The Neutral-Fermion Branch and Carrier-Census Completion Theorem in VERSF

▲ Programme Milestone — Neutral-Fermion and Lepton-Completion Series

Gate NFCC-1R — Carrier-First Census Closure, Neutral Quadratic-Operator Certificate, Pre-Spectral Ordering, Assembly-Invariant Attachment and CP Certificates, and Numerical Non-Insertion Closure

Keith Taylor VERSF Theoretical Physics Programme — Neutral-Fermion and Lepton-Completion Series Final-Eight Paper 2 — three-certificate branch-completion theorem — Tier A conditional discrete branch theorem, with frozen neutral-completion object, carrier-first census, pre-spectral branch labels, invariant orientation certificates and numerical non-insertion audit

Primary predecessors: *The Neutrino Mass and Charge-Conjugation Completion Theorem in VERSF* (NMCC-1); *The PMNS Closure-Hamiltonian Theorem in VERSF* (PMNS-1); *The Generation and Species Census Theorem in VERSF* (SC-1); *The World-to-Framework Occupancy Map for Standard Model Matter in VERSF* (SC-2); *The Closure-Operator Yukawa Construction Theorem in VERSF* (CMSY-1); *The Electroweak Representation and Connection Closure in VERSF*; *The Substrate Anomaly-Inadmissibility Theorem in VERSF*; *The Weak-Commitment Closure Kernel in VERSF*; *Substrate-Generated Sequential Transport in VERSF*; and *The Electroweak Vacuum and Higgs-Potential Completion Theorem in VERSF* (HR-1).

General Reader Summary

Neutrinos are the strangest particles we know. They pour out of the Sun in unimaginable numbers and pass through the Earth — and through you — as if nothing were there. They are the only matter particles with no electric charge, and that one fact creates a mystery the Standard Model has never resolved. Because they carry no charge, the mathematics allows them several completely different ways of having mass. They could be ordinary particles, like electrons, with distinct antiparticles. They could be their own antiparticles — a possibility that exists for no other matter particle. Or their tiny masses could be the visible end of a lever, pushed down by enormously heavy hidden partners. Decades of experiments have measured how neutrinos morph from one type to another in flight, but those measurements, by a proven limitation rather than a lack of effort, cannot decide which of these pictures is true.

Most theories simply pick a picture and adjust it to fit the data. This paper does something different: it **freezes** the choice — conditionally, on stated framework premises — before neutrino

data are consulted, and writes down the exact microscopic tests that must subsequently confirm or overturn it. What has been done here is branch *commitment*: the framework's rules select one picture and stake it publicly. The branch *derivation* — the microscopic calculation that would settle each committed answer unconditionally — is specified in full but not yet performed, and the paper never blurs that line. Within this paper's declared derivation chain, no neutrino measurement is used anywhere; the data are kept downstream, waiting as judges, never consulted as guides.

The reasoning runs in three stages, and the order matters.

Stage one asks: how many neutrino-completing particles must exist? The framework already contains three neutrinos, one per family of matter, and it has a simple rule: no particle line may be left dangling — each must connect to something that finishes it. At the basic level of the theory studied here, the minimal legal finish for a chargeless particle is a partner carrying none of the known force charges — what physicists call an electroweak singlet. One completion partner cannot serve two families — a rule of the framework's own family-structure theorems, not a fact of matrix arithmetic — so three families require three partners. The minimal branch-complete ledger therefore contains exactly three, one for each family key; additional decoupled sterile sectors, disconnected from everything, are not part of this minimal result. Crucially, this count is settled before asking anything about mass. Even if a later calculation returned a strange result for the masses, the three partners would still be there. That ordering forecloses a chicken-and-egg problem that a census-after-symmetry ordering would create.

Stage two asks: what kind of mass do they all have? Here the paper deliberately trades belief for tests. There is a bookkeeping rule (physicists call it B−L) which, if it were a genuine unbreakable law, would force neutrinos to be ordinary particles like electrons. Is it a genuine law? The paper doesn't argue philosophically — it writes down an exact yes/no test that a future calculation must run on the actual mass structure the deep theory produces. Its expectation, argued from what the framework's ledger of real, physically recorded laws actually contains, is that the test will say no: the rule is an accounting habit, not a law. If that expectation holds — and if the operator returns come back as the paper's supporting arguments predict — everything follows in a cascade. The selected branch requires the hidden partners' masses to lie far above the bridge that connects them to ordinary neutrinos — a hierarchy the later microscopic calculation must return, not a number asserted here — while the bridge itself comes from the same Higgs field whose strength the previous paper in this series calculated. Heavy on one side, light on the other: a seesaw. The visible neutrino masses are parametrically suppressed by that lever — which is how they can sit so far below every other particle — although their absolute values remain for a later calculation. And on this branch the light neutrinos are Majorana-type, their own antiparticles, which makes lepton-number-violating searches such as neutrinoless double-beta decay directly relevant, subject to the required operator and nuclear matching. One further safeguard checks that no overlooked back-door contribution could quietly spoil the seesaw picture; that check is one number, and it must come back small.

Stage three commits to three predictions — and takes unusual care that they are real commitments, not word games. First, which neutrino is heaviest? It would be meaningless to compute three masses and then label the biggest one "number three." So the labels are nailed down *first*, using the framework's own built-in ordering of the three families — a first line, a middle line, and a special final line — before any mass is calculated. The prediction is then genuine and could genuinely fail: the neutrino on that special final line is the heaviest. Experiments call this "normal ordering." Second, when the flavours mix, the leading cross-connection runs from the electron family to the tau family — the far endpoint of the chain — not to the muon family in the middle. That pins a key mixing angle above 45 degrees, in the range experiments call the upper octant. Third, the neutrino sector treats matter and antimatter differently, with a specific direction of the difference: negative, in the standard way physicists measure it. For each of these three, the paper defines a single, unambiguous quantity — deliberately built so that no choice of mathematical bookkeeping, coordinates, or sign conventions could ever flip the answer — that a future calculation must return. Until then, each prediction rests on a clearly named supporting argument, and the paper states plainly which of those supports is the weakest.

So the paper's claim, in one sentence: **on the selected branch, neutrinos are their own antiparticles, their tiny masses explained by a seesaw against three far heavier hidden partners, with the heaviest visible neutrino sitting on the final family line, mixing tilted toward the tau family, and matter–antimatter symmetry broken in the negative direction — with no neutrino measurement used anywhere in this paper's declared derivation chain.**

Every one of these claims can die. If experiments confirm the opposite mass ordering, or the lower mixing octant, or the opposite matter–antimatter sign — or if the framework's own future calculations return the wrong answer on any of the tests written down here — the failure will be specific, public, and traceable to a named assumption. The paper ends with a ledger showing that its six conclusions stand or fall independently: a future result could confirm five and destroy one. That is by design. A theory that cannot tell you how to kill it is not making predictions; this one is.

Headline Results and Conditionality

This paper is a **discrete branch-completion theorem**, not a numerical evaluation paper. Its architecture is one new premise and three certificates, evaluated carrier-first.

Premise NT (neutral terminal-carrier completeness) — the only genuinely new premise:

Every independently keyed active neutral carrier must possess a typed, gauge-admissible terminal completion route in the closed carrier ledger.

NT concerns the existence and typing of a completion carrier, not whether the eventual quadratic coefficient on that route is nonzero. Carrier closure is decided before the mass operator is evaluated, and a later zero singular value, null mode or protecting symmetry would change the spectrum without retroactively erasing the route.

Certificate NC-1 — Minimal Neutral-Carrier Completion:

$\mathscr{H}_{N\_R} = \mathcal{N}_1 \oplus \mathcal{N}_2 \oplus \mathcal{N}_3$, $\dim \mathscr{H}_{N\_R} = 3$, $\mathscr{H}_{N\_R} \cong (\mathbf{1}, \mathbf{1}, 0)^{\oplus 3}$ (minimal branch-complete extension) .

Certificate NC-2 — Neutral Quadratic-Operator Completion, five invariant returns required of the substrate operator:

$\nexists$ exact protecting B−L ($[Q_{B-L}, K_N]$ and $Q_{B-L}^T B_N + B_N Q_{B-L}$ do not both vanish) ; rank $M_D = 3$ via the cross-Gram certificate $\sigma_{min}(\mathscr{E}_{\nu L}^\dagger C_Y \mathscr{E}_{N\_R}) \geq \varepsilon_Y > 0$; $\det M_R \neq 0$, $M_R = M_R^T$; $\sigma_{min}(M_R) \gg \sigma_{max}(M_D)$; $\rho_L \equiv \|M_L^{irr}\| / \|S_N\| \ll 1$, with rank-stability companion $(C_\Theta\|\Theta\|^2 + \rho_L)\cdot\kappa_S(S_N) < 1$.

Certificate NC-3 — Projected Spectral and Orientation Branch, three invariant returns:

$m(P_1^\nu) < m(P_2^\nu) < m(P_3^\nu)$ with pre-spectral labels ; $\Delta_{att}^{phys} > 0$; $\mathrm{Im}\,\mathrm{Tr}([H_\ell^L, H_\nu]^3) < 0$ in the declared commutator convention .

Every certificate return is graded. NC-1 is discharged now, conditionally on NT plus the inherited census stack. NC-2 and NC-3 are discharged today by **named conditional stand-ins** — the ledger-completeness argument, the closure-response support expectation, the generation and scale stand-ins, the higher-order suppression expectation, the common-closure-response alignment, endpoint completion, and the forward transport arrow — each demoted from principle to its true role: the current conditional route to a return whose unconditional arbiter is a direct operator evaluation.

Three stand-ins carry disproportionate weight:

1. **The ledger-completeness argument** carries the B−L expectation. It is a graded premise, not an inheritance: the anomaly-inadmissibility theorem closes persistent gauge and source currents but explicitly leaves accidental and global currents incompletely interpreted. The unconditional arbiter is the NC-2A commutator pair on the returned operator.
2. **The common-closure-response alignment** (Lemma 1) carries the ordering. Its unconditional arbiter is the directly computed light spectrum traced through the pre-spectral labels.
3. **The forward transport arrow** carries the attachment and CP signs. Sequential transport derives the oriented chain only up to an unresolved overall orientation, so the arrow is genuinely load-bearing until Δ_{att}^{phys} and the CP trace are evaluated.

Accordingly:

The discrete branch closure passes on the frozen premise-and-stand-in stack, carrier-first.

But:

Unconditional closure is OPEN on the direct operator evaluations: the NC-2A commutator pair; the NC-2B/2C/2D support, singular-value and ρ_L returns; and the NC-3 triple (pre-spectrally labelled spectrum, Δ_{att}^{phys}, $\mathrm{Im}\,\mathrm{Tr}([H_\ell^L, H_\nu]^3)$).

Scope and Closure Box

Required output	Result of NFCC-1R	Grade
Do right-handed neutral carriers exist?	Yes: typed terminal routes of the three keyed active lines (NC-1)	Pass under NT, decided before any mass question
How many?	Exactly three, in the minimal branch-complete extension	Pass (Theorem 1); decoupled non-ledger sterile sectors not excluded, by scope
Electroweak type	$(\mathbf{1}, \mathbf{1}, 0)^{\oplus 3}$ in the $Q = T_3 + Y$ convention	Inherited typing
Exact or protecting B−L?	Not protecting on the returned operator; minimal even violation $\lvert\Delta(B-L)\rvert = 2$	Conditional via ledger stand-in; arbiter = NC-2A commutator pair
Primitive M_L	Zero: $M_L^{[0]} = 0$	Pass, inherited gauge admissibility
Higher-order active block	$\rho_L \ll 1$ required for type-I dominance; rank stability $(C_\Theta\lVert\Theta\rVert^2 + \rho_L)\cdot\kappa_S(S_N) < 1$ keeps all three light masses nonzero	Conditional via suppression stand-in; invariant returns (NC-2D)
M_D	Nonzero, rank three via the cross-Gram certificate $\sigma_{min}(\mathscr{E}_{\nu L}^\dagger C_Y \mathscr{E}_{N\_R}) \geq \varepsilon_Y$	Conditional via support expectation; evaluated certificate open (NC-2B)
M_R	Nonzero, symmetric, nonsingular	Conditional via generation stand-in; return open (NC-2C)
Heavy gap	$\sigma_{min}(M_R) \gg \sigma_{max}(M_D)$	Conditional via scale stand-in; invariant return (NC-2C)
Physical branch	Full-rank type-I-dominant seesaw	Invariant conclusion given NC-2 returns (automatic from NMCC-1)
Three light states exhaust active carrier	Yes in the matched effective carrier; defect $\Theta\Theta^\dagger + O(\Theta^4)$	Automatic
Light mass ordering	Normal: $m(P_1^\nu) < m(P_2^\nu) < m(P_3^\nu)$, labels frozen pre-spectrally	Conditional via alignment + gap bound; non-tautological by the branch-label firewall

Required output	Result of NFCC-1R	Grade
Atmospheric attachment	Electron–tau: $\Delta_{att}^{phys} > 0$	Conditional via endpoint stand-in; arbiter = assembly-invariant Hilbert–Schmidt return (NC-3B)
Atmospheric octant	Upper	Exact given attachment (reflection theorem)
Dirac CP sign	$J_{lep} < 0$ via convention-fixed J_{inv}	Conditional via forward arrow; arbiter = invariant trace (NC-3C)
Independence of the six outputs	Preserved and tabulated, with named interlocks	Section 12
Absolute masses	Not required by this paper	Open by scope
Numerical neutral Hessian and pole matching	Not required to select the branch	Open by scope
Cross-paper vacuum consistency	Same v_{cl} as the electroweak completion gate	Structural; falsifier F17
Numerical non-insertion	No neutrino target data used upstream	Pass

Closure verdict. NFCC-1R meets the Paper-2 brief at the discrete carrier-census and physical-branch level. It freezes the minimal neutral carrier ledger, the uniquely typed physical completion operator, the declared-order block pattern, the B−L-violating type-I-dominant seesaw branch, active-light completeness, normal ordering, electron–tau attachment, upper atmospheric octant and negative leptonic CP branch — without using neutrino observations as inputs. Its branch outputs remain conditional on the named NC-2 and NC-3 stand-ins until the corresponding microscopic operator returns are evaluated. It does not claim the absolute spectrum, heavy eigenvalues, Majorana phases or radiative pole observables.

