The Physical Half-Lazy χ Gate Audit in VERSF

▲ Programme Milestone — χ-Gate Physicality and Audit Series Gate PHχ-1 / Ordered-History χ Readout, Half-Lazy Closure, Non-Degeneracy, and Physical-Bridge Audit

Auditing when χ is a physical commitment gate rather than a convention, hidden half-factor, arbitrary selector, or disguised double-counting rule

Keith Taylor VERSF Theoretical Physics Programme χ-Gate Physicality and Audit Series Tier A gate-audit paper, PHχ-1

General Reader Summary

Earlier VERSF papers introduced χ as a readout of ordered commitment. The key idea was that χ cannot come from a final tally alone. If the theory only knows the final record — what has been committed, but not the order in which commitment occurred — then the arrow of commitment is invisible. A label-blind final-record readout gives no physical χ.

This paper audits the next danger.

Even if χ is allowed to read ordered history, the theory must still prove that the so-called **half-lazy χ gate** is physical. A half-lazy gate can easily become a place where the theory hides a choice:

- a hidden factor of ½;
- an arbitrary active/passive split;
- a convention disguised as physics;
- a branch counted once in one paper and again later in another;
- a sign label inserted by hand;
- a selection rule that is not actually derived.

PHχ-1 closes that loophole.

The core claim is:

A half-lazy χ gate is physically admissible only if the active half, lazy half, ordering rule, quotient normalisation, and readout consequence are all derived from typed VERSF structure.

In plain language:

The lazy half is not ignored. It is closure-present but readout-inactive. It constrains the physical gate, prevents double counting, and supplies the reason why a half-factor is allowed.

The paper separates five objects that must not be confused:

1. **The ordered history H** — the irreversible sequence of commitments.
2. **The final record F(H)** — the order-forgetting tally produced by that history.
3. **The reversal partner H^←** — the same commitments read in reverse order.
4. **The lazy twin LH** — the closure-required partner that is present but not independently counted as active physical support.
5. **The χ readout** — the physical, reversal-sensitive residue that survives only when ordered history is available.

The central audit result is:

χ is physical only when it reads an ordered, irreversible, reversal-sensitive commitment history; the half-lazy structure is physical only when it is an involutive closure quotient that prevents double counting rather than an inserted numerical convenience.

This blocks six characteristic errors.

It blocks: "χ is whatever sign we assign to the branch."

It blocks: "χ can be read from the final tally."

It blocks: "The half factor is just put in because there are two sides."

It blocks: "The lazy side can be ignored now and counted later."

It blocks: "The active/lazy split is a convention."

It blocks: "A nonzero χ can survive if ordering is erased."

PHχ-1 therefore makes the χ gate hard to abuse.

In one sentence:

The Physical Half-Lazy χ Gate Audit proves that a VERSF χ readout is admissible only when it is ordered-history-derived, reversal-sensitive, lazy-quotient-normalised, non-double-counted, and invariant under all permitted relabellings; otherwise χ is not physics but bookkeeping.

Scope Box — What PHχ-1 Closes and What It Does Not

Claim	Status
χ cannot be a raw sign label	Closed
χ cannot be read from final record alone	Closed
Ordered commitment history is necessary for nonzero χ	Closed
Reversal-odd residue supplies the admissible χ carrier	Closed
The factor ½ must arise from a projection or quotient normalisation	Closed
"Half-lazy" cannot mean "half inserted by hand"	Closed
Lazy support must be closure-present, not physically absent	Closed
Lazy support cannot later be counted as independent active support	Closed
Active/lazy split must be invariantly typed	Closed
A lazy involution must be stated and tested	Closed
Half-lazy χ must pass positivity and physical-sector admissibility	Closed
χ must be compatible with ordered-commitment readout	Closed
χ must be compatible with frame/gauge relabelling discipline	Closed
Final-record readout gives $\chi = 0$ under label-blindness	Closed
Reversal-sensitive history readout may give $\chi \neq 0$	Closed
Seven physical χ-gate audit tests supplied	Closed
Lazy fixed-point sectors must be separated from genuine half-lazy pairs	Closed
Reversal is an audit comparison, not physical de-commitment	Closed
Later χ use must provide a bridge map into a downstream physical target	Closed
Composite half-lazy carrier must be non-degenerate on physical support	Closed
Readout functional ω_χ must be typed, not discretionary	Closed
Exact numerical value of a later χ observable	Not closed
Derivation of a specific $\chi_1 = \ln(6/13)$ value	Not closed
Full Yukawa hierarchy from χ alone	Not closed
Full CKM/PMNS prediction from χ alone	Not closed
Full confinement or hadronic calculation	Not closed
Proof that every later VERSF χ use passes this audit	Not closed
Empirical fit of χ to observed masses or mixings	Not closed

PHχ-1 is an **admissibility audit**. It does not numerically derive the Standard Model. It proves what a legitimate VERSF half-lazy χ gate must be before later papers may use χ as physical input.

Abstract

Previous VERSF χ-readout work established that χ cannot be extracted from final-record data alone. A final record F(H) forgets the ordering of an irreversible commitment history $H = (f_1, \dots, f_n)$. Under label-blindness, any readout depending only on F(H) is blind to orientation and therefore gives zero χ. A nonzero χ requires an ordered-history readout sensitive to reversal, with residue of the form

$$\Gamma_\chi(H) = \tfrac{1}{2} (|H\rangle - |H^{\leftarrow}\rangle) ,$$

or an equivalent reversal-odd structure.

This paper audits the additional claim that the χ gate may be **half-lazy**. The term "half-lazy" is dangerous unless carefully typed. It could mean a physical quotient, a paired-sector non-double-counting rule, a dormant closure partner, a gauge redundancy, a convention, or an arbitrary half-factor inserted after the fact. PHχ-1 proves that only derived projection, quotient, or closure-pair structures are physically admissible. Gauge redundancy may be admissible only when explicitly handled as equivalence structure and removed from independent counting. Convention halves, fit halves, and arbitrary active/lazy splits are not physical.

The paper defines five distinct objects:

1. the **ordered commitment history** H;
2. the **final record** F(H), which forgets order;
3. the **reversal partner** $H^{\leftarrow}$;
4. the **lazy twin** LH, generated by an involutive closure pairing;
5. the **χ readout** χ(H), a reversal-sensitive physical residue.

Six theorems are proved.

Theorem 1 — Final-Record Null Theorem. Any label-blind, reversal-odd χ readout depending only on the final record F(H) vanishes.

Theorem 2 — Ordered-History Necessity Theorem. A nonzero χ readout requires ordered, irreversible commitment history and a reversal-sensitive residue.

Theorem 3 — Half-Lazy Projection Theorem. A half-lazy gate is physical only when its half-factor arises from an idempotent projection or involutive quotient normalisation, not from discretionary halving.

Theorem 4 — Lazy Non-Double-Counting Theorem. The lazy half is closure-present but not independently active. It may constrain the gate, but it may not later return as separate counted physical support.

Theorem 5 — Physical χ-Gate Audit Theorem. A χ gate is physically admissible if and only if it passes seven tests: ordered-history, final-record null, reversal-odd residue, lazy-involution, quotient normalisation, non-double-counting, and physical-sector invariance.

Theorem 6 — Half-Lazy χ Closure Theorem. A VERSF half-lazy χ gate is closed as physical only when χ is a history-derived, reversal-sensitive, lazy-quotiented, non-double-counted readout. Any χ use failing this audit is firewalled as convention, bookkeeping, or hidden fit freedom.

PHχ-1 therefore closes the physicality audit for half-lazy χ gates. It permits later VERSF papers to use χ only when the active/lazy split is derived, the ½ factor is projection-normalised, the lazy side cannot be double counted, and the readout remains invariant under permitted relabellings. It forbids arbitrary χ signs, final-record χ, raw half-factors, convention-based active/lazy splits, and lazy-sector re-entry as hidden predictive content.

Contents

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0. Inheritance, Aim, and Firewalls

0.1 Inherited results

PHχ-1 inherits the following VERSF results and disciplines.

I1 — Ordered-commitment χ readout discipline. χ is not a primitive label. It is a readout of ordered commitment history. If the readout is forced to depend only on the final order-forgetting record, χ vanishes.

I2 — Final-record blindness. A final record F(H) contains what has been committed but not the order in which commitment occurred. It cannot by itself carry orientation, arrow, or reversal-sensitive residue.

I3 — History/reversal residue. The minimal reversal-sensitive carrier has the form

$\Gamma\_\chi(H) = ½ (|H\rangle - |H\text{^}\leftarrow\rangle)$,

or an equivalent reversal-odd object.

I4 — Label-blindness. Primitive (+) and (−) labels are not physical unless derived from ordered structure. A fact does not arrive carrying an unearned sign.

I5 — Physical-sector positivity. χ gates may not reintroduce negative-norm leakage, ghost residue, adjoint artifacts, gauge remnants, or nonphysical terminal support.

I6 — Bridge discipline. A later prediction must state the bridge between its internal structure and the physical readout. A symbol alone is not a bridge.

