The Non-Abelian Bath-Transport Theorem, Continuous Colour Gauge Dynamics, and Full Gauge-Product Closure in VERSF

▲ Programme Milestone — Standard Model Non-Abelian Gauge-Origin Series

Deriving SU(3)_C and SU(2)_L as Shared-Bath Transport Groups, Lifting the k = 3 Colour Class to Gluon Connection, Colour Curvature, and Yang–Mills Dynamics, and Completing the Standard Model Gauge Product SU(3)_C × SU(2)_L × U(1)_Y

Keith Taylor *VERSF Theoretical Physics Programme — Standard Model Non-Abelian Gauge-Origin Series Programme-marker paper — working manuscript*

Successor to *The Electroweak Completion-Interface Theorem in VERSF*, *The Broken Electroweak Phase in VERSF*, *The Electroweak Gauge-Curvature Dynamics in VERSF*, *The Electroweak Representation and Connection Closure in VERSF*, and *The Substrate Anomaly-Inadmissibility Theorem in VERSF*. Draws bath-transport support from *The Bath Criterion* and the Born-rule bath/ledger arc.

Why Particles Hold Different Account Types

Before the formal development, one picture organises everything this paper does.

Think of a particle's internal options as accounts at a bank. A quark is not one account — it is an account *holder*, and its three colour possibilities constitute a single account with three names on it. The question that decides the character of the forces is what kind of account the bank has issued.

Sole accounts. Each holder keeps a separately protected balance. The bank may relabel accounts and turn a phase dial on each one, but weight never flows from one balance into another. Sole accounts are rigid: their only continuous activity is the phase turning on each account individually.

Joint accounts. Only the combined balance is protected. Weight flows freely among the named holders — red into green, green into blue — and no individual name has a guaranteed share. Joint accounts are fluid: they support continuous internal reallocation.

The theorem at the centre of this paper says that the rules for moving weight around inside a joint account **are** a force. Count the independent ways of shuffling a k-name joint account without touching the total, and the answer is the gauge structure SU(k). The Standard Model then reads off as a set of banking arrangements:

- **Quark colour** — a three-name joint account. Three names admit eight independent shuffles: the eight gluons, and the strong force.
- **The weak doublet** — a two-name joint account, the up-type and down-type members sharing one pool. Two names admit three shuffles: the three weak directions.
- **Hypercharge** — sole accounts throughout. Nothing to shuffle; the only activity is the individual phase dial. This is why the abelian force is structurally simple and does not interact with itself.
- **Leptons and colour** — leptons hold no colour account at all. Not a sole account: no account at the colour bank. This is why they are entirely deaf to the strong force.

Even chirality finds a place in the picture: the left-handed electron and its neutrino share a joint weak account, while the right-handed electron holds only sole accounts. Left-handedness is, in this reading, a statement about who was issued the joint account — with the reason for that issuing policy inherited from the chirality programme, not derived here.

The deepest question the paper isolates is then a single question about the bank itself: **does it issue joint accounts, or only sole ones?** A sole-accounts-only bank — the ledger reading — permits swaps and phase turns but never a genuine transfer, and no strong or weak force can arise from its identical-sector classes. A bank that issues joint accounts — the bath reading — produces non-abelian forces automatically. The observed world contains strong and weak forces; nature's bank evidently issues joint accounts. What remains owed, and is recorded as this paper's first debt, is deriving that issuing policy from first principles.

Two honest limits of the analogy. Real joint accounts carry no quantum phases between their holders, and it is precisely the sector-resolved phases that make the shuffling group the full complex SU(k) rather than a thinner real structure — that assumption is named and graded in the premise list. And gluon self-interaction — the transfer rules themselves carrying colour, the bank's own machinery holding a balance — has no everyday banking counterpart; it is where the analogy hands over to the curvature theorem. Which holders exist and how many names each account carries — why three colours, why two weak branches — is likewise not decided by the account mechanism: that census is inherited, and the account structure tells us only what those counts become dynamically.

With the picture in place, the paper proceeds: the account dichotomy is formalised as the ledger/bath conservation fork, the shuffling groups are derived as transport theorems, and the joint accounts of the quark and weak sectors are lifted to the connections, curvatures, and dynamics of the strong and weak forces.

General Reader Summary

The preceding VERSF papers build the electroweak half of the Standard Model — why the weak force acts on left-handed pairs, why hypercharge is required, why the photon survives symmetry breaking, and why the W and Z acquire mass. What remains is the deepest structural question about the forces themselves:

Where does non-abelian gauge structure come from?

The strong force is described by SU(3), the weak force by SU(2). In ordinary particle physics these groups are read off from experiment and postulated. This paper reduces their origin to a single conservation-law question about the substrate — the account question of the preceding section, now stated as bookkeeping. Take k identical sectors holding weight. There are two ways a bookkeeping system can treat them.

Ledger. Each account is preserved separately. The system may swap the accounts around and adjust their phases, but it may never move weight from one account into another. Swap-and-phase is everything a ledger allows.

Bath. Only the *total* is preserved. Weight may flow continuously between the accounts, the way water redistributes between connected vessels. Individual accounts have no protected balance — only the pool does.

The theorem at the centre of this paper says: this bookkeeping choice is exactly the difference between having non-abelian forces and not having them. A ledger of k identical sectors generates only discrete swaps and phase turns — mathematically, a structure far too thin to be a force like the strong interaction. A shared bath of k identical sectors, with continuous transport and quantum phases, generates the full continuous mixing group U(k) — and once the shared overall phase is set aside, its non-abelian content is SU(k).

The Standard Model then falls out as two specialisations of one mechanism:

- quarks carry a threefold internal class, so k = 3 gives **SU(3)_C** — the strong force, with its eight gluon directions;
- the weak branch structure is twofold, so k = 2 gives **SU(2)** — which the inherited chirality results restrict to **SU(2)_L**, the weak force.

The number of copies is supplied by the census programme (why three, why two). The bath theorem supplies what those copies *become* dynamically: geometry proposes k; bath transport disposes SU(k). Local comparison then forces the gauge connections, curvatures, and the familiar Yang–Mills dynamics — including the self-interaction of gluons, which traces to the simple fact that continuous blendings of three sectors do not commute.

The paper is candid about its hinge. The bath reading is not yet derived from first principles — it is the named decision node of the wider programme, the same bath/ledger fork that appears in the Born-rule arc. If the substrate takes the ledger reading, this route to the strong and weak forces fails, and the paper says so explicitly. That is a feature: the origin of non-abelian gauge structure has been reduced to one precise, falsifiable substrate decision.

In plain language: **identical sectors sharing one bath become continuously mixable; continuous mixing of k sectors is SU(k); three colours give the strong force, two weak branches give the weak force; and the full Standard Model gauge architecture closes.**

Contents

Why Particles Hold Different Account Types General Reader Summary Abstract

1. Notation, Conventions, and Firewalls 0′. Predictive-Content Ledger 0″. Inherited Infrastructure 0‴. Bath-Transport Support Layer
2. Purpose and Claim Level
3. Non-Abelian Bath-Transport Premises
4. The Non-Abelian Bath-Transport Theorem
5. Census versus Mechanism
6. Standard Model Specialisations — Colour ($k = 3$) and Weak ($k = 2$)
7. Why C_3 Support Is Not Yet SU(3)_C
8. Colour Gauge Dynamics from the $k = 3$ Bath Class
9. Colour Curvature and the Bianchi Identity
10. Colour Kinetic Form
11. Colour Currents and Quark Coupling
12. Colour Anomaly Closure
13. Colour Singlets — Leptons and the Completion Interface
14. Why Electroweak Breaking Does Not Break Colour
15. Weak Gauge Structure as the $k = 2$ Bath Class
16. Full Gauge-Product Closure
17. Ledger Branch and the Non-Abelian Kill Condition
18. Fourteen-Generator Firewall
19. Relation to the Existing Quark-Sector and Flavour Programme
20. What Has Actually Been Derived
21. Open Debts
22. Falsification Conditions
23. Milestone Statement
24. Conclusion

Appendix A — SU(3) Generator Conventions Appendix B — Colour Singlet Audits Appendix C — Excluded Alternatives Appendix D — Claim-Status Ledger Appendix E — Global-Quotient Audit: $\mathbb{Z}_6$ Centre Action and Charge–Triality

---

Abstract

The VERSF Standard Model programme derives the chiral matter skeleton, the electroweak representation algebra $\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$, the local W/B connection, anomaly-admissible source ledgers, unbroken electroweak gauge-curvature dynamics, the broken electroweak phase, and the universal completion interface

$$\Phi_{cl} \sim (1, 2, +½).$$

The present paper supplies the non-abelian gauge-origin layer. The central claim is that non-abelian gauge structure arises when identical closure sectors do **not** retain separate conserved ledgers but share a common transport bath. Under ledger transport, each sector keeps its own account; admissible transformations on the class carrier $\mathscr{H}_k \cong \mathbb{C}^k$ are confined to the monomial floor

$$T^k \rtimes S_k$$

— phases and permutations, with abelian connected component T^k and no continuous off-diagonal mixing. Under bath transport, only the total class weight

$$W_{\mathcal{C}} = \Sigma_i |w_i|^2$$

is conserved; weight flows continuously between indistinguishable sectors, and that flow, together with sector-resolved phases and bath saturation, generates the full class-transport group — with relabelling covariance obtained as a consequence, not assumed. The Non-Abelian Bath-Transport Theorem states:

identical closure class of size k + shared bath transport + continuity + sector phase ⟹ U(k),

with the bath taken saturated — total class weight the only protected sectoral quantity, so no protected sub-account survives — and with non-abelian content SU(k) after removal of the shared abelian phase; no proper closed subgroup suffices. The Standard Model specialisations follow from the inherited census:

$$k = 3 \Longrightarrow SU(3)_C, \quad k = 2 \Longrightarrow SU(2) \rightarrow SU(2)_L \text{ (chirality inherited).}$$

Local distinguishability conservation lifts these class-transport groups to gauge connections. For colour, the connection is

$$A^C_\mu = g_s G^a_\mu T_a, \quad a = 1, \ldots, 8,$$

with covariant derivative $D^C_\mu = \partial_\mu + i g_s G^a_\mu T_a$ and curvature

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu,$$

whose non-abelian self-term is the curvature of bath-transport colour comparison, not an insertion. The leading admissible kinetic form is $\mathcal{L}_C = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$, sourced by quark colour currents only. The pure $SU(3)_C^3$ and mixed $SU(3)_C^2$–$U(1)_Y$ anomalies vanish on the inherited ledger; the completion interface is colourless, so electroweak condensation leaves the gluons massless. With abelian uniqueness supplying a single $U(1)_Y$, the full gauge product closes:

$$SU(3)_C \times SU(2)_L \times U(1)_Y,\ \dim = 8 + 3 + 1 = 12,$$

at the level of the local gauge algebra and product connection; the possible global quotient by a shared discrete centre $\Gamma \subseteq \mathbb{Z}_6$ is a downstream global-structure question, not decided here. The bath premise is not derived here; it remains the programme-critical decision node, and the ledger branch is stated as an explicit kill condition: a ledger substrate yields swap-and-phase only and no non-abelian gauge origin by this route. The paper likewise does not derive confinement, asymptotic freedom, the QCD scale, the hadron spectrum, the value or running of g_s, strong CP structure, chirality, or the census values themselves. It derives the conservation-law origin of non-abelian gauge structure and the leading classical dynamics of its $k = 3$ realisation.

