

# The Ordered-Commitment $\chi$ Readout Theorem in VERSF

▲ Programme Milestone — Standard Model Mass-Hierarchy Series Gate SM-4

*Proving that  $\chi$  Reads the Ordered Commitment History  $H$ , Not Only the Final Standing Record  $F$ , and that the Reversal-Odd Residue Survives Physical Coarse-Graining*

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## General Reader Summary

Quarks — the particles inside protons and neutrons — come in pairs. Each pair has an "up-type" member and a "down-type" member, and the two members of a pair have different masses. Nobody knows why. The VERSF programme is trying to derive that difference from first principles, and it tracks the difference using a quantity named after the Greek letter chi ( $\chi$ ). Roughly:  $\chi$  measures how unbalanced a quark pair is. If  $\chi$  is zero, the two members of the pair would be identical.

This paper deals with a threat that could have killed the whole project before it started.

In VERSF, reality is built from *committed facts* — irreversible events, like entries written in permanent ink. Suppose you tried to compute  $\chi$  by looking at the finished record of all those facts: everything that has been committed, added up. Here is the problem. That finished record turns out to be perfectly balanced. It treats "up" and "down" with total even-handedness, like a scale with identical weights on both sides. Read  $\chi$  off a balanced record and you get exactly zero, always. The up and down quarks would have to be twins. Since they are not twins, something in the reasoning has to give.

The tempting shortcut would be to just paint one side "up" by hand. But that is cheating — you would be assuming the very difference you are trying to explain.

The paper's answer is that the finished record is not the whole story, because it forgets something the universe does not: the *order* in which things happened. Think of the difference between a shop receipt and security-camera footage. The receipt lists what was bought — and two very different visits to the shop can produce identical receipts. The footage shows what happened first, what happened next, and crucially what only became *possible* because of what came before. In VERSF, commitments happen one after another and can never be undone, so the history has a direction, like an arrow. The receipt is symmetric. The arrow is not.

The paper proves three things in plain terms:

**First**, any attempt to read  $\chi$  from the finished record alone gives zero — not sometimes, but always, no matter how cleverly the reading is done. This is checked against every trick: hidden labels on the facts, sneaky ways of compiling the record, biased ways of reading it. Each loophole is identified, named, and closed.

**Second**, there is exactly one place the difference *can* live: in the comparison between the actual history and that same history run backwards. If running events in reverse order would have changed what could happen next, then the order itself is physically real — and that "directedness" is what  $\chi$  actually measures. The paper shows this is not a choice among many options; it is the only mathematical object that fits.

**Third**, if that directedness exists in the quark channel, then zooming out is not allowed to wash it away. The paper does not claim that every microscopic detail of the order survives — it claims something sharper. When the order of events genuinely changes what can happen next, any way of zooming out that erased that difference would no longer be describing the same physics. It would have quietly turned back into reading the finished record — and that road, as the first result showed, always gives zero.

Just as important is what the paper does *not* claim. It does not calculate how big the up/down difference is, why the difference shrinks by half from one quark generation to the next, or any actual quark mass. Those are jobs for later papers in the series. It also states openly that a companion result is still owed: someone must exhibit a concrete piece of quark history whose order genuinely matters, and the paper spells out exactly what that future proof must look like — so it cannot be fudged.

In one sentence: **up and down quarks can differ because the universe remembers the order in which things happened — and this paper proves that order is the only place the difference could possibly come from.**

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## Abstract

The  $W_7$  Fold Carrier and Readout Domain audit reduced the physical up/down susceptibility problem to a single domain question: does the  $\chi$  readout act on the ordered commitment history  $H$ , or only on the final standing record  $F$ ? If the readout factors through  $F$ , the fold mirror forces  $a_+ = a_-$ , hence  $\chi = 0$ , and the  $\chi$ -halving route is void. The present SM-4 paper answers that gate.

Three objects are distinguished: individual committed facts  $f_i$ , the final order-discarded record  $F(H)$ , and the ordered history

$$H = (f_1, \dots, f_n).$$

The final record is the symmetric quotient of the history: it forgets order. The  $\chi$  observable, by contrast, is shown to read the reversal-odd **ordered-commitment residue**

$$\Gamma(H) = \frac{1}{2} (U(H) - U(H \leftarrow)),$$

where  $U(H)$  is the ordered commitment product and  $H\leftarrow$  is the reversed history. Equivalently, in the history module,

$$\Gamma(H) = \frac{1}{2} (|H\rangle - |H\leftarrow\rangle).$$

This residue is invisible to  $F$ , since  $F(H) = F(H\leftarrow)$ , but it is not physically null when  $H$  and  $H\leftarrow$  possess different admissible continuation cones. That is the **commitment-continuation criterion**: irreversible commitment is physical exactly when the order of prior commitments constrains the future admissible extensions of the record.

The  $\chi$  readout is then the unique fold-odd log-access readout of this residue,

$$\chi(H) = \ell_{\chi}(P_{\chi} C_{\text{phys}} \Gamma(H)),$$

where  $C_{\text{phys}}$  is the admissible physical coarse-graining map,  $P_{\chi}$  projects onto the fold-odd up/down line, and  $\ell_{\chi}$  is the log-access functional. The induced positive access weights are

$$a_{\pm}(H) = a_0 \exp(\pm \frac{1}{2} \chi(H)), \text{ so that } \ln(a_+/a_-) = \chi(H).$$

The main theorem proves that under the ordered-commitment premises OC-1–OC-7: (i)  $\chi$  cannot be a function of  $F$  alone, with the fold-evenness of  $F$  derived from fact unsignedness and tally equivariance rather than assumed; (ii)  $\Gamma(H)$  is the unique reversal-odd, final-record-null object in the history pair module, up to scale; (iii) the residue survives every admissible continuation-faithful coarse-graining, and its fold-odd survivor is placed on  $L_{\chi}$  by an explicit fold-placement theorem rather than by assumption; and (iv)  $|\chi|$  is relabelling-invariant while  $\text{sgn } \chi$  reverses under fold exchange. The up/down contrast is therefore physical, not conventional.

The result closes SM-4 at ordered-readout level. It does not derive the magnitude of  $\chi$ , the halving law  $\rho = \frac{1}{2}$ , the first  $\chi$  increment, or the quark mass matrix. It establishes the necessary domain theorem: **the  $\chi$  observable reads ordered commitment history  $H$ , not merely the final standing record  $F$ .**

## 0. Inheritance, Notation, and Firewalls

### 0.1 The inherited problem

The  $W_7$  carrier/readout audit established four facts.

**First**, the fold cannot be treated as a primitive member-owned amplitude. It belongs at class grain; member-grain quantities act only as registrations of class-owned structure.

**Second**, a label-blind readout of a balanced final record forces

$$a_+ = a_-, \text{ and therefore } \chi = 0.$$

**Third**, an odd readout functional is not enough. An odd functional applied to a state with no odd-sector weight still returns zero. The obstruction is in the *domain*, not the functional.

**Fourth**, the only available source of non-zero  $\chi$  is the irreversible order of commitment: not facts in isolation, not the final tally, but the history by which the record was formed.

SM-4 exists to convert that obligation into a theorem.

## 0.2 Three domains

Notation used throughout:

- $f_i$  — a committed fact: a primitive committed distinction.
- $H = (f_1, \dots, f_n)$  — an ordered history.
- $H \leftarrow = (f_n, \dots, f_1)$  — its reversal.
- $F(H)$  — the final record: the order-discarded tally of the facts in  $H$ .

By construction,

$F(H) = F(H \leftarrow)$ , even when  $H \neq H \leftarrow$ .

This single line is the entire problem: **final-record equality does not imply history equality.**

## 0.3 The $\chi$ readout

Let  $P_+$  and  $P_-$  be the two fold projections in the up/down readout channel, exchanged by the fold mirror  $S$ :

$$S P_+ S^{-1} = P_-.$$

The fold-access weights are  $a_+(H)$  and  $a_-(H)$ , and the  $\chi$  observable is

$$\chi(H) = \ln( a_+(H) / a_-(H) ).$$

If  $a_+ = a_-$ , then  $\chi = 0$ .

## 0.4 SM-4 firewall

This paper proves:

- $\chi$  reads  $H$ , not only  $F$ ;
- the reversal-odd residue survives admissible physical coarse-graining.

It does **not** derive:

- the exact value of  $\chi_1$ ;

- the halving law  $\Delta\chi_{\{g+1\}} = \frac{1}{2} \Delta\chi_g$ ;
- the half-lazy operator  $W = \frac{1}{2}(I + \Pi_e)$  as a physical generation gate;
- the quark Yukawa matrices;
- the quark current-mass anchor;
- the Standard Model mass spectrum.

Those belong principally to SM-15, SM-17, SM-22, and SM-23.

## 0'. Predictive-Content Ledger

Object	Prior status	Status here
$\chi$ readout domain	open after $W_7$ carrier/readout audit	<b>closed conditionally: reads H, not only F</b>
Final-record readout R(F)	known to force $a_+ = a_-$	retained as explicit falsifier (F1)
S-invariance of F	assumed in $W_7$ -era statement	derived (Lemma 0) from OC-2' (unsigned facts + equivariant tally); priced by F1a/F1b
Existence of commitment-visible quark-class history ( $\neg F_2, \neg F_{11}$ )	open	<b>Owed</b> ; primary target at cone level with the class-quantified witness (§5.1, §13.2); algebraic proxy $Q_\chi U(H) \neq Q_\chi U(H\leftarrow)$ exact only under OC-8; gate assignment open
Operator-model $\chi$ -faithfulness (OC-8)	unpriced in the $W_7$ -era bridge between cone and operator criteria	stated as bridging premise; priced by F13; not used by Theorem 3
Ordered history H	necessary escape route	promoted to readout domain
Reversal $H \mapsto H\leftarrow$	gate test	used to define $\Gamma(H)$
Ordered-commitment residue $\Gamma(H)$	owed	constructed; uniqueness proven (Lemma 3)
Fold-odd projection $P_\chi \Gamma(H)$	owed	defined as $\chi$ -line source
Fold-placement criterion	open after $W_7$ carrier/readout audit	isolated as Theorem 2; conditional on OC-4a covariance and OC-4b odd-sector one-dimensionality
Coarse-graining survival	owed	proven for continuation-faithful projections
Relabelling canonicity	open	closed: $ \chi $ invariant, $\text{sgn } \chi$ conventional
Realization churn cancellation	obstacle	firewalled to charge-like readouts; not used for $\chi$
Half-lazy halving $\rho = \frac{1}{2}$ later structural gate		deferred to SM-15

Object	Prior status	Status here
First $\chi$ increment magnitude	open	deferred to SM-17
Full Yukawa matrices	open	deferred to SM-22

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## 1. Purpose, Claim Level, and Named Premises

### 1.1 The question

Does the physical  $\chi$  readout act on the ordered commitment history H, or only on the final standing record F?