I7 — Hermitian-lift and frame discipline. Physical claims must not depend on arbitrary coordinate choices, raw matrix entries, or untyped frame conventions.

I8 — Gate-admissibility discipline. A gate is not physical merely because it is useful. It must be typed, invariant, non-arbitrary, and falsifiable.

0.2 The new problem

Earlier χ work established:

χ must read ordered history, not final tally.

PHχ-1 asks the next question:

When is a half-lazy χ gate physical?

The word "half-lazy" is not safe by itself. It may hide several different meanings:

1. **Projection half:** a genuine factor ½ arising from an idempotent projection.
2. **Quotient half:** a paired-sector non-double-counting normalisation.
3. **Dormant half:** a closure partner present but not active in readout.
4. **Gauge half:** a redundant partner removed by equivalence.
5. **Convention half:** a manually inserted numerical factor.
6. **Fit half:** a factor chosen because it makes numbers work.

Only the first three, when properly derived and audited, can be physical. The fourth must be declared as redundancy and handled by equivalence, not by counting. The last two are not physics.

0.3 Core aim

PHχ-1 has one aim:

To prove the audit conditions under which a half-lazy χ gate is a physical VERSF gate rather than a hidden convention.

The paper does not try to compute a final numerical mass, mixing angle, or hierarchy ratio. It fixes the admissibility rules for later papers that want to use χ.

0.4 Firewalls

This paper does **not** claim:

- that every use of χ in VERSF is automatically physical;
- that a numerical χ value is derived here;
- that $\chi_1 = \ln(6/13)$ is derived here;
- that a complete Yukawa hierarchy follows from PHχ-1 alone;
- that CKM or PMNS values are derived here;
- that the half-lazy gate supplies arbitrary fit freedom;
- that the lazy half can be ignored;
- that the lazy half can be counted later as independent physical support;
- that ½ may be inserted because the structure "looks binary";
- that a final-record tally can carry χ;

- that (+) and (−) labels are primitive physical signs;
- that active/lazy is a coordinate convention.

It proves the narrower result:

A half-lazy χ gate is physically admissible only when the half-factor is projection- or quotient-derived, the lazy partner is closure-present but not independently counted, and the χ readout is carried by ordered, reversal-sensitive history rather than by final-record tally or arbitrary sign labels.

0′. Predictive-Content Ledger

Object	Prior status	PHχ-1 status
Ordered history H	Required by χ readout	Made primitive audit target
Final record F(H)	Known order-forgetting object	Proved χ-null under label-blindness
Reversal H^←	Present in ordered χ work	Used to define reversal-odd residue
Reversal residue Γ_χ	Candidate χ carrier	Made admissible χ carrier
Raw (+)/(−) label	Potential convention	Firewalled
Half factor ½	Potential vulnerability	Must arise from projection or quotient
Lazy twin LH	Previously implicit	Defined as closure-present partner
Active/lazy split	Potential convention	Must be invariantly typed
Half-lazy gate	Informal	Audited as physical only under tests
Lazy sector re-entry	Potential double count	Firewalled
Final-record χ	Possible shortcut	Proved zero
Ordered-history χ	Candidate physical readout	Audited as admissible
Projection normalisation	Needed	Made explicit
Quotient non-double-counting	Needed	Made explicit
Half-ledger	Absent	Required for every later χ use
Lazy fixed points	Unaddressed	Separated from half-lazy count
Reversal comparison H^←	Potentially misread as physical undoing	Typed as orientation audit
Bridge map B_χ	Informal expectation	Typed, non-rhetorical, audit-stable

Object	Prior status	PHχ-1 status
Readout functional ω_χ	Unconstrained-freedom risk	Typed via PHχ-3/10/12/13 and bridge
Composite carrier degeneracy	Undetected vacuity risk	Firewalled by PHχ-14
Positivity compatibility	Required	Closed
Frame/gauge compatibility	Required	Closed
Numerical χ value	Not derived	Outside scope
Full hierarchy prediction	Not derived	Outside scope

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1. The Physical χ-Gate Problem

1.1 The central risk

χ is powerful only if it is physical.

If χ is just a sign inserted by notation, it is not a prediction. If χ is a half-factor added because the theory needs halving, it is not a derivation. If χ selects a branch without a physical rule, it is an arbitrary switch.

The danger is subtle because χ can look mathematical while doing hidden bookkeeping.

A paper might write

$\chi = \frac{1}{2} (a_+ - a_-)$

and then silently assume:

- (+) and (−) are physical;
- the half is justified;
- the active branch is selected;
- the lazy branch is inactive;
- the inactive branch cannot be counted again later.

None of those assumptions is free. Each one is a claim, and each claim can fail.

PHχ-1 audits every one of them.

1.2 What χ must not be

χ must not be:

- a raw sign label;
- a naming convention;
- a free binary choice;
- a post-hoc selector;
- a hidden fitting parameter;
- a half-factor inserted by convenience;
- a way to count one branch now and its twin later;
- a readout of an order-forgetting final tally.

1.3 What χ may be

χ may be physical if it is:

- derived from an ordered irreversible history;
- sensitive to reversal;
- blind to arbitrary labels;
- invariant under permitted relabellings;
- compatible with physical-sector positivity;
- normalised by a genuine projection or quotient;
- protected from double counting;
- tied to an observable later gate through a declared bridge.

1.4 Why "half-lazy" must be audited

The phrase "half-lazy" can be legitimate, but only with discipline.

A lazy half is not absent. It is not deleted. It is not ignored.

It is present as a closure partner, but it is not independently active in the physical readout.

Thus:

Lazy means closure-present but readout-inactive.

And:

Half means quotient-normalised, not manually divided.

The physical half-lazy χ gate is therefore a constrained structure:

ordered history + lazy closure twin + projection/quotient normalisation + non-double-counting rule $\rightarrow \chi$.

Every term in this chain is audited below, and every term carries its own falsifier.

2. Named Premises of the Half-Lazy χ Audit

2.1 PHχ-1 — Ordered-history premise

A nonzero χ readout must depend on an ordered commitment history:

$H = (f_1, f_2, \ldots, f\_n)$.

The history records not only what is committed but also the order in which commitment occurred.

Falsifier. χ is computed from an unordered set, tally, final record, or multiset alone.

2.2 PHχ-2 — Final-record blindness premise

The final record F(H) forgets ordering. If

$F(H) = F(H\^{\leftarrow})$,

then any readout depending only on F cannot distinguish H from its reversal.

Falsifier. A nonzero χ is claimed from F(H) alone.

2.3 PHχ-3 — Label-blindness premise

Primitive (+) and (−) labels are not physical unless derived from ordered structure. A raw assignment

$f_i \mapsto +$ or $f_i \mapsto -$

has no physical content by itself.

Falsifier. χ changes under a relabelling of arbitrary (+)/(−) names.

2.4 PHχ-4 — Reversal-sensitivity premise

A physical χ readout must be sensitive to reversal:

$\chi(H\^{\leftarrow}) = -\chi(H)$,

or must transform under an explicitly stated equivalent odd representation.

Falsifier. χ is unchanged by reversal while being claimed as an ordered-commitment asymmetry.

2.5 PHχ-5 — Reversal-residue premise

The minimal admissible χ carrier is the reversal-odd residue

$\Gamma\_\chi(H) = ½ (|H\rangle - |H^\leftarrow\rangle)$,

or an equivalent projection onto the reversal-odd subspace.

Falsifier. χ is asserted without identifying the reversal-odd object that carries it.

2.6 PHχ-6 — Lazy-twin premise

A half-lazy gate requires a lazy-twin operation

$L : H \mapsto LH$,

with

$L^2 = I$.

The lazy twin is closure-present but not independently active in the readout.

Falsifier. The paper uses "lazy" without defining the paired operation or its involutive status.

2.7 PHχ-7 — Half-factor derivation premise

The factor ½ must arise from a projection or quotient normalisation, such as

$P_- = ½ (I - R)$,

or

$W\_L = ½ (I + L)$,

where R is reversal and L is a valid lazy involution.

Falsifier. A factor ½ is inserted without deriving it from an idempotent projection, quotient, or non-double-counting rule.

2.8 PHχ-8 — Uniform binary admissibility premise

If a half-factor is justified by paired support, the two members of the pair must be admissible under the same rule. The pair must be structurally binary, not selectively constructed after the fact.

Falsifier. The theory halves a set whose paired members do not share the same admissibility status.

2.9 PHχ-9 — Non-double-counting premise

The lazy partner may not be counted later as independent active physical support. Once quotient-normalised, the pair contributes one physical support class, not two.

Falsifier. The lazy half is excluded in one calculation but reintroduced as an independent counted degree of freedom in a later one.

2.10 PHχ-10 — Active/lazy invariance premise

The active/lazy distinction must be invariantly typed. It cannot depend on arbitrary coordinate naming, notation, or presentation order.

Falsifier. The active branch and lazy branch can be swapped by a harmless relabelling while leaving a different physical prediction.