0. Notation, Conventions, and Firewalls

0.1 Notation

A class of k indistinguishable closure sectors is carried by

$$\mathscr{H}_k \cong \mathbb{C}^k,$$

with state weights $w = (w_1, \ldots, w_k)$ and total class weight

$$W_{\mathcal{C}} = \Sigma_{i=1}^k |w_i|^2.$$

The **monomial floor** is the group of unitaries with exactly one nonzero unit-modulus entry per row and column,

$$T^k \rtimes S_k,$$

where T^k is the diagonal phase torus and S_k the sector permutations.

The colour specialisation uses $k = 3$ with carrier

$$\mathcal{C} \cong \mathbb{C}^3,$$

colour index q^α, $\alpha = 1, 2, 3$, and generators

$T_a = \lambda_a/2$, $a = 1, \ldots, 8$, $\mathrm{tr}(T_a T_b) = \frac{1}{2} \delta_{ab}$, $[T_a, T_b] = i f^{abc} T_c$,

with λ_a the Gell-Mann matrices and f^{abc} the totally antisymmetric SU(3) structure constants.

The colour connection, covariant derivative, and curvature are

$A^C_\mu = g_s G^a_\mu T_a$, $D^C_\mu = \partial_\mu + i g_s G^a_\mu T_a$, $[D^C_\mu, D^C_\nu] = i F^C_{\mu\nu}$,
$F^C_{\mu\nu} = g_s G^a_{\mu\nu} T_a$, $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu$.

With the opposite sign convention $D_\mu = \partial_\mu - i A_\mu$, the non-abelian self-term flips sign and the physics is unchanged. All continuum statements refer to the emergent 4-manifold of the record layer; no background temporal substance is assumed.

0.2 Representation firewall

This paper does not rederive the chiral matter skeleton. It inherits the quark sectors $Q_L = (u_L, d_L)$, u_R, d_R and the lepton sectors $L_L = (\nu_L, e_L)$, e_R.

0.3 Electroweak firewall

This paper does not rederive $SU(2)_L \times U(1)_Y$ as a representation result, the W/B connection, electroweak gauge curvature, the broken phase, photon emergence, W/Z masses, or the completion interface. Those are inherited. What this paper adds on the weak side is the *transport origin* of the SU(2) factor — why a twofold class mixes continuously at all.

0.4 Chirality firewall

The bath theorem yields SU(2) for a twofold class. It does not derive left-handedness. The selection $SU(2) \to SU(2)_L$ is inherited from the electroweak representation/chirality programme and is not rederived here.

0.5 Abelian firewall

The shared central phase removed from U(k) is not a new independent abelian gauge factor. The single $U(1)_Y$ is inherited from the abelian-uniqueness and hypercharge programme.

0.6 Bath-premise firewall

The bath reading (NAB-4) is used, not derived. It is the named decision node of the wider programme — the same bath/ledger fork that appears in the Born-rule arc. Section 16 states the opposite branch as an explicit kill condition.

0.7 QCD phenomenology firewall

This paper derives the colour gauge architecture and leading classical colour-curvature dynamics. It does **not** derive: confinement; asymptotic freedom; the QCD beta function; Λ_QCD; hadron masses; chiral symmetry breaking; the pion sector; the gluon condensate; the θ_QCD angle; the strong CP problem; lattice dynamics; or the nonperturbative vacuum.

0.8 Coupling firewall

The paper derives where g_s enters. It does not compute its numerical value or running.

0.9 Census firewall

The class sizes k = 3 (colour) and k = 2 (weak) are inherited census results from the internal-symmetry and closure-geometry programme. The transport theorem does not determine k.

0.10 Fourteen-generator firewall

The Standard Model gauge-generator count derived here is 8 + 3 + 1 = 12. The separate fourteen-generator (K = 7) closure-response census belongs to closure architecture and dihedral transport, not to gauge-boson enumeration. See §17.

0.11 Global gauge-group firewall

This paper derives the local gauge algebra 𝔤_SM = 𝔰𝔲(3)_C ⊕ 𝔰𝔲(2)_L ⊕ 𝔲(1)_Y and the product connection. The **global** gauge group may be a quotient of the naive product by a shared discrete centre,

$[SU(3) \times SU(2) \times U(1)] / \Gamma, \Gamma \subseteq \mathbb{Z}_6$,

with the correct Γ determined by which representations of the product act faithfully on the physical matter content. Every local statement in this paper — connections, curvatures, currents, anomaly audits, the broken phase — is insensitive to Γ. The determination of the global quotient is a downstream global-structure question and is not decided here; Appendix E supplies the centre-action audit on the derived ledger, showing $\mathbb{Z}_6$ acts trivially and is therefore the maximal faithful candidate. The word "product" throughout refers to the Lie-algebra direct sum and the product connection, not to a claim about global topology. One Γ-sensitive family deserves explicit note: θ-angle periodicity and the classification of instanton sectors depend on the global topology of the gauge group and hence on Γ — harmless here, because the entire θ/strong-CP layer is already firewalled to §0.7 and Debt 5, and no statement in this paper touches it.

0′. Predictive-Content Ledger

Object	Grade	Status
Ledger transport = monomial floor T^k ⋊ S_k	Derived / Exact	Theorem 1
No continuous non-abelian mixing on ledger branch	Derived / Exact	Corollary 1.1
Anonymous ledgers are ledgers	Derived	Remark 1.2 — any symmetric differentiable invariant collapses to the torus
Invariant-arity completeness	Derived	Corollary 3.0′ — no third escape route at any arity
Bath transport forces off-diagonal generator	Conditional theorem	Theorem 2 — requires bath + continuity
Bath + torus + saturation gives U(k)	Conditional theorem	Theorem 3 via Lemma 3.0 — covariance a consequence
Block-partition lemma	Exact	Lemma 3.0
Central-phase detachment	Derived here	Proposition 3.3
Non-abelian content SU(k)	Exact given U(k)	Shared phase removed — Proposition 3.3
Colour census k = 3	Inherited	Three-body singlet / minimal internal symmetry — strongly supported
Weak census k = 2	Inherited	Weak-branch architecture
k = 3 ⟹ SU(3)_C	Conditional on census + bath	Corollary 3.1
k = 2 ⟹ SU(2)	Conditional on census + bath	Corollary 3.2
SU(2) → SU(2)_L	Inherited	Chirality selection
Single U(1)_Y	Inherited	Abelian uniqueness
Gauge-connection necessity	Inherited	Distinguishability conservation — strongly supported
Colour connection A^C_μ	Derived here	Theorem 4
Colour curvature G^a_μν	Exact given connection	Theorem 5
Colour Bianchi identity	Exact	Corollary 5.1
Colour kinetic form $-\frac{1}{4}G^2$	Conditional	Curvature-norm admissibility — Theorem 6
Quark colour currents	Derived here	Theorem 7
Pure $SU(3)^3$ anomaly closure	Derived / audit	Proposition 8a
Mixed $SU(3)^2$–U(1)_Y anomaly closure	Derived / audit	Proposition 8b
Lepton / interface colourlessness	Inherited / audited	Theorem 8 — audit
Colour unbroken under electroweak breaking	Derived	Theorem 9

Object	Grade	Status
Full gauge-product closure	Conditional synthesis	Theorem 10
Bath premise (NAB-4)	Open / programme-critical	Must be discharged upstream
Bath saturation (NAB-4′)	Conditional	Load-bearing for uniqueness — conservation-residue scope
Global gauge quotient $\Gamma \subseteq \mathbb{Z}_6$	Downstream / audited	$\mathbb{Z}_6$ trivial on ledger — Appendix E
Charge–triality correlation $6Y \equiv 4t + 3d$	Derived / audit	Corollary E.1
Confinement, Λ_{QCD}, hadron spectrum	Owed	Not derived
g_s value / running	Owed	Not derived
Strong CP / θ sector	Owed	Not derived
Fractional charge carrier	Downstream	Carrier-theorem paper
Quark mass hierarchy	Downstream	Assignment/refinement papers
Fourteen-generator relation	Firewalled	Closure architecture, not gauge count

0″. Inherited Infrastructure

0″.1 Threefold support and the n = 3 census anchor

Stable three-body confined structures select a three-dimensional internal support module: $n = 3$ is the unique minimal setting in which a totally antisymmetric three-body singlet exists,

$$3 \wedge 3 \wedge 3 \supset 1, \; \varepsilon_{\alpha\beta\gamma} q^{\alpha} q^{\beta} q^{\gamma},$$

since $\Lambda^3(\mathbb{C}^n)$ is one-dimensional precisely when $n = 3$. This is the census source of $k = 3$ for colour.

0″.2 Gauge-connection necessity

The Distinguishability Conservation programme proves that local comparison of internal quantum states across the continuum requires a connection. Fixed global labels are not physical; only locally transported relations are. This lifts class-transport groups to gauge fields.

0″.3 Product-structure discipline

The gauge-product structure SU(3) × SU(2) × U(1) is inherited at the level of commuting generator algebras and factorised internal distinguishability directions. Quarks carry colour, weak, and hypercharge simultaneously because they live in product representations.

0″.4 Electroweak completion interface

The Electroweak Completion-Interface Theorem derives Φ_cl ~ (1, 2, +½) with no colour support. Electroweak condensation therefore acts on weak and hypercharge directions and cannot break colour.

0″.5 Closure-architecture firewall

The fourteen-generator (K = 7) closure-response structure is inherited as closure architecture, not as gauge algebra.

0‴. Bath-Transport Support Layer

The present paper additionally inherits a bath-transport support layer from the Bath Criterion, the Born-rule bath/ledger arc, and the commitment-bath and spectral-density papers. That layer contributes four facts.

First — the bath/ledger distinction is precise. A ledger preserves each sector's weight account separately. A bath preserves only the total class weight, allowing weight to move between sectors. This is a conservation-law dichotomy, not a metaphor: the spectral-density results treat the bath as a calculable substrate structure with definite transport properties.

Second — ledger transport is mathematically the swap-and-phase floor. On a class carrier ℂ^k, transformations preserving the sector-weight multiset of every state are exactly the monomial unitaries T^k ⋊ S_k — phases plus permutations. This is the normaliser floor, not full non-abelian mixing.

Third — bath transport supplies the missing non-abelian element. If only the total class weight is protected, and transport is continuous, some admissible transformation redistributes weight between sectors. Such a transformation lies outside the monomial floor. With sector phases present, one blend spreads through the class and generates U(k), with SU(k) as the non-abelian structure beyond the shared phase.