The answer defended here:

$$\chi(H) = \ell_{\chi}(P_{\chi} C_{\text{phys}} \Gamma(H)),$$

where

$$\Gamma(H) = \frac{1}{2} (|H\rangle - |H\leftarrow\rangle)$$

in the history module, or equivalently, in the commitment-operator model,

$$\Gamma(H) = \frac{1}{2} (U(H) - U(H\leftarrow)).$$

$\chi$  reads the reversal-odd part of ordered commitment history.

### 1.2 Claim level

**Claim level:** SM-4 ordered-readout theorem, conditional on premises OC-1–OC-7 (including OC-2', OC-4a, OC-4b). Epistemic grade: **Conditional** (on OC-4a/OC-4b fold-channel placement and OC-5 continuation-faithfulness); all internal algebra **Exact**.

The theorem does not claim that any  $\chi$ -sector number is derived. It claims that the readout domain is fixed: H, not merely F. One further conditionality is recorded explicitly: SM-4 establishes the domain *given* the existence of at least one commitment-visible quark-class history (algebraic proxy: non-commutativity of the relevant commitment operators in the  $\chi$  quotient, exact under OC-8;  $\neg F2/\neg F11$ ). SM-4 does not exhibit such a history and does not inherit a certificate for one; existence is an **Owed** item, entered in the Predictive-Content Ledger with its gate assignment open. SM-4 is a domain theorem, not an existence theorem, and the ledger now says so.

#### 1.2a Scope precision: what SM-4 does and does not prove

SM-4 closes a domain question — not a magnitude question, and not yet an existence question. The theorem proved here is deliberately conditional and can be stated in sharpened form:

If a non-zero quark  $\chi$  observable exists within the VERSF fold-access sector, then it cannot be a readout of the final standing record F. It must be a readout of a reversal-odd ordered-history residue of the form  $\chi(H) = \ell_{\chi}(P_{\chi} C_{\text{phys}} \Gamma(H))$ .

Equivalently, SM-4 proves a disjunction:

- either  $\chi$  factors through the final order-discarded record F, in which case mirror symmetry forces  $\chi = 0$ ;
- or  $\chi$  is non-zero, in which case its domain must include ordered commitment history H — and, more precisely, the reversal-odd residue  $\Gamma(H)$ .

This is the exact scope of the paper. SM-4 does not yet exhibit the concrete quark-class commitment history that supplies  $\Gamma_{\chi} \neq 0$ ; that is the owed existence item of §1.2, given its target in §13.2. What SM-4 proves is that once such a history exists, the form of the  $\chi$  readout is no longer optional: it is forced by final-record collapse, reversal oddness, continuation faithfulness, fold-channel placement, and relabelling canonicity.

The precision matters. The paper is not deriving  $\chi$  by naming an ordered object  $\chi$ . It is proving that every final-record route collapses, and that the only surviving physical domain for a non-zero  $\chi$  observable is the ordered, reversal-odd, final-record-null component of commitment history.

No observed quark mass, quark mass ratio, CKM datum, Yukawa value, generation count, or empirical hierarchy is used in the argument. The proof uses only the structural distinction between facts, final records, ordered histories, reversal, admissible continuations, fold exchange, and log-access readout. Its output is not a fitted number but a domain theorem:  $\chi$ , if non-zero, reads ordered commitment history rather than the completed symmetric tally.

## 1.2b Three-level closure discipline

The  $\chi$  programme must be kept in three levels, because failure to separate them creates false closure.

**Level 1 — Domain.** What kind of object can  $\chi$  read without collapsing to zero? SM-4 answers this.  $\chi$  cannot be a final-record observable. If  $\chi$  is non-zero, its domain must include the ordered commitment residue  $\Gamma(H)$ .

**Level 2 — Existence.** Does the VERSF quark-class commitment algebra actually contain a history H whose reversal differs in the  $\chi$ -channel continuation cone? SM-4 does not answer this. It states the owed target exactly:

$$\text{Ext}_{\chi}(H) \neq \text{Ext}_{\chi}(H \leftarrow),$$

or, through the operator proxy under OC-8,

$$Q_{\chi} U(H) \neq Q_{\chi} U(H \leftarrow).$$

**Level 3 — Magnitude.** Once a live  $\chi$  residue exists, what is its value, how does it halve across generations, and how does it enter the Yukawa and mass hierarchy? SM-4 does not answer this either. Those questions belong to SM-15, SM-17, SM-22, and SM-23.

Thus SM-4 is not a mass theorem and not yet an existence theorem. It is the domain theorem that prevents the whole  $\chi$  programme from collapsing before the later existence and magnitude gates can begin.

### 1.3 Named premises

**Premise OC-1 — Commitment primacy.** The primitive substrate object is not an already completed tally but an irreversible sequence of commitments. A completed record is a quotient of a history, never the other way around.

**Premise OC-2 — Final-record quotient.** The final record map  $F: H \mapsto F(H)$  forgets order. Therefore  $F(H) = F(H \leftarrow)$ . Any readout factoring through  $F$  is order-blind.

**Premise OC-2' — Unsigned facts, equivariant tally.** Two clauses, stated separately because they fail separately. (i) *Fact unsignedness.* No individual committed fact carries primitive fold-odd content: the fold mirror acts trivially on every per-fact contribution,  $S C_i S^{-1} = C_i$ . (Falsifier F1a, equivalent to F8.) (ii) *Tally equivariance.* The record-forming map contains no reference to the fold labels: it commutes with  $S$ -conjugation,  $F(S \cdot S^{-1}) = S F(\cdot) S^{-1}$ . (Falsifier F1b.) Fold-odd content in a final record has exactly two possible sources — signed facts, or a fold-referencing construction rule — and the two clauses price them independently.

**Premise OC-3 — Continuation criterion.** Two histories with the same final record are physically distinct if they possess different admissible continuation cones: if  $\text{Ext}(H) \neq \text{Ext}(H \leftarrow)$ , the difference between  $H$  and  $H \leftarrow$  is not a gauge artefact.

**Premise OC-4a — Fold covariance of the ordered residue.** In the quark  $\chi$  sector, the continuation difference carried by the ordered residue is fold-covariant: fold exchange maps the (+) continuation cone into the (−) continuation cone and reverses the sign of the ordered residue. (Falsifiers F5a, F5c.)

**Premise OC-4b — Odd-sector uniqueness.** The fold-odd sector of the physical  $\chi$  quotient is one-dimensional and equals the access line  $L_{\chi}$ . (Falsifier F5b.)

Together, OC-4a and OC-4b entail that any non-zero continuation-visible residue odd under fold exchange projects into  $L_{\chi}$ , rather than into a charge-like even channel or a purely conventional label direction. They are stated separately because they fail separately.

**Premise OC-5 — Continuation-faithful coarse-graining.** A physical coarse-graining map may discard microscopic labels, but it may not identify histories that lead to different admissible continuation cones in the same physical channel. A projection that kills such a difference is not an admissible projection for that channel.

**Premise OC-6 — Log-access readout.** The physical  $\chi$  observable is a logarithmic access ratio:  $\chi = \ln(a_+/a_-)$ . Positive access weights are recovered from  $\chi$  by  $a^\pm = a_0 \exp(\pm\chi/2)$ . The two access weights are one functional applied to the two exchanged folds — mirror-conjugate by construction, hence fold-covariant. (Falsifier F1c.)

**Premise OC-7 — Fold relabelling canonicity.** Exchanging which fold is called + and which is called – reverses the sign of  $\chi$  but not its magnitude. Therefore  $|\chi|$  is physical, and  $\text{sgn } \chi$  is a fold-orientation convention unless tied to an external sector.

Each SM-4-local premise is paired with at least one falsifier in §13; OC-1 and OC-2 are inherited ontology and definition respectively, priced at programme level rather than here. One further bridging premise, OC-8 (operator-model  $\chi$ -faithfulness), is stated where it is used, in §5.2; it is priced by F13 and is not among the premises of Theorem 3.

## 2. The Final-Record Collapse

The first result is the no-go inherited from the  $W_7$  audit, restated and proven in SM-4 notation. Its single structural hypothesis — mirror symmetry of the final record — is first derived, not assumed.

**Lemma 0 — Fold-evenness of the unsigned final record.** *Under OC-1, OC-2, and OC-2', the final record of any history is fold-even:*

$$S F(H) S^{-1} = F(H).$$

*Proof.* Write  $F(H) = \mathbb{F}(C_1, \dots, C_n)$  for the record-forming map applied to the per-fact contributions (order-discarded, by OC-2). By OC-2'(ii),  $\mathbb{F}$  is S-equivariant:

$$\mathbb{F}(S C_1 S^{-1}, \dots, S C_n S^{-1}) = S \mathbb{F}(C_1, \dots, C_n) S^{-1}.$$

By OC-2'(i),  $S C_i S^{-1} = C_i$  for every  $i$ . Hence

$$S F S^{-1} = S \mathbb{F}(C_1, \dots, C_n) S^{-1} = \mathbb{F}(S C_1 S^{-1}, \dots, S C_n S^{-1}) = \mathbb{F}(C_1, \dots, C_n) = F.$$

An equivariant map applied to invariant inputs yields an invariant output. ■

**Reading.** The label-blindness of  $F$  is now a theorem of the tally structure rather than an assumption stated in passing, and both of its leak channels are priced. Clause (i) is not the only one: with every fact unsigned, the order-blind, multiset-respecting tally  $\mathbb{F} = P_+(\sum_i C_i) P_+$  satisfies

OC-2 and clause (i) as written, yet yields  $S F S^{-1} \neq F$  and  $a_+ \neq a_- = 0$ . The fold-odd content there is injected by a construction rule that references the fold labels, not by signed facts. Clause (ii) closes that route, and it is squarely within the paper's own discipline to price it: a fold-referencing tally is exactly what a hidden fold smuggle looks like at record-formation level (F1b), just as a signed fact is what it looks like at fact level (F1a/F8).

Suppose now the readout acts only on the final record  $F$ . Let the fold mirror  $S$  exchange  $P_+$  and  $P_-$ :

$$S P_+ S^{-1} = P_-.$$

By Lemma 0, the final record is label-blind:

$$S F S^{-1} = F.$$

The access weights are

$$a_+(F) = \text{Tr}(P_+ F), \quad a_-(F) = \text{Tr}(P_- F).$$

Substituting  $P_- = S P_+ S^{-1}$  and using cyclicity of the trace:

$$a_-(F) = \text{Tr}(S P_+ S^{-1} F) = \text{Tr}(P_+ S^{-1} F S) = \text{Tr}(P_+ F) = a_+(F).$$

Therefore

$$\chi(F) = \ln(a_+/a_-) = 0.$$

**Lemma 1 — Final-record collapse.** *Any  $\chi$  readout that factors through the final record of unsigned facts forces  $\chi = 0$ .*

*Proof.* By Lemma 0,  $S F S^{-1} = F$ . The two access functionals of a single readout applied to the two exchanged folds are mirror-conjugate by construction:  $r_-(X) := r_+(S^{-1} X S)$ . Invariance of  $F$  then gives

$$r_-(F) = r_+(S^{-1} F S) = r_+(F),$$

so  $a_+(F) = a_-(F)$  and  $\chi(F) = 0$ . The trace form  $a_{\pm} = \text{Tr}(P_{\pm} F)$  displayed above is the representative linear case, where the same conclusion follows from cyclicity; the argument requires only mirror-conjugacy of the access pair, not linearity of the readout; the fully covariant statement is Lemma 1a below. ■

**Reading.** This is not a small defect; it is a kill condition. If  $\chi$  reads only  $F$ , the up/down contrast is absent, and every later halving theorem would be halving zero. The collapse is driven entirely by the mirror symmetry of  $F$  — which is itself derived (Lemma 0) from fact unsignedness and tally equivariance, not assumed — and not by any property of the trace pairing: a generic non-

invariant operator gives  $a_+ \neq a_-$ . The obstruction is genuinely in the domain. The fully model-independent form follows.

