# The Multi-Regulator Full-Faithful Source Assembly, Exact Six-Mode Bath Identification and HM4 Provenance Execution-Boundary Theorem in VERSF

**A source-clean 334-variable edge-field extension, an exact Fourier bath return under circulant-closure premises, a normalized defect-quotient certificate, a conditional fixed-step marked-population descent architecture and the precise current-corpus boundary to physical HFB admission**

Keith Taylor — VERSF Theoretical Physics Programme, Standard Model Closure Series

**Designation:** HM4-EX1 — Full-Faithful Source Assembly and Provenance Execution Boundary, incorporating the HM4-EX2A execution lock and certificate (Parts VIII–IX), the HM4-EX2B structural universality theorem (Part X), the HM4-EX3A/EX3B identifiability and embedding-interval theorems (Parts XI–XII), the HM4-EX4A graded source-jet theorem (Part XIII), and the HM4-EX4B terminal-record and CAR/Fock primitive-leg compatibility certificates (Part XIV); the execution history is preserved as Appendix A **Date:** 25 July 2026

---

## Contents

---

# General Reader Summary

This series asks one question honestly: does the deep bookkeeping of the theory actually produce the forces of particle physics, or does it merely have room for them? An earlier shortcut version carried three adjustable settings per network link where the honest theory needs twelve, and no test run on the shortcut can vouch for what it lacks. This paper counts what honesty requires — exactly 334 continuous source coordinates, a count the declared honest structure forces, even though the particular coordinates realizing it are declared choices — proves that two separate background descriptions of the structure share the same six notes (an exact result under stated symmetry assumptions whose routine machine check at each level is still owed), and proves that the missing pieces can never be certified by proxy.

The honest structure was then actually built and put through its first decisive test, with both possible outcomes contracted in advance. It passed exactly: the way the system commits to events turned out to be a natural resonance of the underlying seven-site wheel, so the descent is provable from the wheel geometry itself rather than inferred from a small numerical residual. An independently built second version reproduced the answer to better than a trillionth of a degree, and a follow-up theorem showed that every balanced commitment pattern is guaranteed to descend through the declared defect architecture — closing that whole simple sector, while honestly conceding that a guaranteed descent cannot discriminate.

The harder sectors were then cornered step by step. The output side of the test was shown to be fixed by the theory's own declared decomposition; the freedom to arrange the same ingredients into a perfect pass or a complete fail was first proven and then eliminated by derivation rather than by choice. The different kinds of physics enter at different depths — slope, curvature, and one level deeper still — and tested each at its proper depth, every tested sector passes: the full gauge machinery, which had no guarantee protecting it; the record sector, for *every* admissible placement, once eighteen supposedly missing directions were shown to be pure relabeling redundancy; and the occupation sector at the level of its primitive connection to the source.

The final position is stated carefully. The theory's declared source laws are compatible with the honest structure in every tested sector — with the occupation sector so far certified at its primitive source connection rather than by its complete three-way tensor, whose last certificate is still to be issued. The remaining foundational question is therefore no longer whether those declared laws can be accommodated, but whether the deepest layer of the theory selects them uniquely — plus the standing items the paper refuses to shortcut: the physical clock, the absolute scale, and the final admission decision. No measured particle-physics number was used to tune any certificate, and every declared convention and imported source law is listed explicitly.

---

# Abstract

PFHBA-1 established recurrent capacity without identifiability: the record-preserving $K = 7 \times$ neutral renewal law has 930 positive edge coordinates and Fisher rank $867 = 216 + 147 + 504$, admits exact 102-dimensional HCMR pullbacks at four regulators, yet carries a 34,152-dimensional non-identifying lift family; EPMD-1's strongest executable source (226 variables, rank 187, nullity 39) lacked 108 faithful gauge tangents. This paper executes and sharply bounds the resulting compatibility programme — closing five graded sectors and reaching a primitive-leg certificate in the sixth — and states its exact boundary.

**Census.** The minimal faithful continuous source has $d_{faith} = 226 - 36 + 12\times12 =$ **334** variables (Theorem A), with the exact per-edge transport $U_e = U_{e,\star}\exp(iE_{SM}(a_e))$ injective on all 144 gauge directions for generator Gram ranks (8, 3, 1), and the reduced embedding $\iota$ declared, not canonical (P-HM2b).

**Bath carrier (conditional exact).** Under declared circulancy and positivity premises (P-HM3, P-HM3b), M, L and the compound-symmetric event covariance are jointly Fourier-diagonal: $Q_6 = u_0^\perp$ is invariant, $R_{inv} = (I-P_Q)M^{-1/2}LM^{-1/2}P_Q = 0$ identically, $\tilde{u}_0 = u_0$, with benchmark nonuniform frequencies $(1.529, 2.181, 2.965)\xi^{-1}$ at $\sigma/d = 0.8$ (Theorem B). Finite-regulator verification of the premises remains open (D-HM13).

**Tangent image.** The faithful codomain $E_D^{faith} = \bigoplus_{\alpha,c} \mathrm{im}_{\mathbb{R}}\, dD_{\alpha,c}(X^\star)$ is canonical relative to the declared six-defect decomposition, carriers and Hilbert–Schmidt metrics (Theorem I), with census 1,125. The frozen-package construction gives $J_D^{faith} \in \mathbb{R}^{1125\times334}$ of rank 309 (singular gap 0.99946 vs $3.86\times10^{-15}$, cutoff-stable $10^{-8}$–$10^{-14}$), $J_{full} \in \mathbb{R}^{1147\times334}$ of rank 313, and the exact GRC kernel closure 19 = 18 (record-stretch) + 1 (completion) + 0 (gauge).

**Factorisation framework.** $J_s = CJ_D \Leftrightarrow \ker J_D \subseteq \ker J_s$ (Theorem D), with the exclusion of reduced-domain certificates (Theorem C), truncated-SVD robustness discipline (Remark D.1), and the declared source/defect metric pair $(G_X, G_D)$ required for every metric-dependent residual, angle and coupling (P-HM16, D-HM31): kernel inclusion is coordinate-invariant, Frobenius residuals are not.

**Sector certificates (final state).** Scalar commitment: exact structural pass — every zero-sum density source descends with explicit witness (Theorem H; Theorem G the $K = 7$ wheel case),

machine values $\varepsilon = 1.51\times10^{-15}$ and $8.68\times10^{-16}$ under blind replication, frozen-package. Faithful gauge, second order: $\varepsilon^{(2)} = 2.53\times10^{-15}$ at stiffnesses (1, 5/2, 298), all three factors, the 144-dimensional carrier in $\mathrm{ran}(J\_D)^T$ to $4.78\times10^{-15}$ — frozen-package machine pass with genuine failure freedom. Bosonic completion: pass on the full action stack ($1.89\times10^{-15}$), the single defect-invisible mode being the potential-detected line. Terminal record: the 18 stretch nulls are exactly three coboundary copies $\ker B\_C = \mathrm{im}\, B^T$ (Theorem L), the RCMO derivative is regulator-covariant at $p = 2.0047$, and the phase-typed pass is universal over every admissible embedding within the declared phase carrier and execution chart, $\varepsilon\hat{}(R) \leq 3.135\times10^{-15}$ (Theorem M). Shared-link CAR/Fock gauge jet: full rank 144, $\varepsilon = 1.79\times10^{-15}$, zero branch difference. CAR/Fock completion: **primitive-leg pass** on the full action stack ($1.149\times10^{-15}$); the complete mixed-tensor certificate $\varepsilon^{(3)}(T) = \|T - T \times_1 P\_D\|\_F/\|T\|\_F$ of Theorem N remains to be issued (D-HM29).

**Identifiability.** Gauge-sensitive capacity exists in an 81,003-dimensional exact family at $n = 3$–$6$ (Theorem J), but the action Gram and Fisher metric select no member (Corollary J.1), and identical selector data span the full first-order residual interval [0, 1] (Theorem K); the freedom is resolved by differential-order grading and physical typing, not by choice.

**Boundary.** Five sectors hold full graded compatibility certificates at their first nonzero differential order; the CAR/Fock completion sector holds a primitive-leg pass, its full mixed-tensor certificate open. Graded compatibility of the declared conditional RCMO/RRHF/FCHI source stack is strongly supported but not yet a complete full-tensor certificate, and primitive provenance is not closed: PRLU-1 shows the published primitive wheel does not uniquely generate the RCMO record action, representation ledger and CAR/Fock amplitudes (D-HM28). All numerical returns carry frozen-package grade pending the reproducibility annex (D-HM30, F-HM22). The physical clock, absolute action scale, neutral/global-measure completion, finite scheme bridge and HFB-1 admission remain open.

# Headline result

**MINIMAL FULL-FAITHFUL SOURCE ASSEMBLY — EXACT PASS. COMMON SIX-MODE BATH CARRIER — CONDITIONAL EXACT THEOREM UNDER THE DECLARED CIRCULANCY AND POSITIVITY PREMISES; FINITE-REGULATOR PREMISE AUDIT OPEN. SCALAR COMMITMENT DESCENT — EXACT STRUCTURAL PASS. FULL-FAITHFUL GAUGE QUADRATIC DESCENT — FROZEN-PACKAGE MACHINE PASS. BOSONIC COMPLETION — PASS ON THE FULL ACTION STACK. TERMINAL-RECORD DESCENT — UNIVERSAL PHASE-SUPPORTED PASS, WITH RECORD-STRETCH NULLS CLASSIFIED AS COBOUNDARY REDUNDANCY. SHARED-LINK CAR/FOCK GAUGE JET — PASS. CAR/FOCK COMPLETION — PRIMITIVE-LEG PASS; COMPLETE MIXED-TENSOR CERTIFICATE PENDING. COMPATIBILITY OF THE DECLARED RCMO/RRHF/FCHI SOURCE STACK — STRONGLY SUPPORTED BUT NOT YET A COMPLETE FULL-TENSOR PROVENANCE CERTIFICATE. PRIMITIVE-SOURCE UNIQUENESS, PHYSICAL CLOCK, ABSOLUTE ACTION SCALE, NEUTRAL/GLOBAL-MEASURE COMPLETION, FINITE SCHEME BRIDGE AND HFB-1 ADMISSION — OPEN.**

Grading of the named results: Theorems A, C, D, G, H and N, and Propositions E and F, are exact algebraic results as stated. Theorem B is exact under P-HM3/P-HM3b/P-HM4, with the finite-regulator premise audit open. Theorems I, J and K combine exact finite-dimensional statements with frozen-package constructions. Theorems L and M combine an exact structural classification (proved algebraically in Theorem L; metric-free formulation in Proposition M.2, its residual kernel classification open, D-HM32) with frozen-package verification of the declared finite carrier. Purely numerical returns are frozen-package results — internally hashed and, for EX2A, independently replicated — and carry that grade until the reproducibility annex is complete (D-HM30, F-HM22).

---

# Final-state audit matrix

Latest verdict per object only. The stage-by-stage history, including superseded interim statuses, is Appendix A.

| Object | Final verdict | Meaning |
|---|---|---|
| PFHBA recurrent capacity (inherited) | MACHINE PASS | 930 coordinates, rank 867 = 216+147+504. |
| Exact-lift identifiability (inherited) | MACHINE FAIL | Broad typed family dimension 34,152. |
| Reduced faithful gauge carrier (inherited) | EXACT FAIL BY 108 | 36 scalar coordinates versus 144 faithful tangents — the deficit this paper repairs. |
| Minimal faithful continuous source | EXACT PASS | 334 = 226 − 36 + 144 (Theorem A); reduced embedding ι declared, not canonical (P-HM2b). |
| Common six-mode bath carrier | CONDITIONAL EXACT THEOREM | $R\_inv = 0$ and $\tilde{u}_0 = u_0$ under P-HM3/P-HM3b (Theorem B); finite-regulator premise audit OPEN (D-HM13). |
| Bath-spectrum benchmark | IMPORTED NUMERICAL RETURN | $(1.529, 2.181, 2.965)\xi^{-1}$ at $\sigma/d = 0.8$; $\Omega\_max\ \xi \in [2.94, 3.24]$ across kernel families. |
| Faithful tangent-image codomain | EXACT PASS | Canonical relative to the declared decomposition, carriers and HS metrics (Theorem I); census 1,125. |
| Faithful Jacobian J_D^faith | FROZEN-PACKAGE MACHINE CONSTRUCTED | 1125×334, rank 309, gap 0.99946 vs $3.86\times10^{-15}$, cutoff-stable $10^{-8}$–$10^{-14}$; J_full 1147×334, rank 313. |
| Rank reconciliation 183/187 | DISCHARGED, INDEPENDENTLY CONFIRMED | 265 = 243 + 22; four potential-detected modes, rediscovered blind to $7.91\times10^{-14}$ degrees. |
| GRC kernel bookkeeping | EXACTLY CLOSED | 19 = 18 record-stretch + 1 completion + 0 gauge; both 0.5 diagnostics exact as $\sqrt{1/4}$. |

| Object | Final verdict | Meaning |
|---|---|---|
| Scalar commitment descent | EXACT STRUCTURAL PASS | Every zero-sum density source descends with explicit witness (Theorems G, H); machine values $1.51\times10^{-15}$ / $8.68\times10^{-16}$ (frozen-package, blind-replicated). |
| Gauge quadratic descent | FROZEN-PACKAGE MACHINE PASS | $\varepsilon^{(2)} = 2.53\times10^{-15}$ at stiffnesses (1, 5/2, 298); factor residuals $3.54/2.56/2.52\times10^{-15}$; 144-dim carrier ⊆ $\mathrm{ran}(J\_D)^T$ to $4.78\times10^{-15}$; no structural protection — genuine failure freedom. |
| Bosonic completion descent | PASS ON FULL ACTION STACK | Single defect-invisible mode = the potential-detected line; $\varepsilon^{(2)}(J\_full) = 1.89\times10^{-15}$; H_potential passes at $2.71\times10^{-15}$ (frozen-package). |
| Record-stretch nulls | EXACT COBOUNDARY CLASSIFICATION | 18 = 3 × dim(ker B_C = im $B^T$); projector error $9.54\times10^{-16}$ (Theorem L) — redundancy, not lost information. |
| RCMO record derivative | FROZEN-PACKAGE REGULATOR PASS | Six-sign derivative, levels 0–6; rank 6/5 with exact radial null and parity identities; convergence order p = 2.0047. |
| Terminal-record HM4 | UNIVERSAL PHASE-SUPPORTED PASS | ε_HM4^(R) ≤ $3.135\times10^{-15}$ for every admissible embedding within the declared phase carrier and G_X = I chart (Theorem M); metric-free formulation stated, final kernel classification open (Proposition M.2, D-HM32). |
| Shared-link CAR/Fock gauge jet | FROZEN-PACKAGE PASS | J_CAR,A full rank 144; $\varepsilon = 1.794\times10^{-15}$; branch difference exactly 0. |
| CAR/Fock completion | PRIMITIVE-LEG PASS ON FULL ACTION STACK | c_H = ½(1,1,1,1); full stack $1.149\times10^{-15}$ for charged RRHF (3–4) and neutral FCHI (3–6) tensors, both branches; complete mixed-tensor $\varepsilon^{(3)}$ certificate OPEN (Theorem N, D-HM29). |
| Gauge-source orientation | EXACT IDENTIFIABILITY NO-GO | 81,003-dimensional exact family (Theorem J); K and −K both members (Corollary J.1). |
| First-order embedding freedom | RESOLVED BY GRADING AND TYPING | Interval [0, 1] proved (Theorem K), then contracted by order typing (Part XIII) and dissolved sector-by-sector (Theorems L, M). |
| EPMD/FRDT independent finite-source CY3′ | FAIL | Residuals 0.37–0.49; unrepaired. |

| Object | Final verdict | Meaning |
|---|---|---|
| Conditional RRHF ordered-parent consistency | FROZEN-PACKAGE PASS | $4.95\times10^{-16}$ / $1.15\times10^{-16}$ — a distinct internal-consistency test, not independent CY3′. |
| Full primitive physical-direct CY3′ | OPEN | Awaits the primitive-source-unique derivation (D-HM28). |
| Declared source metric G_X | OPEN | Per-coordinate scalings for all 334 variables to publish; metric-weighted certificates or invariance proofs owed (P-HM16, D-HM31, F-HM23). |
| Residual kernel sector classification | FROZEN-PACKAGE VERIFIED; EXACT PROOF OPEN | Non-GRC kernel directions verified phase-free at $3.135\times10^{-15}$; the exact classification closes Proposition M.2 (D-HM32). |
| Reproducibility annex | OPEN | Required for external citation of the numerical certificates (D-HM30, F-HM22). |
| Euclidean transfer, Doob, rates, amplitudes | CONDITIONAL / OPEN | (P1)–(P3) gates undischarged; Kraus package open. |
| Neutral tangent repair, completion residue, branch orientation, clock, action unit | OPEN | As specified in Parts V–VI. |
| Primitive-source uniqueness | OPEN — THE FRONTIER | PRLU-1: the published primitive wheel does not uniquely generate the RCMO/RRHF/FCHI structures (D-HM28). |
| HFB-1 | NOT ADMITTED | Faithful compatibility descent executed (Appendix A); uniqueness, clock, scale, measure closure and scheme bridge open. |

# Named premises

**P-HM1 (Frozen reduced census).** The EPMD-1 source is inherited unchanged: 226 real variables, of which 36 are scalar gauge-edge phases (three per edge on twelve edges); residual Jacobian 265×226, rank 187, nullity 39, with the 39 null modes classified exactly.

**P-HM2 (Faithful edge carrier).** The faithful per-edge gauge carrier is $su(3) \oplus su(2) \oplus u(1)$, real dimension $8 + 3 + 1 = 12$, transported by the exact matrix exponential through a fixed representation E_SM with generator Gram ranks (8, 3, 1).

**P-HM2b (Reduced embedding).** Three fixed unit algebra directions $H_C \in su(3)$, $H_W \in su(2)$, $H_Y \in u(1)$ are declared per edge, and the reduced embedding $\iota : X_{red} \hookrightarrow X_{faith}$ identifies each former scalar phase with its span — $\theta_e^C \mapsto \theta_e^C H_C$, $\theta_e^W \mapsto \theta_e^W H_W$, $\theta_e^Y \mapsto \theta_e^Y H_Y$ — acting as the identity on all non-gauge coordinates. G_miss is the declared 108-dimensional complement of $\iota(X_{red})$ in the gauge sector. The old scalar colour phase is *not* a canonically distinguished direction of su(3): the decomposition $X\_faith = \iota(X\_red) \oplus G\_miss$ exists only relative to this declaration.

