The Generation and Species Census Theorem in VERSF

▲ Programme Milestone — Standard Model Census Series Gate SC-1 / Generation and Species-Count Closure

Deriving the minimal anomaly-stable Standard Model matter census from terminal-record species closure, weak attachment, colour branching, and rank-three support-return completion

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General Reader Summary

Before a theory can derive particle masses, mixings, and couplings, it must first answer a simpler question:

What are the things whose masses, mixings, and couplings are being derived?

The Standard Model has a very specific census. It contains quarks and leptons. Quarks come in colour triplets. Left-handed particles participate in weak doublets. Right-handed charged particles are weak singlets. The pattern repeats three times, producing three generations. The minimal Standard Model has no right-handed neutrino, although neutrino-mass extensions may add one or something equivalent.

This paper asks:

Can VERSF explain why this species and generation census is the minimal admissible one?

The answer is yes, conditionally on the VERSF record-sector premises.

The key idea is that a physical particle species is not merely a label. It is an irreducible way for a terminal record to carry gauge attachment, charge, weak transition structure, mass closure, and anomaly-stable ledger membership.

The species census is forced by five requirements:

1. **Weak attachment:** unresolved record distinctions appear as weak doublets.
2. **Terminal mass closure:** weak-doublet components need right-singlet partners if they are to become charged massive terminal records.
3. **Colour branching:** support-coupled matter has a colour-triplet branch, producing the quark sector.

4. **Ledger completion:** the coloured weak sector alone is globally anomalous; a colourless weak doublet is required, producing the lepton sector.
5. **Anomaly stability:** the full package must cancel all local and global gauge anomalies.

These requirements give exactly one minimal generation package:

Q_l, u_r, d_r, L_l, e_r,

with the usual hypercharge assignments under the convention

$Q = T_3 + Y/2$.

The generation count is a separate question. VERSF does not identify generations with colour. Colour is a gauge-facing triplicity inside the quark sector. Generation is a completion-layer triplicity of the whole species package.

The prior anomaly-descent theorem established that terminal physical support has rank three. A complete species package must be repeated once for each independent support-return completion layer. Fewer than three packages leave a completion direction unserved. More than three require an extra support-return layer not present in the minimal physical quotient. Therefore the minimal complete census has exactly three generations.

In one sentence:

VERSF's minimal Standard Model matter census is three complete anomaly-stable generations, each containing one quark weak doublet, two quark singlets, one lepton weak doublet, and one charged-lepton singlet, with the Higgs doublet acting as the universal closure carrier.

Scope Box — What SC-1 Closes and What It Does Not

Claim	Status
Bare particle-list assumption is disallowed	Closed
Species as terminal-record carrier roles	Defined here
One-generation chiral matter package	Derived here, conditional on weak/colour attachment premises
Hypercharge assignments under $Q = T_3 + Y/2$	Derived here from Higgs closure and anomaly cancellation

Claim	Status
Uniqueness of the hypercharge solution within the minimal field content	Derived here from gauge-sector conditions and ordered closure alone; the cubic factorises as a perfect cube, leaving no branch ambiguity
Gravitational–hypercharge anomaly consistency	Emerges as a surplus check, not a determining input
Layer-local terminality and exclusion of inter-layer anomaly borrowing	Stated and used here
Layer-exclusive attachment of terminal carriers	Stated and used here
Family universality of hypercharge across generations	Corollary here
Generation mixing typed as post-census structure	Closed here
Necessity of quark and lepton sectors	Derived here through colour branching and anomaly stability
Electric-charge quantisation and proton–electron charge balance	Corollary here
Absence of ν_r in the minimal census	Closed as minimal; extension remains allowed
One Higgs doublet as minimal closure carrier	Closed as minimal; extensions typed separately
Exactly three generations	Derived here from rank-three support-return completion, layer-local terminality, and layer-exclusive attachment
No fourth generation in the minimal census	Closed conditionally on no fourth support-return layer
Gauge-coupling values	Not closed
Fermion mass values	Not closed
Yukawa hierarchy	Not closed
CKM/PMNS numerical values	Not closed
Neutrino-mass mechanism	Not closed; extension gate
Origin of gauge algebra from deeper VERSF structure	Not closed here if not inherited

SC-1 closes the **census** problem. It does not close the **mass**, **mixing**, **coupling**, or **neutrino-extension** problems.

Abstract

VERSF's quantitative Standard Model programme cannot begin with masses. It must first fix the species and generation census: the list of terminal physical record carriers to which masses, charges, mixings, and couplings attach.

This paper proves a conditional census theorem. Under the VERSF premises of terminal-record species closure, weak binary attachment, colour support branching, Higgs-mediated mass admissibility, anomaly stability, and rank-three support-return completion, the minimal admissible matter census is exactly the Standard Model chiral census with three generations:

$N_g = 3$,

and, for each generation,

$Q_l = (u_l, d_l)^T, u_r, d_r, L_l = (\nu_l, e_l)^T, e_r$.

With the convention

$Q = T_3 + Y/2$,

the unique hypercharge assignments compatible with one Higgs doublet $H = (H^+, H^0)^T$, $Y(H) = +1$, gauge-invariant ordered Yukawa closure, and gauge-sector anomaly cancellation are

$Y(Q_l) = +1/3, Y(u_r) = +4/3, Y(d_r) = -2/3, Y(L_l) = -1, Y(e_r) = -2, Y(H) = +1$.

The determination is gauge-sector only: under ordered closure the cubic hypercharge condition factorises as a perfect cube, $(1 - 3Y(Q_l))^3 = 0$, with the single triple root $Y(Q_l) = +1/3$, and the mixed gravitational–hypercharge condition then closes automatically as a surplus check.

The proof proceeds in two parts.

First, the **species theorem** shows that one complete anomaly-stable terminal generation requires one colour-triplet weak doublet, two colour-triplet right singlets, one colourless weak doublet, and one charged colourless right singlet. The colourless lepton doublet is not optional: the three coloured weak doublets alone give an odd number of SU(2)_L doublets and fail the global weak-anomaly test, and the minimal parity repair — measured in added chiral degrees of freedom and induced anomaly obligations — is a single colourless doublet. The lepton doublet also participates in hypercharge and gravitational anomaly closure. The sterile right-handed neutrino is not forced by gauge closure, anomaly closure, or charged mass closure, so it is absent from the minimal census and belongs to an extension ledger. Its absence also removes the B−L flat direction that would otherwise leave the hypercharge assignment underdetermined.

Second, the **generation theorem** shows that a complete species package must occur once for each independent support-return completion layer. The bridge is layer-local terminality together with layer-exclusive attachment: the projection of the census onto each independent layer must

itself be a terminally admissible census, and each terminal carrier attaches to exactly one layer — so partial generations cannot be repaired by inter-layer anomaly borrowing, mixing structures cannot substitute for missing species, and a single package cannot span multiple layers. The VERSF physical support quotient has rank three. Therefore exactly three complete anomaly-stable packages are required and sufficient. A fourth generation would require either a fourth support-return layer, a new primitive record obligation, or an additional anomaly-stable sector. Without such a bridge, it is not part of the minimal census.

SC-1 therefore closes the Standard Model census gate: VERSF's later mass, Yukawa, CKM, PMNS, coupling, and response papers inherit a fixed list of admissible species and generation slots. They may predict values only after this census is respected.

Contents

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0. Inheritance, Firewalls, and Aim

0.1 Inherited results

SC-1 inherits the following results.

I1 — QN-1 κ-ledger discipline. Bare κ-like symbols are inadmissible. Localization stiffness, compliance, channel-log weights, gap scales, and response couplings are typed separately.

I2 — Substrate anomaly-descent result. The physical support quotient used by VERSF terminal records has rank three, and the canonical localization stiffness $\kappa_\star = 8/3$ arises from a four-obligation substrate commitment descending into that rank-three support quotient. SC-1 uses only the rank statement; the numerical stiffness plays no role in the census.

I3 — Standard Model convention ledger. SC-1 uses the canonical convention

$Q = T_3 + Y/2,$

with fundamental generator normalisation fixed by the Standard Model convention audit. All hypercharge statements below are convention-relative; a change of convention requires the declared conversion map and nothing else.

I4 — Physical-sector positivity. Negative-norm residue, ghost directions, and adjoint leakage do not contribute to the physical species census.

I5 — Minimal Standard Model boundary. Unless explicitly declared, the census is the minimal chiral Standard Model census: one Higgs doublet, no right-handed neutrino, and no extra gauge sectors.

I6 — Bridge discipline. No VERSF object is identified with a Standard Model object by numerical coincidence. Bridges require representation slot, mechanism, domain, dimensional map, and error control.

0.2 Firewalls

This paper does **not** claim:

- that fermion masses are derived;

- that Yukawa eigenvalues are derived;
- that CKM or PMNS numbers are derived;
- that neutrino masses are explained;
- that gauge-coupling values are derived;
- that the Higgs mass is derived;
- that a fourth generation is impossible in every conceivable extension;
- that colour triplicity and generation triplicity are the same thing;
- that anomaly cancellation alone fixes the number of generations;
- that a sterile neutrino is forbidden in extensions;
- that the Standard Model gauge algebra is derived from nothing in this paper.

It proves a narrower census theorem:

Given the active VERSF/Standard Model attachment ledger, the minimal anomaly-stable terminal matter census is exactly three complete Standard Model generations with one Higgs doublet as universal closure carrier.

0.3 Aim

The paper answers:

Why this species list, and why three copies?

It does not answer:

Why these masses, mixings, and couplings?

That is the correct order. A derivation must first know what is being counted.

0.4 Derived, Inherited, and Deferred Claims

The following table separates what SC-1 derives, what it inherits, and what it defers. It prevents a stronger claim from being read into SC-1 than the paper actually proves. The census theorem is strong because its dependencies are explicit.

Claim	Status in SC-1	Dependency
One-generation chiral matter package	Derived conditionally here	Weak attachment, colour branching, terminal charged closure, anomaly stability
Lepton-doublet necessity	Derived conditionally here	Global SU(2) anomaly parity plus minimality
Right charged singlets	Derived conditionally here	Higgs-mediated charged terminal closure
Hypercharge assignments	Derived conditionally here	Neutral Higgs vacuum, ordered closure, anomaly equations, no ν_r in minimal census

Claim	Status in SC-1	Dependency
Electric-charge quantisation	Corollary here	Hypercharge solution
One Higgs doublet	Minimal and sufficient here	Minimal Standard Model boundary plus Higgs-mediated charged closure
Absence of ν_r from minimal census	Derived as minimality result here	No anomaly, weak-parity, or charged-closure need
Three generations	Derived here from inherited support rank	Rank-three support-return completion plus layer-local terminality and layer-exclusive attachment
Rank-three support-return quotient	Inherited, not reproved here	Upstream VERSF anomaly-descent/support theorem
Standard Model gauge algebra	Inherited, not derived here	Gauge-origin gate or prior ledger
Fermion mass values	Deferred	Later Yukawa/completion-density gates
CKM/PMNS numerical values	Deferred	Later mixing and neutrino-extension gates
Neutrino masses	Deferred extension	Requires Majorana, Dirac, seesaw, sterile, or other declared bridge
Gauge-coupling values	Deferred	Later coupling-normalisation gates

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0′. Predictive-Content Ledger

Object	Prior status	Status here
Species	Often treated as particle labels	Defined as terminal-record carrier roles
Generation	Often treated as empirical repetition	Defined as a complete support-return census layer
One-generation matter package	Standard Model input	Derived as minimal anomaly-stable package
Q_l	Standard Model field	Derived as colour-triplet weak attachment carrier
u_r, d_r	Standard Model fields	Derived as terminal mass-closure singlets
L_l	Standard Model field	Derived as colourless anomaly-completion doublet
e_r	Standard Model field	Derived as charged-lepton closure singlet
ν_r	Absent/minimal or extension	Typed as non-minimal sterile extension
Hypercharges	Standard Model inputs	Derived up to convention, fixed by Y(H) = +1; uniqueness proved within the minimal field content from gauge-sector conditions alone

Object	Prior status	Status here
Generation-universal hypercharge	Implicit Standard Model assumption	Derived as a corollary of per-layer closure through the single H
Electric-charge quantisation	Empirical fact in SM	Corollary of the census theorem
One Higgs doublet	Standard Model input	Derived as minimal universal closure carrier
Gauge boson count	Inherited from gauge algebra	Ledgered, not newly derived
Three generations	Empirical input in SM	Derived from rank-three support-return completion plus layer-local terminality and layer-exclusive attachment
Fourth generation	Not part of minimal SM	Excluded unless extra support/obligation bridge supplied
Mass hierarchy	Open	Later gate
CKM/PMNS values	Open	Later gate
Neutrino masses	Open	Extension gate
Gauge-coupling values	Open	Later gate

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1. The Census Problem

1.1 Why census must precede mass

A mass derivation without a species census is under-defined.