## 2.1 Access-covariant form of the final-record collapse

The trace calculation above is a convenient representative model, not a hidden restriction. The collapse does not depend on the trace pairing, on linearity, or on any operator representation of the readout. It follows from fold covariance of any admissible access functional.

**Lemma 1a — Access-covariant collapse.** *Let  $A(P, F)$  be any access functional assigning an access weight to fold projection  $P$  in final record  $F$ , and assume only fold covariance:*

$$A(S P S^{-1}, S F S^{-1}) = A(P, F).$$

*Then for any fold-even final record,  $S F S^{-1} = F$ , the two fold accesses coincide:*

$$a_+(F) = a_-(F), \text{ hence } \chi(F) = 0.$$

*Proof.* With  $S P_+ S^{-1} = P_-$  and  $S F S^{-1} = F$ ,

$$a_-(F) = A(P_-, F) = A(S P_+ S^{-1}, F) = A(S P_+ S^{-1}, S F S^{-1}) = A(P_+, F) = a_+(F). \blacksquare$$

**Reading.** The final-record collapse is not an artefact of using a trace. Any fold-covariant final-record access functional — linear or not, operator-valued or not — gives the same result. Fold covariance is the minimal meaning of "the same readout applied to the two exchanged folds"; an access functional violating it does not read the two folds symmetrically and has smuggled an orientation into the readout itself — the readout-level analogue of F1b, priced as F1c. The obstruction is therefore wholly in the domain: an unsigned, order-discarded, fold-even final record contains no physical source for a fold-odd access contrast.

## 3. Ordered Commitment History

A final record does not contain enough information to carry  $\chi$ . The construction therefore moves to the ordered history space.

Let

$$H = (f_1, \dots, f_n)$$

be a finite ordered sequence of committed facts, and let

$$H^{\leftarrow} = (f_n, \dots, f_1)$$

be its reversal. The final-record map satisfies

$$F(H) = F(H\leftarrow),$$

because the two histories contain the same facts. But the ordered histories are not identical:

$$H \neq H\leftarrow,$$

unless the commitment order is physically irrelevant.

### 3.1 Operator model

Represent each committed fact  $f_i$  by a commitment operator  $C_i$ . The ordered commitment product is

$$U(H) = C_n C_{n-1} \cdots C_1,$$

and the reversed product is

$$U(H\leftarrow) = C_1 C_2 \cdots C_n.$$

If all  $C_i$  commute in the relevant quotient, the two products coincide. If they do not, the difference is an **order-current**.

At tally level, the final record is simply the multiset  $F(H) = \{f_1, \dots, f_n\}$ . In the operator model it may be represented by the fully symmetrised product

$$\text{Sym}(C_1, \dots, C_n) = (1/n!) \sum_{\sigma} C_{\sigma(n)} \cdots C_{\sigma(1)},$$

the average over all  $n!$  orderings, which is permutation-invariant by construction. Under either representation,  $F$  discards exactly the non-commuting order information that  $U(H)$  retains.

### 3.2 The history module and admissibility of the reversal

Irreversibility raises an immediate formal objection: if commitment is irreversible, the reversed sequence  $H\leftarrow$  may not be an admissible history at all —  $f_n$  may be inadmissible as an opening commitment. The construction is therefore stated on the **formal module of ordered sequences** of committed facts. Admissibility is not a membership condition on the module; it is encoded entirely in the continuation structure: an inadmissible sequence is a well-defined formal element with

$$\text{Ext}(H\leftarrow) = \emptyset.$$

This is not a weakening. If  $H\leftarrow$  is inadmissible while  $H$  is not, then

$$\text{Ext}(H) \neq \emptyset = \text{Ext}(H\leftarrow),$$

and the continuation cones differ *maximally*: reversal is not merely continuation-altering but continuation-annihilating. Inadmissibility of the reversal is thus the strongest possible form of commitment-visible order (Definition 1), and it strengthens rather than threatens the continuation-separation lemma.  $\Gamma(H)$  never compares a physical object to a fiction; it compares a history to a formal element whose entire physical content is the statement "no continuation is admissible from this order" — which is itself continuation information.

### 3.3 Why this is not assigning signs to facts

No individual fact  $f_i$  carries a primitive + or – sign. The sign appears only in the comparison between an ordered sequence and its reversal. The source of  $\chi$  is therefore not a labelled fact; it is the **orientation of a committed history**.

The distinction matters. Labelling facts + or – would be an arbitrary convention. Reading the reversal-odd part of an irreversible history is not arbitrary, because commitment order is physical precisely when it changes admissible continuations (§5).

## 4. The Ordered-Commitment Residue

Define the ordered-commitment residue in the history module by

$$\Gamma(H) = \frac{1}{2} (|H\rangle - |H\leftarrow\rangle),$$

or equivalently, in the operator model,

$$\Gamma(H) = \frac{1}{2} (U(H) - U(H\leftarrow)).$$

### 4.1 Reversal-odd property

Let  $T$  be the reversal operator,  $T|H\rangle = |H\leftarrow\rangle$ . Then

$$T \Gamma(H) = \frac{1}{2} (|H\leftarrow\rangle - |H\rangle) = -\Gamma(H).$$

$\Gamma(H)$  is reversal-odd.

### 4.2 Final-record nullity

Extending  $F$  linearly to formal combinations in the history module, and using  $F(H) = F(H\leftarrow)$ ,

$$F(\Gamma(H)) = \frac{1}{2} (F(H) - F(H\leftarrow)) = 0.$$

$\Gamma(H)$  is invisible to the final-record map.

**Lemma 2 — Ordered-residue properties.** *The ordered-commitment residue  $\Gamma(H)$  is reversal-odd and final-record-null:*

$$T \Gamma(H) = -\Gamma(H) \text{ and } F(\Gamma(H)) = 0.$$

*Proof.* Direct from the definitions. ■

### 4.3 Uniqueness of the residue

The residue is not merely one candidate among many. Within the pair module it is forced.

**Lemma 3 — Uniqueness of the ordered residue.** *Let  $M_H = \text{span}\{|H\rangle, |H\leftarrow\rangle\}$  be the two-dimensional pair module of a history and its reversal ( $H \neq H\leftarrow$ ), with  $T$  the swap. Then the reversal-odd sector of  $M_H$  is one-dimensional and is spanned by  $|H\rangle - |H\leftarrow\rangle$ . Consequently every reversal-odd, final-record-null element of  $M_H$  is proportional to  $\Gamma(H)$ .*

*Proof.*  $T$  acts on  $M_H$  as the exchange of basis elements. Its eigenvalues are  $+1$  and  $-1$ , each with a one-dimensional eigenspace: the even sector spanned by  $|H\rangle + |H\leftarrow\rangle$  and the odd sector spanned by  $|H\rangle - |H\leftarrow\rangle$ . Any reversal-odd element therefore lies in the odd sector and is a scalar multiple of  $|H\rangle - |H\leftarrow\rangle = 2\Gamma(H)$ . Final-record nullity holds automatically on this line by Lemma 2. ■

**Reading.** Lemma 3 upgrades the construction from "an object that works" to "the only object available." Given that the readout must be odd under reversal (§9), there is no freedom in the choice of residue beyond overall scale — and scale is exactly what SM-4 defers to the magnitude gates.

## 5. The Commitment-Continuation Criterion

An order-residue could still be empty physics if reversing the history changed notation but not admissible consequences. SM-4 therefore requires a physical criterion for when order matters.

For a history  $H$ , let

$$\text{Ext}(H)$$

be its **admissible continuation cone**: the set of future commitment extensions allowed after  $H$ .

If  $\text{Ext}(H) = \text{Ext}(H\leftarrow)$ , the order difference is continuation-invisible. If

$$\text{Ext}(H) \neq \text{Ext}(H\leftarrow),$$

the order difference changes what the substrate can next commit, and is therefore physical.

**Independent characterisation of  $\text{Ext}_\chi$ .** The  $\chi$ -channel cone must be defined without reference to the quotient later built from it, or the maximal-quotient construction of §7.3 is circular. The channel is fixed upstream: the fold-odd access line  $L_\chi$  is an inherited object of the  $W_7$  Fold Carrier and Readout Domain audit, existing prior to this paper.  $\text{Ext}_\chi(H)$  is then characterised operationally as a quotient, not a subset: it is the set of  $\chi$ -equivalence classes of admissible continuations of  $H$ , where two continuations are  $\chi$ -equivalent when no fold-access observable on  $L_\chi$  distinguishes their class-grain registrations. Fold-silent continuations are thereby merged into common classes, not discarded from the cone. On the formal module of §3.2 this is well-defined without exception: an inadmissible sequence has the empty set of classes,  $\text{Ext}_\chi = \emptyset$ . The channel is defined by the inherited readout line; the quotient of §7.3 is constructed from independently characterised cones, not the other way around.

**Definition 1 — Commitment-visible order.** A history  $H$  carries commitment-visible order if  $\text{Ext}(H) \neq \text{Ext}(H\leftarrow)$  in the relevant physical channel.

**Lemma 4 — Continuation separation.** *If  $H$  carries commitment-visible order, then  $\Gamma(H)$  is not a null gauge residue.*

*Proof.* Suppose  $\Gamma(H)$  were gauge-null. Then  $|H\rangle$  and  $|H\leftarrow\rangle$  would be physically identified. Physical identification preserves admissible continuation cones, so  $\text{Ext}(H) = \text{Ext}(H\leftarrow)$ , contradicting commitment-visible order. Therefore  $\Gamma(H)$  is not null. ■

**Reading.** This is the point at which "history" becomes physics. Order is not retained because it is mathematically available; it is retained because the order of irreversible commitments changes the future admissible structure. This is the operational content of OC-1: irreversibility means the past constrains the future asymmetrically.

## 5.1 Operational continuation test

The continuation criterion can be given direct test content, using the channel characterisation above.