**P-HM3 (Circulant closure).** The projected overlap matrix M and projected stiffness matrix L on the seven-site K = 7 carrier are real symmetric circulants on $\mathbb{Z}_7$.

**P-HM3b (Positivity of the overlap).** $M \succ 0$ at every regulator under consideration. Real symmetric circulancy alone does not imply positive definiteness; the operators $M^{\pm 1/2}$ used in Theorem B exist by this premise, not by P-HM3. Whether the actual finite-regulator matrices satisfy P-HM3 and P-HM3b is a machine question, recorded open at D-HM13.

**P-HM4 (Compound-symmetric event covariance).** The commitment-event covariance is $\Sigma\_\eta = \nu\_\eta^2[(1-\rho)I + \rho\mathbb{1}\mathbb{1}^T]$ with $\rho \in (-1/6, 1)$, so that $\Sigma\_\eta \succ 0$.

**P-HM5 (HMGBC positivity architecture).** Euclidean descent is gated by (P1) coercivity, (P2) order-cone positivity improvement, (P3) Doob-gauge stoquasticity — declared hypotheses, never assumed satisfied.

**P-HM6 (Non-insertion).** No measured Standard Model mass, coupling, mixing angle, CP phase, neutrino quantity or confinement observable enters the faithful-source assembly or the definition of the HM4 test. HCMR arrays are opened only after the forward faithful-source arrays are frozen and hashed.

**P-HM7 (Record/export completion as hypothesis).** The norm-preserving record/export map is carried as a coefficient-free structural completion, not as a native source return.

**P-HM8 (Executable inventory).** The frozen numerical corpus contains: the six-defect reduced Jacobian $J\_D^{red} \in \mathbb{R}^{243\times 226}$ with rank 183; the 226×187 physical quotient basis; the 226×39 analytic null basis; the parametric Jacobian components; the twelve E_SM Lie-algebra generators represented as exact 45×45 matrices, with verified Gram ranks (8, 3, 1) and vanishing traces for every represented generator including total hypercharge; the six-dimensional K = 7 bath basis; and one target-blind scalar commitment-density source direction.

**P-HM9 (Scalar commitment source).** The factor-blind K = 7 density profile is $c = (-6,1,1,1,1,1,1)/\sqrt{42}$ — unit, orthogonal to $\mathbb{1}$, hence $c \in Q_6$ — embedded equally in the real density coordinates of the colour, weak and Abelian source channels: $u\_\kappa = (u\_{\kappa,C} + u\_{\kappa,W} + u\_{\kappa,Y})/\sqrt{3}$, assuming the three channel embeddings orthonormal. The six-mode source Jacobian is $J\_s^{(\kappa)} = -g\_bath\, u\_\kappa^T$ with $g\_bath = (1, 0, 0, 0, 0, 0)^T$ up to a declared orthogonal bath frame. Both vectors are declared unit, $\|g\_bath\| = \|u\_\kappa\| = 1$, and the residual floor satisfies $0 < \varepsilon_0 < 1$, so the Proposition F reduction is never floor-dominated.

**P-HM10 (Whitened source convention).** Per the PFHBA-5 pass, the projected source is evaluated in whitened coordinates, $s\_n = P\_{Q,n}\, W\_n\, S\_n[\Phi]$ with $W\_n = M\_n^{1/2}$, and both $W\_n$ and $P\_{Q,n}$ are frozen at $X^\star$ (fixed-carrier convention). The orthogonal bath frame must be declared before differentiation.

**P-HM11 (Derived-zero hypothesis).** The implemented scalar commitment source functional depends on the factor-blind density coordinates only and carries no $a_e$-dependence, direct or through $\Phi_n$. Because of that independence alone, its derivative on the faithful gauge coordinates vanishes: $J_s^{(\kappa),faith} |_{G_{miss}} = 0$ as a derived, verifiable zero. Generator tracelessness is retained only as a separate full-carrier consistency audit (Remark IX.3.1, D-HM20), not as a co-premise.

**P-HM12 (Scalar incidence subblock).** At the declared zero-current vacuum, the faithful residual contains the real scalar constitutive-current and divergence readouts $D_J = \delta j - \kappa B^T \delta\rho$ and $D_T = B\delta j$ on the connected retained graph with oriented incidence B, with $\kappa \neq 0$, and these scalar readouts have no linear gauge-link component. Theorem H is conditional on exactly this premise; the faithful codomain may otherwise contain arbitrary additional matrix-valued gauge and mixed rows.

**P-HM13 (Selector inputs).** BTAC-1 supplies the six non-common physical source channels — colour, weak, primitive $q = 6Y$, terminal record, completion strain, CAR/Fock participation — on the occupied representation classes Q, u, d, L, e, N, H, with BCB centering and weighted-RMS normalisation and no target-derived weights. RCMO-1 supplies the regulator-covariant role-odd record/completion response, whose blocking residual converges as $O(h^2)$. Their combined six-channel Gram $G_S$ is positive definite. The selector fixes $G_S$; it does not fix the embedding of the six channels in the faithful tangent.

**P-HM14 (RRHF grading).** RRHF-1 types the source channels by first nonzero differential order at the symmetric vacuum: the gauge auxiliary action and the charged bosonic parent are quadratic actions (gradients vanish at the origin; first nonzero returns are Hessians, with reduced stiffnesses $(Z_3, Z_2, Z_q) = (1, 5/2, 298)$); the CAR/Fock completion vertex is trilinear (first nonzero return a mixed third derivative); the scalar commitment density is first order. The master-action architecture places gauge response in a Hessian and transports the CAR/Fock carrier through the same faithful links.

**P-HM15 (Shared link and conditional source laws).** The CAR/Fock matter carrier uses the same physical link $U_e$ as the faithful gauge and closure sectors — $\Gamma_{kin}^{(0)} = \Sigma_{\{e:i\to j\}} \bar{\Psi}_j \gamma_e(\Psi_j - U_e\Psi_i) + \text{h.c.}$, with no separate matter-link normalisation. The record, completion and CAR/Fock certificates of Part XIV are issued for the RCMO/RRHF/FCHI source laws as currently constructed; whether the primitive MASD-HMGBC transfer generates these laws uniquely is a separate, upstream question (D-HM28).

**P-HM16 (Declared metrics).** A frozen positive-definite source metric $G_X$ on the 334-dimensional source tangent and defect metric $G_D$ on $E_D^{faith}$ are part of every metric-dependent certificate. The exact kernel-inclusion statement $\ker J_D \subseteq \ker J_s$ is coordinate-invariant; Frobenius residuals, principal angles, row-space projectors and minimum-norm couplings are not, because the 334 source coordinates mix densities, currents, gauge-algebra coordinates, record phases, record stretches and completion coordinates with potentially different physical normalisations. The weighted objects are

$J_D^\sharp = G_X^{-1} J_D^T G_D$, $P_D^{(G)} = J_D^{+,(G_X,G_D)} J_D$, $\varepsilon\_HM4^{(G)} = \|J_s(I - P_D^{(G)})\|_{G_X \to G_Q} / \max(\|J_s\|_{G_X \to G_Q}, \varepsilon_0)$.

The executed certificates used $G_X = I$ in the frozen coordinate chart — a legitimate choice, but one that must stand as a *declared* physical unit normalisation, with the scaling assigned to each of the 334 coordinates published, and either the metric-weighted values re-issued or invariance under the declared chart proved (D-HM31). The bath codomain carries $G_Q = I_6$ in the frozen whitened Fourier frame of P-HM10; orthogonal changes of that bath frame leave every declared Frobenius certificate unchanged. This completes the metric triple $(G_X, G_D, G_Q)$ and gives the principal angles of Proposition F and Theorems K and N their metric meaning.

# Falsifiers

**F-HM1.** Any faithful implementation whose E_SM generator Gram ranks differ from (8, 3, 1) voids the Theorem A injectivity claim.

**F-HM2.** A measured M or L at any regulator failing real symmetric circulant symmetry beyond declared tolerance voids the Theorem B carrier claim at that regulator, and the invariance defect must then be computed numerically per PFHBA-4.

**F-HM3.** Quoting any residual computed on the 226-variable reduced source as a faithful HM4 certificate fires immediately (Theorem C).

**F-HM4.** Appending zero columns or zero rows to reduced defect arrays in place of rebuilding them from the matrix-valued source fires immediately.

**F-HM5.** Implementing the transport within a non-Abelian factor as a product of per-generator exponentials — $\exp(i \Sigma_a a_3^a T_a)$ replaced by $\Pi_a \exp(i a_3^a T_a)$, and likewise within SU(2) — fires as non-Abelian transport misrepresentation: the ordered product differs from the exact exponential at second order by BCH commutators of the non-commuting generators. Factorisation across the three gauge factors, $e^{iA_3} e^{iA_2} e^{iA_1} = e^{i(A_3+A_2+A_1)}$, is exact in the direct-product representation, since generators of different factors commute, and does not fire.

**F-HM6.** Adjusting or fitting C_n after an above-tolerance ε_HM4 fires as target fitting; the sole admissible response to an above-tolerance residual is STOP. "Exactly zero" is reserved for symbolic identities such as R_inv = 0; numerical residuals are judged against the predeclared tolerance only.

**F-HM7.** Issuing continuous-time marked rates without a passed (P3) stoquasticity certificate, or dropping the Perron-vector ratio from the rates, fires.

**F-HM8.** A benchmark $\Omega_{max}\,\xi$ falling outside [2.94, 3.24] across the declared Gaussian/sech/Lorentzian kernel families voids the imported bath-spectrum inheritance.

**F-HM9.** Opening any HCMR array before the forward faithful arrays are frozen and hashed fires under P-HM6.

**F-HM10.** Declaring an ε_HM4 pass without publishing the singular spectrum and pseudoinverse cutoff used to construct $J\_D^+$ fires: an unpublished cutoff can silently absorb genuine residual.

**F-HM11.** Typing J_D^faith as 243×334 — reusing the scalar residual codomain — before E_D^faith is constructed from declared representation or response carriers fires. Matrix-valued links generically enlarge and rotate the residual space; D_B is the principal direct gauge–record block, and every mixed block touching a link must be recalculated.

**F-HM12.** Asserting the zero source block on G_miss without the P-HM11 derivation, or retaining it after the implemented source acquires any a_e dependence, fires. The derived zero must also be confirmed by differentiating the implemented source on the full 334-variable domain, not only argued.

**F-HM13.** Constructing $(J\_D\text{\^{}faith})^+$ before the 183-versus-187 reduced-stack rank discrepancy is reconciled and the four unclassified quotient directions are typed fires: an unresolved kernel mislabels the residual and can convert a genuine fail into an apparent pass or the reverse.

**F-HM14.** Quoting an EX2A pass as evidence for the unique physical TPB source, the nonlinear identity s(X) = C(D(X)), a source-derived marked decomposition, Kraus amplitudes or HFB-1 admission fires: the pass certifies quadratic compatibility of one declared source branch only.

**F-HM15.** Declaring an ε_HM4 pass or fail at a single cutoff, without a published cutoff-stability interval or a demonstrated singular-value gap, fires (Remark D.1): a verdict that flips within the justified cutoff range is no verdict.

**F-HM16.** Presenting the EX2A/EX2B pass as evidence that the non-Abelian faithful gauge kernel uniquely produces the commitment source, or as a discriminating test of the faithful gauge reconstruction, fires: within the constitutive/divergence class of P-HM12 the pass is structurally forced for the entire zero-sum density carrier (Theorem H), so it cannot distinguish the gauge enlargement from anything that preserves the scalar incidence subblock.

**F-HM17.** Grading any future HM4 gate as *discriminating* when its source directions lie in the zero-sum density carrier fires: by Theorem H such directions pass automatically. A discriminating gate must include source directions with direct gauge-link, record or completion dependence, returned by the finite map X ↦ Φ_n ↦ TPB_n[Φ] ↦ s_n(X), with the discrimination criterion declared before execution.

**F-HM18.** Selecting an orientation K ∈ St(201, 504) — hence a gauge-sensitive J_s^faith — by any criterion other than differentiation of the finite selector map fires as arbitrary selection: by Corollary J.1 the current action Gram and Fisher data cannot prefer any member of the 81,003-dimensional family, so a hand-picked K (by convenience, symmetry, sparsity, or agreement with HCMR) inserts orientation data the corpus does not contain and is target fitting in a new guise.

**F-HM19.** Reporting any single value of the full multi-channel $\varepsilon$_HM4 before $d\Phi_n|_{X\star}$ is derived fires as a manufactured verdict: by Theorem K the same frozen selector data admit the entire interval [0, 1], with the pass and the fail equally constructible from identical selector Grams. Until the embedding derivative exists, the only honest publication of the multi-channel residual is the interval itself. This applies symmetrically — manufacturing a fail is as forbidden as manufacturing a pass.

**F-HM20.** Stacking sectors of different first-nonzero differential order into one ordinary first-derivative HM4 matrix fires as order mistyping: sectors whose gradient vanishes identically at the symmetric vacuum (P-HM14) contribute a structurally zero linear row, and a first-order certificate over such a stack tests nothing about them. Each sector must be certified at its own first nonzero jet — $\varepsilon^{(1)}$ for linear sources, $\varepsilon^{(2)} = \|H - P_D H P_D\|_F/\|H\|_F$ for Hessian sources, and the corresponding mixed-jet criterion at third order — with the order typing published before the run.

**F-HM21.** Presenting the Part XIV all-sector pass as *unconditional* provenance fires: the certificates are conditional on the RCMO/RRHF/FCHI source laws as constructed (P-HM15), and PRLU-1 already shows that the published primitive wheel alone does not uniquely generate those laws. The closed result is graded HM4 *compatibility* of the conditional source stack; primitive-source uniqueness (D-HM28), the physical clock, the absolute action scale, the neutral/global measure completion and the finite scheme bridge remain between it and HFB-1 admission.

**F-HM22.** Publishing or externally citing the Part IX–XIV numerical certificates without the completed reproducibility annex fires: unaccompanied numbers are assertions in a manuscript, not independently auditable certificates (D-HM30).

**F-HM23.** Quoting any metric-dependent residual, principal angle, projector or coupling without the declared metric triple ($G_X$, $G_D$, $G_Q$), or silently changing any coordinate chart or bath frame between runs, fires (P-HM16): an undeclared chart is an unnoticed convention doing physical work.

---

# Principal theorems

## Theorem A — Minimal faithful edge-field assembly

Let the frozen reduced source contain 226 real variables, including three scalar edge-phase fields on twelve edges, hence 36 scalar gauge-link variables (P-HM1). Replace those fields by

$a_e \in \mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$, $\dim_{\mathbb{R}} = 8 + 3 + 1 = 12$ per edge.

Then the minimal faithful continuous source has

$$\mathbf{d\_faith = 226 - 36 + 12{\times}12 = 334}$$

real variables. At the common vacuum,

$$\partial U_e^{faith}/\partial a_e^A \,|_{\{a=0\}} = i\, U_{\{e,\star\}}\, E\_SM(T\_A),$$

and if E_SM has generator Gram ranks (8, 3, 1) this tangent map is injective on the full 144-dimensional gauge-link carrier.

**Proof.** Census: the faithful carrier supplies, across twelve edges, 12×8 = 96 colour, 12×3 = 36 weak and 12×1 = 12 hypercharge directions, total 144; the reduced source supplies 12×3 = 36 scalars; the correction is +108 and the total is 226 − 36 + 144 = 334. Tangent: the derivative of the matrix exponential at the identity is the representation map itself, $U_{\{e,\star\}}$-translated; faithfulness (full Gram rank per factor) gives injectivity on each edge, and edges carry independent coordinates, so the direct sum over twelve edges is injective on all 144 directions. ■

**Remark A.1 (No hidden discrete count).** The 334 count is the minimum *continuous* source dimension. Discrete ledger labels and any enlarged export fibre are carrier extensions, not independent continuous source coordinates, and are excluded from the census by construction rather than by convention.

**Remark A.3 (What is unique).** The minimum *dimension* 334 is unique, given the retained 190 non-gauge variables and the requirement of twelve independent Lie-algebra directions per edge. The enlargement itself is not unique: bases, representations, coordinate charts and the reduced embedding ι (P-HM2b) may all be chosen differently. Every uniqueness claim in this paper refers to the dimension count only.

**Remark A.2 (Exactness of the exponential — corrected scope).** The implementation must use the exact matrix exponential within each non-Abelian factor. Factorisation *across* the three gauge factors is exact: in the direct-product representation, generators of different factors commute, so with $A = A_3 + A_2 + A_1$ and $[A_3, A_2] = [A_3, A_1] = [A_2, A_1] = 0$,

$$e^{iA_3}\, e^{iA_2}\, e^{iA_1} = e^{i(A_3+A_2+A_1)}$$

identically. The genuine obstruction is per-generator factorisation *within* a non-Abelian factor,

$$\exp(i\, \Sigma_a\, a_3^a\, T\_a) \longrightarrow \Pi_a \exp(i\, a_3^a\, T\_a),$$

and likewise within SU(2): the ordered product differs from the exact exponential at second order by Baker–Campbell–Hausdorff commutators of the non-commuting generators. It is this within-factor misrepresentation — collapsing eight colour directions or three weak directions to sequenced scalar phases — that produced the 108-dimensional deficit, now promoted to falsifier F-HM5.

## Theorem B — Exact common six-mode bath carrier

Let M, L and $\Sigma\_\eta$ be respectively the projected overlap matrix, projected closure stiffness and commitment-event covariance on the seven-site K = 7 carrier, with P-HM3, P-HM3b and P-HM4 in force. Then all three matrices are simultaneously diagonal in the discrete Fourier basis on $\mathbb{Z}_7$. The uniform vector

$$u_0 = (1, \ldots, 1)^T/\sqrt{7}$$

is a common eigenvector, and its orthogonal complement $Q_6 = u_0^\perp$ is invariant under $M$, $L$, $M^{\pm 1/2}$, $M^{-1/2} L M^{-1/2}$ and $\Sigma_\eta$. Hence

$$\boldsymbol{\Omega_Q^2 = M^{-1/2} L M^{-1/2} |_{Q_6}}$$

is an exact six-dimensional restriction, and

$$(I - P_Q) M^{-1/2} L M^{-1/2} P_Q = 0$$

identically.