Abstract

NMCC-1 closes the neutral Nambu carrier, complex-symmetric quadratic form, canonical kinetic normalisation, Takagi mass theorem, exact continuous-charge criterion, Dirac–Majorana taxonomy, gauge-admissibility firewall, exact Schur complement, seesaw rank identities and relative-frame mixing architecture, while leaving open the physical sterile census, the fate of B−L, the block pattern, the hierarchy, the ordering, active completeness and the octant/CP branch. PMNS-1 proves the relative-frame and exact μ↔τ reflection structures while leaving the physical attachment channel unresolved.

This paper supplies the missing branch selection through one new premise and three master certificates, evaluated carrier-first. **Premise NT (neutral terminal-carrier completeness)** requires every independently keyed active neutral carrier to possess a typed, gauge-admissible terminal completion route; it concerns route existence and typing, not the eventual mass coefficient, so the census is decided before the operator is evaluated and independently of any symmetry verdict.

NC-1. At primitive electroweak order the active carrier has no gauge-admissible bare self-completion; the minimal terminal route is $L_L \Phi_{cl} N_R$ with $N_R \sim (\mathbf{1}, \mathbf{1}, 0)$. Generation-key preservation and layer-exclusive attachment (inherited from SC-1) forbid one terminal carrier from serving two keys, so at least three keyed terminal addresses are required; the minimal generation-equivariant extension contains precisely the three addresses induced by the active keys. Hence $\mathscr{H}_{N_R} = \mathcal{N}_1 \oplus \mathcal{N}_2 \oplus \mathcal{N}_3 \cong (\mathbf{1}, \mathbf{1}, 0)^{\oplus 3}$ in the minimal branch-complete extension — a ledger census with a stated reopening condition.

NC-2. Five invariant returns are required of the returned neutral operator — the fifth carrying a companion scalar, so that six numbers are returned in all. (A) B−L is decided by the exact inherited criterion — exactness iff $[Q_{B-L}, K_N] = 0$ and $Q_{B-L}^T B_N + B_N Q_{B-L} = 0$, block-wise forcing $M_L = M_R = 0$ — evaluated on the returned operator, never used to license it. (B) rank $M_D = 3$ through the cross-Gram certificate $\sigma_{min}(\mathscr{E}_{\nu L}^\dagger C_Y \mathscr{E}_{N_R}) \geq \varepsilon_Y > 0$, with the support conditions $\dim S_\nu = 3$, rank $C_Y \geq 3$ on the retained directions, rank $\mathscr{E}_{\nu L}$ = rank $\mathscr{E}_{N_R} = 3$ as necessary preliminaries — full supports alone do not exclude determinant cancellation. (C) $M_R = M_R^T$, $\det M_R \neq 0$, and the invariant hierarchy $\sigma_{min}(M_R) \gg \sigma_{max}(M_D)$. (D) The irreducible higher-order direct active block M_L^{irr} — explicitly excluding the Weinberg piece generated by integrating out N_R, which is the seesaw term itself — must satisfy $\rho_L = \|M_L^{irr}\|/\|S_N\| \ll 1$, making "type-I-dominant" an invariant conclusion rather than an operator-order expectation. Given the returns, NMCC-1 automatically yields $M_{light} = S_N + \Delta M$ with $S_N = -M_D M_R^{-1} M_D^T$ and $\|\Delta M\| \leq (C_\Theta\|\Theta\|^2 + \rho_L)\|S_N\|$, the physical rank condition $\|\Delta M\| < \sigma_{min}(S_N)$ secured by the NC-2D rank-stability return, rank $M_{light} = 3$, the 3 + 3 Majorana census, and unitarity defect $\Theta\Theta^\dagger + O(\Theta^4)$.

NC-3. Three invariant returns close the branch. (A) Ordering: the labels 1, 2, 3 are frozen pre-spectrally as the spectral projectors $P_1^\nu, P_2^\nu, P_3^\nu$ of the nondegenerate closure-frame kernel $\mathcal{K}_\nu$, ordered by the substrate continuation structure (initial, intermediate, terminal); under $H_\nu^T = \mu_\nu^2 f_\nu(\mathcal{K}_\nu) + R_\nu$ with f_ν strictly increasing and $2\|R_\nu\| < \mu_\nu^2 g_f$, no crossing occurs, each Takagi projector is identified by continuation, and the non-tautological result is $m(P_1^\nu) < m(P_2^\nu) < m(P_3^\nu)$. (B) Attachment: the assembly-invariant Hilbert–Schmidt difference $\Delta_{att}^{phys} = \|\tilde{P}_e H_W^{full} \tilde{P}_\tau\|^2_{HS} - \|\tilde{P}_e H_W^{full} \tilde{P}_\mu\|^2_{HS}$, built from the complete projected weak operator and the independently fixed charged-lepton projectors transported onto the same canonically normalised weak carrier ($\tilde{P}_\alpha = C_W^\dagger P_\alpha C_W$), invariant under simultaneous unitary carrier changes and independent of any base/leakage decomposition; $\Delta_{att}^{phys} > 0$ selects eτ and, by the exact reflection theorem, $\theta_{23} > 45°$. (C) CP sign: the convention-fixed invariant $J_{inv} = \mathrm{Im}\,\mathrm{Tr}([H_\ell^L, H_\nu]^3)/(6\, \Delta_\ell\, \Delta_\nu)$ with the commutator order and ordered spectral products declared once; $\mathrm{sgn}\,\mathrm{Im}\,\mathrm{Tr}([H_\ell^L, H_\nu]^3) < 0 \Leftrightarrow J_{lep} < 0$. Pending direct evaluation, the alignment lemma, endpoint completion and the forward arrow conditionally select normal ordering, eτ, upper octant and $J_{lep} < 0$.

A convention and operator-type ledger fixes the hypercharge normalisation $Q = T_3 + Y$ and firewalls the three distinct operators $K_N \neq \mathcal{K}_\nu \neq H_\nu^T$. The paper publishes the frozen neutral-completion object $\mathcal{N}_{VERSF}$, an independence ledger with named interlocks, a numerical non-insertion audit and local falsifiers. No observed neutrino quantity appears upstream of any selection.

Contents

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1. Purpose and the Strict Closure Test

1.1 The problem left by the rulebook

On the canonically normalised neutral Nambu carrier, the most general quadratic operator is

$$M_N^c = K_N^{-T/2} \begin{bmatrix} M_L & M_D \\ M_D^T & M_R \end{bmatrix} K_N^{-1/2} .$$

This form permits every familiar neutral-fermion branch; the Takagi factorisation supplies the unique unordered mass multiset. None of it selects a physical branch.

Paper 2 must answer: what does the frozen VERSF substrate require of the neutral sector before the intended neutrino observables are examined?

1.2 The strict pass condition

The paper passes only if all of the following are frozen upstream of neutrino data: the existence and multiplicity of right-handed neutral carriers; the protection status of B−L; the primitive block pattern and the higher-order suppression ρ_L; the completion class and light–heavy census; active-light completeness; the light mass ordering with pre-spectrally frozen labels; the eμ/eτ attachment branch; and the octant and CP sign.

The paper fails if any choice is made because it resembles an oscillation fit, satisfies a cosmological limit, avoids a double-beta bound or reproduces a preferred global-fit branch.

1.3 Proof standard

Every claim carries one of four grades:

1. **Exact:** linear algebra or operator identities on the frozen carrier, or an inherited exact theorem.
2. **Conditional theorem:** follows uniquely from premise NT plus inherited theorems.
3. **Conditional certificate return:** an invariant return of NC-1–NC-3 currently discharged by a named stand-in, with the direct evaluation open.
4. **Open object:** formula and evaluation address known; the microscopic entries are absent.

Observed neutrino values are not permitted upstream at any grade.

1.4 Claim boundary

NFCC-1R does not claim: the three absolute light masses; the three heavy masses; the numerical entries of Y_ν or R_N; the Majorana phases; running, thresholds or pole matching; or empirical confirmation. Those are downstream evaluations of the frozen object constructed here.

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2. Conventions, Operator Types and the Inherited Theorem Stack

2.1 Convention and operator-type ledger

Hypercharge convention

This paper uses

$\mathbf{Q = T_3 + Y}$ **,**

so that

$L\_L \sim (\mathbf{1}, \mathbf{2}, -\tfrac{1}{2})$, $\Phi\_cl \sim (\mathbf{1}, \mathbf{2}, +\tfrac{1}{2})$, $N\_R \sim (\mathbf{1}, \mathbf{1}, 0)$.

Results inherited from papers using $Q = T_3 + Y\_old/2$ are converted through $Y = Y\_old/2$. No representation tuple may combine the two conventions without this conversion. Where the gauge-sector papers employ the primitive integer charge lattice $q = 6Y$, the map is $q(L\_L) = -3$, $q(\Phi\_cl) = +3$, $q(N\_R) = 0$; the lattice convention and the present convention must likewise never be mixed inside one tuple.

Operator-type firewall

Three distinct operators occur and must never share a symbol or be identified:

$\mathbf{K\_N = K\_N^\dagger > 0}$ — the neutral kinetic metric;

$\boldsymbol{\mathcal{K}\_\nu = \mathcal{K}\_\nu^\dagger}$ — the Hermitian weak-commitment / closure-frame kernel;

$\mathbf{H\_\nu\text{^}T = M\_light^\dagger\ M\_light}$ — the physical light-neutrino Takagi mass-squared operator.

$K\_N \neq \mathcal{K}\_\nu \neq H\_\nu\text{^}T$. The ordering bridge has the form $H\_\nu\text{^}T = \mu\_\nu^2\ f\_\nu(\mathcal{K}\_\nu) + R\_\nu$ — a function of the frame kernel $\mathcal{K}\_\nu$, never of the kinetic metric K_N. This preserves the central NMCC-1 distinction between kinetic normalisation, Hermitian frame geometry and the complex-symmetric mass operator.

Norm convention

Throughout this paper, $\|\cdot\|$ denotes the operator (spectral) norm. The Hilbert–Schmidt norm is written $\|\cdot\|\_HS$ and appears only where stated (the NC-3B attachment certificate). Every inequality among ρ_L, $\|\Theta\|$, $\|R\_\nu\|$, $\|\Delta M\|$, σ_min and σ_max is an operator-norm/singular-value statement; mixing norms would silently change the NC-2D thresholds and is forbidden.

2.2 Census and equivariance (SC-1)

SC-1 proves that the three generation layers are independently keyed, that completion is layer-local, and that one completion package cannot serve two keyed layers without a derived quotient. **Generation equivariance and layer-exclusive attachment are inherited, not NFCC premises.** Falsifier imported as F3.

2.3 Occupancy and extension status (SC-2)

SC-2 fixes three independently keyed active neutral addresses,

$$\mathscr{H}\_{\nu L} = \mathscr{H}\_{\nu L}^{(1)} \oplus \mathscr{H}\_{\nu L}^{(2)} \oplus \mathscr{H}\_{\nu L}^{(3)} \cong \mathbb{C}^3 ,$$

and establishes that N_R is an **extension carrier** rather than a member of the minimal Standard-Model occupancy ledger. Once a minimal, generation-equivariant neutral extension is declared, the three inherited generation keys **induce** a canonical family of candidate terminal addresses

$$\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3 ,$$

one for each keyed active neutral line. SC-2 types these addresses; NT and minimal branch completeness populate them. SC-2 itself correctly treats the right-handed neutrino as an extension requiring a separately declared neutrino-completion bridge, rather than as an already occupied minimal slot — and the declaration of that bridge is exactly what NC-1 performs.

2.4 Representation typing

The electroweak and anomaly gates show that, inside the minimal singlet extension class, an admitted right-handed neutral carrier has $N_R \sim (\mathbf{1}, \mathbf{1}, 0)$ and contributes no gauge anomaly. Typing, not occupancy.

2.5 Neutral quadratic mathematics (NMCC-1)

NMCC-1 supplies: the left-handed Nambu carrier and non-doubling theorem; the symmetry of the quadratic bilinear; canonical kinetic normalisation; Takagi masses; **the exact continuous-charge criterion** consumed by NC-2A; the pure-Dirac pair theorem; the completion taxonomy; pre-breaking gauge admissibility of the blocks — in particular that a bare M_L is not gauge admissible at primitive order while the Dirac bridge and the fully neutral sterile self-completion are; the exact Schur complement; the controlled seesaw theorem; rank and determinant identities; and relative-frame mixing.

2.6 Dirac construction and rank bound (CMSY-1)

CMSY-1 supplies the typed construction

$$Y_\nu^c = \mathscr{E}_{\nu L}^\dagger \, C_Y \, \mathscr{E}_{N_R} , \quad C_Y = P_Y K_{cl}^{-1/2} H_{cl} K_{cl}^{-1/2} P_Y \geq 0 ,$$

the rank bound

$$\boldsymbol{\mathrm{rank}\, Y_\nu^c \leq \min\{ \dim S_\nu , \mathrm{rank}\, C_Y , \mathrm{rank}\, \mathscr{E}_{\nu L} , \mathrm{rank}\, \mathscr{E}_{N_R} \} ,}$$

and the warning that a positive closure eigenvalue does not guarantee a mass if the required chiral projection vanishes — the fact that mandates the NC-2B support certificate and forbids any "saturation" principle.