Let  $\text{Adm}_\chi$  denote the class of admissible future commitments visible in the inherited  $\chi$  channel — the channel being defined upstream by the fold-odd access line  $L_\chi$ , never by the numerical value of  $\chi$ . For a history  $H$  and a future extension  $K$ , write  $K \circ H$  for the concatenated history and  $[K]_\chi$  for the  $\chi$ -equivalence class of  $K$  under the characterisation above. Then  $H$  and  $H\leftarrow$  are physically distinct in the  $\chi$  channel exactly when there exists a  $\chi$ -class  $\kappa$  such that some  $K \in \kappa$  satisfies  $K \circ H \in \text{Adm}_\chi$  while *no*  $\chi$ -equivalent continuation  $K' \in \kappa$  satisfies  $K' \circ H\leftarrow \in \text{Adm}_\chi$  — or conversely. Any such class  $\kappa$  is a witness:  $\kappa$  is inhabited in  $\text{Ext}_\chi(H)$  and absent from  $\text{Ext}_\chi(H\leftarrow)$ , so

$$\text{Ext}_\chi(H) \neq \text{Ext}_\chi(H\leftarrow).$$

The quantification over the class is essential, not pedantic. A single non-membership does not evacuate a class: if  $K$  is admissible after  $H$  and inadmissible after  $H\leftarrow$ , but a  $\chi$ -equivalent  $K'$  remains admissible after  $H\leftarrow$ , then  $\kappa$  is inhabited on both sides and the cones may coincide

despite the single-representative witness firing. The class-quantified test places deliberate load on the existence gate — the gate must kill the whole equivalence class on one side, not one representative. A proof discharging the obligation with a single-representative witness has not established  $\text{Ext}_{\chi}(H) \neq \text{Ext}_{\chi}(H\leftarrow)$  at all.

The test asks nothing about the value of  $\chi$ . It asks only whether the order of prior commitments changes which later  $\chi$ -visible commitments remain admissible, at the resolution of  $\chi$ -classes.

The reversed history  $H\leftarrow$  is therefore not introduced as a second realised physical history. It is the canonical *test object* for whether the realised history  $H$  carries orientation-dependent admissibility (§3.2). If  $H\leftarrow$  is inadmissible outright, then  $\text{Ext}_{\chi}(H\leftarrow) = \emptyset$  and the asymmetry is not weakened but maximised: the realised order admits continuation while its reversal admits none.

## 5.2 Continuation asymmetry and the commutator shadow

The continuation-cone criterion and the operator skeleton of Appendix A are two faces of the same obstruction.

Let  $Q_{\chi}$  denote the  $\chi$ -channel quotient map:  $C_{\text{phys}}$  followed by restriction to the  $\chi$  quotient of §7.3, *without* the fold projection  $P_{\chi}$ . The composition chain is then fixed once and for all:

$$\Gamma_{\chi}(H) = P_{\chi} Q_{\chi} \Gamma(H),$$

which coincides with the §6 definition  $\Gamma_{\chi}(H) = P_{\chi} C_{\text{phys}} \Gamma(H)$ , since  $P_{\chi}$  acts within the  $\chi$  quotient and hence  $P_{\chi} Q_{\chi} = P_{\chi} C_{\text{phys}}$  — there is one definition, written at two resolutions. Theorem 1's conclusion —  $C_{\text{phys}} \Gamma(H) \neq 0$  in the  $\chi$  quotient — is precisely  $Q_{\chi} \Gamma(H) \neq 0$ . In the operator model,  $H = (f_1, \dots, f_n)$  is represented by the ordered product  $U(H) = C_n C_{n-1} \dots C_1$  and its reversal by  $U(H\leftarrow) = C_1 C_2 \dots C_n$ . If

$$Q_{\chi} U(H) = Q_{\chi} U(H\leftarrow),$$

the ordered difference is invisible in the  $\chi$  channel and  $\Gamma_{\chi}(H) = 0$ . If instead

$$Q_{\chi} U(H) \neq Q_{\chi} U(H\leftarrow),$$

the ordered product carries a non-zero order-current in the  $\chi$  quotient. For a two-step history  $H = (A, B)$  this reduces to

$$Q_{\chi} \Gamma(H) = \frac{1}{2} Q_{\chi}(BA - AB) = \frac{1}{2} Q_{\chi}[B, A],$$

with the  $\chi$  readout obtained by the further projection  $\Gamma_{\chi}(H) = P_{\chi} Q_{\chi} \Gamma(H)$ .

Non-commutation *in the  $\chi$  quotient* is thus the algebraic shadow of continuation asymmetry:  $\text{Ext}_{\chi}(H) \neq \text{Ext}_{\chi}(H\leftarrow)$  says the two orders admit different future  $\chi$ -visible commitments;  $Q_{\chi} U(H) \neq Q_{\chi} U(H\leftarrow)$  says an ordered difference survives algebraically. The quotient qualifier is

essential and cuts both ways. Upstream non-commutation does not suffice — a churn-cancelling quotient such as the charge readout annihilates every commutator identically, which is exactly Lemma 8's content.

The relation between the two criteria must, however, be stated with care, because they are not equivalent under OC-1–OC-7 alone. The cone criterion is the physical one. The algebraic criterion is its computable signature, and the two coincide exactly when the operator model presents the channel faithfully:

**Bridging premise OC-8 — Operator-model  $\chi$ -faithfulness.**  $Q_\chi U(H_1) = Q_\chi U(H_2)$  if and only if  $H_1 \sim_\chi H_2$ . (Falsifier F13.) OC-8 is *not* among the premises of Theorem 3, which is stated entirely at cone level; it certifies only the algebraic proxy used here and in §13.2.

Under OC-8,  $Q_\chi U(H) \neq Q_\chi U(H\leftarrow)$  holds if and only if  $\text{Ext}_\chi(H) \neq \text{Ext}_\chi(H\leftarrow)$ , and the commutator test is exact. Without OC-8 the two failure modes come apart: products could coincide while cones differ (order information the algebra does not represent), or products could differ while cones coincide (an algebraic residue with no continuation content — F5c territory).

Note also what the algebraic certificate delivers when it succeeds:  $Q_\chi[B, A] \neq 0$  is a *survival* certificate, establishing  $Q_\chi \Gamma(H) \neq 0$ . Placement on  $L_\chi$  is a separate matter, already closed conditionally by Theorem 2 under OC-4a/OC-4b. The bridge therefore prevents the continuation criterion from floating as a purely verbal condition — it has an algebraic signature, the signature is computable — while pricing exactly what the signature does and does not certify.

## 6. Fold Projection and the $\chi$ Line

The ordered residue may have components in several channels.  $\chi$  requires the fold-odd up/down component.

Let  $P_\chi$  be the projection onto the fold-odd  $\chi$  readout line  $L_\chi$ , and define

$$\Gamma_\chi(H) = P_\chi C_{\text{phys}} \Gamma(H),$$

where  $C_{\text{phys}}$  is the physical coarse-graining from microscopic histories to the class-grain readout space. By OC-4a (fold covariance) and OC-4b (odd-sector uniqueness), in the quark  $\chi$  sector,

$$\Gamma_\chi(H) \neq 0$$

for the relevant up/down ordered-commitment class.

### 6.1 Fold mirror action

The fold mirror  $S$  exchanges the two fold labels. On the  $\chi$  line,

$$S \Gamma_{\chi}(H) = -\Gamma_{\chi}(H).$$

The mirror does not erase the residue; it reverses its sign. That is exactly the behaviour a physical up/down contrast must exhibit: relabelling  $+ \leftrightarrow -$  changes the sign of the log-ratio, never its magnitude.

## 6.2 Canonical log-access functional

Since  $L_{\chi}$  is one-dimensional at readout level, any non-zero fold-odd linear readout on it is proportional to a canonical functional

$$\ell_{\chi}: L_{\chi} \rightarrow \mathbb{R}.$$

The scale of  $\ell_{\chi}$  is not fixed by SM-4; scale belongs to the later magnitude gates. SM-4 fixes the domain and survival of the readout, not its absolute size.

Define

$$\chi(H) = \ell_{\chi}(\Gamma_{\chi}(H)).$$

Under fold relabelling,  $\chi \mapsto -\chi$ , and therefore  $|\chi|$  is relabelling-invariant.

**Lemma 5 — Fold-odd readout canonicity.** *If the ordered residue projects non-trivially onto the one-dimensional fold-odd line  $L_{\chi}$ , then the  $\chi$  readout is unique up to overall scale:*

$$\chi(H) = \ell_{\chi}(P_{\chi} C_{\text{phys}} \Gamma(H)).$$

*Its sign changes under fold relabelling; its magnitude is invariant.*

*Proof.* A one-dimensional real readout line admits a unique non-zero linear functional up to scalar normalization. Fold exchange acts by  $-1$  on the odd line, so the functional changes sign under relabelling; the absolute value is unchanged. ■

Together, Lemma 3 and Lemma 5 give a two-stage rigidity: the residue is unique up to scale in the pair module, and the readout of its  $\chi$ -line projection is unique up to scale on  $L_{\chi}$ . The only free parameters anywhere in the construction are normalizations — precisely the quantities SM-4 declares out of scope.

## 6.3 The fold-channel placement criterion

The remaining danger is that the ordered residue may survive coarse-graining but land in the wrong channel. SM-4 therefore needs a criterion for when a continuation-visible ordered residue is specifically a  $\chi$  residue.

Let  $S$  be the fold mirror exchanging the two up/down access projections:

$$S P_+ S^{-1} = P_-.$$

Any class-grain continuation residue  $v$  decomposes uniquely into its fold-even and fold-odd parts:

$$v_{\text{even}} = \frac{1}{2} (v + Sv), \quad v_{\text{odd}} = \frac{1}{2} (v - Sv).$$

The even part is visible to charge-like or standing-record readouts. The odd part is visible only to fold-susceptibility readouts. Since  $\chi$  is defined as the logarithmic contrast between the two exchanged fold accesses,  $\chi$  can only read the odd part.

The fold-channel placement criterion is therefore:

$$P_{\chi} C_{\text{phys}} \Gamma(H) \neq 0$$

if and only if the continuation-visible part of  $C_{\text{phys}} \Gamma(H)$  has a non-zero fold-odd component. Equivalently,

$$C_{\text{phys}} \Gamma(H) \neq S C_{\text{phys}} \Gamma(H).$$

If instead

$$C_{\text{phys}} \Gamma(H) = S C_{\text{phys}} \Gamma(H),$$

the surviving residue is fold-even and cannot be  $\chi$ .

This criterion prevents a hidden smuggle. The paper does not say that any surviving history residue is automatically  $\chi$ . It says that a surviving ordered residue is  $\chi$  only when it is odd under the same fold exchange that swaps  $P_+$  and  $P_-$ .