**Proof.** Every circulant on $\mathbb{Z}_7$ is diagonalized by the discrete Fourier transform; real symmetric circulants have real Fourier eigenvalues and mutually commute, and every measurable function of a circulant (in particular $M^{\pm 1/2}$, defined by P-HM3b) is circulant with the transformed eigenvalues. The compound-symmetric $\Sigma_\eta$ is the circulant $\mathrm{Circ}(\nu_\eta^2(1-\rho)+\nu_\eta^2\rho, \nu_\eta^2\rho, \ldots, \nu_\eta^2\rho)$. The zero-frequency Fourier vector is $u_0$; its complement is the direct sum of Fourier modes $r = 1, \ldots, 6$, invariant under every circulant. The restriction and the vanishing off-diagonal block are immediate. ■

**Corollary B.1 (Real conjugate-pair structure).** Because $M$ and $L$ are real, the Fourier eigenvalues pair as $\hat{M}_r = \hat{M}_{7-r}$, $\hat{L}_r = \hat{L}_{7-r}$; $Q_6$ decomposes into three real two-dimensional joint eigenplanes indexed by the pairs (1,6), (2,5), (3,4), on which

$$\Omega_r^2 = \hat{L}_r / \hat{M}_r, \quad \hat{M}_r = M_0 + 2M_1 \cos(2\pi r/7) + 2M_2 \cos(4\pi r/7) + 2M_3 \cos(6\pi r/7),$$

with the analogous expression for $\hat{L}_r$.

**Corollary B.2 (PFHBA-4 invariance conditional discharged).** PFHBA-4 downgraded $\Omega_Q^2$ to a compressed candidate pending numerical vanishing of the invariance defect $R_{inv} = (I - P_Q) M^{-1/2} L M^{-1/2} P_Q$. Under P-HM3, Theorem B gives $R_{inv} = 0$ as an identity. The corresponding step of the execution protocol becomes an analytic verification of circulant symmetry (F-HM2), not a numerical tolerance test.

**Corollary B.3 (Whitened uniform direction).** Since $u_0$ is an eigenvector of $M$, it is an eigenvector of $M^{1/2}$, so the whitened uniform direction of PFHBA-4,

$$\tilde{u}_0 = M^{1/2}u_0 / \|M^{1/2}u_0\| = u_0,$$

coincides exactly with the coefficient uniform direction. The whitened projector $P_Q = I - \tilde{u}_0\tilde{u}_0^\dagger$ and the coefficient projector onto $Q_6$ are the same operator, and the PFHBA-4 concern that $P_Q\mathbb{1} \neq 0$ is void under P-HM3: $P_Q\mathbb{1} = 0$ exactly.

**Corollary B.4 (Correct canonical-coordinate event covariance).** In coefficient coordinates,

$\Sigma\_\eta\, u_0 = \nu\_\eta^2(1 + 6\rho)\, u_0$, $\Sigma\_\eta\, v = \nu\_\eta^2(1-\rho)\, v$ for every $v \in Q_6$,

so the event forcing is isotropic on $Q_6$ in coefficient coordinates:

$\Sigma\_\eta\, |\{Q_6\} = \nu\_\eta^2(1-\rho)\, I\{Q_6\}$.

In canonical coordinates $z = M^{\{1/2\}}q$ the covariance is $\Sigma\_z = M^{\{1/2\}}\, \Sigma\_\eta\, M^{\{1/2\}}$; since M and $\Sigma\_\eta$ commute, the Fourier basis remains diagonal, with nonuniform eigenvalues

$\Sigma\_\{z,r\} = \nu\_\eta^2(1-\rho)\, \hat{M}\_r$, $r = 1, \dots, 6$.

The canonical-coordinate noise is therefore diagonal but isotropic only if the nonuniform overlap eigenvalues $\hat{M}\_r$ are equal. The PFHBA-4 isotropy test is thereby resolved rather than repealed: isotropy holds exactly in coefficient normalization and fails in canonical normalization exactly to the extent that $\hat{M}_1$ , $\hat{M}_2$ , $\hat{M}_3$ differ — the earlier isotropic covariance and the projected closure spectrum coexist because they refer to different normalizations of one common Fourier carrier.

## Theorem C — Reduced-source HM4 exclusion

Let $\iota$ : X_red ↪ X_faith be the declared reduced embedding of P-HM2b, identify X_red with ι(X_red), and write X_faith = ι(X_red) ⊕ G_miss with dim G_miss = 108. Reduced-source Jacobians J_D^red, J_s^red are defined only on ι(X_red). Then:

(i) the reduced residual R_HM4^red = J_s^red (I − (J_D^red)$^+$ J_D^red) carries no information about ker J_D^faith ∩ G_miss or about J_s^faith |_{G_miss}, and therefore cannot establish ker J_D^faith ⊆ ker J_s^faith on X_faith;

(ii) every extension of (J_D^red, J_s^red) to X_faith either sets both restrictions to zero on G_miss — which *assumes* the 108 directions are simultaneously defect-invisible and source-invisible, the very proposition a provenance certificate must derive — or introduces new source information, in which case the certificate is no longer computed from the reduced source.

Hence no reduced-domain certificate implies the faithful-domain certificate, under any extension convention.

**Proof.** (i) Kernel inclusion is a statement quantified over the domain of the operators. No vector of G_miss lies in X_red, so the reduced residual is a function of the restrictions to X_red only and is invariant under arbitrary modification of the faithful operators on G_miss; in particular it is compatible both with ker J_D^faith ⊆ ker J_s^faith and with its failure. (ii) An extension is a choice of the two linear maps on G_miss. The zero choice is the stated assumption; any nonzero choice is data about the faithful source not contained in (J_D^red, J_s^red). ■

**Remark C.1.** The exclusion is exact and structural. It cannot be repaired by a Fisher isometry, a pseudoinverse identity, a rank argument on X_red, or agreement with HCMR: all of these are silent on G_miss for the same domain reason.

**Remark C.2 (Embedding-relativity).** The direct-sum decomposition is not canonical: it exists only relative to the declared H_C, H_W, H_Y of P-HM2b. The theorem's conclusion, however, is ι-independent — the proof uses only that the reduced Jacobians are defined on a 226-dimensional subspace with a 108-dimensional complement, which holds for every admissible embedding. What changes with ι is which faithful directions are labelled "missing", not the impossibility of certifying them from the reduced arrays. Any implementation must nevertheless publish its declared ι, since the retyped defect blocks and the verification of Lemma E.1 are computed in those coordinates.

**Convention (Declared Euclidean execution chart).** For every numerical certificate in this paper, the 334 source coordinates are assigned the frozen unit metric G_X = I in the frozen coordinate chart, the defect codomain carries the declared Hilbert–Schmidt metric G_D, and every residual, principal angle, projector, orthonormal frame and minimum-norm coupling below is defined relative to that declared pair (P-HM16). Coordinate-invariant content is limited to exact kernel inclusion and factorisation existence. The per-coordinate scaling ledger, and either metric-weighted reissues under a physical G_X or invariance proofs, remain D-HM31.

## Theorem D — HM4 factorisation on the faithful source

For the faithful defect and projected-source derivatives

$J\_D^{faith} : T\_{X\star}\, X\_faith \to E\_D,\quad J\_s^{faith} : T\_{X\star}\, X\_faith \to Q_6,$

a coupling $C : E\_D \to Q_6$ exists with $J\_s^{faith} = C\, J\_D^{faith}$ if and only if

**$\ker J\_D^{faith} \subseteq \ker J\_s^{faith}$.**

The normalized certificate is

$\varepsilon\_HM4 = \| J\_s^{faith} (I - (J\_D^{faith})^{+} J\_D^{faith}) \|\_F / \max(\|J\_s^{faith}\|\_F, \varepsilon_0),$

evaluated under a declared singular-value cutoff, with all norms, pseudoinverses and projectors taken in the declared execution chart of the Convention above; the equivalence $J\_s = CJ\_D \Leftrightarrow \ker J\_D \subseteq \ker J\_s$ is itself chart-independent. When it vanishes,

$C^{min} = J\_s^{faith} (J\_D^{faith})^{+}$

is the minimum-norm representative of the coupling on the reachable defect image.

**Proof.** If $J\_s = C\, J\_D$ then $\ker J\_D \subseteq \ker J\_s$ trivially. Conversely, kernel inclusion means J_s factors through the quotient by ker J_D, hence through $\operatorname{ran} J\_D \cong T\_{X\star}X\_faith / \ker J\_D$; define C on ran J_D by $C(J\_D\, v) = J\_s\, v$ (well defined by the inclusion) and extend by zero on the orthogonal complement — this extension is the minimum-norm one and equals $J\_s J\_D^{+}$. The projector $I - J\_D^{+}J\_D$ is the orthogonal projector onto ker $J\_D$ under the declared cutoff, so $\varepsilon\_HM4 = 0$ is exactly the kernel-inclusion statement at that cutoff. ■

**Remark D.1 (Exact theorem versus truncated certificate).** Three layers must not be conflated. (i) *Exact theorem:* with the genuine Moore–Penrose pseudoinverse, exact kernel inclusion is equivalent to exact factorisation. (ii) *Numerical certificate:* under a singular-value cutoff, $J\_D^{+}J\_D$ is the projector onto the *retained numerical* row space, not necessarily the exact one, and the certificate is a truncated-SVD statement relative to the declared cutoff. (iii) *Robustness requirement:* a numerical verdict must be stable across a justified cutoff interval, preferably separated by a visible singular-value gap; a cutoff-stability ledger must be published with the run (D-HM21, F-HM15). Publishing a single cutoff is necessary but not sufficient — a verdict that flips inside the justified cutoff range is no verdict.

**Remark D.2 (Metric dependence of the numerical certificate).** The Euclidean Moore–Penrose objects $P\_D = J\_D^{+}J\_D$ and $C\text{^}min = J\_sJ\_D^{+}$ presuppose the unit metric on the source tangent. Under a non-orthogonal rescaling of the 334 heterogeneous coordinates, P_D, the principal angles, C^min and the numerical value of ε_HM4 all change even though exact kernel inclusion does not. Every numerical certificate in this paper is therefore issued relative to the declared chart of P-HM16 (currently G_X = I in the frozen coordinates), and D-HM31 records the obligation to publish the per-coordinate scalings and the metric-weighted or invariance statements.

---

# Part I — Source boundary and non-insertion discipline

The paper consumes four classes of result. The PFHBA-1 machine certificate supplies recurrent capacity and exact non-identifiability. The EPMD-1 machine certificate supplies the strongest current-source forward derivative and the 108-dimensional faithful-carrier deficit. The Full Projected Closure Operator supplies M, L, their circulant structure and their benchmark spectrum. HMGBC-1 supplies the conditional positivity, Perron and Doob architecture. HCMR-MR1 supplies the convergent downstream candidate and its remaining physical-admission failures. All inherited numerical certificates are consumed unchanged. New numerical arrays are generated only in the declared execution stages of Appendix A (Parts IX–XIV); each execution freezes its inputs, coordinate chart, response carriers, differentiation method, numerical cutoffs and outputs before any cross-comparison. No new numerical return is introduced in the theorem-only Parts II–VII.

Under P-HM6, no measured Standard Model quantity is used to assemble the faithful source or to define the HM4 test, and HCMR arrays may be opened only after the forward faithful-source arrays are frozen and hashed.

The following implications remain forbidden:

1. carrier dimension does not imply action provenance;

2. source-Hessian compatibility does not imply a probability law;
3. a gauge-group theorem does not itself constitute finite-regulator implementation;
4. a scalar Doob kernel does not determine representation-bearing amplitudes;
5. a reduced-source HM4 residual does not certify omitted source directions (Theorem C);
6. a formal Schur complement is not a numerical completion scale;
7. regulator convergence does not turn an inconsistent independent descent into a physical one;
8. a single-level, single-branch quadratic pass does not certify unique-source, nonlinear, multi-regulator or conjugate-branch provenance (F-HM14, D-HM23).

---

# Part II — The faithful source extension

At each retained edge e, define

$$a_e = a_{3,e}^a T_a + a_{2,e}^i \tau_i + b_e Y,$$

with faithful edge transport

$$U_e^{faith} = U_{e,\star} \cdot \exp(i E_{SM}(a_e)).$$

The implementation must use the exact matrix exponential of the summed algebra element (Remark A.2, F-HM5). The primitive configuration becomes

$$X_n^{faith} = (X_n^{red} \setminus \{\theta_e^C, \theta_e^W, \theta_e^Y\}) \cup \{a_{3,e}^a, a_{2,e}^i, b_e\},$$

with minimum continuous dimension 334 (Theorem A). Discrete ledger labels and the export fibre are carrier extensions, not counted (Remark A.1).

The six MASD defect classes remain typed as

$$D_n : X_n^{faith} \to E_C \oplus E_J \oplus E_T \oplus E_R \oplus E_B \oplus E_H.$$

Each defect must be reevaluated with the full matrix-valued link. It is not legitimate to retain the scalar defect arrays and merely attach 108 zero columns (F-HM4): the matrix-valued transport enters the defects nonlinearly, so the new columns are generically nonzero and the new rows generically couple to the retained variables.

**Record/export carrier.** The minimal norm-preserving record/export map is retained as a source-extension hypothesis (P-HM7):

$$|C_i, \ell\rangle \mapsto e^{-L_i} |A_i, \ell\oplus i\rangle + \sqrt{1 - e^{-2L_i}} |E_i, \ell\oplus i\rangle,$$

with exact ledger identity $e^{-2L_i} + (1 - e^{-2L_i}) = 1$. This closes the disappearance of committed capacity at the structural level. It does not establish that the original MASD action dynamically generates the export fibre; EPMD-1's grading of it as a coefficient-free structural completion, not a native source return, is preserved.

---

# Part III — Exact six-mode bath calculation

The projected action has kinetic matrix M and stiffness matrix L:

$S_{eff} = \frac{1}{2} \int dt\, [\, \dot{q}^T M \dot{q} - q^T L q\, ]$,

with

$M = \mathrm{Circ}(M_0, M_1, M_2, M_3, M_3, M_2, M_1)$, $L = \mathrm{Circ}(L_0, L_1, L_2, L_3, L_3, L_2, L_1)$.

By Corollary B.1,

$\Omega_r^2 = \hat{L}_r / \hat{M}_r$, $\hat{M}_r = M_0 + 2M_1 \cos(2\pi r/7) + 2M_2 \cos(4\pi r/7) + 2M_3 \cos(6\pi r/7)$,

and the commitment-event bath separates into one uniform mode and six nonuniform modes, the uniform mode decoupling from the commitment field.

At $\sigma/d = 0.8$, the inherited benchmark spectrum is:

| Fourier modes | $\Omega_r \xi$ |
|---|---|
| 0 | 1.238 |
| 1, 6 | 1.529 |
| 2, 5 | 2.181 |
| 3, 4 | 2.965 |

The HM4 bath uses the six nonuniform entries; the maximum nonuniform frequency is $\Omega_{max} \xi = 2.965$. Across the harmonic-matched Gaussian range the maximum remains in

$2.937 \leq \Omega_{max} \xi \leq 3.023$,

and across Gaussian, sech and Lorentzian families it remains approximately in

$2.94 \leq \Omega_{max} \xi \leq 3.24$ (F-HM8).

These figures are dimensionless spectral returns. They do not determine ξ in physical units and do not remove the remaining kernel-width ambiguity.

**Event covariance.** In coefficient coordinates,

$\Sigma_\eta = \nu_\eta^2\, [\, (1-\rho)\, I + 7\rho\, u_0 u_0^T\, ]$, so $\Sigma_\eta u_0 = \nu_\eta^2(1+6\rho)\, u_0$ while every $v \in Q_6$ satisfies $\Sigma_\eta v = \nu_\eta^2(1-\rho)\, v$: the event forcing is isotropic on $Q_6$ in coefficient coordinates. In canonical coordinates $z = M^{1/2}q$ the six nonuniform variances become $\nu_\eta^2(1-\rho)\, \hat{M}_r$ (Corollary B.4): diagonal, generally non-isotropic, on the same invariant Fourier carrier.

---

# Part IV — The HM4 provenance calculation

Define the projected commitment source in the whitened frozen-carrier convention (P-HM10):

$$s_n(X) = P_{Q,n}\, W_n\, S_n[\Phi(X)],\quad W_n = M_n^{1/2},$$

with $W_n$ and $P_{Q,n}$ frozen at $X\star$; the moving-carrier terms $(DP_Q)WS + P_Q(DW)S$ are recorded open in the PFHBA ledger and are excluded from the quadratic certificate by declaration. All quantities below are evaluated in the declared Euclidean execution chart of the Convention preceding Theorem D — $G_X = I$, $G_Q = I_6$ in the frozen bath frame, Hilbert–Schmidt $G_D$ (P-HM16).

At the faithful vacuum,

$$J_{D,n}^{faith} = DD_n|_{X\star},\quad J_{s,n}^{faith} = Ds_n|_{X\star}.$$

The full HM4 test must publish: rank $J_{D,n}^{faith}$, dim ker $J_{D,n}^{faith}$, the singular spectrum used to construct the pseudoinverse together with the declared cutoff (F-HM10), and

$$\varepsilon\_HM4^{(n)} = \| J_{s,n}^{faith} (I - (J_{D,n}^{faith})^+ J_{D,n}^{faith}) \|_F / \max(\|J_{s,n}^{faith}\|_F, \varepsilon_0).$$

A pass requires $\varepsilon\_HM4^{(n)} \le \varepsilon_n$ at every regulator, with $\varepsilon_n$ fixed before any HCMR access and decreasing consistently with refinement or with the declared numerical error model. On a pass,

$$C_n^{min} = J_{s,n}^{faith} (J_{D,n}^{faith})^+$$

is frozen. Its regulator transport must satisfy the HMGBC bath compatibility conditions: refinement may not invent a new coupling on the inherited coarse subspace. When the residual exceeds the predeclared tolerance the calculation stops and $C_n$ may not be fitted (F-HM6); a residual below tolerance is a numerical pass, subject to the cutoff and refinement stability of Remark D.1.

**Which numbers are issued, and which are not.** The κ-branch quantities now exist as frozen numerical arrays: $J_D^{faith}$ (1125×334), $J_s^{(\kappa),faith}$, $\varepsilon\_HM4^{(\kappa)}$ and $C_\kappa^{min}$ are issued in Part IX under exactly this Part's test definition, with the singular spectrum, cutoff and stability ledger published. What remains unissued is everything the κ branch does not reach: the residual for the unique physical TPB source, the multi-regulator family $\varepsilon\_HM4^{(n)}$ at n = 3–6, the conjugate-branch run, and a source-derived amplitude operator for every record mark. By Theorem C no reduced-source value may ever be reported in place of any of these, and by F-

HM14 the κ-branch pass may not be reported in their place either: each would answer a different question.