2.7 PMNS frame and reflection theorems (PMNS-1)

PMNS-1 proves that physical mixing is the relative transporter $U_{CC} = U_{\ell L}^\dagger C_W P_A U_N$; that the eτ and eμ breaking kernels are exact μ↔τ reflections with identical θ_{12}, θ_{13} but $\theta_{23} \mapsto 90° - \theta_{23}$ and $J_{lep} \mapsto -J_{lep}$; that the attachment branch is open; and that Frame–Takagi transfer requires a positive injective map with a controlled remainder. PMNS-1 additionally evaluates the CP-quartet orientation of each kernel at fixed half-transport phase; NC-3C's stand-in consumes that evaluation.

2.8 Sequential transport and operational geometry

The sequential-transport programme derives a directed generation chain from substrate response — **but only up to an unresolved overall orientation.** The operational-geometry papers supply the canonical Hilbert metric and geodesic grammar. The unresolved orientation is why the forward arrow survives as a conditional stand-in inside NC-3.

2.9 Anomaly-inadmissibility scope

The anomaly-inadmissibility theorem closes **persistent gauge and source currents**: the frozen connected source ledger is $SU(3)_C \times SU(2)_L \times U(1)_Y$, with no B−L connection or persistent source current occupied. Its own scope statement leaves accidental and global currents incompletely interpreted. The ledger-completeness argument against exact B−L is therefore a *graded premise* — a conditional stand-in — not an inheritance.

2.10 Electroweak vacuum (HR-1)

The Dirac block is generated at the electroweak radial interface and is proportional to the same closure-interface vacuum v_{cl} that HR-1 locks numerically. This paper consumes only its type (post-interface) and cross-sector identity (falsifier F17); no numerical value is used in any selection.

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3. The New Premise, the Certificate Architecture, and Firewalls

3.1 Premise NT — Neutral terminal-carrier completeness

The only genuinely new premise of this paper.

Every independently keyed active neutral carrier must possess a typed, gauge-admissible terminal completion route in the closed carrier ledger.

This premise concerns the **existence and typing of a completion carrier**, not whether the eventual quadratic coefficient on that route is nonzero. Carrier closure is therefore determined before the neutral mass operator is evaluated.

A later calculation may return a zero Dirac singular value, an exact null mode or a protecting symmetry. Such a result changes the rank or mass spectrum, but it does not retrospectively erase the carrier route required to make the neutral completion problem well posed.

At primitive electroweak order, the active carrier has no gauge-admissible bare self-completion. Its minimal terminal route is therefore

L_L Φ_cl N_R , N_R ~ (1, 1, 0) .

The physical occupation and multiplicity of these terminal addresses are determined by NC-1. Their mass coefficients and symmetry status are determined only afterwards, by NC-2.

Falsifier NT-F1 (= F1). A keyed active neutral carrier is shown to require no terminal route whatsoever, or a different same-order terminal carrier is derived.

Physical status of the premise

The premise must be stated at its honest strength. Ordinary quantum field theory does **not** require every left-handed neutrino to possess a right-handed terminal carrier: the minimal Standard Model accommodates massless left-handed neutrinos with no internal inconsistency. The three-N_R result is therefore not a consequence of neutrality or electroweak gauge admissibility alone. It follows from the conjunction

NT + generation-key preservation + layer-exclusive attachment + declared minimality ,

of which NT is the VERSF-native premise and the other three are inherited census structure. Nothing in this paper describes the census as forced by gauge consistency, and no reader should come away believing it is.

Why this form matters

It removes the logical dependence

absence of a protecting symmetry ⟶ existence of N_R .

The carrier census is decided first. The later B−L calculation determines whether the completed carrier is Dirac, Majorana or massless — a verdict about the operator on the carriers, never about the carriers themselves. A census that leaned on the symmetry verdict which the census itself is needed to formulate would be circular; the carrier-first ordering eliminates that circularity by construction.

3.2 The certificate architecture

Each certificate consists of three layers, never conflated:

1. **Required invariant return** — the coordinate-independent property the microscopic neutral operator must possess, stated in advance.
2. **Conditional stand-in** — the named argument currently discharging the return at branch level, each demoted from principle to stand-in and carrying its own falsifier.
3. **Unconditional test** — the direct operator evaluation whose result supersedes the stand-in, in either direction.

The stand-ins select the branch today; the certificates define what would overturn it tomorrow.

3.3 Scope firewall

NC-1 counts the carriers of the **declared minimal branch-complete extension**. It is not a universal impossibility theorem for decoupled heavy sterile sectors outside the declared ledger. Any such sector must be introduced through a separately proved carrier role (F4) and cannot be hidden inside the NFCC ledger.

3.4 Circularity firewall (B−L)

The commutator criterion of NC-2A is evaluated **on the substrate-returned operator**. It is never used in the other direction — assuming non-protection in order to license $M_R \neq 0$, then reading non-protection off the result. The ledger stand-in supplies an *expectation*; the criterion, applied to whatever the calculation returns, is the arbiter. Under NT, the census does not consume this verdict at all, so the firewall now protects a genuinely one-way street.

3.5 Assembly firewall (attachment)

Any attachment diagnostic built from a *decomposition* of the weak operator — a "base" piece versus a "leakage" piece — carries sign dependence on the splitting convention and is inadmissible as a certificate. The NC-3B certificate is built from the **complete assembled** operator H_W^{full} and the independently fixed charged-lepton projectors, and is invariant under every simultaneous unitary change of carrier coordinates. Decomposition-based diagnostics are retained only as convention-tagged downstream consistency checks.

3.6 Label firewall (ordering)

The branch labels 1, 2, 3 are frozen **pre-spectrally** (Section 8.1), by the substrate continuation structure, before any Takagi mass is computed. An ordering statement whose labels are assigned by sorting the computed masses is a tautology and is inadmissible.

3.7 Convention-fixing firewall (CP)

The commutator order, the eigenvalue orderings, and the ordered spectral products entering the CP invariant are declared once (Section 8.3) and may never be silently changed. Reversing the commutator order reverses the numerator and must not be described as a physical sign change.

3.8 Octant-selection firewall

The eμ/eτ, octant and CP-sign branches may not be selected by comparing either branch with oscillation global fits (F16).

3.9 Scale firewall

No absolute heavy scale is used or implied. NC-2C consumes only the invariant inequality $\sigma_{min}(M_R) \gg \sigma_{max}(M_D)$; NC-2D only the invariant ratio ρ_L. Absolute masses are out of scope.

3.10 Quarantine firewall

The conditional kernel *benchmark numbers* of the PMNS programme — $\rho_\odot = \sqrt{3/2}$, $B/A = 6/5$, $\beta = \sqrt{3/20}$ — may be used only after the branch is frozen. The phase $\psi = 3\pi/4$ appears in two distinct roles which this firewall separates explicitly: as the **derived forward half-transport root** of the algebra $h_\nu^2 = e^{i3\pi/2}$ (one root per oriented arc, arrow selects forward), it is structural content of the NC-3C stand-in and is admissible upstream; as a **numerical kernel benchmark** feeding PMNS magnitudes alongside $\rho_\odot$, B/A and β, it is quarantined with them. Consuming the derived root is not consuming the benchmark.

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4. Certificate NC-1 — Minimal Neutral-Carrier Completion

4.1 Inherited inputs

From Sections 2.2–2.5: the three keyed active neutral carriers $\mathscr{H}_{\nu L} = \mathscr{H}_{\nu L}^{(1)} \oplus \mathscr{H}_{\nu L}^{(2)} \oplus \mathscr{H}_{\nu L}^{(3)}$; layer-exclusive attachment and key preservation; the canonical extension type $N_R \sim (\mathbf{1}, \mathbf{1}, 0)$; the extension status of N_R; and the three candidate terminal addresses $\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3$ induced by the generation keys once the minimal equivariant extension is declared.

4.2 The terminal route

At primitive electroweak order, $L\_L \otimes L\_L$ contains no full electroweak singlet of hypercharge zero without two interface fields or an equivalent ultraviolet carrier: no gauge-admissible bare self-completion exists. By NT, each keyed active line requires a typed terminal route, and the minimal admissible route is the singlet bridge $L\_L\ \Phi\_cl\ N\_R$ — the declaration of the neutrino-completion bridge that SC-2 requires. Whether the route ultimately carries a nonzero coefficient is NC-2's question, not this one.

4.3 Theorem 1 — Minimal Neutral-Carrier Completion

Grade: conditional theorem on NT plus the inherited census stack.

Let $\mathscr{H}\_\nu L = \mathscr{H}\_\nu L^{(1)} \oplus \mathscr{H}\_\nu L^{(2)} \oplus \mathscr{H}\_\nu L^{(3)}$ be the three independently keyed active neutral carriers inherited from the matter census, and let

$D\_N : \mathscr{H}\_N\_R \to \mathscr{H}\_\nu L$

denote the primitive terminal-completion map. Under NT, generation-key preservation and layer-exclusive attachment, the minimal branch-complete extension contains one terminal neutral singlet address for each active key:

$$\mathscr{H}\_N\_R = \mathscr{N}_1 \oplus \mathscr{N}_2 \oplus \mathscr{N}_3\ ,\ \dim \mathscr{H}\_N\_R = 3\ ,\ \mathscr{H}\_N\_R \cong (\mathbf{1}, \mathbf{1}, 0)^{\wedge}\oplus 3\ .$$

Proof

Each keyed active neutral line requires a typed terminal route by NT. At primitive electroweak order, the available route is the neutral singlet bridge (Section 4.2). Layer-exclusive attachment forbids one terminal carrier from serving two independent generation keys. The two halves of the count have different standing and are stated separately. **Lower bound (derived):** at least three keyed terminal addresses are required, by NT plus layer exclusivity. **Upper bound (by declared minimality):** the minimal generation-equivariant extension contains precisely the three addresses induced by the active keys; any additional sterile direction lies outside the declared ledger, with F4 as the explicit reopening condition. The minimal branch-complete carrier census is therefore three — 'exactly' meaning derived-≥3 capped by declared scope, not exactness derived from dynamics alone. ■

Remark — matrix rank versus keyed completion

The exclusion of fewer than three terminal carriers is not mass-matrix mathematics. A generic 3×2 Dirac coupling of three active flavours to two sterile carriers is perfectly well defined (its rank simply cannot exceed two). What forbids it here is SC-1's **layer-exclusive attachment theorem**: a terminal completion *package* is keyed to one generation layer and cannot serve two independently keyed layers without a derived quotient. The count is a consequence of keyed-completion structure, inherited from that named theorem, and the paper leans on it explicitly rather than on rank counting.

Scope clause

This theorem does not prohibit all possible decoupled heavy sterile sectors. It fixes the carrier content of the minimal neutral completion consumed by the Standard-Model derivation. An additional sterile sector must be introduced through a separately proved carrier role and cannot be hidden inside the NFCC ledger.

4.4 Why this is not data insertion

The number three enters from the already-derived count of generation keys. The proof does not use the observed count of mass splittings, the rank of a reconstructed PMNS matrix, or any assumption that all three neutrinos are massive — indeed, under NT, the census would survive even a returned massless line.

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5. Electroweak Typing and the Frozen Carrier Ledger

5.1 Singlet rigidity

Within the inherited minimal extension class, anomaly and pairing admissibility return

N_R^{(a)} ~ (𝟏, 𝟏, 0) , a = 1, 2, 3 ,

with no new weak doublet, colour representation or hypercharge source, in the $Q = T_3 + Y$ convention of Section 2.1.

5.2 Charge-conjugate coordinates

The left-handed Nambu description uses 𝒽_{N_R}^c ≅ 𝒽_N_R — the left-handed charge-conjugate description of the same three right-handed carriers, not a second set of particles.

5.3 Frozen neutral carrier ledger

Carrier	Rank	Chirality	Electroweak type	Weak-current status	Completion role
𝒽_νL	3	Left	neutral component of (𝟏, 𝟐, −½)	Active	Occupied source lines

Carrier	Rank	Chirality	Electroweak type	Weak-current status	Completion role
$\mathcal{H}_{N\_R} = \mathcal{N}_1 \oplus \mathcal{N}_2 \oplus \mathcal{N}_3$	3	Right	$(\mathbf{1}, \mathbf{1}, 0)$	Sterile	Keyed terminal routes
$\mathcal{H}_{\{N\_R\}^c}$	3	Left conjugate	$(\mathbf{1}, \mathbf{1}, 0)$	Sterile Nambu coordinate	Charge-conjugate completion coordinate
Additional same-layer sterile carrier	not included in the declared minimal ledger	—	—	—	Scope firewall; F4

$$\mathcal{N}_L^{\{NFCC\}} = \mathcal{H}_{\nu L}^{\{(3)\}} \oplus \mathcal{H}_{\{N\_R\}}^{\{c(3)\}} \cong \mathbb{C}^6 .$$

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6. Certificate NC-2 — Neutral Quadratic-Operator Completion

The census is closed; this certificate now types the operator on the frozen carrier. It replaces the former ledger-completeness, saturation, scale-stratification and implicit higher-order-suppression assumptions with **five required invariant returns — the fifth (NC-2D) carrying a companion scalar, six returned numbers in all** — each with its stand-in and unconditional test.