2.11 PHχ-11 — Commutator-compatibility premise

The reversal operation R and lazy involution L must have a declared compatibility relation. In the standard half-lazy χ gate, they commute:

$$[\, R , L \,] = 0 \, .$$

If they do not commute, the commutator shadow must be declared and shown not to contaminate the χ readout.

Falsifier. The gate uses both reversal and lazy pairing without specifying whether the operations commute, anticommute, or produce a residual obstruction.

2.12 PHχ-12 — Physical-sector positivity premise

A χ gate may not reintroduce negative-norm leakage, ghost support, gauge artifact, unphysical adjoint residue, or nonphysical terminal support.

Falsifier. χ is carried by a sector previously excluded from the physical matter or observable ledger.

2.13 PHχ-13 — Observable-bridge premise

A physical χ gate must state how its readout can feed a later observable target: hierarchy, orientation, projector alignment, admissibility split, or interaction channel. χ without a bridge is internal bookkeeping.

Falsifier. χ is declared physical but no downstream invariant, projector, spectral, or observable role is specified.

2.14 PHχ-14 — Non-degeneracy premise

The composite half-lazy carrier must be nonvanishing on physical χ-support:

$$W_L P_- P_{phys} \neq 0 .$$

In particular, L must not coincide with R on the χ-support subspace. Since R and L are commuting involutions, the coincidence L = R there gives

$$W_L P_- = \tfrac{1}{2}(I + R) \cdot \tfrac{1}{2}(I - R) = \tfrac{1}{4}(I - R^2) = 0 ,$$

and the gate is structurally perfect but identically zero. A vacuous gate is not a physical gate.

More precisely, non-degeneracy must hold on the support actually used by the proposed χ readout. It is not enough for $W_L P_- P_{phys}$ to be nonzero somewhere in the theory if it vanishes on the specific history class being claimed as χ-carrying. The condition is local to the claim, not global to the framework.

Falsifier. L restricted to the χ-support subspace coincides with R, or $W_L P_-$ otherwise vanishes on physical support, while a nonzero χ is claimed.

2.15 Premise-dependency map

Premise	Theorem 1	Theorem 2	Theorem 3	Theorem 4	Theorem 5	Theorem 6
PHχ-1	●	●			●	●
PHχ-2	●	●			●	●
PHχ-3	●	●			●	●
PHχ-4	●	●			●	●
PHχ-5		●			●	●
PHχ-6			●	●	●	●
PHχ-7			●	●	●	●
PHχ-8			●		●	●
PHχ-9				●	●	●

Premise	Theorem 1	Theorem 2	Theorem 3	Theorem 4	Theorem 5	Theorem 6
PHχ-10					●	●
PHχ-11					●	●
PHχ-12					●	●
PHχ-13					●	●
PHχ-14					●	●

Every premise is load-bearing, and each mark corresponds to an explicit citation inside the marked theorem's proof. No premise is decorative, and no mark is decorative either: the dependency map is itself subject to the audit it serves.

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3. Formal Setup: Histories, Records, Reversal, and Lazy Twins

3.1 Ordered history

Let

$H = (f_1, f_2, \ldots, f_n)$

be an ordered commitment history.

The entries f_i are committed facts, folds, closure events, access decisions, or admissibility steps, depending on the later application.

The essential point is that H is ordered.

3.2 Final record

Let F(H) be the final record produced by H. It retains the committed content but forgets order.

Thus

$F(H) = F(H^{\leftarrow})$

whenever $H^{\leftarrow}$ is merely the reverse-order presentation of the same committed content.

F is defined as the order-forgetting quotient and nothing coarser: it retains committed content, multiplicities, and typing data, and forgets exactly the order. Any coarser map that also forgets content or typing is not the F of this paper. This definitional restriction is load-bearing for

Theorem 2, where "the only information forgotten by F is the order" is used as a premise, not an observation.

3.3 Reversal

Define the reversal operation:

$$RH = H^{\leftarrow} = (f_n, \ldots, f_2, f_1) .$$

It satisfies

$$R^2 = I .$$

The object $H^{\leftarrow}$ is a formal orientation-comparison element in the free linear span of ordered presentations; it is not claimed to be a realizable commitment history, and R is defined on that formal span. Forming the difference $\frac{1}{2}(|H\rangle - |H^{\leftarrow}\rangle)$ is licensed by the span, not by any claim that reversed commitment physically occurs. Physical content is extracted from this formal carrier only through the audited readout.

The reversal-even and reversal-odd projectors are:

$$P_+ = \tfrac{1}{2} (I + R) , P_- = \tfrac{1}{2} (I - R) .$$

The reversal-odd residue is:

$$\Gamma_\chi(H) = P_- |H\rangle = \tfrac{1}{2} (|H\rangle - |H^{\leftarrow}\rangle) .$$

3.4 χ readout functional

Let ω_χ be a legitimate readout functional on the reversal-odd history residue. Then:

$$\chi(H) = \langle \omega_\chi , \Gamma_\chi(H) \rangle .$$

This implies

$$\chi(H^{\leftarrow}) = -\chi(H) .$$

If $H = H^{\leftarrow}$ as an ordered object, then $\Gamma_\chi(H) = 0$, and therefore $\chi(H) = 0$.

The functional ω_χ is not discretionary. It must itself be derived from typed VERSF structure: label-blind (PHχ-3), invariant under permitted relabellings (PHχ-10), supported on the physical sector (PHχ-12), and typed by the bridge map of the consuming paper (PHχ-13, §9.4). The audit removes the discretionary sign and the discretionary half; it does not relocate that freedom into the readout functional. An ω_χ chosen to produce a desired number is a fit functional and fails the audit exactly as a raw sign or a raw half does.

3.5 Lazy twin

Define a lazy-twin operation:

$L : H \mapsto LH$.

For a valid half-lazy gate:

$L^2 = I$.

The pair

$\{ H , LH \}$

is a closure pair.

The lazy twin is not absent. It is present as paired support required by closure. But after quotient normalisation, the pair contributes one physical support class.

3.6 Lazy quotient projector

The standard lazy quotient projector is:

$W\_L = \frac{1}{2} (I + L)$.

If $L^2 = I$, then

$W\_L^2 = W\_L$.

The half-factor is therefore not arbitrary. It is forced by projection.

3.7 Compatibility of reversal and lazy pairing

In the standard physical half-lazy χ gate, require:

$[R , L] = 0$.

Then

$P_- W\_L = W\_L P_-$,

and the lazy quotient and reversal-odd χ readout can be applied in either order without ambiguity.

If

$[R, L] \neq 0$,

then the commutator shadow

$C\_RL = [R, L]$

must be shown to vanish on the physical χ-support subspace or must be carried forward as an obstruction. A gate that uses both operations while remaining silent about their commutator has an unaudited degree of freedom, and PHχ-11 forbids that silence.

3.8 Irreversibility versus reversal comparison

A possible misunderstanding must be removed before the theorems are stated.

The reversal object

$H^{\leftarrow} = (f_n, \ldots, f_2, f_1)$

is not the physical undoing of an irreversible commitment history. VERSF does not claim that a completed history can be dynamically reversed after commitment. Rather, H^← is the comparison history used by the audit to test whether a proposed χ readout is sensitive to ordering.

Reversal in PHχ-1 is therefore an orientation test, not a physical time-reversal process.

The distinction has three parts:

- physical commitment is irreversible;
- the audit comparison is reversible as a mathematical operation;
- χ is admissible only if the physical readout distinguishes the irreversible ordering from its reversal comparison.

The use of H^← therefore does not weaken the irreversibility assumption. It formalises the statement that an ordered history has an orientation that a final record has forgotten. The audit needs both objects on the page precisely because the physics permits only one of them to occur.

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4. Final Records Cannot Carry χ

4.1 The final-record temptation

It is tempting to write

$\chi = \chi(F)$.

This would be convenient. A final record is easier to handle than an ordered history.

But this shortcut destroys χ.

A final record does not know which commitment came first. It cannot distinguish

$H = (f_1, f_2, \dots, f\_n)$

from

$H\^\leftarrow = (f\_n, \dots, f_2, f_1)$.

If χ is supposed to read ordered asymmetry, then $F(H)$ is insufficient — not approximately insufficient, but exactly insufficient, as Theorem 1 proves.

4.2 Label-blind collapse

If the readout is label-blind, arbitrary (+) and (−) names cannot rescue χ.

Any final-record readout that claims

$a_+ \neq a_-$

must derive (+) and (−) from physical structure. If it cannot, then label-blindness forces

$a_+ = a_-$,

and therefore

$\chi(F) = 0$.

4.3 Audit rule

Any proposed χ use must first answer:

Does χ read ordered history or only final record?

If it reads only final record, it fails PHχ-1 and PHχ-2, and no later structure can repair the failure.

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5. Ordered History and Reversal-Odd Residue

5.1 Ordered history as the minimal carrier

The minimal carrier of χ is not a fact, not a label, and not a tally.

It is an ordered irreversible commitment history.

The reason is simple:

Only ordered history can distinguish a path from its reversal.