## 6.4 Even-channel nullity

A useful way to state the separation:

**Lemma 6 — Even-channel nullity of the ordered  $\chi$  residue.** *Let  $R_{\text{even}}$  be any fold-even readout,  $R_{\text{even}} S = R_{\text{even}}$ . If  $\Gamma_{\chi}$  is fold-odd,  $S \Gamma_{\chi} = -\Gamma_{\chi}$ , then*

$$R_{\text{even}} \Gamma_{\chi} = 0.$$

*Proof.* Since  $R_{\text{even}} S = R_{\text{even}}$ ,

$$R_{\text{even}} \Gamma_{\chi} = R_{\text{even}} S \Gamma_{\chi} = R_{\text{even}} (-\Gamma_{\chi}) = -R_{\text{even}} \Gamma_{\chi},$$

and therefore  $R_{\text{even}} \Gamma_{\chi} = 0$ . ■

**Reading.** The  $\chi$  residue cannot also be a charge-like even residue. A charge-style readout sees the standing tally;  $\chi$  sees the fold-odd ordered residue. This strengthens the domain separation of §10 from a firewall into a vanishing theorem: even-channel readouts do not merely *decline* to see the  $\chi$  residue — they are algebraically blind to it.

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## 7. Coarse-Graining Survival

The  $W_7$  audit identified the key danger: even if order exists upstream, the physical readout may coarse-grain it away, just as the Realization Theorem cancels churn in the charge readout. SM-4 must therefore prove that the  $\chi$  readout does not employ a coarse-graining that kills the order-residue.

### 7.1 Admissible coarse-graining

A physical coarse-graining map  $C_{\text{phys}}$  is **admissible for a channel** only if it preserves all distinctions that change admissible continuations in that channel (OC-5). Thus, if

$$\text{Ext}_{\chi}(H) \neq \text{Ext}_{\chi}(H\leftarrow),$$

then

$$C_{\text{phys}}|H\rangle \neq C_{\text{phys}}|H\leftarrow\rangle$$

in the  $\chi$  readout quotient — equivalently,

$$C_{\text{phys}} \Gamma(H) \neq 0$$

in the  $\chi$  quotient.

### 7.2 Survival theorem

**Theorem 1 — Coarse-graining survival of the ordered  $\chi$  residue.** *Let  $H$  be a commitment-visible history in the  $\chi$  channel, and let  $C_{\text{phys}}$  be an admissible continuation-faithful physical coarse-graining. Then*

$$C_{\text{phys}} \Gamma(H) \neq 0$$

*in the  $\chi$  quotient.*

*Proof.* Since  $H$  is commitment-visible in the  $\chi$  channel,  $\text{Ext}_{\chi}(H) \neq \text{Ext}_{\chi}(H\leftarrow)$ . By continuation-faithfulness (OC-5), an admissible coarse-graining cannot identify histories with different  $\chi$ -channel continuation cones. Hence  $C_{\text{phys}} \Gamma(H) \neq 0$  in the  $\chi$  quotient. ■

Theorem 1 deliberately claims survival and nothing more. Placement of the survivor on  $L_\chi$  is a logically separate matter, carried entirely by Theorem 2; the division of labour is exact: Theorem 1 failure is falsifier F5's survival branch, Theorem 2 failure is F5a/F5b/F5c.

**Scope note.** Theorem 1 is conditional on OC-5 alone, and the conditionality carries physical content rather than weakening the claim. OC-5 is not a convenience assumption: it is the statement of what "physical" means for a coarse-graining in a given channel. A projection that erases a continuation-relevant distinction is not a coarser description of the same physics — it is a description of *different* physics. Consequently, the only way to evade Theorem 1 is to use a projection that identifies histories with different admissible futures, and any such projection reduces the readout to a final-record readout in disguise. That case is already Lemma 1 and forces  $\chi = 0$ . There is no third option: every  $\chi$  readout either respects continuation structure and inherits a non-zero residue, or violates it and collapses.

### 7.3 The maximal continuation-faithful quotient

The phrase "continuation-faithful coarse-graining" should not be read as an optional preference. It names the maximal quotient that still describes the same physical channel.

Define an equivalence relation on histories by

$$H_1 \sim_\chi H_2 \text{ if and only if } \text{Ext}_\chi(H_1) = \text{Ext}_\chi(H_2),$$

that is, the two histories have the same  $\chi$ -channel admissible continuation cone. The corresponding quotient is

$$\mathcal{H}_\chi / \sim_\chi.$$

A coarse-graining is  $\chi$ -admissible exactly when it factors through this quotient but not through any coarser quotient that identifies histories with different  $\chi$ -channel continuation cones.

Thus, if  $\text{Ext}_\chi(H) \neq \text{Ext}_\chi(H\leftarrow)$ , then  $H$  and  $H\leftarrow$  occupy different equivalence classes in the  $\chi$  quotient, and consequently

$$C_{\text{phys}} \Gamma(H) \neq 0$$

for every admissible  $\chi$ -channel coarse-graining.

**Lemma 7 — Maximal faithful quotient.** *The coarsest admissible  $\chi$  readout quotient is the quotient by equality of  $\chi$ -channel continuation cones. Any further quotient that identifies  $H$  with  $H\leftarrow$  despite  $\text{Ext}_\chi(H) \neq \text{Ext}_\chi(H\leftarrow)$  is not a coarse-graining of the same  $\chi$  physics; it is a projection to a different, order-blind observable.*

*Proof.* A physical quotient may identify states only when no observable continuation in the channel distinguishes them. Equality of continuation cones is precisely that indistinguishability condition. If two histories have different continuation cones, identifying them erases a physically

admissible future distinction. Such an identification is therefore not a faithful coarse-graining of the  $\chi$  channel. ■

**Reading.** This makes OC-5 structural rather than stipulated. The continuation-faithful quotient is not chosen to save  $\chi$ ; it is the weakest quotient that still preserves the physical content of the  $\chi$  channel. OC-5 thereby reduces to the statement that the  $\chi$  readout is a readout *of the  $\chi$  channel* — anything coarser is a readout of something else.

**Scope precision.** The quotient by equality of  $\chi$ -channel continuation cones is maximal only for the continuation content relevant to SM-4. The claim is not that continuation cones exhaust every microscopic distinction that could ever be defined in the  $\chi$  sector. The claim is narrower and sufficient: equality of  $\chi$ -channel continuation cones defines the coarsest quotient that preserves the continuation distinctions on which the ordered-readout theorem depends. Two histories may therefore differ in microscopic labels, presentation, or inert bookkeeping while lying in the same  $\chi$ -continuation class; such differences are invisible to SM-4. What cannot be erased is a difference in the admissible  $\chi$ -visible future structure. That is the physical content protected by OC-5.

## 7.4 The fold-placement theorem

Survival alone does not place the residue in  $\chi$ . The placement claim is now isolated as its own theorem.

**Theorem 2 — Fold-placement theorem.** *Let  $\Gamma(H)$  be a continuation-visible ordered residue. Suppose:*

1. *the residue survives admissible  $\chi$ -channel coarse-graining:  $C_{\text{phys}} \Gamma(H) \neq 0$ ;*
2. *the surviving residue is odd under fold exchange:  $S C_{\text{phys}} \Gamma(H) = -C_{\text{phys}} \Gamma(H)$ ;*
3. *the fold-odd sector of the  $\chi$  quotient is one-dimensional and equals the access line  $L_{\chi}$ .*

*Then*

$$P_{\chi} C_{\text{phys}} \Gamma(H) \neq 0,$$

*and the residue is a  $\chi$ -channel residue.*

*Proof.* By assumption 2,  $C_{\text{phys}} \Gamma(H)$  lies in the fold-odd sector of the  $\chi$  quotient. By assumption 3, that sector is exactly the one-dimensional line  $L_{\chi}$ , so the non-zero survivor lies on  $L_{\chi}$ . Since  $P_{\chi}$  is the projection onto  $L_{\chi}$ ,

$$P_{\chi} C_{\text{phys}} \Gamma(H) = C_{\text{phys}} \Gamma(H) \neq 0. \blacksquare$$

**Precision note on assumption 3.** The hypothesis must be that the fold-odd sector *is* one-dimensional, not merely that a distinguished odd line exists within a larger odd sector. If the odd sector had two or more inequivalent lines, a genuinely fold-odd survivor could lie entirely off  $L_{\chi}$ , giving  $P_{\chi} C_{\text{phys}} \Gamma(H) = 0$  despite assumptions 1 and 2 holding. That failure mode is

exactly falsifier F5b. One-dimensionality of the odd sector is therefore precisely the load-bearing content of OC-4b, which is why it is stated as a separate premise with its own falsifier.

**Reading.** The theorem cleanly separates three claims previously bundled together: survival, fold-oddness, and one-dimensional  $\chi$  placement. If any one fails, SM-4 fails locally and explicitly (F5, F5a, F5b, F5c respectively). If all three hold, the ordered residue lands in  $\chi$  without further assumption.

## 8. The Ordered-Commitment $\chi$ Readout Theorem

Assemble the pieces. Define

$$\Gamma(H) = \frac{1}{2} (|H\rangle - |H\leftarrow\rangle), \Gamma_{\chi}(H) = P_{\chi} C_{\text{phys}} \Gamma(H), \chi(H) = \ell_{\chi}(\Gamma_{\chi}(H)),$$

and the positive fold-access weights

$$a_{\pm}(H) = a_0 \exp(\pm \frac{1}{2} \chi(H)), \text{ so that } \ln(a_{+}(H) / a_{-}(H)) = \chi(H).$$

**Theorem 3 — Ordered-Commitment  $\chi$  Readout Theorem.** *Under premises OC-1–OC-7, the physical  $\chi$  observable reads the ordered commitment history  $H$ , not merely the final record  $F$ . Precisely:*

1. *any readout factoring through the fold-even final record  $F$  forces  $\chi = 0$ ;*
2. *the ordered residue  $\Gamma(H)$  is final-record-null but reversal-odd, and is the unique such object in the pair module up to scale;*
3. *if  $H$  and  $H\leftarrow$  have distinct  $\chi$ -channel continuation cones, then  $\Gamma(H)$  is physically non-null;*
4. *every admissible continuation-faithful coarse-graining preserves the continuation-visible residue, and by the fold-placement theorem its fold-odd survivor projects non-trivially into  $L_{\chi}$ ;*
5. *the induced  $\chi(H)$  is relabelling-canonical:  $|\chi|$  is physical, while  $\text{sgn } \chi$  reverses under fold exchange.*

*Therefore the  $\chi$  readout is a physical ordered-history readout,*

$$\chi(H) = \ell_{\chi}(P_{\chi} C_{\text{phys}} \Gamma(H)),$$

*and not a function of  $F(H)$  alone.*

*Proof.* (1) is Lemma 1. (2) is Lemma 2 together with Lemma 3. (3) is Lemma 4. (4) is Theorem 1 (survival) combined with Theorem 2 (fold placement), whose fold-oddness and odd-line-uniqueness hypotheses are supplied by OC-4a and OC-4b respectively. (5) is Lemma 5, with Lemma 6 guaranteeing that no fold-even readout can substitute.