6.1 The inherited leading form

The declared-order neutral block statement is made with explicit order labels. Let [0] denote primitive local neutral completion at the leading single-interface order. Then the required block pattern is

$$(M_L^{\{[0]\}}, M_D^{\{[0]\}}, M_R^{\{[0]\}}) = (0, \text{rank } 3, \text{rank } 3) ,$$

with

$$B_N^{\{[0]\}} = \begin{bmatrix} 0 & (v_{cl}/\sqrt{2})\, Y_\nu \\ (v_{cl}/\sqrt{2})\, Y_\nu^T & M_R^{\{[0]\}} \end{bmatrix} , \quad M_D^{\{[0]\}} = (v_{cl}/\sqrt{2})\, Y_\nu , \quad M_R^{\{[0]\}} = M_R^{\{[0]T\}} .$$

$M_L^{\{[0]\}} = 0$ because no primitive bare active Majorana bilinear is electroweak gauge admissible — its lowest admissible form, $(c_{ij}/\Lambda)(L_i\, \Phi_{cl})(L_j\, \Phi_{cl})$, carries two interface fields (Section 2.5). This is inheritance, not assumption. The full active block is

$$M_L = M_L^{\{[0]\}} + M_L^{\{[\geq 1]\}} = M_L^{\{[\geq 1]\}} \equiv M_L^{irr} ,$$

where M_L^{irr} is the **irreducible** direct active block: it explicitly **excludes** the effective Weinberg-type contribution generated by integrating out the already-counted N_R, because that piece *is* the seesaw term S_N and counting it again would double-count the same physics. The fate of the irreducible piece is not waved away: it is priced by the invariant return of NC-2D below.

6.2 NC-2A — Direct B−L test

The exact criterion

NMCC-1's continuous-charge criterion is exact. Let Q_{B-L} act on the Nambu carrier as $q_a I_3 \oplus q_s I_3$ with the B−L-compatible assignment $q_a = -q_s \neq 0$. B−L is an exact symmetry of the quadratic completion iff

$$[Q_{B-L}, K_N] = 0 \text{ and } Q_{B-L}^T B_N + B_N Q_{B-L} = 0 .$$

Block-wise the second condition reads $2q_a M_L = 0$, $(q_a + q_s) M_D = 0$, $2q_s M_R = 0$; with $q_a + q_s = 0$ the Dirac bridge is neutral while both Majorana blocks are charged, so **exactness $\Leftrightarrow$ $M_L = M_R = 0$.** Two outcomes only:

both vanish $\Longrightarrow$ B−L exact $\Longrightarrow$ Dirac-class (or massless) neutral sector ;

either fails $\Longrightarrow$ **B−L does not protect the physical quadratic completion**, the returned M_R carrying minimal even violation $|\Delta(B-L)| = 2$, under the standard unit normalisation $B-L = -1$ for leptons — the numerical value is normalisation-dependent and is quoted in that convention throughout.

Grade of the criterion: exact, inherited.

The conditional stand-in

The branch-level expectation that M_R is generated rests on the ledger argument at its honest strength: the frozen connected source ledger is $SU(3)_C \times SU(2)_L \times U(1)_Y$; no B−L connection or persistent source current is occupied; anomaly admissibility of a gauged extension is not occupancy. Against it stands the conventional objection at full force — B−L is an accidental anomaly-free global symmetry of the renormalisable action — and the anomaly-inadmissibility theorem's own scope leaves accidental and global currents incompletely interpreted (Section 2.9). The ledger argument is therefore a graded premise, and under NT it carries **no census weight whatsoever**: it bears only on the operator.

Grade: conditional; falsifier F5.

The unconditional test

The commutator pair on the substrate-returned operator (circularity firewall, Section 3.4). A returned $M_R \neq 0$ settles non-protection as an output. Exact B−L is returned only if **$[Q_{B-L}, K_N] = 0$, $M_L = 0$ and $M_R = 0$** — a returned $M_R = 0$ is insufficient by itself if a nonzero direct active block $M_L^{[\geq 1]}$ survives, since that block is B−L-charged in exactly the same way. A full null return flips the operator branch to Dirac-class (or massless) — with the carrier census of NC-1 standing either way.

The B−L verdict

The selected physical quadratic operator does not possess exact B−L. Its Majorana completion violates the standard grading by

$|\Delta(B-L)| = 2$.

NFCC-1R therefore selects the **B−L-violating branch**. It does not yet determine whether the ultraviolet realisation is explicit breaking, spontaneous breaking through a compensating grade-carrying field, or the absence of B−L as a fundamental symmetry. That ultraviolet distinction does not alter the neutral quadratic branch selected here, and its resolution belongs to a deeper completion paper, not to Paper 2.

6.3 NC-2B — Dirac rank as an evaluated support certificate

The required return

rank $M_D = 3$, certified by the **cross-Gram certificate**

$\sigma_{\min}(\mathcal{E}_{\nu L}^\dagger C_Y \mathcal{E}_{N_R}) \geq \varepsilon_Y > 0$, equivalently $\det(\mathcal{E}_{\nu L}^\dagger C_Y \mathcal{E}_{N_R}) \neq 0$ with a declared quantitative margin ε_Y .

The support conditions

$\dim S_\nu = 3$, rank $C_Y \geq 3$ on the retained closure directions , rank $\mathcal{E}_{\nu L} = 3$, rank $\mathcal{E}_{N_R} = 3$,

with nonvanishing projector-level pairings, are **necessary preliminary tests, not sufficient ones**: two full-rank embedded subspaces can still produce a singular cross-Gram operator, and nonzero individual pairings do not exclude determinant cancellation. The certificate is therefore the smallest singular value of the assembled cross operator itself — one number, evaluated on the returned objects — with the four factors of the CMSY rank bound retained as the diagnostic decomposition of any failure. A positive closure eigenvalue with a vanishing chiral projection yields no mass; a full-rank embedding paired with a kernel that annihilates a retained direction yields none; and full supports with a cancelling determinant yield rank deficit — three distinct failure modes, all caught by ε_Y.

The conditional stand-in

Under NT, terminal-route existence does not by itself deliver a nonzero coefficient — that decoupling is deliberate. The rank-3 expectation therefore rests on two inherited facts and one graded expectation: the keyed chiral embeddings are inherited full rank at the carrier level (the route is three-dimensional by Theorem 1); the closure response C_Y is positive on the retained directions (CMSY-1); and no chiral pairing is expected to vanish without a named protecting mechanism, none being derived. This is deliberately weaker support than a surjectivity argument would provide — NT purchases census independence at exactly this price, and the price is stated openly rather than papered over.

Grade: conditional; falsifiers F6 (a required pairing vanishes), F7 (Y_ν singular).

The unconditional test

Direct evaluation of the assembled cross-Gram operator $\mathscr{E}$_νL† C_Y $\mathscr{E}$_N_R and its smallest singular value against the declared margin ε_Y, from H_cl, P_Y and the chiral embeddings — including the extension of H_cl from its traceless restriction to the trace sector, the same open objects the charged-fermion gates await — with the four support factors evaluated as failure diagnostics, not as the arbiter.

6.4 NC-2C — Sterile block and heavy-gap certificate

The required returns

$$\mathbf{M\_R = M\_R^T , \det M\_R \neq 0 ,}$$

and the invariant hierarchy

$$\boldsymbol{\sigma\_\min(M\_R) \gg \sigma\_\max(M\_D) , \text{ equivalently } \Theta = M\_D\, M\_R^{-1} , \|\Theta\| \ll 1 .}$$

Stated as a singular-value inequality on the returned operator — invariant and convention-free — not as an assertion that "pre-interface scales are heavy."

The conditional stand-ins

The *generation stand-in*: the sterile bilinear N_R^T C N_R is a gauge-ledger singlet, and absent exact protection the closure response generates it; a zero singular direction would be a retained direction with neither terminal coupling nor protecting grade. The *scale stand-in*: the sterile self-completion is typed at the pre-interface neutral scale, the Dirac bridge at the interface vacuum v_cl, and the category separation supplies the hierarchy expectation. Both are expectations about returns, graded as such.

Grade: conditional; falsifiers F7, F9.

The unconditional test

The returned singular values: $\sigma_{min}(M_R)/\sigma_{max}(M_D) \gg 1$ — one number, no interpretation.

6.5 NC-2D — Higher-order active-block suppression

The required return

Define the invariant ratio

$$\boldsymbol{\rho_L \equiv \|M_L^{irr}\| / \|M_D M_R^{-1} M_D^T\| = \|M_L^{irr}\| / \|S_N\| ,}$$

where $S_N \equiv -M_D M_R^{-1} M_D^T$ is the leading seesaw operator (used throughout Section 7). The physical branch is type-I-dominant only if

$$\boldsymbol{\rho_L \ll 1 .}$$

If $\rho_L \sim 1$ the branch is **hybrid**; if $\rho_L \gg 1$ the direct active completion dominates. This is not a fourth master certificate — it is an additional scalar return inside NC-2 — but it is what converts "type-I-dominant" from an operator-order expectation into an invariant conclusion.

Rank-stability companion return

A correction small relative to $\|S_N\|$ does not by itself preserve the smallest singular value when S_N is strongly hierarchical. NC-2D therefore carries a companion scalar return: with $\kappa_S(S_N) = \sigma_{max}(S_N)/\sigma_{min}(S_N)$ the condition number of the leading seesaw operator (the subscript S distinguishing it from the $\mathcal{K}_\nu$ eigenvalues κ_i of Section 8.1),

$$\boldsymbol{(C_\Theta \|\Theta\|^2 + \rho_L) \cdot \kappa_S(S_N) < 1 .}$$

Through the Section 7.2 bound this guarantees $\|\Delta M\| < \sigma_{min}(S_N)$, hence $\eta_{rank} \equiv \|\Delta M\|/\sigma_{min}(S_N) < 1$, and Weyl's singular-value inequality then keeps all three light masses nonzero (Section 7.3). Written out, the return is $C_\Theta\|\Theta\|^2 + \rho_L < \sigma_{min}(S_N)/\sigma_{max}(S_N)$: the ρ_L threshold is not the fixed number 1 but the inverse condition number of the light operator, so a strongly hierarchical light spectrum — which normal ordering permits — tightens the requirement automatically, and the lightest state in particular remains seesaw-dominated rather than captured by the higher-order active block. The hierarchy of S_N is priced by κ_S, not ignored. (For the ordering certificate the same ρ_L-bounded piece sits inside R_ν and is priced there by the gap bound of Section 8.1 — two certificates, two prices, one source.) Companion return inside NC-2D, not a new certificate; falsifier F8.

The conditional stand-in

Interface counting alone does **not** suffice here, and the paper says so: an irreducible Weinberg-type contribution $c_L v_{cl}^2/\Lambda_L$ and the type-I seesaw contribution both scale schematically as v_{cl}^2/(scale), so the number of interface insertions cannot by itself rank them. The stand-in is therefore the ultraviolet inequality

$$\boldsymbol{\|c_L\| / \Lambda_L \ll \|Y_\nu R_N^{-1} Y_\nu^{T}\| / M_\star ,}$$

i.e. any independent ultraviolet channel feeding M_L^{irr} must sit above the sterile completion scale or couple more weakly than the singlet route. Within the frozen corpus this is supported by the census: no independent lepton-number-violating carrier below $M_\star$ is occupied, so the leading available Λ_L *is* $M_\star$ through the N_R route — which is exactly the piece excluded from M_L^{irr} by construction. That support is real but thin, and this is graded as the **weakest stand-in inside NC-2**: an undeclared heavy channel is precisely what the ρ_L return exists to detect.

Grade: conditional; falsifier F8 ($\rho_L \gtrsim 1$). The weakest NC-2 stand-in.

The unconditional test

The returned ρ_L.

6.6 Theorem 2 — Neutral Quadratic-Operator Certificate

Grade: conditional certificate (returns discharged by the named stand-ins; direct evaluations open).

On the frozen carrier of Theorem 1, the required neutral operator returns are

$$\boldsymbol{\nexists \text{ exact protecting } B-L \; ; \; \mathrm{rank}\, M_D = 3 \; ; \; \det M_R \neq 0 \; ; \; \sigma_{min}(M_R) \gg \sigma_{max}(M_D) \; ; \; \rho_L \ll 1 \; ; \; (C_\Theta\|\Theta\|^2 + \rho_L)\cdot\kappa_S(S_N) < 1 ,}$$

with declared-order block pattern $(M_L^{[0]}, M_D^{[0]}, M_R^{[0]}) = (0, \mathrm{rank}\ 3, \mathrm{rank}\ 3)$ and leading completion

$$\boldsymbol{B_N^{[0]} = \begin{bmatrix} 0 & (v_{cl}/\sqrt{2})\, Y_\nu \\ (v_{cl}/\sqrt{2})\, Y_\nu^{T} & M_\star R_N \end{bmatrix} , \; \det Y_\nu \cdot \det R_N \neq 0 \; , \; M_R^{[0]} = M_\star R_N \; .}$$

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7. Automatic Consequences: Seesaw Class, Census and Active Completeness

Given the NC-2 returns, NMCC-1 supplies everything below automatically; this section contains no new premise.

7.1 Exact light completion

M_R invertible gives the exact zero-momentum Schur complement $M_{eff} = M_L - M_D M_R^{-1} M_D^{T}$. Define the **leading seesaw operator**

$$\mathbf{S\_N \equiv -\, M\_D\, M\_R^{-1}\, M\_D^{T}}\,,$$

so that at declared order $M\_eff^{[0]} = S\_N$.

Grade: exact given NC-2.

7.2 Controlled seesaw

$$\mathbf{M\_light = S\_N + \Delta M\,,\ \|\Delta M\| \le (\, C\_\Theta\, \|\Theta\|^2 + \rho\_L\,)\, \|S\_N\|\,,\ M\_heavy = M\_R + O(\|\Theta\|\, \|M\_D\|)}\,,$$

for a declared constant $C\_\Theta = O(1)$ fixed by the block-diagonalisation convention. The error scale is set by the leading operator S_N, not by the corrected operator it perturbs, so the bound cannot circularly use its own output; the remainder carries both the block-diagonalisation and the higher-order active contributions, each priced by its own return.