Before VERSF can claim to derive

$m_e, m_\mu, m_\tau, m_u, m_c, m_t, m_d, m_s, m_b$,

or the CKM matrix, or Yukawa hierarchy exponents, it must explain why the target list contains:

- quarks and leptons;
- left weak doublets and right weak singlets;
- colour triplets and colour singlets;
- exactly three repetitions;
- one Higgs doublet in the minimal closure sector;
- no right-handed neutrino in the minimal chiral census.

SC-1 supplies that target list.

A programme that skips this step and reports numerical agreements against an assumed particle list has smuggled its most structural input. VERSF's later quantitative gates are only as honest as the census they inherit; SC-1 exists so that the census is a theorem with named premises rather than an unexamined assumption.

1.2 The central distinction

There are two different census questions.

Species census: What irreducible terminal-record carrier roles exist inside one complete anomaly-stable package?

Generation census: How many complete copies of that package are required by the support-return completion ledger?

The first question gives

Q_l, u_r, d_r, L_l, e_r.

The second gives

N_g = 3.

The two results must not be conflated. Colour triplicity is not generation triplicity.

1.3 The result in one line

The minimal VERSF/Standard Model matter census is

$3 \times [\ Q_l \oplus u_r \oplus d_r \oplus L_l \oplus e_r\ ]$,

with one universal Higgs closure carrier

$H = (H^+, H^0)^T$,

and no forced sterile right-handed neutrino in the minimal ledger.

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2. Named Premises of the Census Theorem

The theorem rests on named premises. Each is falsifiable. Premises are labelled C1–C15 (census premises) to keep the premise namespace distinct from the gate identifier SC-1.

C1 — Terminal species principle

A physical species is an irreducible terminal-record carrier role. It is not merely a particle name. It must specify:

1. physical support class;
2. gauge attachment;
3. weak transition status;
4. charge assignment;
5. mass-closure possibility;
6. anomaly-stable ledger membership.

Falsifier: a claimed species lacks one of these record roles but is still treated as terminal and physical.

C2 — Weak binary attachment

Unresolved terminal distinction appears minimally as a weak binary attachment. Therefore the left attachment sector is built from SU(2)_L doublets, and no higher weak multiplet is primitive.

Falsifier: terminal weak distinction is represented without a binary doublet, or a higher primitive weak multiplet is required while preserving minimality.

C3 — Right terminal closure singlets

A weak-doublet component that becomes a charged massive terminal record requires a right-chiral weak-singlet closure partner.

This premise is grounded rather than free-standing. A charged Weyl fermion cannot self-close: a Majorana-type mass violates the unbroken U(1)_em charge carried by the record. A mass-closure channel for a charged terminal record therefore requires a right-chiral weak-singlet partner of the same physical electric charge — equivalently, a left-handed conjugate field of opposite charge in the anomaly ledger. C3 thus follows from charge conservation together with the mass-closure-possibility clause of C1(5); it is retained as a named premise so that its falsifier remains independently testable.

Falsifier: a charged massive terminal record is formed without a right singlet or equivalent closure mechanism.

C4 — Colour support branching

Support-coupled matter has a colour-triplet branch. Colourless ledger-completion matter is allowed and required where anomaly stability demands it.

Falsifier: coloured support records are represented without triplet colour branching, or colourless completion records are forbidden despite anomaly failure.

C5 — One-Higgs minimal closure carrier

The minimal charged-fermion closure carrier is one weak doublet $H = (H^+, H^0)^T$, whose lower component is electrically neutral in the broken phase. Under the convention $Q = T_3 + Y/2$, this implies

$Y(H) = +1.$

Together with its conjugate

$\tilde{H} = i\sigma^2 H^*,$

this single doublet supplies the minimal charged-closure channels. The hypercharge value is thus a consequence of the neutral-lower-component condition, derived in §6.1, not an independent stipulation of this premise.

Falsifier: charged mass closure requires either no closure carrier or more than one primitive Higgs doublet in the minimal census.

C6 — Gauge and global anomaly stability

A physical census must cancel all required local gauge anomalies, the mixed gravitational–hypercharge anomaly, and the global SU(2) anomaly. An anomalous census is not a terminal physical ledger.

Falsifier: an anomalous species set is admitted as a terminal physical census.

C7 — Census minimality

No species may be included in the minimal census unless it is required by weak attachment, colour branching, mass closure, anomaly stability, or support-return completion. Minimality is measured in chiral degrees of freedom: among admissible repairs of a ledger failure, the repair adding the fewest Weyl fields and inducing the fewest new anomaly obligations is the minimal one.

Falsifier: an unnecessary species is admitted as minimal, or a smaller admissible repair is exhibited for a ledger failure resolved here.

C8 — Support-return generation principle

A generation is a complete anomaly-stable species package assigned to one independent support-return completion layer.

Falsifier: a generation is treated as a partial species package or as a gauge charge rather than a complete completion layer.

C9 — Rank-three completion

The physical support-return quotient has exactly three independent completion layers. This premise is inherited from I2 and is restated here so that the generation theorem carries its own falsifier.

Falsifier: exhibit a fourth independent support-return layer in the minimal physical quotient, or show that fewer than three layers suffice for terminal support completion.

C10 — Generation completeness

Each support-return completion layer must carry a full anomaly-stable species package. Partial generations are inadmissible because they leave anomaly or mass-closure defects in that layer.

Falsifier: a partial generation is admitted as terminal and anomaly-stable.

C11 — No duplicate layer without bridge

A fourth complete generation is non-minimal unless a fourth support-return layer, a new primitive record obligation, or a new anomaly-stable sector is supplied.

Falsifier: a fourth generation is required without extending the support-return or record-obligation ledger.

C12 — Layer-local terminality

A support-return completion layer is terminal only if its own projected census is closed under weak attachment, colour branching, Higgs-mediated charged closure, electric-charge assignment, and anomaly stability.

Equivalently, if P_i denotes projection onto the i-th support-return layer, then $P_i\mathcal{C}$ must itself be a terminally admissible census. A layer may later mix with other completed layers through Yukawa or response maps, but it may not borrow missing species, anomaly cancellation, or charged-mass closure from another layer in order to become admissible.

Falsifier: a support-return layer is shown to be terminal while containing only a partial generation, or while relying on another layer to cancel its projected anomaly, supply a missing closure partner, or provide its weak-completion doublet.

C13 — Inter-layer mixing is post-census

Generation mixing is not a census-forming operation. CKM and PMNS-type structures are maps between already-populated generation slots. They may rotate, couple, or misalign complete species packages, but they cannot create the species package itself.

Therefore the census must be fixed before mass matrices, mixing matrices, or hierarchy exponents are introduced.

Falsifier: a missing species in one generation slot is made admissible solely by a mixing term with another slot before the source and target packages are independently terminal.

C14 — Ordered weak-component closure

The upper and lower components of a weak doublet carry different T_3 values and therefore different electric charges. Higgs closure must preserve this ordered weak structure.

For the minimal Higgs doublet H, with Y(H) = +1, the lower weak component closes through H, while the upper weak component closes through $\tilde{H} = i\sigma^2 H^*$. Thus the ordered closure channels are

$\bar{Q}_l H d_r$, $\bar{Q}_l \tilde{H} u_r$, $\bar{L}_l H e_r$.

This ordering is a census condition, not a later mass-value assumption. It identifies which right singlet closes the upper and which closes the lower terminal charged record.

Falsifier: an alternative hypercharge branch satisfies all anomaly equations and neutral-Higgs conditions while preserving the same ordered upper/lower weak-component closure and the same charged terminal species labels.

C15 — Layer-exclusive attachment

An irreducible terminal record carrier attaches to exactly one independent support-return completion layer.

Justification: the completion layers are independent as record ledgers. If a single carrier's commitment simultaneously constituted the completion of two layers, those layers would share a constituting entry and would not be independent completion directions — contradicting the inherited rank statement (I2, C9). Sharing structure across layers is therefore not a cheaper census; it is a denial of layer independence under another name.

C15 does work that C12 cannot: layer-local terminality forbids a layer from *lacking* structure, while layer-exclusive attachment forbids layers from *sharing* it. Without C15, a single layer-spanning generation whose projection onto every layer is complete would satisfy C12 with N_g = 1. C15 closes that route.

Falsifier: exhibit a terminal carrier whose projection onto two independent support-return layers is complete in both, while the layers remain independent as record ledgers.

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3. Species, Generations, and Census Ledgers

3.1 Definition — species

A **species** is an irreducible terminal-record carrier role in the physical quotient.

A species is specified by a tuple

$\mathcal{S} = (\mathcal{C}, \mathcal{W}, Y, \chi, \mathscr{M})$,

where:

- $\mathcal{C}$ is colour status;
- $\mathcal{W}$ is weak representation;
- Y is hypercharge;
- χ is chirality/attachment status;
- $\mathscr{M}$ is mass-closure role.

A species is not defined by mass. Mass values are later data. Species are census slots.

3.2 Definition — complete generation package

A **complete generation package** is a minimal anomaly-stable set of species closed under:

1. weak attachment;
2. colour branching;
3. Higgs mass closure;
4. electric-charge assignment;
5. local anomaly cancellation;
6. global SU(2) anomaly cancellation.

The theorem will show that the minimal package is

$\mathcal{G}_1 = \{ Q_l, u_r, d_r, L_l, e_r \}$.

3.3 Definition — generation

A **generation** is one complete copy of $\mathcal{G}_1$ assigned to one independent support-return completion layer.

A generation is therefore not a colour. It is not a single particle. It is a full anomaly-stable species package.

3.4 Firewall — colour triplicity is not generation triplicity

Colour triplicity and generation triplicity are different ledger objects.

- Colour is a gauge-facing internal branch of coloured species.
- Generation is a support-return repetition of the whole species package.

The quark field Q_l carries colour inside each generation. The three generations repeat the entire quark-plus-lepton package. These two threes are not identified by notation, by arithmetic coincidence, or by rhetorical convenience. The falsifier F16 guards this firewall.

3.5 Representation-Role Ledger

The census theorem is not a field-list assertion. Each admitted multiplet closes a specific terminal-record obligation. The ledger below records the role of each minimal species.

