to zero gives the ceiling. ■

The physical content: **screening acts only on the incoherent content of a profile.** The wheel can never relax away a factor's coherent grip. This is the profile branch's exact generalisation of the benchmark's vanishing-Schur theorem, which is the special case of zero incoherent content.

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45. Audit-Locked Normalisation and the Direction of Correction

An immediate danger of the profile branch is scale degeneracy: rescaling $\chi_A \to s_A \chi_A$ rescales $Z_A \to s_A^2 Z_A$ independently per factor, so unnormalised profiles could reproduce any triple — reintroducing exactly the freedom the non-identifiability theorems spent. The shared-link universality audit closes this, conditionally.

Theorem 19 — Audit-Locked Normalisation

Suppose the matter amplitudes occupy the coherent wheel line (as on the benchmark carrier). The shared-link vertex generator is then the coherent projection of the profiled embedding,

$$\langle \mathbb{1} , \chi_A \rangle_\pi \, \rho_A(T_A) = c_{A0} \, \rho_A(T_A) ,$$

so the current-matching coefficient is $\nu_A^{(\text{shared-link})} = c_{A0}$. Coupling universality, $\nu_A = 1$, therefore forces

$$\mathbf{c_{A0} = 1}$$

for every factor. On the audit-locked branch,

$$\mathbf{N_A = 1 + \Sigma_r \, c_{Ar}^2 \, (1 - \mu_r) \geq 1 , \quad 0 \leq \mu_r \leq 1 ,}$$

with μ_r the relaxation fraction of mode r, and hence

$$\mathbf{Z_A \geq Z_A^{benchmark} , \quad g_A \leq g_A^{benchmark} ,}$$

with equality if and only if $\chi_A = \mathbb{1}$.

Proof

The vertex derivative of the shared-link matter action inserts the embedding evaluated on the matter state; for coherent matter this is the π-projection of χ_A onto $\mathbb{1}$, which in the orthonormal expansion is c_{A0}. Universality fixes it to one. Theorem 18 with $c_{A0} = 1$ gives $N_A \in [1, 1 + \Sigma \, c_{Ar}^2]$. ■

Two consequences deserve emphasis.

First, the scale non-identifiability is spent: the audit converts three free normalisations into three exact constraints, leaving only the shape coefficients c_{Ar} free. The certificate of Section 48 is correspondingly sharper.

Second, the direction of physical correction is now two-sided across the programme. The monotonicity corollary of Theorem 4 says enlarging the auxiliary carrier at fixed embedding can only *lower* Z_A. This theorem says profiling the embedding on the audit-locked, coherent-matter branch can only *raise* Z_A. The benchmark sits at the junction of two oppositely directed deformation classes: within the fixed-embedding auxiliary-enlargement class it is the maximum stiffness and minimum coupling; within the audit-locked profile class it is the minimum stiffness and maximum coupling. Any physical branch must declare which side of the benchmark each factor lands on — and by how much — as an output, not a choice.

If the matter carrier itself acquires wheel profile, the vertex projection changes and the lock is modified; that possibility is part of WPE-1's falsifier and belongs to the genuinely nonseparable class.

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46. Exact Deformation Laws

The observables deform through the N_A alone — with the caveat of Section 43 that on the general profiled link branch $N_A = N_A(C_{hol})$, so the laws below are exact in the N_A and unconditionally C_{hol}-free only where the mixed block vanishes:

$$g_s / g = \sqrt{(13/12) \cdot (N_2/N_3)}\ ,\quad g' / g = \sqrt{(13/21) \cdot (N_2/N_q)}\ ,$$

$$1/e^2 = (C_{hol}/2) (13 N_2 + 21 N_q)\ ,\quad \alpha^{-1} = 2\pi C_{hol} (13 N_2 + 21 N_q)\ ,$$

$$\sin^2\theta_W = 13 N_2 / (13 N_2 + 21 N_q) = 13 / (13 + 21 N_q/N_2)\ .$$

The weak-mixing boundary value depends only on the single ratio N_q/N_2: hypercharge screened more stiffly than weak isospin ($N_q > N_2$) lowers $\sin^2\theta_W$ below 13/34; the reverse raises it. The strong sector decouples from the electroweak conversion entirely, entering only through the ratio g_s/g. These are boundary-value laws at $\mu\bigstar$ in the native scheme; comparison with observation remains gated behind Stages 14–17 of the evaluation algorithm (Section 55).

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47. Canonical-Coupling Display Case

The physical relaxation fractions μ_r require the auxiliary derivatives. One declared model closes them for display purposes only.

Premise CRM-0 — Canonical Relaxation Model (display-scoped)

In rescaled (per-C_{hol}) variables: each incoherent mode relaxes against an auxiliary whose substrate stiffness satisfies $H_{\eta\eta,r}^{sub}/C_{hol} = 1$ and whose metric block is $J_\eta^\dagger \mathbb{W} J_\eta = \lambda_r$, so

$H\_{\eta\eta,r}/C\_{hol} = 1 + \lambda_r$, with unit mixed coupling $\bar{b}_{A,r} = c_{Ar}$. CRM-0 is a display convention with its units declared, not a substrate derivation; nothing downstream depends on it.

Under CRM-0 the Schur reduction is exact and closed-form:

$$\mathbf{N_A = c_{A0}^2 + \Sigma_r\, c_{Ar}^2 \cdot \lambda_r/(1 + \lambda_r)\ ,}$$

verified against direct matrix Schur reduction to full precision. The retained fractions are the ledger's last column: {1/3, 1/3, 1/2, 1/2, 5/9, 31/59}. Softer wheel modes screen more; the hub-contrast mode retains 31/59 ≈ 0.5254 of its weight.

For the audit-locked, D_6-symmetric family $\chi_A = \mathbb{1} + s_A \hat{f}_6$ — the minimal case every role assignment contains — $N_A = 1 + (31/59)\, s_A^2$, and (numerical display at $C_{hol} = 1$):

Case (s_3, s_2, s_q)	N_3	N_2	N_q	g_s	g	g'	α^{-1}	$\sin^2\theta_W$
(0, 0, 0) benchmark	1	1	1	0.408248	0.392232	0.308607	213.628	0.382353
(0, 0, 1)	1	1	90/59	0.408248	0.392232	0.249868	282.956	0.288671
(0, 1, 0)	1	90/59	1	0.408248	0.317576	0.308607	256.546	0.485679
(1, 0, 0)	90/59	1	1	0.330544	0.392232	0.308607	213.628	0.382353
(1, 1, 1)	90/59	90/59	90/59	0.330544	0.317576	0.249868	325.874	0.382353

The table is arithmetic, not physics: the s_A are undetermined (Section 48) and CRM-0 is declared. Its purpose is structural — it exhibits, with exact fractions, how each observable moves under each factor's wheel participation: hypercharge participation alone lowers $\sin^2\theta_W$ and g' and raises α^{-1}; weak participation alone does the opposite to $\sin^2\theta_W$; equal participation of all three factors preserves every ratio and $\sin^2\theta_W = 13/34$ while lowering all three couplings together — a pure C_{hol}-like rescaling generated dynamically.

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48. The Factor-Profile Non-Identifiability Certificate

Theorem 20 — Factor-Profile Non-Identifiability

On branch GRCE-1(1), the frozen census, gauge covariance and the full wheel symmetry determine: the admissible profile spaces (Section 42), the response formula (Theorem 17), the coherent floor (Theorem 18) and the audit lock $c_{A0} = 1$ (Theorem 19). They do not determine the shape coefficients c_{Ar}. Explicitly: for any target values $N_A \in [1, \infty)$ there exist admissible audit-locked profiles and positive auxiliary dynamics realising them, so census and symmetry alone leave the triple (Z_3, Z_2, Z_q) undetermined up to the benchmark bound $Z_A \geq Z_A^{benchmark}$. The branch becomes predictive precisely when the corpus supplies:

1. **the role-projector map** — a canonical positive response operator carrying the colour-support, weak-role and primitive-phase projectors to the three wheel profiles, fixing the c_{Ar};
2. **the auxiliary derivatives** $\partial U_{e,\bigstar}/\partial\eta$ on the incoherent components, fixing b_A and C_A, hence the relaxation fractions μ_r.

Proof

The invariant subspaces of Section 42 are the complete constraint symmetry imposes; within them the coefficients transform trivially and no further invariant condition exists. Given any targets, choose c_{Ar} with $\Sigma c_{Ar}^2(1 - \mu_r) = N_A - 1$ for admissible $\mu_r \in [0, 1)$; positivity of the auxiliary block is preserved for small enough coupling. The colour-participation result already establishes the pattern for the colour factor: symmetry fixes equality within the triplet but not the colour-versus-noncolour weight; the same argument applies verbatim to the weak and primitive-phase profiles. ■

This is the gauge analogue of the Yukawa programme's Threefold Non-Identifiability Certificate, and it is sharper than the generic response-map certificate of Section 58: the audit lock has already spent the scale freedom, the coherent floor has already fixed the direction, and the residual freedom is finite-dimensional — at most six shape coefficients per factor, at most one in the fully symmetric row of Section 42.

Of the two outcomes the branch admitted at the outset — the projectors uniquely determine the profiles, or a non-identifiability theorem names the missing map — census and symmetry alone return the second. Sections 49–52 then supply the first forward candidate for the named map: under the declared premise stack MCR-1, SHR-1, AHP-1, ARP-1 and DRF-1, the sectors are fixed uniquely and the triple is evaluated. Theorem 20 is not thereby retired; it is exactly the statement of what those premises are needed *for*, and it becomes the measure of their content.

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49. The Support-Harmonic Selection

The executed numerical realisation of this construction is designated **GRCE-1C(1) — the Support-Harmonic Matter-Census Susceptibility Branch** — the census companion of GRCE-1C(0). It is not an instance of the link-profile branch, and it is not an execution of the unresolved fourteen-channel microscopic response map: its profiles are declared response susceptibilities, and its matter side inherits the census factorisation, under the following premises.

Premise MCR-1 — Census–Susceptibility Factorisation (branch-scoped)

On GRCE-1C(1), the response factorises as wheel susceptibility × Standard Model representation index: the matter trace returns the census indices $I_3 = 6$, $I_2 = 13/2$, $I_Y = 21/2$ of the frozen 47-dimensional carrier, inheriting the matter-census reading of Premise MCR-0. This is a benchmark-class factorisation, not the fourteen-channel map.

Falsifier for MCR-1. The derived microscopic response does not factorise as susceptibility × census index, or its matter side differs from the frozen representation trace.

Premise SHR-1 — Susceptibility-Profile Response (branch-scoped)

On the support-harmonic branch, each factor is assigned a density ϱ_A^resp on the wheel carrier — a susceptibility profile — selecting the wheel sector in which factor-A curvature energy is measured:

Z_A = C_hol I_A Tr(ϱ_A^resp D_cl) .

The susceptibility profile is **not** the link generator: matter transport continues to use the canonically normalised primitive gauge link of Section 23, so the shared-link tree-level audit ν_A^(shared-link) = 1 is untouched by construction. What SHR-1 leaves owing is a bridge theorem: a substrate derivation of why the response susceptibility may differ from the link profile while both are governed by one gauge connection (Section 53).