7.3 Light rank

Since M_D and M_R are invertible,

$$\mathbf{\det S\_N = (-1)^3 \det(M\_D)^2 / \det(M\_R) \neq 0}\,,$$

so S_N has rank three. The physical operator $M\_light = S\_N + \Delta M$ also has rank three provided

$$\mathbf{\eta\_rank \equiv \|\Delta M\| / \sigma\_min(S\_N) < 1}\,,$$

which the NC-2D rank-stability return $(C\_\Theta\|\Theta\|^2 + \rho\_L)\cdot\kappa\_S(S\_N) < 1$ guarantees through the Section 7.2 bound. Under this condition, Weyl's singular-value inequality gives

$$\mathbf{\sigma\_min(M\_light) \ge \sigma\_min(S\_N) - \|\Delta M\| > 0}\,,$$

so all three light Takagi masses remain nonzero after the controlled corrections. A bound relative to $\|S\_N\|$ alone would not suffice for a strongly hierarchical S_N; $\kappa\_S(S\_N)$ is what prices that hierarchy.

7.4 Mode census and completion class

3 light active-dominant Majorana modes ⊕ 3 heavy Majorana modes .

The class is a **full-rank type-I-dominant seesaw** as an invariant conclusion: not pure Dirac ($M\_R \neq 0$), not pseudo-Dirac ($\sigma\_min(M\_R) \gg \sigma\_max(M\_D)$), not active-only Majorana (occupied right routes, $M\_L^{[0]} = 0$), not hybrid ($\rho\_L \ll 1$), not generic one-scale mixed (hierarchy return). The eigenmodes are Majorana-type because the returned M_R breaks the continuous lepton-number grade — the NC-2A verdict read as physics.

7.5 Active completeness and unitarity defect

The Schur operator acts on $\mathscr{H}\_{\nu L} \cong \mathbb{C}^3$; its three Takagi vectors span the low-energy active carrier. The full-theory light eigenvectors carry sterile components of order Θ,

$N_A = (I - \tfrac{1}{2}\Theta\Theta^\dagger)\, U_\nu + O(\Theta^3)$, $\mathbf{I - N_A N_A^\dagger = \Theta\Theta^\dagger + O(\Theta^4)}$ **,**

and the defect pattern $\Theta\Theta^\dagger$ is itself a downstream prediction address.

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8. Certificate NC-3 — Projected Spectral and Orientation Branch

8.1 Pre-spectral branch-label firewall

The labels 1, 2, 3 must be assigned before the Takagi masses are calculated.

Let

$\mathbf{P_1^\nu , P_2^\nu , P_3^\nu}$

be the independently derived one-dimensional spectral projectors of the nondegenerate closure-frame kernel $\mathcal{K}_\nu$, ordered by the **substrate continuation structure** rather than by neutrino mass:

- P_1^ν — the initial keyed neutral line;
- P_2^ν — the intermediate continuation line;
- P_3^ν — the terminal continuation line.

The anchoring conditions are explicit — and they are **continuation-order only**:

$\mathbf{rank\ P_i^\nu = 1\ ,\ [\mathcal{K}_\nu, P_i^\nu] = 0\ ,\ \mathcal{K}_\nu = \kappa_1 P_1^\nu + \kappa_2 P_2^\nu + \kappa_3 P_3^\nu\ ,\ \kappa_i\ pairwise\ distinct\ ,}$

with the assignments initial/intermediate/terminal derived from the continuation structure **before** any mass evaluation, and κ_i denoting the $\mathcal{K}_\nu$ eigenvalue carried by the continuation-labelled line P_i^ν — with **no ordering among the κ_i imposed here**. ("Pre-spectral" means prior to the *mass* spectrum; the P_i^ν are of course spectral projectors of $\mathcal{K}_\nu$ itself.)

The coincidence of the two orderings — that the initial line carries the smallest $\mathcal{K}_\nu$ eigenvalue and the terminal line the largest, $\kappa_1 < \kappa_2 < \kappa_3$ — is deliberately **not** written into the anchoring. That coincidence is substantive physical content, and writing it into the firewall would partially rebuild the tautology the firewall exists to kill. It is carried instead as explicit conditional content of Lemma 1 (Section 8.2), with its own falsifier: a returned label–eigenvalue mismatch fails F11 as an order-coincidence failure, distinct from a descent or crossing failure. These labels are frozen before H_ν^T is evaluated. If

$$H_\nu^T = \mu_\nu^2 f_\nu(\mathcal{K}_\nu) + R_\nu ,$$

with f_ν strictly increasing and the remainder satisfying

$$\mathbf{2\|R_\nu\| < \mu_\nu^2 \cdot \min_{i\neq j} |f_\nu(\kappa_i) - f_\nu(\kappa_j)| ,}$$

then no eigenvalue crossing occurs, and each physical Takagi projector is identified by continuous continuation from its pre-spectral projector P_i^ν. Given, in addition, the order coincidence $\kappa_1 < \kappa_2 < \kappa_3$ of Lemma 1, the statement $m_1 < m_2 < m_3$ then means

$$\mathbf{m(P_1^\nu) < m(P_2^\nu) < m(P_3^\nu) ,}$$

with P_3^ν already identified as the terminal line. It is a physical normal-ordering result, not the tautological sorting of three masses after calculation. Downstream comparison with the experimental labelling proceeds through the same frozen labels carried into U_{CC} by the charged-lepton anchoring, so no relabelling freedom survives to the comparison stage.

8.2 NC-3A — Ordering

The required return

Either the directly computed spectrum of $H_\nu^T = M_{light}^\dagger M_{light}$ traced through the pre-spectral projectors, or the proved functional form $H_\nu^T = \mu_\nu^2 f_\nu(\mathcal{K}_\nu) + R_\nu$ with f_ν strictly increasing and the displayed gap bound — which returns the ordering through Weyl and Davis–Kahan control with no separate "positive descent" premise.

The conditional stand-in — Lemma 1 (Common-Closure-Response Alignment)

Grade: conditional; falsifier F11. The load-bearing wall of the ordering output.

The Dirac block and the weak-commitment frame descend from the same positive scalar-readout closure operator. The lemma carries **three** distinct conditional contents, stated separately so that each can fail separately:

(i) Frame alignment and functional descent. With $\mathcal{K}_\nu = V_\nu \operatorname{diag}(\kappa_1, \kappa_2, \kappa_3) V_\nu^\dagger$ (eigenvalues pairwise distinct, nondegeneracy inherited from the kernel gap discipline), the leading closure-functional identity and key-preserving embeddings give

*$M_D = m_D V_\nu\hat{d} O_R^\dagger + E_D$, $\hat{d} \equiv \operatorname{diag}(d(\kappa_1), d(\kappa_2), d(\kappa_3))$, $d > 0$, $d' > 0$,**

with E_D subordinate. (The diagonal matrix $\hat{d}$ is written out because the operator function $d(\mathcal{K}\_\nu) = V\_\nu \hat{d} V\_\nu^\dagger$ is a different object from the sandwiched expression used here.) Gauge neutrality and key preservation give the generation-isotropic leading sterile block $M\_R = M_\star O\_R^{*}(r_0 I + E\_R) O\_R^\dagger$, $r_0 > 0$, $\mathbf{E\_R = E\_R^T}$ (required for $M\_R = M\_R^T$). The seesaw square yields (Appendix C)

$$\mathbf{H\_\nu^T = (m\_D^4/M_\star^2 r_0^2)\, V\_\nu \hat{d}^4 V\_\nu^\dagger + R\_\nu = \mu\_\nu^2 f\_\nu(\mathcal{K}\_\nu) + R\_\nu\ ,\ f\_\nu(\kappa) = d(\kappa)^4\ ,}$$

strictly increasing.

(ii) Order coincidence. The continuation order coincides with the $\mathcal{K}\_\nu$ eigenvalue order:

$$\boldsymbol{\kappa(\text{initial}) < \kappa(\text{intermediate}) < \kappa(\text{terminal})\ ,\ \text{i.e. } \kappa_1 < \kappa_2 < \kappa_3}$$

in the continuation labelling of Section 8.1. This is the substantive claim the label firewall deliberately excludes from the anchoring conditions: it is supported recursively — one positive rank-one continuation per chain stage — under the two explicit stage-overlap conditions of Appendix C ($\eta\_\{k+1\} > (1 + \sqrt{2})\, \Delta_0^{(k)}$, $k = 1, 2$), which order initial against intermediate and intermediate against terminal separately. It fails independently of (i), and it fails per stage.

(iii) Single-operator origin. Both structures consume the same substrate operator; if the neutrino Dirac response consumed a different operator than the weak-commitment kernel, neither (i) nor (ii) would follow.

Failure of (i) or (iii) — nonpositive, nonmonotone, or foreign-operator response — is F11 (alignment failure). Failure of (ii) — a returned label–eigenvalue mismatch — is likewise F11, recorded as an **order-coincidence failure**, distinct from the crossing and degeneracy failures of F12. Sterile anisotropy dominating is F10.

The conditional output

Positive one-extra continuation adds the terminal continuation line as a positive rank-one update of $\mathcal{K}\_\nu$; interlacing places each updated spectrum on or above its base, and the two stage-overlap conditions of Appendix C ($\eta\_\{k+1\} > (1 + \sqrt{2})\, \Delta_0^{(k)}$, ordering initial-vs-intermediate and intermediate-vs-terminal separately) discharge the order coincidence (ii); the increasing descent and gap bound then carry the terminal line to the largest mass. Therefore

$$\mathbf{m(P_1\^\nu) < m(P_2\^\nu) < m(P_3\^\nu)}\ \text{—}$$

the physical state attached to the terminal continuation projector is the heaviest light neutrino. The theorem orders the spectrum; it does not spread it.

Phenomenological identification

Establishing $m(P_1\^\nu) < m(P_2\^\nu) < m(P_3\^\nu)$ is a substrate ordering. The experimental phrase "normal ordering" carries additional labelling content: the conventional ν_1, ν_2 are tied to the solar pair, and ν_3 is identified through the oscillation structure and its electron-flavour content — not by ascending mass alone. The identification of the two orderings therefore requires, and NC-3A's return includes, the map through the charged-current transporter: with the charged-lepton frame independently anchored (F15), the electron-row content $|U_{e i}|$ of the three frozen lines fixes which pair is the solar pair and which line is the conventional third state, **independently of the sign of the atmospheric splitting**. On the selected eτ branch, the terminal line P_3^ν carries the small electron content of the conventional ν_3, so the substrate ordering coincides with phenomenological normal ordering. This identification is a conditional output of the same kernel structure that fixes the attachment; its full proof travels with the direct kernel evaluation, and its failure — frozen lines whose electron-content labelling cannot be fixed independently of the splitting sign — falls under F15.

The unconditional test

The direct spectrum of H_ν^T, traced through the frozen projectors, together with the electron-row identification above.

8.3 NC-3B — Physical attachment invariant

The required return

Let $\mathbf{H_W^{full}}$ be the complete projected weak closure Hamiltonian after all base, leakage and response contributions at the declared order have been assembled, and let P_e, P_μ, P_τ be the independently fixed charged-lepton spectral projectors (F15).

All operators entering the certificate are represented on one and the same canonically normalised weak charged-current carrier. If the charged-lepton projectors are initially defined in the charged-lepton mass carrier, they are transported into the weak carrier through the independently derived current bridge,

$$\boldsymbol{\tilde{P}_\alpha = C_W^\dagger P_\alpha C_W , \alpha = e, \mu, \tau ,}$$

so that no piece of the certificate lives in a different space. The $\tilde{P}_\alpha$ are genuine orthogonal projectors because C_W is unitary on the matched carrier — a property inherited from the relative-frame theorem, which requires the charged-current transporter to be an isometry for U_{CC} to be unitary. Should a future return supply only a non-isometric C_W, the transport must use its unitary polar factor; without that step the transported objects are positive operators, not projectors, and the certificate is not defined. Define the Hilbert–Schmidt attachment difference

$$\boldsymbol{\Delta_{att}^{phys} = \|\tilde{P}_e H_W^{full} \tilde{P}_\tau\|^2_{HS} - \|\tilde{P}_e H_W^{full} \tilde{P}_\mu\|^2_{HS} .}$$

This quantity is invariant under every simultaneous unitary change of carrier coordinates,

$$H_W^{full} \mapsto U^\dagger H_W^{full} U , \tilde{P}_\alpha \mapsto U^\dagger \tilde{P}_\alpha U ,$$

because the Hilbert–Schmidt norm is unitarily invariant — and it is **independent of how the full operator is decomposed** into "base," "leakage" or "correction" pieces. Only the assembled physical operator enters the certificate. The branch selection:

$\Delta_{att}^{phys} > 0 \Longrightarrow e\tau$ attachment ; $\Delta_{att}^{phys} < 0 \Longrightarrow e\mu$ attachment ,

subject to one **branch-family condition**: a larger electron–tau block norm implies $\theta_{23} > 45°$ not for an arbitrary Hermitian operator, but exactly when the returned operator lies within the two reflected kernel families proved by PMNS-1,

$H_W^{full} \in \mathcal{K}_{e\tau} \cup \mathcal{K}_{e\mu}$, with remainder below the reflection gap .

Within that family pair, the exact reflection theorem converts the sign of Δ_{att}^{phys} into the octant; outside it, the sign selects nothing. Family membership is itself a required return of the direct evaluation. The equality case $\Delta_{att}^{phys} = 0$ — and, realistically, any returned $|\Delta_{att}^{phys}|$ below the declared numerical control of the evaluation — leaves the reflected branches unresolved and fails Paper 2's attachment-closure requirement (F13). Given the exact reflection theorem: $e\tau \Longrightarrow \theta_{23} > 45°$, $e\mu \Longrightarrow \theta_{23} < 45°$.