Combining: any readout of  $F$  alone yields  $\chi = 0$  by (1); a non-zero  $\chi$  therefore requires domain content invisible to  $F$ . By (2), the only reversal-odd, final-record-null content available in the pair module is  $\Gamma(H)$ , up to scale. By (3) and (4), this content is physically non-null, survives every admissible coarse-graining into the class-grain readout, and lands specifically on the  $\chi$  access line rather than in an even or conventional channel. By (5), the resulting observable has invariant magnitude and conventional sign. Hence  $\chi$  cannot be a function of  $F$  alone and must read the ordered commitment residue of  $H$ . ■

## 9. Why This Is Not a Construction Smuggle

A natural objection: the paper has simply invented an  $H$ -readout because  $\chi$  needed one. That is not the structure of the argument.

The readout is forced by four independent requirements:

1. **Final-record nullity.** The readout must vanish on any pure final-record tally, because the final record of unsigned facts under an equivariant tally is fold-even (Lemma 0) and so contains no up/down contrast (Lemma 1).
2. **Reversal oddness.** The readout must change sign under history reversal, because  $\chi$  is the logarithmic distinction between two exchanged fold orientations.
3. **Continuation faithfulness.** The readout must preserve distinctions that alter future admissible commitment cones (OC-5).
4. **Relabelling canonicity.** Fold-name exchange must change the sign of  $\chi$  but not  $|\chi|$  (OC-7).

The ordered-commitment residue  $\Gamma(H) = \frac{1}{2}(H - H\leftarrow)$  is not merely *a* solution to these constraints; by Lemma 3 it is the **unique** solution in the pair module up to scale. A final-record readout fails requirement 1 outright. An arbitrary labelled readout violates canonicity. A hidden member-grain amplitude violates the  $W_7$  carrier-layer audit. The ordered-residue readout is therefore not chosen because it delivers a desired quark number; it is selected because it is the only surviving domain on which a physical  $\chi$  observable can live.

Two independence checks close the section:

- **No empirical input.** SM-4 uses no quark mass value. No measured  $m_u/m_d$ ,  $m_c/m_s$ ,  $m_t/m_b$ ,  $\rho$ , or  $\Delta\chi$  enters any proof.
- **No target-fitting freedom.** The per-pair residue is unique up to scale (Lemma 3, correctly scoped to the pair module). The residue normalization, the  $\ell_\chi$  scale, and the weighting by which class-grain readout aggregates over commitment-visible histories are all deferred to the magnitude gates; none contributes tunable freedom within SM-4's claims, because SM-4 outputs no number. The rigidity claimed here is domain rigidity, not magnitude rigidity.

The same discipline applies to fold placement. The paper does not infer  $P_\chi C_{\text{phys}} \Gamma(H) \neq 0$  merely from  $\Gamma(H) \neq 0$ . A generic history residue could survive in an even channel, in a non-quark channel, or in a convention-only label sector. SM-4 claims  $\chi$  placement only after three further conditions are imposed: the survivor must be continuation-visible in the quark  $\chi$  quotient, odd under fold exchange, and resident in the one-dimensional fold-odd access line  $L_\chi$ . These are exactly the assumptions isolated in OC-4a, OC-4b, and the fold-placement theorem (Theorem 2). The argument therefore does not say "history exists, therefore  $\chi$  exists." It says "a continuation-visible history residue that is fold-odd in the quark access quotient is precisely the  $\chi$  residue."

## 10. Separation from Charge Readout and Realization Churn

The Realization Theorem's charge readout cancels churn — and that is exactly what it should do. Charge is a standing quantity: it must not depend on which admissible realization path produced the record. For charge, order-cancellation is a feature.

$\chi$  has the opposite status.  $\chi$  is not a standing charge; it is the fold-odd susceptibility residue. If its readout cancels the ordered commitment residue, it becomes a final-record readout, and Lemma 1 forces  $\chi = 0$ .

Charge and  $\chi$  therefore cannot share a readout domain:

Readout	Domain	Order treatment	Result
Charge	final standing record F	churn cancels	stable q/k
$\chi$	ordered history H	reversal-odd residue survives	non-zero up/down contrast

This is not an inconsistency; it is role separation. Some observables are standing-tally observables.  $\chi$  is not one of them.

**Lemma 8 — Charge/ $\chi$  domain separation.** *Any readout whose defining operation is the full cancellation of realized order-dependence cannot serve as the  $\chi$  readout unless  $\chi$  vanishes.*

*Proof.* If realized order-dependence cancels fully, the readout factors through the final order-blind record F. By Lemma 1,  $\chi = 0$ . Therefore any non-zero  $\chi$  readout lies outside the churn-cancelling charge-readout class. ■

Lemma 6 (even-channel nullity) supplies the converse direction of the firewall: not only does a churn-cancelling readout kill  $\chi$ , but any fold-even readout — whatever its construction — returns exactly zero on the  $\chi$  residue. The separation is thus symmetric: charge readouts cannot see  $\chi$ , and the  $\chi$  readout does not contribute to any even standing quantity.

The separation also explains, rather than merely tolerates, the Realization Theorem: the same substrate that must forget order for charge stability must remember order for fold susceptibility. The two theorems are complementary halves of one domain discipline.

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## 11. Relation to the Half-Lazy $\chi$ Programme

This paper is upstream of the half-lazy  $\chi$ -halving theorem. The half-lazy programme seeks to prove

$$\Delta\chi_{\{g+1\}} = \frac{1}{2} \Delta\chi_g, \text{ i.e. } \rho = \frac{1}{2},$$

which requires additional structure:

- a genuine even/odd fold decomposition;
- a closure-order twin pair;
- a uniform binary admissibility gate;
- a complete evenisation branch;
- log-access intertwining across generation refinement;
- generation-independent readout.

SM-4 proves something earlier and more basic: that any non-zero  $\chi$  observable cannot be a final-record observable, and that a continuation-visible ordered-history residue is the necessary domain for the later operators to act upon.

Strictly, SM-4 supplies the domain and survival theorem. The existence of the relevant quark-class order-current remains the next owed gate (§13.2, §13.3). SM-15 should therefore not be read as acting on an already-computed  $\chi$  magnitude; it acts only after the existence gate supplies a live  $\Gamma_\chi$  object.

The dependency order is:

SM-4 (domain of  $\chi$ )  $\rightarrow$  existence gate (live  $\Gamma_\chi$ )  $\rightarrow$  SM-15 (half-lazy gate)  $\rightarrow$  SM-17 (first increment magnitude).

If SM-4 failed, SM-15 would operate on zero. Because SM-4 succeeds, SM-15 still has all of its own work to do — the success here is necessary, not sufficient (F12).

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## 12. Exclusion Ladder

### 12.1 Why not individual facts?

A single committed fact has no primitive + or – fold sign. Assigning one would be a label choice, not physics.  $\chi$  is not a property of isolated facts.

## 12.2 Why not the final record F?

Because  $F(H) = F(H\leftarrow)$ , and F is fold-even (Lemma 0), so any final-record readout with a mirror-conjugate access pair forces  $a_+ = a_-$ , hence  $\chi = 0$  (Lemma 1).

## 12.3 Why not arbitrary history dependence?

Arbitrary history dependence could encode convention. The physical history dependence must satisfy reversal oddness, continuation faithfulness, and fold relabelling canonicity. By Lemma 3, those constraints select  $\Gamma(H)$  — not an arbitrary function of the sequence.

### 12.3a Why not any surviving ordered residue?

A surviving ordered residue is not automatically  $\chi$ . It becomes  $\chi$  only if its survivor is fold-odd under the mirror exchanging  $P_+$  and  $P_-$ . A continuation-visible residue that is fold-even belongs to a standing-record channel. A residue that is odd under some unrelated involution but not under the up/down fold mirror belongs elsewhere. The  $\chi$  residue is specifically the component  $P_\chi C_{\text{phys}} \Gamma(H)$ , not the whole of  $C_{\text{phys}} \Gamma(H)$ . This is the content of Theorem 2 and falsifiers F5a–F5c.

## 12.4 Why not a member-owned amplitude?

The  $W_7$  carrier audit demotes member-owned fold amplitudes. The fold structure lives at class grain; member-grain records register class-owned structure but do not own independent standing fold content.

## 12.5 Why not the charge readout?

The charge readout correctly cancels realized churn.  $\chi$  requires exactly the part of the ordered history that does not cancel. A charge-style readout reduces  $\chi$  to F and kills it (Lemma 8).

## 12.6 Why not convention?

Under fold relabelling,  $\chi \mapsto -\chi$  but  $|\chi| \mapsto |\chi|$ . The sign is conventional until oriented externally; the magnitude is not.

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# 13. Falsification Conditions

#	Failure	Consequence
F1	$\chi$ readout factors through final record F	$a_+ = a_-$ , $\chi = 0$ ; SM-4 fails
F1a	an individual fact carries primitive fold-odd content (OC-2'(i) fails; equivalent to F8)	Lemma 0 fails; the final-record collapse is no longer forced
F1b	the tally construction references the fold labels (OC-2'(ii) equivariance fails)	Lemma 0 fails; fold-odd content is injected at record formation
F1c	the access functional violates fold covariance (OC-6 mirror-conjugacy fails; Lemma 1a inapplicable)	orientation smuggled at readout level; collapse evadable only by a readout that does not read the two folds symmetrically
F2	H and $H\leftarrow$ have identical continuation cones in the $\chi$ sector	ordered residue is gauge-null
F3	ordered residue survives but projects outside the fold-odd $\chi$ line	$\chi$ not derived from ordered commitment; fold placement (OC-4a/OC-4b) fails
F4	physical coarse-graining is allowed to identify histories with different $\chi$ -channel continuations	continuation-faithfulness fails; ordered readout collapses
F5	$P_\chi C_{\text{phys}} \Gamma(H) = 0$ for the quark $\chi$ class	no physical up/down residue
F5a	$C_{\text{phys}} \Gamma(H) \neq 0$ but $S C_{\text{phys}} \Gamma(H) = C_{\text{phys}} \Gamma(H)$	survivor is fold-even; not $\chi$
F5b	survivor is fold-odd but the fold-odd sector of the $\chi$ quotient has dimension greater than one	$L_\chi$ placement is not forced; fold-placement theorem inapplicable
F5c	survivor is odd under history reversal but not under the up/down fold mirror S	ordered residue exists but belongs outside the $\chi$ channel
F6	fold relabelling changes $ \chi $	canonicity fails; $\chi$ is convention-laden
F7	the churn-cancelling readout used for charge is forced for $\chi$	$\chi$ collapses to final-record readout
F8	individual facts carry primitive $+/-$ labels	construction smuggles the fold assignment
F9	$\ell_\chi$ cannot be defined as a log-access functional with positive $a_\pm$	$\chi$ not a physical access ratio
F10	the ordered residue exists only transiently and cannot persist to class-grain readout	class readout route fails; SM-4 survives only as a transient branch
F11	all admissible histories commute in the $\chi$ quotient	$U(H) = U(H\leftarrow)$ , so $\Gamma = 0$
F12	SM-15 later shows no half-lazy gate despite SM-4	$\chi$ exists but the halving route fails
F13	the operator model is not $\chi$ -faithful (OC-8 fails): $Q_\chi U$ equal while cones differ, or $Q_\chi U$ differ while cones coincide	algebraic proxy decouples from the physical target; the existence gate must work at cone level

F1 is the primary kill condition; F1a, F1b, and F1c price the three leak channels of the collapse theorem — signed facts (fact level), fold-referencing record formation (tally level, Lemma 0),

and a fold-non-covariant access functional (readout level, Lemma 1a). F2–F5 mark where the ordered-history route could fail; F5a–F5c decompose the placement failure isolated by Theorem 2, falsifying OC-4a (F5a, F5c) and OC-4b (F5b) independently. F6 guards convention. F7 prevents accidental reuse of the charge readout. F11 is the algebraic degeneracy check. F12 records that SM-4 success is necessary but not sufficient for the mass-hierarchy programme. F13 prices the bridging premise OC-8 that certifies the algebraic proxy of §5.2/§13.2; it is a proxy-exactness falsifier, not a falsifier of Theorem 3.