Object	Representation role	Why admitted	Why no smaller substitute works
$Q_l = (u_l, d_l)^T$	Colour-triplet weak attachment carrier	Required by colour support branching plus weak binary attachment	A colour singlet would not represent support-coupled coloured matter; weak singlets would not carry unresolved weak binary attachment
u_r	Upper charged coloured terminal closure singlet	Required to close the upper component of Q_l through $\tilde{H}$	Without it, the upper coloured charged record has no minimal right-singlet closure channel
d_r	Lower charged coloured terminal closure singlet	Required to close the lower component of Q_l through H	Without it, the lower coloured charged record has no minimal right-singlet closure channel
$L_l = (\nu_l, e_l)^T$	Colourless weak-completion doublet	Required as the minimal repair of the odd weak-doublet parity of coloured matter	A second coloured doublet adds more Weyl fields and new colour obligations; higher weak multiplets violate minimal weak-binary attachment
e_r	Lower charged colourless terminal closure singlet	Required to close the charged lower component of L_l through H	Without it, the charged lepton has no minimal right-singlet closure channel
H	Universal weak-doublet closure carrier	Required by Higgs-mediated charged terminal closure under the minimal Standard Model boundary	H and $\tilde{H}$ close all charged channels; extra scalar doublets are therefore extensions

Object	Representation role	Why admitted	Why no smaller substitute works
ν_r	Sterile extension singlet	Not admitted in the minimal census	Not required by anomaly cancellation, weak parity, charged closure, or hypercharge uniqueness

This table is also a guard against numerology. A multiplet is admitted only when a named ledger failure exists without it. A multiplet is excluded when no such failure exists.

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4. Weak Attachment and Colour Branching

4.1 Weak binary attachment

A terminal record must be distinguishable, and in the weak sector the minimal unresolved distinction is binary. Therefore the left weak attachment sector is represented by doublets.

For coloured support-coupled matter, the minimal weak doublet is

$Q_l = (u_l, d_l)^T$.

For colourless ledger-completion matter, the minimal weak doublet is

$L_l = (\nu_l, e_l)^T$.

4.2 Why coloured weak matter alone is not enough

A colour-triplet weak doublet contains three weak doublets, one for each colour:

$Q_l : 3 \times 2$.

That gives an odd number of SU(2)_L doublets.

The global SU(2) anomaly condition — the Witten condition — is an index condition: the sum $\Sigma\ 2T(\mathcal{W})$ over left-handed Weyl fermions must be even, where $T(\mathcal{W})$ is the SU(2) representation index normalised to $T = 1/2$ for the doublet. For doublets $2T = 1$, so within doublet-only content the condition reduces exactly to counting doublets. The coloured weak doublet contributes $3 \times 1 = 3$, which is odd. Therefore the coloured weak sector alone is not a terminal physical census.

The minimal repair is one colourless weak doublet:

$L_l = (\nu_l, e_l)^T$.

Then the number of weak doublets per generation is

$3 + 1 = 4,$

which is even.

This is the first appearance of leptons in the census theorem: the lepton doublet is not decorative. It is the minimal colourless weak-completion partner required by anomaly-stable terminality.

4.3 Lemma 1 — Weak-doublet completion

Lemma 1. Given one colour-triplet weak doublet Q_l, anomaly-stable weak attachment minimally requires exactly one colourless weak doublet L_l.

Proof. The colour-triplet weak doublet Q_l contributes three left-handed weak doublets, one per colour branch. The global SU(2) anomaly requires Σ 2T to be even over left-handed Weyl fermions; for doublets $2T = 1$, so the coloured content contributes 3, which is odd and therefore inadmissible.

The parity defect must be repaired by an addition contributing an odd number of weak doublets. Three candidate repairs exist:

1. **One colourless weak doublet.** Adds one doublet (parity restored) and two Weyl fields. Induces one new hypercharge obligation, resolved in §6.
2. **A second colour-triplet weak doublet.** Adds three doublets (parity restored) but six Weyl fields, together with new $SU(3)^3$ and $SU(3)^2U(1)$ obligations and two further right-singlet closure obligations under C3.
3. **A higher weak multiplet.** For an SU(2) multiplet of isospin j, the Witten index is $2T(j) = 2j(j + 1)(2j + 1)/3$. The first half-integer multiplet above the doublet, $j = 3/2$, has $2T = 10$, which is even and cannot repair the parity at all. The smallest higher multiplet with odd index is $j = 5/2$, with $2T = 35$ and six components — already more chiral structure than the entire colourless-doublet repair. All such multiplets are in any case excluded as primitive by weak-binary minimality (C2).

Two further candidates are excluded at once. A colour representation of dimension d entering with a weak doublet shifts the parity by d mod 2, so only odd colour dimensions can repair it; the triplet is the smallest coloured option and already costs six Weyl fields, as in repair 2. And the Higgs doublet cannot serve as the repair: the global anomaly counts Weyl fermions, and H is a scalar, contributing nothing to the index.

By C7, minimality selects the repair with the fewest added Weyl fields and fewest induced obligations. Repair 1 adds two Weyl fields; repair 2 adds six and cascades. Therefore exactly one colourless weak doublet is the minimal admissible completion. ■

Remark. The lemma does more than admit the lepton doublet; it *forces* it, and forces it to be colourless. The lepton sector is the cheapest anomaly-stable repair of the quark sector's global weak defect.

4.4 Boundary of the Witten-anomaly argument

Lemma 1 proves the necessity of a minimal colourless weak doublet. It does not, by itself, determine the full lepton sector or the full hypercharge assignment. Those are fixed only after Higgs closure, ordered weak-component closure, and the anomaly equations are imposed.

The logic is sequential:

$Q_l \Rightarrow$ odd weak-doublet count

$\Rightarrow L_l$ as minimal colourless weak repair

$\Rightarrow e_r$ by charged lower-component closure

$\Rightarrow Y(Q_l), Y(u_r), Y(d_r), Y(L_l), Y(e_r)$ by Higgs closure and anomalies.

This ordering prevents the lepton doublet from being inserted by charge intuition. It first enters as the cheapest weak-anomaly repair, and only later becomes the familiar lepton doublet once charge and closure are solved.

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5. The Minimal One-Generation Species Package

5.1 Coloured sector

The coloured weak doublet is

$Q_l = (u_l, d_l)^T$.

The upper and lower components have different electric charges. To become terminal charged massive records, they require right weak-singlet closure partners:

u_r, d_r.

C1(5) requires the closure *channel* to exist; it does not require the realised mass value to be nonzero. A census containing u_r with a vanishing realised mass is a value statement for later gates. A census lacking u_r altogether has no channel at all and fails mass-closure possibility. The census question is the existence of the slot, not the number inside it.

Thus the minimal coloured package is

Q_l, u_r, d_r.

5.2 Colourless sector

The weak-completion doublet is

$L_l = (\nu_l, e_l)^T$.

The lower charged component requires a right weak-singlet closure partner:

e_r.

The upper neutral component ν_l does not require a charged right-singlet partner for minimal gauge, anomaly, or charged-mass closure. A sterile ν_r may be added in a neutrino-mass extension, but it is not forced by the minimal census.

Thus the minimal colourless package is

L_l, e_r.

5.3 Lemma 2 — Minimal one-generation matter package

Lemma 2. Under weak attachment (C2), colour branching (C4), right terminal closure (C3), anomaly stability (C6), and census minimality (C7), the minimal one-generation matter package is

$\mathcal{G}_1 = \{ Q_l, u_r, d_r, L_l, e_r \}$.

Proof. Weak binary attachment in the coloured support sector gives Q_l. Right terminal closure for the two charged coloured components gives u_r and d_r. Lemma 1 forces exactly one colourless weak doublet L_l. Right terminal closure for the charged lower component of L_l gives e_r. No sterile right-handed neutrino is required by gauge anomaly cancellation, global weak-anomaly cancellation, or charged mass closure (Lemma 5). By census minimality, no further species is admitted. ■

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6. Higgs Closure and Hypercharge Determination

6.1 Higgs closure carrier

The minimal closure carrier is a weak doublet

$H = (H^+, H^0)^T$,

with

$Y(H) = h$.

The neutral vacuum condition requires

$Q(H^0) = 0$.

Since H^0 is the lower component with $T_3 = -1/2$,

$0 = -1/2 + h/2$,

so

$h = 1$.

Thus

$Y(H) = +1$.

The conjugate doublet

$\tilde{H} = i\sigma^2 H^*$

has

$Y(\tilde{H}) = -1$

and supplies the upper-type closure channel. One doublet therefore serves three closure channels; no second primitive scalar is required (C5).

6.2 Yukawa closure equations

Let

$Y(Q_l) = Y\_Q$, $Y(L_l) = Y\_L$.

Gauge-invariant Higgs closure requires:

$\bar{Q}_l H d_r : -Y\_Q + Y\_H + Y\_d = 0$,

$\bar{Q}_l \tilde{H} u_r : -Y\_Q - Y\_H + Y\_u = 0$,

$\bar{L}_l H e_r : -Y\_L + Y\_H + Y\_e = 0$.

Therefore

$Y_d = Y_Q - Y_H$, $Y_u = Y_Q + Y_H$, $Y_e = Y_L - Y_H$.

With $Y_H = 1$,

$Y_d = Y_Q - 1$, $Y_u = Y_Q + 1$, $Y_e = Y_L - 1$.

6.3 Gauge-sector determination of hypercharge

The $SU(2)^2U(1)$ anomaly gives

$3Y_Q + Y_L = 0$,

so

$Y_L = -3Y_Q$, and hence $Y_e = Y_L - 1 = -3Y_Q - 1$.

Under ordered closure the $SU(3)^2U(1)$ condition, $2Y_Q - Y_u - Y_d = 0$, is satisfied identically:

$2Y_Q - (Y_Q + 1) - (Y_Q - 1) = 0$.

The cubic $U(1)^3$ condition over the left-handed ledger,

$6Y_Q^3 - 3Y_u^3 - 3Y_d^3 + 2Y_L^3 - Y_e^3 = 0$,

becomes after substitution

$-27Y_Q^3 + 27Y_Q^2 - 9Y_Q + 1 = 0$,

which factorises as a perfect cube:

$(1 - 3Y_Q)^3 = 0$.

The unique root is $Y_Q = +1/3$, and it is a triple root: there is no second real branch and no complex branch to exclude.

Hence

$Y_L = -1$, $Y_u = +4/3$, $Y_d = -2/3$, $Y_e = -2$.

The mixed gravitational–hypercharge condition of the continuum description,

$6Y_Q - 3Y_u - 3Y_d + 2Y_L - Y_e = 0$,

then closes automatically. Under ordered closure it reduces to precisely the linear factor of the perfect cube,

$1 - 3Y_Q = 0$,

so the surplus check and the determining cubic share the single root by construction. It is a consistency check, not a determining input.

This ordering matters for ledger licensing. VERSF treats geometry as emergent and defers gravitational response (§13.2), so a census theorem should not lean on consistency of coupling to a sector the programme has not yet derived. It does not: the census is fixed by gauge-sector conditions alone, and gravitational-anomaly consistency emerges free of charge.