Fourth — the fork is not invented for gauge theory. The same bath/ledger decision node appears in the Born-rule arc. The gauge question therefore introduces no new ad hoc device; it reuses the substrate fork already identified in the probability and commitment-dynamics programme.

This permits a clean three-layer separation:

1. **Census layer** — which multiplicities k exist;
2. **Transport layer** — whether those multiplicities remain ledgered or form a shared bath;
3. **Gauge layer** — the local connection and curvature forced when bath transport is made local.

Colour is the k = 3 specialisation of this architecture. The weak factor is the k = 2 specialisation, with chirality inherited.

A scope note on two neighbouring papers: the Carrier Theorem is treated as *downstream* support for fractional quark charge, not as a load-bearing colour premise; the quark mass-hierarchy and assignment/refinement results are likewise downstream and enter only as future integration (§18, §20).

---

1. Purpose and Claim Level

The purpose of this paper is to derive the origin of non-abelian gauge structure and to complete the Standard Model gauge product.

The target chain is

identical closure class (size k) → bath transport → continuous class mixing → U(k) → SU(k) → local connection → curvature → Yang–Mills dynamics,

specialised as

k = 3 → SU(3)_C (colour, eight gluons), k = 2 → SU(2) → SU(2)_L (weak, chirality inherited),

and closed as

SU(3)_C × SU(2)_L × U(1)_Y.

Central Claim

Non-abelian gauge structure is the continuous transport structure of a shared bath of indistinguishable closure sectors. A ledger substrate admits only swap-and-phase transport and yields no non-abelian gauge origin by this route. A bath substrate, with continuity and sector phase, yields U(k) class transport, whose non-abelian content SU(k) lifts under local comparison to a gauge connection with curvature and leading Yang–Mills dynamics.

Claim level

The derivation carries four grades.

First — the ledger characterisation (Theorem 1) is exact: a translation lemma between conservation structure and group structure.

Second — the bath consequences (Theorems 2–3) are conditional theorems: exact given the bath premise NAB-4 with its saturation clause NAB-4′, continuity NAB-5, and sector phase NAB-6. The bath premise itself is open and programme-critical.

Third — the specialisations are conditional on the inherited census ($k = 3$ colour, $k = 2$ weak) and, for the weak factor, on inherited chirality.

Fourth — the connection and curvature are exact given gauge-connection necessity; the kinetic form is conditional on the curvature-norm admissibility principle already deployed in the electroweak gauge-curvature paper.

---

2. Non-Abelian Bath-Transport Premises

NAB-1 — Identical closure class

There exists a class of k indistinguishable closure sectors, represented by a carrier $\mathscr{H}_k \cong \mathbb{C}^k$.

Non-circularity note. The indistinguishability of the sectors is inherited from the closure-census programme — the sectors are identical as closure sectors, prior to and independently of any transport structure. It is not inferred from the SU(k) symmetry this paper derives; the would-be circle bath ⟹ SU(k) ⟹ sectors equivalent ⟹ bath never closes, because the equivalence enters upstream as census input, not downstream as a symmetry consequence.

Status: Inherited / census-dependent. *Falsifier:* no such identical-sector class exists at the relevant layer.

NAB-2 — Unitary class transport

Admissible reversible transport on the class carrier preserves total class weight $W_\mathcal{C} = \Sigma_i |w_i|^2$.

Status: Inherited from the transport and Born-measure reconstruction programme. *Falsifier:* class transport is not unitary or does not preserve total class weight.

NAB-3 — Ledger/bath dichotomy

Either each sector-weight account is separately preserved — the ledger reading — or only the class total is preserved — the bath reading.

Status: Inherited structural dichotomy. *Falsifier:* a third conservation structure exists, neither ledger nor bath, that still supports the required transport.

NAB-4 — Bath premise

At the non-abelian gauge-origin layer, identical closure sectors share one transport bath: total class weight is conserved, individual sector weights are reallocable.

Status: **Conditional / programme-critical.** *Falsifier:* the substrate takes the ledger reading for identical closure classes.

NAB-4′ — Bath saturation (no protected sub-account)

Total class weight is the **only** protected weight quantity. No sectoral sub-account is separately conserved by admissible transport: no individual sector weight $|w_i|^2$, no sub-total $\Sigma\_\{i\in b\}\ |w_i|^2$ over a proper subset b of sectors, no directional account $|\langle e, w\rangle|^2$ for a distinguished vector e, no frozen inter-sector phase relation, and no anonymous sectoral invariant — no symmetric function of the sector weights beyond the class norm, such as $\Sigma_i |w_i|^4$ (Remark 1.2). Any such quantity would constitute a residual ledger hidden inside the bath, and the bath reading excludes it: a bath with a protected sub-account is not one bath but two.

Scope. This premise governs conservation residues — protected weight quantities — only. Exclusions that are not weight quantities in any bookkeeping sense, such as invariant real structures or symplectic tensors, are not its business: those are owned by NAB-6 through the rank argument of Corollary 3.0. Each exclusion in this paper has exactly one owner. *Status:* Conditional — the saturation clause of NAB-4, stated separately because it is load-bearing for the uniqueness of the transport group (Theorem 3, Corollary 3.0). For the quantities within its scope, it is the requirement that NAB-4 be taken exactly, without residue. *Falsifier:* admissible transport conserves a sectoral weight quantity beyond the class total; by Lemma 3.0 the transport algebra is then the block or stabiliser algebra of that quantity rather than the full class algebra.

NAB-5 — Continuity of transport

Bath transport is continuous in the pre-commitment regime. Weight reallocation produces a continuous generator, not merely a discrete jump.

Status: Conditional / inherited from the bath and Born-rule arc. *Falsifier:* weight reallocation exists only discretely and does not generate continuous transport.

NAB-6 — Sector phase

The class carrier admits sector-resolved relative phase structure. This is required for complex U(k) rather than a thinner real mixing group.

Support. This premise is not free-standing; it is inherited from the quantum-reconstruction layer of the programme. The class carrier is a complex Hilbert module because interference among distinguishable support alternatives is physical and norm-preserving reversible composition on a real carrier cannot reproduce it — the same result that fixes complex amplitudes in the Born-rule arc. On a complex carrier, the relative phase between two sector directions is an observable-in-principle quantity (witnessed by interference between the sectors), so admissible transport must include the diagonal phase torus T^k that reparametrises those phases. The premise therefore reduces to the claim that the sectors of a bath class are genuine quantum support alternatives — which is what NAB-1 asserts them to be. A consistency cross-check runs in the other direction: the matter skeleton requires complex fundamental representations (a 3 of colour, a 2 of weak), and a real mixing group of orthogonal type admits no such complex fundamentals — so a failure of NAB-6 would already have collided with the inherited representation programme (Appendix C.7).

Status: Inherited / conditional. *Falsifier:* relative phases are not sector-resolved; the result falls back to real mixing rather than full complex SU(k).

NAB-7 — Census specialisation

The Standard Model gauge factors use class sizes k = 3 for colour and k = 2 for weak structure.

Status: Inherited from the internal-symmetry / closure-census programme. *Falsifier:* the colour or weak census is not 3 or 2, respectively.

NAB-8 — Abelian uniqueness

The shared central U(1) phase is not counted as a new non-abelian gauge factor. The independent abelian sector is already accounted for by the U(1)_Y / phase-hypercharge programme.

Status: Inherited from the abelian-uniqueness / hypercharge programme. *Falsifier:* a second independent fundamental abelian gauge factor is required.

NAB-9 — Anomaly-admissible ledgers

Whatever gauge structure the bath layer produces must sit on anomaly-admissible source ledgers.

Status: Inherited from the Substrate Anomaly-Inadmissibility Theorem. *Falsifier:* an anomalous source sector persists physically.

---

3. The Non-Abelian Bath-Transport Theorem

Theorem 1 — Ledger Transport Gives Only the Monomial Floor

Let $\mathscr{H}_k \cong \mathbb{C}^k$ carry a class of k indistinguishable closure sectors. If admissible transport preserves the sector-weight multiset

$\{|w_1|^2, \ldots, |w_k|^2\}$

for every state, then every admissible unitary is monomial, and the full ledger group is

$T^k \rtimes S_k$.

Its connected component is the abelian torus T^k. There is no continuous off-diagonal non-abelian mixing.

Proof

Let U be admissible. The basis vector e_i has weight multiset $\{1, 0, \ldots, 0\}$; its image $U e_i$ must have the same multiset, hence all its weight in a single sector:

$e_i \mapsto e^{i\theta_i} e_{\pi(i)}$

for some phase θ_i and some assignment π, which unitarity forces to be a permutation. U is therefore monomial — exactly one nonzero unit-modulus entry per row and column — and the monomial unitaries form $T^k \rtimes S_k$. The identity component of this group is T^k, which is abelian and diagonal. ■

Corollary 1.1 — Strict-ledger refinement

If the ledger is strict — each sector weight $|w_i|^2$ preserved individually, with no indistinguishability swaps — the admissible group is only the diagonal torus T^k. Indistinguishability of sectors restores the permutations, giving the full monomial floor. In neither reading does a continuous off-diagonal generator exist.

Remark 1.2 — Anonymous ledgers are ledgers

A critic might concede labelled accounts and retreat to an anonymous conserved quantity — a symmetric function of the sector weights carrying no sector labels, such as $\Sigma_i |w_i|^4$. The retreat fails twice over. Conceptually, any such quantity is defined only relative to the sector projector family $\{P_i\}$: symmetry in the labels does not remove the dependence on the frame that produces the $|w_i|^2$, so its conservation is the conservation of a frame-anchored structure — a hidden projector ledger. Mechanically, the exclusion holds for every conserved symmetric differentiable function of the sector weights, not merely polynomials: for $P(|w_1|^2, \ldots, |w_k|^2)$, the first-order drift under an (i, j) blend generator is $(\partial_i P - \partial_j P) \cdot 2\,\mathrm{Re}(\bar{w}_i w_j)$; vanishing on all states forces $\partial_i P \equiv \partial_j P$ identically, symmetry propagates the identity to every pair, so ∇P has equal components everywhere and P is a function of the class norm alone. Any symmetric conserved quantity beyond the class norm therefore excludes every blend — the quartic $\Sigma_i |w_i|^4$, with drift $2(|w_i|^2 - |w_j|^2) \cdot 2\,\mathrm{Re}(\bar{w}_i w_j)$, is the simplest instance — and the connected stabiliser collapses to the torus: the anonymous ledger reinstates the monomial floor of Theorem 1. Anonymous or labelled, a ledger is a ledger, and NAB-4′ excludes both.

Reading

A ledger can swap sectors and rotate their phases. It cannot blend them continuously — and dressing the accounts anonymously does not change this. Ledger conservation is structurally incapable of generating non-abelian force structure.

Theorem 2 — Shared Bath Transport Forces Continuous Mixing

Let $\mathcal{H}_k$ satisfy the bath reading: only the total $W_{\mathcal{C}}$ is conserved, and at least one admissible process reallocates weight between sectors. If at least one such weight-reallocating transformation is reachable from the identity along a continuous path of admissible transformations (NAB-5), then the Lie algebra $\mathfrak{g}$ of the identity component G_0 of the admissible group contains a non-diagonal generator.