Falsifier for SHR-1. The substrate derivation forces the response carrier to coincide with the link profile — whose zero coherent component would then break the shared-link audit — or no consistent susceptibility–link bridge exists.

Premise AHP-1 — Arity–Harmonic Correspondence (branch-scoped)

A gauge sector supported by an irreducible n-fold commitment structure occupies the lowest nontrivial wheel harmonic of rotational order n. The corpus inputs invoking it: the confined threefold support of colour, and the two-branch support of weak-active matter, whose roles the projected weak-doublet construction distinguishes by $\tau_3 = \pm 1$. AHP-1 selects the colour and weak sectors only; it does not select the Abelian profile, which carries no support arity.

Falsifier for AHP-1. The derived microscopic role projectors select a different harmonic sector for colour or weak isospin, or a support arity without a matching wheel harmonic.

Premise ARP-1 — Abelian Relative-Phase Selection (branch-scoped)

The primitive Abelian susceptibility is the unique D_6-invariant profile orthogonal to the coherent phase. The two inputs are distinct from arity: D_6-invariance of the Abelian response, and physical nullity of the coherent common phase — the latter supplied by the Dirichlet reading.

Falsifier for ARP-1. The derived Abelian response is not D_6-invariant, or carries a coherent component. Note the dependence structure: ARP-1's nullity input is supplied by the Dirichlet reading, so a stationary-norm resolution of the wheel-metric fork removes that input and reopens the Abelian selection — ARP-1 partially falls with DRF-1.

Theorem 21 — Uniqueness of the Support-Harmonic Sectors

Under AHP-1 and ARP-1, the weak and Abelian profiles are unique up to phase, and the colour object is the unique isotypic projector onto the two-dimensional period-three sector — AHP-1 does **not** select a unique oriented colour vector, and general linear combinations within the sector remain admissible. On the boundary Fourier ladder of Section 41:

Colour (n = 3). The period-three sector is the two-dimensional conjugate pair k = 2, 4, with $\hat{T}$-eigenvalue 0. Normalised to unit π-norm, with $\omega = e^{2\pi i/3}$,

$$\chi_{3,\pm} = \sqrt{31/24}\,(\,0\,;\,1,\,\omega^{\pm 1},\,\omega^{\pm 2},\,1,\,\omega^{\pm 1},\,\omega^{\pm 2}\,)\,,\quad \hat{T}\,\chi_{3,\pm} = 0\,.$$

The canonical colour object is the isotypic projector onto the full period-three subspace,

$$P_3^{(E_2)} = |\chi_{3,+}\rangle\langle\chi_{3,+}|_\pi + |\chi_{3,-}\rangle\langle\chi_{3,-}|_\pi\,,$$

and the response density is its trace-normalised, conjugation-symmetric form

$$\varrho_3 = \tfrac{1}{2}\,P_3^{(E_2)}\,.$$

The trace normalisation and the orientation neutrality — conjugation symmetry of the response on the degenerate pair — are conventions of SHR-1, declared, not derived; because the two harmonics are degenerate under D_{cl}, they leave the stiffness s_3 unaffected.

This does not identify the two-dimensional harmonic space with the eight gluons; it is the wheel susceptibility profile shared by all eight colour generators, per Theorem 16. The gauge-algebra count and the closure-response count remain separate explanatory layers.

Weak isospin (n = 2). The unique nonconstant period-two profile is the alternating harmonic k = 3,

$$\chi_2 = \sqrt{31/24}\,(\,0\,;\,1,\,-1,\,1,\,-1,\,1,\,-1\,)\,,\quad \hat{T}\,\chi_2 = -(1/4)\,\chi_2\,,\quad \varrho_2 = |\chi_2\rangle\langle\chi_2|_\pi\,.$$

This is the alternating closure-competition mode selected elsewhere by spectral, variational and symmetry arguments — not an arbitrary sign pattern.

Primitive hypercharge. The coherent constant profile is the global phase-null direction and cannot carry physical Abelian stiffness. The unique D_6-invariant profile π-orthogonal to it is the hub–boundary contrast — the mode $\hat{f}_6$ of the ledger:

$$\chi_q = \sqrt{7/24}\,(\,-24/7\,;\,1,\,1,\,1,\,1,\,1,\,1\,)\,,\quad \langle\mathbb{1},\chi_q\rangle_\pi = 0\,,\quad \hat{T}\,\chi_q = -(3/28)\,\chi_q\,,\quad \varrho_q = |\chi_q\rangle\langle\chi_q|_\pi\,.$$

Primitive hypercharge is thereby the relative phase response between the hub commitment and the democratic boundary ledger — not the unobservable common phase.

Proof

Each selected sector is an isotypic component of the D_6 action resolved by the boundary Fourier index: the period-three space is exactly span{k = 2, 4}, the nonconstant period-two space is exactly span{k = 3}, and the D_6-invariant complement of $\mathbb{1}$ is exactly span{f_6} — each read off the classification of Section 42. The one-dimensional sectors give profiles unique up to phase; the two-dimensional colour sector gives a unique isotypic projector but no preferred oriented vector. Unit π-norms, eigenvalue relations and π-orthogonality to $\mathbb{1}$ and to one another are verified identically. ■

All three profiles have zero coherent component, $c_{A0} = 0$: the support-harmonic branch sits at the opposite extreme from the benchmark, whose profiles were purely coherent. Section 53 addresses what this does to the vertex audit.

Optional lift to the fourteen-channel carrier

On the oriented-doubling reading $\mathscr{H}_{14} \simeq \mathscr{H}_7 \otimes \mathbb{C}^2_{or}$, the orientation-neutral lift is $\tilde{\varrho}_A = \varrho_A \otimes (\mathbb{I}_{or}/2)$. This lifts each seven-state susceptibility profile equally over the two closure orientations; it does not distribute twelve gauge generators among fourteen closure directions, and the category firewall of Section 39 is untouched.

---

50. The Dirichlet Response and the Wheel-Metric Fork

Premise DRF-1 — Dirichlet Response (branch-scoped)

The energetic cost of a wheel-profile deformation is the reversible refinement Dirichlet form generated by the wheel:

$$D_{cl} = \mathbb{I} - \hat{T}\ , \quad D[f] = \langle f, D_{cl}\, f\rangle_\pi = \tfrac{1}{2} \Sigma_{\kappa,\kappa'}\ \pi_\kappa\ \hat{T}_{\kappa\kappa'}\ |f_\kappa - f_{\kappa'}|^2\ .$$

Equivalently, in the CPF-1 response-map form, $A_{cl} = (\mathbb{I} - \hat{T})^{1/2}$ and $W_{cl} = A_{cl}^\dagger A_{cl}$.

Falsifier for DRF-1. The substrate returns a different positive wheel-response operator at the same order, or a nonzero cost for the coherent mode.

The form is positive, annihilates the coherent constant mode, and measures exactly the mismatch destroyed by one refinement step — the phase-difference cost, which is the physically correct currency for gauge response, since gauge phases are relative and the common phase is null.

For a unit π-normalised eigenprofile of $\hat{T}$-eigenvalue λ_A,

$$\mathbf{s_A = Tr(\,\varrho_A\, D_{cl}\,) = 1 - \lambda_A\ ,}$$

so the ledger of Section 41 supplies the stiffnesses directly:

Factor	Sector	λ_A	s_A = 1 − λ_A
SU(3)_C	k = 2, 4	0	1
SU(2)_L	k = 3	−1/4	5/4
U(1)_q	radial $\hat{f}_6$	−3/28	31/28

The selected sectors are mutually π-orthogonal and, on this minimal symmetry-resolved branch, are *retained* gauge-response directions rather than integrated-out auxiliaries. The only eliminated wheel modes are the k = 1, 5 pair, which are orthogonal to all three profiles; hence

H_Fη = 0

within this restricted branch, exactly. Additional microscopic bath or completion modes could change that result, and remain the open relaxation debt.

The wheel-metric fork

DRF-1 is not a refinement of the benchmark's wheel metric; it is an alternative to it. The benchmark GRCE-1C(0) weighted the wheel factor with the stationary norm, under which the coherent profile has unit weight and N = 1. Under DRF-1 the coherent profile has *zero* cost — the benchmark's embedding would return Z_A = 0 and infinite couplings, which is precisely why the branch forces the profiles off the coherent line. The two readings are mutually exclusive constitutive choices for the wheel factor of the response metric:

- **stationary-norm reading** — coherent transport is costly; the benchmark is well-defined; wheel participation is a deformation (Sections 43–47);
- **Dirichlet reading** — only phase *differences* cost; the coherent mode is null; wheel participation is mandatory (this section).

This fork joins the constitutive fork of Section 63 as a sub-tier of its Tier 1 (shape): HRF-1's stationary-record norm on the full carrier versus the Dirichlet-projected norm on the wheel factor. Deciding it is a substrate derivation task, not a preference. One output is invariant across the two displayed readings: the colour sector has s_3 = 1 under DRF-1 and N_3 = 1 on the benchmark, so **Z_3 = 6 C_hol and g_s = 1/√6 are identical on both**. This is a two-branch invariance, not full-theory robustness: a genuine fourteen-channel embedding or colour-specific relaxation could shift it.

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51. The Support-Harmonic Triple

Theorem 22 — Support-Harmonic Response Coefficients (Branch GRCE-1C(1))

Under MCR-1, SHR-1, AHP-1, ARP-1 and DRF-1, with the census indices of MCR-1, $I_3 = 6$, $I_2 = 13/2$, $I_Y = 21/2$ and primitive charge $q = 6Y$,

$$Z_A = C_{hol}\, I_A\, \mathrm{Tr}(\varrho_A^{resp} D_{cl}) = C_{hol}\, I_A\, s_A :$$

$$Z_3 = 6\, C_{hol},\ Z_2 = (13/2)(5/4)\, C_{hol} = (65/8)\, C_{hol},\ Z_Y = (21/2)(31/28)\, C_{hol} = (93/8)\, C_{hol},\ Z_q = 36\, Z_Y = (837/2)\, C_{hol}.$$

At the literal unit-action reading $C_{hol} = 1$:

$$g_s = 1/\sqrt{6} = 0.408248,\ g = \sqrt{(8/65)} = 0.350823,\ g' = \sqrt{(8/93)} = 0.293294,$$

$$1/e^2 = Z_2 + Z_Y = 79/4,\ e = 2/\sqrt{79} = 0.225018,\ \alpha^{-1} = 79\pi = 248.186,$$

$$\sin^2\theta_W = Z_2/(Z_2 + Z_Y) = 65/158 = 0.411392,$$

with the C_{hol}-independent ratios

$$g_s/g = \sqrt{(65/48)} = 1.163687,\ g'/g = \sqrt{(65/93)} = 0.836017.$$

Proof

Premise MCR-1 factorises each response into the frozen matter-census index I_A and a wheel susceptibility. SHR-1 identifies the susceptibility with $\mathrm{Tr}(\varrho_A^{resp} D_{cl})$; AHP-1 and ARP-1 select the three sectors uniquely (Theorem 21); DRF-1 evaluates each stiffness as $s_A = 1 - \lambda_A$ on the unit-normalised densities. Multiplication gives $Z_A = C_{hol}\, I_A\, s_A$, and the electroweak conversion chain of Section 29 does the rest. ■

Consistency with the deformation laws

The support-harmonic result sits exactly inside Section 46's deformation framework with $N_A = s_A$: $\sin^2\theta_W = 13\, s_2 / (13\, s_2 + 21\, s_q) = (65/4)/(158/4) = 65/158$ and, at $C_{hol} = 1$, $\alpha^{-1} = 2\pi\,(13\, s_2 + 21\, s_q) = 2\pi \cdot 79/2 = 79\pi$, identically. The direction is immediate: $s_A = 1 - \lambda_A \geq 1$ because every selected harmonic has $\lambda_A \leq 0$, so the derived point lies on the weaker-coupling side of the benchmark for the electroweak factors and exactly on it for colour.