### 13.1 Load-bearing status of OC-4a and OC-4b

Two premises carry the fold-placement burden and are to be read as explicit gate assumptions, not background decoration.

OC-4a states that the surviving ordered residue is covariant under the same fold mirror  $S$  that exchanges  $P_+$  and  $P_-$ . Without it, the residue may be odd under reversal but not odd under the physical up/down fold exchange. Such a residue would be an order-current — but not yet a  $\chi$  order-current (F5c).

OC-4b states that the fold-odd sector of the  $\chi$  quotient is one-dimensional and equals the access line  $L_\chi$ . Without it, a surviving fold-odd residue could lie in a different odd direction, leaving  $P_\chi C_{\text{phys}} \Gamma(H) = 0$  even though some odd ordered residue survives (F5b).

These premises are exactly what converts a surviving ordered residue into *the  $\chi$  residue*. The proof must not be read as: history exists, therefore  $\chi$  exists. It must be read as: a continuation-visible ordered residue that survives coarse-graining, transforms oddly under the up/down fold mirror, and lies in the one-dimensional  $\chi$  odd sector is precisely the  $\chi$  residue. That is why F5a, F5b, and F5c are separate falsifiers: each identifies a distinct way in which ordered history could survive without yielding quark  $\chi$ .

### 13.2 Next-gate obligation

The next required step is not another restatement of ordered readout. It is the exhibition of the owed quark-class source.

The primary owed object is stated at cone level, with the class-quantified witness of §5.1: the programme must show that within the VERSF quark-class commitment algebra there exists at least one ordered commitment history  $H$  and a  $\chi$ -class  $\kappa$  inhabited in  $\text{Ext}_\chi(H)$  and evacuated in  $\text{Ext}_\chi(H\leftarrow)$  (or conversely), so that

$$\text{Ext}_\chi(H) \neq \text{Ext}_\chi(H\leftarrow).$$

The computable proxy is the  $\chi$ -quotient order-current,

$$Q_\chi U(H) \neq Q_\chi U(H\leftarrow), \text{ in the two-step skeleton } Q_\chi[B, A] \neq 0,$$

which certifies the cone statement exactly when the operator model is  $\chi$ -faithful (OC-8, §5.2). The gate therefore has one primary target and one proxy route: establish the cone inequality directly at class level, or establish the algebraic inequality together with OC-8. Discharging the proxy without OC-8 proves survival of an algebraic residue but does not establish the physical target. In either route, what is delivered is *survival*: placement on  $L_\chi$  remains supplied by Theorem 2 under OC-4a/OC-4b and requires no further work from the existence gate.

This is the concrete existence obligation left open by SM-4 (§1.2, §1.2a). Until it is discharged, SM-4 is closed as a readout-domain theorem but remains conditional as a quark-sector existence theorem. Once it is discharged, the ordered  $\chi$  readout is no longer merely an admissible escape from final-record collapse; it becomes the forced readout of a concrete VERSF quark-class order-current.

The distinction is central to the integrity of the programme:

- SM-4 proves that  $\chi$  cannot live on F and must live on ordered history if it is non-zero.
- The next existence gate must prove that the relevant VERSF quark history actually supplies a non-zero  $\Gamma_\chi$ .
- Later magnitude gates must then compute the size, halving behaviour, and mass-hierarchy consequences of that  $\Gamma_\chi$ .

Domain first, existence second, magnitude third.

### 13.3 Acceptance criteria for the existence gate

The next gate is accepted only if it supplies one of the following two certificates.

**Route A — Direct cone certificate.** Exhibit a concrete quark-class ordered history H and a  $\chi$ -equivalence class  $\kappa$  such that  $\kappa$  is inhabited in  $\text{Ext}_\chi(H)$  and absent from  $\text{Ext}_\chi(H\leftarrow)$ , or conversely. This directly proves

$$\text{Ext}_\chi(H) \neq \text{Ext}_\chi(H\leftarrow).$$

This is the strongest certificate, because it works at the physical continuation-cone level and does not depend on the operator model.

**Route B — Operator-proxy certificate.** Exhibit a concrete quark-class ordered history H such that

$$Q_\chi U(H) \neq Q_\chi U(H\leftarrow),$$

or, in the two-step skeleton,

$$Q_\chi[B, A] \neq 0,$$

and also prove OC-8: that equality of  $Q_\chi$  U-products is equivalent to equality of  $\chi$ -channel continuation cones. This route proves the same target, but only because the operator model has been shown to be  $\chi$ -faithful.

The following do **not** discharge the existence gate:

- showing non-commutation upstream, before  $\chi$ -channel quotienting (Lemma 8's cancellation territory);
- showing that a single representative continuation  $K$  differs while its  $\chi$ -equivalence class remains inhabited on both sides (§5.1);
- showing that an ordered residue survives in a fold-even or non- $\chi$  channel (F5a, F5c);
- showing  $P_\chi C_{\text{phys}} \Gamma(H) \neq 0$  by assumption rather than by cone survival plus fold placement;
- computing a desired mass ratio without first exhibiting the live  $\chi$  order-current.

The existence gate must deliver a concrete quark-class order-current, not a suggestive commutator. It must show that the order of commitment changes the admissible  $\chi$ -visible future structure of the quark class.

## 14. SM-4 Closure Certificate

SM-4 condition	Result
Fold-evenness of final record	closed (Lemma 0), from OC-2' clauses (i)+(ii); priced by F1a/F1b
Final-record collapse shown	closed: $R(F) \Rightarrow \chi = 0$ , hypothesis derived not assumed
Ordered history $H$ defined	closed
Reversed history $H \leftarrow$ defined	closed
Ordered residue $\Gamma(H)$ constructed	closed
$\Gamma(H)$ final-record-null	closed
$\Gamma(H)$ reversal-odd	closed
Uniqueness of $\Gamma(H)$ in the pair module	closed (Lemma 3)
Physical non-nullity criterion	closed via continuation cones
Fold-channel placement	closed via Theorem 2, conditional on OC-4a and OC-4b
Coarse-graining survival	closed for continuation-faithful projections
Maximal faithful quotient	closed (Lemma 7): OC-5 structural, not stipulated
Even-channel nullity	closed (Lemma 6): fold-even readouts identically blind to $\Gamma_\chi$
$\chi$ readout functional	closed up to magnitude scale
Positive access weights $a_\pm$	closed by log-access parametrisation
Relabelling canonicity	closed: $ \chi $ invariant, $\text{sgn } \chi$ conventional

<b>SM-4 condition</b>	<b>Result</b>
Charge/ $\chi$ readout separation	closed
Existence of commitment-visible quark history	<b>owed</b> ; primary cone-level target, proxy route under OC-8 (§13.2); gate assignment open
Algebraic proxy for continuation asymmetry	stated (§5.2); exact under OC-8, priced by F13
Domain/existence/magnitude separation	closed: SM-4 closes domain only; existence and magnitude explicitly deferred (§1.2b)
Next-gate acceptance criteria	closed: direct cone certificate, or operator-proxy certificate plus OC-8 (§13.3)
Exact $\chi$ magnitude	outside SM-4
Exact halving $\rho = \frac{1}{2}$	outside SM-4; SM-15
First increment $\Delta\chi_1$	outside SM-4; SM-17
Full Yukawa construction	outside SM-4; SM-22

**SM-4 closure grade:** closed at ordered-readout-domain level. The internal algebra of final-record fold-evenness (Lemma 0), final-record collapse, residue uniqueness, reversal oddness, even-channel nullity, and relabelling canonicity is exact. Physical closure remains conditional on three stated items: OC-4a/OC-4b fold-channel placement, OC-5 continuation-faithful coarse-graining (structural via the maximal-quotient lemma, Lemma 7), and the owed existence of at least one commitment-visible quark-class history ( $\neg F2/\neg F11$ ), whose gate assignment is open.

## 15. Conclusion

The  $W_7$  carrier/readout audit left the  $\chi$  programme balanced on a knife edge. If  $\chi$  reads only the final standing record, the mirror symmetry of the completed tally forces

$$a_+ = a_-, \text{ and therefore } \chi = 0,$$

killing the  $\chi$ -halving route before any magnitude theorem could begin.

This paper proves the ordered-readout alternative, and the result is sharp but not overstated.

The completed record  $F$  is too symmetric. It remembers what has been committed but forgets the irreversible order in which commitment occurred. Because individual facts carry no primitive up/down sign and the tally construction references no fold label (Lemma 0), the final record is fold-even — and every fold-covariant final-record readout, linear or not (Lemma 1a), gives  $a_+ = a_-$  and hence  $\chi = 0$ .

The only possible non-zero source is the ordered residue

$$\Gamma(H) = \frac{1}{2} (|H\rangle - |H\leftarrow\rangle) = \frac{1}{2} (U(H) - U(H\leftarrow)),$$

the unique reversal-odd, final-record-null object in the  $H/H\leftarrow$  pair module (Lemma 3). This residue becomes physical when  $H$  and  $H\leftarrow$  possess different  $\chi$ -channel continuation cones — a condition with a computable algebraic signature,  $Q_\chi U(H) \neq Q_\chi U(H\leftarrow)$ , exact under operator-model  $\chi$ -faithfulness (OC-8, §5.2). It becomes specifically  $\chi$  when its continuation-faithful survivor is odd under the up/down fold mirror and lies on the one-dimensional access line  $L_\chi$  (Theorem 2). The readout is then

$$\chi(H) = \ell_\chi(P_\chi C_{\text{phys}} \Gamma(H)),$$

with fold-access weights

$$a_\pm(H) = a_0 \exp(\pm \frac{1}{2} \chi(H)), \text{ so that } \ln(a_+/a_-) = \chi(H).$$

Thus SM-4 closes the readout-domain gate:  $\chi$ , if non-zero, reads ordered commitment history  $H$  and not merely the final standing record  $F$ .

SM-4 does not yet derive the numerical quark hierarchy, the half-lazy ratio, the first  $\chi$  increment, or the Yukawa matrices. It proves the prior fact without which all of those later gates would be empty: a non-zero  $\chi$  observable cannot be a final-record observable.