Proof

By the bath reading there exists an admissible U, reachable from the identity by a continuous admissible path, and a state w whose sector-weight multiset changes under U. By Theorem 1, any unitary preserving every weight multiset is monomial; hence U is not monomial. The path from the identity to U lies in G_0. Suppose $G_0 \subseteq T^k \rtimes S_k$; then G_0, being connected and containing the identity, lies in the identity component of the monomial group, which is the diagonal torus T^k — contradicting $U \in G_0$, since U is not monomial. Hence $G_0 \not\subset T^k \rtimes S_k$, and since G_0 is a connected Lie group not contained in the monomial floor, $\mathfrak{g}$ contains a generator with a nonvanishing off-diagonal component — one that transfers weight between at least two sector directions. ■

Scope note

Only the reallocating process itself is required to be continuously reachable from the identity; no assumption is made that the full admissible set is path-connected. All subsequent statements about "the transport group" refer to the identity component G_0. The theorem concerns conserved accounts assignable to a single class state w on the pure class carrier; multi-copy invariants, density-matrix invariants, and relational invariants involving external systems are not addressed here and belong to a separate composite-state audit. By Corollary 3.0′, the deferral is harmless: no invariant of any arity can lower the transport group below $U(k)$ without violating NAB-4′ or NAB-6, so that audit is confirmatory, not load-bearing.

Reading

The bath reading is exactly the condition that turns identical copies from a swap-and-phase list into a continuously mixable class.

Lemma 3.0 — Block-Partition Lemma

Let $\mathfrak{g} \subseteq \mathfrak{u}(k)$ be a Lie subalgebra containing the full diagonal torus $\mathfrak{t}^k$. Then $\mathfrak{g}$ is the block algebra of a partition of the k sectors,

$$\mathfrak{g} = \bigoplus_b \mathfrak{u}(k_b),$$

equivalently $\mathfrak{g} = \mathfrak{t}^k \oplus \bigoplus P_{ij}$ over the pairs (i, j) lying in a common block, where $P_{ij} = \mathrm{span}\{i(E_{ij} + E_{ji}), (E_{ij} - E_{ji})\}$ is the (i, j) root plane. If the partition is proper, every block sub-total

$$W_b = \Sigma_{i \in b} |w_i|^2$$

is conserved by the transport $\mathfrak{g}$ generates.

Proof

Since $\mathfrak{t}^k \subseteq \mathfrak{g}$ and $\mathfrak{g}$ is closed under brackets, $\mathfrak{g}$ is invariant under $\mathrm{ad}(\mathfrak{t}^k)$. The adjoint torus action decomposes $\mathfrak{u}(k)$ into $\mathfrak{t}^k$ plus the root planes P_{ij}, and the planes carry pairwise-distinct torus characters: the element with diagonal phases $(\theta_1, \ldots, \theta_k)$ acts on P_{ij} as a rotation by $\theta_i - \theta_j$, and these functionals differ for distinct pairs. By complete reducibility of the compact torus action, any $\mathrm{ad}(\mathfrak{t}^k)$-invariant subspace therefore decomposes as the direct sum of its intersections with $\mathfrak{t}^k$ and with the individual planes — no cross-plane component survives invariance, so a vector lying in several planes lands each component in $\mathfrak{g}$ individually. Each plane is irreducible over $\mathbb{R}$ (the rotation by $\theta_i - \theta_j$ sweeps it), so each intersection is zero or the whole plane. Hence $\mathfrak{g} = \mathfrak{t}^k \oplus \bigoplus_{(i,j)\in S} P_{ij}$ for some set S of pairs. Bracket closure makes S transitive: $[E_{ij} - E_{ji}, E_{jl} - E_{lj}] = E_{il} - E_{li}$, a nonzero element of P_{il}, so $(i, j), (j, l) \in S$ forces $(i, l) \in S$. With symmetry automatic, S is the same-block relation of a partition of the sectors, and $\mathfrak{g}$ is the corresponding block algebra. Exponentiating a block algebra gives block-diagonal unitaries, which preserve the norm of each block: every W_b is conserved. ■

Corollary 3.0′ — Invariant-Arity Completeness

Whatever conserved quantity a substrate might protect — single-copy, multi-copy, density-matrix, or relational — its effect on the connected transport group G_0 is exhausted by two cases. If the Lie algebra of G_0 contains the full torus $\mathfrak{t}^k$, Lemma 3.0 forces it to be a block-partition algebra, and every proper block conserves single-copy sub-totals — caught by NAB-4′. If it does not contain the full torus, NAB-6 is violated directly. No conserved quantity of any arity opens a third route below U(k); the composite-state audit deferred in Theorem 2's scope note is therefore confirmatory, not load-bearing.

Theorem 3 — Bath Transport Generates U(k), and SU(k) Beyond the Shared Phase

Given (1) a class of k indistinguishable closure sectors (NAB-1); (2) shared bath transport (NAB-4) taken saturated (NAB-4′); (3) continuous transport (NAB-5); (4) sector-resolved relative phases (NAB-6) — the connected admissible class-transport group is

U(k),

and factoring out the shared central phase leaves the non-abelian structure

SU(k).

Relabelling covariance is not assumed: it is a consequence, since 𝔲(k) is invariant under sector permutations.

Proof

By NAB-6 the diagonal torus t^k lies in the transport algebra 𝔤 — a withheld diagonal direction would be a frozen inter-sector phase relation, a protected sectoral quantity excluded by NAB-4′ — and by NAB-2, 𝔤 ⊆ 𝔲(k). Lemma 3.0 therefore applies: 𝔤 is the block algebra of some partition of the sectors, and every proper partition conserves its block sub-totals. By Theorem 2 (from NAB-4 and NAB-5), 𝔤 contains an off-diagonal generator, so at least one block contains two or more sectors. If the partition were proper, some W_b over a proper subset b would be conserved — a protected sub-account beyond the class total, violating NAB-4′. Hence the partition is the single block and 𝔤 = 𝔲(k): the connected transport group is U(k). The centre acts as a shared scalar phase and distinguishes no sector direction; by Proposition 3.3 it generates no comparison obligation of its own, and its gauge realisation is the inherited abelian layer (NAB-8). Removing it leaves 𝔰𝔲(k), whose simply connected group is SU(k). ■

Remark — constructive generation

The block mechanics can also be exhibited by hand: a blend's (i, j) component spans the root plane P_ij under the torus sweep of Lemma 3.0, brackets [P_ij, P_jl] chain the blend outward pair by pair, and adjacent 𝔰𝔲(2) blocks close to 𝔰𝔲(k). Appendix A displays the k = 3 case in the Gell-Mann basis. The lemma supersedes this construction as proof — in particular, extracting a blend's root components requires no separate justification, being the ad(t^k) decomposition itself — but the construction remains the clearest picture of how one blend becomes eight gluon directions.

Discipline note

The natural smuggling objection — that full diagonal phase freedom and full relabelling covariance would assume most of U(k) — is not answerable here but unformulable: no permutation covariance appears anywhere in the premises or the proof. The inputs are exactly four, each with one owner: unitarity (NAB-2); one continuous blend (Theorem 2, from NAB-4 + NAB-5); the phase torus (NAB-6); and no protected sub-account (NAB-4′). The narrowing of NAB-4′ to conservation residues also keeps genuine distance between premise and conclusion: the premise forbids protected weight quantities, and the theorem still does work — through the block-partition structure — to reach U(k). A substrate protecting a hidden sub-account escapes the conclusion by failing NAB-4′, not by evading the proof, and its escape route is exhibited concretely by the lemma: it is the block algebra of the protected partition. The anonymous variant of the objection — conserving an unlabelled symmetric function of the weights — is closed separately by Remark 1.2.

Reading

This is the central result. Non-abelian structure is not selected from a catalogue of observed gauge groups. It is the transport group of a shared bath of indistinguishable closure sectors — a conservation-law consequence, conditional on one named substrate decision.

Corollary 3.0 — No Proper Subgroup Suffices, One Owner per Exclusion

The transport group cannot be a proper closed connected subgroup of SU(k), and each exclusion has a single named owner. Subgroups whose algebra contains the torus are block algebras (Lemma 3.0): every proper block partition conserves its sub-totals, and the invariant-line stabiliser S(U(1) × U(k−1)) is the two-block case, conserving the directional account $|\langle e, w\rangle|^2$ — all excluded by NAB-4′. Subgroups whose algebra does not contain the torus — the real forms and the symplectic stabilisers — are excluded by NAB-6 on rank grounds: rank $\mathfrak{so}(k) = \lfloor k/2 \rfloor < k$ and rank $\mathfrak{sp}(k/2) = k/2 < k$, so no stabiliser of a real or symplectic form can contain the full torus $\mathfrak{t}^k$; their invariant tensors are not weight quantities and are not NAB-4″s business. Finite subgroups — including C_k and the centre $\mathbb{Z}_k$ — supply no continuous transport and are excluded by NAB-5. Nothing smaller survives; nothing larger than U(k) is available by NAB-2.

Proposition 3.3 — Central-Phase Detachment

The centre of U(k) generates no class-specific comparison obligation, and the gauged non-abelian content of bath transport is SU(k).

Proof

A central element $e^{i\theta}\mathbb{1}$ acts identically on every sector: every inter-sector weight ratio, every relative phase, and every sectoral distinguishability relation on the class carrier is invariant under it. The gauge-connection necessity theorem (§0″.2) obligates local transport structure precisely for comparisons of distinguishability across the record layer; a transformation that alters no sectoral distinguishability relation creates no such comparison to transport, hence no class-specific connection. The central phase is instead an instance of the universal abelian phase grade, whose unique gauge realisation is the inherited $U(1)_Y$ layer (NAB-8) — a second abelian connection duplicating it is excluded there. The transformations that do alter sectoral distinguishability relations, and therefore do generate a comparison obligation, are exactly those acting nontrivially on the projective class structure — U(k) modulo its centre, realised on the matter representations through SU(k). ■

Reading

This supplies mechanism, not merely authority, for gauging SU(k) rather than U(k). The passage U(k) → SU(k) discards nothing by fiat: the discarded centre carries no inter-sector comparison content, and its gauge realisation already exists as the abelian layer. Appendix C.4 applies this at k = 3.

---

4. Census versus Mechanism

The theorem does not determine k. It says:

shared bath class of size k ⟹ SU(k) non-abelian structure.

The census programme determines which k values occur; the bath-transport theorem determines what those k-classes become dynamically. In slogan form:

geometry/census proposes k; bath transport disposes SU(k).

For the Standard Model: k = 3 is supplied by the threefold colour-support / baryonic-singlet architecture (§0″.1), yielding SU(3)_C; k = 2 is supplied by the twofold weak-branch architecture, yielding SU(2). The chirality restriction that makes this SU(2)_L rather than SU(2)_R or SU(2)_L × SU(2)_R is inherited from the electroweak/chirality programme (§0.4), not rederived here.