This is a genuine forward return. No observed coupling entered the selection of the harmonics, the choice of the stiffness operator, or the arithmetic — under the declared premise stack, every number above is a ratio of small integers fixed by the wheel spectrum and the frozen census.

These remain boundary values at $\mu^\star$ in the native scheme; comparison with observation stays gated behind Stages 14–17 of the evaluation algorithm (Section 55).

52. Planck Anchoring of the Substrate Unit and the Matching Scale

The open-debt ledger carried the dimensionful debt $(v_c\, \ell_{ref})^{-1}$ as a single unknown. The certification corpus splits it and discharges half.

VERSF contains two distinct length scales: the Planck certification scale — the floor of metric admissibility — and the proposed mesoscopic closure-coherence scale of roughly 30–100 μm, which is a threshold for intrinsic closure stability, not the microscopic metric spacing of the refinement foam. A third possibility cannot yet be excluded: the gauge-response matching carrier could emerge after several refinement levels, above the metric floor but below the mesoscopic coherence scale. The identification with the certification floor is therefore a declared premise, not a forced inference.

Premise PSA-1 — Planck Substrate Anchoring (branch-scoped)

The elementary refinement length entering the gauge holonomy action is identified with the minimum metric-certification length, $\ell_{ref} = \sqrt{(\kappa/\eta)}\ \ell_P$.

Falsifier for PSA-1. The derived gauge-matching lattice spacing differs from the certification floor — for example, the matching carrier emerges at a coarser refinement level.

Without PSA-1, the certification theorem supplies a lower bound — a reference floor — for ℓ_{ref}, not necessarily the exact gauge matching spacing. Under PSA-1, the certification calculation $\Delta t_{sub} \geq \sqrt{(\kappa/\eta)}\ t_P$ gives

$$\boldsymbol{\ell_{sub} = \ell_{ref} = \sqrt{(\kappa/\eta)}\ \ell_P\ ,\quad E_{sub} = \hbar c/\ell_{sub} = \sqrt{(\eta/\kappa)}\ E_P\ ,}$$

with κ the quantum-speed-limit coefficient and η the no-trapping compactness coefficient. The Planck scaling is derived; the exact order-one prefactor depends on the certification convention and safety margin.

Sharp-boundary evaluation. With $\kappa = \pi/2$ (orthogonalisation) and $\eta = 1/2$ (the limiting no-trapping boundary $2GE/(c^4R) = 1$):

$$\boldsymbol{\ell_{sub}^{sharp} = \sqrt{\pi}\ \ell_P = 2.865 \times 10^{-35}\ \text{m}\ ,\quad t_{sub}^{sharp} = \sqrt{\pi}\ t_P = 9.556 \times 10^{-44}\ \text{s}\ ,}$$
$$\boldsymbol{E_{sub}^{sharp} = E_P/\sqrt{\pi} = 6.89 \times 10^{18}\ \text{GeV}\ .}$$

"Sharp" is the limiting boundary of outward certification; any nonzero safety margin lengthens ℓ_{sub} and lowers E_{sub}.

Matching surface. With the band-broadened, saturation-corrected rule of Section 25, the eliminated modes on this branch are the $k = 1, 5$ pair (wheel centre 1/2), and

$$\mu\bigstar = E_{sub} \cdot \min[\, 1 , (1 - 4v_c^\infty)/(4v_c^\infty) \,] , \quad 0 < v_c^\infty < 1/4 .$$

On the sharp PSA-1 branch:

- **local-cutoff regime, $v_c^\infty \le 1/8$:** $\mu\bigstar = E_{sub}^{sharp} = 6.89 \times 10^{18}$ GeV exactly — and throughout the fully band-separated regime $v_c^\infty < 3/56$ this holds automatically, so the exact value of v_c^∞ drops out of the matching surface;
- **extended regime, $1/8 < v_c^\infty < 1/4$:** $\mu\bigstar = E_{sub}^{sharp} (1 - 4v_c^\infty)/(4v_c^\infty)$, decreasing to zero as the first gap closes.

The critical-locality candidate $v_c^\infty = 1/8$ (Premise CLP-1, Section 61) sits exactly at the regime boundary and also returns $\mu\bigstar = E_{sub}^{sharp}$.

The anchoring fork now has numbers. The three scales — Planck certification, mesoscopic closure coherence, record-layer gauge-response refinement — are governed by different functionals and may not be identified automatically. On the critical branch $v_c^\infty = 1/8$: Planck-sharp anchoring (PSA-1) gives $\mu\bigstar \simeq 6.89 \times 10^{18}$ GeV; record-coherence anchoring at 5.8 TeV gives $\mu\bigstar \simeq 5.8$ TeV. Both follow from the same dimensionless calculation; the difference is entirely the unresolved identification of ℓ_{ref}. Producing a 5.8 TeV matching scale from a Planckian cutoff through the interband gap instead would require $v_c^\infty = 1/[4(1 + \varepsilon)]$ with $\varepsilon \simeq 8.42 \times 10^{-16}$ — within about 2.1×10^{-16} of 1/4 — an extreme critical tuning unless separately derived.

This sharpens Premise AGI-0 (Section 37.3): the threshold ledger on this branch contains only the $k = 1, 5$ pair (the retained harmonics are response directions, not thresholds); the substrate unit is anchored conditionally on PSA-1; and the two remaining unknowns are the identification of ℓ_{ref} and, in the extended regime only, v_c^∞ — which Proposition 27 (Section 61) proves the current axioms cannot fix.

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53. The Susceptibility–Link Bridge

The link-profile and susceptibility-profile constructions are now cleanly separated, and the separation resolves the apparent audit conflict — at the price of one new, sharply stated theorem obligation.

Link-profile branch (WPE-1). χ_A enters the shared-link embedding; on wheel-coherent matter the vertex generator is $c_{A0}\, \rho_A$, and universality locks $c_{A0} = 1$ (Theorem 19). The support-harmonic profiles, with $c_{A0} = 0$ exactly, are inadmissible as link profiles: as an actual shared-link embedding they would give a vanishing matter vertex.

Susceptibility-profile branch (SHR-1). ϱ_A^{resp} selects where curvature energy is measured; matter transport uses the canonically normalised primitive gauge link throughout, so $\nu_A^{(shared\text{-}link)} = 1$ holds by construction, not by audit. The support-harmonic response therefore coexists with exact tree-level universality.

The bridge obligation. What this coexistence costs is a theorem the corpus does not yet contain:

Susceptibility–link bridge. Derive, from the closure dynamics, why the response susceptibility ϱ_A^{resp} may differ from the link profile while both are governed by the same gauge connection — equivalently, why the sector in which the substrate registers curvature energy need not be the sector through which the gauge phase is transported.

Until that bridge is derived, the support-harmonic couplings are consistently normalised response coefficients whose relation to the link sector is declared (SHR-1) rather than proved. This is a more precise statement than "the vertex audit is unfinished": the tree-level audit is closed on this branch; what is open is the constitutive right to separate the two roles. Failure of the bridge is Falsifier F34. The scale freedom, as before, is fully spent: AHP-1 and DRF-1 fix the susceptibility profiles' shapes and norms outright, so what is unverified is the bridge's existence, not any residual continuous freedom.

Part VII — Blind Evaluation and Non-Identifiability

54. Frozen Input Manifest

Before measured couplings are accessible, freeze:

1. the substrate background $\rho^\star$ on the commitment foam $\mathfrak{S}_n$;
2. the gauge algebra and generator normalisation;
3. the primitive Abelian lattice $q = 6Y$;
4. the holonomy-normalised connection;
5. the functional $\Gamma_{GRCE}^{(n)}$, including the logarithm branch Log_0;
6. the canonical wheel matrix $\hat{T}$ and its stationary measure π;
7. the fourteen-dimensional embeddings $\mathscr{E}_3$, $\mathscr{E}_2$, $\mathscr{E}_q$ with every multiplicity-space direction resolved;
8. the frozen edge transports $U_{e,\star}$ and their auxiliary derivatives;
9. the substrate auxiliary Hessian $H_{\eta\eta}^{sub}$;
10. the dual-cell geometry $|p|$, $|p^\star|$;
11. the complete auxiliary carrier $\mathscr{H}_\eta$;
12. gauge and exact-null projectors;
13. the regulator or blocking sequence and refinement limit;

14. the auxiliary spectral distances δ_r, their certified identification with the physical mode gaps, and the conversion $(v_c \ell_{ref})^{-1}$, hence $\mu★$;
15. the record-to-Fock current map;
16. the derivative truncation;
17. the momentum-extraction prescription;
18. the scheme bridge;
19. the threshold ledger;
20. the solver and precision;
21. the positivity and isotropy thresholds;
22. the publication repository and hash process.

Every item receives a content hash before comparison.

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55. Evaluation Algorithm

Stage 0 — Data quarantine. Remove measured coupling values, weak angle, gauge-boson masses and unification plots from the computational environment.

Stage 1 — Background. Solve the frozen substrate equations for $\rho★$ on $\mathfrak{S}_n$.

Stage 2 — Matching surface. Evaluate the band-edge spectral distances δ_r (certifying their identification with the auxiliary spectrum, cf. Premise AGI-0), apply $M_r = \ell_{ref}^{-1} \min(1, \delta_r/2v_c^\infty)$, take $\mu★ = \min_{r\in\eta} M_r$, and freeze $\mu★$.

Stage 3 — Curvature embedding. Construct $\mathscr{E}_{SM}$ on the holonomy-normalised connection; resolve and certify every multiplicity direction.

Stage 4 — Auxiliary census. Construct $\mathscr{H}_\eta$, remove redundancies and classify every null direction.

Stage 5 — Full Hessian. Evaluate $H_{FF}(q) = C_{hol} J_F^\dagger \mathbb{W}_n J_F$, $H_{F\eta}(q) = C_{hol} J_F^\dagger \mathbb{W}_n J_\eta$, $H_{\eta\eta}(q) = H_{\eta\eta}^{sub} + C_{hol} J_\eta^\dagger \mathbb{W}_n J_\eta$.

Stage 6 — Locality test. Verify analyticity of $H_{\eta\eta}(q)^{-1}$ near $q = 0$.

Stage 7 — Schur reduction. Calculate $\mathcal{K}_g(q)$.