6.4 Lemma 3 — Unique minimal hypercharge solution

Lemma 3. Under the convention $Q = T_3 + Y/2$, one Higgs doublet with neutral vacuum, gauge-invariant ordered Yukawa closure, and gauge-sector anomaly stability fix the minimal one-generation hypercharges uniquely as

$Y(Q_l) = +1/3,\ Y(u_r) = +4/3,\ Y(d_r) = -2/3,\ Y(L_l) = -1,\ Y(e_r) = -2,\ Y(H) = +1.$

Proof. The neutral Higgs vacuum fixes $Y_H = 1$. Ordered closure gives $Y_d = Y_Q - 1$, $Y_u = Y_Q + 1$, and $Y_e = Y_L - 1$. The $SU(2)^2U(1)$ anomaly gives $Y_L = -3Y_Q$. Under these substitutions the $SU(3)^2U(1)$ condition is satisfied identically, and the cubic $U(1)^3$ condition factorises as the perfect cube $(1 - 3Y_Q)^3 = 0$, whose only root — a triple root — is $Y_Q = +1/3$. Hence $Y_L = -1$, and the singlet hypercharges follow from ordered closure. The mixed gravitational–hypercharge condition then closes automatically as a surplus check. Appendix C shows that no second solution branch survives within the minimal field content: the triple root leaves no branch ambiguity, the unordered singlet labelling is fixed by ordered weak-component closure (C14), and without a sterile ν_r there is no B−L flat direction. ■

6.5 Electric charges

Using

$Q = T_3 + Y/2$,

the species charges are:

u : $+1/2 + 1/6 = +2/3$,

d : $-1/2 + 1/6 = -1/3$,

ν : $+1/2 - 1/2 = 0$,

$e : -1/2 - 1/2 = -1$.

Thus the charge census is recovered, not separately inserted.

6.6 Corollary — Charge quantisation and proton–electron balance

The hypercharge solution of Lemma 3 quantises electric charge across the minimal census. Every species charge is an integer multiple of one third of the electron charge:

$Q(u) = +2/3$, $Q(d) = -1/3$, $Q(e) = -1$, $Q(\nu) = 0$.

The quark and lepton charges are not independently assigned. They are locked by the same anomaly and Higgs-closure equations. In particular,

$2Q(u) + Q(d) = 2\cdot(+2/3) + (-1/3) = +1$,

so

$2Q(u) + Q(d) + Q(e) = 0$.

Thus the charge balance required for proton–electron neutrality follows from the census closure. SC-1 does not derive QCD confinement, baryon formation, or the atomic bound state of hydrogen. It derives the charge relation that makes proton–electron neutrality possible once the later QCD and atomic sectors are supplied.

In VERSF terms, the same terminal ledger conditions that force the colourless weak doublet also lock the coloured charge ledger to the colourless charged-lepton ledger.

---

7. Anomaly Stability of One Generation

7.1 Local gauge anomalies

The one-generation package is anomaly-stable.

Using left-handed fields only, replace right-handed physical fields by their conjugates:

$u_r \to u_l^c$, $d_r \to d_l^c$, $e_r \to e_l^c$.

Their hypercharges flip sign.

The left-handed anomaly ledger is:

Field	Multiplicity	Hypercharge
Q_l	3 × 2	+1⁄3
u_l^c	3	−4⁄3
d_l^c	3	+2⁄3
L_l	2	−1
e_l^c	1	+2

Then:

$[SU(3)]^3 : 2 - 1 - 1 = 0.$

$[SU(3)]^2U(1) : 2\cdot(1/3) - 4/3 + 2/3 = 0.$

$[SU(2)]^2U(1) : 3\cdot(1/3) - 1 = 0.$

$[grav]^2U(1) : 6\cdot(1/3) + 3\cdot(-4/3) + 3\cdot(2/3) + 2\cdot(-1) + 2 = 2 - 4 + 2 - 2 + 2 = 0.$

$[U(1)]^3 : 6\cdot(1/3)^3 + 3\cdot(-4/3)^3 + 3\cdot(2/3)^3 + 2\cdot(-1)^3 + 2^3 = 0.$

The cubic sum is verified digit by digit in Appendix B.

7.2 Global weak anomaly

The number of SU(2)_L doublets per generation is

3 + 1 = 4.

Equivalently, Σ 2T = 4 over the left-handed Weyl fermions of one generation, since each doublet carries 2T = 1. The Higgs doublet, being scalar, contributes nothing to the count. The sum is even, so the global weak anomaly is absent.

7.3 Lemma 4 — One-generation anomaly closure

Lemma 4. The species package

$\mathcal{G}_1 = \{ Q_l, u_r, d_r, L_l, e_r \}$

with the hypercharges of Lemma 3 cancels all required perturbative gauge anomalies, the mixed gravitational–hypercharge anomaly, and the global SU(2) anomaly.

Proof. The five anomaly sums above vanish. The number of weak doublets is even. Therefore the one-generation package is anomaly-stable. ■

8. Why the Sterile Neutrino Is Not Minimal

8.1 The sterile singlet

A right-handed neutrino would have the representation

$\nu_r \sim (1, 1, 0)$

under (SU(3)_c, SU(2)_L, U(1)_Y).

It is neutral under colour, weak isospin, and hypercharge. Therefore it does not affect any gauge anomaly cancellation equation, does not alter the weak-doublet parity, and does not participate in charged mass closure.

8.2 Minimality

A sterile singlet may be physically important in a neutrino-mass extension. SC-1 does not deny that. It says only that ν_r is not forced by the minimal species census.

The minimal census already has:

- weak attachment closure;
- charged mass closure;
- electric-charge closure;
- anomaly cancellation;
- global weak-anomaly stability.

Adding ν_r introduces a new sterile species role not required by any of those conditions. Its inclusion would also weaken the census theorem itself: with ν_r present, the anomaly system acquires a B−L flat direction, and hypercharge is no longer uniquely determined without additional input (Appendix C). Minimality and uniqueness therefore reinforce one another.

8.3 Lemma 5 — Sterile neutrino extension status

Lemma 5. A right-handed neutrino is not part of the minimal SC-1 census. It is an admissible extension species only when a neutrino-mass or sterile-sector bridge is declared.

Proof. ν_r is a gauge singlet with $Y = 0$. It is not required for anomaly cancellation. It is not required for charged fermion mass closure. It is not required to make the weak-doublet count even. Therefore, by census minimality (C7), it is not admitted into the minimal species package. Since it can participate in Dirac or Majorana neutrino-mass extensions, it is not forbidden; it is typed as an extension. ■

9. The Generation Count

9.1 What a generation is

A generation is not an individual particle. It is one complete anomaly-stable copy of the species package:

$\mathcal{G}_i = \{ Q_{li}, u_{ri}, d_{ri}, L_{li}, e_{ri} \}$.

Each generation is gauge-identical at the census level. Differences in masses and mixings belong to later completion-density and hierarchy gates.

9.2 Why anomaly cancellation alone does not fix N_g

Because one generation is anomaly-stable by itself (Lemma 4), anomaly cancellation permits

$N_g = 1, 2, 3, \ldots$

at the purely gauge-anomaly level.

Therefore VERSF must not pretend that anomaly cancellation alone proves three generations. Any programme making that claim is overreaching, and SC-1 explicitly disowns it (Firewalls, §0.2).

The generation count requires the VERSF support-return completion principle.

9.3 Rank-three support-return completion

The inherited support-return theorem (I2, restated as C9) states that the physical support quotient has exactly three independent completion layers:

$d_{comp} = 3$.

SC-1 uses only this rank statement. It does not reuse the numerical stiffness value, nor does it infer generation count from colour triplicity or from anomaly cancellation alone.

Let the three independent completion layers be denoted

R_1, R_2, R_3.

A completion layer is not merely a label. It is an independently terminal support-return direction. Therefore the projection of the physical census onto each layer must be terminally admissible (C12).

This gives the central bridge:

$R_i \Longrightarrow$ one complete anomaly-stable species package.

By layer-exclusive attachment (C15), the assignment is one-to-one: each terminal carrier attaches to exactly one layer, so packages are not shared between layers.

The bridge is not a numerical identification. It is a terminality condition: an independent support-return layer cannot be populated by a partial generation, because partial packages fail at least one of weak attachment, Higgs closure, charge assignment, or anomaly stability.

9.4 Lemma 6 — Layer-local generation realisation

Lemma 6. Each independent support-return completion layer requires one complete anomaly-stable species package.

Proof. Let R_i be an independent support-return completion layer, and let P_i be projection onto that layer. By layer-local terminality (C12), $P_i\mathcal{C}$ must itself be closed under the terminal census conditions: weak attachment, colour branching, charged closure, electric-charge assignment, and anomaly stability.

Suppose $P_i\mathcal{C}$ contains only part of the species package.

If Q_l is present without the required right singlets, at least one charged coloured terminal record lacks Higgs-mediated closure. If Q_l is present without L_l, the layer contains three weak doublets and fails the global weak-anomaly parity test. If L_l is present without e_r, the charged lower colourless component lacks closure. If the hypercharge assignments are not those fixed by ordered Higgs closure and the anomaly equations, the layer fails charge or anomaly terminality.

Therefore a partial projected package is not terminal. It cannot be made terminal by borrowing a missing species or anomaly cancellation from another layer, because such borrowing would make terminality depend on the unprojected aggregate rather than on the independent completion direction itself. That violates C12.

By Lemmas 1–5, the minimal projected package satisfying the terminality conditions is

$\mathcal{G}_i = \{ Q_{li}, u_{ri}, d_{ri}, L_{li}, e_{ri} \}$.

Therefore every independent support-return layer requires one complete anomaly-stable species package. ■

9.5 Lemma 7 — No inter-layer anomaly borrowing

Lemma 7. A partial generation cannot be made admissible by cancelling its anomaly or closure defect against another support-return layer.

Proof. Gauge anomaly cancellation is necessary for the full physical census, but VERSF terminality is stronger: each independently projected support-return layer must itself define an admissible terminal record ledger (C12). If a projected layer is anomalous, incomplete, or lacks a charged closure partner, then the layer is not terminal. Adding another incomplete layer with an opposite defect may cancel the aggregate anomaly polynomial, but it does not repair the projected failure.

Inter-layer mixing is a later map between completed packages (C13). It cannot supply the species needed to define the source and target packages in the first place. Therefore anomaly or closure defects cannot be repaired by cross-layer borrowing at the census stage. ■

9.6 Lemma 8 — Generation necessity

Lemma 8. At least three complete generations are required.

Proof. The physical support-return quotient has three independent completion layers:

R_1, R_2, R_3.

By Lemma 6, each such layer requires one complete anomaly-stable species package. By layer-exclusive attachment (C15), a package of terminal carriers can serve at most one layer: each carrier attaches to exactly one completion direction, so a single package cannot simultaneously constitute the completion of two independent layers. Therefore, if fewer than three packages are present, at least one independent completion layer is unserved.

An unserved layer is not an innocuous absence. It means that an independent terminal support-return direction lacks the species package required to carry colour, weak attachment, charged closure, and anomaly-stable gauge membership. This contradicts terminal support completion.

Therefore

$N_g \geq 3$. ■

9.7 Lemma 9 — Generation sufficiency and exclusion of a fourth minimal generation

Lemma 9. Three complete generations are sufficient, and a fourth generation is not part of the minimal census.

Proof. Three complete packages,

$\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3$,

can be assigned one-to-one to the three support-return completion layers,

R_1, R_2, R_3.

Each package is anomaly-stable by Lemma 4. Each package has the minimal species content of Lemma 2. Each layer is therefore terminally populated.

Thus

$N_g = 3$

is sufficient.

A fourth generation would require a fourth independently terminal support-return layer, a new primitive record obligation, or a new anomaly-stable sector that cannot be represented inside the existing three-layer quotient. None is present in the minimal ledger (C11). A fourth package would therefore duplicate an already completed support-return direction without closing any new primitive obligation. By census minimality (C7), it is excluded from the minimal census.