5.2 Reversal-odd residue

The minimal reversal-odd residue is:

$\Gamma\_\chi(H) = ½ (|H\rangle - |H^\leftarrow\rangle)$.

This object satisfies

$\Gamma\_\chi(H^\leftarrow) = -\Gamma\_\chi(H)$,

and

$\Gamma\_\chi(H) = 0$

whenever H is reversal-symmetric.

5.3 Why the factor ½ is legitimate here

The factor ½ is not inserted for convenience.

It is the normalisation of the idempotent projection

$P_- = ½ (I - R)$.

Since $R^2 = I$,

$P_-^2 = ¼ (I - R)(I - R) = ¼ (I - 2R + R^2) = ¼ (2I - 2R) = ½ (I - R) = P_-$.

The half is therefore mathematically forced by projection onto the reversal-odd subspace. Any other coefficient would break idempotence.

5.4 Ordered χ is not yet half-lazy χ

The reversal-odd projection establishes the ordered-history χ carrier.

It does not yet establish the lazy half.

Half-laziness requires an additional structure:

$L^2 = I$,

plus quotient normalisation and non-double-counting.

PHχ-1 therefore keeps two facts separate:

1. the reversal half in $P_- = ½(I − R)$;
2. the lazy quotient half in $W_L = ½(I + L)$.

These are different operators, acting for different reasons, on different pairings. A later paper may show they align in a given application. PHχ-1 forbids silently treating them as the same operation, and the half-ledger of §7.4 enforces the separation.

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6. The Half-Lazy Gate

6.1 Definition

A **half-lazy χ gate** is a gate in which ordered active commitment is paired with a closure-required lazy twin, but the pair contributes one physical support class after quotient normalisation.

Formally, it consists of:

1. an ordered history H;
2. a reversal operation R;
3. a lazy involution L;
4. a reversal-odd residue $P_-|H\rangle$;
5. a lazy quotient projector W_L;
6. a non-double-counting rule;
7. a physical readout functional ω_χ.

The standard half-lazy χ carrier is:

$\Gamma_{\chi,L}(H) = W_L P_- |H\rangle$.

When

$[R , L] = 0$,

this is equivalent to:

$\Gamma\_\chi,L(H) = P_- W\_L |H\rangle$.

6.2 What "lazy" means

"Lazy" means:

closure-present but readout-inactive as an independent count.

It does not mean:

- nonexistent;
- discarded;
- unphysical garbage;
- available for later double counting;
- arbitrary;
- optional.

The lazy partner contributes by constraining the closure class. It does not contribute as an additional active readout branch.

6.3 What "half" means

"Half" means:

projection-normalised or quotient-normalised over an involutive pair.

It does not mean:

- divided by two because there are two things;
- divided by two because the result needs halving;
- divided by two as an aesthetic symmetry;
- divided by two as a fitting device.

The half must be derived.

6.4 Standard admissible half-lazy form

The standard admissible form is:

$\Gamma\_\chi,L(H) = \frac{1}{4} (I + L)(I - R) |H\rangle$.

This expression contains two half factors, but they do different jobs:

- $\frac{1}{2}(I - R)$ extracts the reversal-odd χ residue;
- $\frac{1}{2}(I + L)$ quotient-normalises lazy closure support.

A later paper must not collapse these into one vague "half-lazy" factor. The ¼ is the product of two independently derived halves, and each half must be traceable to its own involution.

Reversal-oddness survives the composite. Since $[R, L] = 0$ and $P_- R = -P_-$:

$\Gamma\_\chi,L(H^\leftarrow) = W\_L\ P_-\ R\ |H\rangle = -W\_L\ P_-\ |H\rangle = -\Gamma\_\chi,L(H)$.

The quotient therefore does not dilute the χ-carrying property established for the un-quotiented residue.

6.5 Warning on double halving

If a later derivation uses both P_- and W_L, it must show that both operations are independently required.

It may not use ¼ where only one physical quotient exists, and it may not use ½ where two independent quotients exist.

PHχ-1 therefore requires a **half-ledger**: each half-factor must state its origin, its involution, and its idempotence proof.

6.6 Non-degeneracy of the composite carrier

Two correctly derived halves can still compose to nothing.

If L coincides with R on the χ-support subspace — which is structurally possible, since both are commuting involutions — then:

$W\_L\ P_- = \frac{1}{2}(I + R) \cdot \frac{1}{2}(I - R) = \frac{1}{4}(I - R^2) = 0$,

and Γ_χ,L vanishes identically. Such a gate has an ordered history, defined involutions, derived halves, and a correct commutator, yet carries no χ at all. It is structurally perfect and physically vacuous.

PHχ-14 therefore requires:

$W\_L\ P_-\ P\_phys \neq 0$.

The lazy-even subspace must intersect the reversal-odd subspace nontrivially on physical support — and, per PHχ-14, on the specific history class the readout actually claims as χ-carrying, not merely somewhere in the theory. A gate failing this condition may not claim a nonzero χ, however impeccable its ledger. This condition also sharpens F14: conflating P_- and W_L is one route to degeneracy, but identifying L with R is a distinct route, and F28 records it separately.

---

7. Projection, Quotient, and the Origin of the Half

7.1 Projection half

If an operation Π satisfies

$\Pi^2 = I$,

then

$P\pm = \frac{1}{2} (I \pm \Pi)$

are idempotent projectors:

$P\pm^2 = P\pm$.

The factor ½ is forced: it is the unique coefficient making $P\pm$ idempotent.

This is a legitimate origin of a half-factor.

7.2 Quotient half

If two members of a pair are related by an involution,

$x \sim Lx$, $L^2 = I$,

then quotienting by the pair counts the equivalence class once:

$[x] = \{ x , Lx \}$.

The normalised representative is:

$\frac{1}{2} (x + Lx)$.

This is also a legitimate origin of a half-factor.

A binary-counting half is a special case of this construction, not a third independent origin. A counting rule is admissible only when the pairing it invokes can be exhibited as an involution L, at which point the counting half is the quotient half above. PHχ-8 supplies the uniformity requirement; this reduction supplies the rigour. A counting half whose pairing cannot be exhibited as an involution is forbidden.

7.3 Invalid half

The following is invalid:

$\chi = ½ \cdot$ (chosen branch)

unless the half is tied to a projection or quotient.

The following is also invalid:

$\chi = ½ (a_+ - a_-)$

unless a_+ and a_- are derived physical branches and the half is projection-normalised.

7.4 Half-ledger requirement

Every later VERSF use of half-lazy χ must include a half-ledger:

Half factor	Source	Status
½ from reversal-odd projection	$P_- = ½(I - R)$	admissible if $R^2 = I$
½ from lazy quotient	$W\_L = ½(I + L)$	admissible if $L^2 = I$
½ from binary counting	paired support class with exhibited pairing involution	admissible only if it factors through an involutive quotient (PHχ-8, §7.2)
½ inserted by convenience	none	forbidden

A ledger entry without an involution, quotient, or uniform binary rule in its Source column is an automatic failure (F7).

7.5 Free-orbit condition and fixed-point exception

The lazy quotient half is cleanest on the free lazy-orbit sector, where

$H \neq LH$

and

[H]_L = { H , LH }

is a genuine two-member closure class.

PHχ-1 therefore adds the following admissibility condition, sharpening PHχ-8:

A half-lazy quotient is physically admissible as a half only on the sector where the lazy involution acts freely, or where fixed points are separately identified and removed from the half-lazy count.

If a history satisfies

H = LH ,

then it is a lazy fixed point, not a two-branch half-lazy pair. Such a fixed point may still be physically meaningful, but it cannot justify the same half-factor as a genuine two-member quotient class: W_L acts on it as the identity, and no halving occurs.

Accordingly, every later half-lazy χ use must state one of the following:

- the relevant χ-support lies in the free lazy-orbit sector;
- lazy fixed points are absent by construction;
- lazy fixed points are projected out before the half-lazy readout;
- lazy fixed points are retained but treated as a separate non-half-lazy sector.

This prevents a hidden miscounting error. The lazy quotient may not apply a two-branch normalisation to a one-branch fixed point.

The half-ledger is therefore amended with a sector column:

Sector	Lazy action	Half-lazy status
Free lazy orbit	H ≠ LH	admissible two-member quotient
Fixed point	H = LH	not half-lazy unless separately justified
Mixed support	both free and fixed components	must be decomposed before counting

This strengthens the audit: the half-factor must be not only involution-derived but also correctly applied to the sector where a half-count is meaningful.

7.6 Minimal worked example

The audit becomes concrete on the smallest nontrivial history.

Ordered history and reversal. Let the history commit two distinct facts A then B:

$H = (A, B)$, $H^{\leftarrow} = (B, A)$.

Final-record null. The final record forgets order:

$F(H) = \{A, B\} = F(H^{\leftarrow})$.

Any readout depending only on F therefore assigns the same value to H and $H^{\leftarrow}$, and, for a reversal-odd readout, by Theorem 1:

$\chi(F(H)) = 0$.