The conditional stand-in — endpoint completion

The keyed oriented chain $g_1 \rightarrow g_2 \rightarrow g_3$ carries the unique terminal continuation on g_3 — the same line NC-3A sends to the heaviest state. Weak neutrality suppresses a first-order diagonal μ–τ split, so the first breaking is an electron-to-non-electron attachment. In an oriented three-address chain with a unique terminal continuation at g_3, the minimal nonlocal attachment closing the initial line to that continuation is the endpoint link $g_1 \leftrightarrow g_3$ — after charged-lepton anchoring, $e \leftrightarrow \tau$. This is the current conditional reason to expect $\Delta_{att}^{phys} > 0$, hence conditionally

$\chi_{att} = e\tau$, $\theta_{23} > 45°$,

the octant step being exact given the attachment.

Grade: conditional; falsifier F13. The softest stand-in in the paper.

The unconditional test

The returned sign of Δ_{att}^{phys} — the full-operator invariant is the arbiter.

8.4 NC-3C — Convention-fixed invariant CP sign

The required return

Let $H_\ell^L = M_\ell M_\ell^\dagger$ in the canonically normalised charged-lepton left carrier, and let

$H_\nu = C_W H_\nu^T C_W^\dagger$

be the light-neutrino mass-squared operator transported into the same charged-current carrier. Fix the commutator order **once and permanently** as

$$[H_\ell^L, H_\nu] = H_\ell^L H_\nu - H_\nu H_\ell^L ,$$

and define the positive ordered spectral products

$$\Delta_\ell = (m_\mu^2 - m_e^2)(m_\tau^2 - m_e^2)(m_\tau^2 - m_\mu^2) > 0 ,$$

and, **after NC-3A has fixed the neutrino branch labels**,

$$\Delta_\nu = (m_2^2 - m_1^2)(m_3^2 - m_1^2)(m_3^2 - m_2^2) > 0 .$$

Define

$$\mathbf{J_{inv} \equiv Im\,Tr([H_\ell^L, H_\nu]^3) / (6\, \Delta_\ell\, \Delta_\nu) .}$$

With this declared commutator and ordering convention, $J_{inv} = J_{lep}$, and the CP-sign branch is

$$\mathbf{Im\,Tr([H_\ell^L, H_\nu]^3) < 0 \Leftrightarrow J_{lep} < 0 .}$$

Reversing the commutator order would reverse the numerator and must not be described as a physical sign change; the definition above permanently fixes that convention (firewall 3.7).

The conditional stand-in — forward half-transport

The half-transport element satisfies $h_\nu^2 = e^{i3\pi/2}$, with roots $e^{i3\pi/4}$ and $e^{i7\pi/4}$, exactly one on each oriented arc; any arrow-respecting rule selects the forward root $\psi = 3\pi/4$, so the entire conditional content is the arrow — genuinely load-bearing, because sequential transport fixes the chain only up to an unresolved overall orientation (Section 2.8). With the forward root and the inherited PMNS-1 quartet-orientation evaluation of the eτ kernel at $\psi = 3\pi/4$ in the declared orderings, the stand-in predicts

$$\mathbf{J_{inv} < 0 , \text{ i.e. } J_{lep} < 0 .}$$

Reversing the physical commitment arrow selects the conjugate root and flips every CP-odd invariant — a physical reversal, not a rephasing, once the orderings are fixed.

Grade: conditional; falsifier F14.

The unconditional test

The returned sign of $\mathrm{Im\,Tr}([H_\ell^L, H_\nu]^3)$ in the declared convention.

8.5 Theorem 3 — Projected Branch Certificate

Grade: conditional certificate (returns discharged by the named stand-ins; direct evaluations open).

The required orientation returns are

$$m(P_1^\nu) < m(P_2^\nu) < m(P_3^\nu)\ ,\ \Delta_{att}^{phys} > 0\ ,\ \mathrm{Im}\,\mathrm{Tr}([H_\ell^L, H_\nu]^3) < 0\ ,$$

and the conditionally selected physical branch is

$$\text{normal ordering}\ ,\ \chi_{att} = e\tau\ ,\ \theta_{23} > 45°\ ,\ J_{lep} < 0\ .$$

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9. The Neutral-Fermion Branch and Carrier-Census Completion Theorem

Theorem 4 — Neutral-Fermion Branch and Carrier-Census Completion

Grade: conditional theorem on NT, the inherited stack, and the NC-2/NC-3 certificate returns as currently discharged by their stand-ins.

Under the inherited three-key active-neutral census; the inherited electroweak and anomaly typing; premise NT; the NC-2 neutral-operator returns; and the NC-3 spectral and orientation returns:

The minimal branch-complete neutral carrier is

$$\mathcal{N}_L^{NFCC} = \mathcal{H}_{\nu L}^{(3)} \oplus \mathcal{H}_{N_R}^{c(3)} \cong \mathbb{C}^6\ ,\ N_R^{(a)} \sim (\mathbf{1}, \mathbf{1}, 0)\ ,\ a = 1, 2, 3\ .$$

At declared primitive order, $M_L^{[0]} = 0$ by inherited gauge admissibility, and the Dirac and sterile blocks occupy their uniquely typed locations with the NC-2B/2C rank returns

$$(M_L^{[0]}, M_D^{[0]}, M_R^{[0]}) = (0, \text{rank } 3, \text{rank } 3)\ .$$

The returned operator does not preserve an exact protecting B−L grade, satisfies

$$\sigma_{min}(M_R) \gg \sigma_{max}(M_D)\ ,\ \rho_L \ll 1\ ,$$

and therefore lies in the **full-rank type-I-dominant seesaw regime**. Its physical spectrum contains

3 light active-dominant Majorana modes ⊕ 3 heavy Majorana modes ,

the three light modes spanning the complete effective active carrier with full-theory unitarity defect

$I - N_A N_A^\dagger = \Theta\Theta^\dagger + O(\Theta^4)$, $\Theta = M_D M_R^{-1}$.

Using the independently fixed pre-spectral neutral projectors,

$\mathbf{m(P_1^\nu) < m(P_2^\nu) < m(P_3^\nu)}$ **,**

so the physical ordering is normal. Finally,

$\mathbf{\Delta_{att}^{phys} > 0}$ **,** $\mathbf{J_{inv} < 0}$ **,**

selecting

eτ attachment , $\mathbf{\theta_{23} > 45°}$ **,** $\mathbf{J_{lep} < 0}$ **.**

No oscillation splitting, neutrino mass, PMNS angle, cosmological mass limit, beta-decay result or neutrinoless-double-beta-decay result is used in this selection.

The theorem accomplishes branch **commitment**: the frozen premise-and-stand-in stack selects and publicly stakes one branch. Microscopic branch **derivation** — the direct evaluation of the certificate returns — is fully specified by NC-2 and NC-3 and remains the downstream obligation. ■

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10. The Frozen Neutral-Completion Object

10.1 Carrier ledger

$\mathscr{H}_{\nu L} \cong \mathbb{C}^3$, $\mathscr{H}_{N_R} = \mathcal{N}_1 \oplus \mathcal{N}_2 \oplus \mathcal{N}_3 \cong \mathbb{C}^3$, $N_R^{(a)} : (\mathbf{1}, \mathbf{1}, 0)$.

10.2 Operator-symmetry ledger

- Connected gauge ledger: $SU(3)_C \times SU(2)_L \times U(1)_Y$ (convention $Q = T_3 + Y$).
- Exact protecting B−L on the returned operator: absent (conditional; arbiter NC-2A). The selected branch is B−L-violating, with $|\Delta(B-L)| = 2$; whether the ultraviolet realisation is explicit breaking, spontaneous breaking through a compensating carrier, or absence of fundamental B−L is left to a deeper completion paper and does not alter the quadratic branch.

10.3 Block ledger

$M_L^{[0]} = 0$, rank $M_D = 3$, rank $M_R = 3$, $\sigma_{min}(M_R) \gg \sigma_{max}(M_D)$, $\rho_L \ll 1$.

10.4 Canonical operators — leading and physical

The object publishes both the leading and the full physical raw completions, because the physical branch permits a suppressed higher-order active block:

$$B_N^{[0]} = \begin{bmatrix} 0 & M_D \\ M_D^T & M_R \end{bmatrix}, \quad B_N^{phys} = \begin{bmatrix} M_L^{[\geq 1]} & M_D \\ M_D^T & M_R \end{bmatrix}, \quad \rho_L \ll 1 .$$

With $K_N = K_N^\dagger > 0$ the neutral kinetic metric at the declared matching scale, the canonical operator is

$$M_N^c = K_N^{-T/2} B_N^{phys} K_N^{-1/2} , \quad (M_N^c)^T = M_N^c , \quad M_D = (v_{cl}/\sqrt{2})\, Y_\nu , \quad M_R = M_\star R_N .$$

Entries of K_N, Y_ν, R_N and $M_L^{[\geq 1]}$ are downstream calculations; carrier types, block locations, rank returns, hierarchy and ρ_L bound are frozen here.

10.5 Completion-class ledger

- Leading class: full-rank type-I-dominant seesaw (invariant conclusion).
- Light spectrum: three active-dominant Majorana Takagi modes.
- Heavy spectrum: three Majorana Takagi modes.
- Additional same-layer light sterile mode: none in the declared ledger.
- Effective active carrier: exhausted by the three light modes; defect $\Theta\Theta^\dagger + O(\Theta^4)$.

10.6 Ordering and orientation ledger

$\mathcal{O}_N : m(P_1^\nu) < m(P_2^\nu) < m(P_3^\nu)$ (pre-spectral labels) , $\chi_{att} = e\tau$, $\varepsilon_{oct} = +1$, $\varepsilon_{CP} = -1$.

10.7 Master downstream object

$$\mathcal{N}_{VERSF} = (\mathcal{N}_L^{NFCC}, K_N, B_N^{[0]}, B_N^{phys}, \rho_L, P_A, P_S, \{P_i^\nu\}, \mathcal{O}_N, \chi_{att}, \varepsilon_{CP}) .$$

The pre-spectral projectors $\{P_i^\nu\}$ are published as part of the object, so no downstream paper can relabel the branches. Every downstream paper that would otherwise assume a neutrino Yukawa matrix or reopen a neutral branch consumes $\mathcal{N}_{VERSF}$. A later microscopic paper may overturn a component by returning a certificate value against its stand-in; it may not silently choose a different branch because it fits data better.

11. Alternative-Branch Elimination

Candidate branch	Certificate ground for exclusion	Exact reopening condition
No N_R	Fails NT: keyed active lines require typed terminal routes; no bare self-completion admissible	Show a keyed line requires no terminal route, or derive a different same-order terminal carrier (F1)
One or two N_R	Layer-exclusive attachment: one terminal carrier cannot serve two keys	Derive a quotient identifying active addresses (F3)
More than three N_R	Outside the declared minimal ledger	Derive an independent extension role (F4)
Exact protecting B−L	NC-2A expectation: no persistent ledger; commutator pair expected nonvanishing	NC-2A returns $[Q_{B-L}, K_N] = 0$, $M_L = 0$, $M_R = 0$ (F5); carrier census stands, operator branch flips to Dirac-class or massless
Pure Dirac	NC-2C: sterile self-completion generated, $\det M_R \neq 0$	Return $M_R = 0$
Active-only Majorana	Occupied right routes and $M_L^{[0]} = 0$ by gauge admissibility	Derive no terminal route plus a primitive active channel
Hybrid seesaw	NC-2D: $\rho_L \ll 1$	Return $\rho_L \sim 1$ (F8)
Direct-dominated active	NC-2D	Return $\rho_L \gg 1$ (F8)
Pseudo-Dirac	NC-2C hierarchy	Return $\sigma_{min}(M_R) \lesssim \sigma_{max}(M_D)$ (F9)
Generic one-scale mixed	NC-2C hierarchy plus $M_L^{[0]} = 0$	Return $M_R \sim M_D$ or a same-order M_L (F2, F9)
Inverted ordering	NC-3A stand-in: increasing descent, gap control, pre-spectral labels	Direct spectrum returns $m(P_3^\nu)$ not maximal (F10–F12)
Electron–muon attachment	NC-3B stand-in: endpoint completion	Return $\Delta_{att}^{phys} < 0$ (F13)
Lower atmospheric octant	Exact reflection of the excluded eμ branch	$\Delta_{att}^{phys} < 0$
Positive J_{lep}	NC-3C stand-in: forward arrow + quartet orientation	Return $\mathrm{Im}\,\mathrm{Tr}([H_\ell^L, H_\nu]^3) > 0$ (F14)

12. Independence Ledger

The compact certificate presentation must not conceal that the branch consists of logically independent outputs; a future calculation can confirm five and overturn one. The six outputs:

Output	Certificate	Conditional stand-in	Unconditional test	Falsifier
B−L non-protection	NC-2A	Ledger-completeness premise	Commutator pair on returned operator	F5
rank $M_D = 3$	NC-2B	Support expectation (carrier-rank + positive response)	Cross-Gram certificate $\sigma_{min} \geq \varepsilon_Y$ (support factors as diagnostics)	F6, F7
$\det M_R \neq 0$, heavy gap, $\rho_L \ll 1$, rank stability	NC-2C/2D	Generation + scale + suppression stand-ins	Returned singular values, ρ_L and $\kappa_S(S_N)$	F7, F8, F9
Normal ordering (pre-spectral labels)	NC-3A	Alignment Lemma 1 + gap bound	Direct spectrum through frozen projectors	F10–F12
eτ attachment ($\Longrightarrow$ upper octant)	NC-3B	Endpoint completion (forward arrow)	sgn Δ_{att}^{phys}	F13
$J_{lep} < 0$	NC-3C	Forward root + quartet orientation	sgn Im $\mathrm{Tr}([H_\ell^L, H_\nu]^3)$, declared convention	F14

Named interlocks (one-directional, no cycles): the Section 7 automatics require the NC-2 rows; NC-3A's stand-in uses NC-2C's isotropy structure; NC-3C's sign identification $J_{inv} = J_{lep}$ consumes NC-3A's label ordering through $\Delta_\nu > 0$ — if the returned spectrum were inverted, the sign is read from the actual ordered products, and only the identification with the frozen labels changes; NC-3B and NC-3C share the forward arrow, so a reversed arrow flips both together — their joint reversal is the reflected branch, one failure, not two. Under NT, no row feeds back into the census: the carriers stand whatever the operator returns.