Stage 8 — Zero-momentum response. Extract $\mathcal{K}_g^{(0)}$ and the density $\mathcal{K}_g^{(0)}$.

Stage 9 — Factorwise audit. Verify product-factor orthogonality and isotropy; publish Δ_A^{iso}.

Stage 10 — Coefficient extraction. Calculate Z_3, Z_2, Z_q, and separately publish (w_A, $b_A^\dagger C_A^{-1} b_A$).

Stage 11 — Current audit. Evaluate ν_3, ν_2, ν_q against the shared-link prediction $\nu_A = 1$.

Stage 12 — Primitive couplings. Calculate g_s, g, g_q, g'.

Stage 13 — Freeze publication. Publish the boundary result before running.

Stage 14 — Scheme bridge. Calculate the finite conversion.

Stage 15 — Threshold matching. Apply the frozen threshold ledger.

Stage 16 — Quantum transport. Run using QGC-1.

Stage 17 — Broken phase. Calculate e, α, $\sin^2\theta_W$, m_W, m_Z using independently derived scalar data.

Stage 18 — Observation. Only now open experimental values.

Branch GRCE-1C(0) of Part V executes Stages 0–13 exactly on the factorised carrier under declared Premises MCR-0 and AGI-0, defines the native VBF scheme at Stage 14, demonstrates Stage 16 transport, and halts before Stage 17 for want of the absolute normalisation, the physical scale and the broken-phase ledger.

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56. Numerical Conditioning and Certification

Publish, for every factor and regulator:

$$\kappa_\eta = \lambda_{max}(H_{\eta\eta}) / \lambda_{min}(H_{\eta\eta}) ,$$

and the Schur sensitivity

$$\mathfrak{s}_A = \|H_{F\eta} H_{\eta\eta}^{-1} H_{\eta F}\| / \|H_{FF}\| .$$

A large $\mathfrak{s}_A$ means the physical coupling is sensitive to the auxiliary spectrum. Since $Z_A = w_A - b_A^\dagger C_A^{-1} b_A$, near-cancellation between the two published terms is itself a certified conditioning statement.

Publish the isotropy residual, factor-mixing norm and precision stability.

The final Z_A must carry: floating-point value; certified numerical interval; regulator dependence; truncation error; auxiliary-census convergence (monotone from above, per Corollary 17.2); matching-scale uncertainty; scheme uncertainty; threshold uncertainty.

A coefficient whose sign, rank or isotropy changes under precision refinement is unresolved.

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57. Matching-Scale Non-Identifiability

Suppose a downstream value $g_A(\mu_0)$ is fixed and the beta function is known. At one loop,

$$1/g_A^2(\mu\star) = 1/g_A^2(\mu_0) + (b_A/8\pi^2) \ln(\mu\star/\mu_0) - \Delta_A .$$

For every admissible $\mu\star$, this equation defines a corresponding boundary value.

Proposition 23 — Scale–Boundary Non-Identifiability

Without an independently derived matching-scale selector, the downstream coupling and known beta function determine only a one-parameter family $\{ \mu\star, g_A(\mu\star) \}$. They do not determine the substrate boundary value.

Proof

The displayed RG relation gives a distinct boundary value for every choice of $\mu\star$. All members run to the same downstream value. ■

Consequence

Selecting $\mu\star$ because the resulting Z_A resembles a desired substrate number is reverse engineering. The spectral-gap rule of Section 25 exists precisely to spend this freedom before output inspection.

---

58. Response-Map Non-Identifiability

Assume the desired factorwise physical response is any positive operator $\mathcal{Z}$. Choose no auxiliary response and set $W_{rec} = \mathbb{I}$, $J_F = \mathcal{Z}^{(1/2)}$. Then

$$J_F^\dagger W_{rec} J_F = \mathcal{Z} .$$

More generally, choose any positive decomposition $\mathcal{Z} = X^\dagger X - B^\dagger C^{-1} B$ for which the full block Hessian is positive.

Proposition 24 — Gauge-Response Non-Identifiability

If J_F, J_η and W_{rec} were left free, every positive factorwise coupling triple could be realised by infinitely many holonomy-response systems.

Proof

The square-root construction realises any positive response directly. Unitary transformations on the response carrier and arbitrary enlargement by orthogonal auxiliary sectors produce infinitely many distinct microscopic realisations of the same $\mathcal{Z}$. ■

Consequence

A numerical triple reconstructed from observed couplings proves only representability. It does not prove microscopic derivation.

This proposition is why the constructions of Part II are load-bearing. In Γ_{GRCE} the objects J_F, J_η and W_{rec} are not free: J_F is the frozen embedding $\mathscr{E}_{SM}$, J_η is the auxiliary derivative of the frozen edge transports, and W_{rec} is the Haar completion of the normalised wheel measure. The freedom that Proposition 24 exploits has been spent — conditional on those objects being supplied from the substrate and not tuned. The executed benchmark of Part V demonstrates the collapse concretely: once the embedding, metric and auxiliary class were frozen, the branch returned a unique triple up to the single scalar C_{hol}. Representability freedom, infinite-dimensional in the free case, contracted to one dimension — and that dimension is now a named debt rather than a hidden fit.

---

59. Current-Normalisation Non-Identifiability

Let the record-to-Fock matching remain free: $J_{rec} = \nu_A J_{Fock}$. Then a measured matter vertex determines only $g_A \nu_A$. For any nonzero c,

$g_A \to c\, g_A$, $\nu_A \to \nu_A/c$

leaves the matter interaction unchanged.

Proposition 25 — Current–Coupling Non-Identifiability

A matter-vertex measurement cannot determine g_A separately from the current normalisation unless ν_A is fixed independently.

Proof

Immediate from invariance of the product $g_A \nu_A$. ■

Non-Abelian rescue test

The gauge self-interaction depends on g_A but not on ν_A. Therefore agreement between the matter vertex and self-coupling tests the current matching. It cannot be assumed in advance.

The shared-link construction (Section 23.1) fixes $\nu_A = 1$ structurally at the level of the displayed action. The audit of Section 23.2 remains mandatory because the descent from the microscopic record current to the retained Fock current is a separate calculation.

---

60. Auxiliary-Census Incompleteness Certificate

Suppose a candidate calculation includes only $\mathscr{H}_{\eta,1}$, while the true carrier is

$$\mathscr{H}_{\eta,2} = \mathscr{H}_{\eta,1} \oplus \mathscr{H}_{miss} .$$

By Theorem 4, $Z_A^{true} \leq Z_A^{candidate}$.

Proposition 26 — Omitted-Mode Bias

A missing physical auxiliary mode can never be justified by claiming that its inclusion would only produce a small upward correction to Z_A. Its exact effect is non-positive:

$$\delta Z_A \leq 0 .$$

Proof

Auxiliary enlargement can only lower the minimised quadratic response. ■

Completeness certificate

A valid census must prove either:

1. all same-order modes are included; or
2. every omitted mode has vanishing mixed response with curvature — for the constructed functional, vanishing $\partial\mathscr{R}\_p/\partial\eta$ on that mode; or
3. its effect is rigorously bounded below the declared uncertainty.

---

61. Transport-Velocity Non-Identifiability

The continuum transport velocity is defined by

$$v\_c^\infty = \lim\_{n\to\infty} \gamma_n \rho(A_n) .$$

The refinement-holonomy theorem assumes this limit exists in $0 < v\_c^\infty < 1/2$; it does not select its value.

Proposition 27 — Transport-Velocity Non-Identifiability

Every value in the permitted interval defines a refinement-compatible transport sequence: the current axioms admit a continuous one-parameter family of v_c^∞.

Proof

Choose any fixed $v \in (0, 1/2)$ and set $\gamma_n^{(v)} = v / \rho(A_n)$. Then $\gamma_n^{(v)} \rho(A_n) = v$ at every refinement level, so $v\_c^\infty = v$. The refinement-contraction results determine contraction constants, admissibility gaps, Lipschitz response and continuum regularity, but none of them constrains the independent spatial coefficient γ_n. ■

A new microscopic equation fixing γ_n from the explicit refinement update is therefore unavoidable: any exact-looking value of v_c^∞ adopted without it would be a choice disguised as a calculation.

The critical-locality candidate

One clean conditional value can nevertheless be isolated.

Premise CLP-1 — Critical Locality (branch-scoped)

The least-stiff eliminated gauge-response mode lies exactly at the transition between one-cell Combes–Thomas localization and extended response.

Falsifier for CLP-1. The derived γ_n equation returns $v\_c^\infty \neq 1/8$, or the least-stiff eliminated mode is proved strictly localized or strictly extended.

Under CLP-1, with the band-edge gap of Section 25, $\delta_1 = 1/2 - 2v\_c^\infty$ and the saturation ratio $\delta_1/(2v\_c^\infty) = 1$ give

$$\mathbf{v\_c^\infty = 1/8 \,,\ \delta_1 = 1/4 \,,\ \xi\_aux = 1 \,,\ \mu\bigstar = \ell\_ref^{-1} \,.}$$

Nothing in the present corpus proves the criticality premise: $v\_c^\infty = 1/8$ is a sharp candidate branch, not a current result. It requires only separation of the coherent and first incoherent bands, and it is incompatible with complete separation of all five transport bands, which demands $v\_c^\infty < 3/56 < 1/8$.

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62. Finite-Bridge Non-Identifiability

The exact bridge is defined by the finite transverse background-field response:

$$\Delta\_A^{\{VBF\to M\bar{S}\}} = [\ \Pi\_\{A,T\}^{\{M\bar{S}\}}(-\mu^2) - \Pi\_\{A,T\}^{\{VBF\}}(-\mu^2)\ ]\_\text{finite} \,.$$

The holonomy corpus makes the VBF observable concrete — vacuum-subtracted loop holonomy, refinement convergence, continuum small-loop curvature density, a natural background-field response observable — but it remains classical and operator-theoretic. It does not supply R_VERSF, L_gf, L_ghost, ξ_gauge, or the finite subtraction condition; the electroweak gauge-curvature result likewise derives the classical W/B action while leaving gauge fixing, ghosts, perturbative quantisation and renormalisation open.

Proposition 28 — Finite-Bridge Non-Identifiability

The finite bridge $\Delta\_A^{\{VBF\to M\bar{S}\}}$ is not determined by the present data. Its strongest admissible form at equal matching scale is

$$\mathbf{\Delta\_A^{\{VBF\to M\bar{S}\}} = k\_A^{\{VBF\}} / 8\pi^2 \,,}$$

with k_3^{VBF}, k_2^{VBF}, k_Y^{VBF} undefined regulator-dependent constants.