The remaining owed item is correspondingly precise. The next gate must exhibit the concrete VERSF quark-class commitment history for which the  $\chi$ -channel continuation cones differ, with a class-quantified witness:  $\text{Ext}_\chi(H) \neq \text{Ext}_\chi(H\leftarrow)$ . The computable proxy route shows instead

$$Q_\chi U(H) - Q_\chi U(H\leftarrow) \neq 0, \text{ in the two-step case } Q_\chi[B, A] \neq 0,$$

together with OC-8, which certifies the proxy's exactness (§13.2). Once that object is supplied, the later halving and mass-hierarchy papers have a live  $\chi$  residue to compute. Without it, the programme has a domain theorem. With it, the ordered-commitment route becomes the necessary foundation of the VERSF Standard Model mass-hierarchy derivation.

The programme consequence is precise. SM-4 has removed the final-record route from the  $\chi$  programme. It has also identified the only admissible replacement: an ordered, reversal-odd, continuation-visible residue that survives  $\chi$ -faithful coarse-graining and lands on the fold-odd access line. The next paper cannot reopen the domain question; it must supply the concrete quark-class witness. Later papers cannot compute a meaningful  $\chi$  hierarchy until that witness exists.

The result is therefore neither too weak nor too strong. It is not merely a formal construction, because every step from final-record collapse to fold placement is carried by named premises with priced falsifiers, and the acceptance criteria for the next gate are stated in advance. And it is not an overclaim, because the existence of the quark-class order-current and every magnitude question remain explicitly owed.

In one sentence:

$\chi$  is non-zero only when the irreversible order-current of commitment history survives continuation-faithful coarse-graining as a fold-odd up/down access residue; a completed symmetric tally alone always gives  $\chi = 0$ .

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## Appendix A — Minimal Algebraic Skeleton

Let a two-step history be  $H = (A, B)$ , with reversal  $H \leftarrow = (B, A)$ . The final record is

$$F(H) = \{A, B\} = F(H \leftarrow),$$

so a final-record readout cannot distinguish the two.

The ordered products are

$$U(H) = BA \text{ and } U(H \leftarrow) = AB,$$

giving

$$\Gamma(H) = \frac{1}{2} (BA - AB) = \frac{1}{2} [B, A].$$

If  $A$  and  $B$  commute in the  $\chi$  quotient,  $\Gamma(H) = 0$  (this is F11). If they do not commute,  $\Gamma(H)$  is a genuine order-current.

Under reversal,

$$\Gamma(H \leftarrow) = \frac{1}{2} (AB - BA) = -\Gamma(H).$$

Project onto the  $\chi$  line and read out:

$$\Gamma_{\chi}(H) = P_{\chi} C_{\text{phys}} \Gamma(H), \quad \chi(H) = \ell_{\chi}(\Gamma_{\chi}(H)).$$

Access weights:

$$a_{+} = a_0 e^{\chi/2}, \quad a_{-} = a_0 e^{-\chi/2}, \quad \text{so } \ln(a_{+}/a_{-}) = \chi.$$

If the readout factors through  $F$ ,  $\Gamma$  is invisible and  $\chi = 0$ .

This two-step model is not the full VERSF quark mechanism. It is the minimal algebra exhibiting why order matters and why a final-record readout kills  $\chi$ . Note that in this skeleton the ordered residue is literally a commutator — the elementary object that measures failure of order-indifference — which is the correct minimal avatar of an order-current.

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## Appendix B — Referee Objections and Replies

**Objection 1 — You are just defining  $\chi$  to read history because the final record gives zero.**

Response: The paper does not choose history arbitrarily. It identifies the object satisfying four physical constraints — final-record nullity, reversal oddness, continuation faithfulness, and fold relabelling canonicity — and proves (Lemma 3) that  $\Gamma(H) = \frac{1}{2}(H - H\leftarrow)$  is the *unique* such object in the pair module up to scale. A final-record readout fails the first constraint outright by forcing  $\chi = 0$ . Uniqueness, not convenience, drives the selection.

**Objection 2 — Individual facts do not have an up/down sign.** Response: Correct, and the paper agrees. No sign is assigned to any individual fact. The sign appears only in the comparison between an irreversible history and its reversal.  $\chi$  is an order-current, not a primitive fact-label.

**Objection 3 — A standing class state should be reflection-symmetric, so the odd residue should vanish.** Response: Standing means persistent; it does not mean order-blind. A final tally is reflection-symmetric because it has forgotten order. A class record containing an order-imprint can be standing and still carry reversal-odd information. The physical question is whether admissible continuation cones distinguish  $H$  from  $H\leftarrow$ . If they do, erasing the difference is not admissible coarse-graining (OC-5) — it is a change of physics, not a coarser description of the same physics.

**Objection 4 — The Realization Theorem cancels churn, so this residue should cancel too.**

Response: The Realization Theorem cancels order-dependence for charge-like standing readouts, and that is correct for charge.  $\chi$  is not charge. If the  $\chi$  readout used the same cancellation, it would factor through  $F$  and Lemma 1 would give  $\chi = 0$ . A non-zero  $\chi$  observable must lie outside the charge-style churn-cancelling class (Lemma 8), and by even-channel nullity (Lemma 6) every fold-even readout returns identically zero on the  $\chi$  residue. The two theorems are complementary, not in tension: one domain must forget order for charge stability, the other must remember it for fold susceptibility.

**Objection 5 — The ordered residue may exist mathematically but vanish under physical projection.**

Response: That is exactly the F5 family. Theorem 1 proves survival alone, conditional on OC-5 continuation-faithfulness; placement on  $L_\chi$  is carried separately by Theorem 2, conditional on OC-4a and OC-4b. A projection that kills a distinction between histories with different  $\chi$ -channel continuation cones is not an admissible  $\chi$  projection; using one is equivalent to adopting the final-record readout, which is already excluded. A survivor that evades placement instead falls to F5a, F5b, or F5c.

**Objection 6 — The sign of  $\chi$  depends on which fold is called plus.** Response: Yes, and the paper says so. The sign reverses under fold relabelling. The physical object at SM-4 level is  $|\chi|$  — equivalently, the existence of a non-zero fold-odd contrast. The sign becomes physical only when tied to an external orientation convention or sector.

**Objection 7 — Does this prove the half-lazy halving law?** Response: No. It proves the readout domain and the survival of the ordered residue. The half-lazy law requires additional gates:

closure-order twins, a uniform binary admissibility gate, complete evenisation, and log-access intertwining. Those belong to SM-15.

**Objection 8 — Does this prove the up/down quark mass ratio?** Response: No. SM-4 computes no  $\Delta\chi_1$ , no  $m_u/m_d$ , and no absolute mass. It prevents the  $\chi$  observable from collapsing to zero. Magnitudes are later gates, and SM-4 contains no tunable parameter with which to anticipate them.

**Objection 9 — Could all commitment operators commute, making  $\Gamma = 0$ ?** Response: Yes. That is F11. If all relevant commitments commute in the  $\chi$  quotient, then  $U(H) = U(H\leftarrow)$  and the ordered residue vanishes identically. The theorem requires commitment-visible order in the  $\chi$  channel; F11 is the algebraic form of that requirement's failure.

**Objection 10 — Is this circular because it assumes the quark  $\chi$  channel exists?** Response: The paper inherits the existence of the fold-odd  $\chi$  readout line from the  $W_7$  carrier/readout framework and then proves what domain that line must read to avoid collapse. It does not assume the measured quark masses, the halving profile, or the magnitude of  $\chi$ . The fold-channel placement is explicitly recorded as premises OC-4a and OC-4b and is falsified by F3/F5/F5a–F5c — the assumptions are priced, not hidden.

**Objection 11 — You have shown that an ordered residue can survive, but not that it is the quark  $\chi$  residue.** Response: Correct, unless the fold-placement criterion is satisfied. SM-4 does not identify every surviving order-residue with  $\chi$ . It identifies  $\chi$  with the fold-odd survivor in the quark access quotient:  $P_\chi C_{\text{phys}} \Gamma(H)$ . The fold-placement theorem (Theorem 2) states the required conditions explicitly: the residue must survive  $\chi$ -channel coarse-graining, transform oddly under the fold mirror that exchanges  $P_+$  and  $P_-$ , and lie in the fold-odd sector of the  $\chi$  quotient, which must be one-dimensional and equal to  $L_\chi$ . If the survivor is fold-even (F5a), if the odd sector has more than one inequivalent line (F5b), or if the residue is odd under the wrong involution (F5c), the SM-4 placement fails. OC-4a and OC-4b are thus not hidden; each is isolated, priced, and falsifiable independently.

**Objection 12 — Under irreversibility,  $H\leftarrow$  may not be an admissible history at all, so  $\Gamma(H)$  compares a physical object to a fiction.** Response: The construction anticipates this and does not require  $H\leftarrow$  to be admissible. The history module is the formal module of ordered sequences; admissibility lives entirely in the continuation structure  $\text{Ext}$  (§3.2). If  $H\leftarrow$  is inadmissible, then  $\text{Ext}(H\leftarrow) = \emptyset$  while  $\text{Ext}(H) \neq \emptyset$ , and the continuation cones differ maximally — the strongest possible form of commitment-visible order. Far from being a fiction, the inadmissible reversal carries exact physical content: the statement that no continuation is admissible from that order. Inadmissibility of the reversal does not threaten Lemma 4; it saturates it.

**Objection 13 — Fact unsignedness alone does not make the final record fold-even; the tally itself could inject fold bias.** Response: Correct, which is why OC-2' has two clauses. Fact unsignedness blocks primitive fold signs at fact level. Tally equivariance blocks fold labels from entering through the record-forming map. Lemma 0 requires both. If the tally construction references  $P_+$  or  $P_-$ , fold bias has been inserted at record formation, not derived — that is falsifier F1b, and the explicit counterexample is recorded in the Reading of Lemma 0.

**Objection 14 — A non-covariant access functional could read different values from a fold-even final record.** Response: Yes, but that would not be the same physical readout applied to the two exchanged folds. Lemma 1a assumes fold covariance,  $A(S P S^{-1}, S F S^{-1}) = A(P, F)$ , which is the minimal formal expression of using one mirror-conjugate access rule for both folds (OC-6). A readout violating this condition has smuggled orientation into the measurement rule itself. That is falsifier F1c.

**Objection 15 — The commutator condition  $Q_{\chi}[B, A] \neq 0$  is being treated as equivalent to continuation asymmetry.** Response: Not under OC-1–OC-7 alone. The paper explicitly distinguishes the physical cone criterion from the algebraic proxy. The physical target is  $\text{Ext}_{\chi}(H) \neq \text{Ext}_{\chi}(H \leftarrow)$ . The commutator proxy is exact only under OC-8, operator-model  $\chi$ -faithfulness. Without OC-8, the algebraic and continuation criteria can come apart: the algebra may miss continuation information, or it may retain a residue with no continuation content. That is falsifier F13, and §13.3 accordingly requires Route B to prove OC-8, not assume it.