This separation is load-bearing for honesty: a critic who rejects the census argument for k = 3 has not touched the transport theorem, and a critic who rejects the bath premise has not touched the census. The two failure modes are independent and independently falsifiable (§21).

---

5. Standard Model Specialisations — Colour (k = 3) and Weak (k = 2)

Corollary 3.1 — Colour as the k = 3 Bath-Transport Specialisation

For a threefold colour-support class, $k = 3$. By Theorem 3 the connected class-transport group is U(3); removing the shared abelian phase leaves

$SU(3)\_C$,

with Lie algebra $\mathfrak{su}(3)\_C$ and

$\dim \mathfrak{su}(3) = 3^2 - 1 = 8$.

These eight directions are the gluon directions.

Programme reading

The threefold support programme supplies the number three. The bath theorem supplies continuous non-abelian transport. Gauge necessity supplies the local connection. Together they derive the continuous colour gauge sector — §§7–13 develop its dynamics.

Corollary 3.2 — Weak Structure as the k = 2 Bath-Transport Specialisation

For a twofold weak-branch class, $k = 2$. By Theorem 3 the connected class-transport group is U(2); removing the shared abelian phase leaves SU(2). With the inherited chirality restriction this becomes

$SU(2)\_L$,

with $\dim \mathfrak{su}(2) = 3$ — the three weak directions W^1, W^2, W^3.

Programme reading

The weak group is not merely a two-state label. It is the bath transport group of a twofold weak-branch class. Chirality is a further inherited selection restricting this group to the left-handed Weyl sector — §14 develops the interface with the inherited electroweak stack.

---

6. Why C_3 Support Is Not Yet SU(3)_C

A threefold support structure is not yet a continuous gauge theory. A C_3-style label or cyclic support pattern establishes that the quark-support architecture is threefold. SU(3)_C asserts strictly more:

1. colour support is represented by a three-dimensional complex module;
2. local colour basis choices are conventional;

3. arbitrary continuous norm-preserving colour blends are admissible;
4. comparison between neighbouring colour frames requires a connection;
5. the connection has curvature;
6. curvature carries field dynamics.

The bath framing makes the gap exact. C_3 — indeed the entire monomial floor $T^3 \rtimes S_3$ — is precisely what a *ledger* of three identical sectors provides: swaps and phases. The passage to SU(3)_C is precisely the passage from ledger to bath (Theorems 1–3). The correct claim is therefore

C_3-style threefold support (census) + bath transport ⟹ $\mathcal{C} \cong \mathbb{C}^3$ with SU(3)_C class transport.

The discrete structure fixes the support count; the conservation structure fixes whether that count becomes a continuous force. Asserting C_3 = SU(3) would be false; deriving SU(3) from C_3-census plus bath conservation is the content of this paper.

---

7. Colour Gauge Dynamics from the k = 3 Bath Class

The colour class is the k = 3 case of the general theorem. The class carrier is

$\mathcal{C}$_x ≅ $\mathbb{C}^3$

at each record-layer point x, with quark colour support q(x) = q^α(x) e_α in a local basis {e_1, e_2, e_3}. The basis is not physical: by Corollary 3.1 the admissible local frame relations form SU(3)*C, and a colour vector at x and one at x + dx live in different fibres $\mathcal{C}$_x, $\mathcal{C}$*{x+dx}. Differentiating a quark field requires comparing these fibres, and by gauge-connection necessity (§0″.2) that comparison requires a connection.

Theorem 4 — Local Colour Connection Theorem

Given local colour comparison and the class-transport algebra 𝔰𝔲(3)_C, the local colour connection has the form

A^C_μ = g_s G^a_μ T_a,

with covariant derivative

D^C_μ = ∂_μ + i g_s G^a_μ T_a,

and transformation law

$A^C_\mu \to U A^C_\mu U^\dagger + i (\partial_\mu U) U^\dagger$, $U(x) \in SU(3)_C$.

Proof

A local comparison rule between neighbouring colour fibres is, infinitesimally, an $\mathfrak{su}(3)$-valued one-form on the record layer. Expanding in the basis T_a gives coefficient fields G^a_μ; conventional normalisation separates the coupling g_s. The covariant derivative is the ordinary derivative corrected by the comparison connection, and the inhomogeneous law follows from demanding that $D^C_\mu q$ transform in the same representation as q. ■

Reading

The gluon field is the connection required to compare bath-transported colour frames locally. It is not a substance painted onto quarks; it is the transport structure of colour distinguishability.

8. Colour Curvature and the Bianchi Identity

Theorem 5 — Colour Curvature Theorem

The local colour connection has curvature

$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu$.

Proof

Compute $[D^C_\mu, D^C_\nu] = i F^C_{\mu\nu}$. The derivative terms give $\partial_\mu G^a_\nu - \partial_\nu G^a_\mu$. The connection commutator gives

$i [g_s G^a_\mu T_a, g_s G^b_\nu T_b] = -g_s^2 G^a_\mu G^b_\nu f^{abc} T_c$;

relabelling indices yields the stated component form with $F^C_{\mu\nu} = g_s G^a_{\mu\nu} T_a$. ■

Corollary 5.1 — Colour Bianchi Identity

$D_{[\lambda} G^a_{\mu\nu]} = 0$

identically, from the Jacobi identity for covariant derivatives. This is structural, not dynamical: it constrains admissible colour-field configurations independently of any action principle.

Reading

Theorem 5 is the origin of gluon self-interaction, and the bath framing explains it: continuous blendings of three sectors do not commute, so the curvature contains a field–field term and the gluon itself carries colour. The non-abelian self-term is not added by hand — it is the curvature of bath-transport colour comparison. The strong force is self-interacting because the record layer remembers the order of local colour blends.

9. Colour Kinetic Form

The leading local colour gauge action must be: (1) local; (2) Lorentz-compatible; (3) gauge-covariant; (4) built from curvature, not bare connection; (5) of lowest nontrivial derivative order; (6) positive in energy.

Theorem 6 — Colour Curvature-Norm Kinetic Theorem

Given the colour connection, colour curvature, and conditions (1)–(6), the unique leading colour-gauge kinetic form is

$\mathcal{L}_C = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$,

up to normalisation and topological total-derivative terms.

Proof

Gauge covariance excludes bare-connection terms. The first nontrivial gauge-covariant local object is the curvature. A Lorentz scalar linear in curvature either vanishes ($\mathrm{tr}\, F = 0$ on a simple algebra) or is a total derivative. The leading parity-even local scalar is the invariant quadratic curvature norm; on the simple algebra $\mathfrak{su}(3)$ the invariant symmetric bilinear form is unique up to scale, so no inequivalent quadratic invariant exists. Canonical normalisation gives $-\frac{1}{4} G^2$; positivity of the energy density fixes the sign. ■

Scope note

The theorem derives the leading classical Yang–Mills form, and with it the three- and four-gluon vertices implicit in expanding $-\frac{1}{4} G^2$ through the self-term. It does not derive the beta function, asymptotic freedom, confinement, instantons, or the θ-term coefficient. The topological density $G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$, with $\tilde{G}^{a\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma}$, is a total derivative at the classical record-layer level; its quantum role belongs to the owed strong-CP layer.

10. Colour Currents and Quark Coupling

The quark kinetic term uses the full Standard Model covariant derivative; its colour part is $\bar{q}\, i \gamma^\mu D^C_\mu q$, giving the interaction

$$\mathcal{L}^C_{\text{int}} = -g_s G^a_\mu J^{a\mu}_C, \quad J^{a\mu}_C = \Sigma_q :\bar{q} \gamma^\mu T_a q:,$$

the sum running over the colour-supported sectors Q_L, u_R, d_R.

Theorem 7 — Colour Source-Current Theorem

The colour connection couples to quark colour currents and not to lepton currents:

$$J^{a\mu}_C = \Sigma_{q \in \{Q_L, u_R, d_R\}} :\bar{q} \gamma^\mu T_a q:,$$

with sourced field equations $(D_\nu G^{\nu\mu})^a = g_s J^{a\mu}_C$ and covariant conservation $D_\mu J^{a\mu}_C = 0$.

Proof

The colour generators act nontrivially only on colour-supported sectors: Q_L, u_R, d_R transform in the fundamental 3; L_L, e_R are colour singlets, so $T_a = 0$ on leptons. Variation of the matter kinetic term with respect to G^a_μ returns the quark colour current; covariant conservation follows from the antisymmetry of $G^{\nu\mu}$ and the Bianchi structure. ■

Reading

Colour is vectorlike with respect to chirality: both left- and right-handed quark sectors carry colour support, in sharp contrast to the chiral weak sector. In bath language, the colour bath is chirality-blind while the weak bath is chirality-selected — the selection living in the inherited chirality layer, not in the transport theorem. The same contrast reappears in the anomaly ledger of §11.

---

11. Colour Anomaly Closure

The colour sector must sit on an anomaly-admissible ledger (NAB-9). In left-handed Weyl language, the colour-charged sectors per generation are

$$Q_L : (3, 2)_{+1/6}, \quad u_L^c : (\bar{3}, 1)_{-2/3}, \quad d_L^c : (\bar{3}, 1)_{+1/3}.$$

Proposition 8a — Pure SU(3)$_C^3$ Anomaly Closure

Proof

The cubic coefficients satisfy A(3̄) = −A(3). Q_L contributes two colour fundamentals; u_L^c and d_L^c contribute two anti-fundamentals:

2·A(3) + A(3̄) + A(3̄) = 2A(3) − 2A(3) = 0. ■

Proposition 8b — Mixed SU(3)_C²–U(1)_Y Anomaly Closure

Proof

The coefficient is the hypercharge sum over colour triplets (equal Dynkin index for 3 and 3̄):

Σ Y = 2·(+⅙) + (−⅔) + (+⅓) = ⅓ − ⅔ + ⅓ = 0. ■

Reading

Proposition 8a is why colour is a consistent gauge sector on the inherited chiral matter skeleton: the strong force is vectorlike even though the weak force is chiral. Proposition 8b confirms that attaching the derived colour layer to the inherited hypercharge grading introduces no new inadmissibility — the colour completion is compatible with the full audit table of the Substrate Anomaly-Inadmissibility Theorem, used here but not repeated.

---

12. Colour Singlets — Leptons and the Completion Interface

Theorem 8 — Colour-Singlet Lepton and Interface Audit

Leptons and the electroweak completion interface Φ_cl ~ (1, 2, +½) carry the trivial colour representation.

Grade note. The colourlessness of both objects is inherited (§0.2, §0″.4); this theorem records the audit that the colour layer derived here is consistent with it, and derives nothing not already inherited.