Therefore

$N_g \leq 3$

inside the minimal physical quotient. Combining this with Lemma 8 gives

$N_g = 3$. ■

9.8 Corollary — Mixing does not weaken the generation theorem

The existence of CKM and PMNS-type mixing does not undermine the generation theorem. Mixing matrices are defined only after there are complete generation slots to mix (C13).

In census language, the generation theorem supplies the basis size:

$N_g = 3$.

Later mass and mixing gates may introduce non-diagonal Yukawa matrices acting across these three slots. Such matrices can rotate between completed layers, but they cannot explain away the need for the layers themselves.

Thus the order of derivation is:

support-return layers ⟹ complete generation slots ⟹ Yukawa matrices ⟹ mass eigenstates and mixing.

SC-1 closes only the first two arrows.

9.9 Epistemic note

The generation theorem is exactly as strong as premise C9. SC-1 does not restate the derivation of rank-three support; it inherits that result through I2 and exposes it to falsification through C9's own falsifier. The nontrivial content added here is the bridge, not the rank: layer-local terminality (C12) and layer-exclusive attachment (C15) are what convert "three layers" into "three complete generations" — C12 forbids a layer from lacking structure, and C15 forbids layers from sharing it. If the rank-three result fails upstream, the generation count fails here, and the failure propagates transparently through the ledger. This is the intended behaviour of a gated programme: no result is stronger than its weakest declared premise, and every dependence is named.

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10. The Full Minimal Census

10.1 One-generation multiplet census

Representations are written as triples ($\mathcal{C}$, $\mathcal{W}$, Y) under (SU(3)_c, SU(2)_L, U(1)_Y).

One generation contains the five chiral matter multiplets:

Multiplet	Representation	Hypercharge
$Q_l = (u_l, d_l)^T$	(3, 2)	+1⁄3
u_r	(3, 1)	+4⁄3
d_r	(3, 1)	−2⁄3
$L_l = (\nu_l, e_l)^T$	(1, 2)	−1
e_r	(1, 1)	−2

This is equivalent to fifteen left-handed Weyl fields per generation when right-handed fields are written as left-handed conjugates.

10.2 Three-generation matter census

The full minimal matter census is

$3 \times [\ (3, 2, +1/3) \oplus (3, 1, +4/3) \oplus (3, 1, -2/3) \oplus (1, 2, -1) \oplus (1, 1, -2)\ ]$.

Equivalently, in left-handed conjugate notation:

$3 \times [\ (3, 2, +1/3) \oplus (\bar{3}, 1, -4/3) \oplus (\bar{3}, 1, +2/3) \oplus (1, 2, -1) \oplus (1, 1, +2)\ ]$.

The total minimal chiral matter count is

$3 \times 15 = 45$

left-handed Weyl fields, excluding sterile right-handed neutrinos.

10.3 Higgs closure carrier

Given the minimal Standard Model boundary (I5) and Higgs-mediated charged-fermion closure (C5), one weak doublet

$H = (H^+, H^0)^T \sim (1, 2, +1)$

is sufficient to close all charged fermion channels. The conjugate doublet $\tilde{H} = i\sigma^2 H^*$ closes the upper-type channel, while H closes the lower-type and charged-lepton channels:

$\bar{Q}_l \tilde{H} u_r$, $\bar{Q}_l H d_r$, $\bar{L}_l H e_r$.

Therefore no second primitive scalar doublet is forced at the census level. Additional Higgs doublets, composite-Higgs mechanisms, or alternative electroweak-breaking sectors are not ruled out absolutely; they are extension ledgers. They require additional bridge structure because they introduce new scalar species, new flavour obligations, or new closure maps not needed by the minimal census.

The precise SC-1 claim is therefore:

one Higgs doublet is minimal and sufficient under Higgs-mediated charged closure,

not

all conceivable scalar or electroweak-breaking extensions are impossible.

10.4 Gauge carrier census

Given the inherited local gauge algebra,

SU(3)_c × SU(2)_L × U(1)_Y,

the gauge carrier census is:

Sector	Gauge carriers
SU(3)_c	8 gluons
SU(2)_L	3 weak carriers
U(1)_Y	1 hypercharge carrier

Before electroweak symmetry breaking, there are therefore

$8 + 3 + 1 = 12$

gauge carriers.

After electroweak symmetry breaking, the weak and hypercharge carriers reorganise into

W^+, W^-, Z, γ.

The census transformation is a field-basis change plus Higgs closure, not a change in the underlying gauge-carrier count.

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11. The Generation and Species Census Theorem

11.1 Statement

Theorem 1 — Generation and Species Census Theorem. Under premises C1 through C15, the minimal VERSF/Standard Model terminal matter census is uniquely:

$N_g = 3$

complete generations, each with the species package

$\mathcal{G}_i = \{ Q_{li}, u_{ri}, d_{ri}, L_{li}, e_{ri} \}$,

with hypercharges

$Y(Q_l) = +1/3,\ Y(u_r) = +4/3,\ Y(d_r) = -2/3,\ Y(L_l) = -1,\ Y(e_r) = -2,$

and one universal Higgs closure carrier

$H = (H^+, H^0)^T,\ Y(H) = +1.$

The sterile right-handed neutrino is not part of the minimal census. It is an admissible extension species only when a neutrino-mass bridge is supplied.

11.2 Proof

By weak binary attachment (C2) and colour support branching (C4), the coloured matter sector requires a colour-triplet weak doublet Q_l. By right terminal closure (C3), its upper and lower charged components require weak-singlet partners u_r and d_r.

The coloured weak doublet contributes three left-handed weak doublets, one per colour. The global SU(2) anomaly requires an even number of weak doublets. By Lemma 1, exactly one colourless weak doublet L_l is the minimal repair. By right terminal closure, the charged lower component requires e_r. No sterile right-handed neutrino is required by anomaly closure or charged mass closure (Lemma 5). Thus the minimal one-generation species package is

$\mathcal{G}_1 = \{ Q_l, u_r, d_r, L_l, e_r \}$.

The neutral Higgs vacuum fixes Y(H) = 1. Ordered closure and the gauge-sector anomaly conditions then uniquely determine the hypercharges of Lemma 3 through the perfect-cube factorisation $(1 - 3Y\_Q)^3 = 0$, with the mixed gravitational–hypercharge condition closing automatically as a surplus check, and Appendix C shows no second solution branch exists within the minimal field content. Lemma 4 verifies that the one-generation package cancels all required anomalies.

By layer-local terminality (C12) and support-return generation completeness (C8, C10), each independent completion layer must carry one full anomaly-stable species package (Lemma 6); a partial package cannot be repaired by inter-layer anomaly borrowing or mixing (Lemma 7, C13); and by layer-exclusive attachment (C15) a single package cannot serve more than one layer. The physical support-return quotient has rank three (C9). Therefore at least three generations are required (Lemma 8). Three are sufficient because they fill the three layers with complete anomaly-stable packages, and a fourth generation would require a fourth completion layer or an extended ledger (Lemma 9, C11). Therefore the minimal generation count is

N_g = 3.

Combining the species package with the generation count gives the stated census. ■

11.3 Corollary 1 — No partial generations

A partial generation is inadmissible. Since anomaly closure, mass closure, and weak-doublet parity are package-level properties, and since layer-local terminality forbids borrowing them across layers (Lemma 6, Lemma 7, C12), any generation-like layer must contain the full package

Q_l, u_r, d_r, L_l, e_r.

11.4 Corollary 2 — No fourth generation without extension

A fourth generation is not forbidden by notation. It is excluded from the minimal census because there is no fourth support-return completion layer. A fourth generation becomes admissible only if a later theory supplies new support rank, new record obligation, or a new sector requiring a fourth complete package.

11.5 Corollary 3 — Charge quantisation

Electric-charge quantisation across the census, including exact proton–electron charge balance, follows from the theorem (§6.6). It is not an additional assumption.

11.6 Corollary 4 — Mass hierarchy is downstream

The theorem gives three generation slots. It does not give the mass values inside those slots.

Mass ordering, hierarchy, CKM mixing, PMNS mixing, and CP phases require later completion-density and Yukawa gates.

11.7 Corollary 5 — Family universality of hypercharge

With three generations, aggregate anomaly cancellation alone admits generation-non-universal hypercharge assignments: anomaly-free deformations can redistribute charges between generations while preserving the total anomaly polynomial. The census theorem excludes these at the census level. By layer-local terminality (C12) and layer-exclusive attachment (C15), the hypercharge determination of Proposition C.1 applies to each support-return layer separately; and because every layer closes through the single universal carrier H (C5), each layer is forced to the identical assignment of Lemma 3. Hypercharge is therefore generation-universal in the minimal census — not as an assumption, but as a corollary.

Falsifier (F25): exhibit a generation-non-universal hypercharge assignment that is admissible layer by layer under ordered closure through the single Higgs doublet.

---

12. Why This Is Not Numerology

12.1 The obvious criticism

A critic might say:

You wanted the Standard Model census, so you created premises that reproduce it.

That would be valid if the paper merely listed the Standard Model fields and declared them necessary. It does not.

It gives each species a job:

- Q_l: coloured weak attachment carrier;
- u_r, d_r: coloured terminal mass-closure singlets;
- L_l: colourless weak anomaly-completion doublet;
- e_r: charged-lepton mass-closure singlet;
- H: universal weak-doublet closure carrier;
- three generations: rank-three support-return completion layers.

A field is admitted only when it closes a specific ledger failure, and the failure it closes is named before the field is admitted. The representation-role ledger of §3.5 records this discipline multiplet by multiplet, including the "why no smaller substitute works" column that a curated list could not fill.

12.2 The single standard

The census uses one rule:

Admit only species required for terminal record closure; reject species not required unless an extension bridge supplies new structure.

That rule admits e_r because charged-lepton closure requires it.

It rejects ν_r from the minimal census because no anomaly or charged mass closure requires it.

It admits L_l because coloured weak matter alone fails the global weak-anomaly parity test, and Lemma 1 shows the colourless doublet is the cheapest admissible repair.

It rejects a fourth generation because no fourth support-return layer exists in the minimal physical quotient.

The standard is applied uniformly. There is no field admitted for one reason and another field of the same type rejected under the same reason. That uniformity is what separates a census theorem from a curated list.

12.3 The three is not colour

The theorem uses two different triplicities:

1. colour triplicity inside the quark sector;
2. generation triplicity across support-return completion layers.

They are not identified.

This prevents a common false shortcut:

three colours ⇒ three generations.

SC-1 does not make that argument. Colour is a gauge-facing branch. Generation is a complete package repetition. The two threes have different origins, different falsifiers, and different downstream consumers.

12.4 The overdetermination check

A curated list can always be made anomaly-free by construction. The SC-1 census is not merely consistent; it is overdetermined. The hypercharges are fixed by gauge-sector conditions alone — neutral vacuum, ordered closure, $SU(2)^2U(1)$, and the cubic $U(1)^3$, which factorises as the perfect cube $(1 - 3Y(Q_l))^3 = 0$ — and the surplus constraints then close automatically: $SU(3)^2U(1)$ identically, and the mixed gravitational–hypercharge condition at the root, where it reduces to the linear factor of the cube itself. The gravitational check is genuinely surplus: the census predicts consistency of the continuum-coupled anomaly rather than assuming it, which matters in a programme where geometry is emergent and gravitational response is a deferred gate. Overdetermined closure is a structural signature, not a fitting outcome: a wrong census generically fails the surplus constraints.

12.5 The fourth-generation falsifier

The theorem is falsifiable.

A fourth generation would be allowed if a later result showed:

- a fourth support-return layer;
- a fifth primitive record obligation;
- a new anomaly-stable sector demanding another complete package;
- or an empirical terminal record that cannot be placed inside the three-generation census.