---

# Part V — Euclidean transfer and marked populations

Given a faithful coupling $C_n$, define

$$\mathscr{H}_n = -\Delta_n + V_{loc}^{(n)} + \tfrac{1}{2} q^T \Omega_Q^2 q + q^T C_n D_n.$$

A transfer $T_n = e^{-a_t \mathscr{H}_n}$ may be used only after two gates pass (P-HM5):

**(P1) Coercivity.** $W_{core}^{(n)} - C_n^\dagger \Omega_Q^{-2} C_n \succeq 0$, or more generally $\mathscr{H}_n \geq -c_n I$.

**(P2) Cone positivity.** The semigroup must be positivity preserving, and preferably positivity improving, in a declared physical mark cone. Operator positivity $T_n \succeq 0$ alone is insufficient: HMGBC explicitly separates Hessian positivity, measure positivity and order-cone positivity.

For a positivity-improving transfer, let $T_n r_n = \lambda_{0,n} r_n$ and $\ell_n^\dagger T_n = \lambda_{0,n} \ell_n^\dagger$. The Doob kernel

$$K_n(y|x) = T_n(x,y)\, r_n(y) / (\lambda_{0,n}\, r_n(x))$$

is exactly stochastic.

**Primary fixed-step marked law.** Partition the kernel by record mark, $K_n = \Sigma_\alpha K_{n,\alpha}$. The primary physical output is the fixed-step marked-population law $K_{n,\alpha}(y|x)$. This avoids claiming a classical continuous-time limit when the faithful generator has not been shown stoquastic.

**Conditional continuous-time rates.** If the Doob-transformed generator is Metzler (P3), then for $y \neq x$,

$$q_{n,\alpha}(y|x) = -\mathscr{H}_{n,\alpha;xy} \cdot r_n(y)/r_n(x).$$

The Perron-vector ratio is part of the rate and cannot be dropped (F-HM7).

**Amplitude return.** The scalar population law does not determine the representation-bearing partial isometries. A source amplitude operator must return

$$M_{n,\alpha;yx} = P_y A_{n,\alpha} P_x,\quad V_{n,\alpha;yx} = \mathrm{Pol}(M_{n,\alpha;yx}),$$

with fixed-step instrument

$$L_{n,\alpha;yx} = \sqrt{K_{n,\alpha}(y|x)} \cdot V_{n,\alpha;yx}$$

and actual completeness condition

$$\Sigma_{\alpha,y} K_{n,\alpha}(y|x) V_{n,\alpha;yx}^\dagger V_{n,\alpha;yx} = P_x.$$

The simpler resolution of the identity follows only when every outgoing partial isometry has initial projector $P_x$. The visible/exported terminal pair is structurally known; the active-to-singlet and singlet-to-record amplitude maps remain source-open.

---

# Part VI — Neutral release and completion response

At fixed step, define the interaction residual

$$\Xi_X^{(K)} = \log [ K_X(y, j \mid x, i) \cdot K_X(\cdot, \cdot \mid x, i) / ( K_{X,H}(y \mid x, i) \cdot K_{X,N}(j \mid x, i) ) ],$$

and, when a continuous-time limit exists, the rate residual

$$\Xi_X^{(q)} = \log [ q_X(y, j \mid x, i) \cdot \Gamma_X(x, i) / ( q_{X,H}(y \mid x, i) \cdot q_{X,N}(j \mid x, i) ) ].$$

Both vanish exactly when history and neutral destinations factorize.

The common-bath cross response is

$$B_n = C_{neutral,n}^\dagger \Omega_{Q,n}^{-2} C_{hist,n} : H_{12} \to N_{24}.$$

Let $P_{miss}$ project onto the three-dimensional neutral complement not reached by the strict shared-neutral tangent. The release condition is stable surjectivity:

**(T1)** $\mathrm{ran}(P_{miss} B_n) = \mathrm{ran}\, P_{miss}$; **(T2)** $\sigma_3(P_{miss} B_n) > \varepsilon_{reg}$; **(T3)** projection-leakage check only.

This repairs the three-dimensional tangent deficit only. It does not return $Y_\nu$, $M_R$, $M_L$ or the complete physical neutral Hessian.

**Completion response.** For the completion column $c_{H,n}$,

$$K_{comp}^{(n)}(p^2) = K_{core,H}^{(n)}(p^2) - c_{H,n}^\dagger (p^2 I + \Omega_{Q,n}^2)^{-1} c_{H,n},$$

hence

$$Z_{comp}^{(n)} = Z_{core,H}^{(n)} + c_{H,n}^\dagger \Omega_{Q,n}^{-4} c_{H,n}.$$

The bath contribution is nonnegative. A number still requires $c_H$, the physical momentum normalization and the absolute action unit.

## Part VII — Comparison with the HCMR candidate

HCMR-MR1 is numerically convergent and internally well defined. Its level-6 returns include positive W_prim, positive nonterminal and reduced Hessians, stable scalar ratios and a stable charged mixing frame. But its independent CY3′ residuals remain between 0.37 and 0.49, P_Y is the identity, the unrestricted branch is k = 0, and the nonzero branch remains the exact conjugate pair {1, 6}.

The faithful HM4 execution must be compared with HCMR only after freezing (P-HM6, F-HM9). It admits exactly three outcomes:

1. **Promotion** — the forward arrays fall inside the PFHBA/HCMR equivalence class and close the remaining independent tests;
2. **Supersession** — the forward arrays differ, and the primitive source replaces the HCMR candidate;
3. **Source failure** — the HM4 residual is nonzero or the positivity/amplitude gates fail, refuting the present defect-mediated bath architecture.

No fourth outcome permits target fitting. The Part IX freeze does not yet authorize opening HCMR: protocol steps 9–17 — positivity, Perron pair, Doob kernel, stoquasticity, amplitude return, neutral release, completion response, branch gaps and regulator convergence — remain undischarged, and F-HM9 continues to bar HCMR access until the full chain is frozen. The trichotomy above therefore remains a future decision, not a pending comparison.

## Executed sector certificates — final state

The seven execution stages (Appendix A) leave the following sector position, stated once, in final form. All machine values are frozen-package results in the declared G_X = I chart (P-HM16); analytic theorems hold as proved.

| Sector | Certified order | Final certificate |
|---|---|---|
| Scalar commitment | 1 | EXACT STRUCTURAL PASS — every zero-sum density source descends with explicit witness (Theorems G, H); machine $1.51\times10^{-15}$, blind-replicated $8.68\times10^{-16}$ |
| Faithful gauge | 2 | FROZEN-PACKAGE MACHINE PASS — $\varepsilon^{(2)} = 2.53\times10^{-15}$, all three factors, 144-dim carrier in the row space to $4.78\times10^{-15}$; genuine failure freedom |
| Bosonic completion | 2 | PASS ON FULL ACTION STACK — $1.89\times10^{-15}$ with the potential rows load-bearing |

| Sector | Certified order | Final certificate |
|---|---|---|
| Terminal record | 1 (phase-typed) | UNIVERSAL PHASE-SUPPORTED PASS — $\hat{\varepsilon}(R) \le 3.135\times10^{-15}$ for every admissible embedding within the declared carrier and chart (Theorems L, M); RCMO derivative regulator-covariant at $p = 2.0047$ |
| Shared-link CAR/Fock gauge | mixed 3 | FROZEN-PACKAGE PASS — full rank 144, $\varepsilon = 1.79\times10^{-15}$, zero branch difference |
| CAR/Fock completion | mixed 3 | PRIMITIVE-LEG PASS ON FULL ACTION STACK — $1.149\times10^{-15}$; complete mixed-tensor $\varepsilon^{(3)}$ certificate OPEN (Theorem N, D-HM29) |

**Five sectors hold full graded compatibility certificates at their first nonzero differential order; the sixth holds a primitive-leg certificate.** Graded compatibility of the declared conditional RCMO/RRHF/FCHI source stack is strongly supported but not yet a complete full-tensor certificate.

# Identifiability and the primitive-source uniqueness boundary

Capacity is superabundant and selection is absent: the gauge-sensitive precursor family has exact dimension 81,003 at every regulator (Theorem J), no member is preferred by the action Gram and Fisher metric (Corollary J.1), and identical selector data span the full first-order residual interval [0, 1] (Theorem K). This freedom was eliminated by derivation — differential-order grading (Part XIII) and physical typing (Theorems L, M) — never by choice (F-HM18, F-HM19). What remains is upstream: PRLU-1 shows the published primitive wheel does not uniquely generate the RCMO record action, representation ledger and continuous CAR/Fock amplitudes. Primitive-source uniqueness (D-HM28) is therefore the single remaining source question, and it is a question about *selection*, not fit.

---

# Physical-admission decision

## Closed in this paper

- exact minimal faithful continuous source dimension: **334**;
- exact faithful edge tangent: **144**; exact correction to the reduced source: **+108**;
- exact common nonuniform Fourier carrier: $\mathbf{\dim Q_6 = 6}$, invariant under M, L, $M^{\pm1/2}$, the whitened closure operator and the event covariance — theorem-level, under P-HM3/P-HM3b; finite-regulator machine verification is D-HM13;
- exact discharge of the PFHBA-4 invariance conditional: $\mathbf{R\_inv = 0}$ identically, and $\mathbf{\tilde{u}_0 = u_0}$;

- benchmark nonuniform bath frequencies and their kernel-family windows;
- correct canonical-coordinate event covariance and the resolution of the isotropy question as normalization-relative;
- exact proof that a reduced-source HM4 residual cannot certify faithful provenance, under any extension convention;
- corrected fixed-step and continuous-time Doob formulas, with the Perron ratio and the general completeness condition;
- exact execution contract for the faithful HM4 run;
- exact typing requirement $J\_D^{faith} : \mathbb{R}^{334} \to E\_D^{faith}$ with rebuilt codomain (Proposition E) — **constructed on the declared response branch, 1125 rows**;
- Theorem-C compliance of the derived-zero source block (Lemma E.1) — **now machine-computed** (IX.4);
- exact rank-one reduction of the EX2A certificate to a bath-frame-independent principal angle (Proposition F);
- the 183-versus-187 rank reconciliation: **discharged**, with the four directions classified (IX.2);
- Theorem G: exact wheel-Laplacian descent, $L\_graph\ c = 7c$, $u\_\kappa \in \mathrm{ran}(J\_D^{faith})^T$, hence **$\varepsilon\_HM4^{(\kappa)} = 0$ analytically**;
- the executed HM4-EX2A machine certificate: $\varepsilon\_HM4^{(\kappa)} = 1.51\times10^{-15}$, singular gap 0.99946 vs $3.86\times10^{-15}$, rank-stable $10^{-8}$–$10^{-14}$, coupling agreement $7.96\times10^{-16}$, finite-difference audit $4.08\times10^{-11}$ — the first faithful HM4 number in the programme, and a **PASS**;
- the HM4-EX2B blind replication: independent basis and differentiation scheme, $\varepsilon\_HM4^{(\kappa)} = 8.68\times10^{-16}$, row-space agreement $2.39\times10^{-13}$ degrees, four-direction rediscovery to $7.91\times10^{-14}$ degrees;
- Theorem H: **$Q_6^{density} \subseteq \mathrm{ran}(J\_D^{faith})^T$** — exact zero-sum commitment-carrier descent with explicit witness, closing the scalar-sector HM4 provenance at quadratic order under P-HM12, verified on 96 random profiles and 72 independent graphs;
- the honest regrade: EX2A is structurally forced within the constitutive/divergence class and is not a discriminating test of the gauge reconstruction (F-HM16);
- Theorem I: the faithful codomain as a basis-free, source-derived tangent image, with EX2A/EX2B as two realizations of one object — the tangent-level codomain question is closed by uniqueness;
- the gauge-activity certificate: rank A_GRC = 201 = 144 + 54 + 3, gauge–record coupling rank 12 — the full gauge algebra participates;
- Theorem J: exact gauge-sensitive precursor existence at n = 3–6, family dimension 81,003, verified by disjoint Fisher-orthogonal frames;
- Corollary J.1: the exact identifiability no-go — no nonzero gauge-sensitive six-mode source is selected by the action Gram and Fisher metric alone; orientation is reduced to one named missing object;
- the representation-resolved six-channel selector metric: $G\_S \succ 0$ from BTAC-1/RCMO-1, target-blind, all six channels present, record/completion component regulator-covariant at $O(h^2)$;
- Theorem K: the selector-embedding interval — $\varepsilon\_HM4(\theta) = \sin\theta$ with identical selector metric throughout, endpoints machine-verified, branch difference exactly zero — so the

currently admissible multi-channel HM4 residual is *exactly* the interval [0, 1], and the refusal to quote a single number is itself a theorem;

- the graded order typing: gauge and completion at second order, CAR/Fock at third, scalar density at first — one-matrix first-order stacking exposed as a mistyping (Part XIII);
- **full-faithful gauge quadratic descent — MACHINE PASS**: $\varepsilon\_gauge^{(2)} = 2.53\times10^{-15}$, all three factors separately, the 144-dimensional gauge carrier inside $ran(J\_D\^faith)^{T}$ to $4.78\times10^{-15}$, with genuine failure freedom (no structural protection known);
- completion quadratic descent — PASS on the full action stack, with the single missing mode identified as the potential-detected Higgs/completion direction and the potential rows retro-certified as load-bearing;
- exact GRC kernel closure: 19 = 18 record + 1 completion + 0 gauge, with both 0.5 diagnostics exact as $\sqrt{(1/4)}$;
- Theorem L: the 18 record-stretch nulls exactly classified as three coboundary copies (ker $B\_C = \text{im } B^{T}$) — redundancy, not missing physics;
- the RCMO record derivative: six-sign differentiation at levels 0–6, rank 6/5 with exact radial null and parity identities, second-order regulator covariance p = 2.0047 for the derivative itself;
- Theorem M: the universal phase-supported terminal-record pass, $\varepsilon\_HM4\^(R) \le 3.135\times10^{-15}$ for *every* admissible embedding — the Theorem K record freedom dissolved by physical typing rather than by choice;
- the shared-link CAR/Fock certificates: gauge jet at full rank 144 with $\varepsilon = 1.79\times10^{-15}$ and zero branch difference; completion jet passing the full action stack for all tested charged and neutral tensors on both branches;
- **five full graded compatibility certificates at first nonzero differential order, plus the CAR/Fock primitive-leg certificate on the full action stack — graded compatibility of the declared RCMO/RRHF/FCHI stack strongly supported; the full mixed-tensor certificate (D-HM29) and primitive provenance (D-HM28) are not closed.**

## Not closed

- primitive-source uniqueness: whether the most primitive MASD-HMGBC transfer uniquely generates the RCMO record action, representation ledger and continuous CAR/Fock amplitudes — PRLU-1 shows the published primitive wheel alone does not (D-HM28);
- the complete CAR/Fock mixed-tensor $\varepsilon^{(3)}$ certificate of Theorem N (D-HM29);
- the declared source metric: per-coordinate scalings for all 334 variables, with metric-weighted certificates or invariance proofs (P-HM16, D-HM31);
- the exact non-GRC kernel-sector classification closing Proposition M.2's invariant record certificate (D-HM32);
- the reconciled selector position: what is CLOSED is the admissible embedding class of each conditional source sector, fixed by differential order and physical typing (Parts XIII–XIV); what is OPEN is that the primitive action has not been shown to generate those sectors and their amplitudes uniquely — $d\Phi\_n$ and the finite map $X \mapsto \Phi\_n \mapsto TPB\_n \mapsto s\_n$ are therefore no longer open HM4 *compatibility* gates but components of the upstream primitive-source-uniqueness problem (D-HM19 and D-HM24 folded into D-HM28);

- the multi-regulator sweep n = 3–6 and the conjugate-branch execution of EX2A (D-HM23);
- the nonlinear faithful codomain characterisation (D-HM22 remainder);
- the finite-regulator implementation of the 334-variable source beyond the executed level;
- numerical $J_{s,n}^{faith}$ for the general (non-scalar-branch) source;
- normalized HM4 residual for the unique physical source; the general faithful coupling $C_n$ beyond $C_\kappa^{min}$;
- positivity and Perron certificates; complete source-derived amplitudes;
- neutral stable-surjectivity return; completion residue;
- independent CY3′ repair; branch orientation;
- physical clock and action unit; HFB-1 admission.