Proof

Add a finite gauge-invariant local term

$$\delta\Gamma\_\text{finite} = \tfrac{1}{4}\, \Sigma\_A\ c\_A \int d^4x\ F\_\{A\mu\nu\}\ F\_A^{\{\mu\nu\}} \,.$$

This preserves gauge invariance, preserves the one-loop beta functions, and preserves the classical field equations after coupling redefinition, while shifting

$$1/g\_\{A,VBF\}^2 \to 1/g\_\{A,VBF\}^2 + c\_A \,,\ \Delta\_A^{\{VBF\to M\bar{S}\}} \to \Delta\_A^{\{VBF\to M\bar{S}\}} - c\_A \,.$$

Since the current VBF definition does not fix the three c_A, the finite bridge carries exactly this three-parameter freedom. ■

What is fixed is the logarithmic transport, $(b_Y, b_2, b_3) = (41/6, -19/6, -7)$. The native identity $\Delta^{V\to VBF} = 0$ stands by definition of the boundary scheme — but declaring $\Delta^{VBF\to M\bar{S}} = 0$ would *define* VBF to be $M\bar{S}$; it would not calculate the bridge. The missing object is the quantum subtraction prescription: the gauge-fixed, ghost-completed VERSF regulator whose finite determinant returns the three k_A^{VBF}.

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63. The Constitutive Fork

The earlier form of this gate faced a completeness fork: either the substrate action contained no curvature-dependent holonomy residual — in which case $J_F = 0$, $\mathcal{K}_g = 0$, $g_A = \infty$ and the gauge sector fails — or the action was incomplete and the missing term had to be added from substrate principles.

The construction of Part II resolved that fork at the constitutive level, and the execution of Part V refines it further. The fork now has three tiers.

Tier 1 — Shape

Either the quadratic stationary-record form of HRF-1 is the physical leading cost, or the substrate returns a different convex functional. In the second case HRF-1 is falsified; the replacement must again be derived from substrate principles, must repass the admissibility conditions of Section 12, and its coefficients may not be selected from measured couplings. The evaluation theorem, monotonicity results, extraction rules, normalisation audits and non-identifiability certificates of this paper apply unchanged to any replacement.

Tier 2 — Normalisation

Within the HRF-1 shape, either the literal unit-action equality $C_{hol} = 1$ holds, or C_{hol} must be derived from an upstream closure-normalisation principle. The benchmark shows this is one universal scalar, common to all three factors: it moves all three couplings together and leaves the ratios $g_s/g = \sqrt{13/12}$ and $g'/g = \sqrt{13/21}$ untouched. Fitting C_{hol} to a measured coupling is forbidden; deriving it from the same principle that fixes the other sectors' action normalisations is the open task.

Tier 3 — Embedding

Within the HRF-1 shape, either the physical gauge-to-record embedding is genuinely factorised — in which case the census triple $(6, 13/2, 378)\, C_{hol}$ is final and the wheel spectrum is invisible to the couplings — or the physical embedding is nonfactorising, or the fourteen-channel map is nontrivial. In the latter cases, factor-dependent relaxation corrections $b_A^\dagger C_A^{-1} b_A$ and modified direct weights w_A deform the census values, and the deformation must be published against the benchmark.

Corollary — Direction of correction

By Theorem 4, any physical branch that retains the direct embedding and enlarges the auxiliary response satisfies

$Z_A^{phys} \leq Z_A^{benchmark}$ at equal C_{hol}, hence $g_A^{phys} \geq g_A^{benchmark}$.

Within that fixed-embedding, fixed-metric, fixed-C_{hol} auxiliary-enlargement class the benchmark is a floor, not an estimate. Outside it no floor property holds: a branch that changes the embedding changes the direct weights w_A in either direction and must republish both terms of the normal form.

Sub-tier of Tier 1 — the wheel-metric fork

The wheel-profile branch adds a sub-tier to Tier 1: the wheel factor of the response metric is read either as the stationary norm — under which the coherent profile has unit weight and the benchmark is well-defined — or as the reversible Dirichlet form — under which the coherent mode is null and wheel participation is mandatory. The two readings are mutually exclusive constitutive choices, developed in Section 50. The colour coefficient $Z_3 = 6\, C_{hol}$ and hence $g_s = 1/\sqrt{6}$ are identical on both displayed readings — a two-branch invariance that a fourteen-channel embedding or colour-specific relaxation could still shift.

Corollary — No architecture-only escape

The existence of a theorem saying what Z_A would mean does not supply a nonzero Z_A. The action must contain the response — it now does, conditionally on HRF-1 — and the response must be sensitive to the substrate, which the factorised branch provably is not.

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64. Publication Package and Non-Insertion Firewall

The blind package must contain:

- frozen background and foam data;
- wheel matrix, stationary measure and Haar-completed metric;
- gauge embeddings with multiplicity certificates;
- scale-selector inputs (δ_r, v_c) and output $\mu\star$;

- the full functional with its frozen logarithm branch;
- auxiliary census;
- complete Hessian blocks;
- gauge and null projectors;
- Schur complement;
- factorwise response matrices;
- isotropy and mixing residuals;
- (w_A, $b_A^\dagger C_A^{-1} b_A$) and Z_3, Z_2, Z_q;
- current-matching coefficients ν_A;
- primitive couplings;
- finite scheme bridge;
- threshold ledger;
- running output;
- broken-phase conversion;
- uncertainty budget;
- hashes and timestamps.

Forbidden operations include:

1. choosing Z_A from measured g_A;
2. choosing $\mu\star$ by fit;
3. rescaling W_{rec}, or any factor block of it, after output inspection;
4. rotating a gauge embedding or choosing a multiplicity direction from coupling ratios;
5. attaching separate coefficients to the three factor terms of the functional;
6. omitting an auxiliary mode because it worsens agreement;
7. defining the current unit through the desired matter vertex;
8. selecting the logarithm branch after output inspection;
9. counting the flat Gate-3 class as local curvature energy;
10. inserting 5/3;
11. treating a VERSF coefficient as $\overline{\mathrm{MS}}$ without a bridge;
12. omitting thresholds;
13. using m_W, m_Z to infer g, g';
14. comparing couplings at different scales;
15. calling an inverse reconstruction a prediction.

Part VIII — Closure

65. Current Corpus Evaluation

65.1 Available

The present VERSF corpus, including this paper, supplies:

- the local Standard Model gauge algebra;
- the primitive connection and curvature carriers;
- the product-factor decomposition;
- the primitive charge lattice $q = 6Y$;
- an explicit gauge-covariant, positive, reversal-even, refinement-compatible holonomy-response functional, conditional on one falsifiable premise;
- explicit gauge-perturbed edge transport and vacuum-relative plaquette residuals;
- the stationary-record metric with no free relative weights — isotropy-tested against the frozen embeddings, Haar-completed only as a licensed gauge-orbit quotient — and its single universal conversion C_{hol} declared at the level of the action;
- explicit formulas for every Hessian block as derivatives of the displayed functional;
- the exact auxiliary Schur-reduction theorem and residual-projector form;
- positivity, locality and zero-mode classification rules;
- auxiliary-census monotonicity with fixed bias direction;
- factorwise adjoint isotropy, tested against the frozen embeddings rather than manufactured, with Haar completion as a licensed quotient;
- typed extraction of Z_A and the normal form $Z_A = w_A - b_A^\dagger C_A^{-1} b_A$;
- the explicit Abelian reduction through the $\mathbb{Z}_7$ closure connection, with the flat Gate-3 sector separated;
- an executed blind benchmark under Premises MCR-0 and AGI-0: the canonical wheel matrix, its stationary measure and spectrum, the 47-dimensional census carrier, exact product-factor intertwining, exact vanishing of the Schur correction, and the census triple $(6, 13/2, 378)\ C_{hol}$;
- the C_{hol}-independent ratios $g_s/g = \sqrt{13/12}$ and $g'/g = \sqrt{13/21}$;
- exact shared-link tree-level current matching $v_A^{(\text{shared-link})} = (1, 1, 1)$;
- the native VBF scheme with $\Delta^{V\to VBF} = 0$ and demonstrated one-loop transport;
- exact reproduction of the QGC beta census from the same embedding;
- the wheel-gap ratios $\{1, 1, 2, 2, 31/14, 5/2\}$ as raw local gaps, together with the superseding band-edge, saturation-corrected matching rule $\mu\bigstar = \ell_{ref}^{-1} \min[1, (1 - 4v_c^{\infty})/(4v_c^{\infty})]$ and its two regimes;
- a provisional charged-fermion threshold ledger from the MCHY audit branch;
- an executed wheel-profile rulebook: the exact wheel-mode ledger with D_6 types, gaps and screening factors; the coherent floor; the audit-locked normalisation on the stationary reading; the exact deformation laws; the Factor-Profile Non-Identifiability Certificate;
- an executed support-harmonic branch GRCE-1C(1) under Premises MCR-1, SHR-1, AHP-1, ARP-1 and DRF-1: unique sectors, exact stiffnesses (1, 5/4, 31/28), the triple $(6, 65/8, 837/2)\ C_{hol}$, $\sin^2\theta_W = 65/158$, and both ratios $\sqrt{65/48}$ and $\sqrt{65/93}$, C_{hol}-free by the vanishing restricted mixed block;
- a Planck-anchored substrate unit conditional on PSA-1, with sharp values $\sqrt{\pi}\,\ell_P$ and $E_P/\sqrt{\pi}$, and $\mu\bigstar$ equal to the sharp cutoff throughout the band-separated regime;
- exact non-identifiability proofs for the transport velocity v_c^{∞} (Proposition 27) and the finite bridge constants k_A^{VBF} (Proposition 28), converting two vague debts into two named missing equations;
- a conditionally assembled structural threshold ledger: the electroweak and scalar block, the charged-fermion boundary outputs with exact one-loop removal increments, and the substrate band table (Section 27.1);

- the blind non-insertion discipline.

65.2 Not yet available

The corpus does not yet supply, for the physical branch:

- substrate derivations of the census factorisation (MCR-1), the arity–harmonic assignment (AHP-1), the Abelian relative-phase selection (ARP-1) and the Dirichlet response operator (DRF-1) from the closure dynamics, in place of their declaration;
- the susceptibility–link bridge theorem: the derivation permitting ϱ_A^{resp} to differ from the link profile under one gauge connection;
- resolution of the wheel-metric fork: stationary norm versus Dirichlet form as the physical reading of the wheel factor;
- the genuine fourteen-to-twelve substrate map with resolved multiplicity directions (the orientation-neutral lift does not discharge it);
- the auxiliary derivatives of the frozen edge transports beyond the coherent-orthogonal class — the microscopic source of relaxation from deeper bath and completion modes;
- factor-dependent physical auxiliary Hessians C_A distinct from the wheel form C_0;
- an upstream derivation of the universal action-normalisation constant C_{hol} — now localised to the holonomy–Wilson bridge theorem and the universal-measure premise (Section 34.2);
- the microscopic equation deriving γ_n from the explicit refinement update — without it v_c^∞ is constitutively free (Proposition 27), though it drops out of μ★ under full band separation;
- the quantum subtraction prescription (R_VERSF, gauge fixing, ghosts, finite determinant) fixing the three bridge constants k_A^{VBF} (Proposition 28);
- the identification of ℓ_{ref} among the Planck certification, record-coherence and intermediate-refinement candidates (PSA-1 versus the 5.8 TeV alternative);
- the common-scheme conversion of the charged-fermion ledger, the neutrino thresholds, the substrate defect catalogue (Birman–Schwinger roots) and the κ-coherence formula resolution;
- a derived (non-provisional) identification of the auxiliary spectral distances with the physical mode gaps (discharging AGI-0);
- the certified independent record-to-Fock current descent;
- the finite VBF→$\overline{\mathrm{MS}}$ bridge coefficients;
- the complete broken-phase threshold ledger (Higgs, W, Z, Goldstone, heavy neutral and completion modes) from Papers 1–3.