Without such evidence, a fourth generation is not minimal.

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13. Relation to Later Mass, Mixing, and Coupling Gates

13.1 What later papers inherit

Later VERSF Standard Model derivations inherit:

$N_g = 3$,

$\mathcal{G}_i = \{ Q_{li}, u_{ri}, d_{ri}, L_{li}, e_{ri} \}$,

$H = (H^+, H^0)^T$, $Y(H) = +1$.

They may no longer treat the list of species as an empirical accident or adjustable input. Any later paper that adds a species, removes a species, or repopulates a generation slot must declare an extension bridge and accept the corresponding ledger debt.

13.2 What later papers must still derive

SC-1 does not derive:

- fermion masses;
- Yukawa matrices;
- CKM angles;
- CKM CP phase;
- neutrino masses;
- PMNS angles;
- gauge couplings;
- Higgs quartic;
- electroweak scale;
- gravitational response.

It supplies the census foundation for those derivations.

13.3 CKM and PMNS

With three generations, the CKM matrix has exactly one physical CP-violating phase slot. With fewer than three generations, no such quark-sector CP phase exists in the charged-current mixing matrix (Appendix D).

SC-1 therefore supplies the minimal generation count compatible with a nontrivial CKM phase. It does not predict the value of that phase. This is a genuine consistency dividend: the same rank-three completion that fixes N_g = 3 opens exactly the CP-orientation slot that the observed quark sector occupies.

PMNS belongs to the neutrino extension ledger. The minimal census contains ν_l inside L_l, but no sterile ν_r. A neutrino-mass gate must state whether it uses Majorana mass, Dirac mass, seesaw structure, sterile species, or another VERSF bridge, and must accept the loss of hypercharge uniqueness that a sterile singlet reintroduces (Appendix C) unless the bridge closes it.

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14. Falsification Conditions

#	Failure	Consequence
F1	Species are treated as labels rather than terminal-record roles	Census theorem under-specified
F2	Weak distinction does not require binary attachment	SU(2)_L doublet premise fails
F3	Charged massive records do not require right-singlet closure	u_r, d_r, e_r derivation fails
F4	Colour support branching is not triplet	Quark-sector census fails
F5	Global SU(2) anomaly need not cancel	Lepton-doublet necessity weakens
F6	An anomalous census is physically admissible	Anomaly-stability premise fails
F7	Hypercharge convention changes without conversion	Charge derivation invalid
F8	One Higgs doublet is not sufficient for minimal closure	Higgs minimality fails
F9	More than one Higgs doublet is required before extensions	Minimal Higgs census fails
F10	ν_r is required by minimal anomaly or charged closure	Sterile-extension status fails
F11	A partial generation is anomaly-stable and terminal	Generation completeness fails
F12	Physical support-return rank is not three	N_g = 3 fails
F13	Fewer than three completion layers suffice	Generation necessity fails
F14	A fourth completion layer exists in the minimal quotient	No-fourth-generation result fails
F15	A fourth generation is empirically required without an extension ledger	SC-1 incomplete or falsified
F16	Colour triplicity and generation triplicity are identified	Ledger collision
F17	Mass hierarchy is inferred from census alone	Overclaim
F18	PMNS is derived without declaring neutrino extension	Sector boundary violation
F19	A smaller admissible repair of the weak-parity defect than one colourless doublet is exhibited	Lemma 1 minimality fails
F20	A second hypercharge solution branch survives ordered closure within the minimal field content	Lemma 3 / Proposition C.1 uniqueness fails
F21	A support-return layer is terminal while relying on another layer for anomaly cancellation or closure	Layer-local terminality (C12) fails; Lemmas 6–7 fail
F22	A mixing term makes a missing species admissible before both slots are independently terminal	Post-census mixing premise (C13) fails
F23	Ordered upper/lower weak-component closure is violated while the charged terminal census is preserved	Ordered-closure premise (C14) fails; up/down hypercharge ordering unfixed
F24	A terminal carrier's projection onto two independent support-return layers is complete in both	Layer-exclusive attachment (C15) fails; Lemma 8 necessity fails

#	Failure	Consequence
F25	A generation-non-universal hypercharge assignment is admissible layer by layer under ordered closure through the single H	Family-universality corollary fails

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15. SC-1 Closure Certificate

SC-1 condition	Result
Species defined as terminal-record carrier roles	Closed
Generation defined as complete support-return package	Closed
Colour/generation firewall stated	Closed
Weak binary attachment premise stated	Closed
Coloured weak doublet role derived	Closed, conditional on colour support branching
Right-singlet closure roles derived	Closed
Lepton-doublet necessity derived from weak anomaly parity	Closed
Minimality of the colourless repair proved	Closed
Minimal one-generation package derived	Closed
Higgs closure carrier stated	Closed
Hypercharge assignments derived from gauge-sector conditions and ordered closure	Closed
Gravitational–hypercharge consistency verified as surplus check	Closed
Hypercharge uniqueness within minimal field content proved	Closed
Family universality of hypercharge derived	Closed
Local anomaly cancellation verified	Closed
Global SU(2) anomaly cancellation verified	Closed
Overdetermination consistency checks verified	Closed
Charge quantisation corollary stated	Closed
Sterile ν_r typed as extension	Closed
Rank-three support-return generation principle stated	Closed
Layer-local terminality stated and applied	Closed
Layer-exclusive attachment stated and applied	Closed
Inter-layer anomaly borrowing excluded	Closed
Mixing typed as post-census structure	Closed
Ordered weak-component closure stated and applied	Closed

SC-1 condition	Result
Anomaly branch structure recorded and non-SM branches excluded	Closed
Representation-role ledger supplied	Closed
$N\_g \geq 3$ necessity proved	Closed
$N\_g \leq 3$ minimality proved	Closed
$N\_g = 3$ derived	Closed
Fourth generation excluded from minimal ledger	Closed, conditional on no fourth support-return layer
Full minimal matter census supplied	Closed
Gauge carrier census ledgered	Closed, inherited from local gauge algebra
Mass values	Outside scope
Yukawa hierarchy	Outside scope
CKM values	Outside scope
PMNS/neutrino-mass mechanism	Extension gate
Gauge-coupling values	Outside scope
Origin of gauge algebra	Outside scope unless inherited

SC-1 closure grade: conditionally closed as a minimal generation and species census theorem.

SC-1 closes the matter-census problem inside the active VERSF/Standard Model attachment ledger. It derives the minimal one-generation chiral package, fixes the hypercharge assignments under the declared convention and Higgs-closure mechanism, excludes ν_r from the minimal census, and derives N_g = 3 from inherited rank-three support-return completion plus layer-local terminality and layer-exclusive attachment. The hypercharge determination uses gauge-sector conditions only; the mixed gravitational–hypercharge condition is verified as a surplus check, so the census does not presuppose the deferred gravitational-response sector.

The closure is conditional in three explicit ways:

1. the Standard Model gauge algebra is inherited, not derived here;
2. rank-three support-return completion is inherited, not reproved here;
3. Higgs-mediated charged closure is the minimal Standard Model boundary condition, not a proof against all electroweak-breaking extensions.

Within those conditions, SC-1 fixes the species list that later VERSF gates must use. A later paper may add species or sectors only by declaring an extension bridge and servicing the corresponding anomaly, closure, and support-return debts.

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16. Conclusion

SC-1 closes the census gate.

The Standard Model is not just a set of numerical parameters. It is first a census: a specific list of admissible matter species, repeated three times, with one Higgs closure carrier and a fixed gauge-attachment structure.

This paper derives that census inside VERSF.

One complete generation is forced by weak attachment, colour branching, terminal mass closure, and anomaly stability:

Q_l, u_r, d_r, L_l, e_r.

The Higgs doublet supplies the minimal closure carrier:

$H = (H^+, H^0)^T$, $Y(H) = +1$.

Gauge-invariant closure and anomaly cancellation fix the hypercharges uniquely:

+1⁄3, +4⁄3, −2⁄3, −1, −2.

The generation count follows from rank-three support-return completion:

N_g = 3.

A sterile right-handed neutrino is not minimal, though it remains admissible in a declared neutrino-mass extension. A fourth generation is not minimal, though it remains admissible only if a fourth support-return layer, new primitive record obligation, or new anomaly-stable sector is supplied. Electric-charge quantisation, including the proton–electron charge balance, emerges as a corollary rather than an input.

SC-1 therefore advances VERSF from convention discipline into Standard Model census content.

It does not yet derive mass values, hierarchy exponents, mixing angles, CP phases, gauge-coupling values, or neutrino masses. It does something prior and necessary: it fixes the terminal matter ledger on which those later derivations must operate:

$3 \times \{ Q_l, u_r, d_r, L_l, e_r \}$,

with

$H = (H^+, H^0)^T$, $Y(H) = +1$,

and hypercharges

$Y(Q_l) = +1/3$, $Y(u_r) = +4/3$, $Y(d_r) = -2/3$, $Y(L_l) = -1$, $Y(e_r) = -2$.

In one sentence:

Given weak binary attachment, colour support branching, ordered Higgs-mediated charged closure, anomaly stability, rank-three support-return completion, layer-local terminality, and layer-exclusive attachment, VERSF's minimal Standard Model matter census is exactly three complete anomaly-stable generations, each containing Q_l, u_r, d_r, L_l, e_r, with one Higgs doublet as the universal closure carrier.

Appendix A — Minimal Algebraic Skeleton

Representations are triples $(\mathcal{C}, \mathcal{W}, Y)$ under (SU(3)_c, SU(2)_L, U(1)_Y).

One generation:

$Q_l = (u_l, d_l)^T \sim (3, 2, +1/3)$,

$u_r \sim (3, 1, +4/3)$,

$d_r \sim (3, 1, -2/3)$,

$L_l = (\nu_l, e_l)^T \sim (1, 2, -1)$,

$e_r \sim (1, 1, -2)$.

Higgs:

$H = (H^+, H^0)^T \sim (1, 2, +1)$, $\tilde{H} = i\sigma^2 H^*$.

Yukawa closure:

$\bar{Q}_l H d_r$, $\bar{Q}_l \tilde{H} u_r$, $\bar{L}_l H e_r$.

Generation count:

N_g = d_comp = 3.

Full matter census:

$3 \times [\ (3, 2, +1/3) \oplus (3, 1, +4/3) \oplus (3, 1, -2/3) \oplus (1, 2, -1) \oplus (1, 1, -2)\ ]$.

Appendix B — Hypercharge and Anomaly Calculations

B.1 Higgs vacuum

Let

$Y(H) = h$.

The neutral Higgs condition gives

$0 = Q(H^0) = -1/2 + h/2$,

so

$h = 1$.

B.2 Yukawa closure

$Y_u = Y_Q + h$, $Y_d = Y_Q - h$, $Y_e = Y_L - h$.

B.3 Gauge-sector determination

Ordered closure with $h = 1$ gives

$Y_u = Y_Q + 1$, $Y_d = Y_Q - 1$, $Y_e = Y_L - 1$.

The $SU(2)^2U(1)$ anomaly gives

$3Y_Q + Y_L = 0$, so $Y_L = -3Y_Q$ and $Y_e = -3Y_Q - 1$.

The $SU(3)^2U(1)$ condition is then satisfied identically:

$2Y_Q - (Y_Q + 1) - (Y_Q - 1) = 0$.

The cubic $U(1)^3$ condition becomes

$6Y_Q^3 - 3(Y_Q + 1)^3 - 3(Y_Q - 1)^3 + 2(-3Y_Q)^3 - (-3Y_Q - 1)^3 = 0$.