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13. Numerical Non-Insertion and Blind Protocol

13.1 Forbidden target set

$\mathcal{T}_\nu$ = { Δm_{ij}^2, m_i, Σm_i, θ_{12}, θ_{13}, θ_{23}, δ_{CP}, J_{lep}, m_β, $m_{\beta\beta}$, cosmological limits } , $\boldsymbol{\mathcal{T}_\nu \cap}$ **Inputs($\mathcal{N}$_VERSF) = ∅ .**

13.2 Blind sequence

1. Freeze the three active occupancy addresses and the convention ledger.
2. Apply NT; publish the carrier census and typing (NC-1) — before any mass question.
3. Publish the required operator returns and their stand-ins (NC-2, including ρ_L).
4. Freeze the pre-spectral branch labels $\{P_i^\nu\}$ from the continuation structure.
5. Construct and canonically normalise K_N, Y_ν, R_N.
6. Evaluate the NC-2 returns: commutator pair; cross-Gram σ_{min} against ε_Y (support factors as diagnostics); singular values and hierarchy; ρ_L and the rank-stability companion.
7. Compute the Takagi spectrum through the frozen projectors; compute Δ_{att}^{phys} and Im $\mathrm{Tr}([H_\ell^L, H_\nu]^3)$.
8. Publish the branch output.
9. Only then compare with oscillation, beta-decay, lepton-number-violation and cosmological data.

Steps 1–4 and the conditional outputs of steps 6–8 are completed by this paper; the direct evaluations in steps 5–7 are the downstream microscopic gates.

13.3 Inputs actually used

Three keyed active occupancy layers; layer-exclusive keyed completion (inherited); primitive electroweak representation and operator order (inherited); the declared minimal gauge-singlet extension; premise NT; the ledger, support, generation, scale, suppression, alignment, endpoint and forward-arrow stand-ins, each named and graded; and the exact inherited criterion, rank-bound, Schur, reflection and invariant identities. No measured neutrino property appears in that list.

13.4 Downstream benchmark quarantine

The kernel benchmark numbers $\rho_\odot = \sqrt{(3/2)}$, $B/A = 6/5$, $\beta = \sqrt{3}/20$ — and $\psi = 3\pi/4$ in its benchmark role, as distinguished in Section 3.10 — may be used only after the branch is frozen; the derived forward root of Section 8.4 is structural and is not part of this quarantine.

13.5 What non-insertion does not prove

A full provenance audit must still demonstrate that the sequential-transport orientation was derived before octant comparison, that the weak-commitment kernel structure was not tuned to PMNS magnitudes, and that the scale categories were fixed before any light-mass window was examined. This paper certifies its own chain; the provenance of the inherited stack is certified, or not, by its own papers.

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14. Falsification Conditions

F1 — Terminal-route failure (NT-F1). A keyed active neutral carrier is shown to require no terminal route whatsoever, or a different same-order terminal carrier is derived.

F2 — Primitive active completion. A same-order gauge-admissible bare active Majorana block is derived.

F3 — Generation quotient. A substrate quotient identifies two neutral generation addresses at the completion layer (inherited SC-1 falsifier).

F4 — Extension-role sterile. An additional sterile carrier with an independently derived role is admitted, enlarging the declared ledger.

F5 — B−L certificate reversal. NC-2A returns $[Q_{B-L}, K_N] = 0$, $M_L = 0$ and $M_R = 0$ on the physical operator. The operator branch reopens as Dirac-class or massless; the carrier census stands.

F6 — Cross-Gram failure. $\sigma_{min}(\mathcal{E}_{\nu L}^\dagger C_Y \mathcal{E}_{N\_R}) = 0$ (or below the declared margin ε_Y): a support factor fails — $\dim S_\nu < 3$, $\operatorname{rank} C_Y < 3$ on the retained directions, $\operatorname{rank} \mathcal{E}_{\nu L} < 3$, $\operatorname{rank} \mathcal{E}_{N\_R} < 3$ — or the supports are full but the cross-Gram determinant cancels. Either way $\operatorname{rank} M_D < 3$.

F7 — Rank failure. Y_ν or R_N is returned singular.

F8 — Dominance or rank-stability failure. $\rho_L \gtrsim 1$: an irreducible higher-order direct active block M_L^{irr} competes with or dominates the seesaw-induced term, so the branch is hybrid or direct-dominated rather than type-I-dominant; or the rank-stability return fails, $(C_\Theta\|\Theta\|^2 + \rho_L)\cdot\kappa_S(S_N) \geq 1$, permitting a light singular value to be driven to zero.

F9 — Hierarchy failure. $\sigma_{min}(M_R) \gg \sigma_{max}(M_D)$ fails on the returned operator.

F10 — Sterile-anisotropy failure. Leading generation anisotropy in M_R dominates the common closure ordering.

F11 — Alignment or order-coincidence failure. The neutrino Dirac response is nonpositive or nonmonotone, or does not descend from the same closure spectral algebra as $\mathcal{K}_\nu$ (Lemma 1(i)/(iii)); or the continuation order does not coincide with the $\mathcal{K}_\nu$ eigenvalue order — including failure of the Appendix C overlap condition — so the terminal line does not carry the largest eigenvalue (Lemma 1(ii)).

F12 — Crossing or degeneracy failure. $2\|R_\nu\| \geq \mu_\nu^2 g_f$, permitting an ordering reversal by remainder-driven crossing; or $\mathcal{K}_\nu$ is returned degenerate, so the pre-spectral labels cannot be assigned. (A returned label–eigenvalue mismatch with intact gaps is the order-coincidence branch of F11, not F12.)

F13 — Attachment reversal or irresolution. $\Delta_{att}^{phys} < 0$; or $|\Delta_{att}^{phys}|$ at or below the declared numerical control of the evaluation, leaving the reflected branches unresolved and failing attachment closure; or H_W^{full} lies outside both reflected kernel families beyond the declared reflection gap, so the sign selects no branch; or the physical transport fails to orient the chain; or the terminal continuation sits on the intermediate address.

F14 — CP reversal. $\mathrm{Im}\,\mathrm{Tr}([H_\ell^L, H_\nu]^3) > 0$ in the declared commutator convention.

F15 — Charged-frame or identification failure. The charged-lepton left frame and address ordering are not independently fixed; or the electron-row identification of the frozen lines with the phenomenological labels (solar pair, conventional third state) cannot be fixed independently of the sign of the atmospheric splitting.

F16 — Numerical insertion. Any neutrino measurement is used to choose a carrier, rank, symmetry verdict, block pattern, hierarchy, suppression ratio, ordering, attachment or CP branch.

F17 — Cross-sector vacuum failure. The neutral Dirac block requires an interface vacuum different from the v_{cl} that closes the charged-fermion and gauge sectors.

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15. Closure Certificate

Paper-2 discrete carrier-census closure: PASS under NT.

The minimal generation-equivariant neutral extension contains three right-handed electroweak-singlet carriers, decided before and independently of any mass or symmetry question.

Declared-order block typing: PASS.

$M_L^{[0]} = 0$, while the Dirac and sterile blocks occupy the uniquely typed locations

$$M_D : \mathscr{H}_{N_R} \to \mathscr{H}_{\nu L} ,\quad M_R : \mathscr{H}_{\{N_R\}^c} \to \mathscr{H}_{N_R} ,$$

by inherited gauge admissibility and the typed completion routes. Gauge admissibility fixes the block types and locations; it does not by itself prove the nonzero ranks.

Declared-order block ranks: CONDITIONAL PASS.

$$\mathrm{rank}\, M_D = 3 ,\quad \mathrm{rank}\, M_R = 3 ,$$

pending the NC-2B and NC-2C direct evaluations, currently discharged by the support and generation stand-ins.

Physical completion-class closure: CONDITIONAL PASS.

Type-I dominance additionally requires the directly testable returns

$$\sigma\_min(M\_R) \gg \sigma\_max(M\_D)\ ,\ \rho\_L \ll 1\ ,\ (C\_\Theta\|\Theta\|^2 + \rho\_L)\cdot\kappa\_S(S\_N) < 1\ ,$$

currently discharged by the scale and suppression stand-ins, the last return guaranteeing that all three light masses survive the controlled corrections.

Ordering and orientation closure: CONDITIONAL PASS.

The current stand-ins select normal ordering (through pre-spectrally frozen labels), electron–tau attachment, the upper atmospheric octant and J_lep < 0. Unconditional closure awaits direct evaluation of

$$\mathrm{ord}\ \sigma(H\_\nu\hat{}T)\ \text{through}\ \{P\_i\hat{}\nu\}\ ,\ \Delta\_att\hat{}phys\ ,\ \mathrm{Im\ Tr}([H\_\ell\hat{}L, H\_\nu]^3)\ .$$

Numerical non-insertion: PASS.

No neutrino observation is used to populate the carrier ledger, choose the operator branch or select its ordering, octant or CP sign.

Gate verdict:

Carrier-first discrete branch closure on one new premise plus named conditional stand-ins: PASS.

Unconditional closure: OPEN on the direct operator evaluations — the NC-2A commutator pair; the NC-2B/2C/2D support, singular-value and ρ_L returns; and the NC-3 triple.

The selected branch is three right-handed electroweak singlets, non-protecting B−L, primitive (M_L, M_D, M_R) = (0, rank 3, rank 3) with ρ_L ≪ 1, a full-rank type-I-dominant seesaw, three light and three heavy Majorana modes, effective active completeness, normal ordering on pre-spectral labels, electron–tau attachment, upper atmospheric octant and negative leptonic Dirac CP invariant.

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16. Conclusion

The neutrino sector presents a sharper test of a derivation programme than almost any other part of the Standard Model. Neutrality opens several mathematically consistent mass routes; a theory that merely lists them has not explained which one nature realises.

NFCC-1R converts the VERSF neutral rulebook into a frozen branch, with its machinery reduced to the honest minimum and the logic running carrier-first. The single new premise, neutral terminal-carrier completeness, asks only that every keyed neutral line possess a typed completion route. It deliberately does not ask that the route carry mass. The census is thereby decided before the operator is examined; a carrier count that depended on the B−L verdict would be quietly circular, and the carrier-first ordering forecloses that by construction. If the operator calculation one day returns exact B−L and a Dirac sector, the three carriers stand; only the physics on them changes.

The operator certificate then demands five coordinate-independent returns: the two-commutator B−L test, evaluated on the returned operator and never used to license it; the cross-Gram certificate behind rank $M_D = 3$, because a positive response with a dead projection yields nothing and full supports do not exclude determinant cancellation; the nonsingular symmetric sterile block; the singular-value hierarchy; and the suppression ratio $\rho_L \ll 1$ with its rank-stability companion, which closes the last escape route by which a higher-order direct active block could quietly turn the branch hybrid. "Type-I-dominant" is thereby an invariant conclusion, not an operator-order hope. Given the returns, the rulebook delivers the seesaw, the 3 + 3 Majorana census and active completeness automatically.

The orientation certificate makes the three remaining predictions non-tautological. The branch labels are frozen pre-spectrally, on the substrate's own continuation structure, so "normal ordering" asserts something that can fail: the state on the terminal projector is the heaviest. The attachment is decided by a Hilbert–Schmidt invariant of the complete assembled weak operator — immune to coordinates and to every base/leakage split, retiring the decomposition-dependent diagnostics for good. The CP sign is defined through a fully invariant trace with its commutator order and spectral products declared once, permanently, so that no bookkeeping reversal can ever masquerade as physics. Pending those evaluations, the alignment lemma, endpoint completion and the forward arrow — the honest residue of sequential transport's unresolved overall orientation — conditionally select normal ordering, eτ, the upper octant and $J_{lep} < 0$.

The result is protected by a blind derivation order, an independence ledger with named interlocks, and per-output falsifiers, not by numerical agreement. A future calculation may overturn any single row — the commutators may vanish, a pairing may die, ρ_L may be large, the hierarchy may collapse, the spectrum may invert on its frozen labels, the invariant signs may flip — and each failure would be precise and scientifically useful. What no future calculation may do is reopen the branch merely because another option fits the data better.

That is the role of the second paper of the final eight: to replace an open neutrino menu with one frozen VERSF neutral-completion object, selected carrier-first by the least machinery that honestly suffices, with every remaining open condition expressed as a coordinate-independent operator return rather than a narrative principle.

Appendix A — Minimal Algebraic Skeleton

A.1 Carriers

$\mathscr{H}_{\nu L} \cong \mathbb{C}^3$, $\mathscr{H}_{N_R} = \mathcal{N}_1 \oplus \mathcal{N}_2 \oplus \mathcal{N}_3 \cong \mathbb{C}^3$, $\mathcal{N}_L = \mathscr{H}_{\nu L} \oplus \mathscr{H}_{\{N_R\}^c} \cong \mathbb{C}^6$.

A.2 Raw completion

$B_N^{[0]} = \begin{bmatrix} 0 & M_D \\ M_D^T & M_R \end{bmatrix}$, $M_D = (v_{cl}/\sqrt{2})\, Y_\nu$, $M_R = M_\star R_N$, $M_L = M_L^{[\geq 1]} \equiv M_L^{irr}$.