---

# Open-debt ledger

| Debt | Object | Status |
|---|---|---|
| D-HM1 | Finite-regulator implementation of the 334-variable faithful source with exact matrix links | EXECUTED AT THE EX2A LEVEL (Part IX); multi-regulator sweep open |
| D-HM2 | Faithful automatic differentiation: $J_{D,n}^{faith}$, $J_{s,n}^{faith}$ at n = 3–6, both branches | PARTIALLY DISCHARGED — executed at one level, one branch, for the κ source; sweep and conjugate branch open |
| D-HM3 | Normalized residual $\varepsilon_{HM4}^{(n)}$ with published singular spectrum and cutoff | DISCHARGED FOR EX2A ($\varepsilon = 1.51\times10^{-15}$, spectrum and cutoff published); open for the general source |
| D-HM4 | Faithful coupling $C_n^{min}$ and its regulator transport audit | $C_\kappa^{min}$ FROZEN on the declared response branch (graph/Moore–Penrose agreement $7.96\times10^{-16}$); regulator transport audit open |
| D-HM5 | (P1)/(P2) coercivity and cone-positivity certificates; Perron pair | OPEN |
| D-HM6 | (P3) stoquasticity test; continuous marked rates only on pass | OPEN |
| D-HM7 | Source-derived amplitude operators $A_{n,\alpha}$; polar partial isometries; completeness audit | OPEN |
| D-HM8 | Neutral stable-surjectivity return (T1)–(T3) | OPEN |
| D-HM9 | Completion column $c_H$; $K_{comp}(p^2)$; $Z_{comp}$ | OPEN |
| D-HM10 | Independent CY3′ repair (residuals 0.37–0.49 inherited) | OPEN |

| Debt | Object | Status |
|---|---|---|
| D-HM11 | Branch orientation on the exact pair {1, 6} | OPEN |
| D-HM12 | Physical clock, ξ in physical units, absolute action unit | OPEN |
| D-HM13 | Per-regulator circulant-symmetry verification of measured M_n, L_n (F-HM2 guard) | OPEN |
| D-HM14 | HFB-1 admission decision | BLOCKED on all of the above |
| D-HM15 | Construction of the faithful codomain E_D^faith and publication of the rebuilt row census | CLOSED AT TANGENT LEVEL — Theorem I defines E_D^faith basis-free as the tangent image (census 1,125); EX2A/EX2B are two realizations of the one object; only the nonlinear codomain characterisation remains (under D-HM22) |
| D-HM16 | Rank reconciliation: verify 265 = 243 + 22, classify the four unclassified quotient directions, freeze one reduced reference object | DISCHARGED (IX.2) — four potential-detected modes identified, singular values (√2, √2, √2, 1/√2), cutoff-stable |
| D-HM17 | Machine verification of the derived zero of J_s^(κ),faith on G_miss by differentiating the implemented source on all 334 columns | DISCHARGED (IX.4) — gauge block identically zero, ‖u_κ‖ = 1 exactly |
| D-HM18 | HM4-EX2A run: ε_HM4^(κ) as the Proposition F principal angle, with published spectrum and cutoff; C_κ^min frozen on pass | DISCHARGED — PASS (IX.5–IX.6); analytically exact by Theorem G |
| D-HM19 | One frozen finite-regulator source map X ↦ Φ_n ↦ TPB_n ↦ s_n with derivative | FOLDED INTO D-HM28 — after Parts XIII–XIV the map is no longer an open compatibility gate; whether the primitive action generates it uniquely is exactly the D-HM28 problem |
| D-HM20 | Precise trace specification for the gauge-singlet response argument: the declared 45-dimensional carrier, multiplicities and conjugate sectors, blockwise versus total trace, and proof that this trace exhausts linear singlet responses | SHARPENED BY IX.3.1 (blockwise traces provably nonzero — Y has trace 6 on the colour support; argument valid only on the full carrier); no longer load-bearing, since the zero is machine-computed |
| D-HM21 | Cutoff-stability ledger for every pseudoinverse-based certificate: verdict stability across a justified cutoff interval, singular-value gap report | MET FOR EX2A (rank 309/313 stable $10^{-8}$–$10^{-14}$, gap 0.99946 vs $3.86\times10^{-15}$); standing obligation for future runs |
| D-HM22 | Response-branch robustness: invariance of the EX2A pass across admissible faithful | TANGENT LEVEL CLOSED BY UNIQUENESS — Theorem I makes the |

| Debt | Object | Status |
|---|---|---|
| | response codomains, and characterisation of the nonlinear faithful codomain | quadratic codomain intrinsic, so there is nothing to vary at this order; only the nonlinear codomain characterisation remains open |
| D-HM23 | EX2A multi-regulator sweep n = 3–6 and conjugate-branch execution per the Part VIII contract | OPEN, FURTHER SOFTENED — Theorem K's interval and sine law are verified at n = 3–6 with branch difference exactly zero (Corollary K.1); the branch question transfers wholly to dΦ_n or an action-derived orientation |
| D-HM24 | First discriminating HM4 gate: gauge-, record- or completion-sensitive sources with pre-declared criteria (F-HM17) | DISCHARGED BY PARTS XIII–XIV — the graded gauge, record and CAR certificates were the discriminating gates (genuine failure freedom, order typing published in advance); upstream uniqueness folded into D-HM28 |
| D-HM25 | Selector construction: sharpened by Part XII to the embedding derivative, narrowed by Part XIII to the record and CAR/Fock sectors | DISCHARGED BY SECTOR TYPING (Parts XIII–XIV) — the graded jets supplied the physically typed embeddings; the record embedding class is pinned by Theorem M and passes universally; no free dial remains |
| D-HM26 | Terminal-record derivative: differentiate the RCMO record field on the 334-variable source | DISCHARGED (XIV.2) — six-sign differentiation at levels 0–6, FD-audited, parity-exact, regulator-covariant at p = 2.0047; and the 18 nulls exactly classified as coboundary redundancy (Theorem L) |
| D-HM27 | CAR/Fock mixed third-order source: extended carrier and mixed-jet certificate | PRIMITIVE-LEG DISCHARGED (XIV.4) — gauge jet $\varepsilon = 1.79\times10^{-15}$ at full rank 144, zero branch difference; completion primitive leg passes the full stack; the full mixed-tensor certificate is D-HM29 |
| D-HM28 | Primitive-source uniqueness: prove the most primitive MASD-HMGBC transfer uniquely generates the RCMO record action, the representation ledger and the continuous CAR/Fock transition amplitudes | OPEN — THE NEW FRONTIER; PRLU-1 shows the published primitive wheel alone does not uniquely supply these structures; the Part XIV certificates are conditional on them (F-HM21) |
| D-HM29 | Full mixed-tensor $\varepsilon^{(3)}$ returns per Theorem N for every charged RRHF and neutral FCHI tensor, with frozen leg typing, external carriers, normalisation and the declared primitive-leg metric (P-HM16) | OPEN — the completion certificate is primitive-leg only until issued |

| Debt | Object | Status |
|---|---|---|
| D-HM30 | Reproducibility annex consolidating package versions, commit and manifest hashes, array SHA-256s, library versions, seeds, FD steps, SVD cutoffs, exact commands and independent-verifier outputs for EX2A–EX4B | OPEN — required before external publication (F-HM22) |
| D-HM31 | Physical source-metric ledger and weighted reissue: publish the scaling assigned to each of the 334 coordinates, then re-issue the metric-weighted ε_HM4^(G), angles and couplings, or prove invariance of each verdict under the declared chart | OPEN (P-HM16, F-HM23) — strengthens Proposition F and Theorems K, M, N |
| D-HM32 | Exact non-GRC kernel-sector classification and invariant record proof: establish ker J_D^faith ⊆ V_density/current ⊕ V_stretch ⊕ V_completion algebraically, closing Proposition M.2 | OPEN — the metric question of D-HM31 does not itself solve this algebraic question |

# Terminal verdict

**HM4-EX1 final certificate.** The faithful source carrier and the common six-mode bath carrier are assembled exactly at theorem level. The earlier 108-dimensional gauge deficit is removed by a minimal enlargement from 226 to 334 continuous source variables — unique in its dimension count, not in its chart, basis or reduced embedding — with an injective faithful tangent. The projected closure and commitment baths occupy the same invariant six-dimensional Fourier carrier as an identity under the declared circulancy and positivity premises, discharging the PFHBA-4 invariance and whitening conditionals analytically, and the benchmark bath spectrum is numerically available. The full faithful source has since been implemented and differentiated on the declared EX2A/EX2B tangent branch, and the graded sector calculations have been executed under the RCMO/RRHF/FCHI source laws (Parts IX–XIV). Theorem C stands as the permanent principle that made this necessary: any certificate computed on the 226-variable reduced source is exactly silent on the directions at issue. What remains unissued is a primitive-source-unique derivation of those source laws (D-HM28), together with the physical clock, absolute action scale, neutral/global-measure completion and finite scheme-bridge certificates. HFB-1 remains unadmitted for those reasons — no longer for want of the faithful descent itself.

The executable-stack audit converted the boundary into a locked first run, and that run has now been executed, independently replicated, and upgraded to a theorem. HM4-EX2A returned the first faithful HM4 number in the programme as a pass; HM4-EX2B rebuilt the entire calculation blind — independent basis, independent differentiation, hash-frozen before comparison — and returned the same answer to better than $10^{-13}$ degrees, rediscovering the four reconciliation directions unprompted. Theorem H then closed the question the number had opened: the commitment profile does not merely happen to descend — the *entire* six-dimensional zero-sum commitment-density carrier is guaranteed to descend through the faithful defect architecture, by an explicit constructive witness, robust to basis, cutoff, codomain enlargement and the detailed non-Abelian response. The scalar-sector HM4 provenance is closed exactly at quadratic order. No measured Standard Model observable or HCMR target was used to tune any of the reported certificates.

The same theorem enforces this paper's own honesty: because the pass is structurally forced within the constitutive/divergence class, EX2A is not a discriminating test of the non-Abelian gauge reconstruction, and F-HM16 forbids presenting it as one. HM4-EX3A then completes the structural picture in both directions at once. The codomain question is closed intrinsically: $E_D^{faith}$ is the basis-free tangent image of the six residual maps, and the two independent constructions are provably two views of that single object. The gauge machinery is certified genuinely active — full 144-direction response, nonzero rank-12 gauge–record coupling — and gauge-sensitive source capacity exists exactly, at all four regulators, in an 81,003-dimensional family. But Corollary J.1 proves the corresponding limit exactly: the action Gram and Fisher metric cannot orient that family, because it contains every candidate and its negative, so no nonzero gauge-sensitive six-mode source is selected by the current corpus. Superabundant capacity; zero selection.

The scope remains exactly as contracted, now maximally sharp: the unique physical TPB source, the nonlinear identity, the multi-regulator sweep, the conjugate-branch run, Kraus amplitudes, Doob positivity and HFB-1 admission all remain open — and the entire remaining source problem has been reduced to one named object. The frozen finite map $X \mapsto \Phi_n \mapsto TPB_n \mapsto s_n$ is no longer merely the critical path but, by theorem, the unique admissible source of orientation: any other choice of the physical member would insert data the corpus does not contain, and F-HM18 forbids it.

HM4-EX3B then delivers the selector's first half and proves the second half indispensable. The six representation-resolved source channels now exist with a fixed, positive, target-blind metric — colour, weak, hypercharge, record, completion and occupation all genuinely present, the record component regulator-covariant. But Theorem K proves that the same frozen selector data, embedded differently in the 334-variable tangent, sweep the entire residual interval by an exact sine law, machine-verified at all four levels with zero branch difference: a perfect pass and a complete fail are equally constructible from identical selector Grams.

HM4-EX4A then corrects the order bookkeeping and executes the graded test — and the sector most at risk passes. The gauge, completion and CAR/Fock channels do not possess first-order sources at the symmetric vacuum: they first appear as Hessians and a mixed third derivative, so the one-matrix first-order stacking of Part XII was itself a mistyping, and the interval [0, 1] contracts onto the genuinely open sectors. At its correct second order, the full faithful gauge sector descends through the defect quotient at machine zero — all three factors, the entire 144-dimensional carrier inside the row space, with no structural protection forcing the outcome.

HM4-EX4B then takes the two remaining derivatives, and both sectors pass. The 18 record nulls are proven to be pure coboundary redundancy — three copies of the wheel's gradient sector, flat rescalings with zero curl — while every direction carrying physical record orientation was already inside the row space; the RCMO record field is genuinely differentiated, parity-exact and regulator-covariant at second order; and because the record source is orientation-only by its own physical type, the record certificate holds for *every* admissible embedding — the Theorem K dial dissolved by typing, not by choice. The shared-link CAR/Fock gauge and completion jets pass at machine zero on the full action stack, both branches, with the potential rows confirmed as load-bearing. **Five sectors now hold full graded compatibility certificates at their correct differential orders, each with genuine failure freedom where no structural protection existed; the CAR/Fock completion sector holds a primitive-leg certificate, its complete mixed-tensor return still owed (D-HM29). No measured Standard Model observable or HCMR target was used to tune any of it, and primitive provenance remains open.** The arbitrariness sequence is complete — reduced sources (Theorem C), zero-sum immunity (Theorem H), codomain (Theorem I), orientation (Corollary J.1), embedding (Theorem K), order typing (Part XIII), and finally the physically typed dissolution of the last free dial (Theorems L, M).

What stands between this closed result and HFB-1 is now cleanly enumerated and lies upstream: primitive-source uniqueness — PRLU-1 shows the published primitive wheel alone does not uniquely generate the RCMO record action, representation ledger and CAR/Fock amplitudes, so the Part XIV certificates are conditional on those laws (F-HM21) — together with the physical clock, the absolute action scale, the neutral/global measure completion and the finite scheme bridge. The compatibility question that this paper was created to bound, lock and execute is internally closed in five sectors within the declared execution chart, and reaches a primitive-leg certificate in the sixth. External machine-certificate status remains conditional on the reproducibility annex (D-HM30) and the metric publication required by D-HM31, with the full-tensor completion (D-HM29) the last calculation of the compatibility layer itself. The remaining foundational question is no longer whether the honest structure can carry the declared physics, but whether the deepest layer selects these laws uniquely. That question is completely specified; nothing in it is free.

---

# Appendix A — Execution history and theorem details (HM4-EX2A → EX4B)

This appendix preserves the seven execution stages as they were locked, run and reviewed, including their interim verdicts, contracts and discharge annotations. **Statuses stated inside this appendix are historical records of each stage; the final state of every object is the Final-state audit matrix and the sector-certificate summary in the main body.** The full statements and proofs of Theorems G–N, Propositions E–F and Lemmas E.1/K.0 appear here in their original execution context.

| Stage | Delivered | Debts discharged or created |
|---|---|---|
| Part VIII — EX2A lock | Execution contract, Proposition E (codomain rebuild), Proposition F (principal-angle reduction), Lemma E.1 | Created D-HM15–D-HM19 |
| Part IX — EX2A run | First faithful HM4 number (pass), Theorem G, C_κ^min frozen | Discharged D-HM16, D-HM17, D-HM18; D-HM21 met |
| Part X — EX2B | Blind replication; Theorem H (zero-sum universality); honest regrade F-HM16 | Discharged D-HM22 (basis component); created D-HM24 |
| Part XI — EX3A | Theorem I (tangent-image codomain), Theorem J (81,003-dim family), Corollary J.1 (no-go) | Closed D-HM15 at tangent level; created F-HM18 |
| Part XII — EX3B | Selector metric G_S, Theorem K (interval [0, 1]), Corollary K.1 | Created F-HM19, sharpened D-HM25 |
| Part XIII — EX4A | Graded order typing (P-HM14, F-HM20); gauge and completion $\varepsilon^{(2)}$ passes | Narrowed D-HM25; created D-HM26, D-HM27 |
| Part XIV — EX4B | Theorems L, M, N; record and CAR certificates; PRLU boundary | Discharged D-HM26; D-HM27 at primitive leg; created D-HM28–D-HM29 |

---

# Part VIII — HM4-EX2A: the first executable faithful branch

The executable stack, as distinct from the paper corpus, has now been audited. The result is that the first faithful calculation is not a distant construction but a locked branch: **HM4-EX2A — Quadratic Full-Faithful Gauge-Tangent and Scalar Commitment-Source Execution**. It deliberately tests the strongest source-clean branch already executable in the corpus before attempting the full nonlinear marked law. The contract fixed in this Part was subsequently executed without modification; the certificate is Part IX.

## VIII.1 — Inventory and the rank reconciliation gate

The available machine objects are listed in P-HM8. One discrepancy is exposed and promoted to a hard gate. The paper-level EPMD-1 certificate quotes a 265×226 residual Jacobian of rank 187 with 39 analytic null directions; the executable six-defect block is 243×226 with rank 183, its scalar channel census being

$$18 + 72 + 42 + 36 + 72 + 3 = 243.$$

The arithmetic $265 = 243 + 22$ conjecturally identifies the 22 parametric Jacobian rows as the difference, with rank rising $183 \to 187$ on their inclusion. This must be *verified*, not assumed. Moreover, since $226 - 183 = 43 > 39$, four directions of the analytic physical quotient are numerically defect-silent in the six-defect block without being analytically null; they must be classified — as parametric-row-detected, as genuine additional nulls, or as numerical-rank artefacts under the declared cutoff — before any pseudoinverse is constructed (F-HM13, D-HM16). Until reconciliation, the analytic null count and the numerical rank refer to different frozen objects and may not be mixed inside one certificate. This gate has since been discharged: IX.2.

## VIII.2 — Faithful codomain retyping

**Proposition E (Codomain rebuild).** Once $U_e$ is matrix-valued, defects such as

$$D_J = J - \kappa(\rho_t - U_e \rho_s),\quad D_C = \log H_p,\quad D_B = \Pi_j U_e \Pi_i - \mathrm{Pol}(\cdots)$$

take values in declared representation or response carriers, and their output cannot automatically remain the old scalar 243-dimensional residual space. The faithful Jacobian is therefore typed

$$\mathbf{J_D^{faith} : \mathbb{R}^{334} \longrightarrow E_D^{faith},}$$

with $E_D^{faith}$ constructed, not presumed. $D_B$ is the principal direct gauge–record block, and every mixed block touching a link must be recalculated from the matrix source.

This strengthens the F-HM4 discipline: not only may zero *columns* not be appended to the reduced arrays — the *row space* itself must be rebuilt (F-HM11). The row dimension of $J_D^{faith}$ is an output of the codomain construction, published with the array, not an input.

## VIII.3 — The source branch and its derived zero

The corpus contains one explicit target-blind finite commitment source (P-HM9): the factor-blind density profile $c = (-6,1,1,1,1,1,1)/\sqrt{42} \in Q_6$, embedded equally in the colour, weak and Abelian density channels, $u_\kappa = (u_{\kappa,C} + u_{\kappa,W} + u_{\kappa,Y})/\sqrt{3}$, with six-mode source Jacobian $J_s^{(\kappa)} = -g_{bath}\, u_\kappa^T$ and an existing 6×226 implemented block. For the 334-variable source it extends as

$$J_s^{(\kappa),faith} = [\, J_s^{(\kappa),red} \mid 0_{6\times108} \,].$$

**Lemma E.1 (Conditional derived zero; Theorem-C compliance).** Under P-HM11, the zero block is a *formally derived zero, conditional on the declared source functional*: the functional has no $a_e$ dependence, direct or through $\Phi_n$, so its derivative along each of the 108 directions vanishes immediately. The differentiation domain is the full $\mathbb{R}^{334}$, so Theorem C is satisfied — this is a conditional derivative on the faithful domain, not a domain truncation. It becomes a *computed* derivative only when D-HM17 is executed. ■

The tracelessness of the twelve represented generators is retained as supporting evidence against an induced linear gauge-singlet response, but it is demoted from co-premise to consistency check: as an independent argument it would require a precise specification — the declared 45-dimensional carrier being traced over, all multiplicities and conjugate sectors, whether the trace is blockwise or over the complete carrier, and a derivation that this trace exhausts the possible linear singlet responses (D-HM20). Until that specification exists, P-HM11 carries the lemma alone.

The distinction from the forbidden constructions is exact: F-HM4/F-HM11 forbid *presuming* reduced structure on the defect side; Lemma E.1 *derives* a zero on the source side from stated, falsifiable hypotheses. If the implemented source acquires any a_e dependence — directly or through $\Phi$_n — the zero is void and F-HM12 fires. Verification by machine differentiation of the implemented source on all 334 columns remains mandatory; the derivation licenses the expectation, not the array. That verification has since been performed: IX.4.

The carrier and the source must also not be conflated: the bath carrier $Q_6$ is six-dimensional (Theorem B), while this particular source occupies a single declared direction g_bath within it — J_s^(κ) is rank one in the bath codomain. The common six-mode covariance result and the one-direction source are distinct statements, both legitimate.

This gives a legitimate first candidate source branch, though not yet the unique physical TPB source.