Expanding term by term:

$(Y_Q + 1)^3 + (Y_Q - 1)^3 = 2Y_Q^3 + 6Y_Q$, so $-3[(Y_Q + 1)^3 + (Y_Q - 1)^3] = -6Y_Q^3 - 18Y_Q$,

$2(-3Y_Q)^3 = -54Y_Q^3$,

$-(-3Y_Q - 1)^3 = (3Y_Q + 1)^3 = 27Y_Q^3 + 27Y_Q^2 + 9Y_Q + 1$.

Summing:

$$6Y_Q^3 - 6Y_Q^3 - 18Y_Q - 54Y_Q^3 + 27Y_Q^3 + 27Y_Q^2 + 9Y_Q + 1 = -27Y_Q^3 + 27Y_Q^2 - 9Y_Q + 1,$$

which factorises as a perfect cube:

$$(1 - 3Y_Q)^3 = 0.$$

The unique (triple) root is

$$Y_Q = +1/3,$$

hence

$$Y_L = -1,\ Y_u = +4/3,\ Y_d = -2/3,\ Y_e = -2.$$

B.4 Surplus check — mixed gravitational–hypercharge condition

The condition

$$6Y_Q - 3Y_u - 3Y_d + 2Y_L - Y_e = 0$$

reduces, under the ordered closure and $SU(2)^2U(1)$ substitutions, to

$$1 - 3Y_Q = 0,$$

which is precisely the linear factor of the perfect cube in B.3. The surplus check and the determining cubic therefore share the single root $Y_Q = 1/3$ by construction. Numerically, over the left-handed ledger at the solution:

$$6\cdot(1/3) + 3\cdot(-4/3) + 3\cdot(2/3) + 2\cdot(-1) + 2 = 2 - 4 + 2 - 2 + 2 = 0. ✓$$

This condition is not used as a determining input anywhere in SC-1.

B.5 Numerical verification — cubic at the root

Over the left-handed ledger:

$6\cdot(1/3)^3 + 3\cdot(-4/3)^3 + 3\cdot(2/3)^3 + 2\cdot(-1)^3 + 1\cdot(2)^3$.

Term by term:

$6\cdot(1/27) = 6/27$,

$3\cdot(-64/27) = -192/27$,

$3\cdot(8/27) = 24/27$,

$2\cdot(-1) = -2 = -54/27$,

$1\cdot(8) = +8 = +216/27$.

Sum:

$(6 - 192 + 24 - 54 + 216)/27 = 0/27 = 0$. ✓

The cubic is a determining input in SC-1; its numerical closure at the root verifies the perfect-cube factorisation of B.3.

Appendix C — Hypercharge Uniqueness, Anomaly Branches, and Ordered Closure

C.1 Why anomaly cancellation alone is not enough

Hypercharge uniqueness is sometimes misstated in two opposite directions. In one direction, anomaly cancellation alone is claimed to fix the Standard Model hypercharges; it does not, because the pure anomaly system leaves residual freedom before a mass-closure mechanism is declared. In the other direction, the derivation is made to depend on the mixed gravitational–hypercharge anomaly; SC-1 avoids this, because a programme in which geometry is emergent should not rest its census on consistency of coupling to a sector it has not yet derived.

The SC-1 claim is:

neutral Higgs vacuum + ordered Higgs closure + minimal field content + gauge-sector anomaly cancellation ⇒ unique Standard Model hypercharge assignment.

The gravitational–hypercharge condition is then a surplus check that closes automatically.

C.2 Variables and gauge-sector anomaly equations

Let

$Y(Q_l) = a$, $Y(u_r) = b$, $Y(d_r) = c$, $Y(L_l) = \ell$, $Y(e_r) = e$.

The gauge-sector anomaly equations, with right-handed contributions entering with the opposite sign in the left-handed ledger, are:

$[SU(3)]^2U(1) : 2a - b - c = 0,$

$[SU(2)]^2U(1) : 3a + \ell = 0,$

$[U(1)]^3 : 6a^3 - 3b^3 - 3c^3 + 2\ell^3 - e^3 = 0.$

These are three conditions on five unknowns. Before a closure mechanism is declared, the system is underdetermined: gauge anomaly cancellation alone does not fix hypercharge. This is why the census theorem imposes ordered Higgs closure as a census condition rather than an afterthought.

C.3 Ordered closure and the perfect-cube determination

The neutral Higgs vacuum fixes $Y(H) = h = 1$ (§6.1). Ordered closure (C14) gives

$b = a + 1$, $c = a - 1$, $e = \ell - 1$.

The $SU(2)^2U(1)$ condition gives $\ell = -3a$, hence $e = -3a - 1$.

Under these substitutions the $SU(3)^2U(1)$ condition is satisfied identically:

$2a - (a + 1) - (a - 1) = 0.$

The cubic condition becomes

$6a^3 - 3(a + 1)^3 - 3(a - 1)^3 + 2(-3a)^3 - (-3a - 1)^3 = -27a^3 + 27a^2 - 9a + 1,$

which factorises as a perfect cube:

$(1 - 3a)^3 = 0.$

The unique root is

$a = +1/3,$

and it is a triple root: there is no second real branch and no complex branch to exclude. Hence

$\ell = -1$, $b = +4/3$, $c = -2/3$, $e = -2$,

which is the assignment of Lemma 3. The degenerate $a = 0$ possibility of the unconstrained anomaly system never arises here: $1 - 3 \cdot 0 \neq 0$.

C.4 The gravitational–hypercharge condition as a surplus check

The mixed gravitational–hypercharge condition of the continuum description is

$$6a - 3b - 3c + 2\ell - e = 0.$$

Under the ordered-closure and $SU(2)^2U(1)$ substitutions it reduces to

$$1 - 3a = 0,$$

which is precisely the linear factor of the perfect cube in C.3. The surplus check and the determining cubic share the single root $a = 1/3$ by construction. SC-1 therefore does not use this condition as an input; it records it as a surplus consistency check. The dependency order matters: the census is fixed by gauge-sector conditions alone, and consistency of the continuum-coupled anomaly emerges as a prediction rather than an assumption.

For comparison with the literature, if the gravitational condition is instead imposed on the pre-closure anomaly system, it gives $e = -6a$, and the cubic then factorises as

$$18a(8a^2 + bc) = 0,$$

with a non-degenerate branch $\{ b, c \} = \{ 4a, -2a \}$ and a degenerate branch $a = 0$. That is the classical branch analysis. SC-1 does not rely on it, and both residual branches are in any case eliminated by the closure route of C.3, where the perfect cube admits only $a = 1/3$ and the up/down labelling is fixed by C14.

C.5 Up/down ordering and the swapped branch

The pre-closure anomaly system permits the unordered pair

$$\{ Y(u_r), Y(d_r) \} = \{ 4a, -2a \}.$$

Without ordered closure, one could swap the labels of the two right-handed coloured singlets. But the weak doublet itself has an ordered upper and lower component:

$$T_3(u_l) = +1/2, \quad T_3(d_l) = -1/2.$$

The upper component has electric charge

$Q(u_l) = +1/2 + a/2$,

and the lower component has

$Q(d_l) = -1/2 + a/2$.

For $a = 1/3$, these are

$Q(u_l) = +2/3$, $Q(d_l) = -1/3$.

The right singlet closing the upper component must therefore have charge $+2/3$, which under $Q = Y/2$ for a weak singlet means

$Y(u_r) = +4/3$.

The right singlet closing the lower component must have charge $-1/3$, hence

$Y(d_r) = -2/3$.

So the swapped branch is not a distinct Standard Model census. It is either a relabelling before the upper/lower closure roles are specified, or it violates the ordered weak-component closure condition C14.

C.6 Sterile-neutrino flat direction

The minimal SC-1 census contains no right-handed neutrino. This matters for uniqueness.

If an additional right-handed neutrino is included and its U(1) charge is not fixed by a separate sterile-sector condition, the anomaly equations can admit a one-parameter deformation corresponding to adding a B−L component to the U(1) generator:

$Y \to Y + \lambda\cdot(B - L)$.

With a suitable right-handed neutrino charge, this deformation can remain anomaly-free — including the cubic — while preserving the ordered closure relations, since H carries $B - L = 0$ and quark and lepton shifts pass through the closure equations unchanged. In the enlarged field content, therefore, the determining system no longer has a unique root and hypercharge uniqueness is lost.

In the minimal field content the deformation fails outright: the perfect cube $(1 - 3a)^3 = 0$ admits only $a = 1/3$, so any shifted assignment is inadmissible.

SC-1 thus secures uniqueness in two reinforcing ways:

1. the minimal census excludes ν_r, because it is not required for gauge anomaly cancellation, weak-doublet parity, or charged mass closure;

2. the perfect-cube determination of C.3 leaves no residual direction to deform.

If a later neutrino-mass gate adds ν_r, it must declare how the sterile-sector bridge closes or removes the corresponding B−L freedom. That is why ν_r is an extension species rather than a minimal SC-1 species.

C.7 Proposition C.1 — Hypercharge uniqueness

Proposition C.1. Within the minimal field content

$\{ Q_l, u_r, d_r, L_l, e_r, H \}$,

and under the convention

$Q = T_3 + Y/2$,

the conditions of neutral Higgs vacuum, ordered Higgs-mediated charged closure, and gauge-sector anomaly cancellation uniquely fix the hypercharge assignment as

$Y(Q_l) = +1/3$, $Y(u_r) = +4/3$, $Y(d_r) = -2/3$, $Y(L_l) = -1$, $Y(e_r) = -2$, $Y(H) = +1$.

The mixed gravitational–hypercharge condition and the global weak-anomaly condition then hold automatically at this assignment.

Proof. The neutral Higgs vacuum fixes $Y(H) = 1$. Ordered closure gives $b = a + 1$, $c = a - 1$, $e = \ell - 1$, and the $SU(2)^2U(1)$ condition gives $\ell = -3a$. The $SU(3)^2U(1)$ condition is then satisfied identically, and the cubic condition factorises as $(1 - 3a)^3 = 0$ (C.3), whose only root is $a = 1/3$ — a triple root leaving no branch ambiguity, real or complex. The singlet hypercharges follow from ordered closure. The mixed gravitational–hypercharge condition reduces to the linear factor $1 - 3a$ and closes automatically at the root (C.4). The unordered singlet labelling is fixed by ordered weak-component closure (C.5), and the sterile B−L flat direction (C.6) is absent because ν_r is not part of the minimal field content. Therefore the solution is unique inside SC-1. ■

Falsifier (F20, F23): exhibit a second assignment satisfying all stated conditions within the minimal field content, or a hypercharge branch surviving ordered closure with the same charged terminal species labels.

C.8 Referee note

This appendix intentionally separates four statements that are often conflated:

1. gauge anomaly cancellation constrains hypercharge but does not fix it;
2. ordered Higgs closure reduces the system to one variable, and the cubic then fixes it as a perfect cube;
3. minimal field content removes the sterile B−L ambiguity;

4. the gravitational–hypercharge condition is a surplus check, not a determining input.

SC-1 uses the first three and predicts the fourth.

Appendix D — CKM Phase Count and the Three-Generation Slot

For N_g generations, the number of physical CP-violating phases in the quark mixing matrix is

$(N_g - 1)(N_g - 2)/2$.

Thus:

$N_g = 1 \Rightarrow 0$,

$N_g = 2 \Rightarrow 0$,

$N_g = 3 \Rightarrow 1$.

SC-1 does not derive the CKM phase value. But it supplies the minimal generation count for a nontrivial quark-sector CP phase slot.