65.3 Current numerical verdict

Matter-census benchmark: returned exactly on branch GRCE-1C(0) — $(Z_3, Z_2, Z_q) = (6, 13/2, 378)\, C_{hol}$; at $C_{hol} = 1$, $(g_s, g, g') = (0.408248, 0.392232, 0.308607)$, $\alpha^{-1} = 213.628$, $\sin^2\theta_W = 13/34$ at μ★.

Support-harmonic branch GRCE-1C(1): returned conditionally on MCR-1, SHR-1, AHP-1, ARP-1 and DRF-1 — (Z_3, Z_2, Z_q) = (6, 65/8, 837/2) C_hol; at C_hol = 1, (g_s, g, g′) = (0.408248, 0.350823, 0.293294), $\alpha^{-1} = 79\pi = 248.186$, $\sin^2\theta$_W = 65/158 at μ★; g_s invariant across the wheel-metric fork.

Matching surface: μ★ = ℓ_ref^{-1} min[1, (1 − 4v_c^∞)/(4v_c^∞)]; exactly 6.89 × 10^{18} GeV throughout the band-separated regime under the full stack (PSA-1; κ = π/2; η = 1/2; sharp certification boundary), where v_c^∞ drops out; the anchoring of ℓ_ref (Planck versus record-coherence, 6.89 × 10^{18} GeV versus 5.8 TeV on the critical branch) is the decisive open identification.

Physical gauge-coupling gate: not yet passed.

65.4 What has nevertheless been closed

The unknown is no longer "the gauge couplings," no longer "the response functional," and no longer even "any numerical output."

It is the finite correction set

{ derivations of MCR-1, SHR-1, AHP-1, ARP-1, DRF-1 (with the susceptibility–link bridge) and PSA-1 , the wheel-metric fork , ℰ: 𝔤_SM → $\mathscr{H}_{14}$, physical intertwining of U_{e,★} , ∂U_{e,★}/∂η beyond the coherent-orthogonal class , C_A per factor , C_hol , the γ_n equation behind v_c^∞ , the quantum subtraction behind k_A^{VBF} , the identification of ℓ_ref , completed 𝒯 , ν_A^(record→Fock) } .

Once these objects are supplied, the physical calculation is a finite deformation of two already executed returns — the exact matter-census benchmark, which within the fixed-embedding auxiliary-enlargement class is also a coupling lower bound, and the conditional support-harmonic triple GRCE-1C(1), whose profiles are unique under the named premise stack — and no further freedom remains at leading order.

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66. Falsification Conditions

F1 — No holonomy response. The frozen substrate generates no nonzero quadratic cost for an admissible gauge curvature.

F2 — Nonlocal reduction. Auxiliary elimination produces unresolved poles or nonanalyticity at the matching surface.

F3 — Negative gauge stiffness. A propagating gauge direction has Z_A < 0, i.e. b_A† C_A^{-1} b_A > w_A.

F4 — Unexplained zero stiffness. A Standard Model gauge factor has $Z_A = 0$ with no protected mechanism.

F5 — Factor mixing. A same-order colour–weak, colour–Abelian or weak–Abelian kinetic term survives.

F6 — Isotropy failure. Generators inside one unbroken simple factor receive different physical coefficients.

F7 — Incomplete auxiliary census. A newly admitted same-order mode materially changes a published Z_A.

F8 — Embedding failure. No equivariant rank-twelve descent exists from the fourteen-channel carrier.

F9 — Multiplicity ambiguity. A continuous embedding angle remains undetermined.

F10 — Current mismatch. The record and canonical Fock currents return $\nu_A \neq 1$.

F11 — Coupling non-universality. Matter and self-couplings differ after all normalisations.

F12 — Abelian lattice failure. No primitive charge unit is selected.

F13 — Scale-selector failure. $\delta_{min} = 0$, so $\xi_{aux} = \infty$ and no matching surface exists; or the selector output is non-unique without a pre-registered branch set.

F14 — Scheme-bridge failure. No finite connection to a physical running-coupling scheme exists.

F15 — Threshold incompleteness. The result changes materially when a previously omitted threshold is included.

F16 — Derivative-hierarchy failure. Higher-curvature operators dominate over F^2.

F17 — Quantum Ward failure. The scheme cannot preserve or restore coupling universality.

F18 — Numerical nonconvergence. The factor coefficients fail regulator, precision or auxiliary-census convergence, or the census sequence fails monotone convergence from above.

F19 — Data insertion. A measured coupling or weak angle selects an upstream object.

F20 — Constitutive failure. The physical substrate returns a leading local cost that is not the stationary-record quadratic norm: HRF-1 is falsified.

F21 — Metric texture. The stationary metric, projected through the frozen embeddings, requires rescaling to render the factor responses isotropic — indicating the Haar completion conceals a physical anisotropy.

F22 — Branch dependence. The published coefficients depend on the choice of logarithm branch within the admissible neighbourhood of the identity.

F23 — Topological miscount. The flat Gate-3 class contributes to a published local Z_A.

F24 — Normalisation non-derivation. No substrate principle returns the universal constant C_{hol}; the absolute couplings remain conventional rather than derived.

F25 — Sterile factorisation. Every admissible physical gauge-to-record embedding is provably factorised and every same-order auxiliary block is provably coherent-orthogonal, so no substrate mechanism can move the couplings off the census triple — the substrate contributes nothing to the coupling pattern beyond the matter census.

F26 — Empirical failure. After blind evaluation, matching and running, the derived couplings disagree with observation beyond the declared uncertainty and higher-order freedom.

F27 — Profile-class failure. The derived physical embedding is block-dependent, nonseparable, or carries coherent auxiliary components at leading order, falsifying WPE-1's homogeneity and the branch's carrier typing.

F28 — Audit-lock failure. The matter carrier is wheel-coherent yet the derived profiles return $c_{A0} \neq 1$, breaking shared-link universality; or the matter carrier itself acquires profile and the modified lock is violated.

F29 — Floor violation. A derived physical branch with wheel-incoherent auxiliaries returns $N_A < c_{A0}^2$, contradicting Theorem 18 and hence the positivity or orthogonality structure of the response.

F30 — Role-projector non-derivation. No canonical positive map from the role projectors to the wheel profiles exists in the substrate: the profile coefficients are constitutively free, and the gauge couplings are not fully derivable on any wheel-profile branch.

F31 — Symmetry-row failure. The derived role projectors select a residual symmetry row of Section 42 whose invariant space excludes the profiles actually returned by the substrate dynamics.

F32 — Harmonic-selection failure. The derived microscopic role projectors contradict AHP-1 or ARP-1: colour does not occupy the period-three sector, the weak roles do not occupy the alternating sector, the Abelian response is not D_6-invariant or carries a coherent component, or the response does not factorise as census × susceptibility (MCR-1).

F33 — Dirichlet failure. The substrate returns a wheel-response operator other than $\mathbb{I} - \hat{T}$ at leading order — in particular, a nonzero cost for the coherent mode or relative reweighting of the harmonic sectors — falsifying DRF-1 and the stiffness triple (1, 5/4, 31/28).

F34 — Susceptibility–link bridge failure. No substrate derivation permits the response susceptibility ϱ_A^{resp} to differ from the link profile under one gauge connection; or the derived bridge forces the two to coincide, whose zero coherent component then breaks the shared-link audit on the support-harmonic branch.

F35 — Anchoring failure. Premise PSA-1 fails — the gauge-matching spacing is not the certification floor — or the derived (κ, η) place ℓ_{sub} outside the certified band, or the transport constant is derived with $v_c^\infty \geq 1/4$, closing the first interband gap.

F36 — Transport-coefficient non-derivation. No microscopic equation derived from the explicit refinement update fixes the spatial coefficient γ_n; the transport velocity v_c^∞ remains constitutively free (Proposition 27) and no matching surface in the extended regime can be certified.

F37 — Band-structure failure. The derived global transport contradicts the coherent-band analysis: the wheel bands do not broaden with half-width $\gamma\rho(A_X)$, the Combes–Thomas rate does not saturate at one refinement unit, or the first interband gap differs from $1/2 - 2v_c^\infty$.

F38 — Subtraction non-derivation. No gauge-fixed, ghost-completed VERSF regulator and finite subtraction prescription is derived; the three constants k_A^{VBF} remain free (Proposition 28) and the finite VBF→$\overline{\text{MS}}$ bridge cannot be calculated — declaring it zero merely defines VBF to be $\overline{\text{MS}}$.

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67. The Gauge-Response and Coupling Evaluation Theorem

Theorem 29 — GRCE-1

Assume:

1. a frozen stable homogeneous VERSF substrate background $\rho\star$ on the closure-compatible $K = 7$ commitment foam;
2. the inherited holonomy-normalised Standard Model connection and primitive charge lattice $q = 6Y$;
3. the constructed holonomy-response functional read parametrically in a declared response operator, $\Gamma_{GRCE}^{(n)}[W_{phys}] = \Gamma_{sub}^{(n)} + (C_{hol}/2)\, \Sigma_p |p||p\star|\, \langle \mathscr{R}_p, W_{phys}\, \mathscr{R}_p \rangle$,

with frozen logarithm branch and W_phys a frozen positive-semidefinite gauge-invariant response operator declared before evaluation, the curvature image $\mathscr{E}$_SM(𝔤_SM) certified off ker W_phys — equivalently, W_phys strictly positive on the curvature image; failure of that certification returns Z_A = 0 and fires F4;
4. the complete equivariant curvature embedding $\mathscr{E}$_SM with resolved multiplicity spaces;
5. a complete physical auxiliary carrier;
6. a positive analytic auxiliary Hessian after gauge and null removal;
7. a declared frozen reading of W_phys: W_phys = W_rec, the Haar-completed stationary-record metric, defines the HRF-1 stationary branch; W_phys = D_cl ⊗ 𝕀, the reversible Dirichlet form, defines the susceptibility branch (relative weights rigid in each reading; overall conversion C_hol carried explicitly);
8. factorwise gauge covariance and product-factor independence;
9. the band-saturated substrate-gap matching scale $\mu\star = \min_{r\in\eta} \ell_{ref}^{-1} \min(1, \delta_r/2v_c^\infty)$ with $\delta_{min} > 0$;
10. the shared-link record-to-Fock current identity;
11. a finite declared scheme bridge;
12. a complete threshold ledger;
13. a Ward-identity-preserving quantum running scheme;
14. blind freezing before observation.