Ordered-history residue. The reversal-odd carrier is:

$\Gamma_\chi(H) = ½ (|A,B\rangle - |B,A\rangle)$,

which satisfies $\Gamma_\chi(H^{\leftarrow}) = -\Gamma_\chi(H)$. If instead the history's ordered presentation coincided with its reversal — a reversal-symmetric history — then $\Gamma_\chi(H) = 0$: no χ is available, exactly as Theorem 2 requires. (Whether a repeated-content history such as two commitments of the same fact literally equals its reversal depends on how fact tokens are individuated; the audit condition is coincidence of ordered presentation, not content repetition.)

Legal lazy quotient (free orbit). Let each fact carry a twin label, $A \leftrightarrow A'$ and $B \leftrightarrow B'$, and define the lazy involution entry-wise:

$L(A, B) = (A', B')$, $L^2 = I$.

Since L acts entry-wise and R reverses order, the two operations commute: $[R, L] = 0$, satisfying PHχ-11. On this free orbit, $H \neq LH$, and the half-lazy carrier is:

$\Gamma_{\chi,L}(H) = W_L P_- |H\rangle = ¼ (|A,B\rangle - |B,A\rangle + |A',B'\rangle - |B',A'\rangle)$.

The class { (A,B), (A′,B′) } is counted once, both halves are ledgered (one from P_-, one from W_L), and no later paper may count (A′, B′) again as independent support.

In a physical application, the primed twin labels must not be arbitrary names; they must stand for a typed lazy-pairing operation already derived in the consuming VERSF gate. The toy notation only displays the quotient logic — it does not license label invention, which PHχ-3 forbids.

Illegal half (fixed point). Suppose instead the twin labels coincide: $A' = A$, $B' = B$. Then $LH = H$, the orbit is a fixed point, and

$W_L |H\rangle = |H\rangle$.

No halving occurs, because there is no pair to quotient. Inserting ½ here by hand would misweight a one-member class as if it were two — precisely the error the sector table of §7.5 forbids.

Non-degeneracy. The example also passes PHχ-14: the composite carrier displayed above is manifestly nonzero, because the twin involution L (label swap) and the reversal R (order swap) act on different data and do not coincide on the support.

The example shows each audit outcome operationally: the final record is provably χ-blind, the ordered residue carries χ, the free-orbit quotient legitimately earns its half, the fixed point earns none, and the composite carrier is verifiably non-degenerate.

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8. Non-Double-Counting and Lazy-Sector Exclusion

8.1 The double-counting problem

The most dangerous misuse of half-lazy χ is this:

1. use the lazy half to justify a quotient or half-factor;
2. later count the lazy half as an independent physical branch.

That is double counting: the pair is charged once through the quotient and again through enumeration.

If the lazy half has already supplied a quotient normalisation, it cannot later return as a new counted degree of freedom.

8.2 Closure-present is not count-present

The lazy twin is closure-present. It may be required to define the pair, the quotient, or the symmetry.

But closure-present does not mean independently count-present.

The physical support is the equivalence class

[H]_L = { H , LH } .

The class is counted once.

8.3 Re-entry rule

A lazy branch may re-enter a later calculation only as:

- part of the same quotient class;
- a constraint;
- a covariance partner;
- a cancellation partner;
- a boundary condition;
- a non-independent shadow.

It may not re-enter as:

- a second active branch;
- an additional physical generation;
- an extra multiplicity;
- an independent spectral line;
- an untracked contribution to a numerical coefficient.

8.4 Audit statement

A later paper that uses half-lazy χ must state:

Where did the lazy half go?

The only admissible answer is:

It remains in the quotient class and cannot be counted independently.

Any other answer — "it was deleted," "it cancelled," "it will be counted in a later paper" — fails the audit.

---

9. Physical-Sector Compatibility

9.1 Positivity

The half-lazy χ gate must preserve physical-sector positivity.

If P_phys is the physical-sector projector, then an admissible χ carrier must satisfy:

$P_{phys}\, \Gamma_{\chi,L} = \Gamma_{\chi,L}$.

If the χ carrier lies partly outside the physical sector, the nonphysical component must be projected out before any physical claim is made.

9.2 Frame and gauge discipline

χ must not depend on arbitrary presentation, coordinate ordering, raw labels, or gauge convention.

A permitted relabelling U must preserve the physical χ readout:

$\chi(UH) = \chi(H)$,

or transform it under a declared physical representation.

If U is merely a notational relabelling and χ changes, the gate is not physical.

9.3 Observable bridge

A χ gate is not physical merely because it is internally definable.

It must feed a later physical target, such as:

- a hierarchy ordering;
- a projector selection;
- a weak-orientation sign;
- a spectral admissibility condition;
- a closure-density halving;
- a non-double-counting quotient;
- a transition-channel selection;
- a physical-sector positivity filter.

Without such a bridge, χ is an internal ledger mark, not a physical gate.

9.4 Bridge-map requirement

The observable-bridge premise PHχ-13 is now given its explicit form. Every later physical use of χ must provide a bridge map

$B\_\chi : \Gamma\_\chi,L(H) \to O\_phys$,

where $\Gamma\_\chi,L(H)$ is the audited half-lazy χ carrier and O_phys is a stated physical target: a hierarchy ordering, sign orientation, projector selection, spectral split, coupling asymmetry, admissibility filter, or transition-channel constraint.

The bridge must satisfy three conditions.

First, it must be typed:

domain(B_χ) = physical χ-support , codomain(B_χ) = declared physical observable or invariant sector .

Second, it must be non-rhetorical. It is not enough to say that χ "influences" a later result. The later paper must state where χ enters the calculation, which invariant it changes, and what value or structural feature would be lost if χ were set to zero.

Third, it must be audit-stable. If χ is removed, replaced by a raw sign, read from final record, or allowed to double count lazy support, the downstream result must either vanish, become ambiguous, or fail the declared physical-sector test. A downstream result that survives all such corruptions was never using χ as physical input.

This gives PHχ-1 a sharper falsifier:

A χ gate with no bridge map is not yet physical input. It is only an internal bookkeeping residue.

The observable bridge is therefore part of the physicality audit itself, not a later explanatory preference.

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10. Theorem 1 — Final-Record Null Theorem

10.1 Statement

Theorem 1 — Final-Record Null Theorem. Let F(H) be an order-forgetting final record with

$F(H) = F(H^\leftarrow)$.

Let χ_F be a label-blind, reversal-odd readout — a physical χ readout in the sense of PHχ-4 — depending only on F(H). Then:

$\chi_F(F(H)) = 0$.

Therefore no nonzero physical χ can be derived from final-record data alone.

10.2 Proof

The objects H and H^← are ordered presentations (PHχ-1). Since F(H) forgets order (PHχ-2),

$F(H) = F(H^\leftarrow)$.

A readout depending only on F therefore assigns equal values to H and H^←. Label-blindness (PHχ-3) closes the escape route of attaching raw (+)/(−) names to F-entries: permutable labels cannot reintroduce the forgotten orientation. Hence:

$\chi\_F(F(H^\leftarrow)) = \chi\_F(F(H))$.

By hypothesis the readout is reversal-odd (PHχ-4):

$\chi(H^\leftarrow) = -\chi(H)$,

so, applied through F,

$\chi\_F(F(H^\leftarrow)) = -\chi\_F(F(H))$.

Combining the two displayed equalities:

$\chi\_F(F(H)) = -\chi\_F(F(H))$,

hence

$2 \cdot \chi\_F(F(H)) = 0$, $\chi\_F(F(H)) = 0$.

A final-record-only readout is therefore forced to zero: it cannot simultaneously satisfy order-forgetting equality and reversal-oddness except at the zero value.

The reversal-odd hypothesis is essential and is carried explicitly. Without it the statement fails: a constant functional, or any reversal-even readout of F such as total committed content, is final-record-measurable and generally nonzero. The exact content of the theorem is that no functional is simultaneously final-record-measurable and reversal-odd except the zero functional. ■

10.3 Corollary — No tally χ

A tally, multiset, final ledger, unordered occupancy count, or final support record cannot by itself produce nonzero χ.

10.4 Corollary — No primitive sign rescue

Assigning (+) and (−) labels to final-record entries does not rescue χ unless those labels are derived from ordered physical structure. Under label-blindness the labels are permutable, and permutable labels carry no orientation.

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11. Theorem 2 — Ordered-History Necessity Theorem

11.1 Statement

Theorem 2 — Ordered-History Necessity Theorem. A nonzero physical χ readout requires an ordered irreversible commitment history H and a reversal-odd residue. The minimal admissible carrier is:

$$\Gamma\_\chi(H) = \tfrac{1}{2} \left(|H\rangle - |H^{\leftarrow}\rangle \right) ,$$

or an equivalent reversal-odd projection.

11.2 Proof

By Theorem 1, final-record data alone produce $\chi = 0$ (PHχ-2). A nonzero χ must therefore depend on information not contained in F(H).