Proof

Leptons do not participate in three-body confined colour support and carry no colour index; their comparison structure is exhausted by the weak and hypercharge layers — in bath language, leptons are not members of the k = 3 colour class, so the colour bath transports nothing of theirs. The completion interface is inherited as the weak-active, colourless bridge between left weak doublets and right weak singlets (§0″.4). The audit is consistency, not derivation: a coloured interface would make charged-lepton attachment colour-nontrivial — the singlet channel $\bar{L}_L \Phi e_R$ would fail to close on the derived matter skeleton — so the inherited assignment and the colour layer derived here are mutually consistent. Both objects transform as colour representation 1. ■

---

13. Why Electroweak Breaking Does Not Break Colour

The electroweak vacuum is

$$\langle \Phi_{cl} \rangle = (v/\sqrt{2})\,(0, 1)^T,$$

in colour representation 1, so for every colour generator T^C_a,

$$T^C_a \langle \Phi_{cl} \rangle = 0.$$

Theorem 9 — Colour Preservation Under Electroweak Breaking

The electroweak completion-interface condensate leaves $SU(3)_C$ unbroken.

Proof

Gauge bosons acquire condensate stiffness only if their generators act nontrivially on the vacuum order parameter — the mass matrix is proportional to $\langle \Phi \rangle^\dagger T_A T_B \langle \Phi \rangle$ over the full generator set. The interface is a colour singlet, so every colour generator annihilates it, and every colour–colour and colour–electroweak entry of the mass matrix vanishes. No gluon mass term and no colour–electroweak mixing arise. ■

Reading

The object that gives the photon, W, and Z also protects the gluons: the completion interface is electroweak-active but colour-inert. Gluon masslessness at this layer is derived, not assumed — though gluon non-observation as free states belongs to the owed confinement layer.

---

14. Weak Gauge Structure as the k = 2 Bath Class

The weak branch is the $k = 2$ case of the same transport principle. A twofold weak commitment class has carrier

$\mathcal{W} \cong \mathbb{C}^2$;

bath transport gives U(2), and removal of the shared abelian phase gives SU(2) (Corollary 3.2), with three generators τ_i and connection contribution $g W^i_\mu \tau_i$.

The electroweak representation programme then supplies two further facts, inherited and not rederived:

1. this SU(2) acts only on left-handed weak doublets — the chirality selection $SU(2) \rightarrow SU(2)_L$;
2. the independent abelian direction is $U(1)_Y$ — not an extra duplicate of the removed class phase (NAB-8).

The weak gauge group is therefore $SU(2)_L$ and the electroweak product $SU(2)_L \times U(1)_Y$, exactly as the inherited stack requires. The contribution of the present paper on the weak side is the transport origin: the weak group is not a two-state label but the bath transport group of a twofold weak-branch class. One mechanism, two censuses: the strong and weak forces are the $k = 3$ and $k = 2$ readings of a single conservation-law theorem.

---

15. Full Gauge-Product Closure

Theorem 10 — Full Gauge-Product Closure from Census, Bath Transport, and Abelian Uniqueness

Given (1) the colour census $k = 3$; (2) the weak census $k = 2$; (3) the bath-transport theorem; (4) local distinguishability conservation requiring gauge connections; (5) abelian uniqueness supplying one $U(1)_Y$; (6) chirality selecting $SU(2)_L$ — the Standard Model gauge product is

$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$,

with local connection

$A^{SM}_\mu = g_s G^a_\mu T_a + g W^i_\mu \tau_i + g' B_\mu Y$, and generator count

$$\dim \mathfrak{su}(3) + \dim \mathfrak{su}(2) + \dim \mathfrak{u}(1) = 8 + 3 + 1 = 12.$$

Proof

The $k = 3$ census plus bath transport gives SU(3)_C (Corollary 3.1). The $k = 2$ census plus bath transport gives SU(2), and the inherited chirality theorem selects SU(2)_L (Corollary 3.2, §0.4). The abelian-uniqueness programme supplies a single U(1)_Y (NAB-8). Distinguishability conservation lifts each internal comparison group to a local gauge connection (§0″.2, Theorem 4 and its electroweak counterpart). The colour, weak, and hypercharge generators act on distinct commuting tensor factors of the internal representation space (§0″.3), so the full gauge algebra is the direct sum

$$\mathfrak{g}\_\mathrm{SM} = \mathfrak{su}(3)\_\mathrm{C} \oplus \mathfrak{su}(2)\_\mathrm{L} \oplus \mathfrak{u}(1)\_\mathrm{Y},$$

and the corresponding group is the product SU(3)_C × SU(2)_L × U(1)_Y. ■

Corollary 10.1 — Gauge-boson count

Twelve gauge directions before electroweak breaking. After breaking, the four electroweak directions reorganise as W^+, W^-, Z, and the photon A, while the eight colour directions remain massless gluons (Theorem 9).

Remark 10.2 — Global-structure scope

Theorem 10 is a statement about the local gauge algebra and the product connection. The global gauge group may be the quotient $[SU(3) \times SU(2) \times U(1)] / \Gamma$ with $\Gamma \subseteq \mathbb{Z}_6$ a shared discrete centre, fixed by the faithfulness of the physical representation content — for the derived matter skeleton, $\Gamma = \mathbb{Z}_6$ acts trivially on every ledger entry (Appendix E), making it the maximal faithful candidate, but the determination — contingent on the completeness of the ledger — is not decided here. All results here are Γ-insensitive (§0.11); the quotient is recorded as a downstream global-structure debt (§20).

---

16. Ledger Branch and the Non-Abelian Kill Condition

The bath premise is load-bearing, and the opposite branch must be stated with equal clarity.

If the substrate takes the ledger reading for identical closure sectors, admissible class transport is confined to the monomial floor

$$T^k \rtimes S_k$$

(Theorem 1). The connected component is the abelian torus T^k. There is no continuous off-diagonal blend, no U(k), and therefore no derived SU(k) non-abelian structure from class transport:

ledger ⟹ swap-and-phase only ⟹ no non-abelian gauge origin by this route.

This is a strength, not a weakness: it makes the theorem falsifiable inside the programme. The bath reading is not decorative — it is exactly the condition on which the strong and weak non-abelian gauge sectors depend.

The observed world contains non-abelian gauge structure. Therefore the VERSF programme must ultimately discharge the bath premise from commitment dynamics, BCB balance, or the Born-rule/refinement obstruction — or admit that non-abelian gauge structure enters by another route. That obligation is recorded as Debt 1 (§20).

---

17. Fourteen-Generator Firewall

The programme contains an inherited fourteen-generator response structure attached to the $K = 7$ closure architecture, $14 = 2 \times 7$. That census belongs to closure architecture, not to gauge-boson enumeration. This paper's gauge count is $8 + 3 + 1 = 12$; there is no tension, because the two counts live at different layers.

Proposition 11 — Gauge Count / Closure Count Separation

The $8 + 3 + 1 = 12$ gauge-generator count and the $14 = 2K$ closure-generator census are distinct layer counts and must not be identified.

Proof

The gauge count follows from Lie-algebra dimensions: $\dim \mathfrak{su}(3) = 8$, $\dim \mathfrak{su}(2) = 3$, $\dim \mathfrak{u}(1) = 1$ — each factor now sourced by the bath-transport theorem or abelian uniqueness. The fourteen-generator census arises from a $K = 7$ closure-response architecture with dihedral transport structure; it is not the dimension of any factor of the gauge algebra, and no group in the Standard Model product has dimension 14 acting on the derived matter representations. The counts refer to different objects at different explanatory layers. ■

Reading

Without this firewall, one might attempt to identify the closure-response count with gauge bosons — corrupting both derivations. (G_2 has dimension 14, which makes the temptation concrete; but G_2 is not a bath-transport group of any ℂ^k class, has no complex k-dimensional unitary fundamental suitable for colour, and plays no role in the gauge layer. The coincidence of the number 14 is exactly that — a coincidence across layers.)

---

18. Relation to the Existing Quark-Sector and Flavour Programme

The existing quark-sector work — Yukawa hierarchy, CKM curvature, flavour-frame misalignment, quark-sector closure ledgers, the assignment/bath/refinement results isolating the susceptibility $\chi(g)$ — is downstream of this paper. The relationship is

census (k = 3) → bath transport → SU(3)_C gauge dynamics → quark mass/flavour modules.

CKM mixing remains a weak charged-current flavour phenomenon: colour does not create CKM mixing, and the colour generators are flavour-blind — they commute with every flavour-space operator in the Hessian programme. The Carrier Theorem's route from class-level quantised response to member-grain fractional charge is likewise downstream: it is supported by, but not load-bearing for, the k = 3 transport result.

The colour closure sharpens the mass-attachment audits: charged leptons are colour singlets; down- and up-quark attachment operators must be colour singlets; the completion interface carries no colour; and quark colour indices contract internally in every admissible mass channel (Appendix B).

---

19. What Has Actually Been Derived

Conditionally on the inherited census, the bath premise NAB-4, and premises NAB-1–NAB-9, this paper derives:

1. the exact equivalence of ledger conservation with the monomial floor T^k ⋊ S_k, and the absence of continuous non-abelian mixing on the ledger branch;
2. the forcing of a continuous off-diagonal generator by bath conservation plus continuity;
3. the block-partition lemma — every torus-containing subalgebra of $\mathfrak{u}(k)$ is a block-partition algebra, and every proper partition conserves its sub-accounts — and through it the generation of U(k) from one blend, the phase torus, and the saturated bath, with relabelling covariance obtained as a consequence rather than assumed; SU(k) as the non-abelian content, with proper subgroups excluded, one owner per exclusion, and the central-phase detachment mechanism (Proposition 3.3) grounding the passage U(k) → SU(k);

4. colour as the k = 3 specialisation — SU(3)_C with eight gluon directions;
5. the weak factor as the k = 2 specialisation — SU(2), restricted to SU(2)_L by inherited chirality;
6. the local colour connection with its inhomogeneous transport law;
7. the colour curvature, its Bianchi identity, and the non-abelian self-term as bath-transport curvature;
8. the leading colour Yang–Mills kinetic form;
9. the quark colour current and its covariant conservation; the absence of colour coupling to leptons;
10. pure $SU(3)^3$ and mixed $SU(3)^2$–U(1)_Y anomaly closure;
11. colour preservation under electroweak breaking — massless gluons at this layer;
12. the full gauge-product closure with generator count 12;
13. the layer separation between the twelve gauge generators and the fourteen closure-response generators;
14. the explicit kill condition on the ledger branch.

This paper does not derive:

1. the bath premise itself; 2. the census values k = 3, k = 2; 3. chirality; 4. hypercharge or abelian uniqueness; 5. the global gauge-group quotient $\Gamma \subseteq \mathbb{Z}_6$; 6. the numerical value or running of g_s; 7. asymptotic freedom; 8. confinement; 9. Λ_QCD; 10. hadron masses; 11. chiral symmetry breaking; 12. the QCD vacuum, instantons, θ_QCD, or the strong CP solution; 13. the full nonperturbative path integral; 14. three generations; 15. the quark mass hierarchy or CKM angles; 16. fractional charge at member grain.