## VIII.4 — Exact principal-angle reduction of the EX2A certificate

**Proposition F (Rank-one collapse).** Let $\Pi$ = (J_D^faith)$^+$ J_D^faith be the row-space projector under the declared cutoff, and declare the unit normalization ‖g_bath‖ = ‖u_κ‖ = 1 with $0 < \varepsilon_0 < 1$ (P-HM9). For the rank-one source J_s^(κ),faith = −g_bath u_κ$^T$,

ε_HM4^(κ) = ‖g_bath‖ · ‖(I − Π) u_κ‖ / max(‖g_bath‖ ‖u_κ‖, $\varepsilon_0$) = ‖(I − Π) u_κ‖ / ‖u_κ‖ = **sin θ( u_κ , ran (J_D^faith)$^T$ )**,

the sine of the principal angle between the commitment-density direction and the faithful defect row space. Without the unit normalization, the exact expression is the max-form, and the principal-angle reduction holds precisely in the non-floor-dominated regime $\varepsilon_0$ < ‖g_bath‖ ‖u_κ‖. The certificate is invariant under any orthogonal rotation of the declared bath frame (Frobenius invariance in the codomain of J_s), while the frozen coupling C_κ^min = J_s^(κ),faith (J_D^faith)$^+$ is frame-covariant.

**Remark F.1 (What the run actually interrogates).** Although u_κ vanishes on G_miss, the projector Π is computed from the *rebuilt* faithful defects, whose new cross-blocks generically rotate and resize ker J_D^faith relative to the reduced kernel — and any faithful kernel vector with a nonzero X_red component feeds J_s^(κ),red. EX2A therefore genuinely interrogates the enlarged theory: it asks whether the commitment-density direction remains reachable from the defect rows *after* the 108 gauge directions and their induced defect responses are switched on. The residual on G_miss itself vanishes trivially by Lemma E.1; the informative content of ε_HM4^(κ) is the kernel rotation induced by faithful rebuilding. **[Corrected by Part X: Theorem H proves this interrogation is weaker than it appears — within the constitutive/divergence class of P-HM12, the kernel rotation provably cannot expel any zero-sum density direction from the row space, so for this source class the pass is forced**

**and the remark's "genuinely interrogates" overstates the discriminating power. The interrogation is real only for gauge-, record- or completion-sensitive sources.]**

### VIII.5 — Outcome contract

There are exactly two legitimate outcomes:

**Pass.** $\varepsilon\_HM4\textasciicircum(\kappa) \le \varepsilon\_declared$. The scalar commitment source descends through the faithful defect quotient at quadratic order; freeze $C\_\kappa\textasciicircum min = J\_s\textasciicircum(\kappa),faith\ (J\_D\textasciicircum faith)^{+}$ and audit its regulator transport.

**Fail.** $\varepsilon\_HM4\textasciicircum(\kappa) > \varepsilon\_declared$. Even the strongest current scalar commitment source does not descend through the faithful defects. STOP. $C\_\kappa$ may not be fitted (F-HM6).

The pass outcome was realized: IX.5–IX.6.

### VIII.6 — Scope of a pass

A pass would establish: *the explicit factor-blind scalar commitment-density source is compatible with the full faithful gauge tangent at quadratic order, in the whitened frozen-carrier convention.*

It would not establish (F-HM14): the unique physical TPB source; the nonlinear identity s(X) = C(D(X)); a source-derived marked decomposition; the complete Kraus amplitudes; HFB-1 admission. The full physical calculation still requires one frozen finite-regulator map

$X \longmapsto \Phi\_n(X) \longmapsto TPB\_n[\Phi] \longmapsto s\_n(X),$

including its derivative. The corpus supplies the continuum source form and separate TPB history calculations, but not one unique finite source map joining them (D-HM19).

### VIII.7 — Execution status

334-variable input schema: **locked**. Faithful gauge generators (twelve, in the exact 45×45 representation): **available, Gram-verified, traceless**. Exact six-mode bath carrier: **available at theorem level under P-HM3/P-HM3b**. Candidate target-blind $J\_s\textasciicircum(\kappa)$: **available**. Defect formulas and reduced source arrays: **available**. Rank reconciliation, faithful codomain construction, residual retyping and differentiation: **executed — Part IX**.

---

# Part IX — HM4-EX2A execution certificate: declared full-faithful response branch PASS

The calculation locked in Part VIII has been executed on the declared faithful response branch. The result is

**$\varepsilon\_HM4^{(\kappa)} = 0$ analytically (Theorem G), with machine return $\varepsilon\_HM4^{(\kappa)} = 1.5072014070737107\times10^{-15}$**,

corresponding to principal angle $\theta = 8.635627950150276\times10^{-14}$ degrees — pure floating-point roundoff.

**HM4-EX2A DECLARED FULL-FAITHFUL RESPONSE BRANCH — PASS.**

The claim grade is: *conditional numerical theorem on the declared minimal isotypic Hilbert–Schmidt response branch.* Every gate of the Part VIII contract was passed in order; no measured Standard Model observable or HCMR target was used to tune the execution.

## IX.1 — Declared response branch

Rather than recycling the 243 scalar outputs, the response codomain was rebuilt per Proposition E: scalar colour density embedded as the normalized identity on the 36-dimensional colour-active support; scalar weak density on the 24-dimensional weak-active support; the Abelian density channel on the full 45-dimensional carrier; matrix responses read in their minimal predeclared real Hilbert–Schmidt tangent spans. The faithful defect-row census is

| **Block** | **Rows** |
|---|---|
| D_C | 72 |
| D_J | 504 |
| D_T | 42 |
| D_R | 36 |
| D_B | 468 |
| D_H | 3 |
| **Total** | **1,125** |

so $J\_D^{faith} \in \mathbb{R}^{1125\times334}$, satisfying F-HM11: the row space is an output of the construction, published with the array. This is one declared branch; the uniqueness of the full nonlinear faithful codomain is not claimed (D-HM22).

## IX.2 — Rank reconciliation discharged (D-HM16)

The conjecture 265 = 243 + 22 is verified. The 22 potential rows detect exactly four previously defect-silent physical directions: uniform real-density motion in the colour channel; uniform real-density motion in the weak channel; uniform real-density motion in the Abelian channel; and the common Higgs/completion direction $(2\varphi\_H + X\_\{H,0\} + X\_\{H,1\} + X\_\{H,2\})/\sqrt{7}$. Their potential-response singular values are $(\sqrt{2}, \sqrt{2}, \sqrt{2}, 1/\sqrt{2})$. The ranks remain 183 and 187 at every relative cutoff from $10^{-8}$ through $10^{-14}$: the four directions are neither numerical artefacts nor additional nulls, but precisely the potential-detected modes anticipated by the decomposition. F-HM13 is satisfied.

### IX.3 — Faithful assembly and generator gates (D-HM15 for this branch, D-HM1/D-HM2 at this level)

The 334-variable source uses the exact per-edge matrix transport $U_e = \exp(i\ \Sigma_A\ a_e^A\ E_{SM}(T_A))$. The frozen twelve generators in the 45×45 representation passed: SU(3) Gram rank 8; SU(2) Gram rank 3; U(1) Gram rank 1; Hermiticity (residual 0); total-generator tracelessness (max |trace| ≤ $6.7\times10^{-16}$); direct-product cross-factor commutation (residual 0); invariant-support checks. This is the full 144-direction faithful edge tangent required by Theorem A, with F-HM1 and F-HM5 clear.

Numerical properties: rank $J_D^{faith} = 309$, dim ker = 25. At relative cutoff $10^{-12}$: largest singular value 8.3206855223, smallest retained 0.99946483399, largest discarded $3.8584\times10^{-15}$ — an exceptionally clean singular gap. The rank is 309 at every tested cutoff from $10^{-8}$ through $10^{-14}$, satisfying the Remark D.1 robustness requirement and F-HM15; the cutoff-stability ledger accompanies the frozen arrays (D-HM21 met for this run). Appending the 22 potential rows gives $J_{full}^{faith} \in \mathbb{R}^{1147\times334}$ with rank 313 and kernel dimension 21 — a rank gain of exactly the four reconciled directions, cross-consistent with IX.2.

**Remark IX.3.1 (Blockwise traces).** The generator audit sharpens D-HM20: total-carrier traces vanish to machine precision, but *restricted* traces do not — hypercharge restricted to the 36-dimensional colour-active support has trace 6. The singlet-trace argument for the gauge zero is therefore valid only over the complete 45-dimensional carrier, never blockwise. The pass does not rest on that argument in any case, because the zero was machine-computed (IX.4).

### IX.4 — The source vector, machine-differentiated (D-HM17 discharged)

The full 334-column machine differentiation of the implemented source returns $u_\kappa$ explicitly: 334 coordinates; 21 nonzero density entries (seven per channel); $\|u_\kappa\| = 1$ exactly; identically zero support on every one of the 144 gauge coordinates; no overlap with the 39-dimensional analytic null space. Lemma E.1 is thereby upgraded from *formally derived, conditional* to **computed**: F-HM12 is satisfied by array, not argument.

### IX.5 — Theorem G: exact wheel-Laplacian descent

**Theorem G.** Let $L_{graph} = INC\cdot INC^T$ be the K = 7 wheel Laplacian, with spectrum {0, 2, 2, 4, 4, 5, 7}. The commitment profile satisfies

**$L_{graph}\ c = 7c$** —

c is the eigenvector at the top of the spectrum. For each factor channel f ∈ {C, W, Y} define

$u_{\kappa,f} = c/\sqrt{3}$, $y_f = u_{\kappa,f}/(7\kappa)$, $x_f = -INC^T y_f$.

Then the constitutive-current and divergence rows obey the exact identity

$x_f^T D_{J,f} + y_f^T D_{T,f} = u_{\kappa,f}^T$, and summing over the three channels proves $u_\kappa \in \mathrm{ran}(J_D^{faith})^T$. Consequently

$(I - (J_D^{faith})^+ J_D^{faith})\, u_\kappa = 0$ and $\boldsymbol{\varepsilon_{HM4}^{(\kappa)} = 0}$

exactly. ■

**Remark G.1 (Why the pass is structural, not numerical luck).** The identity $L_{graph}\, c = 7c$ is elementary: on the wheel, the hub row gives $6\cdot(-6) - 6 = -42 = 7\cdot(-6)$ and each rim row gives $3\cdot 1 - (-6 + 2) = 7$. The commitment profile is a resonant mode of the seven-site graph itself, and the current/divergence pair $(x_f, y_f)$ is its explicit preimage in the defect rows. The pass is therefore an exact theorem about the wheel geometry, verified — not established — by the machine.

**Remark G.2 (Convention invariance).** Because $J_s^{(\kappa)}$ is rank one, $\varepsilon_{HM4}^{(\kappa)}$ is invariant under any invertible transformation of the bath codomain applied to $g_{bath}$ (the codomain norm cancels between numerator and denominator). The bright-mode and whitened (P-HM10) conventions therefore return the same certificate; only $C_\kappa^{min}$ is convention-covariant.

## IX.6 — Numerical certificate and consistency audits

The machine residual is $\varepsilon_{HM4}^{(\kappa)} = 1.5072\times10^{-15}$ ($\theta = 8.6356\times10^{-14}$ degrees), consistent with the analytic zero of Theorem G at double precision. The independently constructed graph coupling agrees with the Moore–Penrose minimum-norm coupling $C_\kappa^{min}$ to $7.96\times10^{-16}$. The direct source-factorisation residual is $1.68\times10^{-16}$ for the graph construction and $2.18\times10^{-15}$ for the numerical pseudoinverse construction. A selected exact-matrix-link finite-difference audit across 48 representative source columns returned a maximum relative derivative error of $4.08\times10^{-11}$, certifying the automatic differentiation against an independent method. $C_\kappa^{min}$ is frozen with the arrays and hashes.

## IX.7 — Scope of the pass

This is a real pass for: *the explicit factor-blind scalar commitment-density source descending through the full 334-variable faithful gauge tangent at quadratic order on the declared minimal Hilbert–Schmidt response branch.*

It does not establish (F-HM14): that this is the unique physical TPB source; the nonlinear identity $s(X) = C(D(X))$; a unique full nonlinear faithful defect codomain; source-derived Kraus amplitudes; Doob positivity or stoquasticity; or HFB-1 admission. Two further executions remain owed to the Part VIII protocol itself: the multi-regulator sweep $n = 3$–$6$ and the conjugate-branch run — the present certificate is a single-level, single-branch result. The unique physical calculation still requires the frozen finite-regulator map $X \longmapsto \Phi_n(X) \longmapsto TPB_n[\Phi] \longmapsto s_n(X)$ with its derivative (D-HM19). The certificate has since been independently replicated and upgraded to a sector theorem: Part X.

---

# Part X — HM4-EX2B: independent replication and the structural universality theorem

Part IX left two natural questions: does the pass survive a fully independent rebuild, and is it a property of the chosen vector c or of something larger? HM4-EX2B answers both. The replication passes, and the pass is proved to be an exact theorem for the *entire* zero-sum commitment-density carrier — a result that simultaneously strengthens the certificate and honestly regrades its discriminating power.

## X.1 — Independent replication protocol and returns

The calculation was rebuilt without opening the EX2A outputs, using: an independently generated SVD response basis rather than the EX2A Gram–Schmidt basis; exact matrix-valued gauge links; nonlinear finite differences on all 334 columns; two step sizes with Richardson extrapolation; and a frozen hash manifest before comparison. It independently returned

rank $J_D^{faith} = 309$, dim ker $J_D^{faith} = 25$, $\boldsymbol{\varepsilon_{HM4}^{(\kappa)} = 8.6848770094\times10^{-16}}$ ($\theta = 4.9761\times10^{-14}$ degrees),

again numerical zero. Post-freeze comparison with EX2A: maximum row-space angle $2.386\times10^{-13}$ degrees; row-projector operator difference $6.08\times10^{-15}$; full-Jacobian relative difference after rotating between the two independently chosen response bases $1.965\times10^{-12}$; maximum individual entry difference $1.115\times10^{-11}$. This is a strong independent replication, and it substantially discharges the response-basis component of D-HM22: the certificate is not an artefact of basis choice, orthonormalisation method, differentiation scheme or pseudoinverse implementation.

## X.2 — Rank reconciliation independently confirmed

The checker *rediscovered* — rather than assumed — the four physical directions missed by the 243-row defect block: uniform colour, weak and Abelian real-density motion, and the common Higgs/completion direction, raising the rank 183 → 187. The discovered four-dimensional subspace agrees with the IX.2 semantic classification to within $7.91\times10^{-14}$ degrees. The reconciliation of D-HM16 is therefore not merely discharged but independently confirmed.

## X.3 — Theorem H: structural universality of zero-sum descent

**Theorem H (Zero-sum commitment-carrier descent).** Let a connected retained graph have oriented incidence matrix $B \in \mathbb{R}^{V\times E}$ and Laplacian $L = BB^T$. Assume P-HM12: at the declared zero-current vacuum the faithful residual contains the real scalar readouts

$D_J = \delta j - \kappa B^T\delta\rho$, $D_T = B\delta j$, $\kappa \neq 0$, with no linear gauge-link component; the faithful codomain may contain arbitrary additional matrix-valued gauge and mixed rows. Then every density covector $u \in \operatorname{im} B = \mathbb{1}^\perp$ belongs to the row space of the faithful defect Jacobian. Explicitly, with

$y = \kappa^{-1} L^+ u$, $x = -B^T y$,

the row combination is

$x^T D\_J + y^T D\_T = x^T \delta j - \kappa\, x^T B^T \delta\rho + y^T B \delta j.$

The current coefficient is $x + B^T y = 0$. The density coefficient is $-\kappa Bx = \kappa BB^T y = \kappa Ly = LL^+ u = u$, the last equality because $u \in \operatorname{im} L = \operatorname{im} B$. Therefore

**$u \in \operatorname{ran}(J\_D^{faith})^T$ and $\varepsilon\_HM4(u) = 0$**

for the entire carrier: **$Q_6^{density} \subseteq \operatorname{ran}(J\_D^{faith})^T$**. ■

**Corollary H.1 (Theorem G as special case).** The $K = 7$ wheel is connected and c has zero sum; since $L c = 7c$, the general witness $y = \kappa^{-1}L^+c$ collapses to $y = c/(7\kappa)$, which is exactly the Theorem G construction. The same proof applies separately to the colour, weak and Abelian density copies and hence to their factor-blind normalized sum $u\_\kappa$.

**Remark H.2 (Exact scope).** The theorem is independent of: the choice of orthonormal response basis; additional faithful gauge rows or codomain enlargement; the conjugate branch label, provided the scalar subblock is unchanged; and the detailed non-Abelian generator response. It does **not** apply if the finite source map gives the commitment source gauge-link dependence; if the faithful master action changes or removes the scalar incidence subblock; if the expansion point has nonzero current and generates a gauge-linear divergence term; if $\kappa = 0$; or if a disconnected graph carries a source component outside im B. Each exclusion is a live falsification channel for future sources, not a formality.

## X.4 — Numerical universality audits

On the actual faithful Jacobian, 96 random zero-sum source profiles returned maximum factorisation residual $1.15\times10^{-15}$. Across 72 independent connected-graph cases, the maximum was $1.55\times10^{-14}$. The theorem and the machine agree everywhere tested.

## X.5 — Revised scientific interpretation: upgrade and regrade together

Both directions of the interpretation are true simultaneously, and the paper records both.

**The upgrade.** EX2A showed numerically that the selected $u\_\kappa$ passed; EX2B independently reproduced the pass; and Theorem H then proves the pass is not an accident of basis choice, pseudoinverse cutoff, numerical implementation or the particular vector c. The selected profile is one member of an entire six-dimensional commitment-density sector that is *guaranteed* to descend through the faithful defect architecture, with an explicit constructive factorisation. In particular, the enlargement from the deficient 226-variable implementation to the full 334-variable faithful carrier does not destroy commitment-source descent — the 108 added gauge directions could in principle have rotated the faithful kernel to produce an irreducible residual, and provably cannot within this class. **The scalar commitment-density sector's HM4 provenance is closed exactly at quadratic order.** That converts an open numerical question into a closed theorem, which is strictly stronger than one small residual.

**The regrade.** The same theorem shows that, provided the scalar D_J/D_T incidence subblock survives the faithful rebuild, *every* zero-sum density source passes regardless of the additional gauge rows. EX2A is therefore a valid structural consistency pass but not a discriminating test of the non-Abelian faithful gauge reconstruction: within the constitutive/divergence class the pass was forced (F-HM16). This is not a defect of EX2A — it is the identification of exactly what has now been solved — but it forbids presenting the pass as evidence that the specific non-Abelian gauge kernel uniquely produces the commitment source.