By the definitional restriction of §3.2, the only information forgotten by F is the order of commitment. Hence a nonzero χ must depend on ordering data: the ordered history H itself (PHχ-1), together with its reversal $H^{\leftarrow}$. No raw label assignment can substitute for this ordering data (PHχ-3): permutable labels carry no orientation.

The reversal operation satisfies

$$R^2 = I .$$

The reversal-odd projector

$$P_- = \tfrac{1}{2} (I - R)$$

is idempotent:

$$P_-^2 = \tfrac{1}{4} (I - R)(I - R) = \tfrac{1}{4} (I - 2R + R^2) = \tfrac{1}{4} (2I - 2R) = \tfrac{1}{2} (I - R) = P_- .$$

Therefore

$$\Gamma\_\chi(H) = P_- |H\rangle$$

is a legitimate projected carrier. It satisfies

$$\Gamma\_\chi(H^{\leftarrow}) = P_- R |H\rangle = -\Gamma\_\chi(H) ,$$

since $P_-R = \tfrac{1}{2}(R - R^2) = \tfrac{1}{2}(R - I) = -P_-$.

Thus Γ_χ carries exactly the reversal-odd content, and any readout $\langle \omega\_\chi, \Gamma\_\chi(H)\rangle$ inherits $\chi(H^{\leftarrow}) = -\chi(H)$, as reversal-sensitivity requires (PHχ-4).

Nonzero χ therefore requires ordered history and a reversal-odd residue, and $\Gamma\_\chi(H)$ is the minimal such carrier — the object posited by PHχ-5.

This claim is relative to the PHχ-1 definition of F as the order-forgetting final record; no broader claim is made that all possible coarse-grainings forget only order. ■

11.3 Corollary — χ is history-owned

χ belongs to the ordered history class, not to isolated facts.

11.4 Corollary — Reversal-symmetric histories give zero χ

If $H = H^{\leftarrow}$, then $\Gamma_\chi(H) = 0$, so no nonzero χ readout is available.

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12. Theorem 3 — Half-Lazy Projection Theorem

12.1 Statement

Theorem 3 — Half-Lazy Projection Theorem. A half-lazy gate is physically admissible only if its half-factor arises from a valid projection or quotient normalisation. In the standard case, this requires an involutive lazy operation

$$L^2 = I ,$$

and the lazy quotient projector

$$W_L = \tfrac{1}{2} (I + L) .$$

If no such projection or quotient is supplied, the half-lazy gate is not physical.

12.2 Proof

Assume $L^2 = I$ (PHχ-6). Define

$$W_L = \tfrac{1}{2} (I + L) .$$

Then:

$$W_L^2 = \tfrac{1}{4} (I + L)(I + L) = \tfrac{1}{4} (I + 2L + L^2) = \tfrac{1}{4} (2I + 2L) = \tfrac{1}{2} (I + L) = W_L .$$

Thus W_L is an idempotent projector, and the factor ½ is forced by the projection structure: replacing ½ by any other coefficient c gives $(c(I+L))^2 = 2c^2(I+L)$, which is idempotent only for c

= ½ or c = 0. The coefficient c = 0 gives only the trivial zero projector and cannot support a nonzero physical χ gate; the unique nontrivial normalised projector of this form uses c = ½.

Conversely, if no involution L is defined, or if $L^2 \neq I$, then W_L is not a valid projector of this form and the factor ½ has no established physical origin. It is then either a convention or a fit factor, and both are forbidden by PHχ-7.

If the half is instead justified by binary counting, PHχ-8 requires the paired members to be uniformly admissible, and a legitimate counting half must exhibit its pairing as an involution — at which point it factors through the quotient construction above. A counting half that cannot exhibit its involution is inadmissible.

Therefore a half-lazy gate is physically admissible only when the half-factor is projection- or quotient-derived. ■

12.3 Corollary — No arbitrary half

A half-factor without an involution, quotient, projection, or uniform binary admissibility rule is not physical.

12.4 Corollary — Half-lazy does not mean half-ignored

The lazy half is present in the quotient. It is not deleted.

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13. Theorem 4 — Lazy Non-Double-Counting Theorem

13.1 Statement

Theorem 4 — Lazy Non-Double-Counting Theorem. In a physical half-lazy χ gate, the lazy twin is closure-present but not independently count-present. Once a lazy pair has been quotient-normalised, the lazy partner may not later be counted as an additional active physical support without undoing the quotient and invalidating the half-lazy gate.

13.2 Proof

Let the lazy equivalence class be

[H]_L = { H , LH } ,

generated by the involutive lazy operation (PHχ-6). The quotient-normalised representative is

$$W\_L |H\rangle = \tfrac{1}{2} (|H\rangle + |LH\rangle) .$$

This object counts the pair as one physical class. That single counting is precisely what licenses the factor ½ (PHχ-7): the normalisation and the count are two faces of the same quotient.

Suppose a later derivation counts both H and LH as independent active supports after having already used W_L. Then the pair has been counted once through the quotient and again through independent branch enumeration — a total weight of two where the quotient definition assigns one, in direct violation of the non-double-counting rule (PHχ-9).

The later derivation is then inconsistent with the quotient it inherited. Either the quotient must be abandoned (forfeiting the half-factor and the half-lazy gate), or the independent count must be withdrawn. Both cannot stand.

Therefore the lazy twin may appear later only as part of the same quotient class, a covariance partner, a constraint, or a non-independent shadow. It may not re-enter as separately counted active support. ■

13.3 Corollary — Lazy-sector re-entry is firewalled

Any later paper that reintroduces the lazy branch as independent support must explicitly abandon the half-lazy quotient. It cannot keep both.

13.4 Corollary — Non-double-counting is load-bearing

The physical meaning of "half-lazy" depends on the lazy half not returning as hidden extra content. The half-factor and the single count are jointly earned or jointly lost.

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14. Theorem 5 — Physical χ-Gate Audit Theorem

14.1 The seven tests

A VERSF χ gate is physically admissible only if it passes the following seven tests.

Test 1 — Ordered-history test

The gate must identify the ordered commitment history:

$H = (f_1, \ldots, f_n)$.

A final record alone is insufficient.

Test 2 — Final-record null test

The gate must show that the corresponding final-record-only readout gives:

$\chi(F(H)) = 0$.

This proves χ is not secretly being read from an order-forgetting tally.

Test 3 — Reversal-odd residue test

The gate must identify the reversal operation R and the reversal-odd carrier:

$\Gamma_\chi(H) = \frac{1}{2} (I - R) |H\rangle$,

or must supply an equivalent reversal-odd structure.

Test 4 — Lazy-involution test

If the gate is half-lazy, it must define the lazy operation L, prove

$L^2 = I$,

and verify non-degeneracy of the composite carrier on physical χ-support:

$W_L P_- P_{phys} \neq 0$.

A gate whose composite carrier vanishes identically — for instance because L coincides with R on the support — is vacuous and fails this test (PHχ-14). The verification must address the specific history class claimed as χ-carrying, not global existence elsewhere in the theory.

Test 5 — Quotient-normalisation test

The half-factor must be derived from a projection or quotient, such as

$W_L = \frac{1}{2} (I + L)$,

with its half-ledger entry stated, and the χ-support must be declared as free-orbit, fixed-point-free, fixed-point-projected, or sector-decomposed per §7.5. A raw ½ is insufficient, and a two-branch half applied to a fixed-point sector fails this test.

Test 6 — Non-double-counting test

The lazy partner must not later be counted as independent active physical support. The quotient class counts once, and the paper must say where the lazy half went.

Test 7 — Physical-sector invariance test

The χ carrier must preserve positivity, remain invariant under permitted relabellings, satisfy the declared R/L compatibility relation, and feed a declared downstream physical target through a typed bridge map B_χ satisfying the three conditions of §9.4.

14.2 Statement

Theorem 5 — Physical χ-Gate Audit Theorem. A VERSF χ gate is physically admissible if and only if it passes Tests 1 through 7. Equivalently, χ must be ordered-history-derived, final-record-null, reversal-odd, lazy-involution-supported, quotient-normalised, non-double-counted, and physical-sector invariant.

14.3 Proof

Necessity. Each test transcribes one or more named premises, and each premise carries an explicit falsifier.

Test 1 follows from PHχ-1. Without ordered history, no nonzero χ can be carried.

Test 2 follows from PHχ-2 and PHχ-3, through Theorem 1. A final-record χ must vanish, label rescue is barred, and a gate that has not verified this may be reading a tally without knowing it.

Test 3 follows from PHχ-4 and PHχ-5. A χ carrier must be reversal-sensitive.

Test 4 follows from PHχ-6 and PHχ-14. A half-lazy gate requires a lazy involution, and the composite carrier must not vanish on physical χ-support: a structurally perfect but identically zero gate is vacuous, not admissible.

Test 5 follows from PHχ-7 and PHχ-8, sharpened by the free-orbit condition of §7.5. The half-factor must be derived, paired supports must be uniformly admissible, and the half may be charged only on sectors where a genuine two-member quotient exists.

Test 6 follows from PHχ-9. The lazy half cannot be counted twice.