---

20. Open Debts

Debt 1 — Bath premise discharge

The largest remaining debt is not colour algebra; it is the bath/ledger decision. This paper proves that bath transport yields non-abelian structure and ledger transport does not. A future substrate paper must derive the bath reading from commitment dynamics, BCB balance, or the Born-rule/refinement obstruction.

Debt 2 — Census closure

The values k = 3 (colour) and k = 2 (weak) are inherited. The transport theorem does not determine multiplicities; the census remains sourced to the internal-symmetry and closure-geometry programme.

Debt 3 — Chirality

The bath theorem gives SU(2) for a twofold class. The selection SU(2) → SU(2)_L remains inherited from the electroweak representation/chirality programme.

Debt 4 — Abelian uniqueness and hypercharge

The shared phase removed from U(k) is not a new abelian gauge factor. The single U(1)_Y remains inherited from the abelian-uniqueness and hypercharge programme.

Debt 4′ — Global gauge-group quotient

The global gauge group may be [SU(3) × SU(2) × U(1)] / Γ with $\Gamma \subseteq \mathbb{Z}_6$. Appendix E audits the centre action on the derived ledger: $\mathbb{Z}_6$ acts trivially, so $\Gamma = \mathbb{Z}_6$ is the maximal faithful candidate. What remains owed is the determination itself, which is contingent on the completeness of the ledger — a single physical state with $2t + 3d + q \not\equiv 0 \pmod 6$ would force a smaller Γ. Every result of this paper is Γ-insensitive (§0.11, Remark 10.2).

Debt 5 — QCD dynamics beyond leading curvature

Confinement, asymptotic freedom, the running coupling, Λ_QCD, the hadron spectrum, strong CP, and the nonperturbative vacuum remain owed.

Debt 6 — Fractional charge

The Carrier Theorem suggests a route from class-level quantised response to member-grain fractional charge, especially for k = 3. This belongs in a separate downstream fractional-charge/carrier paper.

Debt 7 — Quark mass/flavour integration

The assignment/bath/refinement results — quark masses not separable as assignment × generation, the susceptibility χ(g) — sit downstream of the gauge-product closure and must be audited against the full gauge background.

Debt 8 — Three generations

The transport structure is generation-independent. The existence of three generations remains a separate programme layer.

---

21. Falsification Conditions

F1 — Ledger substrate

If the substrate takes the ledger reading for identical closure classes, NAB-4 fails and no non-abelian gauge structure arises by this route (§16).

F2 — Discontinuous transport

If weight reallocation exists only discretely, NAB-5 fails and no continuous generator arises.

F3 — Phase failure

If relative phases are not sector-resolved, NAB-6 fails and the result falls back to a real mixing group rather than complex SU(k).

F4 — Census failure

If the colour census is not 3 or the weak census is not 2, NAB-7 fails and the specialisations detach.

F5 — Third conservation structure

If a transport-supporting conservation structure exists that is neither ledger nor bath, NAB-3 fails.

F6 — Wrong colour group

If local colour comparison requires a group other than SU(3) — larger, smaller, or a proper subgroup preserving a sector, a real structure, or a frame — Theorem 3 / Corollary 3.0 fails.

F7 — Wrong gluon count

If the low-energy colour gauge sector contains other than eight gauge bosons, the $k = 3$ specialisation fails.

F8 — Colour mass leakage

If electroweak condensation gives gluons mass, Theorem 9 fails.

F9 — Lepton colour coupling

If leptons carry fundamental colour support, the representation skeleton used here is incomplete.

F10 — Anomaly leakage

If either the pure $SU(3)^3$ or the mixed $SU(3)^2$–U(1)_Y anomaly fails to cancel on the physical ledger, NAB-9 fails.

F11 — Duplicate abelian factor

If a second independent fundamental abelian gauge factor is required, NAB-8 fails.

F12 — Gauge-count conflation

If the fourteen-generator closure-response structure is identified with the Standard Model gauge count, the layer firewall fails.

F13 — QCD overclaim

If the paper is read as deriving confinement, asymptotic freedom, hadron masses, chirality, the census, or the global gauge quotient Γ, its claim is overstated.

F14 — Protected sub-account

If admissible transport conserves a sectoral weight quantity beyond total class weight — a preferred sector weight, a sub-total over a proper subset of sectors, a directional account $|\langle e, w\rangle|^2$, a frozen inter-sector phase relation, or an anonymous symmetric invariant of the weights beyond the class norm — NAB-4′ fails, and by Lemma 3.0 the transport algebra collapses to the block or stabiliser algebra of the protected quantity rather than the full $\mathfrak{su}(k)$.

---

22. Milestone Statement

The result of this paper is

identical closure class + saturated bath transport + continuity + sector phase ⟹ U(k) ⟹ SU(k).

With inherited census values,

k = 3 ⟹ SU(3)_C, k = 2 ⟹ SU(2)_L (after chirality selection),

and together with the inherited abelian sector U(1)_Y, the Standard Model gauge product closes:

SU(3)_C × SU(2)_L × U(1)_Y.

The conceptual advance is that non-abelian gauge structure is no longer treated as a catalogue of abstract groups. It is reduced to a conservation-law question:

Does closure transport conserve separate ledgers, or one shared bath?

If bath — non-abelian gauge structure follows, with the strong force as k = 3 and the weak force as k = 2. If ledger — this route to the strong and weak forces fails, and the programme owes another. That is the sharpened Standard Model milestone.

---

23. Conclusion

The VERSF programme has derived the chiral matter skeleton, the electroweak representation algebra, the local W/B connection, anomaly-admissible source ledgers, electroweak gauge-curvature dynamics, the broken electroweak phase, and the universal electroweak completion interface. The present paper supplies the non-abelian gauge-origin layer and closes the Standard Model gauge product.

The structural content is a single theorem with two readings. On a class of k indistinguishable closure sectors, ledger conservation admits only the monomial floor T^k ⋊ S_k — swaps and phases, with no continuous mixing and no non-abelian force structure. Bath conservation, with continuity and sector phase, forces a continuous off-diagonal blend; with the phase torus present, the transport algebra is a block algebra of the sectors, saturation forbids every proper block partition, and the algebra closes to U(k), with SU(k) as its non-abelian content — relabelling covariance emerging as a consequence rather than an assumption. Non-abelian gauge structure is thereby reduced to a conservation-law decision about the substrate.

Colour is the k = 3 realisation. The census supplies three; the bath theorem supplies continuous SU(3)_C transport; gauge necessity supplies the gluon connection; the curvature of that connection is the colour field strength, with a self-term because bath blends do not commute; and the leading curvature norm gives the classical Yang–Mills form. Quarks source the colour field; leptons do not; the completion interface is colourless, so electroweak breaking leaves the gluons massless; and both the pure colour anomaly and the mixed colour–hypercharge anomaly cancel on the inherited ledger. The weak factor is the k = 2 realisation of the same mechanism, restricted to SU(2)_L by inherited chirality. With a single inherited U(1)_Y, the full gauge product closes:

SU(3)_C × SU(2)_L × U(1)_Y.

The paper is explicit about its hinge. The bath premise is not yet discharged from first principles; it remains the programme-critical decision node, shared with the Born-rule arc, and the ledger branch is stated as a kill condition rather than hidden. The gain is nonetheless substantial: the origin of the strong and weak forces has been reduced to one precise substrate question —

separate ledgers, or one shared bath — and everything downstream of that question is now theorem rather than postulate.

The programme chain now reads:

fermionic matter → chiral skeleton → electroweak representation → anomaly-admissible ledger → electroweak gauge dynamics → completion interface → broken electroweak phase → **non-abelian bath transport → continuous colour gauge dynamics → full Standard Model gauge product**.

That is the milestone.

Appendix A — SU(3) Generator Conventions

The Gell-Mann basis λ_a satisfies $\mathrm{tr}(\lambda_a \lambda_b) = 2\,\delta_{ab}$. With $T_a = \lambda_a/2$,

$\mathrm{tr}(T_a T_b) = \frac{1}{2}\,\delta_{ab}$, $[T_a, T_b] = i f^{abc} T_c$.

The eight generators organise as: λ_1, λ_2, λ_3 acting within the first two colour components (an $\mathfrak{su}(2)$ subalgebra); λ_4, λ_5 mixing the first and third; λ_6, λ_7 mixing the second and third; $\lambda_8 = \mathrm{diag}(1, 1, -2)/\sqrt{3}$ as the second diagonal traceless generator. The diagonal pair λ_3, λ_8 spans the rank-two Cartan subalgebra of $\mathfrak{su}(3)$: internal directions within the single non-abelian colour algebra, not independent U(1) sectors. Nonvanishing structure constants include $f^{123} = 1$ and $f^{458} = f^{678} = \sqrt{3}/2$; only the totally antisymmetric pattern matters for the theorems above.

In bath language, Lemma 3.0's block mechanics are visible in this basis: a λ_1/λ_2-type blend between sectors 1 and 2 spans its root plane under the torus sweep, brackets chain it to the (1,3) plane (λ_4, λ_5) and the (2,3) plane (λ_6, λ_7), and commutators close the Cartan directions λ_3, λ_8 — generating all of $\mathfrak{su}(3)$ from one blend, the phase torus, and nothing else. Any proper block partition of the three sectors would instead terminate at $\mathfrak{u}(2) \oplus \mathfrak{u}(1)$, conserving the excluded sector's weight — precisely the protected sub-account that saturation forbids.

Appendix B — Colour Singlet Audits

B.1 Baryon singlet

$B = \varepsilon_{\alpha\beta\gamma}\, q^\alpha q^\beta q^\gamma$; under $U \in SU(3)$, $\varepsilon_{\alpha\beta\gamma}\, U^\alpha{}_\delta\, U^\beta{}_\varepsilon\, U^\gamma{}_\zeta = (\det U)\, \varepsilon_{\delta\varepsilon\zeta} = \varepsilon_{\delta\varepsilon\zeta}$.

B is a colour singlet — determinant neutrality earning its keep: the baryon channel is invariant because $\det U = 1$.

B.2 Meson singlet

$M = \bar{q}_\alpha q^\alpha$ — the trace contraction $\bar{3} \otimes 3 \supset 1$.

B.3 Charged-lepton mass channel

$\bar{L}_L \Phi_{cl} e_R$ — no colour index; singlet.

B.4 Down-quark mass channel

$\bar{Q}_{L\alpha} \Phi_{cl} d_R^\alpha$ — contracts as $\bar{3} \otimes 3 \supset 1$; singlet.

B.5 Up-quark mass channel

$\bar{Q}_{L\alpha} \tilde{\Phi}_{cl} u_R^\alpha$, $\tilde{\Phi}_{cl} = i\sigma_2 \Phi_{cl}^*$ — contracts as $\bar{3} \otimes 3 \supset 1$; singlet.

Every admissible mass channel closes its colour indices internally, consistent with Theorem 8 and the colourlessness of the completion interface.

Appendix C — Excluded Alternatives

C.1 Ledger colour

A ledger of three identical sectors gives $T^3 \rtimes S_3$ — the monomial floor. Connected component T^3, abelian, no gluon self-interaction, no eight-direction non-abelian algebra. Excluded as the strong force by Theorem 1 and observation; recorded as the kill condition of §16.