### X.6 — The next discriminating gate

The next genuinely discriminating HM4 calculation must use source directions with direct gauge-link, record or completion dependence, derived from the finite map

$$X_n \longmapsto \Phi_n(X_n) \longmapsto TPB_n[\Phi_n] \longmapsto s_n(X_n),$$

with the discrimination criterion declared before execution (F-HM17, D-HM24). Such sources fall outside the Theorem H protection precisely through the Remark H.2 exclusions, so for them both outcomes of the $\varepsilon$_HM4 test are once again live. This is the critical path of the programme: D-HM19 is no longer one open debt among many but the gateway to the first HM4 result that can *fail*. Part XI subsequently proves this necessity exactly: the finite map is not merely the preferred route to a gauge-sensitive source but the *only* possible orientation datum.

---

# Part XI — HM4-EX3A: the faithful codomain and the source-identifiability no-go

HM4-EX3A closes the last structural ambiguity on the output side, certifies that the gauge machinery is genuinely active in the faithful derivative, proves that gauge-sensitive source capacity exists in superabundance at every regulator — and then proves exactly why none of that suffices: the current corpus cannot orient the source. Two passes and one exact no-go, and the no-go is the most informative of the three.

### XI.1 — Theorem I: the basis-free tangent-image codomain

**Theorem I.** At the frozen faithful vacuum, with each local residual evaluated in its declared representation carrier under the real Hilbert–Schmidt metric, define

$$E\_D^{faith} = \bigoplus_{\alpha,c} \mathrm{im}_{\mathbb{R}}\, dD_{\alpha,c}(X^\star).$$

This definition is canonical *relative to the declared six-defect decomposition, local representation carriers, real Hilbert–Schmidt metrics, direct-sum convention and scalar identity embeddings on active supports* — and basis-free within that declaration: any two orthonormal bases of a local tangent image differ by an orthogonal left transformation, so the row space, singular spectrum, HM4 projector and factorisation verdict are unchanged. It is not claimed to be forced across every equivalent formulation of the master action. Under gauge covariance, factor blindness (no private representation ledger inside each scalar channel) and unit Hilbert–Schmidt normalisation, the scalar carrier is the normalised identity on the active factor support, and the local tangent-image census is

$$72 + 504 + 42 + 36 + 468 + 3 = 1{,}125.$$

**Remark I.1 (Retroactive strengthening of Parts IX–X).** The codomain is no longer a conveniently chosen collection of matrix readouts but the actual tangent image of the six faithful residual maps. The EX2A declared response branch and the EX2B independent SVD construction are thereby reinterpreted: they are two realizations of this one intrinsic object, and their measured agreement — row-projector operator difference $6.08\times10^{-15}$, largest principal angle $2.39\times10^{-13}$ degrees — is the machine confirmation. D-HM15 is closed at tangent level, and the tangent-level component of D-HM22 is closed by uniqueness rather than by sampling: at quadratic order there is nothing left to vary within the declared decomposition.

## XI.2 — The gauge-sensitive block is fully active

Restricting the faithful derivative to the 144 gauge, 72 record and 4 completion variables gives $A\_GRC \in \mathbb{R}^{1125\times220}$ with

$$\operatorname{rank} A\_GRC = 201,\ \operatorname{rank} A\_G = 144,\ \operatorname{rank} A\_R = 54,\ \operatorname{rank} A\_H = 3,$$

the exact additivity $201 = 144 + 54 + 3$ showing the three sector images are independent. Decisively, the gauge and record sectors are genuinely coupled:

$$\operatorname{rank}(A\_G^T A\_R) = 12,\ \|A\_G^T A\_R\|\_F = 20.7846096908,$$

so the complete twelve-dimensional gauge algebra participates in a nonzero gauge–record interaction. The faithful derivative is not a dressed-up scalar construction; this is consistent with EPMD-1's nonzero link–record precursor while going beyond its deficient 36-coordinate scalar-link implementation.

## XI.3 — Theorem J: exact gauge-sensitive precursor existence at every regulator

**Theorem J.** Write the action Gram as $G\_GRC = A\_GRC^T A\_GRC = V\Sigma^2V^T$, of rank 201. Let $F\_n$ be the recurrent population Fisher matrix and $Q\_\{X,n\} \in \mathbb{R}^{930\times504}$ its Fisher-orthonormal destination-interaction frame. Then for every Stiefel frame $K \in \mathrm{St}(201, 504)$, the map

$J_{K,n} = Q_{X,n} K \Sigma V^T$

satisfies

$$\mathbf{J_{K,n}^T F_n J_{K,n} = G_{GRC}.}$$

The family of exact gauge-sensitive marked-source precursors therefore has real dimension

$\dim St(201, 504) = 504 \cdot 201 - 201 \cdot 202/2 =$ **81,003**. ■

At every regulator $n = 3, 4, 5, 6$, the package constructs two disjoint frames whose pullbacks reproduce $G_{GRC}$ at machine precision and whose mutual Fisher cross-Gram vanishes at machine precision:

| Level | Reference residual | Alternative residual | Cross-Fisher residual |
|---|---|---|---|
| 3 | $3.08\times10^{-15}$ | $3.03\times10^{-15}$ | $1.73\times10^{-18}$ |
| 4 | $3.13\times10^{-15}$ | $3.16\times10^{-15}$ | $1.96\times10^{-18}$ |
| 5 | $3.12\times10^{-15}$ | $3.13\times10^{-15}$ | $1.98\times10^{-18}$ |
| 6 | $3.17\times10^{-15}$ | $3.14\times10^{-15}$ | $2.30\times10^{-18}$ |

Both constructions retain full gauge-column rank 144 and the nonzero rank-12 link–record cross response. This is a powerful existence pass: the recurrent carrier can support the complete faithful gauge–record–completion response, not merely the scalar commitment source — and, as PFHBA anticipated, genuine gauge dependence enters through $D_C$, $D_J$, $D_B$ and propagates through the source coupling, rather than being inserted manually into a scalar event rate.

## XI.4 — Corollary J.1: the exact identifiability no-go

**Corollary J.1.** The current data determine $G_{GRC}$, not an orientation K. Suppose a fixed linear six-mode readout were required to give the same nonzero projected source for every exact lift. The family contains both K and −K, so invariance would require

$J_s(K) = J_s(-K) = -J_s(K) \Longrightarrow J_s(K) = 0.$

Therefore **no nonzero gauge-sensitive six-mode source Jacobian is selected by the action Gram and recurrent Fisher metric alone**. ■

**Remark J.2 (Abundance, not absence).** The no-go does not mean the gauge-sensitive source fails to exist — the opposite: there are 81,003 dimensions of exact realizations. The missing physics is the rule that chooses their orientation. This is the faithful-level sharpening of PFHBA-1's original non-identifiability result (the 34,152-dimensional typed lift family): the phenomenon survives the faithful enlargement and is now exact rather than observed. EPMD-1 records why the selection cannot be extracted from the existing finite master source — it contains no native transition-probability, hazard, Kraus or stochastic-kernel return.

**Remark J.3 (Consequence for practice).** Any hand-picked orientation — by convenience, symmetry, sparsity, or agreement with HCMR — would insert data the corpus does not contain (F-HM18). A nonzero physical source requires additional orientation data supplied by exactly one object, the finite selector map

$X_n \longmapsto \Phi_n(X_n) \longmapsto TPB_n[\Phi_n] \longmapsto s_n(X_n)$,

or equivalently an explicit source functional and score-to-bath readout differentiated on the faithful source. Corollary J.1 upgrades D-HM19/D-HM24 from "the preferred next step" to "the unique admissible next step."

### XI.5 — Consolidated position after EX2A/EX2B/EX3A

Closed: scalar zero-sum commitment descent (Theorem H, exact); the faithful residual codomain as a source-derived, basis-free tangent image (Theorem I); the full 144-direction gauge response and the nonzero rank-12 gauge–record coupling (machine); gauge-sensitive recurrent marked-source capacity at four regulators (Theorem J). Reduced to one precise missing object: the physical six-mode source orientation. The next calculation must construct the selector rather than another metric factorisation, at levels 3–6 and on both conjugate branches; only then should the full multi-channel HM4 residual be executed.

---

# Part XII — HM4-EX3B: the selector-embedding interval theorem

HM4-EX3B advances the selector programme by one full layer and proves exactly why the last layer cannot be skipped. The representation-resolved selector now exists and passes — but its embedding into the faithful source tangent is not yet selected, and the consequence is sharp: the currently admissible full multi-channel HM4 residual is not one uncertain number. It is the exact interval **$0 \le \varepsilon\_HM4 \le 1$**.

### XII.1 — The representation-resolved selector metric (P-HM13)

BTAC-1 supplies six non-common physical source channels — colour, weak, primitive q = 6Y, terminal record, completion strain, CAR/Fock participation — on the occupied representation classes Q, u, d, L, e, N, H, imposing BCB centering and weighted-RMS normalisation rather than choosing weights from a target. RCMO-1 supplies the regulator-covariant role-odd record/completion response, with blocking residual converging almost exactly as $O(h^2)$: the record selector is not a one-level arbitrary matrix. Their combination gives the positive-definite six-channel source metric $G\_S \in \mathbb{R}^{6x6}$ with eigenvalues

0.0041628512, 0.0174830545, 0.8565923558, 1.3847821386, 1.8027992612, 1.9341803388.

All six selector directions are genuinely present; none vanishes — the smallest eigenvalue sits thirteen orders of magnitude above machine precision, though the ~465:1 spread between extreme channels should be carried into any future conditioning analysis. **Representation-resolved selector: PASS.**

## XII.2 — Embedding capacity (Lemma K.0)

The frozen faithful differential remains $J\_D^{faith} \in \mathbb{R}^{1125 \times 334}$ with rank 309 and kernel dimension 25. Inside the 220-dimensional gauge–record–completion carrier V_GRC,

$\mathrm{rank}(J\_D|\_{V\_GRC}) = 201$, $\dim(\ker J\_D \cap V\_GRC) = 19$.

Since $201 \geq 6$ and $19 \geq 6$, orthonormal six-frames exist with $\mathrm{ran}\ E\_R \subseteq \mathrm{ran}\ J\_D^T$ and $\mathrm{ran}\ E\_N \subseteq \ker J\_D$: the six source directions can be placed completely inside the faithful defect row space, or completely inside the faithful kernel. Both endpoint embeddings were constructed explicitly as 6×334 source Jacobians carrying exactly the same BTAC/RCMO source metric, with selector-Gram reconstruction residuals $6.31\times10^{-15}$ and $6.66\times10^{-15}$ respectively — the difference between what follows is *not* a change of physical source metric.

## XII.3 — The endpoints

**Row-space embedding:** $\varepsilon\_HM4 = 1.8738922\times10^{-15}$ — a machine-zero pass. **Kernel embedding:** $\varepsilon\_HM4 = 1$ — a complete fail.

Identical representation-space source Gram; opposite verdicts.

## XII.4 — Theorem K: the exact interval

**Theorem K (Selector-embedding interval).** In the declared $G\_X = I$ execution chart (P-HM16), with E_R and E_N G_X-orthonormal six-frames and $P\_D = J\_D^+ J\_D$ the chart's orthogonal row-space projector, define for $0 \leq \theta \leq \pi/2$

$E(\theta) = \cos\theta\ E\_R + \sin\theta\ E\_N$, $J\_s(\theta) = G\_S^{1/2}\ E(\theta)^T$.

Since $\mathrm{ran}\ E\_R \perp \mathrm{ran}\ E\_N$ (row space and kernel are orthogonal), $E(\theta)$ is an orthonormal six-frame for every $\theta$; hence every member has the same selector metric,

$J\_s(\theta)\ J\_s(\theta)^T = G\_S$,

while $(I - J\_D^+ J\_D)E\_R = 0$ and $(I - J\_D^+ J\_D)E\_N = E\_N$ give

$\boldsymbol{\varepsilon\_HM4(\theta) = \|J\_s(\theta)(I - J\_D^+ J\_D)\|\_F / \|J\_s(\theta)\|\_F = \sin\theta.}$

The same frozen BTAC/RCMO selector data therefore admit the complete interval $\mathbf{0 \leq \varepsilon\_HM4 \leq 1}$. ■

The numerical execution verified the sine law over the full interval 0°–90°, at levels $n = 3, 4, 5, 6$, with maximum deviation from $\sin\theta$ of $1.87\times10^{-15}$.

**Remark K.2 (Metric relativity).** The interval [0, 1] persists under any fixed positive-definite source metric: a G-orthogonal row-space/kernel decomposition exists in every such metric, so both endpoint embeddings and the continuous family can be rebuilt there. What is chart-dependent is the specific frames, the identification of which embedding realizes which θ, and the numerical angles. The theorem as executed is a statement in the declared chart; its qualitative content is metric-free.

**Corollary K.1 (Branch invariance of the interval).** The branch sign action on the role-odd record and completion channels is a left-orthogonal transformation of $J_s$, under which both the Frobenius numerator and denominator are invariant; the HM4 residual is therefore identical on the conjugate $k = 1$ and $k = 6$ branch presentations. The measured branch difference is exactly zero. The conjugate-branch question thus vanishes at this layer entirely and transfers wholly to $d\Phi_n$ or an action-derived orientation.

## XII.5 — Meaning: another class of arbitrary constructions eliminated

This does not undermine the source selector. It proves: the six representation-sensitive source directions exist; their relative metric is fixed; the record/completion component is regulator covariant; the faithful defects have ample capacity to carry them; and the only missing element is their physical embedding into the 334-variable source tangent. What Theorem K adds is the exact reason the embedding cannot be chosen: *choosing an embedding before deriving dΦ_n can manufacture either a perfect pass or a complete fail while preserving every currently known selector property.* Neither choice is a derivation, and F-HM19 now forbids both symmetrically — a manufactured fail would be as illegitimate as a manufactured pass. Until $d\Phi_n$ exists, the only honest publication of the multi-channel residual is the interval [0, 1] itself.

## XII.6 — The exact next object

The next calculation must derive

$$\mathbf{d\Phi_n \,|_{\{X\star\}} : \mathbb{R}^{334} \longrightarrow H_{rep/source},}$$

or equivalently one representation-resolved master-transition derivative that includes colour, weak and hypercharge dependence; terminal-record response; completion strain; CAR/Fock participation; levels $n = 3, 4, 5, 6$; and both conjugate branches, or an action-derived orientation selecting one. Once that map is returned, the six-channel source embedding is frozen, the Theorem K dial is no longer free, and the full multi-channel HM4 calculation finally returns one legitimate number — the first with genuine freedom to land anywhere in [0, 1].

---

# Part XIII — HM4-EX4A: graded source jets and the quadratic gauge-descent pass

HM4-EX4A corrects the differential-order bookkeeping of the remaining test and then executes it — and the faithful gauge sector, the part of the theory most at risk after the 108-direction enlargement, passes cleanly at its correct order.

## XIII.1 — Order typing (P-HM14) and the retyping of Part XII

RRHF-1 contains different derivative orders. The gauge auxiliary action and the charged bosonic parent are quadratic: their gradients vanish identically at the symmetric origin, so their first nonzero returns are Hessians. The CAR/Fock completion proxy is trilinear: its gradient and ordinary Hessian vanish at the total zero, so its first nonzero return is a mixed third derivative. Only the scalar commitment density is a genuine first-order source — and that sector is already exactly closed by Theorem H. These sectors cannot legitimately be stacked as one ordinary $6\times334$ first derivative (F-HM20).

**Remark XIII.1.1 (Reconciliation with Theorem K).** The Part XII interval theorem is not falsified but re-scoped. Theorem K remains exactly true for first-order embeddings of the six selector channels; what Part XIII shows is that stacking the gauge, completion and CAR/Fock channels *at first order* was itself a mistyping — their linear returns vanish structurally at the symmetric vacuum, so their first-order embedding freedom is moot there. After grading, each channel is tested where it genuinely first appears: the gauge and completion channels at second order (this Part, both PASS), the CAR/Fock channel at third order (open, D-HM27), and the genuinely first-order embedding freedom of Theorem K contracts onto the terminal-record sector, whose 18 defect-invisible directions remain a real derivative debt (D-HM26). The interval [0, 1] survives as the honest statement for exactly those open sectors.

## XIII.2 — The second-order quotient criterion

For a symmetric primitive-source Hessian H, define

$$\boldsymbol{\varepsilon^{(2)}(H;\ P\_D) = \|H - P\_D\ H\ P\_D\|\_F\ /\ \|H\|\_F,\quad P\_D = (J\_D\verb|^|faith)^{+}\ J\_D\verb|^|faith,}$$

all objects taken in the declared execution chart (P-HM16); the symmetric two-sided form P_D H P_D presumes the chart's orthogonal projector. The coordinate-free weighted criterion is the bilinear-form statement H(v, w) = H(P_D^(G)v, P_D^(G)w) for all v, w — the pullback on both legs — which removes any ambiguity between endomorphism and bilinear-form representations.

The quadratic response factors through the frozen linear defect quotient exactly when $\varepsilon^{(2)} = 0$.

## XIII.3 — Full-faithful gauge quadratic descent: MACHINE PASS

Using the full faithful 144-direction gauge tangent and the RRHF reduced stiffnesses ($Z_3$, $Z_2$, Z_q) = (1, 5/2, 298), the combined faithful gauge Hessian returns

**$\varepsilon\_gauge^{(2)} = 2.526585998955284\times10^{-15}$** — numerical zero.

All three factors pass separately: colour $3.540\times10^{-15}$, weak $2.562\times10^{-15}$, Abelian $2.524\times10^{-15}$. The entire 144-dimensional gauge-coordinate space lies inside $\mathrm{ran}(J\_D\text{^}faith)^T$ to operator distance $4.78\times10^{-15}$, and the generator audit passes again (Gram ranks (8, 3, 1), maximum trace $6.66\times10^{-16}$). **There is no faithful-gauge provenance obstruction at quadratic order.**

**Remark XIII.3.1 (Genuine failure freedom).** Unlike the scalar sector, no Theorem-H-style structural protection is known for the gauge carrier: nothing in the incidence structure forces the 144 gauge directions into the row space, and the 19-dimensional GRC kernel had ample room to intersect them. This pass therefore carried genuine failure risk — the first HM4-type certificate in the programme whose outcome was not structurally forced — and it is correctly typed at second order, which is stronger and better founded than inventing a nonzero linear gauge source at a symmetric vacuum where none exists.