Test 7 follows from PHχ-10, PHχ-11, PHχ-12, and PHχ-13, with PHχ-13 given its explicit form by the bridge map B_χ of §9.4. The gate must be invariant, commutator-compatible, positive-sector admissible, and physically bridged through a typed, non-rhetorical, audit-stable map.

All seven tests are therefore necessary, and jointly they exhaust the premise ledger of §2: every load-bearing premise is enforced by at least one test.

Sufficiency for admissibility. If all seven tests pass, the gate has:

- an ordered history carrier;
- proof that final-record data alone cannot supply χ;
- a reversal-odd residue;
- a defined lazy involution;
- a derived half-factor with a half-ledger;
- a non-double-counting rule with a declared lazy destination;
- physical-sector compatibility, commutator compatibility, and a downstream bridge.

No raw sign, arbitrary half, convention-based branch split, final-record shortcut, or hidden double count remains anywhere in the gate. Every discretionary element has been either derived or eliminated.

Therefore the gate is physically admissible.

The equivalence is deliberately a consolidation rather than an independent structural result: admissibility is defined by the premise ledger, and the tests transcribe the premises. The theorem's content is completeness — the verification that no premise escapes enforcement and no test enforces nothing — and it should be read as a closure check on the audit itself, not as circularity.

The "if and only if" is therefore internal to the PHχ-1 admissibility standard. It does not claim that every mathematically admissible χ gate is realised in nature, only that no PHχ-1 physicality objection remains once the seven tests are passed.

This is sufficiency for admissibility, not sufficiency for numerical truth. ■

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15. Theorem 6 — Half-Lazy χ Closure Theorem

15.1 Statement

Theorem 6 — Half-Lazy χ Closure Theorem. The physical half-lazy χ gate closes in VERSF if and only if the χ readout factors through:

┌──────────────────────────────────────┐
│ ordered history → reversal-odd residue → │ │ lazy involutive quotient → │ │ non-double-counted physical readout │
└──────────────────────────────────────┘

A χ use that skips any stage is not a physical half-lazy χ gate.

15.2 Proof

By Theorem 1, final-record-only χ vanishes. Ordered history is therefore required as the first stage.

By Theorem 2, ordered-history χ must be carried by a reversal-odd residue. The second stage is therefore required.

By Theorem 3, half-lazy structure requires a valid lazy involution and projection or quotient normalisation. The third stage is therefore required.

By Theorem 4, the lazy partner cannot be counted independently after quotient normalisation. The fourth stage is therefore required.

By Theorem 5, the seven audit tests are jointly necessary and sufficient for physical admissibility, and the four stages implement exactly those tests: stage one implements Tests 1–2, stage two implements Test 3, stage three implements Tests 4–5, and stage four implements Tests 6–7.

The half-lazy χ gate therefore closes precisely when it passes through ordered history, reversal-odd projection, lazy quotient, and non-double-counted physical readout.

Each stage inherits its premise support through the cited results: stage one carries PHχ-1, PHχ-2, and PHχ-3 through Theorems 1–2; stage two carries PHχ-4 and PHχ-5 through Theorem 2 (PHχ-4 also entering stage one through Theorem 1's hypothesis); stage three carries PHχ-6, PHχ-7, PHχ-8, and PHχ-14 through Theorem 3 and Test 4; stage four carries PHχ-9 through Theorem 4, and PHχ-10, PHχ-11, PHχ-12, and PHχ-13 through Test 7. The full-column marking of Theorem 6 in the dependency map records this inherited support.

Any skipped stage creates a specific and independent failure:

- no ordered history gives final-record nullity;
- no reversal residue gives no χ carrier;
- no lazy involution gives an arbitrary half;
- no quotient gives a hidden convention;
- a degenerate composite carrier gives a vacuous gate (F28);
- a two-branch half applied to a fixed-point sector gives a miscounted quotient (F26);
- no non-double-counting gives overcounted physical support;
- no physical-sector bridge gives internal bookkeeping only.

The stated factorisation is thus necessary and sufficient for half-lazy χ-gate admissibility. ■

15.3 Corollary — χ is not a switch

A physical χ gate is not an on/off selector. It is a derived readout of ordered commitment structure.

15.4 Corollary — Half-lazy is not a fit parameter

The half-lazy structure supplies no free numerical adjustment. The half is either derived from projection/quotient structure or it is forbidden.

15.5 Corollary — Lazy support remains accountable

The lazy half remains in the ledger as a quotient partner. It cannot disappear from the audit trail.

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16. Falsification Conditions

#	Failure	Consequence
F1	χ is assigned as a primitive sign	Raw-label failure
F2	χ is read from final record F(H) alone	Final-record null failure
F3	Ordered history H is not specified	No χ carrier
F4	Reversal $H^\leftarrow$ is not defined	No orientation audit
F5	$\chi(H^\leftarrow) \neq -\chi(H)$ for an ordered-asymmetry claim	Reversal failure
F6	No reversal-odd residue is supplied	Carrier failure
F7	A factor ½ is inserted without projection or quotient	Hidden half-factor
F8	Lazy operation L is not defined	Undefined lazy structure
F9	$L^2 \neq I$ but involutive quotient is claimed	Invalid projection
F10	Active/lazy split changes under harmless relabelling	Convention failure
F11	Lazy partner is ignored rather than quotient-accounted	Missing support
F12	Lazy partner is counted later as independent active support	Double-counting failure
F13	Reversal and lazy operations conflict without commutator audit	Commutator-shadow failure
F14	P_- and W_L are conflated	Half-ledger failure
F15	Two half-factors are used where only one projection exists	Over-halving
F16	A half-lazy gate feeds no physical target	Bookkeeping-only failure
F17	χ depends on coordinate notation	Frame failure
F18	χ depends on gauge convention	Gauge failure
F19	χ carrier includes nonphysical support	Positivity failure

#	Failure	Consequence
F20	A numerical hierarchy is claimed from PHχ-1 alone	Scope violation
F21	$\chi_1 = \ln(6/13)$ is claimed without additional derivation	Scope violation
F22	Lazy branch reappears as an extra generation/species/channel	Multiplicity failure
F23	Ordered history is replaced by a final tally mid-proof	Arrow-erasure failure
F24	Passing audit is treated as empirical confirmation	Truth/admissibility confusion
F25	The half-lazy gate is used as discretionary fit freedom	Physicality failure
F26	Two-branch half-lazy quotient is applied to a lazy fixed point (H = LH)	Fixed-point miscount
F27	χ is claimed as physical input with no typed bridge map B_χ	Bridge-map failure
F28	L coincides with R on χ-support, or W_LP_- vanishes on the claimed physical χ-support, while nonzero χ is claimed	Degenerate-gate failure

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17. PHχ-1 Closure Certificate

PHχ-1 condition	Result
Ordered history H defined	Closed
Final record F(H) distinguished from history	Closed
Reversal H^← defined	Closed
Final-record χ shown to vanish	Closed
Label-blindness enforced	Closed
Primitive (+)/(−) signs firewalled	Closed
Reversal-odd residue supplied	Closed
Projection $P_- = ½(I − R)$ supplied	Closed
Physical origin of reversal half supplied	Closed
Lazy twin L defined	Closed
Lazy involution $L^2 = I$ required	Closed
Lazy quotient W_L = ½(I + L) supplied	Closed
Physical origin of lazy half supplied	Closed
Half-ledger requirement supplied	Closed
Uniform binary admissibility required	Closed
Active/lazy invariance required	Closed
Non-double-counting rule supplied	Closed
Lazy-sector re-entry firewalled	Closed
Reversal/lazy commutator compatibility required	Closed
Positivity compatibility required	Closed

PHχ-1 condition	Result
Frame/gauge invariance required	Closed
Observable bridge required	Closed
Typed bridge map B_χ with three conditions supplied	Closed
Reversal typed as audit comparison, not de-commitment	Closed
Free-orbit condition and fixed-point exception supplied	Closed
Sector-decomposition rule for mixed lazy support supplied	Closed
Minimal worked example supplied	Closed
Non-degeneracy $W\_LP_{-} \neq 0$ on physical χ-support required	Closed
ω_χ typing requirement supplied	Closed
Theorem 1 reversal-odd hypothesis stated explicitly	Closed
Seven audit tests supplied	Closed
Full numerical χ value	Outside scope
Full Yukawa hierarchy	Outside scope
CKM/PMNS numerical prediction	Outside scope
Empirical confirmation	Outside scope

PHχ-1 closure grade: closed as a physical half-lazy χ-gate admissibility audit.

It proves that a half-lazy χ gate is physically admissible only when χ is ordered-history-derived, reversal-sensitive, lazy-quotient-normalised, non-double-counted, and invariant under permitted relabellings.

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18. Conclusion

PHχ-1 closes a subtle but important loophole in the VERSF χ programme.

A χ gate is not physical just because it is mathematically convenient. It is not physical because a sign has been assigned. It is not physical because a branch has been called active and another branch has been called lazy. It is not physical because a factor of ½ appears in a formula.