Then the physical unbroken gauge-response operator is

$$\boldsymbol{\mathcal{K}_g(q) = H_{FF}(q) - H_{F\eta}(q)\, H_{\eta\eta}(q)^{-1}\, H_{\eta F}(q)\,,}$$

with every block the second derivative of $\Gamma\_GRCE^{(n)}$:

$$H_{FF} = C_{hol}\, J_F^\dagger\, \mathbb{W}_n\, J_F\,,\quad H_{F\eta} = C_{hol}\, J_F^\dagger\, \mathbb{W}_n\, J_\eta\,,\quad H_{\eta\eta} = H_{\eta\eta}^{sub} + C_{hol}\, J_\eta^\dagger\, \mathbb{W}_n\, J_\eta\,,$$

with $\mathbb{W}_n = \bigoplus_p |p\rangle\langle p\star|\, W_{phys}$ in the declared reading. Its leading local block is

$$\mathcal{K}_g^{(0)} = Z_3\, \mathbb{I}_{\Lambda^2} \otimes \mathbb{I}_8 \oplus Z_2\, \mathbb{I}_{\Lambda^2} \otimes \mathbb{I}_3 \oplus Z_q\, \mathbb{I}_{\Lambda^2}\,,$$

where

$$\boldsymbol{Z_A(\mu\star) = (1/(6\, d_A))\, \mathrm{Tr}(\, P_A\, \mathcal{K}_g^{(0)}\, P_A\,) = w_A - b_A^\dagger\, C_A^{-1}\, b_A\,,}$$

in the action-scaled convention, with $w_A = C_{hol}\, \bar{w}_A$ and $\bar{w}_A = (1/d_A)\, \mathrm{Tr}(\mathscr{E}_A^\dagger\, W_{phys}\, \mathscr{E}_A)$ in the declared reading.

The primitive matching-scale couplings are uniquely determined:

$$\boldsymbol{g_s(\mu\star) = Z_3(\mu\star)^{(-1/2)}\,,\quad g(\mu\star) = Z_2(\mu\star)^{(-1/2)}\,,\quad g'(\mu\star) = 6\, Z_q(\mu\star)^{(-1/2)}\,.}$$

After finite scheme and threshold matching, QGC-1 transports the values. Following electroweak completion,

$e = g g' / \sqrt{(g^2 + g'^2)}$, $\alpha = e^2/4\pi$, $\sin^2\theta\_W = 36 Z_2 / (Z\_q + 36 Z_2)$.

The same factorwise coupling governs the canonical matter current and the non-Abelian self-interactions if and only if the current-matching coefficient satisfies $\nu\_A = 1$ — the value the shared-link construction predicts and the audit must confirm.

No independent gauge coupling, vertex normalisation, hypercharge scaling, metric weight, plaquette coefficient, matching scale or threshold coefficient remains admissible.

Proof

Gauge covariance, positivity and reversal invariance of the constructed functional are Theorems of Section 12. Exact auxiliary reduction gives the Schur complement with the displayed blocks. Analyticity yields the local zero-momentum block. Gauge covariance, factor simplicity and the isotropy test against the frozen embeddings — with Haar completion where gauge-orbit reduction licenses it — give factorwise isotropy. The typed trace gives Z_A; the residual-projector decomposition gives the normal form. Canonical field normalisation gives the primitive couplings. The primitive lattice gives the factor of six. The shared-link construction and current audit establish universality. The declared finite bridge, thresholds and QGC-1 flow produce the running scheme values. Electroweak rotation gives the photon and weak-angle relations. ■

Corollary 29.1 — Executed matter-census benchmark

Instantiating the stationary reading W_phys = W_rec — realised here as $W_0 = \mathrm{diag}(\pi) \otimes \mathbb{I}_{47}$ — under Premise MCR-0, on the product carrier $L^2(\pi) \otimes \mathscr{H}\_SM$ with $\mathscr{E}\_A = \mathbb{I}_7 \otimes \rho\_A$, hypotheses 4–8 hold exactly with vanishing mixed block (Theorems 13 and 14), and the theorem's extraction returns

$Z_3 = 6$ C_hol , $Z_2 = (13/2)$ C_hol , Z_q = 378 C_hol ,

hence, at the unit-action reading C_hol = 1, in the native VBF scheme at the matching surface μ★ of Section 25:

$(g\_s, g, g') = (0.408248, 0.392232, 0.308607)$, $\alpha^{-1} = 213.628$, $\sin^2\theta\_W = 13/34$,

with ν_A^(shared-link) = (1, 1, 1) at the shared-link tree level; the independent physical current audit is not discharged. ■

Corollary 29.2 — Support-harmonic conditional return

On branch GRCE-1C(1), instantiating the susceptibility reading W_phys = D_cl ⊗ 𝕀 under Premises MCR-1, SHR-1, AHP-1, ARP-1 and DRF-1, the sectors of Theorem 21 with the Dirichlet stiffnesses $(s_3, s_2, s\_q) = (1, 5/4, 31/28)$ return

$Z_3 = 6$ C_hol , $Z_2 = (65/8)$ C_hol , Z_q = (837/2) C_hol , hence, at the unit-action reading $C_{hol} = 1$:

$(g_s, g, g') = (0.408248, 0.350823, 0.293294)$, $\alpha^{-1} = 79\pi = 248.186$, $\sin^2\theta_W = 65/158$,

with C_{hol}-independent ratios $g_s/g = \sqrt{65/48}$ and $g'/g = \sqrt{65/93}$, and $g_s = 1/\sqrt{6}$ identical on the two displayed wheel-metric readings. The susceptibility–link bridge theorem remains an open obligation (Section 53). ■

Current-corpus clause

The theorem's constitutive hypothesis (HRF-1) is not discharged; its numerical hypotheses are discharged exactly on the factorised benchmark and conditionally on the support-harmonic branch. What remains open for the physical result: the constitutive equality $C_{hol} = 1$ or its derivation; substrate derivations of the census factorisation (MCR-1), the arity–harmonic assignment (AHP-1), the Abelian relative-phase selection (ARP-1), the Dirichlet response operator (DRF-1), the susceptibility–link bridge (SHR-1) and the Planck anchoring (PSA-1); resolution of the wheel-metric fork; the dimensionless transport constant v_c^∞; deeper factor-dependent auxiliary relaxation from bath and completion modes; the finite $\overline{\text{VBF}\to\text{M}\bar{\text{S}}}$ bridge; the completed threshold ledger; the independent record-to-Fock descent; and the fourteen-channel map $\mathfrak{g}_{SM} \to \mathscr{H}_{14}$. GRCE-1 is therefore a closed conditional evaluation theorem with a constructed response functional, an executed exact benchmark, an executed conditional support-harmonic derivation, and a sharpened numerical non-closure certificate — not a completed physical coupling derivation.

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68. GRCE-1 Closure Certificate

Closed

- gauge-response target identified;
- curvature and auxiliary carriers typed;
- explicit holonomy-response functional constructed (conditional on HRF-1);
- leading-order minimality and canonical selection of the functional established (unique within the inherited-metric class);
- gauge covariance, positivity, reversal invariance and refinement compatibility proved for the functional;
- every Hessian block displayed as a derivative of the functional;
- exact Schur reduction installed; residual-projector form derived;
- positivity and locality conditions fixed;
- auxiliary-census monotonicity proved; incomplete-census bias direction fixed;
- factorwise isotropy tested, not manufactured; extraction fixed; Haar completion applied only as a licensed gauge-orbit quotient;

- susceptibility–relaxation normal form fixed, with w_A a projection of one stationary metric;
- fourteen-to-twelve descent typed; multiplicity firewall fixed;
- explicit Abelian reduction fixed; flat Gate-3 sector separated;
- **blind benchmark GRCE-1C(0) executed exactly under declared Premises MCR-0 and AGI-0**: product-factor intertwining exact, Schur correction exactly zero, census triple (6, 13/2, 378) C_{hol} returned;
- scale-free ratios g_s/g and g'/g returned unconditionally;
- shared-link tree-level current matching returned exactly: $\nu_A^{(\text{shared-link})} = (1, 1, 1)$;
- QGC beta census reproduced exactly from the same embedding;
- wheel-gap ratio ledger returned as raw local gaps; band broadening and Combes–Thomas saturation supersede its direct threshold reading (Section 25);
- native VBF scheme identity closed by definition;
- primitive connection normalisation, hypercharge lattice and coupling conversion fixed;
- substrate-gap matching-scale rule fixed; scale–boundary non-identifiability proved;
- response-map and current–coupling non-identifiability proved, and their freedoms spent by the construction;
- conversion to e, α, $\sin^2\theta_W$ fixed;
- **wheel-profile branch GRCE-1(1) executed**: exact wheel-mode ledger with symmetry types, gaps and π-norms; one profile per simple factor; profile response formula; unconditional coherent floor; audit-locked normalisation on the stationary-norm reading; exact deformation laws for every boundary observable;
- Factor-Profile Non-Identifiability Certificate proved (Theorem 20), with the missing constitutive map named and its target reduced to the finite symmetry rows;
- **support-harmonic branch GRCE-1C(1) executed under declared Premises MCR-1, SHR-1, AHP-1, ARP-1 and DRF-1**: sectors unique (Theorem 21), stiffnesses (1, 5/4, 31/28) exact, triple (6, 65/8, 837/2) C_{hol} returned (Theorem 22), with $\sin^2\theta_W = 65/158$ and both ratios C_{hol}-free by the vanishing restricted mixed block;
- colour output $Z_3 = 6\, C_{hol}$ invariant across the wheel-metric fork;
- substrate unit Planck-anchored conditionally on PSA-1: $\ell_{sub} = \sqrt{\kappa/\eta}\, \ell_P$, sharp value $\sqrt{\pi}\, \ell_P$;
- band-edge, saturation-corrected matching rule closed: $\mu\star = \ell_{ref}^{-1} \min[1, (1 - 4v_c^\infty)/(4v_c^\infty)]$, with $\mu\star = \ell_{ref}^{-1}$ exactly throughout the band-separated regime $v_c^\infty < 3/56$;
- transport-velocity and finite-bridge non-identifiability proved (Propositions 27 and 28);
- structural threshold ledger assembled conditionally (Section 27.1);
- numerical and publication protocol fixed;
- falsifiers fixed.

Conditional

- HRF-1: the stationary-record quadratic cost is the physical leading functional;
- the wheel-metric reading of the response — stationary norm versus Dirichlet form (the Tier-1 sub-fork);
- MCR-1: the census–susceptibility factorisation of the support-harmonic response;

- AHP-1: the arity–harmonic correspondence assigning the colour and weak sectors their wheel harmonics;
- ARP-1: the Abelian relative-phase selection of the hub–boundary contrast;
- DRF-1: the reversible Dirichlet form as the profile cost operator;
- SHR-1: the susceptibility-profile reading of the support-harmonic response, pending the susceptibility–link bridge theorem;
- PSA-1: the Planck anchoring of the gauge-matching spacing at the certification floor, against the record-coherence and intermediate-refinement alternatives;
- CLP-1: the critical-locality selection $v_c^\infty = 1/8$;
- the certification premises anchoring ℓ_{sub} at the Planck layer, and the sharp-boundary values of (κ, η);
- the unit-action equality $C_{hol} = 1$;
- MCR-0: the matter-census carrier as benchmark response carrier (benchmark-scoped);
- AGI-0: identification of the wheel gaps with the auxiliary spectral distances (benchmark-scoped);
- physicality of the factorised branch as more than a benchmark;
- completeness of the auxiliary carrier;
- positivity and analyticity of every physical auxiliary block;
- existence and resolution of the rank-twelve equivariant fourteen-channel embedding;
- the independent record-to-Fock descent returning $\nu_A = 1$;
- $\delta_{min} > 0$, so a matching surface exists — the raw local wheel gap is 1/2 under AGI-0; the physical band-edge gap is $1/2 - 2v_c^\infty$, positive only for $v_c^\infty < 1/4$;
- the finite scheme bridge;
- the complete threshold spectrum;
- Ward-preserving quantum transport.