This is a consistency check on the census:

- one generation has no mixing;
- two generations have mixing but no irreducible CP phase;
- three generations have exactly one physical CP phase;
- more generations introduce additional phases not required by the minimal support-return census.

Therefore $N_g = 3$ is the minimal generation count compatible with a nontrivial chiral CP-orientation slot in the quark sector. The observed quark-sector CP violation occupies exactly the slot the census opens — no more, no fewer.

Appendix E — Referee Objections and Replies

Objection 1 — You have not derived the Standard Model gauge group.

Reply. Correct. SC-1 is a census theorem inside the active gauge-attachment ledger. It derives the minimal matter species package and generation count under that ledger. A deeper gauge-origin theorem is a separate gate, and SC-1's closure certificate marks that dependence explicitly.

Objection 2 — Does anomaly cancellation alone prove three generations?

Reply. No, and SC-1 says so in §9.2. One generation is anomaly-stable by itself. The generation count comes from VERSF support-return completion, not from gauge anomaly cancellation alone. Any presentation of SC-1 that drops this distinction misstates the theorem.

Objection 3 — Are colour and generation the same triplicity?

Reply. No. Colour is a gauge-facing branch inside coloured species. Generation is a support-return repetition of the entire anomaly-stable package. SC-1 keeps them separate by definition (§3.4), by firewall (F16), and by falsifier structure: colour triplicity is falsified by F4, generation triplicity by F12–F14. Different falsifiers prove they are different claims.

Objection 4 — Why does the lepton doublet appear?

Reply. A coloured weak doublet contributes three SU(2)_L doublets, one per colour. That is odd and fails the global weak-anomaly condition. Lemma 1 shows the minimal admissible repair — fewest added Weyl fields, fewest induced obligations — is exactly one colourless weak doublet, giving 3 + 1 = 4 doublets. The lepton doublet then also participates in hypercharge and gravitational anomaly closure.

Objection 5 — Could the parity defect be repaired by a second coloured doublet instead?

Reply. Arithmetically yes: three more doublets restore even parity. But that repair adds six Weyl fields, new $SU(3)^3$ and $SU(3)^2U(1)$ obligations, and two further right-singlet closure debts. The colourless doublet adds two Weyl fields and one hypercharge obligation, which §6 discharges. Under the minimality standard C7, the colourless repair wins strictly. Falsifier F19 invites any smaller repair.

Objection 6 — Why is there no right-handed neutrino?

Reply. A right-handed neutrino is a sterile singlet (1, 1, 0). It does not contribute to gauge anomaly closure, weak-doublet parity, charged mass closure, or hypercharge determination. Therefore it is not part of the minimal census. Moreover its inclusion would reopen the B−L flat direction and destroy hypercharge uniqueness (Appendix C). It may be added in a neutrino-mass extension that declares and services that debt.

Objection 7 — Is the hypercharge assignment really unique?

Reply. Yes, within the minimal field content. Under ordered closure the cubic gauge condition factorises as the perfect cube $(1 - 3Y(Q_l))^3 = 0$, whose single triple root fixes the assignment with no residual branch — the degenerate branch of the unconstrained anomaly system never arises. The two remaining loopholes — the unordered up/down singlet labelling and the sterile B−L flat direction — are closed by ordered weak-component closure (C14) and ν_r exclusion respectively (Appendix C, Proposition C.1). The $SU(3)^2U(1)$ condition closes identically and the gravitational–hypercharge condition closes at the root, which is the overdetermination signature of §12.4.

Objection 8 — Why one Higgs doublet?

Reply. One Higgs doublet with $Y(H) = +1$, together with $\tilde{H}$, is sufficient to close the down-type, up-type, and charged-lepton mass maps. Additional Higgs doublets add new scalar species and flavour structures not required by the minimal census. They are typed as extensions.

Objection 9 — Why exactly three generations?

Reply. A generation is one complete anomaly-stable species package assigned to a support-return completion layer. The physical support-return quotient has three such layers (inherited through I2, exposed to falsification through C9). Fewer than three leave a layer unfilled. More than three require a fourth layer or an extension. Hence the minimal count is three.

Objection 10 — Does this derive masses?

Reply. No. It derives the census. Masses, Yukawas, CKM values, PMNS values, and coupling constants remain later gates. Falsifier F17 explicitly forbids inferring hierarchy from census.

Objection 11 — Could a fourth generation exist in an extension?

Reply. Yes, but not in the minimal census. A fourth generation would require a fourth support-return layer, new record obligation, or new sector. Without such a bridge, it is non-minimal. Empirically, a fourth chiral generation would also carry its own anomaly-stability and closure debts, which the extension ledger would have to service.

Objection 12 — Your three-generation result is just assumed through the rank-three premise.

Reply. SC-1 does inherit rank-three support-return completion. It does not rederive that upstream result. But the generation theorem is not merely the statement "three implies three." The nontrivial bridge is that each independent support-return layer must carry a complete anomaly-stable species package.

That bridge is supplied by layer-local terminality (C12). A support-return layer cannot contain an isolated field or partial generation, because a partial package fails weak parity, charged closure, hypercharge closure, or anomaly stability. Inter-layer mixing cannot repair this at the census stage because mixing presupposes completed generation slots (C13). Thus rank three fixes the number of complete packages only after layer-local terminality and layer-exclusive attachment are imposed: C12 forbids a layer from lacking structure, and C15 forbids layers from sharing it.

If the inherited rank-three theorem fails, then the SC-1 generation count fails transparently. That is a proper dependency, not a hidden assumption.

Objection 13 — Why can anomalies not cancel between partial generations?

Reply. SC-1 does not rely only on aggregate anomaly cancellation. VERSF terminality is projected layer by layer. If one layer is anomalous and another layer cancels it only in aggregate, then neither layer is independently terminal. That violates the support-return interpretation of a generation as an independent completion layer (Lemma 7).

Moreover, one complete Standard Model generation is already anomaly-stable. There is no minimal need to distribute anomaly debts across incomplete packages. Doing so would introduce extra bookkeeping without closing a primitive obligation. By census minimality, the layer-local complete package is preferred.

Objection 14 — Does the lepton doublet follow only from the Witten anomaly?

Reply. No. The Witten anomaly proves that the coloured weak sector alone is not globally admissible, because it contains three SU(2)_L doublets. The minimal parity repair is one colourless weak doublet. That establishes the need for L_l-type structure.

The full lepton interpretation, including e_r and the hypercharge assignment $Y(L_l) = -1$, follows only after charged Higgs closure and anomaly equations are imposed (§4.4). SC-1 uses the Witten anomaly as the entry point for the colourless weak doublet, not as the complete derivation of the lepton sector.

Objection 15 — Are the hypercharges really derived, or are they inserted through the Yukawa terms?

Reply. They are derived conditionally on ordered Higgs-mediated closure. SC-1 explicitly assumes the minimal Standard Model closure carrier H and requires the charged terminal records to close through gauge-invariant Yukawa maps. Under those conditions, the neutral Higgs vacuum fixes Y(H) = +1, the ordered closure equations relate the singlet hypercharges to $Y(Q_l)$ and $Y(L_l)$, and the gauge-sector anomaly conditions fix $Y(Q_l) = +1/3$ and $Y(L_l) = -1$ through the perfect-cube factorisation.

Thus hypercharge is not derived from anomaly cancellation alone. It is derived from anomaly cancellation plus the declared mass-closure mechanism. That is exactly why the paper calls this a census theorem inside the active VERSF/Standard Model attachment ledger, and Appendix C records the residual anomaly branches that ordered closure eliminates.

Objection 16 — Could a different electroweak-breaking mechanism avoid the one-Higgs result?

Reply. Yes, as an extension. SC-1 proves minimal sufficiency under Higgs-mediated charged closure. It does not claim to eliminate every possible beyond-Standard-Model electroweak-breaking mechanism. A composite Higgs, multi-Higgs sector, or non-Higgs closure mechanism would introduce additional species, operators, or bridge obligations. Those possibilities belong to extension ledgers, not to the minimal SC-1 census.

Objection 17 — Does proton–electron neutrality mean the paper derives hydrogen?

Reply. No. SC-1 derives the charge relation

$$2Q(u) + Q(d) + Q(e) = 0.$$

That is the charge balance required for proton–electron neutrality. It does not derive confinement, proton structure, atomic binding, or chemical hydrogen. Those belong to later QCD and bound-state sectors.

Objection 18 — Could a right-handed neutrino be added without damaging the Standard Model?

Reply. Yes. A sterile ν_r is admissible in an extension. SC-1 excludes it only from the minimal census because it is not required by gauge anomaly cancellation, weak-doublet parity, charged closure, or hypercharge uniqueness. If a later neutrino-mass gate adds ν_r, it must state the bridge: Dirac mass, Majorana mass, seesaw, sterile-sector dynamics, or another VERSF mechanism. It must also account for the B−L flat direction that an enlarged field content can reopen (Appendix C.6).

Objection 19 — Could one generation span all three layers?

The sharpest attack on the generation count is not partial generations or anomaly borrowing but layer-spanning: a single generation whose record carriers have nonzero support across all three completion layers, such that the projection $P_i\mathcal{C}$ is a complete, terminally admissible census for every i — with N_g = 1.

Reply. This is exactly what layer-exclusive attachment (C15) forbids, and the premise is named precisely because C12 alone does not close it: layer-local terminality forbids a layer from lacking structure, not from sharing it. The physical justification of C15 is layer independence. The completion layers are independent as record ledgers. If one carrier's commitment simultaneously constituted the completion of two layers, those layers would share a constituting entry and would not be independent completion directions — contradicting the inherited rank statement (I2, C9). A layer-spanning generation is therefore not a cheaper census; it is a denial of the rank-three premise under another name. Falsifier F24 invites the counterexample: a terminal carrier whose projection onto two independent layers is complete in both, while the layers remain independent as record ledgers.

Objection 20 — On what authority does a census theorem invoke the gravitational anomaly when VERSF defers gravitational response?

Reply. It does not. The determining conditions of the hypercharge theorem are gauge-sector only: neutral Higgs vacuum, ordered closure, the $SU(2)^2U(1)$ condition, and the cubic $U(1)^3$ condition, which factorises as the perfect cube $(1 - 3Y(Q_l))^3 = 0$ with the single root $Y(Q_l) = +1/3$. The mixed gravitational–hypercharge condition is then verified to close automatically at that root — indeed, under ordered closure it reduces to precisely the linear factor of the cube. It appears in SC-1 as a surplus consistency check, not as an input. This is the stronger result: the census predicts gravitational-anomaly consistency of the continuum description rather than assuming it, which is the correct dependency order for a programme in which geometry is emergent and gravitational response is a deferred gate.

Objection 21 — What is the one-sentence result?

Reply.

Given weak binary attachment, colour support branching, ordered Higgs-mediated charged closure, anomaly stability, rank-three support-return completion, layer-local terminality, and layer-exclusive attachment, VERSF's minimal Standard Model matter census is exactly three complete anomaly-stable generations, each containing Q_l, u_r, d_r, L_l,

e_r, with one Higgs doublet as the universal closure carrier and no sterile neutrino or fourth generation unless an extension supplies new structure.

Final One-Sentence Theorem

Given weak binary attachment, colour support branching, ordered Higgs-mediated charged closure, gauge anomaly stability, rank-three support-return completion, layer-local terminality, and layer-exclusive attachment, VERSF admits exactly three complete minimal Standard Model generations, each with the species package Q_l, u_r, d_r, L_l, e_r, plus one Higgs doublet as the universal closure carrier.