A physical χ gate must earn all of that structure.

First, χ must read ordered commitment history, not final record. A final record forgets ordering, and under label-blindness a final-record readout gives:

$\chi = 0$.

Second, χ must be carried by a reversal-odd residue:

$\Gamma\_\chi(H) = ½ (|H\rangle - |H^\leftarrow\rangle)$.

The half in this expression is not arbitrary. It is the normalisation of the projection

$P_- = ½ (I - R)$,

and idempotence permits no other coefficient.

Third, half-lazy structure requires a lazy involution

$L^2 = I$,

and a quotient projector

$W\_L = ½ (I + L)$.

The lazy half is not absent. It is closure-present but not independently count-present. It contributes through the quotient class and cannot later be reintroduced as separate active physical support: the half-factor and the single count are jointly earned or jointly lost.

Fourth, the physical χ gate must preserve positivity, remain invariant under permitted relabellings, and feed a downstream observable target through a declared bridge.

The final result is:

A half-lazy χ gate is physical only when ordered history supplies the arrow, reversal-odd projection supplies the χ carrier, lazy involution supplies the quotient half, and non-double-counting prevents the lazy side from returning as hidden extra content.

PHχ-1 therefore blocks the main abuse modes:

- no primitive signs;
- no final-record χ;
- no arbitrary half;
- no convention-based active/lazy split;
- no lazy-sector double counting;
- no two-branch half applied to a lazy fixed point;
- no χ without a typed bridge into a physical target;
- no χ as hidden fit freedom.

It does not derive the numerical hierarchy. It does something prior and necessary:

It proves when χ is allowed to be used as physics at all.

Appendix A — Minimal Algebraic Skeleton

Ordered history:

$H = (f_1, f_2, \ldots, f_n)$.

Final record:

$F(H)$.

Order forgetting:

$F(H) = F(H^{\leftarrow})$.

Reversal:

$\mathsf{R}H = H^{\leftarrow}$, $\mathsf{R}^2 = I$.

Reversal-even projector:

$P_+ = \frac{1}{2} (I + \mathsf{R})$.

Reversal-odd projector:

$P_- = \frac{1}{2} (I - \mathsf{R})$.

χ residue:

$\Gamma_\chi(H) = P_- |H\rangle = \frac{1}{2} (|H\rangle - |H^{\leftarrow}\rangle)$.

χ readout:

$\chi(H) = \langle \omega_\chi , \Gamma_\chi(H) \rangle$.

Reversal property:

$\chi(H^{\leftarrow}) = -\chi(H)$.

Final-record null (for reversal-odd χ):

$\chi(F(H)) = 0$.

Lazy involution:

$\mathsf{L}^2 = I$.

Lazy quotient projector:

$W\_L = ½ (I + L)$.

Idempotence:

$W\_L^2 = W\_L$, $P_-^2 = P_-$.

Standard half-lazy χ carrier:

$\Gamma\_\chi,L(H) = W\_L P_- |H\rangle$.

Commutation:

$[R , L] = 0 \Rightarrow W\_L P_- = P_- W\_L$.

Expanded form:

$\Gamma\_\chi,L(H) = ¼ (I + L)(I - R) |H\rangle$.

Lazy equivalence class:

$[H]\_L = \{ H , LH \}$.

Free-orbit condition:

$H \neq LH$ on the half-lazy sector; fixed points $H = LH$ are separated before counting .

Non-double-counting rule:

$[H]\_L$ counts once .

Non-degeneracy condition:

$W\_L P_- P\_phys \neq 0$.

Bridge map:

$B\_\chi : \Gamma\_\chi,L(H) \to O\_phys$, typed, non-rhetorical, audit-stable .

Appendix B — χ-Gate Audit Table

Object	Meaning	Admissible role	Failure if confused
H	Ordered commitment history	χ carrier source	No ordered χ
F(H)	Final order-forgetting record	Null test	False χ if used alone
H^←	Reversed history	Orientation audit	No reversal test
R	Reversal operation	Defines P_-	No χ residue
P_-	Reversal-odd projector	Carries χ	Raw sign if absent
Γ_χ	χ residue	Physical readout carrier	No invariant carrier
L	Lazy involution	Defines closure pair	Undefined laziness
W_L	Lazy quotient projector	Supplies lazy half	Arbitrary half
[H]_L	Lazy equivalence class	Counts once	Double counting
ω_χ	Readout functional	Converts residue to χ	Internal mark only
(+)/(−) labels	Possible derived signs	Allowed only if derived	Primitive-label failure
Half factor	Projection/quotient normalisation	Allowed if derived	Hidden fit freedom
P_phys	Physical-sector projector	Positivity filter	Nonphysical χ support
B_χ	Bridge map to physical target	Makes χ physical input	Bookkeeping-only χ
Fixed point	H = LH, one-member lazy orbit	Separate non-half sector	Fixed-point miscount

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Appendix C — Referee Objections and Replies

Objection 1 — Is PHχ-1 just saying "do not double count"?

Reply. No. Non-double-counting is one part of the audit. The full result also proves that final-record χ vanishes, ordered history is necessary, reversal-odd residue is required, the lazy half must come from an involutive quotient, and physical-sector invariance must be preserved. Theorem 1 alone — the forced nullity of any final-record χ — is independent of counting entirely.

Objection 2 — Why can't χ simply be a sign convention?

Reply. Because a sign convention has no predictive content. If (+) and (−) can be swapped by relabelling without changing the underlying physics, then any claimed χ depending on those labels is not physical. χ must be derived from ordered structure, and PHχ-3 makes the swap test explicit.

Objection 3 — Why does final-record χ vanish?

Reply. Because the final record forgets order. If F(H) = F(H^←), then any readout depending only on F cannot distinguish H from its reversal. But χ must be reversal-odd. The only value equal to its own negative is zero.

Objection 4 — Does PHχ-1 prove a numerical χ value?

Reply. No. It proves when χ is physically admissible. Numerical values require later derivations, and F20 and F21 record the scope boundary as explicit falsification conditions.

Objection 5 — Is the lazy half unphysical?

Reply. No. The lazy half is closure-present. It is not independently active as a counted readout branch. It remains part of the quotient class and constrains the gate. "Unphysical" and "readout-inactive" are different properties, and PHχ-1 keeps them separate.

Objection 6 — Why is the factor ½ allowed?

Reply. Only when derived from projection or quotient structure. For example,

$$P_- = \tfrac{1}{2} (I - R)$$

is forced by reversal projection, and

$$W\_L = \tfrac{1}{2} (I + L)$$

is forced by lazy-involution quotienting. In each case idempotence permits no other coefficient. A bare ½ is not allowed.

Objection 7 — Can the lazy side ever reappear?

Reply. Yes, but only as part of the same quotient class, a constraint, a covariance partner, a cancellation partner, or a non-independent shadow. It cannot reappear as separately counted active support. Theorem 4 shows that both counting modes cannot coexist.

Objection 8 — What happens if reversal and lazy operations do not commute?

Reply. Then the commutator shadow

$$[R , L]$$

must be carried forward as an obstruction or shown to vanish on the physical χ-support subspace. The gate cannot ignore it, and the two orderings W_LP_- and P_-W_L must then be distinguished explicitly.

Objection 9 — Does this make χ too restrictive?

Reply. It makes χ honest. A weaker rule would leave χ vulnerable to the objection that it is a switch, convention, or hidden fit parameter. The audit costs nothing to a legitimate gate: every test is passable by structure that is actually derived.

Objection 10 — Has the arbitrariness merely relocated into ω_χ?

Reply. This is the sharpest available objection, and the paper answers it directly rather than by silence. The readout functional ω_χ is constrained on four independent axes: it must be label-blind (PHχ-3), invariant under permitted relabellings (PHχ-10), supported on the physical sector (PHχ-12), and typed through the bridge map B_χ of §9.4 (PHχ-13) — where audit-stability requires that corrupting χ corrupts the downstream result. The residual freedom is therefore not sign freedom or half freedom: those are eliminated by projection and quotient structure. What remains is the obligation, placed on the consuming paper, to derive ω_χ from typed VERSF structure and to demonstrate through the bridge that the result depends on the genuine χ content. An ω_χ selected to produce a desired number fails audit-stability by construction. The freedom is not removed by fiat; it is converted into a declared, falsifiable obligation.

Objection 11 — What is the one-sentence result?

Reply.

A half-lazy χ gate is physical only when χ reads ordered reversal-sensitive commitment history, the lazy partner is an involutive quotient twin, the half-factor is projection-derived, and the lazy side cannot later be double-counted as independent physical support.

Appendix D — Final One-Sentence Theorem

A VERSF half-lazy χ gate is physically admissible if and only if χ is derived from ordered irreversible commitment history through a reversal-odd residue, the lazy half is supplied by an involutive quotient rather than by convention, the resulting half-factor is projection-normalised, and the lazy partner remains closure-present but non-independently counted; therefore final-record χ, raw sign labels, arbitrary halving, convention-based active/lazy splits, and lazy-sector double counting are firewalled from physical prediction.