Open numerical objects

$\mathscr{E}: \mathfrak{g}_{SM} \to \mathscr{H}_{14}$, substrate derivations of MCR-1, SHR-1, AHP-1, ARP-1, DRF-1 and PSA-1 , the susceptibility–link bridge ,

$\partial U_{\{e,\star\}}/\partial\eta$ beyond the coherent-orthogonal class (deeper bath and completion relaxation) , factor-dependent C_A ,

C_{hol} , the γ_n equation (hence v_c^∞) , the quantum subtraction (hence k_A^{VBF} and $\Delta_A^{VBF\to M\bar{S}}$) , the identification of ℓ_{ref} , completed $\mathcal{T}$, $\nu_A^{(record\to Fock)}$,

and therefore the physical

Z_3 , Z_2 , Z_q , g_s , g , g' , e , α , $\sin^2\theta_W$ — now referenced against two explicit executed branches rather than unconstrained.

Numerical closure verdict

Gauge-response evaluation theorem: closed conditionally.

Holonomy-response functional: constructed, conditional on HRF-1.

Blind benchmark GRCE-1C(0): executed exactly; relative triple (6, 13/2, 378) C_hol returned.

Wheel-profile branch GRCE-1(1): rulebook closed; Factor-Profile Non-Identifiability Certificate proved.

Support-harmonic branch GRCE-1C(1): executed conditionally on MCR-1, SHR-1, AHP-1, ARP-1 and DRF-1; triple (6, 65/8, 837/2) C_hol returned; $\sin^2\theta_W = 65/158$; susceptibility–link bridge open.

Matching surface: band-saturated rule closed; $\mu\bigstar = 6.89 \times 10^{18}$ GeV throughout the band-separated regime under the full stack (PSA-1; $\kappa = \pi/2$; $\eta = 1/2$; sharp boundary); v_c^∞ and k_A^{VBF} proved non-identifiable; ℓ_{ref} identification open.

Absolute normalisation: conditional on C_hol = 1; upstream derivation open.

Physical gauge-coupling gate: not yet passed.

No measured coupling has been used as a response input.

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69. Conclusion

The gauge programme derived, from finite distinguishability and closure structure alone, that the world's internal interactions are governed by U(1) × SU(2) × SU(3), acting on exactly the Standard Model representation content, with electroweak breaking and one narrow admissible window beyond. That derivation answered *which* forces exist.

This paper closes the architecture — and, for the first time, executes two constitutive branches of the calculation — for *how strong* those forces are.

Three questions structure the result.

What would a coupling mean? A coupling is the inverse stiffness of the frozen substrate against sourcing gauge curvature: $Z_A = w_A - b_A^\dagger C_A^{-1} b_A$, the direct stationary-metric weight of the curvature image minus the exact relaxation the auxiliary record modes provide. That meaning is now theorem, not metaphor: every block is a displayed second derivative of one constructed functional, the vacuum-relative plaquette holonomy cost in the Haar-completed stationary-record metric.

What does the simplest admissible calculation return? Branch GRCE-1C(0) tensors the canonical seven-state wheel with the frozen 47-dimensional matter-plus-Higgs census in the maximally independent way. Everything about it is exact: the gauge embeddings commute with the product-carrier transport algebraically (by construction, not as a physical intertwining theorem); the curvature image sits on the coherent wheel line while every wheel auxiliary is π-orthogonal to it, so the Schur correction vanishes identically; and the census traces return

$Z_3 = 6$ C_hol , $Z_2 = (13/2)$ C_hol , Z_q = 378 C_hol .

One universal scalar — the conversion between the stationary-record norm and the dimensionless Euclidean action — is all that stands between these and absolute numbers. Reading HRF-1 literally sets C_hol = 1 and gives (g_s, g, g′) = (0.408248, 0.392232, 0.308607), $\alpha^{-1} = 213.628$ and $\sin^2\theta$_W = 13/34 at the matching surface, in the native background-field scheme, before thresholds and running. Two ratios, g_s/g = √(13/12) and g′/g = √(13/21), are unconditional. Current matching is exactly (1, 1, 1); the same embedding reproduces the QGC beta census exactly; the wheel gaps return the raw ratio ledger {1, 1, 2, 2, 31/14, 5/2}, whose threshold reading is superseded by the band-edge, saturation-corrected rule of Section 25. No measured coupling entered anywhere.

Why is that not yet the physical answer? Because the benchmark proves its own insufficiency. Factorisation cancels the wheel dynamics out of the coupling pattern entirely: the three strengths depend only on the matter census, and the substrate contributes nothing distinctive. By the monotonicity theorem the benchmark is a coupling lower bound within its class — any branch that keeps the direct embedding, metric and C_hol and adds genuine relaxation can only soften the stiffnesses and raise the couplings; a changed embedding escapes the bound entirely. The physical branch must therefore return at least one of: a nonfactorising gauge-to-record embedding; the genuine fourteen-to-twelve substrate map; factor-dependent auxiliary relaxation b_A ≠ 0 or Hessians $C_A \neq C_0$; or the upstream derivation of C_hol. The missing microscopic structures no longer have vague roles. They have jobs.

What happens when the wheel is allowed to grip? Branch GRCE-1(1) executes those jobs as far as symmetry reaches, and then one step further. The complete equivariant profile extension yields six exact wheel channels sorted by symmetry type, a coherent floor that no relaxation can breach, an audit lock that spends the profile-scale freedom, and exact deformation laws for every boundary observable — with the Factor-Profile Non-Identifiability Certificate proving that census and symmetry stop exactly there. A declared, falsifiable premise stack — the census factorisation, the susceptibility reading, the arity–harmonic correspondence, the Abelian relative-phase selection and the Dirichlet pricing — then finishes the selection: colour takes the threefold harmonic, weak isospin the alternating one, primitive hypercharge the hub–boundary contrast, at stiffnesses (1, 5/4, 31/28). The triple becomes (6, 65/8, 837/2) C_hol; at unit action, g = 0.350823, g′ = 0.293294, $\alpha^{-1} = 79\pi$ and $\sin^2\theta$_W = 65/158 — granting the premise stack, every entry a ratio of small integers untouched by measurement, with C_hol = 1 for the absolute values. The strong coupling, identical on the two displayed readings of the wheel metric, is the calculation's steadiest single number so far; under the declared Planck-anchoring premise the substrate unit is √π ℓ_P and, throughout the band-separated regime, the matching surface is exactly the cutoff, 6.89×10^{18} GeV under the full anchoring stack (PSA-1; $\kappa = \pi/2$; $\eta = 1/2$; sharp boundary) — the transport constant drops out of it entirely. The branch's decisive open obligation is internal: the susceptibility–link bridge theorem — deriving why the sector that registers curvature energy may differ from the sector that transports the gauge phase — must be proved, or the branch falls to F34.

The non-identifiability theorems remain the paper's discipline. A matching scale left free, a response map left free, a current normalisation left free, or an auxiliary census left open can each absorb the answer — so each was fixed from substrate structure, or spent by the construction, before any number was computed. The one freedom the benchmark could not spend, C_hol, is named, universal, ratio-invisible and forbidden from being fitted.

The correct claim, in one paragraph:

The first blind gauge-response benchmark has been executed. On the canonical factorised wheel–matter carrier, declared as a matter-census induced-response benchmark (Premise MCR-0), product-factor intertwining is exact, the incoherent wheel modes are orthogonal to the gauge-curvature image, auxiliary relaxation vanishes, and the minimal three-generation-plus-Higgs census returns $Z_3 = 6C\_hol$, $Z_2 = (13/2)C\_hol$, $Z\_q = 378C\_hol$ — under the literal unit-action convention, $(g\_s, g, g') = (0.408248, 0.392232, 0.308607)$, $\alpha^{-1} = 213.628$ and $\sin^2\theta\_W = 13/34$ at $\mu\bigstar$. The branch is an exact, unfitted benchmark rather than the physical coupling derivation: its factorisation makes the response blind to the wheel spectrum, and one universal action-normalisation constant remains unresolved. The coupling calculation now has its equivalent of the Yukawa programme's blind baseline, and the benchmark identifies precisely why the simplest microscopic realisation is insufficient.

The path to full closure is unchanged in kind and shortened in length:

1. derive the census factorisation, the harmonic assignments and the Dirichlet response operator from the closure dynamics, replacing Premises MCR-1, AHP-1, ARP-1 and DRF-1 with theorems — and thereby resolve the wheel-metric fork;
2. prove the susceptibility–link bridge: derive why the response susceptibility may differ from the link profile under one gauge connection (discharging SHR-1);
3. derive the fourteen-to-twelve substrate map from the closure dynamics (the orientation-neutral lift is not it);
4. compute the auxiliary derivatives of the frozen edge transports beyond the coherent-orthogonal class — the deeper bath and completion relaxation that could shift the support-harmonic stiffnesses;
5. prove the holonomy–Wilson action-normalisation bridge and the universal-measure unit-fixing, closing C_hol (Section 34.2);
6. derive the microscopic γ_n equation from the explicit refinement update — the only route to v_c^∞ (Proposition 27) — and decide the identification of ℓ_ref among the Planck, record-coherence and intermediate candidates;
7. fix the quantum VBF subtraction prescription — gauge fixing, ghosts, regulator and finite determinant — the only route to the three bridge constants k_A^{VBF} (Proposition 28);
8. complete the numerical side of the structural threshold ledger of Section 27.1: common-scheme charged masses, the neutral block, the substrate defect catalogue and the κ-coherence resolution;
9. certify the independent record-to-Fock current descent;

10. re-execute the evaluation algorithm and publish the physical triple with its decomposition against both executed returns;
11. only then run, convert and compare with observation.

What Gate GRCE-1 delivers now:

A complete gauge-response evaluation theorem, an explicitly constructed holonomy-response functional, an executed exact blind benchmark, an executed wheel-profile rulebook with its non-identifiability certificate, an executed conditional support-harmonic derivation with unique profiles and a Planck-anchored matching surface, an exact normalisation and matching protocol, a band-saturated matching rule with a conditionally assembled structural threshold ledger, exact non-identifiability proofs for the transport velocity and the finite bridge, and a numerical non-closure certificate sharpened to named premises, one universal scalar, one missing microscopic equation and one missing subtraction prescription.

The Standard Model's gauge couplings are, in this framework, no longer three numbers to be measured, no longer three functionals to be evaluated, and no longer even three free profiles on a wheel. They are two explicit reference branches — an exact census benchmark and a symmetry-selected harmonic derivation — against which the physical result must be decomposed and audited, its remaining freedom finite, named and falsifiable.

The gate stands, the functional is built, the benchmark is run, the profiles are derived — and the substrate now owes the proof that it had no other choice.