A.3 Canonical completion

$M_N^{c,[0]} = K_N^{-T/2} B_N^{[0]} K_N^{-1/2}$, $M_N^{c,phys} = K_N^{-T/2} B_N^{phys} K_N^{-1/2}$, $(M_N^{c,phys})^T = M_N^{c,phys}$.

A.4 Certificates

NC-1: typed terminal routes for all keys $\Longrightarrow n_R = 3$ (minimal extension). NC-2: ∄ exact protecting B−L ; rank $M_D = 3$ ($\sigma_{min}(\mathscr{E}_\nu L^\dagger C_Y \mathscr{E}_N R) \geq \varepsilon_Y$) ; $\det M_R \neq 0$; $\sigma_{min}(M_R) \gg \sigma_{max}(M_D)$; $\rho_L \ll 1$; $(C_\Theta \|\Theta\|^2 + \rho_L)\cdot\kappa_S(S_N) < 1$. NC-3: $m(P_1^\nu) < m(P_2^\nu) < m(P_3^\nu)$; $\Delta_{att}^{phys} > 0$; $\mathrm{Im}\,\mathrm{Tr}([H_\ell^L, H_\nu]^3) < 0$.

A.5 Takagi spectrum

$U_N^T M_N^{c,phys} U_N = \mathrm{diag}(m_1, m_2, m_3, M_1, M_2, M_3)$, three light/three heavy under the gap theorem.

A.6 Light operator and mixing

$S_N = - M_D M_R^{-1} M_D^T$, $M_{light} = S_N + \Delta M$, $\|\Delta M\| \leq (C_\Theta \|\Theta\|^2 + \rho_L)\|S_N\|$, $(C_\Theta\|\Theta\|^2 + \rho_L)\kappa_S(S_N) < 1 \Longrightarrow \sigma_{min}(M_{light}) > 0$, $\Theta = M_D M_R^{-1}$, $I - N_A N_A^\dagger = \Theta\Theta^\dagger + O(\Theta^4)$.

A.7 Ordering bridge

$H_\nu^T = \mu_\nu^2 f_\nu(\mathcal{K}_\nu) + R_\nu$, $f_\nu' > 0$, $2\|R_\nu\| < \mu_\nu^2 g_f$, labels $\{P_i^\nu\}$ frozen pre-spectrally .

A.8 PMNS branch

$U_{CC} = U_{\ell L}^\dagger C_W P_A U_N$, $\Delta_{att}^{phys} > 0$ ($e\tau$) , $\theta_{23} > 45°$, $J_{inv} = J_{lep} < 0$.

Appendix B — Rank, Criterion and Seesaw Proofs

B.1 Census

Each key requires a typed terminal route (NT); layer exclusivity forbids shared routes (SC-1); the minimal equivariant extension carries exactly the three induced addresses; extra directions require a declared role (scope firewall). Hence $\dim \mathscr{H}_{N\_R} = 3$. ■

B.2 Block form of the B−L criterion

With $Q = q_a I_3 \oplus q_s I_3$ on the Nambu carrier, $Q^T B_N + B_N Q$ has blocks $2q_a M_L$, $(q_a + q_s) M_D$, $2q_s M_R$. For $q_a = -q_s \neq 0$ the Dirac block is neutral and exactness $\Leftrightarrow$ $M_L = M_R = 0$. A returned $M_R \neq 0$ fails the criterion in the M_R block with $|\Delta(B-L)| = 2$. ■

B.3 Light determinant

$\det(- M_D M_R^{-1} M_D^T) = (-1)^3 \det(M_D)^2 \det(M_R)^{-1} \neq 0$ for invertible square blocks; rank three. ■

B.4 Hierarchy and dominance

$\|M_D M_R^{-1}\| \leq \sigma_{max}(M_D)/\sigma_{min}(M_R) \ll 1$ by NC-2C; the controlled seesaw theorem applies. The higher-order active contribution enters M_{light} additively as $M_L^{[\geq 1]}$, bounded in norm by $\rho_L \|S_N\|$ by NC-2D, hence absorbed into ΔM within the declared bound $\|\Delta M\| \leq (C_\Theta \|\Theta\|^2 + \rho_L)\|S_N\|$, with $C_\Theta = O(1)$ fixed by the block-diagonalisation convention. ■

Appendix C — Functional-Descent and Ordering Proof

Let $M_D = m_D V^* \hat{d} O^\dagger + E_D$ with $\hat{d} \equiv \mathrm{diag}(d(\kappa_1), d(\kappa_2), d(\kappa_3))$, and $M_R = M_\star O^*(r_0 I + E_R) O^\dagger$ with $E_R = E_R^T$, where V unitarily diagonalises $\mathcal{K}_\nu$ (eigenvalues pairwise distinct), $d > 0$, $d' > 0$, and E_D, E_R are subordinate. At zeroth order $M_{light}^{[0]} = -(m_D^2/M_\star r_0) V^* \hat{d}^2 V^\dagger$, complex symmetric since $(V^* \hat{d}^2 V^\dagger)^T = V^* \hat{d}^2 V^\dagger$. Hence

$$H^{(0)} = (M_{light}^{[0]})^\dagger M_{light}^{[0]} = (m_D^4/M_\star^2 r_0^2) V \hat{d}^2 (V^T V^*) \hat{d}^2 V^\dagger = (m_D^4/M_\star^2 r_0^2) V \hat{d}^4 V^\dagger = \mu_\nu^2 f_\nu(\mathcal{K}_\nu),$$

using $V^T V^* = (V^\dagger V)^* = I$ and $f_\nu(\kappa) = d(\kappa)^4$. With f_ν strictly increasing, the perturbations E_D, E_R, higher functional terms, the $O(\Theta^2)$ block-diagonalisation remainder and the ρ_L-bounded active piece collect into R_ν. Weyl bounds each eigenvalue shift by $\|R_\nu\|$; the gap condition $2\|R_\nu\| < \mu_\nu^2 g_f$ prevents crossings; Davis–Kahan bounds projector rotation, so each Takagi projector continues uniquely from its pre-spectral P_i^ν.

Overlap conditions (order coincidence, Lemma 1(ii)) — recursive, two stages. The generation chain is built by successive positive rank-one continuations (Substrate-Generated Sequential Transport): the intermediate line is the continuation of the initial line, and the terminal line the continuation beyond it. The overlap argument therefore applies once per stage, and **both** stages are priced — establishing only the terminal stage would order the terminal line against the rest while leaving $\kappa_1 < \kappa_2$ unsupported.

Single-stage bound. Write one continuation update as $\mathcal{K} = K_0 + \eta P\_c$, $\eta > 0$, rank $P\_c = 1$, with base spread $\Delta_0 \equiv \lambda\_max(K_0) - \lambda\_min(K_0)$. If $\eta > \Delta_0$, the top eigenvalue of $\mathcal{K}$ exceeds every eigenvalue of the compression of $\mathcal{K}$ to the orthogonal complement of the continuation line (Weyl plus Cauchy interlacing: $\lambda\_top \geq \lambda\_min(K_0) + \eta > \lambda\_max(K_0) \geq \lambda\_max(\mathcal{K}|\_\{c\perp\})$), and Davis–Kahan with gap $\gamma \geq \eta - \Delta_0$ gives

$$\sin \theta(P\_top , P\_c) \leq \Delta_0 / (\eta - \Delta_0) .$$

Majority support ($\cos^2\theta > ½$) requires $\sin \theta < 1/\surd 2$, i.e.

$$\boldsymbol{\eta > (1 + \surd 2)\ \Delta_0\ ,}$$

which is the constant quoted as the priced return; $\eta > 2\Delta_0$ makes the bound merely nontrivial, not majority-securing.

Stage 1 (initial → intermediate). With base spread $\Delta_0^{(1)}$ the spread of the pre-continuation kernel on the initial sector and update strength η_2, the condition $\boldsymbol{\eta_2 > (1 + \surd 2)\ \Delta_0^{(1)}}$ places the intermediate continuation line on the top eigenvector of the two-address kernel: $\kappa(\text{initial}) < \kappa(\text{intermediate})$.

Stage 2 (→ terminal). With base spread $\Delta_0^{(2)} = \kappa(\text{intermediate}) - \kappa(\text{initial})$ and update strength η_3, the condition $\boldsymbol{\eta_3 > (1 + \surd 2)\ \Delta_0^{(2)}}$ places the terminal continuation line on the top eigenvector of the full kernel: $\kappa(\text{intermediate}) < \kappa(\text{terminal})$.

Both stage conditions are returns the kernel calculation must satisfy — two numbers, not a gesture — and failure of either is the order-coincidence branch of F11. Under them, $\kappa_1 < \kappa_2 < \kappa_3$ in the continuation labelling; the increasing descent and gap bound then give $m(P_1\hat{}\nu) < m(P_2\hat{}\nu) < m(P_3\hat{}\nu)$. ■

Appendix D — Endpoint and Phase-Branch Proof

D.1 Endpoint branch (stand-in)

Initial address 1, intermediate 2, unique terminal continuation 3. Minimal nonlocal closure from initial to terminal is $1\leftrightarrow3$; after charged-lepton anchoring, $e\leftrightarrow\tau$. The reflected $e\leftrightarrow\mu$ branch reaches the continuation only by reversing the orientation or relocating the continuation role. ■

D.2 Half-transport branch (stand-in)

The positively oriented arc from $\pi/2$ to π has midpoint $3\pi/4$; the root $7\pi/4$ lies on the reverse arc; exactly one root per arc, so any arrow-respecting rule selects $3\pi/4$. The residual premise is the arrow itself. ■

D.3 Reflection output (exact)

The $\mu\leftrightarrow\tau$ reflection swaps $|U_{\mu3}|$ and $|U_{\tau3}|$ and conjugates the CP quartet: eτ/forward $\Longrightarrow$ upper octant, $J < 0$; eμ/reverse $\Longrightarrow$ lower octant, $J > 0$. ■

D.4 Certificate invariances (exact)

Δ_{att}^{phys} is defined with every operator on one canonically normalised weak carrier ($\tilde{P}_\alpha = C_W^\dagger P_\alpha C_W$), is invariant under simultaneous unitary carrier changes (Hilbert–Schmidt norm), and is independent of operator decomposition (only H_W^{full} enters). $I_{CP} = \mathrm{Im}\,\mathrm{Tr}([H_\ell^L, H_\nu]^3)$ is basis-invariant; with the declared commutator order and positive ordered products Δ_ℓ, Δ_ν, $J_{inv} = I_{CP}/(6\Delta_\ell\Delta_\nu) = J_{lep}$. ■

Appendix E — Dependency and Source Map

E.1 Dependency chain

three keyed active addresses → [NT] → NC-1: three typed terminal routes → [ledger stand-in / NC-2A arbiter] → non-protecting B−L → [support + generation + scale + suppression stand-ins / NC-2B–D returns] → $(0, M_D, M_R)$, hierarchy, $\rho_L \ll 1$ → [NMCC-1, automatic] → type-I seesaw, 3+3 census, $\Theta\Theta^\dagger$ defect → [pre-spectral labels; alignment + gap / direct spectrum] → normal ordering → [endpoint + arrow / Δ_{att}^{phys}, I_{CP}] → (eτ, upper octant, $J_{lep} < 0$) .

The census node has no incoming edge from any symmetry or mass node: carrier-first.

E.2 Principal inherited papers

SC-1 — The Generation and Species Census Theorem in VERSF. Keyed layers, layer-local terminality, layer-exclusive attachment, no-fourth minimality; source of the inherited equivariance.

SC-2 — The World-to-Framework Occupancy Map for Standard Model Matter in VERSF. Three keyed active neutral addresses; N_R as an extension carrier requiring a declared completion bridge; the induced candidate terminal addresses typed by the generation keys.

The Electroweak Representation and Connection Closure in VERSF. Doublet typing, Higgs-conjugate bridge, minimal neutral singlet representation.

The Substrate Anomaly-Inadmissibility Theorem in VERSF. Persistent record-current closure for gauge/source currents; scope leaving accidental and global currents incompletely interpreted — the reason the ledger argument is a stand-in.

NMCC-1 — The Neutrino Mass and Charge-Conjugation Completion Theorem in VERSF. Nambu operator, Takagi theorem, exact continuous-charge criterion (NC-2A), block taxonomy, gauge admissibility of the blocks, Schur complement, controlled seesaw, rank identities, operator-type distinctions preserved by the Section 2.1 firewall.

CMSY-1 — The Closure-Operator Yukawa Construction Theorem in VERSF. Typed Dirac construction, rank bound, vanishing-projection warning mandating the NC-2B support certificate.

PMNS-1 — The PMNS Closure-Hamiltonian Theorem in VERSF. Relative-frame theorem, exact attachment reflection, Frame–Takagi transfer requirements, CP invariants, quartet-orientation evaluation (NC-3C stand-in).

The Weak-Commitment Closure Kernel in VERSF. Paired weak-neutral core, half-transport phase grammar, octant–phase correlation; source of the frame kernel $\mathcal{K}_\nu$.

The Weak-Commitment Leakage Trace in VERSF. Off-diagonal attachment structure; its decomposition-tagged diagnostics are superseded as certificates by Δ_{att}^{phys} and retained as convention-tagged consistency checks.

Substrate-Generated Sequential Transport in VERSF. Directed generation chain up to an unresolved overall orientation — the precise residue carried by the forward-arrow stand-in.

Operational Geometry / Minimal Hermitian Lift papers. Canonical operational metric and geodesic branch grammar.

Fold, Fact, Planck and Closure / Planck-certification papers. Pre-interface/post-interface scale typing consumed by the NC-2C stand-in.

HR-1 — The Electroweak Vacuum and Higgs-Potential Completion Theorem in VERSF. The closure-interface vacuum v_{cl} consumed by type and cross-sector identity (F17).