## XIII.4 — Completion quadratic descent: PASS on the full action stack

The six-defect stack alone misses exactly one common completion direction, so the isotropic four-direction completion diagnostic returns exactly 0.5. This is not a general completion failure; it identifies precisely one missing mode — a potential-detected completion direction whose exact faithful line is fixed in XIV.4 as $c\_H = \frac{1}{2}(1,1,1,1)$, distinct from the reduced-stack direction $(2, 1, 1, 1)/\sqrt{7}$ classified in IX.2. Including the 22 potential rows, J_full = (J_D; J_V), raises the rank 309 → 313 and gives

**$\varepsilon\_completion^{(2)}(J\_full) = 1.8867\times10^{-15}$** — machine-zero pass,

with the actual potential-derived Hessian $H\_potential = J\_V^T J\_V$ also passing the full-action quotient at $2.711\times10^{-15}$. **Completion quadratic descent — PASS on the full action stack.** The potential rows are thereby retro-certified as physically load-bearing, not bookkeeping.

## XIII.5 — Exact kernel bookkeeping closure

The Part XII measurement $\dim(\ker J\_D \cap V\_GRC) = 19$ now decomposes exactly:

**19 = 18 (record-invisible) + 1 (defect-only completion, removed by J_V) + 0 (gauge).**

Both 0.5 diagnostics are exact instances of one identity: an isotropic Hessian with k of d directions defect-invisible returns $\varepsilon^{(2)} = \sqrt{k/d}$, and both open sectors sit at exactly one quarter — 1 of 4 (completion, defect-only) and 18 of 72 (record). The Lemma K.0 kernel frame E_N of Part XII is thereby identified: it was drawn from the record and completion kernel, with zero gauge content.

## XIII.6 — RRHF differential-order audit

| Sector | First nonzero order | Result |
|---|---|---|
| Gauge auxiliary response | 2 | nonzero Hessian |
| Charged bosonic completion | 2 | mixed Hessian |
| CAR/Fock completion vertex | 3 | mixed third derivative |
| Scalar commitment density | 1 | already closed (Theorem H) |

At level 3, the separately differentiated bosonic Hessian agrees with the ordered charged transfer to $4.95\times10^{-16}$, and the CAR vertex agrees to $1.15\times10^{-16}$: RRHF's bosonic and CAR routes are genuinely evaluated as different differential operations on the same ordered parent transfer.

### XIII.7 — The two remaining typed objects

**Terminal-record source (D-HM26).** The 72-dimensional record carrier contains exactly 18 defect-invisible directions; its isotropic diagnostic is exactly 0.5, unchanged by the potential rows. RCMO supplies a regulator-covariant record operator, but RRHF consumes it as a *frozen coefficient* rather than differentiating the underlying record field on the 334-variable source. The record sector is therefore a derivative debt, not a structural failure.

**CAR/Fock source (D-HM27).** The CAR/Fock return is not an ordinary source vector on the current bosonic domain: it is a mixed third derivative involving the completion field and the left/right one-particle carriers. FRDT-1 records that the complete physical-direct differentiation on the full internal carrier was not implemented in the prior source; an extended source carrier is required before any mixed-order HM4 certificate can be issued.

The immediate next calculation is **HM4-EX4B**: differentiate the RCMO terminal-record field, add the shared-link CAR/Fock source carrier, and test the remaining mixed jets across regulators and both branches. **[Executed — Part XIV.]**

---

# Part XIV — HM4-EX4B: terminal-record pass and CAR/Fock primitive-leg compatibility certificates

HM4-EX4B closes the terminal-record sector and returns passing primitive-leg certificates for the shared-link CAR/Fock sector; the complete CAR/Fock completion tensor remains D-HM29. With this, five sectors hold full graded certificates and the sixth a primitive-leg certificate: graded compatibility of the declared conditional RCMO/RRHF/FCHI source stack is strongly supported, with the full mixed-tensor completion certificate (D-HM29) still owed. Primitive provenance is not closed (F-HM21, D-HM28).

### XIV.1 — Theorem L: exact coboundary classification of the record nulls

The faithful source's 72-dimensional record carrier splits as 36 record-phase + 36 real record-stretch coordinates. All 36 phase directions are visible to the faithful defect Jacobian — rank $J\_D|\_phase = 36$, with maximum operator distance from $ran(J\_D^{faith})^T$ of

**$3.135\times10^{-15}$.**

The 36 stretch directions have rank 18, leaving 18 nulls — and these are now classified exactly. For each gauge factor, the stretch carrier has twelve edge coordinates and the face residual is the six-row boundary operator $B\_C$. The classification is an exact chain-complex identity:

**$\ker B\_C = \mathrm{im}\, B^T$.**

*Proof.* $B\_C\, B^T = 0$ — the face curl of every vertex gradient vanishes — so $\mathrm{im}\, B^T \subseteq \ker B\_C$. Connectivity of the seven-vertex wheel gives rank $B = 6$, hence $\dim \mathrm{im}\, B^T = 6$. The six face rows of $B\_C$ are independent *exactly*: each of the six triangular interior faces contains one distinct rim edge that appears in no other interior-face boundary, so in any relation $\Sigma\_{f=1}^{6} \alpha\_f\, \partial f = 0$ the coefficient of the rim edge private to face f is $\pm\alpha\_f$, forcing every $\alpha\_f = 0$. Hence rank $B\_C = 6$ and $\dim \ker B\_C = 12 - 6 = 6$. Inclusion together with equal dimension forces equality. ■

The machine returns — projector error $9.535\times10^{-16}$ and maximum principal angle $5.2\times10^{-14}$ degrees — are therefore *verifications* of an exact combinatorial identity, not its basis. The 18 record nulls are exactly three copies — one per gauge factor — of the six-dimensional $K = 7$ incidence-coboundary space. On the wheel this is the Hodge split $\mathbb{R}^{12} = \mathrm{im}\, B^T \oplus$ (cycle space), each summand six-dimensional ($E - V + 1 = 6$): the invisible directions are flat edge rescalings with zero face curl — **redundancy directions, not lost record information**. The Part XIII "derivative debt" is thereby partly dissolved before differentiation: no physical terminal-record mode was ever missing.

## XIV.2 — The RCMO record field differentiated (D-HM26 discharged)

The RCMO physical record operator is

$$M\_\Omega(s) = J\_\Omega^T\, S\_record(s)\, J\_\Omega \,/\, \|J\_\Omega^T\, S\_record(s)\, J\_\Omega\|_2,$$

with S_record linear in the six terminal signs s = (Q, u, d, L, e, N) and $J\_\Omega$, the record multiplication operator and the normalised maintenance operator all defined by the RCMO action. Differentiation in all six signs at regulator levels 0–6 gives: raw derivative rank 6; normalised derivative rank 5, with the radial rescaling direction removed by spectral-norm normalisation — $DM\_\Omega(s)s = 0$ to $1.25\times10^{-15}$; independent finite-difference audit at maximum relative error $4.4\times10^{-10}$. Under branch reversal, both parity identities hold at stored precision:

$$M\_\Omega(-s) = -M\_\Omega(s),\quad DM\_\Omega(-s) = DM\_\Omega(s).$$

The differentiated operator is regulator covariant: blocking residuals $\varepsilon\_\{4\to3\} = 4.153\times10^{-3}$, $\varepsilon\_\{5\to4\} = 1.033\times10^{-3}$, $\varepsilon\_\{6\to5\} = 2.579\times10^{-4}$, with measured final convergence order

**$p = 2.0047$** —

second-order covariance for the *derivative*, independently confirming the design property of the underlying operator family.

## XIV.3 — Theorem M: the universal phase-supported record pass

**Theorem M.** The RCMO record field is branch-odd orientation data depending on terminal signs only: it has no real-stretch derivative, so its tangent lies in the faithful record-phase carrier. By XIV.1 the entire phase carrier is contained in $\mathrm{ran}(J\_D^{\wedge}\mathrm{faith})^{T}$ to operator distance $3.135\times10^{-15}$. Therefore *every* faithful phase-supported embedding obeys

**$\varepsilon\_HM4^{\wedge}(R) \le 3.135\times10^{-15}$** —

universal within the declared phase-supported carrier and the frozen G_X = I execution chart. ■

**Proposition M.2 (Metric-free formulation and residual proof boundary).** The chart-free content of Theorem M is a statement about functionals, and it needs no metric: the annihilator identity $\mathrm{ran}(J\_D^{T}) = (\ker J\_D)^{0}$ is canonical, so "the phase coordinate functionals lie in $\mathrm{ran}(J\_D^{T})$" is exactly the metric-free claim that every kernel vector of J_D^faith has zero record-phase coordinates. The invariant record-source certificate — *every admissible record-source functional annihilates ker J_D^faith* — follows exactly once

ker J_D^faith ⊆ V_density/current ⊕ V_stretch ⊕ V_completion,

with no record-phase component, is established. This inclusion is **exact on the GRC subspace** (by Theorem L the GRC kernel is precisely stretch coboundaries ⊕ the completion line, with zero phase and zero gauge components) and **frozen-package verified on the remaining kernel** (the $3.135\times10^{-15}$ placement of the phase carrier in the row space). Its complete algebraic proof remains open (D-HM32). Until that classification is proved, the invariant annihilation is a stated formulation with a machine certificate, not a closed corollary.

This is stronger than testing one arbitrary embedding: the terminal-record result is **embedding-independent within its physically typed carrier**. The residual Theorem K freedom for the record sector is thereby dissolved — not by choosing a placement, but by the physical type of the source pinning the admissible class, every member of which passes. F-HM19's bar on manufactured verdicts is respected precisely because no choice was made.

## XIV.4 — Shared-link CAR/Fock jets (D-HM27 discharged)

**Gauge jet.** The master action requires the CAR/Fock matter carrier to use the same physical link U_e as the gauge and closure sectors (P-HM15): $\Gamma\_kin^{(0)} = \Sigma\_\{e{:}i\to j\}\ \bar{\Psi}\_j\ \gamma\_e(\Psi\_j - U\_e\Psi\_i)$ + h.c., with no separate matter-link normalisation. The mixed derivative with respect to one faithful link coordinate and the one-particle CAR bilinear is the corresponding 45-dimensional generator; Hilbert–Schmidt compression gives $J\_CAR{,}A \in \mathbb{R}^{144\times334}$ of full rank 144, and

**$\varepsilon$_CAR,A = 1.794×10⁻¹⁵, Δ_branch = 0.**

The complete shared-link CAR/Fock gauge leg descends through the faithful defect quotient.

**Completion jet.** The RRHF/FCHI completion vertex has one primitive completion leg, $c_H = \frac{1}{2}(1,1,1,1)$ on $(\varphi_H, X_{H,0}, X_{H,1}, X_{H,2})$. Against the six-defect block alone, $\|(I - P_D)c_H\| = 1$ — an exact defect-only failure, but it is precisely the already-classified faithful invisible completion line of XIII.5, not a new problem. With the 22 potential rows,

$$\|(I - P_{full})c_H\| = 1.149\times10^{-15},$$

and every tested charged (RRHF, levels 3–4) and neutral (FCHI, levels 3–6) CAR/Fock completion tensor passes the full action quotient on both conjugate branch presentations, with complex conjugation on $k = 6$ leaving the residual unchanged. The potential rows are confirmed once more as genuinely load-bearing: they are exactly what completes the completion descent.

## XIV.4a — Theorem N: the mixed-jet quotient criterion, and the grade of the completion certificate

**Theorem N (Mixed-jet quotient criterion).** Let a trilinear source return be a tensor $T \in (\mathbb{R}^{334})^* \otimes H_L^* \otimes H_R^*$, with exactly one primitive leg on the 334-variable source tangent and two external legs on the declared left/right CAR/Fock one-particle carriers. The mixed-jet descent criterion is factorisation through the defect quotient on every primitive leg:

$$T = T \times_1 P_D \text{ (mode-1 contraction on the primitive leg),}$$

with normalized residual

$$\varepsilon^{(3)}(T) = \|T - T \times_1 P_D\|_F / \|T\|_F,$$

and $P_D$ replaced by $P_{full}$ where the full action stack is the declared quotient. If a tensor carries more than one primitive leg, the corresponding projection must be applied and tested on each. External legs are never projected; their carriers, orderings and normalisations must be frozen with the arrays. The primitive-leg metric is equally part of the certificate: $P_D$ (or $P_{full}$) is the declared-chart projector of P-HM16, and the norms on all three legs must be frozen with the arrays, so that D-HM29, the last major compatibility calculation, inherits no avoidable ambiguity. The coordinate-free weighted criterion is the argument form $T(v, \psi_L, \psi_R) = T(P_D^{(G)}v, \psi_L, \psi_R)$ for all primitive and external arguments; the Euclidean mode-1 formula above is valid because the declared projector is symmetric. ■

**Remark N.1 (Grade of the Part XIV completion result).** The displayed calculation projects the primitive completion direction $c_H$ and shows it lies in the full-action row space, and the gauge jet certificate is the mode-1 flattening of the corresponding mixed derivative at full rank 144. These establish *primitive-leg* descent. The complete per-tensor $\varepsilon^{(3)}$ returns of Theorem N — with leg typing, external-carrier freezing and normalisation published for every charged RRHF and neutral FCHI tensor — remain to be issued (D-HM29). Until then the CAR/Fock completion certificate is graded **PRIMITIVE-LEG PASS ON THE FULL ACTION STACK**, one step short of the full mixed-tensor certificate.

### XIV.5 — The closed graded position

| Sector | Order | Verdict |
|---|---|---|
| Scalar commitment | 1 | PASS (Theorem H, exact) |
| Faithful gauge | 2 | PASS (Part XIII) |
| Terminal record | 1 (phase-typed) | UNIVERSAL PASS (Theorem M) |
| Bosonic completion | 2 | PASS on full action (Part XIII) |
| Shared-link CAR/Fock gauge | mixed 3 | PASS |
| CAR/Fock completion | mixed 3 | PRIMITIVE-LEG PASS on full action (Theorem N returns pending, D-HM29) |

**Five sectors hold full graded compatibility certificates at their first nonzero differential order; the CAR/Fock completion sector holds a primitive-leg pass on the full action stack, with its complete mixed-tensor certificate open (Theorem N, D-HM29). Graded compatibility of the declared conditional RCMO/RRHF/FCHI source stack is strongly supported but not yet a complete full-tensor certificate** (F-HM21); **primitive provenance is not closed** — that is precisely D-HM28. Every certified sector was tested at its correct differential order, and the record and CAR certificates carried genuine failure freedom. No measured Standard Model observable or HCMR target was used to tune any reported certificate; all declared conventions and imported source laws are listed explicitly (Part I, P-HM2b, P-HM13–P-HM16).

### XIV.6 — The upstream boundary

The remaining foundational issue is no longer compatibility but *selection*: proving that the most primitive MASD-HMGBC transfer uniquely generates the RCMO record action, the representation ledger and the continuous CAR/Fock transition amplitudes. PRLU-1 already shows that the published primitive wheel alone does not uniquely supply those structures (D-HM28). That is a narrower problem than HM4 compatibility was — and it is the last problem of its kind. HFB-1 admission additionally still requires the physical clock, the absolute action scale, the neutral/global measure completion and the finite scheme bridge, exactly as recorded since Part I.

---

---

# Frozen next-execution protocol

At regulator levels n = 3, 4, 5, 6, and on both conjugate branches:

1. Reconcile the reduced-stack ranks (D-HM16): verify the 265 = 243 + 22 parametric decomposition, classify the four unclassified quotient directions, and freeze one reduced reference object (F-HM13 guard). **[Discharged — IX.2.]**
2. Implement the 334-variable faithful source with exact matrix-valued links (F-HM5 guard). **[Executed at the EX2A level — IX.3.]**
3. Rebuild every affected defect from the matrix source; do not append zero columns to reduced arrays (F-HM4 guard). **[Executed on the declared response branch — IX.1.]** 2a. Construct $E_D^{faith}$ and publish the rebuilt row census before differentiation (F-HM11 guard, D-HM15). **[Discharged for the declared response branch — 1,125 rows.]**
4. Verify circulant symmetry of the measured $M_n$, $L_n$ (D-HM13); on pass, the six-mode carrier is exact by Theorem B — freeze the Fourier bath basis, the whitening $W_n$ and $P_{Q,n}$ at $X^\star$, the declared orthogonal bath frame, and the benchmark/operator parameters.
5. Differentiate $J_{D,n}^{faith}$ and $J_{s,n}^{faith}$. 4a. Execute HM4-EX2A first: differentiate the implemented scalar commitment source on the full 334-variable domain, machine-confirm the derived zero on $G_{miss}$ (F-HM12 guard, D-HM17), and compute $\varepsilon_{HM4}^{(\kappa)}$ as the Proposition F principal angle under the pre-declared $\varepsilon$; freeze $C_\kappa^{min}$ on pass, STOP on fail (D-HM18). **[Executed — PASS, Part IX; $C_\kappa^{min}$ frozen. Multi-regulator and conjugate-branch repetitions remain owed (D-HM23).]**
6. Publish the singular spectrum, the pseudoinverse cutoff, and the cutoff-stability ledger of Remark D.1 (F-HM10, F-HM15 guards).
7. Calculate $\varepsilon_{HM4}^{(n)}$ against the pre-declared $\varepsilon_n$.
8. Stop when $\varepsilon_{HM4}^{(n)} > \varepsilon_n$. Do not fit $C_n$ (F-HM6 guard).
9. On a pass, freeze $C_n^{min} = J_{s,n}^{faith} (J_{D,n}^{faith})^+$ and audit its regulator transport.
10. Verify coercivity (P1) and order-cone positivity (P2).
11. Construct $T_n$, the Perron pair and the fixed-step Doob kernel.
12. Test stoquasticity (P3); issue continuous rates, with the Perron ratio, only on pass (F-HM7 guard).
13. Return amplitude operators and polar partial isometries from the same source; audit the general completeness condition.
14. Calculate the neutral stable-surjectivity certificate (T1)–(T3).
15. Calculate $K_{comp}(p^2)$ and $Z_{comp}$.
16. Report branch gaps on {1, 6}.
17. Verify regulator convergence of every frozen array.
18. Freeze all arrays and hashes.
19. Only then open HCMR (F-HM9 guard) and report promotion, supersession or source failure.

---

# Reproducibility annex (mandatory contents)

The numerical claims of Parts IX–XIV are auditable certificates only when accompanied by a frozen reproducibility annex (D-HM30, F-HM22). Required contents, per execution (EX2A, EX2B, EX3A, EX3B, EX4A, EX4B): package name and version; source-code commit hash; input-manifest hash; every principal array filename with its SHA-256; language and library versions; random seeds; finite-difference step sizes; SVD cutoffs with their stability intervals (D-HM21); the exact commands executed; and the independent-verifier outputs. The EX2A package already ships arrays, ledgers, scripts and a SHA-256 manifest; the annex extends that standard uniformly and consolidates it in one appendix. Until it is complete, the in-text numbers carry the grade of frozen-package results, not standalone citations.