C.2 SU(2) colour

A twofold module admits a totally antisymmetric singlet only at the two-body level ($\Lambda^2(\mathbb{C}^2) = 1$); $\Lambda^3(\mathbb{C}^2) = 0$. Fails the three-body baryon anchor — a census exclusion.

C.3 SU(4) colour

$\Lambda^3(\mathbb{C}^4) = \bar{4}$ is not a singlet; the minimal totally antisymmetric singlet needs four fundamentals ($\Lambda^4(\mathbb{C}^4) = 1$). Contradicts three-body support and adds unused structure — census plus minimality exclusion.

C.4 U(3) colour

The central direction of U(3) alters no inter-sector distinguishability relation and therefore generates no class-specific comparison obligation (Proposition 3.3 — mechanism, not fiat). Retaining it as a colour gauge factor would duplicate the accounted abelian structure and add a colour-phase boson coupling to baryon number, excluded by NAB-8.

C.5 Proper subgroups of SU(3)

Each exclusion has one owner (Corollary 3.0). Torus-containing proper subalgebras are proper block-partition algebras conserving sub-accounts — the line stabiliser S(U(1) × U(2)) conserves $|\langle e, w\rangle|^2$ — excluded by NAB-4′. The real form SO(3) (rank $1 < 3$) cannot contain the torus and is excluded by NAB-6. Finite subgroups — including C_3 and the centre $\mathbb{Z}_3$ — supply no continuous transport and are excluded by NAB-5.

C.6 Discrete C_3 as final colour

Support architecture, not gauge group: no continuous comparison, no eight directions, no curvature. The bath framing identifies it exactly as (part of) the ledger floor.

C.7 Real mixing without sector phase

Without NAB-6 the transport group falls back to a real orthogonal-type mixing structure, too thin for the complex fundamental representations the matter skeleton requires. This is the falsifier of NAB-6, not a viable alternative.

C.8 Coloured completion interface

Would break colour when condensed and obstruct colourless lepton attachment. Excluded by the audit of Theorem 8 and by Theorem 9.

Appendix D — Claim-Status Ledger

Claim	Status	Comment
Ledger transport = monomial floor T^k ⋊ S_k	Derived / Exact	Theorem 1 — translation lemma
Ledger branch has no continuous non-abelian mixing	Derived / Exact	Corollary 1.1 — connected part is torus
Anonymous ledgers are ledgers	Derived	Remark 1.2 — symmetric-function theorem
Invariant-arity completeness	Derived	Corollary 3.0′ — composite-state audit confirmatory

Claim	Status	Comment
Bath branch supplies off-diagonal mixing	Conditional theorem	Theorem 2 — requires bath + continuity
Bath + torus + saturation gives U(k)	Conditional theorem	Theorem 3 via Lemma 3.0
Block-partition lemma	Exact	Lemma 3.0
Central-phase detachment	Derived	Proposition 3.3
Non-abelian content is SU(k)	Exact given U(k)	Shared phase removed
No proper subgroup suffices	Derived	Corollary 3.0 — NAB-4′ + NAB-6, one owner per exclusion
k = 3 ⟹ SU(3)_C	Conditional on census + bath	Corollary 3.1
k = 2 ⟹ SU(2)	Conditional on census + bath	Corollary 3.2
SU(2) → SU(2)_L	Inherited	Chirality selection
Single U(1)_Y	Inherited	Abelian uniqueness
$C_3 \neq SU(3)$ distinction	Derived / clarified	§6 — ledger floor vs bath transport
Colour connection	Derived	Theorem 4
Colour curvature	Exact given connection	Theorem 5
Bianchi identity	Exact	Corollary 5.1
Yang–Mills kinetic form	Conditional	Curvature-norm principle — Theorem 6
Quark colour current	Derived	Theorem 7
Pure colour anomaly closure	Derived / audit	Proposition 8a
Mixed $SU(3)^2$–U(1)_Y closure	Derived / audit	Proposition 8b
Lepton / interface colourlessness	Inherited / audited	Theorem 8 — audit
Colour preserved under EWSB	Derived	Theorem 9
Full gauge product	Conditional synthesis	Theorem 10
Fourteen-generator separation	Derived / firewall	Proposition 11
Bath premise NAB-4	Open / programme-critical	Must be discharged upstream
Bath saturation NAB-4′	Conditional	Conservation-residue scope — load-bearing for uniqueness
Global gauge quotient Γ	Downstream / audited	$\mathbb{Z}_6$ trivial on ledger (Appendix E) — determination not decided here
Charge–triality correlation	Derived / audit	Corollary E.1 — $6Y \equiv 4t + 3d \pmod 6$
Confinement, Λ_QCD, hadron spectrum	Owed	Not derived

Claim	Status	Comment
Running g_s	Owed	Not derived
Strong CP	Owed	Not derived
Fractional charge carrier	Downstream	Carrier-theorem paper
Quark mass hierarchy	Downstream	Assignment/refinement papers

Appendix E — Global-Quotient Audit: $\mathbb{Z}_6$ Centre Action and Charge–Triality

This appendix supplies the centre-action audit promised in §0.11 and Remark 10.2. It does not discharge Debt 4′ — the determination of Γ is contingent on the completeness of the ledger — but it establishes that $\mathbb{Z}_6$ acts trivially on every representation derived or inherited in this paper, making $\Gamma = \mathbb{Z}_6$ the maximal faithful candidate.

E.1 Triality, duality, and the hypercharge grade

Define, for any representation of the gauge product:

- **colour triality** $t \in \{0, 1, 2\}$: $t = 0$ for colour singlets, $t = 1$ for fundamentals 3, $t = 2$ for anti-fundamentals $\bar{3}$ — the $\mathbb{Z}_3$ centre grade of $SU(3)_C$;
- **weak duality** $d \in \{0, 1\}$: $d = 0$ for $SU(2)_L$ singlets, $d = 1$ for doublets — the $\mathbb{Z}_2$ centre grade of $SU(2)_L$;
- **hypercharge grade** $q \equiv 6Y \pmod 6$ — integral on the derived ledger, since $6Y \in \mathbb{Z}$ for every entry.

Both chirality conventions give the same audit: conjugation fixes d and sends $(t, q) \to (-t, -q)$ mod $(3, 6)$, hence $2t + q \to -(2t + q)$; since the invariance condition $2t + 3d + q \equiv 0 \pmod 6$ forces $2t + q \equiv 3d \in \{0, 3\} \pmod 6$, and both values are fixed under negation mod 6, a representation and its conjugate satisfy the condition together or not at all.

E.2 The generator and the six-element table

Let $\omega = e^{2\pi i/3}$ and define

$$\zeta = (\omega \mathbb{1}_3, -\mathbb{1}_2, e^{i\pi/3}) \in SU(3) \times SU(2) \times U(1).$$

On a representation with grades (t, d, q), ζ acts by the phase

$$\zeta \cdot (t, d, q) = e^{2\pi i (2t + 3d + q)/6}.$$

The powers of ζ enumerate all six elements of $\mathbb{Z}_3 \times \mathbb{Z}_2$, confirming that ζ generates a $\mathbb{Z}_6$ subgroup of the centre:

n	a = n mod 3	b = n mod 2	θ = nπ/3	ζ^n
0	0	0	0	$(\mathbb{1}_3, \mathbb{1}_2, 1)$
1	1	1	π/3	$(\omega\, \mathbb{1}_3, -\mathbb{1}_2, e^{i\pi/3})$
2	2	0	2π/3	$(\omega^2\, \mathbb{1}_3, \mathbb{1}_2, e^{2i\pi/3})$
3	0	1	π	$(\mathbb{1}_3, -\mathbb{1}_2, -1)$
4	1	0	4π/3	$(\omega\, \mathbb{1}_3, \mathbb{1}_2, e^{4i\pi/3})$
5	2	1	5π/3	$(\omega^2\, \mathbb{1}_3, -\mathbb{1}_2, e^{5i\pi/3})$

The pairs (a, b) exhaust $\mathbb{Z}_3 \times \mathbb{Z}_2$, so $\langle\zeta\rangle \cong \mathbb{Z}_6$.

E.3 Ledger audit

ζ acts trivially on a representation exactly when

$2t + 3d + q \equiv 0 \pmod 6$.

The audit over the full derived ledger:

Field	t	d	q = 6Y mod 6	2t + 3d + q mod 6	ζ-action
Q_L	1	1	1	0	trivial
u_R	1	0	4	0	trivial
d_R	1	0	4	0	trivial
L_L	0	1	3	0	trivial
e_R	0	0	0	0	trivial
Φ_cl	0	1	3	0	trivial
gluons	0	0	0	0	trivial
W	0	0	0	0	trivial
B	0	0	0	0	trivial

All six powers of ζ act trivially on every entry — confirmed by numerical audit over all six powers and all nine entries. The gauge fields, transforming in adjoints with $t = d = q = 0$, are trivially invariant.

E.4 The $\mathbb{Z}_2$ and $\mathbb{Z}_3$ sub-quotients

The $\mathbb{Z}_2$ subgroup generated by $\zeta^3 = (\mathbb{1}_3, -\mathbb{1}_2, -1)$ acts on a representation (t, d, q) by $(-1)^{d+q}$. The derived ledger has $d + q$ even for every admissible matter and interface representation, so this $\mathbb{Z}_2$ acts trivially. Quotienting by it alone removes the order-two part and leaves a residual $\mathbb{Z}_3$, generated by $\zeta^2 = (\omega^2 \mathbb{1}_3, \mathbb{1}_2, e^{2i\pi/3})$, which likewise acts trivially on the ledger ($n = 2$ row of the audit). The full faithful-maximal candidate is therefore the whole $\mathbb{Z}_6$, not either sub-quotient alone.

E.5 Corollary E.1 — Charge–Triality Correlation

$\mathbb{Z}_6$-invariance of a representation is equivalent to

$6Y \equiv 4t + 3d \pmod{6}$.

On the derived ledger this correlation holds exactly, tying the hypercharge grade to colour triality and weak duality: colour triplets that are weak singlets carry $6Y \equiv 4$ ($Y \in \{\ldots, -1/3, 2/3, \ldots\}$), colour-singlet weak doublets carry $6Y \equiv 3$ ($Y \in \{\ldots, -1/2, 1/2, \ldots\}$), and colour-singlet weak singlets carry integral Y. This is the structural pattern behind fractional quark charge sitting exactly where it does — the class-level counterpart of the member-grain fractional-charge question deferred to the downstream Carrier-theorem paper (Debt 6).

Grade: Derived / audit — exact on the derived ledger; its extension to all physical states is precisely the ledger-completeness question of Debt 4′. *Falsifier:* a physical state with $2t + 3d + q \not\equiv 0 \pmod{6}$ — equivalently, a violation of the charge–triality correlation — would force $\Gamma \subsetneq \mathbb{Z}_6$ and break the correlation as a law.